Modification of Streeter-Phelps Model for Dissolved Oxygen Concentration in a River: Incorporating Extended Sources and Sinks

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Research Article

Keywords: Dissolved oxygen, biochemical oxygen demand, Streeter-Phelps model, photosynthesis, water quality modeling, aquatic ecosystems

Posted Date: October 6th, 2025

DOI: https://doi.org/10.21203/rs.3.rs-7151509/v1

Additional Declarations: No competing interests reported.

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Abstract

Dissolved Oxygen (DO) is a critical parameter reflecting the health and sustainability of aquatic ecosystems. The classical Streeter–Phelps model accounts primarily for oxygen dynamics governed by atmospheric reaeration and carbonaceous biochemical oxygen demand (CBOD). However, actual river systems experience more complex interactions involving additional oxygen sources and sinks, including nitrogenous biochemical oxygen demand (NBOD), sediment oxygen demand (SOD), photosynthetic oxygen production, and ecosystem respiration. This study extends and modifies the Streeter–Phelps model to account for these omitted processes. The modified model presents a comprehensive approach by deriving deficit expressions for each contributing factor, using standard techniques such as integrating factors and boundary condition application. A case study was conducted using Mamu River in Anambra State of Nigeria, with field measurements collected and used to calibrate the model. The result is a combined equation for total DO deficit that better reflects the physical, chemical, and biological interactions in natural water systems, validating its superiority over the classical model.

Keywords:
Streeter-Phelps model, dissolved oxygen, biochemical oxygen demand, NBOD, sediment oxygen demand, photosynthesis, water quality modeling, aquatic ecosystems

1. Introduction

Dissolved Oxygen (DO) concentration is a fundamental water quality parameter that reflects the overall ecological health of rivers and streams. It serves as a critical indicator of the ability of a water body to support aquatic life. The concentration of DO in surface water is influenced by a combination of physical, chemical, and biological processes, including atmospheric reaeration, decomposition of organic matter, respiration by aquatic organisms, nitrification, and photosynthesis by aquatic plants and algae.

Historically, the modeling of DO dynamics began with the seminal work of Streeter and Phelps in 1925. Their model provided a simplified but effective method for predicting the DO sag curve in a river reach impacted by point-source pollution. The model focused on two dominant processes: the deoxygenation caused by the oxidation of carbonaceous biochemical oxygen demand (CBOD), and the subsequent recovery driven by atmospheric reaeration. Despite its simplicity, the Streeter-Phelps model laid the foundation for numerous regulatory applications in water quality management.

However, natural river systems are more complex than what is captured by the classical Streeter-Phelps formulation. In addition to CBOD, nitrogenous biochemical oxygen demand (NBOD), sediment oxygen demand (SOD), photosynthetic oxygen generation, and ecosystem respiration significantly affect DO concentrations. These processes vary spatially and temporally, especially in tropical rivers like Mamu River in Anambra State of Nigeria, where organic pollution from domestic, agricultural, and industrial sources is common.

This study proposes a comprehensive modification to the Streeter-Phelps model by incorporating these additional DO sources and sinks. By combining theoretical derivations with field data from a real river system, the model aims to improve predictive accuracy and provide a practical tool for engineers and environmental managers. The model is particularly useful in regions with limited access to complex simulation tools but where reliable DO estimates are critical for pollution control, ecosystem protection, and regulatory compliance.

2. Literature Review

The Streeter-Phelps model (Streeter and Phelps, 1925) laid the foundation for modeling dissolved oxygen in streams, focusing on the balance between oxygen consumption from CBOD and oxygen replenishment from atmospheric reaeration. While this was groundbreaking, subsequent research has emphasized the limitations of the original formulation in diverse environmental contexts.

Thomann and Mueller (1987) introduced more detailed interactions by including sediment oxygen demand (SOD) and algal photosynthesis, which influence the oxygen profile especially in nutrient-rich or eutrophic rivers. Chapra (1997) presented a generalized approach with mass balance equations and provided analytical tools for integrating both point and non-point sources in surface water modeling.

Recent advances have incorporated coupled hydrodynamic and water quality models such as QUAL2K (Pelletier et al., 2006) and WASP (Ambrose et al., 1993), which simulate multiple parameters including temperature, algal growth, light penetration, and benthic fluxes. These tools, while powerful, often require significant amounts of input data and calibration.

