

Recurrence dynamics and nonlinear system analysis of choral singing

Viktor Müller¹, Julia A. M. Delius¹, and Ulman Lindenberger^{1 2 3}

¹Center for Lifespan Psychology, Max Planck Institute for Human Development, Lentzeallee 94,
14195 Berlin, Germany

²Max Planck UCL Centre for Computational Psychiatry and Ageing Research, Berlin, Germany

³Max Planck UCL Centre for Computational Psychiatry and Ageing Research, London, England

NONLINEAR DYNAMICS

Supplementary Material

Supplementary Figures and Tables

Table 1. Description of the recurrence quantification analysis (RQA) measures.

Measure	Name	Definition
Measures based on the recurrence density		
<i>RR</i>	<i>Recurrence Rate</i>	The density of recurrence points in the recurrence plot (RP). It reflects the probability that a state will recur to its ε -neighborhood in phase space.
Measures based on diagonal lines		
<i>DET</i>	<i>Determinism</i>	The ratio of recurrence points forming diagonal structures (of at least length l_{min}) to all recurrence points. It indicates the predictability of the system.
<i>L_{avg}</i>	<i>Average diagonal line length</i>	The average time two trajectory segments remain close in phase space; interpreted as mean prediction time.
<i>ENTR_L</i>	<i>Entropy (diagonal lines)</i>	Shannon entropy of the distribution of diagonal line lengths. It reflects system complexity with respect to diagonal structures.
<i>RATIO</i>	<i>Ratio</i>	The ratio between <i>DET</i> and <i>RR</i> , indicating the presence of transitions in system dynamics.
Measures based on vertical lines		
<i>LAM</i>	<i>Laminarity</i>	The ratio of recurrence points forming the vertical structures (of at least length l_{min}) to the total recurrence points. It represents the occurrence of laminar (stationary) states.
<i>TT</i>	<i>Trapping Time</i>	The average length of vertical lines, indicating how long the system remains in a specific state or how long the state is ‘trapped’.
<i>ENTR_V</i>	<i>Entropy (vertical lines)</i>	Shannon entropy of the distribution of vertical line lengths, reflecting system complexity in terms of vertical structures.
Measures based on topological features of recurrence networks		
<i>DC</i>	<i>Degree Centrality</i>	The number of connections (edges) per node.
<i>CC</i>	<i>Clustering Coefficient</i>	The likelihood that two neighbors (recurrences) of a given state are also neighbors themselves.
<i>CPL</i>	<i>Characteristic Path Length</i>	The average shortest path length between all pairs of nodes. Indicates the integrative capacity of the RP/network.
<i>E_{local}</i>	<i>Local Efficiency</i>	A measure of the segregation of a network, indicating information transfer efficiency in the local neighborhood of each node
<i>E_{global}</i>	<i>Global Efficiency</i>	A measure of the integration, indicating efficiency of information exchange across the entire network or phase space.
<i>CoC</i>	<i>Coreness Centrality</i>	A measure based on k -core decomposition, identifying the most influential nodes in the network core.
<i>BC</i>	<i>Betweenness Centrality</i>	The number of shortest paths passing through a node. It indicates fragmentation within the attractor.
<i>$\mathcal{A}$</i>	<i>Assortativity</i>	A preference for network nodes to connect to other nodes of similar degree, indicating continuity in state density.
<i>$\mathcal{T}$</i>	<i>Transitivity</i>	The overall probability that adjacent nodes in the network are interconnected, indicating the presence of tightly connected communities.

Supplementary Table 2. LMM results of the JRP-based RQA measures comparing singing condition vs. resting state.

Ms/Fs	System			Condition			System x Condition		
	<i>F</i> -value	<i>p</i> -value	<i>df</i>	<i>F</i> -value	<i>p</i> -value	<i>df</i>	<i>F</i> -value	<i>p</i> -value	<i>df</i>
Measures based on diagonal lines									
<i>RR</i>	152.15	<0.001	2,546	452.79	<0.001	1,525.6	235.87	<0.001	2,525.6
<i>DET</i>	38.57	<0.001	2,534.5	40.23	<0.001	1,523.9	52.02	<0.001	2,523.9
<i>L_{avg}</i>	68.09	<0.001	2,532.2	4.92	0.027	1,524.2	40.63	<0.001	2,524.2
<i>ENTR_L</i>	3.92	0.020	2,533.8	92.90	<0.001	1,524.3	48.96	<0.001	2,524.3
<i>RATIO</i>	49.82	<0.001	2,525.6	494.16	<0.001	1,524.0	171.78	<0.001	2,524.0
Measures based on vertical lines									
<i>LAM</i>	197.26	<0.001	2,531.4	161.89	<0.001	1,524.4	94.36	<0.001	2,524.4
<i>TT</i>	4.44	0.012	2,533.7	54.44	<0.001	1,524.1	44.69	<0.001	2,524.1
<i>ENTR_V</i>	n.s.			120.45	<0.001	1,524.2	53.92	<0.001	2,524.2
Measures based on topological features of recurrence networks									
<i>CC</i>	240.65	<0.001	2,532.8	716.14	<0.001	1,523.4	332.21	<0.001	2,523.4
<i>CPL</i>	33.83	<0.001	2,533.2	435.00	<0.001	1,524.2	90.30	<0.001	2,524.2
<i>E_{local}</i>	274.76	<0.001	2,532.6	805.24	<0.001	1,523.5	356.24	<0.001	2,523.5
<i>E_{global}</i>	47.59	<0.001	2,532.9	748.13	<0.001	1,523.8	171.72	<0.001	2,523.8
<i>CoC</i>	145.41	<0.001	2,529.3	429.80	<0.001	1,524.2	240.05	<0.001	2,524.2
<i>BC</i>	27.17	<0.001	2,534.5	170.76	<0.001	1,523.9	8.79	<0.001	2,523.9
<i>A</i>	25.99	<0.001	2,526.1	n.s.			3.03	0.049	2,525.0
<i>T</i>	3.45	0.033	2,533.7	50.02	<0.001	1,524.5	40.10	<0.001	2,524.5

JRP, joint recurrence plot; Ms, measures; Fs, factors; *RR*, recurrence rate; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *A*, assortativity; *T*, transitivity.

Supplementary Table 3. Results of the Sidak-adjusted post-hoc tests for within-system JRP-based RQA measures, comparing differences across the levels of the *System* factor.

	M_{Diff}	P -value	df	M_{Diff}	P -value	df	M_{Diff}	P -value	df
		R vs. H			R vs. R-H			H vs. R-H	
<i>RR</i>	0.006	<0.001	525.6	0.006	<0.001	546	n.s.		
<i>DET</i>	-0.003	<0.001	523.9	n.s.			0.002	<0.001	545.0
L_{avg}	-1.056	<0.001	524.2	n.s.			0.896	<0.001	545.8
$ENTR_L$	n.s.			0.032	0.020	545.4	n.s.		
<i>RATIO</i>	-7.859	<0.001	524.0	-9.100	<0.001	541.9	n.s.		
<i>LAM</i>	-0.051	<0.001	524.4	-0.018	<0.001	539.2	0.033	<0.001	539.2
<i>TT</i>	n.s.			0.168	0.012	540.6	n.s.		
$ENTR_V$	n.s.			n.s.			n.s.		
<i>CC</i>	-0.072	<0.001	523.4	-0.017	<0.001	565.6	0.055	<0.001	541.5
<i>CPL</i>	2.097	<0.001	524.2	n.s.			-1.742	<0.001	540.1
E_{local}	-0.093	<0.001	523.5	-0.022	<0.001	540.0	0.070	<0.001	540.0
E_{global}	0.009	<0.001	523.8	0.015	<0.001	538.5	0.006	<0.001	538.5
<i>CoC</i>	3.240	<0.001	524.2	3.383	<0.001	544.6	n.s.		
<i>BC</i>	-905.2	<0.001	523.9	-874.3	<0.001	543.0	n.s.		
$\mathcal{A}$	-0.039	<0.001	525.0	-0.034	<0.001	537.5	n.s.		
$\mathcal{T}$	n.s.			n.s.			n.s.		

