Supplementary Information - Planar Fault-Tolerant Quantum Computation with Low Overhead

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I. LOGICAL BASIS OPTIMIZATION

The pseudocode for logical basis optimization is provided in Algorithm 1.

Algorithm 1 Logical Basis Optimization

Input: Check matrices of the original code H_X and H_Z; distances s_{xx} and s_{zz} characterizing the horizontal and vertical separation between two BB-code blocks.

Output: X logical operators $\{j_{X,1}, j_{X,2}, \ldots, j_{X,k}\}$ and Z logical operators $\{j_{Z,1}, j_{Z,2}, \ldots, j_{Z,k}\}$ of the original code satisfying $j_{X,a} j_{Z,b}^{\mathrm{T}} = \delta_{a,b}$ for all $a, b = 1, 2, \ldots, k$.

Construct a deformed code by horizontally connecting two blocks of the original code separated by s_{xx}. The resulting check matrices are:

$$\bar{H}_X^{(xx)} = \begin{pmatrix} H_X & 0 & 0 \\ S_1 & H_1^{\mathrm{T}} & S_2 \\ 0 & 0 & H_X \end{pmatrix} \text{ and } \bar{H}_Z^{(xx)} = \begin{pmatrix} H_Z & T_1 & 0 \\ 0 & H_2 & 0 \\ 0 & T_2 & H_Z \end{pmatrix}.$$

Find a vector of the form $(j_{X,1}, 0, j_{X,2})$ such that:

(i) $j_{X,1}, j_{X,2} \in \ker H_Z$;

(ii) $j_{X,1}, j_{X,2}$ and the rows of H_X are linearly independent;

(iii) $(j_{X,1}, 0, j_{X,2}) \in \mathrm{rowsp}(\bar{H}_X^{(xx)})$.

▷ If no such vector is found, try other values of s_{xx}.

Construct a deformed code by vertically connecting two blocks of the original code separated by s_{zz}. The resulting check matrices are:

$$\bar{H}_X^{(zz)} = \begin{pmatrix} H_X & T_3 & 0 \\ 0 & H_3 & 0 \\ 0 & T_4 & H_X \end{pmatrix} \text{ and } \bar{H}_Z^{(xx)} = \begin{pmatrix} H_Z & 0 & 0 \\ S_3 & H_4^{\mathrm{T}} & S_4 \\ 0 & 0 & H_Z \end{pmatrix}.$$

Find a set of linearly independent vectors of the form

$$A_l = (A_{L,l}, 0, A_{R,l})$$

satisfying:

(i) $A_{L,l}, A_{R,l} \in \ker H_X$;

(ii) $(A_{L,l}, 0, A_{R,l}) \in \mathrm{rowsp}(\bar{H}_Z^{(zz)})$.

Form a matrix A with rows A_l, so that

$$A = (A_L, 0, A_R).$$

Solve the system

$$\begin{cases} qA_L j_{X,1}^{\mathrm{T}} = 0, \\ qA_L j_{X,2}^{\mathrm{T}} = 0, \\ qA_R j_{X,1}^{\mathrm{T}} = 0, \\ qA_R j_{X,2}^{\mathrm{T}} = 0, \end{cases}$$

and choose a solution q such that the vectors qA_L, qA_R, and the rows of H_Z are linearly independent. ▷ If no suitable solution is found, try different $(j_{X,1}, 0, j_{X,2})$ or vary s_{xx} and s_{zz}.

Set $j_{Z,3} \leftarrow qA_L$ and $j_{Z,4} \leftarrow qA_R$.

Complete the basis construction for the logical operators.

