Supplementary information: Competing effect of activity and non-inert crowding on the dynamics of self-propelled tracer particles

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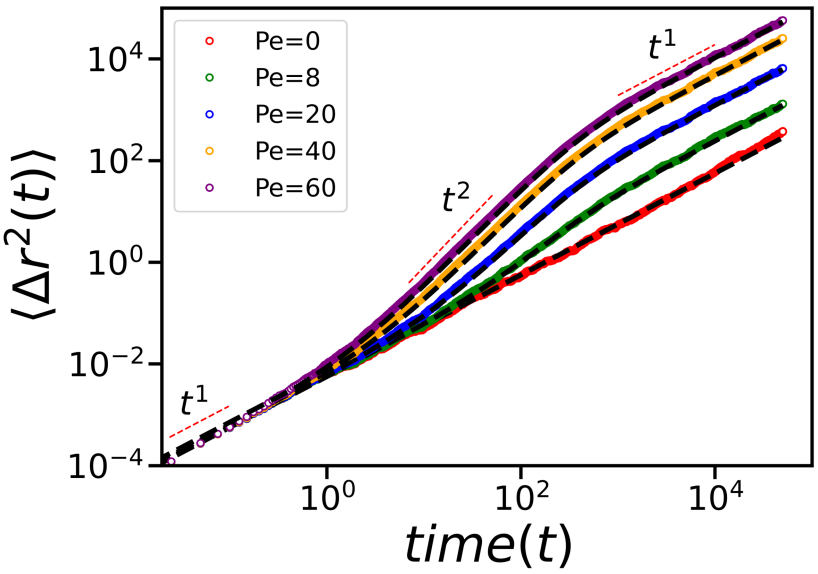

Figure S1: Ornstein-Uhlenbeck solution for self-prpelled particles (dotted lines) shows an excellent agreement with the MSD obtained from our simulation (solid lines) for self-propelled particle in free space across various Peclet numer (Pe) values.

For validation of our model of a self-propelled particle (SPP), we first simulate an SPP in free space. Then, we calculate the mean squared displacement (MSD) of the particle for various propulsion strengths (Pe). In free space, the analytical expression for the active Ornstein-Uhlenbeck (OU) process is

$$\langle \Delta r^2(t) \rangle = 6D_t t + \frac{2F_a^2 t_R}{\gamma^2}[t + t_R(e^{-\frac{t}{t_R}} - 1] \tag{1}$$

For a passive particle in free space (i.e., with Pe=0), the MSD is always diffusive $\Delta r^2(t) = t^1$. For the self-propelled tracer particle, the MSD shows a three-step variation. At the shorter time scale, the MSD shows diffusive behaviour (t^1); at the intermediate time scale, it is superdiffisive (t^2) and finally, at the longer time scale, it becomes diffisive again (t^1).

To verify our simulation, in Fig. S1, we reproduced the solution of the Ornstein-Uhlenbeck (OU) process. Our simulation data perfectly match the analytical expression of OU solution for self-propelled particles for different Pe[Fig. S1].

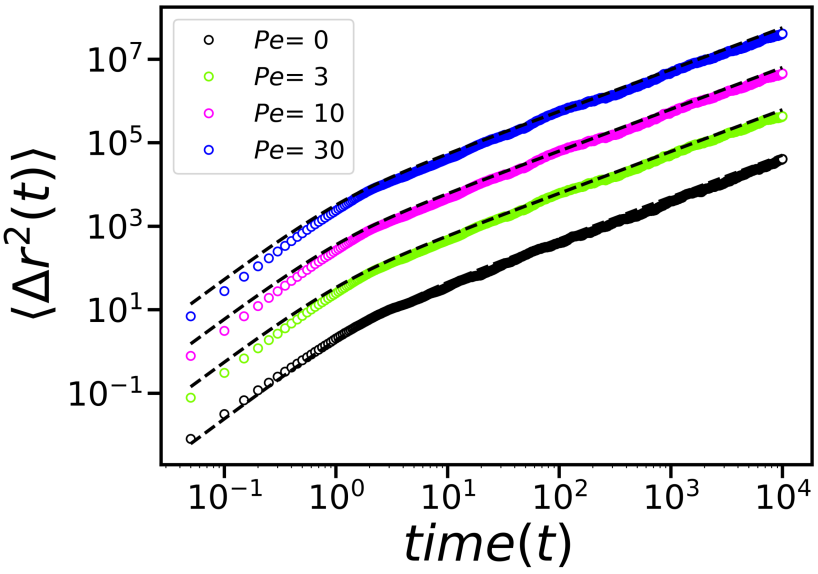

Figure S2: Ornstein-Uhlenbeck solution for self-prpelled particles (dotted lines) shows an excellent agreement with the MSD obtained from our simulation (solid lines) for self-propelled particle in free space across various Peclet numer (Pe) values in the Under damp limit.

