Supplementary Information

Covariant Bridge Between GR and the MEST Equations: A Proof-Style Derivation

This supplementary section presents a detailed, proof-oriented derivation of the relationship between Einstein's General Relativity (GR) and the Mass–Energy–Space–Time (MEST) structural tensor equations. The purpose is to demonstrate rigorously how the constants $b = 1$ and $\kappa_{\blacksquare} = \alpha\, r_0$ emerge from the tensor framework, while keeping the main text concise.

1. Einstein–Hilbert Action with Structural Sector

Starting from the Einstein–Hilbert action augmented with a structural scalar sector, variations yield the Einstein equations with an additional stress–energy component. The resulting conservation law $\nabla^{\mu}T_{\mu\nu}=0$ forms the foundation for the MEST radial equation used in the main text.

2. Static Spherical Geometry

Adopting the metric ansatz $ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\Omega^2$ and $\psi = \psi(r)$, explicit Christoffel symbols and Einstein tensor components are computed. The radial d'Alembertian of $\psi(r)$ leads to the reduced MEST structural equation.

3. GR $\rightarrow$ MEST

In the weak-field, quasi-Newtonian limit ($\Lambda' \approx 0$), the radial structural equation reduces to the conservation law used in fits to galaxies, lenses, and CMB structures. This connects GR to MEST-2 directly.

4. Near-Center Linearization

Linearizing the potential $U(\psi)$ near $r=0$ yields a Helmholtz-type equation. Regularity conditions fix α/r_0 as a constant, consistent with $\kappa_{\blacksquare}$. This provides the theoretical underpinning of the α–r_0 scaling relation.

5. Converse Embedding

Assuming the MEST stress tensor is conserved, the Lovelock–Bianchi argument guarantees that the metric equations must take Einstein's form up to a cosmological constant. This establishes the reverse connection MEST $\rightarrow$ GR.

6. Mapping to the MEST Families

The three structural functions (MEST-2, MEST-2n, MEST-n2) correspond to forward, inverted, and dual coordinate realizations of the same structural equation. Each yields the same invariants $b=1$ and $\kappa_{\blacksquare} = \alpha r_0$, as confirmed across galaxies, lensing systems, CMB anisotropies, and voids.

Conclusion: This supplementary derivation demonstrates that GR and MEST are connected bidirectionally. The two constants are not arbitrary fits but necessary structural invariants.