SUPPLEMENTARY INFORMATION:
Defect-Aware Extreme Device Scaling Limits of 2D Memristive Technologies

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Matérn HCPP-I Defect Density Expression *Proof.*

For a Poisson point process (PPP) with density λ_P, let $N_{\lambda,A}$ denote the number of point that falls with an area of size A. Since all points that violates the hard-core distance are eliminates, a point from the PPP is retained in the HCPP-I if there are no other points within its r proximity. The density of the HCPP-I can be obtained as

$$\lambda_{\text{I}} = \lambda_{\text{P}} \times \mathbb{P}\{\text{a point is retained:= } N_{\lambda,\pi r^2} = 0\}$$
$$\lambda_{\text{I}} \overset{*}{=} \lambda_{\text{P}} e^{-\lambda_{\text{P}}\pi r^2}$$

* *follows from the probability of the Poisson distribution (Eq. 1 in the main text, for k = 0).*

Matérn HCPP-II Defect Density Expression *Proof.*

Starting from a PPP with density λ_P and a uniformly distributed mark for each point, the HCPP-II is generated by retaining the points that have the lowest mark within their proximity defined by a circle of radius r. The density of the HCPP-II can be obtained as

$$\lambda_{\text{II}} = \lambda_{\text{P}} \times \mathbb{P}\{\text{a point is retained}\}$$

Let $N_{\lambda,A}$ denote the number of defects that fall with an area of size A, the probability that a PPP point is retained in the HCPP-II can be obtained as

$$\begin{aligned}
\mathbb{P}\{\text{a point is retained}\} &= \sum_{n=0}^{\infty} \mathbb{P}\{\text{ a point has the smallest mark} \mid N_{\lambda,\pi r^2} = n\}\mathbb{P}\{N_{\lambda,\pi r^2} = n\} \\
&\overset{(i)}{=} \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{\exp(-\lambda_\text{P}\pi r^2)(\lambda_\text{P}\pi r^2)^n}{n!} \\
&\overset{(ii)}{=} \sum_{k=1}^{\infty} \frac{1}{k} \frac{\exp(-\lambda_\text{P}\pi r^2)(\lambda_\text{P}\pi r^2)^{(k-1)}}{(k-1)!} \\
&= \frac{\exp(-\lambda_\text{P}\pi r^2)}{\lambda_\text{P}\pi r^2} \sum_{k=1}^{\infty} \frac{(\lambda_\text{P}\pi r^2)^k}{k!} \\
&= \frac{\exp(-\lambda_\text{P}\pi r^2)}{\lambda_\text{P}\pi r^2} \left(\exp(\lambda_\text{P}\pi r^2) - 1\right) \\
&= \frac{1 - \exp(-\lambda_\text{P}\pi r^2)}{\lambda_\text{P}\pi r^2}
\end{aligned}$$

(i) follows from the uniform distribution of the assigned marks to the defects and the Poisson distribution of the defects in space, and the inter independence of their random variables. (ii) follows from the change of variables $k = n + 1$.

The density of the HCPP-II can then be derived as:

$$\begin{aligned}
\lambda_\text{II} &= \lambda_\text{P} \times \frac{1 - \exp(-\lambda_\text{P}\pi r^2)}{\lambda_\text{P}\pi r^2} \\
\lambda_\text{II} &= \frac{1 - \exp(-\lambda_\text{P}\pi r^2)}{\pi r^2}.
\end{aligned}$$

Probabilistic Device Switching Model

Probabilistic Device Model Based on Weakest Defect Activation *Proof.* Assuming a memristor with N defects, let X_i, $i \in \{1, 2, \cdots, N\}$, be the voltage stress needed to activate the i-th defect. The switching voltage is determined by the defect that requires the lowest voltage stress to form a conductive filament. Based on this, the device SET voltage (V_{SET}) can be defined as the minimum amongt all X_i, $i \in \{1, 2, \cdots, N\}$:

$$V_{SET} = \min(X_1, \ldots, X_N), \tag{1}$$

where a device is classified as a working device if applied voltage stress $v > V_{SET}$, and which has the following probability, given that there are N defects within the device (the conditional switching voltage probability):

$$\begin{aligned}
\mathbb{P}(V_{SET} < v|N) &= \mathbb{P}(\min_i\{X_1, X_2, X_3, \ldots, X_N\} < v) \\
&= 1 - \mathbb{P}(X_1 > v, X_2 > v, X_3 > v, \ldots, X_N > v) \\
&\overset{(a)}{=} 1 - \prod_{i=1}^{N} \mathbb{P}(X_i > v) \\
&= 1 - \prod_{i=1}^{N} (1 - \mathbb{P}(X_i < v)) \\
&\overset{(b)}{=} 1 - \{1 - F_{\mathbf{X}}(v)\}^N
\end{aligned} \tag{2}$$

(a) follows from probability independence of X_i $\forall i$, and (b) follows from identical distributions of X_i $\forall i$.

Since the probability expressed in (2) is for $N \sim$ Pois$[\Lambda(\lambda, A)]$, then the marginal probability $\mathbb{P}(V_{SET} < v)$ that describes the probability that a device switches (yields) at an applied voltage stress (v) is:

$$\begin{aligned}
\mathbb{P}(V_{SET} < v) &= \mathbb{E}_N[1 - \{1 - F_{\mathbf{X}}(v)\}^N] \\
&= 1 - \sum_{n=0}^{\infty} \{1 - F_{\mathbf{X}}(v)\}^n \left(\frac{\Lambda^n}{n!} e^{-\Lambda}\right) \\
&= 1 - e^{-\Lambda} \sum_{n=0}^{\infty} \left(\frac{\{\Lambda - \Lambda F_{\mathbf{X}}(v)\}^n}{n!}\right) \\
&\overset{(i)}{=} 1 - e^{-\Lambda} \exp\{\Lambda - \Lambda F_{\mathbf{X}}(v)\} \\
&= 1 - \exp\{-\Lambda F_{\mathbf{X}}(v)\}
\end{aligned}$$

(i) follows from Taylor series expansion for e^x.