Supplementary Table 4. Results of the Sidak-adjusted post-hoc tests for JRP-based measures, comparing rest versus singing conditions across different subsystems.

	M_{Diff}	P -value	df	M_{Diff}	P -value	df	M_{Diff}	P -value	df
		R			H			R-H	
<i>RR</i>	-0.015	<0.001	525.6	n.s.			-0.003	<0.001	525.6
<i>DET</i>	-0.006	<0.001	523.9	0.002	0.002	523.9	-0.001	0.005	523.9
L_{avg}	-1.092	<0.001	524.2	0.694	<0.001	524.2	n.s.		
$ENTR_L$	-0.230	<0.001	524.3	n.s.			-0.062	<0.001	524.3
<i>RATIO</i>	38.06	<0.001	524.0	n.s.			12.33	<0.001	524.0
<i>LAM</i>	-0.063	<0.001	524.4	n.s.			-0.016	<0.001	524.4
<i>TT</i>	-1.030	<0.001	524.1	n.s.			-0.205	<0.001	524.1
$ENTR_V$	-0.448	<0.001	524.2	n.s.			-0.129	<0.001	524.2
<i>CC</i>	-0.163	<0.001	523.4	0.014	0.004	523.4	-0.057	<0.001	523.4
<i>CPL</i>	8.115	<0.001	524.2	n.s.			4.001	<0.001	524.2
E_{local}	-0.205	<0.001	523.5	0.016	0.006	523.5	-0.074	<0.001	523.5
E_{global}	-0.066	<0.001	523.8	-0.008	<0.001	523.8	-0.025	<0.001	523.8
<i>CoC</i>	-9.207	<0.001	524.2	n.s.			-1.544	<0.001	524.2
<i>BC</i>	2027.8	<0.001	523.9	851.5	<0.001	523.9	1271.6	<0.001	523.9
$\mathcal{A}$	n.s.			n.s.			-0.013	0.019	525.0
$\mathcal{T}$	-0.072	<0.001	524.5	n.s.			-0.016	<0.001	524.5

RR, recurrence rate; *DET*, determinism; L_{avg} , average diagonal line length; $ENTR_L$, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; $ENTR_V$, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; E_{local} , local efficiency; E_{global} , global efficiency; *CoC*, coreness centrality; $\mathcal{A}$, assortativity; $\mathcal{T}$, transitivity.

Supplementary Table 5. LMM results of the RP-based RQA measures comparing the singing conditions among each other.

Ms/Fs	System			Condition			System x Condition		
	F-value	p-value	df	F-value	p-value	df	F-value	p-value	df
Measures based on diagonal lines									
<i>DET</i>	289.03	<0.001	2,46.47	n.s.			3.71	0.011	4,46.47
<i>L_{avg}</i>	97.68	<0.001	2,52.83	n.s.			n.s.		
<i>ENTR_L</i>	103.43	<0.001	2,44.06	n.s.			n.s.		
Measures based on vertical lines									
<i>LAM</i>	729.40	<0.001	2,60.05	5.24	0.008	2,64.72	2.92	0.028	4,60.05
<i>TT</i>	128.56	<0.001	2,61.02	n.s.			n.s.		
<i>ENTR_V</i>	141.56	<0.001	2,59.96	3.26	0.043	2,85.75	2.96	0.027	4,59.96
Measures based on topological features of recurrence networks									
<i>CC</i>	164.66	<0.001	2,65.52	n.s.			n.s.		
<i>CPL</i>	57.94	<0.001	2,41.20	n.s.			n.s.		
<i>E_{local}</i>	74.67	<0.001	2,51.32	n.s.			n.s.		
<i>E_{global}</i>	57.12	<0.001	2,56.25	n.s.			n.s.		
<i>CoC</i>	17.98	<0.001	2,68.76	n.s.			n.s.		
<i>BC</i>	39.65	<0.001	2,51.33	n.s.			n.s.		
<i>A</i>	46.70	<0.001	2,63.06	n.s.			n.s.		
<i>T</i>	33.14	<0.001	2,54.16	n.s.			n.s.		

RP, recurrence plot; Ms, measures; Fs, factors; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *A*, assortativity; *T*, transitivity.

Supplementary Table 6. Results of the Sidak-adjusted post-hoc tests for RP-based measures, comparing differences between the levels of the *System* factor.

	<i>M_{Diff}</i>	<i>P</i> -value	<i>df</i>	<i>M_{Diff}</i>	<i>P</i> -value	<i>df</i>	<i>M_{Diff}</i>	<i>P</i> -value	<i>df</i>
		R vs. H			R vs. V			H vs. V	
<i>DET</i>	-0.002	<0.001	40.0	0.004	<0.001	52.93	0.006	<0.001	39.37
<i>L_{avg}</i>	-0.929	<0.001	40.57	1.603	<0.001	54.78	2.532	<0.001	54.15
<i>ENTR_L</i>	-0.090	<0.001	33.85	0.184	<0.001	61.11	0.274	<0.001	60.12
<i>LAM</i>	-0.008	<0.001	33.87	0.013	<0.001	61.34	0.021	<0.001	34.69
<i>TT</i>	-1.084	<0.001	50.58	1.624	<0.001	60.54	2.708	<0.001	45.13
<i>ENTR_V</i>	-0.182	<0.001	52.17	0.379	<0.001	64.17	0.561	<0.001	49.34
<i>CC</i>	0.014	0.027	41.60	0.071	<0.001	45.49	0.057	<0.001	55.27
<i>CPL</i>	-0.255	0.026	51.63	0.534	<0.001	61.66	0.788	<0.001	57.96
<i>E_{local}</i>	0.009	0.012	50.16	0.039	<0.001	49.90	0.031	<0.001	50.09
<i>E_{global}</i>	n.s.			-0.046	<0.001	53.90	-0.049	<0.001	39.23
<i>CoC</i>	3.305	<0.001	37.70	3.130	<0.001	58.54	n.s.		
<i>BC</i>	-218.6	<0.001	44.0	534.6	<0.001	63.19	753.2	<0.001	56.23
<i>A</i>	n.s.			0.117	<0.001	59.29	0.143	<0.001	53.13
<i>T</i>	0.036	0.002	37.02	0.103	<0.001	58.74	0.067	<0.001	37.33

Supplementary Table 7. LMM results of the within-system JRP-based RQA measures, comparing the singing conditions among each other.

Ms/Fs	System			Condition			System x Condition		
	F-value	p-value	df	F-value	p-value	df	F-value	p-value	df
Measures based on diagonal lines									
<i>RR</i>	419.41	<0.001	2,563.3	58.81	<0.001	2,563.3	16.39	<0.001	4,563.3
<i>DET</i>	612.11	<0.001	2,566.3	14.21	<0.001	2,566.3	22.58	<0.001	4,566.3
<i>L_{avg}</i>	353.61	<0.001	2,565.3	109.41	<0.001	2,565.3	40.01	<0.001	4,565.3
<i>ENTR_L</i>	279.87	<0.001	2,566.4	24.48	<0.001	2,566.4	25.13	<0.001	4,566.4
<i>RATIO</i>	959.46	<0.001	2,564.1	46.19	<0.001	2,564.1	12.79	<0.001	4,564.1
Measures based on vertical lines									
<i>LAM</i>	942.84	<0.001	2,563.8	96.64	<0.001	2,563.8	56.98	<0.001	4,563.8
<i>TT</i>	362.63	<0.001	2,565.8	20.44	<0.001	2,565.8	30.13	<0.001	4,565.8
<i>ENTR_V</i>	442.68	<0.001	2,565.1	35.95	<0.001	2,565.1	47.82	<0.001	4,565.1
Measures based on topological features of recurrence networks									
<i>CC</i>	16.02	<0.001	2,565.6	51.23	<0.001	2,565.6	35.11	<0.001	4,565.6
<i>CPL</i>	404.66	<0.001	2,563.6	3.562	0.029	2,560.4	17.50	<0.001	4,563.6
<i>E_{local}</i>	18.50	<0.001	2,565.5	38.78	<0.001	2,565.5	38.67	<0.001	4,565.5
<i>E_{global}</i>	1128.4	<0.001	2,562.4	43.25	<0.001	2,562.4	10.69	<0.001	4,562.4
<i>CoC</i>	376.68	<0.001	2,563.3	52.79	<0.001	2,563.3	16.06	<0.001	4,563.3
<i>BC</i>	1077.1	<0.001	2,562.8	40.91	<0.001	2,562.8	6.51	<0.001	4,562.8
<i>A</i>	360.65	<0.001	2,562.3	25.06	<0.001	2,562.3	9.36	<0.001	4,562.3
<i>T</i>	537.96	<0.001	2,565.7	n.s.			30.58	<0.001	4,565.7

JRP, joint recurrence plot; Ms, measures; Fs, factors; *RR*, recurrence rate; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *A*, assortativity; *T*, transitivity.