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Single-block X measurement																				
Ancilla size	X_1X_2		X_1X_3		X_1X_4		X_1X_5		X_2X_3		X_2X_4		X_2X_5		X_3X_4		X_3X_5		X_4X_5	
32	2	6	3	6	2	5	2	5	2	7	3	7	2	5	6	6	2	5	2	5
56	2	8	6	8	2	7	2	7	2	9	3	9	2	6	7	7	2	8	2	7
80	2	9	9	9	2	8	2	8					3	9	2	8	2	9	2	7
104					2	9	2	9							2	8			3	8
128															2	9			2	9

Single-block Z measurement																				
Ancilla size	Z_1Z_2		Z_1Z_3		Z_1Z_4		Z_1Z_5		Z_2Z_3		Z_2Z_4		Z_2Z_5		Z_3Z_4		Z_3Z_5		Z_4Z_5	
27	4	5	4	5	4	5	3	6	4	5	3	6	3	4	3	4	4	4	4	5
42	5	6	3	6	5	5	4	6	6	7	5	7	4	5	5	5	2	5	4	6
57	6	6	6	6	6	7	2	7	7	8	5	8	2	6	2	6	3	6	3	7
72	7	7	2	7	7	8	4	9	9	9	5	8	4	7	3	8	4	6	2	8
87	9	9	4	9	9	9					5	9	4	8	4	8	4	6	3	9
102													4	9	5	8	4	8		
117															5	8	4	8		
132															5	9	4	9		

TABLE I. Supplementary data on code distances for the [[180,6,9]] BB code.

Single-block X measurement																				
Ancilla size	X_0		X_1		X_2		X_3		X_4		X_5		X_0X_1		X_0X_2		X_0X_3		X_0X_4	X_0X_5
14	2	2	3	3	2	3	2	2	2	4	2	3	2	2	3	4	2	4	2	2	2	3
26	2	4	2	3	2	4	2	3			2	4	2	3					2	4	2	3
38			2	4			2	3					2	4							2	3
50							2	4													2	4
	X_1X_2		X_1X_3		X_1X_4		X_1X_5		X_2X_3		X_2X_4		X_2X_5		X_3X_4		X_3X_5		X_4X_5	
14	3	4	2	3	2	3	2	3	2	3	2	2	2	3	2	2	2	2	2	3		
26			2	4	2	4	3	3	2	4	2	4	2	4	2	3	2	4	2	3		
38							3	3							2	4			2	4		
50							3	4														

Single-block Z measurement																				
Ancilla size	Z_0		Z_1		Z_2		Z_3		Z_4		Z_5		Z_0Z_1		Z_0Z_2		Z_0Z_3		Z_0Z_4	Z_0Z_5
15	4	4	4	4	3	3	2	2	4	4	3	3	2	2	3	3	3	3	2	2	3	4
24					4	4	4	4			3	3	2	2	2	4	2	4	2	3		
33											3	4	3	3					3	4		
42													4	4								
	Z_1Z_2		Z_1Z_3		Z_1Z_4		Z_1Z_5		Z_2Z_3		Z_2Z_4		Z_2Z_5		Z_3Z_4		Z_3Z_5		Z_4Z_5	
5	3	3	2	2	4	4	3	3	4	4	4	4	3	4	2	2	3	4	3	3		
24	4	4	4	4			2	3							2	2			2	3		
33							3	4							3	3			3	4		
42															2	4						

Two-block X measurement		
Ancilla size	$X_0 \otimes X_1$	
78	2	4

Two-block Z measurement		
Ancilla size	$Z_2 \otimes Z_3$	
54	4	4

TABLE II. Code distances of deformed codes for the [[54,6,4]] BB code.

II. CODE DISTANCES OF DEFORMED CODES

Supplementary data are provided in Tables I, II, and III.

Single-block X measurement

Ancilla size	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_0X_1	X_0X_2	X_0X_3	X_0X_4
28	2 3	2 4	2 2	2 2	2 3	2 4	2 4	3 4	2 3	2 4	5 5	3 4
46	2 4	2 4	2 3	2 3	2 4	2 5	2 5	5 5	2 4	2 4	5 5	2 4
64	3 5	4 6	2 6	2 6	2 6	3 6	3 6	6 6	2 4	2 6	7 7	2 6
82	6 7	4 6	3 6	2 7	2 6	4 7	3 6	7 7	3 6	2 6		3 6
100		6 7	3 6		3 6		3 7		2 7	3 6		4 7
118			3 6		2 7					2 6		
136			3 7							3 7		