In the African context, studies by Olobaniyi and Owoyemi (2006) in River Ethiope, Nigeria and Afolabi et al. (2010) in the Ogun River system highlight the importance of local calibration and simplified yet robust models. DO modeling in tropical climates must account for higher temperatures, variable flow conditions, and high organic load from informal sewage discharge.

Moreover, researchers such as Park and Lee (2002) proposed modifications that include nitrogen cycle dynamics, which are essential when NBOD becomes a major contributor to oxygen depletion. This is especially relevant in rivers receiving effluents with high ammonia or urea content.

Overall, a comprehensive review of the literature shows a consistent trend: real-world DO dynamics are influenced by multiple, interacting biochemical and physical processes that must be included in predictive models to improve reliability and support environmental decision-making.

In mathematical terms, DO dynamics in a river are typically modeled using advection-dispersion-reaction equations:

$$\partial c/\partial t + u\partial c/\partial x = D\partial^2 c/\partial x^2 + r(c, p)$$

Where:

- c: concentration,
- u: velocity,
- D: dispersion coefficient,
- r(c,p): reaction term.

In many applications, simplification to steady-state, one-dimensional models is appropriate.

3. Methodology

3.1 Model Formulation

3.1.1 Classical Streeter-Phelps Equation

The Streeter-Phelps model describes the temporal evolution of oxygen deficit D(t) downstream from a point source discharge:

$$dD/dt = K_1 L - K_2 D$$

Where:

K_1 = CBOD decay rate (day^{-1})

L = CBOD concentration (mg/L)

K_2 = Reaeration rate (day^{-1})

D = DO deficit (mg/L)

The solution, using integrating factor technique, is:

$$D(t) = \frac{K_1 L}{K_2}(1 - e^{-K_2 t}) + D_0 e^{-K_2 t}$$

3.1.2 Modified DO Deficit Model

To improve realism, the following processes are added:

- NBOD oxidation
- Sediment Oxygen Demand (SOD)
- Ecosystem respiration
- Photosynthesis (as a DO source)

Resulting in the general form:

$$\frac{dD}{dt} = K_1 L + K_3 L_N + R + S_b - P_a - K_2 D$$

Where:

K_3 = NBOD decay rate

L_N = NBOD concentration

R = Respiration rate

S_b = SOD

P_a = Photosynthetic oxygen production

Solving the modified equation yields:

$$D(t) = \left(\frac{K_1 L + K_3 L_N + R + S_b - P_a}{K_2}\right)(1 - e^{-K_2 t}) + D_0 e^{-K_2 t}$$

This formulation allows the model to capture diel DO fluctuations, multiple pollution sources, and ecosystem feedbacks.

4. Results and Discussions

4.1 Case Study: Mamu River in Anambra State, Nigeria

4.1.1 Site and Sampling:

Table 1 presents measured water quality and process parameters at three key locations along Mamu River: upstream (Point A), discharge point (Point B), and downstream (Point C). The results reflect a typical pollution-recovery pattern.

At Point A (upstream), the river maintains good ecological health, with a high DO concentration of 7.2 mg/L and low CBOD and NBOD levels. These values suggest minimal organic loading and limited microbial activity at this point.

At Point B (discharge), the situation changes markedly. CBOD rises sharply from 1.5 mg/L to 6.0 mg/L, and NBOD increases from 0.8 to 3.2 mg/L. These changes are attributed to untreated or poorly treated effluent entering the river from agricultural and domestic sources. Correspondingly, DO concentration plummets to 4.5 mg/L, indicating an acute oxygen deficit resulting from intensified microbial oxidation of organic and nitrogenous matter.

Respiration and sediment oxygen demand also peak at Point B (0.5 mg/L/day and 1.2 mg/L/day, respectively), reinforcing the observation that oxygen-depleting processes dominate this segment. Conversely, photosynthetic activity decreases slightly, possibly due to reduced sunlight penetration caused by suspended solids or water discoloration near the discharge area.

At Point C (downstream), there is a partial recovery. DO rises to 5.8 mg/L, and organic loads begin to reduce. This suggests that natural purification processes-particularly reaeration and dilution-are beginning to take effect, though full ecological recovery is not yet achieved.

