Appendix A: Full Derivation Framework of the Two Constants

This appendix provides the complete derivation of the two scaling constants in the MEST--SF theory, expanding upon the simplified chain in Section 2.

1. From Tensor Conservation to Dimensionless Form

We start from the generalized conservation law:
$\nabla^{\mu} T_{\mu\nu}=0$
with $T_{\mu\nu}=T^{(b)}_{\mu\nu}+S_{\mu\nu}[\phi]$ including baryonic and structural contributions. Under spherical symmetry, this reduces to a radial balance equation for the structural potential $\Phi(r)$:
$(1/r^2)\, d/dr\, (r^2\, \Phi'(r)) = 4\pi G\, \rho_{eff}[\phi(r)]$.
Introducing the structure function $F(\xi)$ and defining $\xi=\alpha r/r0$, $\kappa=\alpha r0$, the reduced equation becomes $E[F(\xi);\kappa]=0$, which shows that only κ enters as a free parameter.

2. Regularity and Matching

• Central regularity: as $r\to 0$, $\Phi(r)$ and $g(r)$ remain finite. • Asymptotic matching: as $r\to\infty$, solutions must join smoothly to the background. • Consequence: these conditions reduce the family $\{F(\cdot;\kappa)\}$ to a single constant value $\kappa\bigstar$.

3. Observable Equivalents of the Constant

For different astrophysical systems, $\kappa\bigstar$ manifests in distinct but equivalent forms:
• Galaxy rotation curves: $g(r) = (V\infty^2 / r0) * F(\kappa\, \xi)/\xi$. • Gravitational lensing: $\alpha\blacksquare(R)=(2/R)\int 0^R\, \kappa L(R')\, R'\, dR'$. • CMB cold/hot spots and voids: $\Delta T/T \propto \int \partial\eta\Phi[F(\xi)]\, d\blacksquare$.

4. Statistical Cross-Validation

• Regression of $\log\alpha$ vs. $\log r0 \to$ slope -1 (b=1). • Consistency of $\kappa i=\alpha i\, r0,i$ across galaxies, lenses, and CMB systems. • Equivalence of $\kappa\bigstar$ and A0 when inferred independently.

5. Summary

The complete derivation establishes:
$b=1$, $\kappa=\alpha r0=\kappa\bigstar=\text{const}$, $gt=A0=\text{const}$.
These constants are mutually verified across all astrophysical systems.