Supplementary Table 8. Results of the Sidak-adjusted post-hoc tests for within-system JRP-based RQA measures, comparing differences across the levels of the *System* factor.

	M_{Diff}	P -value	df	M_{Diff}	P -value	df	M_{Diff}	P -value	df
		R vs. H			R vs. V			H vs. V	
<i>RR</i>	0.013	<0.001	563.3	-0.003	<0.001	563.3	-0.016	<0.001	563.3
<i>DET</i>	n.s.			0.009	<0.001	566.3	0.009	<0.001	566.3
<i>L_{avg}</i>	n.s.			1.621	<0.001	565.3	1.784	<0.001	565.3
<i>ENTR_L</i>	0.150	<0.001	566.4	0.233	<0.001	566.4	0.083	<0.001	566.4
<i>RATIO</i>	-26.44	<0.001	564.1	4.709	<0.001	564.1	31.15	<0.001	564.1
<i>LAM</i>	-0.016	<0.001	563.8	0.038	<0.001	563.8	0.054	<0.001	563.8
<i>TT</i>	0.627	<0.001	565.8	1.216	<0.001	565.8	0.589	<0.001	565.8
<i>ENTR_V</i>	0.187	<0.001	565.1	0.455	<0.001	565.1	0.267	<0.001	565.1
<i>CC</i>	0.016	<0.001	565.6	0.048	<0.001	565.6	0.031	<0.001	565.6
<i>CPL</i>	-1.614	<0.001	560.3	1.526	<0.001	560.4	3.140	<0.001	560.4
<i>E_{local}</i>	0.018	<0.001	565.5	0.015	<0.001	565.5	n.s.		
<i>E_{global}</i>	0.038	<0.001	562.4	-0.065	<0.001	562.4	-0.103	<0.001	562.4
<i>CoC</i>	7.855	<0.001	563.3	n.s.			-8.617	<0.001	563.3
<i>BC</i>	-1493.3	<0.001	562.8	1923.3	<0.001	562.8	3416.7	<0.001	562.8
<i>A</i>	-0.028	<0.001	562.3	0.113	<0.001	562.3	0.141	<0.001	562.3
<i>T</i>	0.054	<0.001	565.7	0.128	<0.001	565.7	0.074	<0.001	565.7

Supplementary Table 9. Results of the Sidak-adjusted post-hoc tests for within-system JRP-based RQA measures, comparing differences across the levels of the *Condition* factor.

	M_{Diff}	P -value	df	M_{Diff}	P -value	df	M_{Diff}	P -value	df
		C_{un} vs. C_{eo}			C_{un} vs. C_{ec}			C_{eo} vs. C_{ec}	
<i>RR</i>	0.006	<0.001	563.3	0.004	<0.001	563.3	-0.003	<0.001	563.3
<i>DET</i>	0.001	0.011	566.3	-0.001	0.050	566.3	-0.002	<0.001	566.3
<i>L_{avg}</i>	-0.220	0.009	565.3	-0.808	<0.001	565.3	-0.588	<0.001	565.3
<i>ENTR_L</i>	n.s.			-0.062	<0.001	566.4	-0.059	<0.001	566.4
<i>RATIO</i>	-7.369	<0.001	564.1	-3.641	<0.001	564.1	3.728	<0.001	564.1
<i>LAM</i>	0.016	<0.001	563.8	0.015	<0.001	563.8	n.s.		
<i>TT</i>	0.185	<0.001	565.8	0.284	<0.001	565.8	n.s.		
<i>ENTR_V</i>	0.073	<0.001	565.1	0.130	<0.001	565.1	0.057	<0.001	565.1
<i>CC</i>	n.s.			-0.024	<0.001	565.6	-0.024	<0.001	565.6
<i>CPL</i>	n.s.			-0.267	<0.001	560.4	n.s.		
<i>E_{local}</i>	n.s.			-0.023	<0.001	562.4	-0.025	<0.001	562.4
<i>E_{global}</i>	0.019	<0.001	562.4	0.015	<0.001	562.4	n.s.		
<i>CoC</i>	3.571	<0.001	563.3	1.849	<0.001	563.3	-1.722	<0.001	563.3
<i>BC</i>	-517.5	<0.001	562.8	-624.0	<0.001	562.8	-106.5	<0.001	562.8
<i>A</i>	-0.029	<0.001	562.3	-0.037	<0.001	562.3	n.s.		
<i>T</i>	n.s.			n.s.			n.s.		

JRP, joint recurrence plot; Ms, measures; Fs, factors; *RR*, recurrence rate; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *A*, assortativity; *T*, transitivity.

Supplementary Table 10. Results of the Sidak post-hoc test or within-system JRP-based RQA measures, comparing different singing conditions across various systems.