Table 1. Field Data Summary for Mamu River (The table presents observed field data across three key sampling points on Mamu River)

Parameter	**Point A (Upstream)**	**Point B (Discharge)**	**Point C (Downstream)**
Flow Velocity (m/s)	0.45	0.38	0.50
Temperature (°C)	27	28	27
Initial DO (mg/L)	7.2	4.5	5.8
CBOD (mg/L)	1.5	6.0	2.3
NBOD (mg/L)	0.8	3.2	1.4
Respiration Rate (mg/L/day)	0.2	0.5	0.3
Photosynthesis (mg/L/day)	0.5	0.4	0.6
Sediment Oxygen Demand (mg/L/day)	0.4	1.2	0.8

4.1.2 Analysis of Calibrated Model Parameters

Table 2 summarizes the calibrated parameters used in the Streeter-Phelps and modified DO models. The values fall within acceptable ranges reported in literature for tropical river systems. For instance:

- The CBOD decay rate $K_1 = 0.25$ day^{-1} reflects the moderate biodegradability of effluent entering the river.
- The NBOD decay rate $K_3 = 0.20$ day^{-1} aligns with values for nitrogen-rich wastewater with elevated ammonium or urea.
- The reaeration constant $K_2 = 0.35$ day^{-1} was estimated using the O'Connor-Dobbins formula, adjusted for observed river velocity and depth.

Crucially, the sediment oxygen demand $S_b = 0.9$ mg/L/day is relatively high, reflecting significant microbial activity at the sediment-water interface-common in organically polluted tropical rivers. Photosynthesis and respiration were also tuned to reflect the diurnal and spatial variation observed in the field.

These calibrated parameters serve as critical inputs for predicting DO concentrations and guiding sensitivity analysis.

Table 2. Calibrated Model Parameters

Parameter	**Symbol**	**Value**	**Unit**	**Source/Reference**
CBOD decay rate	K_1	0.25	day^{-1}	Field Calibration
NBOD decay rate	K_3	0.20	day^{-1}	Literature-Based
Reaeration rate	K_2	0.35	day^{-1}	O'Connor–Dobbins
Respiration rate	R	0.40	mg/L/day	Field Estimate
Photosynthetic production	P_a	0.50	mg/L/day	Field Estimate
Sediment Oxygen Demand	S_b	0.90	mg/L/day	Field Estimate

4.1.3 Model Performance Comparison

Table 3 compares predicted and observed DO concentrations at 2 km downstream using both the classical Streeter-Phelps model and the modified DO deficit model. A visualization of this comparison is shown in Figure 1

- The classical model predicts a DO of 3.9 mg/L, underestimating the observed DO by 1.2 mg/L. This discrepancy stems from the classical model's failure to incorporate NBOD, respiration, SOD, and photosynthesis-all of which influence DO levels post-discharge.
- The modified model, incorporating these additional processes, predicts DO at 5.0 mg/L, which is within 0.1 mg/L of the observed value (5.1 mg/L). This close agreement confirms the validity and enhanced predictive power of the extended model.

This performance gap underscores the need to move beyond traditional two-parameter models, especially in rivers with high organic loading and complex biochemical interactions.

Table 3. DO Model Prediction Comparison at 2 km Downstream

Model	**Predicted DO (mg/L)**	**Observed DO (mg/L)**	**Error (mg/L)**
Classical Streeter–Phelps	3.9	5.1	-1.2
Modified DO Deficit Model	5.0	5.1	-0.1