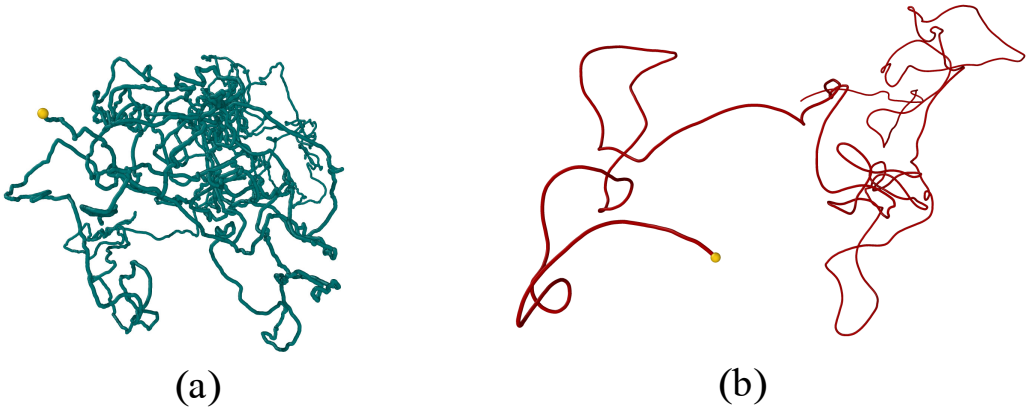

Figure S3: Trajectories of (a) Passive brownian particle and (b) Self-propelled particle in a free space.

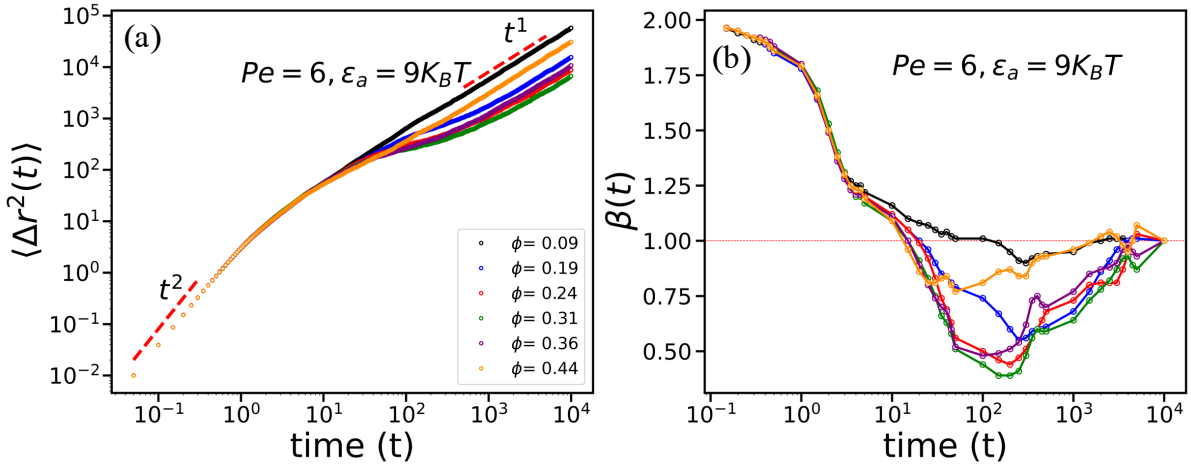

Figure S4: Plot shows the Mean Squared displacement of Self-propelled tracer ($Pe = 6$) at attraction strength ($\varepsilon_a = 9K_BT$) and the corresponding logerthemic exponent.