Measures	Levels	Respiration (R)		HRV (H)		Voice (V)	
		M_{Diff}	p -value	M_{Diff}	p -value	M_{Diff}	p -value
<i>RR</i> df=563.3	C_{un} vs. C_{eo}	0.007	<0.001	n.s.		0.012	<0.001
	C_{un} vs. C_{ec}	0.004	0.002	n.s.		0.007	<0.001
	C_{eo} vs. C_{ec}	-0.003	0.004	n.s.		-0.005	<0.001
<i>DET</i> df=566.3	C_{un} vs. C_{eo}	n.s.		n.s.		0.004	<0.001
	C_{un} vs. C_{ec}	-0.004	<0.001	n.s.		0.001	0.033
	C_{eo} vs. C_{ec}	-0.003	<0.001	n.s.		-0.002	<0.001
<i>L_{avg}</i> df=565.3	C_{un} vs. C_{eo}	-0.486	<0.001	n.s.		n.s.	
	C_{un} vs. C_{ec}	-1.686	<0.001	n.s.		-0.981	<0.001
	C_{eo} vs. C_{ec}	-1.20	<0.001	0.528	<0.001	-1.091	<0.001
<i>ENTR_L</i> df=566.4	C_{un} vs. C_{eo}	n.s.		n.s.		0.064	<0.001
	C_{un} vs. C_{ec}	-0.172	<0.001	n.s.		n.s.	
	C_{eo} vs. C_{ec}	-0.133	<0.001	0.052	0.008	-0.095	<0.001
<i>RATIO</i> df=564.1	C_{un} vs. C_{eo}	-7.56	<0.001	n.s.		-13.825	<0.001
	C_{un} vs. C_{ec}	n.s.		n.s.		-6.315	<0.001
	C_{eo} vs. C_{ec}	4.722	<0.001	n.s.		7.51	<0.001
<i>LAM</i> df=563.8	C_{un} vs. C_{eo}	0.008	<0.001	n.s.		0.039	<0.001
	C_{un} vs. C_{ec}	n.s.		0.006	0.030	0.037	<0.001
	C_{eo} vs. C_{ec}	n.s.		0.006	0.017	n.s.	
<i>TT</i> df=565.8	C_{un} vs. C_{eo}	n.s.		n.s.		0.655	<0.001
	C_{un} vs. C_{ec}	n.s.		n.s.		0.942	<0.001
	C_{eo} vs. C_{ec}	n.s.		n.s.		0.287	<0.001
<i>ENTR_V</i> df=565.1	C_{un} vs. C_{eo}	n.s.		n.s.		0.267	<0.001
	C_{un} vs. C_{ec}	n.s.		n.s.		0.419	<0.001
	C_{eo} vs. C_{ec}	n.s.		n.s.		0.152	<0.001
<i>CC</i> df=565.6	C_{un} vs. C_{eo}	n.s.		n.s.		n.s.	
	C_{un} vs. C_{ec}	-0.031	<0.001	0.012	0.026	-0.052	<0.001
	C_{eo} vs. C_{ec}	-0.030	<0.001	0.014	0.011	-0.055	<0.001
<i>CPL</i> df=560.3	C_{un} vs. C_{eo}	-0.673	<0.001	n.s.		0.710	<0.001
	C_{un} vs. C_{ec}	-1.179	<0.001	n.s.		0.446	0.034
	C_{eo} vs. C_{ec}	-0.506	0.011	n.s.		n.s.	
<i>E_{local}</i> df=565.5	C_{un} vs. C_{eo}	n.s.		n.s.		n.s.	
	C_{un} vs. C_{ec}	-0.018	0.003	0.018	0.003	-0.068	<0.001
	C_{eo} vs. C_{ec}	-0.027	<0.001	0.015	0.022	-0.063	<0.001
<i>E_{global}</i> df=562.4	C_{un} vs. C_{eo}	0.026	<0.001	n.s.		0.028	<0.001
	C_{un} vs. C_{ec}	0.031	<0.001	n.s.		0.012	0.005
	C_{eo} vs. C_{ec}	n.s.		n.s.		-0.016	<0.001
<i>CoC</i> df=563.3	C_{un} vs. C_{eo}	3.762	<0.001	n.s.		6.857	<0.001
	C_{un} vs. C_{ec}	1.530	0.034	n.s.		3.686	<0.001
	C_{eo} vs. C_{ec}	-2.232	0.001	n.s.		-3.171	<0.001
<i>BC</i> df=562.8	C_{un} vs. C_{eo}	-629.654	<0.001	-559.46	<0.001	-363.475	0.014
	C_{un} vs. C_{ec}	-1110.72	<0.001	-508.911	<0.001	n.s.	
	C_{eo} vs. C_{ec}	-481.062	<0.001	n.s.		n.s.	
<i>Ass</i> df=562.3	C_{un} vs. C_{eo}	-0.049	<0.001	n.s.		-0.034	0.0076
	C_{un} vs. C_{ec}	-0.077	<0.001	n.s.		-0.041	0.0012
	C_{eo} vs. C_{ec}	-0.028	0.010	n.s.		n.s.	
<i>Trans</i> df=565.7	C_{un} vs. C_{eo}	-0.033	<0.001	n.s.		0.038	<0.001
	C_{un} vs. C_{ec}	-0.063	<0.001	n.s.		0.032	<0.001
	C_{eo} vs. C_{ec}	-0.030	<0.001	0.015	0.0060	n.s.	

M_{Diff} , mean difference; *RR*, recurrence rate; *DET*, determinism; L_{avg} , average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; $\mathcal{A}$, assortativity; $\mathcal{T}$, transitivity.

Supplementary Table 11. LMM results of the between-system JRP-based RQA measures comparing the singing conditions among each other.

Ms/Fs	System			Condition			System x Condition		
	F-value	p-value	df	F-value	p-value	df	F-value	p-value	df
Measures based on diagonal lines									
<i>RR</i>	1153.5	<0.001	2,1265	71.88	<0.001	2,1265	31.10	<0.001	4,1265
<i>DET</i>	507.98	<0.001	2,1265	17.33	<0.001	2,1265	16.55	<0.001	4,1265
<i>L_{avg}</i>	413.87	<0.001	2,1265	22.64	<0.001	2,1265	53.00	<0.001	4,1265
<i>ENTR_L</i>	370.26	<0.001	2,1265	24.80	<0.001	2,1265	18.60	<0.001	4,1265
<i>RATIO</i>	1137.9	<0.001	2,1265	62.88	<0.001	2,1265	11.61	<0.001	4,1265
Measures based on vertical lines									
<i>LAM</i>	663.40	<0.001	2,1265	10.58	<0.001	2,1265	17.74	<0.001	4,1265
<i>TT</i>	395.38	<0.001	2,1265	28.57	<0.001	2,1265	12.50	<0.001	4,1265
<i>ENTR_V</i>	437.64	<0.001	2,1265	34.92	<0.001	2,1265	15.44	<0.001	4,1265
Measures based on topological features of recurrence networks									
<i>CC</i>	588.02	<0.001	2,1265	55.50	<0.001	2,1265	48.31	<0.001	4,1265
<i>CPL</i>	224.24	<0.001	2,1265	25.49	<0.001	2,1265	19.91	<0.001	4,1265
<i>E_{local}</i>	480.99	<0.001	2,1265	65.86	<0.001	2,1265	52.43	<0.001	4,1265
<i>E_{global}</i>	845.74	<0.001	2,1265	24.77	<0.001	2,1265	14.94	<0.001	4,1265
<i>CoC</i>	1136.7	<0.001	2,1265	73.21	<0.001	2,1265	32.50	<0.001	4,1265
<i>BC</i>	554.56	<0.001	2,1265	n.s.			9.81	<0.001	4,1265
<i>$\mathcal{A}$</i>	283.38	<0.001	2,1265	n.s.			2.87	0.022	4,1265
<i>$\mathcal{T}$</i>	475.31	<0.001	2,1265	6.27	0.002	2,1265	14.82	<0.001	4,1265

JRP, joint recurrence plot; Ms, measures; Fs, factors; *RR*, recurrence rate; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *$\mathcal{A}$* , assortativity; *$\mathcal{T}$* , transitivity.

Supplementary Table 12. Results of the Sidak-adjusted post-hoc tests for between-system JRP-based RQA measures, comparing differences across the levels of the *System* factor.

	M_{Diff}	P -value	df	M_{Diff}	P -value	df	M_{Diff}	P -value	df
		R-H vs. H-V			R-H vs. R-V			H-V vs. R-V	
<i>RR</i>	0.001	<0.001	1265.0	-0.007	<0.001	1265.0	-0.009	<0.001	1265.0
<i>DET</i>	0.006	<0.001	1265.1	0.004	<0.001	1265.1	-0.002	<0.001	1265.1
<i>L_{avg}</i>	1.082	<0.001	1265.0	0.612	<0.001	1265.0	-0.470	<0.001	1265.0
<i>ENTR_L</i>	0.154	<0.001	1265.0	0.028	<0.001	1265.0	-0.125	<0.001	1265.0
<i>RATIO</i>	-3.730	<0.001	1265.0	17.24	<0.001	1265.0	20.97	<0.001	1265.0
<i>LAM</i>	0.033	<0.001	1265.0	0.023	<0.001	1265.0	-0.010	<0.001	1265.0
<i>TT</i>	0.624	<0.001	1265.0	0.407	<0.001	1265.0	-0.217	<0.001	1265.0
<i>ENTR_V</i>	0.255	<0.001	1265.0	0.168	<0.001	1265.0	-0.088	<0.001	1265.0
<i>CC</i>	0.052	<0.001	1265.0	n.s.			-0.051	<0.001	1265.0
<i>CPL</i>	n.s.			1.450	<0.001	1265.0	1.428	<0.001	1265.0
<i>E_{local}</i>	0.046	<0.001	1265.0	-0.019	<0.001	1265.0	-0.066	<0.001	1265.0
<i>E_{global}</i>	-0.013	<0.001	1265.0	-0.042	<0.001	1265.0	-0.028	<0.001	1265.0
<i>CoC</i>	0.888	<0.001	1265.0	-4.110	<0.001	1265.0	-4.999	<0.001	1265.0
<i>BC</i>	608.0	<0.001	1265.0	1695.2	<0.001	1265.0	1087.2	<0.001	1265.0
<i>A</i>	0.070	<0.001	1265.0	0.078	<0.001	1265.0	n.s.		
<i>T</i>	0.073	<0.001	1265.0	0.036	<0.001	1265.0	-0.037	<0.001	1265.0

Supplementary Table 13. Results of the Sidak-adjusted post-hoc tests for between-system JRP-based RQA measures, comparing differences across the levels of the *Condition* factor.