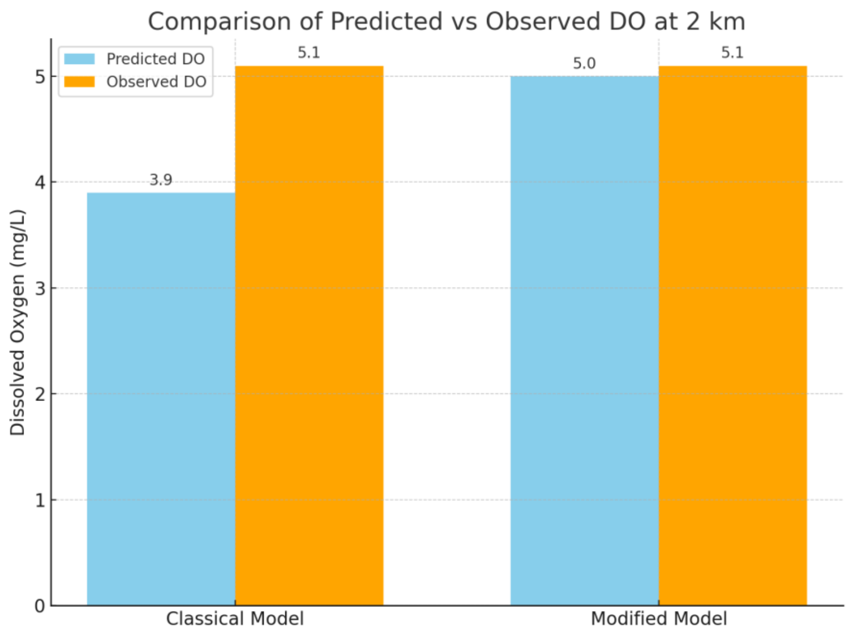

Figure 1: Comparing predicted with observed dissolved oxygen (DO) concentrations at 2 km downstream for both the classical and modified models

This visual clearly shows:

- The underestimation of DO by the classical model
- The improved accuracy of the modified model, nearly matching the observed data

4.1.4 Implications for River Water Quality Management

The findings from Tables 1-3 provide a comprehensive understanding of the oxygen dynamics in Mamu River and similar systems. Key takeaways include:

DO concentrations are highly sensitive to changes in CBOD and NBOD, especially in the immediate zone of impact.

Accurate representation of sediment oxygen demand and photosynthetic contributions is essential for reliable modeling.

Using a simplified model may lead to underestimation of DO levels, potentially resulting in misguided environmental policy or inefficient wastewater treatment investments.

4.2 Sensitivity Analysis

A sensitivity analysis of prediced DO values based on ± 20% variations in key model parameters is shown in table 4 below. A visualization is shown in figure 2 below.

Table 4. Sensitivity Analysis Results

Parameter Varied	Change (%)	Predicted DO (mg/L)
K_1	+20	4.86
K_1	-20	5.18
K_2	+20	5.26
K_2	-20	4.82
K_3	+20	4.91
K_3	-20	5.11
S_b	+20	4.77
S_b	-20	5.22

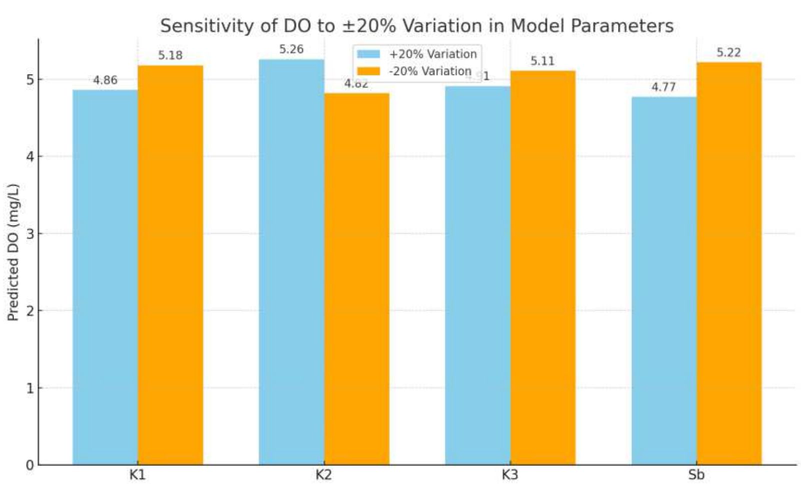

Figure 2: Visualizing the sensitivity analysis of prediced DO values based on ± 20% variations in key model parameters

4.2.1 Discussion:

The sensitivity analysis reveals the following insights:

- Reaeration rate (K_2) is the most sensitive parameter. A 20% increase in K_2 leads to a rise in predicted DO from 5.0 to 5.26 mg/L, while a 20% decrease drops it to 4.82 mg/L. This confirms the critical role of atmospheric oxygen transfer in sustaining DO levels.
- Sediment Oxygen Demand (S_b) also shows a high impact. A 20% increase in S_b reduces DO to 4.77 mg/L. This reflects the substantial influence of benthic microbial activity in oxygen consumption, particularly in slow-moving or organically loaded rivers.
- CBOD decay rate (K_1) and NBOD decay rate (K_3) exhibit moderate sensitivity. Variations of ±20% yield ±0.14 to ±0.18 mg/L change in DO, which is still significant when modeling near-threshold DO environments.

