

Supplementary Materials for "Supercritical dynamics governs extreme seismic hazard"

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ABSTRACT

Supporting materials for the main text

Comments on the seismogenetic potential of the Tohoku-Oki region

The pre-Tohoku 2011 state of knowledge

Before the 2011 Mw 9.1 Tohoku earthquake, the geodetic moment release suggested $M_x \leq 8.3$ for Tohoku. At the same time, the corner magnitude estimated from geodetic data (Figure 1 of the main text and Table 1 in the Supplementary Material, was close to 8.1). Also, that subduction interface was supposed to be strongly segmented, hosting weak rheology rocks highlighted by average low seismic coupling. The 2011 event showed that all these assumptions might be not reliable:

1. Segmentation did not impede the rupture to propagate: pre-Tohoku models assumed tectonic segments would stop ruptures, but the 2011 event jumped multiple at least five macroscopic subduction segments.
2. Violation of the tapered-Gutenberg-Richter law: the corner seismic moment M_{0x} was underestimated because the estimation of the seismic coupling and the largest area of the seismogenetic source were poorly constrained.
3. Incomplete paleoseismic records: the 869 Jogan earthquake (likely $\sim M9$) was deemed to be a $\sim M 8.3$ until 2014¹

Using the pre-Tohoku underestimates:

$$\dot{M}_0 = \mu A v \chi = 30 \text{ GPa} \times 10^5 \text{ km}^2 \times 0.08 \text{ m/yr} \times 0.5 \approx 1.2 \times 10^{20} \text{ Nm/yr} \quad (1)$$

Using the simplified moment rate equation²

$$\dot{M}_0 \approx \frac{\alpha_0 M_{0c}^\beta M_{0x}^{1-\beta}}{1-\beta} \Gamma(2-\beta) \quad (2)$$

and by solving for the corner seismic moment, we get

$$M_{0x} \approx \left[\frac{\dot{M}_0 (1-\beta)}{\alpha_0 M_{0c}^\beta \Gamma(2-\beta)} \right]^{1/(1-\beta)} = \left[\frac{1.2 \times 10^{20} \times 0.33}{(10^{16})^{2/3} \times 0.893} \right]^3 \approx 2 \times 10^{21} \text{ Nm} \quad (3)$$

where $\beta \approx 0.67$ and $\alpha_0 \approx 1$ for our target event. Then, the maximum likely magnitude was supposed to be close to the corner magnitude given by $M_x \approx 8.4^*$.

Post-Tohoku 2011 assumptions

The Tohoku-Oki subduction zone is now usually modeled as a convergent margin characterized by a relative plate velocity $v = 8 \text{ cm/yr}$ with average seismic coupling $\chi \approx 0.9$ instead of 0.5^{3,4}, seismic and a seismic width of about $W = 200 \text{ km}$ to be considered along a interface of about $L = 500 \text{ km}$ that can break during a single seismic event. For calculation, the standard

shear modulus $\mu = 30$ GPa can be used with a b-value $b \approx 1.0$ ($\beta \approx 0.67$) and $M_{0c} \approx 5$ ($\alpha \approx 1$). Under these reasonable hypotheses, the tectonic moment rate is

$$\dot{M}_0 = \mu A v \chi = 30 \times 10^{10} \times (200 \times 500 \times 10^6) \times 0.08 \times 0.5 \approx 2 \times 10^{20} \text{ Nm/yr.} \quad (4)$$

Using the Kagan's formula and solving for the corner moment M_{0x} , it gives an estimate of the maximum likely earthquake

$$M_{0x} \approx \left(\frac{\dot{M}_0(1-\beta)}{\alpha_0 M_{0c}^\beta \Gamma(2-\beta)} \right)^{1/(1-\beta)} \quad (5)$$

This predicts a maximum moment around $\sim 5 \times 10^{22}$ N m, corresponding to $M_x \approx 9.0$, consistent with the 2011 Tohoku earthquake ($M_w 9.1$).

Table 1 summarizes the differences between the assumptions before and after the Tohoku earthquake.

Table 1. Parameters for the Tohoku-Oki region (pre- and post- 2011 event).

Parameter	Pre-Tohoku	Post-Tohoku
Fault Area (A)	$500 \times 200 \text{ km}^2$	$500 \times 200 \text{ km}^2$
Seismic Coupling (χ)	0.5	0.9–1.0 (?)
Corner Magnitude (M_x)	Mw 8.4	Mw 9.0
Segmentation	Rigid	Dynamic

Assumptions in the light of the new model

I add a Gaussian-distributed extreme component ($M_e > M_x$) to the tapered Gutenberg-Richter as

$$N(> M) = \left(\frac{M}{M_{0c}} \right)^{-\beta} e^{(M_{0c}-M)/M_{0x}} + \frac{1}{\sqrt{2\pi}\sigma^2} e^{-(M-M_e)^2/2\sigma^2} \quad (6)$$

$M_e > M_x \approx 9.1$ compatible with the Jogan 869 earthquake with 1σ uncertainty (≈ 9.1).

The seismic moment is now partitioned as

$$\dot{M}_0^{\text{total}} = \dot{M}_0^{\text{GR}}(M_x = 8.4^*) + \dot{M}_0^{\text{ext}}(M_e = 9.1) \quad (7)$$

Using $\sigma = 0.2$ (standard for the uncertainty of large magnitudes, the extreme peak is strict and the Gaussian Integral is roughly 1), using $\varepsilon \approx 1/1150 \text{ yr}^{-1}$ (roughly the interevent time between the Jogan 869 and the Tohoku-Oki earthquake 2011 as in the main text and methods):

$$\dot{M}_0^{\text{ext}} \approx \varepsilon \cdot 10^{1.5(M_e+6.1)} \cdot e^{(1.5\sigma \ln 10)^2/2} \quad (8)$$

$$= \frac{1}{1150} \times 4.5 \times 10^{22} \times 1.12 \approx 5 \times 10^{19} \text{ Nm/yr} \quad (9)$$

So that the total seismic moment rate is given by

$$\dot{M}_0^{\text{total}} \approx 1.2 \times 10^{20} + 0.5 \times 10^{20} = 1.7 \times 10^{20} \text{ Nm/yr} \quad (10)$$

which matches modern geodetic estimates (2×10^{20} Nm/yr) when including both components and without using the “new” hypotheses biased by the occurrence of the Tohoku-Oki event.

In this framework, the pre-Tohoku data was not wrong since the 1.2×10^{20} Nm/yr rate correctly described the tapered Gutenberg-Richter component. They were just incomplete. Indeed, the assumption of a fully coupled subduction interface for thousand years and over the whole structure is not feasible and still poorly supported by geodetic investigations both in Tohoku and worldwide.

Anyway, the extreme term (5×10^{19} Nm/yr) was overlooked.

This is possible because

$$\varepsilon M_{0e} \approx \frac{4.5 \times 10^{22} \text{ N m}}{1150 \text{ yr}} \approx 4 \times 10^{19} \ll 1.2 \times 10^{20} = \dot{M}_0^{\text{GR}}, \quad (11)$$

even though its contribution **dominates** seismic hazard.

This observation solves the paradox of the “missing super-critical term”.

References

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