	M_{Diff}	P -value	df	M_{Diff}	P -value	df	M_{Diff}	P -value	df
		C_{un} vs. C_{eo}			C_{un} vs. C_{ec}			C_{eo} vs. C_{ec}	
<i>RR</i>	-0.001	0.005	1265.0	-0.002	<0.001	1265.0	0.002	<0.001	1265.0
<i>DET</i>	-0.001	0.010	1265.0	-0.001	<0.001	1265.0	-0.001	0.010	1265.0
<i>L_{avg}</i>	n.s.			-0.235	<0.001	1265.0	-0.201	<0.001	1265.0
<i>ENTR_L</i>	-0.016	0.024	1265.0	-0.042	<0.001	1265.0	-0.026	<0.001	1265.0
<i>RATIO</i>	1.687	<0.001	1265.0	5.158	<0.001	1265.0	3.471	<0.001	1265.0
<i>LAM</i>	n.s.			0.003	<0.001	1265.0	0.004	<0.001	1265.0
<i>TT</i>	-0.109	<0.001	1265.0	0.059	<0.001	1265.0	0.168	<0.001	1265.0
<i>ENTR_V</i>	-0.040	<0.001	1265.0	0.033	<0.001	1265.0	0.073	<0.001	1265.0
<i>CC</i>	n.s.			-0.018	<0.001	1265.0	-0.014	<0.001	1265.0
<i>CPL</i>	0.481	<0.001	1265.0	0.489	<0.001	1265.0	n.s.		1265.0
<i>E_{local}</i>	-0.006	0.034	1265.0	-0.024	<0.001	1265.0	-0.018	<0.001	1265.0
<i>E_{global}</i>	-0.004	<0.001	1265.0	-0.007	<0.001	1265.0	-0.003	0.002	1265.0
<i>CoC</i>	-0.338	0.008	1265.0	-1.304	<0.001	1265.0	-0.966	<0.001	1265.0
<i>BC</i>	n.s.			n.s.			n.s.		
<i>A</i>	n.s.			n.s.			n.s.		
<i>T</i>	-0.008	0.002	1265.0	n.s.			0.006	0.025	1265.0

JRP, joint recurrence plot; Ms, measures; Fs, factors; *RR*, recurrence rate; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *A*, assortativity; *T*, transitivity.

Supplementary Table 14. Results of the Sidak post-hoc test for between-system JRP-based RQA measures, comparing different singing conditions across various subsystem interactions.

Measures	Conditions	Respiration-HRV (R-H)		Respiration-Voice (R-V)		HRV-Voice (H-V)	
		M _{Diff}	p-value	M _{Diff}	p-value	M _{Diff}	p-value
<i>RR</i> df=1265	C _{un} vs. C _{eo}	n.s.		-0.001	0.017	n.s.	
	C _{un} vs. C _{ec}	n.s.		-0.005	<0.001	-0.001	<0.001
	C _{eo} vs. C _{ec}	n.s.		-0.004	<0.001	n.s.	
<i>DET</i> df=1265	C _{un} vs. C _{eo}	-0.001	0.019	n.s.		n.s.	
	C _{un} vs. C _{ec}	-0.002	<0.001	-0.003	<0.001	n.s.	
	C _{eo} vs. C _{ec}	n.s.		-0.002	<0.001	0.001	0.003
<i>L_{avg}</i> df=1265	C _{un} vs. C _{eo}	n.s.		n.s.		n.s.	
	C _{un} vs. C _{ec}	n.s.		-0.872	<0.001	0.187	0.0013
	C _{eo} vs. C _{ec}	n.s.		-0.89	<0.001	0.189	0.0012
<i>ENTR_L</i> df=1265	C _{un} vs. C _{eo}	-0.028	0.024	n.s.		n.s.	
	C _{un} vs. C _{ec}	-0.030	0.011	-0.096	<0.001	n.s.	
	C _{eo} vs. C _{ec}	n.s.		-0.093	<0.001	n.s.	
<i>RATIO</i> df=1265	C _{un} vs. C _{eo}	n.s.		n.s.		2.566	0.005
	C _{un} vs. C _{ec}	2.027	0.038	8.875	<0.001	4.574	<0.001
	C _{eo} vs. C _{ec}	n.s.		7.131	<0.001	2.008	0.040
<i>LAM</i> df=1265	C _{un} vs. C _{eo}	n.s.		n.s.		n.s.	
	C _{un} vs. C _{ec}	0.005	0.015	-0.006	<0.001	0.011	<0.001
	C _{eo} vs. C _{ec}	0.006	<0.001	-0.004	0.031	0.010	<0.001
<i>TT</i> df=1265	C _{un} vs. C _{eo}	-0.183	<0.001	n.s.		n.s.	
	C _{un} vs. C _{ec}	-0.157	<0.001	0.121	0.006	0.212	<0.001
	C _{eo} vs. C _{ec}	n.s.		0.213	<0.001	0.264	<0.001
<i>ENTR_V</i> df=1265	C _{un} vs. C _{eo}	-0.064	<0.001	-0.040	0.028	n.s.	
	C _{un} vs. C _{ec}	-0.056	<0.001	0.050	0.003	0.105	<0.001
	C _{eo} vs. C _{ec}	n.s.		0.089	<0.001	0.122	<0.001
<i>CC</i> df=1265	C _{un} vs. C _{eo}	n.s.		n.s.		n.s.	
	C _{un} vs. C _{ec}	n.s.		-0.047	<0.001	-0.010	0.002
	C _{eo} vs. C _{ec}	n.s.		-0.041	<0.001	n.s.	
<i>CPL</i> df=1265	C _{un} vs. C _{eo}	n.s.		0.686	<0.001	0.937	<0.001
	C _{un} vs. C _{ec}	-0.402	0.009	0.631	<0.001	1.239	<0.001
	C _{eo} vs. C _{ec}	n.s.		n.s.		n.s.	
<i>E_{local}</i> df=1265	C _{un} vs. C _{eo}	n.s.		-0.011	0.0010	-0.012	0.007
	C _{un} vs. C _{ec}	0.013	<0.001	-0.060	<0.001	-0.025	<0.001
	C _{eo} vs. C _{ec}	n.s.		-0.049	<0.001	-0.013	0.0011
<i>E_{global}</i> df=1265	C _{un} vs. C _{eo}	n.s.		-0.007	<0.001	-0.007	<0.001
	C _{un} vs. C _{ec}	n.s.		-0.011	<0.001	-0.014	<0.001
	C _{eo} vs. C _{ec}	n.s.		-0.005	0.028	-0.007	<0.001
<i>CoC</i> df=1265	C _{un} vs. C _{eo}	n.s.		n.s.		n.s.	
	C _{un} vs. C _{ec}	n.s.		-2.917	<0.001	-0.624	0.004
	C _{eo} vs. C _{ec}	n.s.		-2.461	<0.001	n.s.	
<i>BC</i> df=1265	C _{un} vs. C _{eo}	-293.824	0.003	n.s.		n.s.	
	C _{un} vs. C _{ec}	-432.439	<0.001	n.s.		244.96	0.018
	C _{eo} vs. C _{ec}	n.s.		n.s.		295.454	0.0030
<i>Ass</i> df=1265	C _{un} vs. C _{eo}	n.s.		n.s.		n.s.	
	C _{un} vs. C _{ec}	n.s.		n.s.		n.s.	
	C _{eo} vs. C _{ec}	n.s.		n.s.		0.017	0.017
<i>Trans</i> df=1265	C _{un} vs. C _{eo}	-0.022	<0.001	n.s.		n.s.	
	C _{un} vs. C _{ec}	-0.021	<0.001	n.s.		0.020	<0.001
	C _{eo} vs. C _{ec}	n.s.		n.s.		0.021	<0.001

M_{Diff}, mean difference; *RR*, recurrence rate; *DET*, determinism; *L_{avg}*, average diagonal line length; *ENTR_L*, entropy based on the diagonal line structure; *LAM*, laminarity; *TT*, trapping time; *ENTR_V*, entropy based on the vertical line structure; *CC*, clustering coefficient; *CPL*, characteristic path length; *E_{local}*, local efficiency; *E_{global}*, global efficiency; *CoC*, coreness centrality; *A*, assortativity; *T*, transitivity.