	X_0X_5	X_0X_6	X_0X_7	X_1X_2	X_1X_3	X_1X_4	X_1X_5	X_1X_6	X_1X_7	X_2X_3	X_2X_4	X_2X_5
28	3 3	2 4	2 4	2 2	2 2	2 3	2 4	2 4	3 4	2 4	2 5	2 2
46	4 5	2 4	3 4	2 3	2 3	2 4	2 5	3 4	2 5	2 4	2 5	2 3
64	3 5	2 6	3 6	3 6	2 4	2 6	2 6	4 6	2 6	2 6	2 7	3 6
82	3 7	2 6	5 6	5 6	2 5	2 6	4 7	3 6	2 7	2 7		2 6
100		2 6	5 6	6 7	2 7	2 7		3 7				2 6
118		2 7	7 7									3 7

	X_2X_6	X_2X_7	X_3X_4	X_3X_5	X_3X_6	X_3X_7	X_4X_5	X_4X_6	X_4X_7	X_5X_6	X_5X_7	X_6X_7
28	2 4	2 2	6 6	3 4	2 4	4 4	3 4	2 3	3 4	2 2	2 4	2 4
46	2 4	2 3	6 6	2 4	2 4	3 5	3 4	2 3	3 4	2 3	3 4	2 6
64	3 6	2 5	7 7	3 6	2 6	3 6	2 6	2 4	5 6	2 5	4 6	2 6
82	5 7	2 6		3 6	2 6	5 7	3 6	2 5	5 7	2 5	5 6	2 7
100		3 7		5 6	3 6		5 7	2 7		2 7	4 6	
118				5 6	3 6						3 7	
136				7 7	3 7							

Single-block Z measurement

Ancilla size	Z_0	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_0Z_1	Z_0Z_2	Z_0Z_3	Z_0Z_4
30	2 4	2 3	2 5	2 5	3 6	2 5	2 5	2 4	2 4	2 5	2 4	2 3
48	2 5	2 5	2 6	2 6	4 7	5 6	4 6	2 5	5 5	2 6	2 5	2 4
66	2 6	2 5	3 6	2 7		6 6	4 6	2 6	4 6	2 6	2 6	2 5
84	3 6	2 6	2 7			7 7	5 6	2 6	2 6	2 6	3 7	2 6
102	4 7	3 6					6 7	2 6	7 7	2 6		3 6
120		2 7						2 7		3 7		4 6
138												4 7

	Z_0Z_5	Z_0Z_6	Z_0Z_7	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_1Z_5	Z_1Z_6	Z_1Z_7	Z_2Z_3	Z_2Z_4	Z_2Z_5
30	3 4	4 4	2 4	2 3	2 4	3 4	2 2	2 4	2 4	2 4	2 5	2 5
48	2 5	4 6	4 4	2 4	2 6	2 4	2 3	2 5	2 5	2 5	2 5	2 6
66	2 6	4 6	2 6	3 5	4 6	2 5	2 5	2 7	5 6	4 6	3 6	3 6
84	3 7	4 6	3 7	4 6	5 7	3 6	3 6		6 7	4 6	3 7	4 6
102		4 7		5 7		4 6	2 7			5 6		2 6
120						2 7				6 7		2 6
138												2 7

	Z_2Z_6	Z_2Z_7	Z_3Z_4	Z_3Z_5	Z_3Z_6	Z_3Z_7	Z_4Z_5	Z_4Z_6	Z_4Z_7	Z_5Z_6	Z_5Z_7	Z_6Z_7
30	2 3	2 3	2 4	2 5	2 5	2 5	2 5	2 4	2 5	3 5	2 6	4 5
48	2 4	2 4	2 5	2 6	2 5	3 6	2 6	3 5	2 5	3 6	2 7	2 6
66	2 5	3 6	3 6	2 6	2 6	5 7	3 6	2 5	2 6	2 6		2 6
84	2 7	2 6	2 7	4 7	2 6		5 7	2 6	3 7	2 7		3 7
102		2 6			2 6			3 7				
120		2 7			2 7							

Two-block X measurement

Ancilla size	$X_0 \otimes X_1$
162	3 7

Two-block Z measurement

Ancilla size	$Z_2 \otimes Z_3$
144	3 7

TABLE III. Code distances of deformed codes for the [[162,8,7]] BB code.