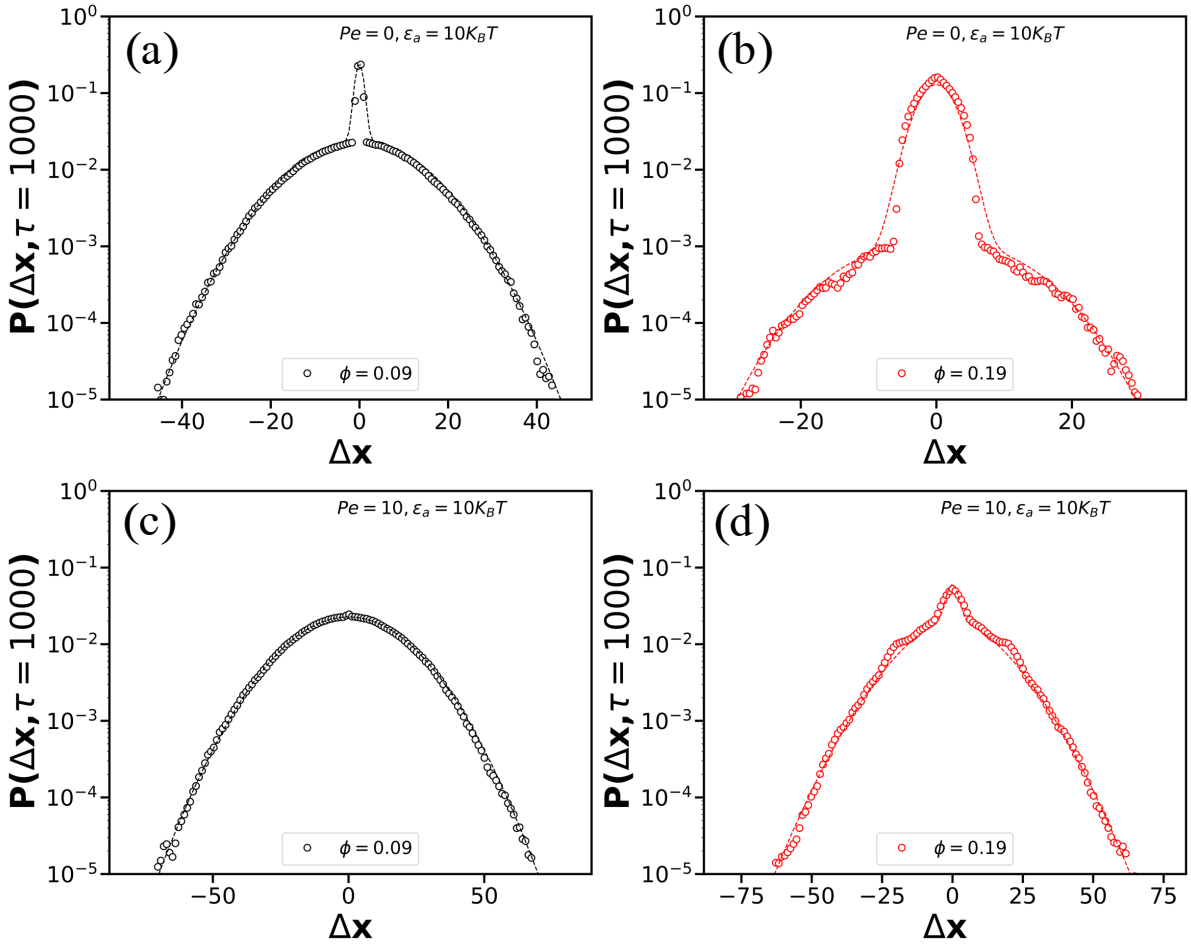

Figure S5: Plot shows the double non-gaussian fitting with the the analytical expression. (a) Shows the fitting of both stiffer and broader gaussian occur at $\phi = 0.09$, $\varepsilon_a = 10K_BT$, (b) Shows fitting of slightly broader inner non-gaussian profile as well as outer non-gaussian profile. (c) For $Pe = 10$ the profile becomes gaussian even there is significant attraction strength and minimal packing fraction. (d) For intermediate value of packing fraction ($\phi = 0.19$), the resultant ripple profile fit with proper analytical expression.

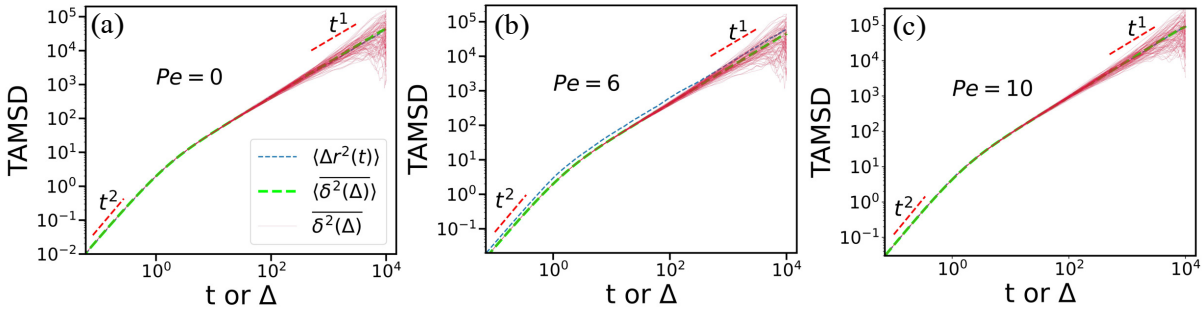

Figure S6: Plot shows the Time averaged MSD of each trajectory, Ensemble average of TAMSD and the Ensemble averaged MSD of (a)Passive (Pe=0) (b) SPP (Pe=6) and (c) SPP (Pe=10) for model parameters $\phi = 0.09$, $\varepsilon_a = 2K_BT$. The three properties plot on same panel to show the comparision.