Supplementary Table 15. Results of the Sidak-adjusted post-hoc test for *MCR*, comparing *System* and *Condition* levels within and between systems.

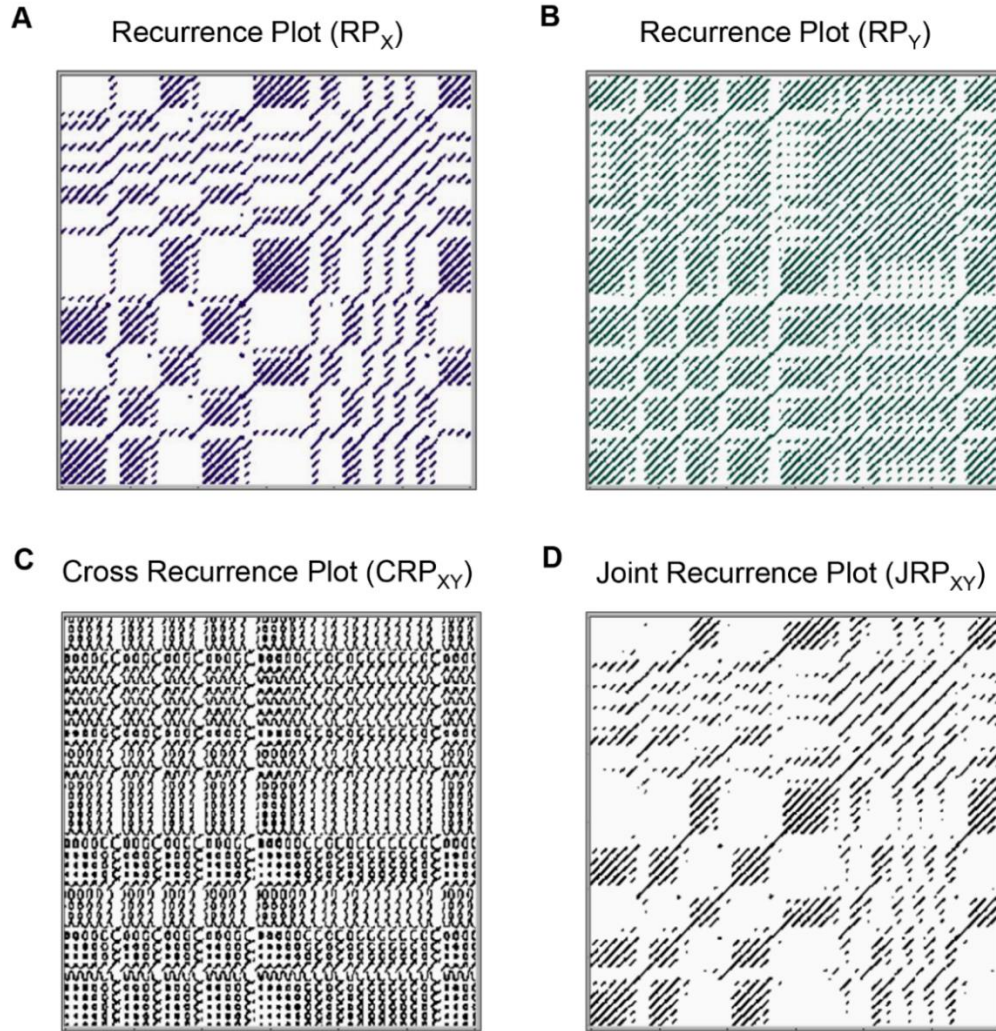
Levels	M _{Diff}	p-value	Levels	M _{Diff}	p-value	Levels	M _{Diff}	p-value
Within Systems (df = 1156.6)								
R vs. H	0.099	<0.001	R vs. V	-0.064	<0.001	H vs. V	-0.163	<0.001
C _{un} vs. C _{eo}	0.049	<0.001	C _{un} vs. C _{ec}	0.017	<0.001	C _{eo} vs. C _{ec}	-0.032	<0.001
Between Systems								
R-H vs. H-R	n.s.		R-V vs. V-R	0.025	<0.001	H-V vs. V-H	-0.010	<0.001
df = 2517.2			df = 2517.2			df = 2552.0		
R-H vs. R-V	-0.102	<0.001	R-V vs. H-V	0.112	<0.001	H-V vs. V-R	-0.087	<0.001
df = 2547.1			df = 2562.3			df = 2489.7		
R-H vs. H-V	0.010	<0.001	R-V vs. H-R	0.108	<0.001	H-R vs. V-R	-0.084	<0.001
df = 2562.3			df = 2517.2			df = 1770.1		
R-H vs. V-R	-0.077	<0.001	R-V vs. V-H	0.102	<0.001	H-R vs. V-H	n.s.	
df = 2517.2			df = 2562.3			df = 2489.7		
R-H vs. V-H	n.s.		H-V vs. H-R	n.s.		V-R vs. V-H	0.077	<0.001
df = 2562.3			df = 2489.7			df = 2489.7		
C _{un} vs. C _{eo}	0.004	0.034	C _{un} vs. C _{ec}	-0.016	<0.001	C _{eo} vs. C _{ec}	-0.019	<0.001
df = 2552.0			df = 2552.0			df = 2552.0		

MCR, mean conditional probability of recurrence; M_{Diff}, mean difference; R, respiration; H, heart rate variability; V, voice. Subsystem interactions: R-H, respiration-HRV; R-V, respiration-voice; H-V, HRV-voice; H-R, HRV-respiration; V-R, voice-respiration; V-H, voice-HRV.

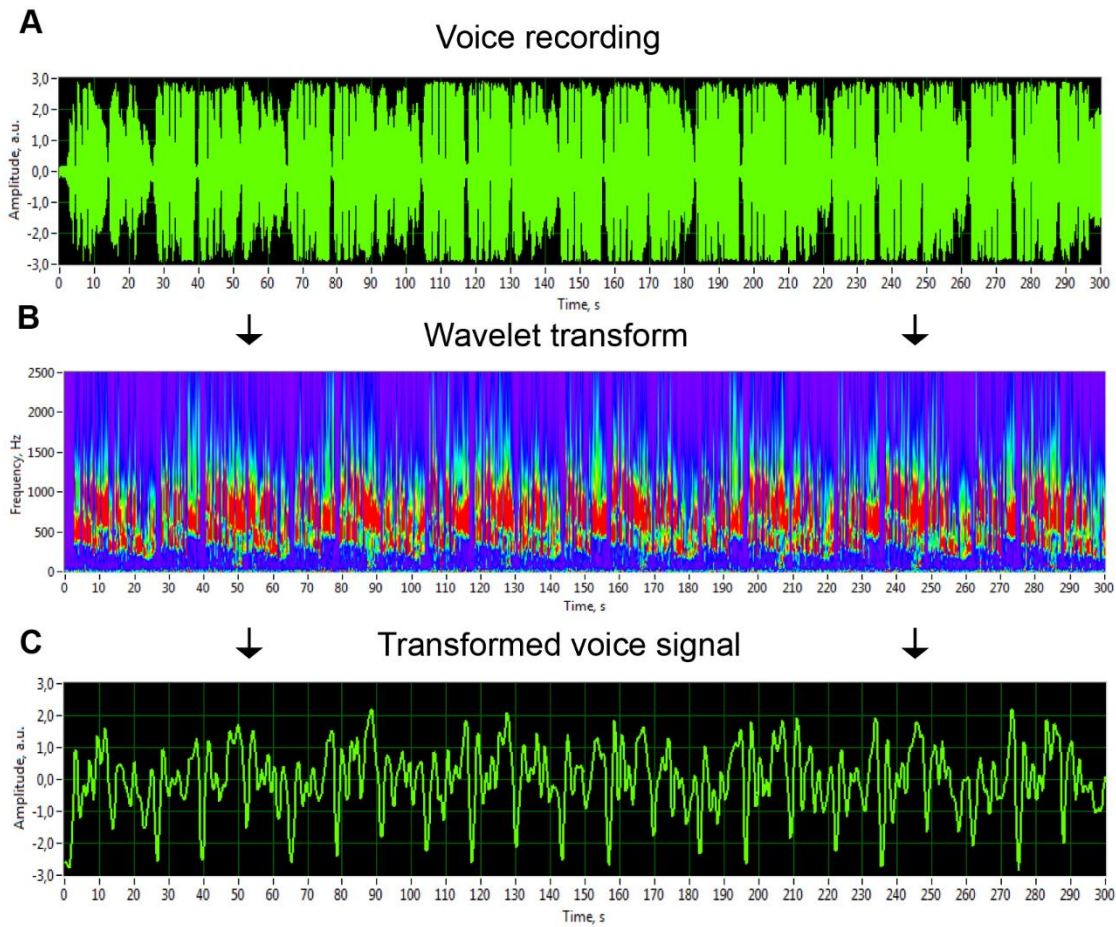
Supplementary Table 16. Results of the Sidak-adjusted post-hoc tests for *MCR*, comparing singing conditions across different subsystems and their interactions.