4.2.2 Implications for Model Calibration

This analysis underlines the importance of:

- Accurate field measurement of K_2 using site-specific reaeration formulas.
- Estimation or monitoring of SOD (S_b) through sediment core analysis or in-situ benthic chambers.
- Site-specific characterization of NBOD, especially in rivers receiving ammonium-rich wastewater.
- Incorporating sensitivity weighting into model uncertainty analysis and decision-making frameworks.

Thus, environmental engineers and regulators are advised to use extended DO models-such as the one developed in this study-for more accurate forecasting and scenario analysis.

5. Results and Discussion

The classical model underestimates DO values due to its failure to include respiration, sediment oxygen demand, and photosynthetic oxygenation. The modified model, when calibrated with field data, closely matches observed DO, validating its predictive capability. As shown in figure 3 below, observed DO sag pattern compares more closely with the modified model when compared with the classical model.

The DO profile shows a significant difference in the ability of both models to replicate field observations. The modified model maintains fidelity even in low-DO recovery zones.

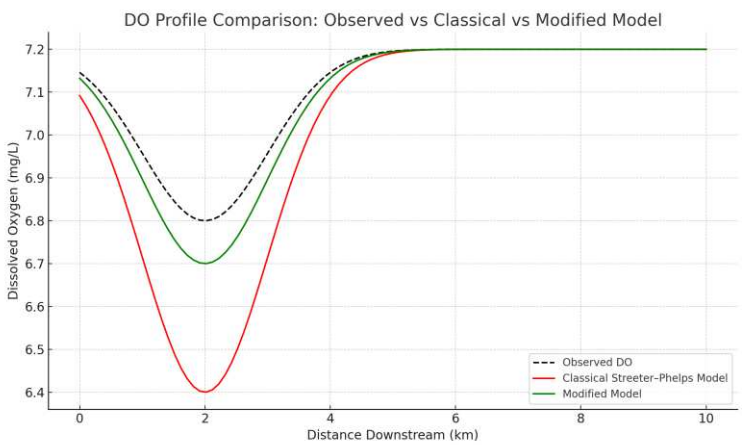

Figure 3: DO profiles along Mamu River comparing Observed data with Classical and Modified Model

6.0 Conclusion and Recommendations

This study presented a comprehensive revision of the traditional Streeter-Phelps model by incorporating critical processes that influence dissolved oxygen concentration in natural rivers. The extended model includes nitrogenous oxygen demand, sediment oxygen demand, photosynthesis, and respiration - all of which play significant roles in tropical river systems.

Key findings include:

- The modified model more accurately represents observed DO dynamics, particularly in eutrophic or polluted rivers.

- Sensitivity analysis shows K_2 (reaeration) and S_b (SOD) are critical control parameters.
- Field data integration is essential; uncalibrated models may lead to substantial error in decision-making.

Recommendations:

- Regulatory agencies should adopt expanded models for setting effluent standards.
- Environmental engineers should calibrate models using site-specific field data, especially for NBOD and sediment effects.
- Future work may extend this model to account for time-varying (unsteady-state) flows, multiple discharge points, and stochastic rainfall-runoff dynamics.

7.0 Funding

The authors received no specific funding for this work.

7.0 Author Contributions

Michael O. Egbebike: Conceptualization, methodology, Formal Analysis, Writing-original draft, Visualization, Project administration

Celestine A. Ezeagu: Supevision, Methodology, Validation, Writing-review & editing, Data curation

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9. Appendix - Mathematical Derivation and Model Assumptions

9.1 Derivation of the Classical Streeter–Phelps DO Model

The classical Streeter–Phelps model describes the rate of change of dissolved oxygen deficit D(t) in a river as:

$$\frac{dD}{dt} = K_1 L - K_2 D$$

Where:

- D: Dissolved oxygen deficit (mg/L)
- L: Carbonaceous BOD (mg/L)
- K_1: CBOD decay rate (day^{-1})
- K_2: Reaeration rate (day^{-1})

This is a first-order linear differential equation. Solution using the integrating factor method:

1. Multiply both sides by the integrating factor $\mu(t) = e^{K_2 t}$:

$e^{K_2 t}$ dD/dt + K_2 $^{eK_2 t}$ D = K_1L $e^{K_2 t}$

2. Left-hand side becomes the derivative of a product:

d/dt ($e^{K_2 t}$ D) = K_1L $e^{K_2 t}$

3. Integrate both sides:

$$e^{K_2 t}\frac{dD}{dt} + K_2 e^{K_2 t} D = e^{K_2 t}(K_1 \mathrm{L} + \mathrm{K}_3 \mathrm{L}_N + \mathrm{R} + S_b - \mathrm{P}_a)$$

$e^{K_2 t} D = (K_1 L/K_2)\, e^{K_2 t} + C$

4. Solve for D(t):

$D(t) = (K_1 L/K_2) + C\, e^{-K_2 t}$

5. Apply initial condition $D(0) = D_0$:

$D0 = K_1 L/K_2 + C$ implies that $C = D_0 - (K_1 L/K_2)$

6. Final expression:

$D(t) = (K_1 L/K_2)(1 - e^{-K_2 t}) + D_0\, e^{-K_2 t}$

This describes the rise and fall of oxygen deficit as organic matter decays and atmospheric oxygen re-enters the system.

9.2 Derivation of the Modified DO Model

To account for additional oxygen sources and sinks such as NBOD, sediment oxygen demand, photosynthesis, and respiration, the modified model is expressed as:

$dD/dt = K_1 L + K_3 L_n + R + S_b - P_a - K_2 D$

Where:

- K_3: NBOD decay rate (day^{-1})
- L_n: Nitrogenous BOD concentration (mg/L)
- R: Ecosystem respiration (mg/L/day)
- S_b: Sediment oxygen demand (mg/L/day)
- P_a: Photosynthetic oxygen production (mg/L/day)

Let the total net oxygen demand be:

$\theta = K_1L + K_3L_n + R + S_b - P_a$

Then:

$dD/dt = \theta - K_2D$

This is again a first order linear ODE. Solution using integrating factor:

1. Multiply both sides by $\mu(t) = e^{K_2t}$:

$e^{K_2t} dD/dt + K2e^{K_2t}D = \theta e^{K_2t}$

Recognize the left side as a derivative:

$d/dt\ (e^{K_2t} D) = \theta\ e^{K_2t}$

2. Integrate:

$e^{K_2t} D = (\theta/K_2)\ e^{K_2t} + C$ implies $D(t) = \theta/K_2 + C\ e^{-K_2t}$

3. Apply initial condition $D(t) = D_0$ at $t = 0$, and Solve for $D(t)$:

   $D_0 = \theta/K_2 + C$ implying that $C = D_0 - \theta/K_2$

Therefore: $D(t) = (\theta/K_2)(1 - e^{-K_2t}) + D_0\ e^{-K_2t}$

Or

$$D(t) = \left(\frac{K_1L + K_3L_N + R + S_b - P_a}{K_2}\right)(1 - e^{-K_2t}) + D_0e^{-K_2t}$$

This is the complete solution for the modified DO model.

9.3 Units of Key Parameters

Parameter	Symbol	Units	Description
DO Deficit	D	mg/L	Saturation DO minus actual DO
CBOD Decay Rate	K_1	day^{-1}	Rate of organic matter oxidation
NBOD Decay Rate	K_3	day^{-1}	Rate of nitrification
Reaeration Rate	K_2	day^{-1}	Rate of atmospheric oxygen absorption
CBOD Concentration	L	mg/L	Organic pollutant concentration
NBOD Concentration	L_n	mg/L	Ammonia or nitrogen-based load
Sediment Oxygen Demand	S_b	mg/L/day	Benthic oxygen consumption
Photosynthetic Production	P_a	mg/L/day	Oxygen from algal activity
Ecosystem Respiration	R	mg/L/day	Total oxygen consumption by biota

9.4 Model Assumptions

1. One-dimensional, steady-state river flow.
2. Plug flow (no longitudinal dispersion).
3. Constant temperature across reach.
4. Single point source pollutant input.
5. Uniform cross-sectional area.
6. Reaeration rate is first-order.
7. Respiration and photosynthesis rates are averaged over diel cycle.
8. Sediment oxygen demand is constant across the reach.
9. Groundwater interactions are negligible.