Supplement file: Galaxy Rotation Curve Fits and Gravitational Lensing Fits

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1 Galaxy Rotation Curve Fits

To complement the main analysis, we summarize representative MEST$_2$ fits to galaxy rotation curves from the SPARC sample. The structural form is

$$V(r) = V_{\max}\left(\frac{r}{\sqrt{r^2+r_0^2}}\right)^{\alpha}, \tag{1}$$

with $V_{\max}$ the asymptotic velocity, r_0 the structural scale radius, and α the slope parameter.

Table 3 reports the root-mean-square errors (RMSE) of MEST fits versus the best-fitting empirical models, along with ΔRMSE. Positive ΔRMSE indicates that MEST provides a better fit. The results are consistent with the $\alpha \propto r_0^{-1}$ scaling law ($b = 1$) and yield $k_\alpha \simeq 0.043$ Mpc^{-1}.

Table 1: Final Galaxy Fit Summary: Comparison Between MEST-SF and Best Empirical Models

Galaxy	RMSE (MEST)	Best Empirical	RMSE (Empirical)	ΔRMSE
DDO 154	2.14	ISO	2.21	0.07
NGC 3198	3.60	Polyex	4.20	0.60
M31	7.34	NFW	8.13	0.79
UGC 128	4.87	Einasto	5.60	0.73
NGC 2841	5.62	Polyex	6.87	1.25
F568-v1	3.91	Polyex	4.75	0.84
F579-v1	4.44	Polyex	5.36	0.92
NGC 2403	3.76	Polyex	4.38	0.62
NGC 6503	2.98	Polyex	3.64	0.66
IC 2574	2.24	Polyex	3.12	0.88

Table 2: Performance summary of MEST kernels (tanh, logistic, arctan) compared to empirical baselines.

Galaxy	MEST-tanh RMSE	MEST-logistic RMSE	MEST-arctan RMSE	Best Empirical	Empirical RMSE	ΔRMSE
DDO 154	2.11	2.15	2.20	ISO	2.21	+0.10
NGC 3198	3.58	3.65	3.70	Polyex	4.20	+0.62
M31	7.30	7.41	7.52	NFW	8.13	+0.83
UGC 128	4.84	4.90	5.01	Einasto	5.60	+0.76
NGC 2841	5.60	5.65	5.74	Polyex	6.87	+1.23
F568-v1	3.90	3.95	4.02	Polyex	4.75	+0.85
F579-v1	4.43	4.47	4.55	Polyex	5.36	+0.93
NGC 2403	3.75	3.80	3.88	Polyex	4.38	+0.63
NGC 6503	2.97	3.02	3.08	Polyex	3.64	+0.67
IC 2574	2.23	2.28	2.34	Polyex	3.12	+0.89

2 Gravitational Lensing Fits

We provide representative strong lensing fits using the MEST$_{2n}$ framework. Results are presented in three complementary views: (i) best-fit parameters and residual statistics, (ii) comparison of observed and predicted Einstein radii, and (iii) baseline comparison with common empirical models. Together, these support the universality of the $\alpha \propto r_0^{-1}$ scaling law.

Table 3: Final Galaxy Fit Summary Table: Comparison Between MEST-SF and Best Empirical Models

Galaxy	**RMSE (MEST)**	**Best Empirical Model**	**RMSE (Empirical)**	ΔRMSE	**Preferred Model**
DDO 154	2.14	ISO	2.21	0.07	MEST
NGC 3198	3.60	Polyex	4.20	0.60	MEST
M31	7.34	NFW	8.13	0.79	MEST
UGC 128	4.87	Einasto	5.60	0.73	MEST
NGC 2841	5.62	Polyex	6.87	1.25	MEST
F568-v1	3.91	Polyex	4.75	0.84	MEST
F579-v1	4.44	Polyex	5.36	0.92	MEST
NGC 2403	3.76	Polyex	4.38	0.62	MEST
NGC 6503	2.98	Polyex	3.64	0.66	MEST
IC 2574	2.24	Polyex	3.12	0.88	MEST

Table 4: Best-fit parameters and residual statistics for representative lensing systems.

System	r_0 [kpc]	α	χ^2_ν	RMSE	Preferred
A1689	4.20	1.20	1.12	0.18	MEST
CL0024+17	8.30	1.50	1.21	0.20	MEST
SDSS J2141	5.20	1.40	1.08	0.12	MEST

Table 5: Observed vs. predicted Einstein radii.

System	Observed (arcsec)	Predicted (arcsec)	Error (arcsec)
SDSS J2141	1.25	1.22	0.03
A1689	45.00	44.60	0.40
CL0024+17	30.20	29.90	0.30

Table 6: Comparison of MEST$_{2n}$ and empirical models.

System	RMSE (MEST)	Best Empirical	RMSE (Emp.)	Preferred
Abell 1689	0.032	SIS	0.045	MEST
CL0024	0.028	SIE	0.031	MEST
A370	0.036	NFW	0.041	MEST