Levels	M _{Diff}	p-value	M _{Diff}	p-value	M _{Diff}	p-value
Within Systems (<i>df</i> = 1156.6)						
	Respiration (R)			HRV (H)		Voice (V)
C _{un} vs. C _{eo}	0.046	<0.001	n.s.		0.102	<0.001
C _{un} vs. C _{ec}	n.s.		n.s.		0.045	<0.001
C _{eo} vs. C _{ec}	-0.037	<0.001	n.s.		-0.057	<0.001
Between Systems (<i>df</i> = 2552.0)						
	R-H			R-V		H-V
C _{un} vs. C _{eo}	n.s.		0.022	<0.001	0.013	<0.001
C _{un} vs. C _{ec}	-0.012	0.004	-0.013	0.001	0.019	<0.001
C _{eo} vs. C _{ec}	-0.011	0.009	-0.035	<0.001	n.s.	
	H-R			V-R		V-H
C _{un} vs. C _{eo}	n.s.		n.s.		n.s.	
C _{un} vs. C _{ec}	n.s.		-0.059	<0.001	-0.021	<0.001
C _{eo} vs. C _{ec}	n.s.		-0.055	<0.001	-0.017	<0.001

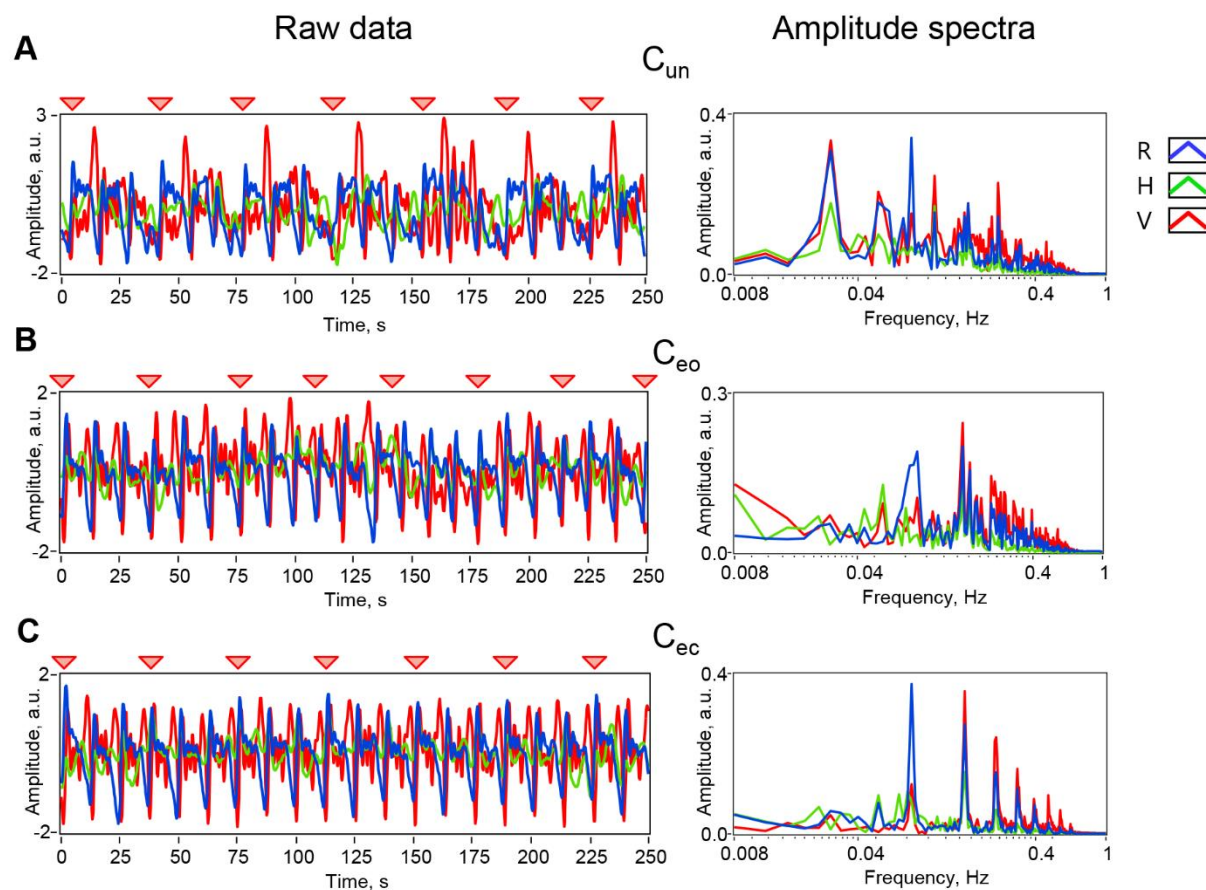
MCR, mean conditional probability of recurrence; M_{Diff}, mean difference.



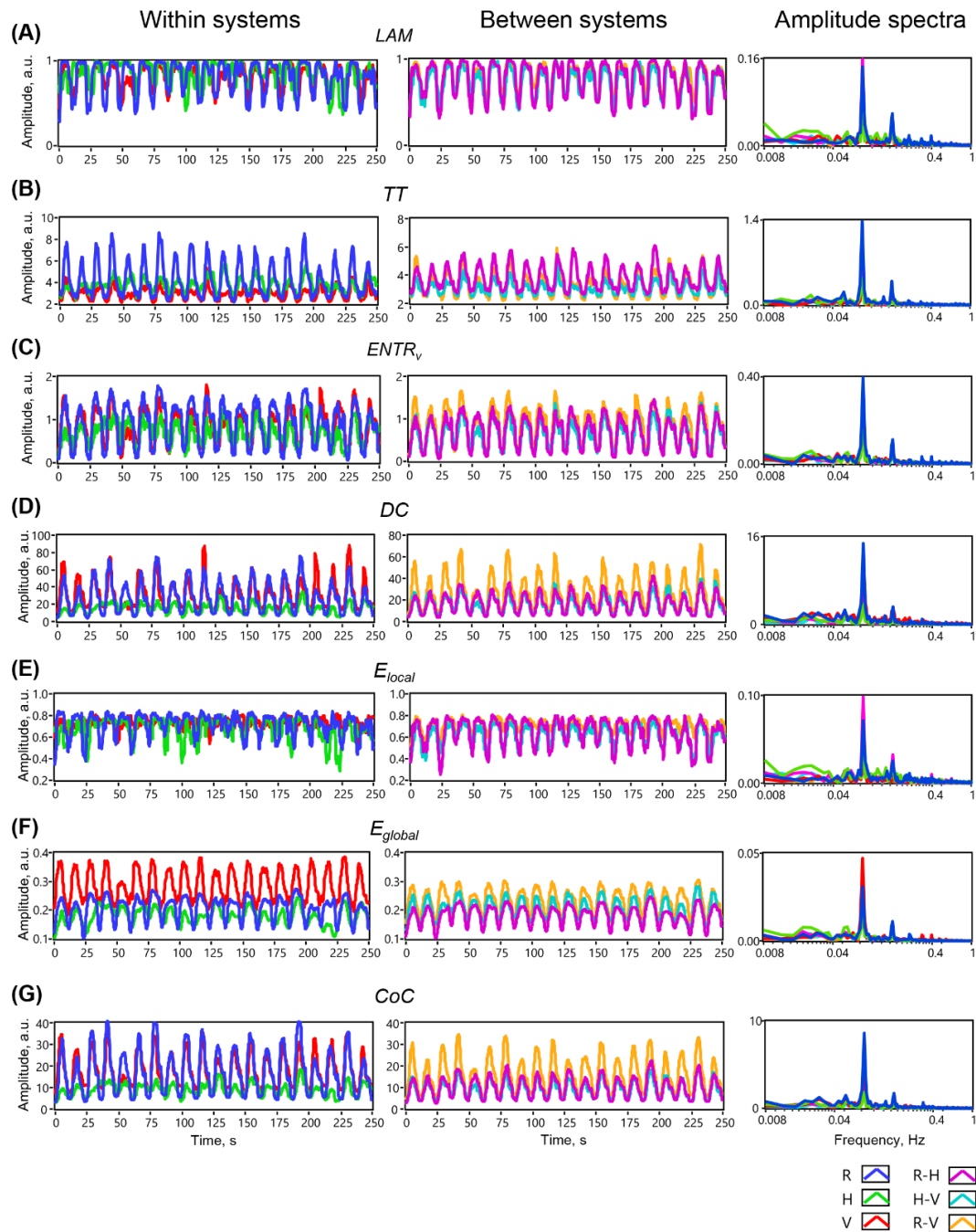
Supplementary Figure 1. Exemplary representation of two recurrence plots (RPs) and their corresponding cross-recurrence plot (CRP) and joint recurrence plot (JRP). **A** RP of signal X, illustrating its recurrent states over time. **B** RP of signal Y, depicting its temporal recurrence patterns. **C** CRP, representing the interdependencies between signals X and Y by highlighting when the states of both signals align. **D** JRP, showing the simultaneous recurrence of signals X and Y in their respective phase spaces, indicating moments when both systems revisit similar states at the same time.



