Supplement 3. The framework of the hierarchical Bayesian conjugate Dirichlet-multinomial model for the estimation of levels of vision impairment

$\vec{Y}$: estimated number of vision impairment of different severity: no, mild moderate, sever and blindness (vector of 5)
N: Population number of specific gender, age group and hospital level, derived from National Health Insurance Research Database (NHIRD)

Three levels of hierarchical structure: gender, age group, hospital level

$$\overrightarrow{Y_{ijk}} \sim Multinomial\left(N_{ijk},\ \overrightarrow{p_{ijk}}\right)$$

i=1,2 (gender: male, female)
j=1,2,3,4 (age group: age 20-39, age 40-54, age 55-69, age ≥70)
k= 1,2,3 (hospital level: medical center, regional hospital, local hospital/clinic)

$$\overrightarrow{\theta_i} \sim Dirichlet\ (\overrightarrow{\alpha_{all}})$$
$$\overrightarrow{\alpha_{all}} = \overrightarrow{\theta_{all}} * \omega_s$$

$\overrightarrow{\theta_{all}}$: overall proportion (overall expectation value)
ω_s: strength between sex level

$$\overrightarrow{\theta_{ij}} \sim Dirichlet\ (\overrightarrow{\alpha_i})$$
$$\overrightarrow{\alpha_i} = \overrightarrow{\theta_i} * \omega_A$$

$\overrightarrow{\theta_i}$: proportion in each sex level
ω_A: strength among age_group level

$$\overrightarrow{p_{ijk}} \sim Dirichlet\ (\overrightarrow{\alpha_{ij}})$$
$$\overrightarrow{\alpha_{ij}} = \overrightarrow{\theta_{ij}} * \omega_L$$

$\overrightarrow{\theta_{ij}}$: proportion in each age_group level
ω_L: strength among hospital_level level

$$\omega_s \sim gamma(0.01,0.01)$$
$$\omega_A \sim gamma(0.01,0.01)$$
$$\omega_L \sim gamma(0.01,0.01)$$