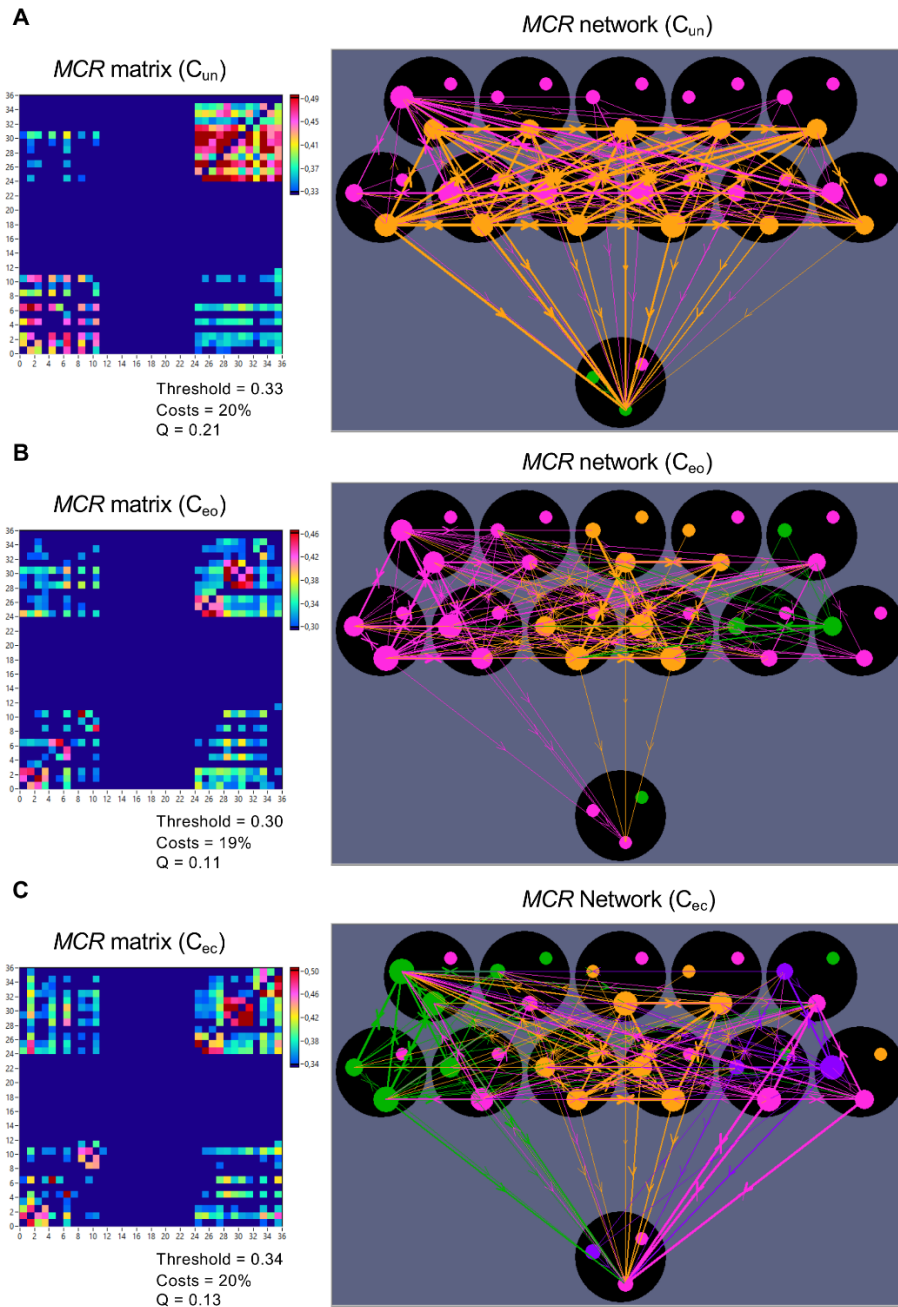
Supplementary Figure 2. Transformation of a voice signal. **A** Raw signal of a voice recording; **B** Analytic wavelet transform of the voice signal with energy normalization; **C** New voice signal derived from the wavelet-transformed signal by averaging the wavelet power across the frequency bins.



Supplementary Figure 3. Raw data (left) and corresponding amplitude spectra (right) averaged across participants for respiration (R), heart rate variability (H), and voice (V) subsystems during different singing conditions: **A** singing the canon in unison (C_{un}); **B** singing the canon in parts with eyes open (C_{eo}); **C** singing the canon in parts with eyes closed (C_{ec}). The left panels display the raw physiological and vocal signals, while the right panels present the amplitude spectra, offering insights into the frequency distribution of each subsystem under the various conditions. Small triangles above the diagrams on the left indicate the onset of each three-line canon sequence within the 250-second analysis window.



Supplementary Figure 4. Time course of different measures averaged across participant pairs within subsystems (left) and between subsystems (middle), along with the corresponding amplitude spectra (right), when singing the canon in parts with eyes closed (C_{ec}): **A** *LAM*, laminarity; **B** *TT*, trapping time; **C** *ENTR_v*, entropy (vertical lines); **D** *DC*, degree centrality; **E** *E_{local}*, local efficiency; **F** *E_{global}*, global efficiency; **G** *CoC*, coreness centrality. Subsystems: R, respiration; H, HRV; V, voice. Subsystem interactions: R-H, respiration-HRV; R-V, respiration-voice; H-V, HRV-voice.



Supplementary Figure 5. Modularity analysis of MCR-based directional coupling across different singing conditions. **A** MCR matrix heatmap (left) with 36 nodes (12 choir members across 3 subsystems: R (1-12), H (13-24), and V (25-36), and the corresponding choir network (right), depicting singers and conductor (3 nodes each, colored according to modules identified by modularity analysis) along with their outgoing links when singing the canon in unison (C_{un}). **B** and **C** show equivalent visualizations for the conditions of singing the canon in parts with eyes open (C_{eo}) and eyes closed (C_{ec}), respectively.