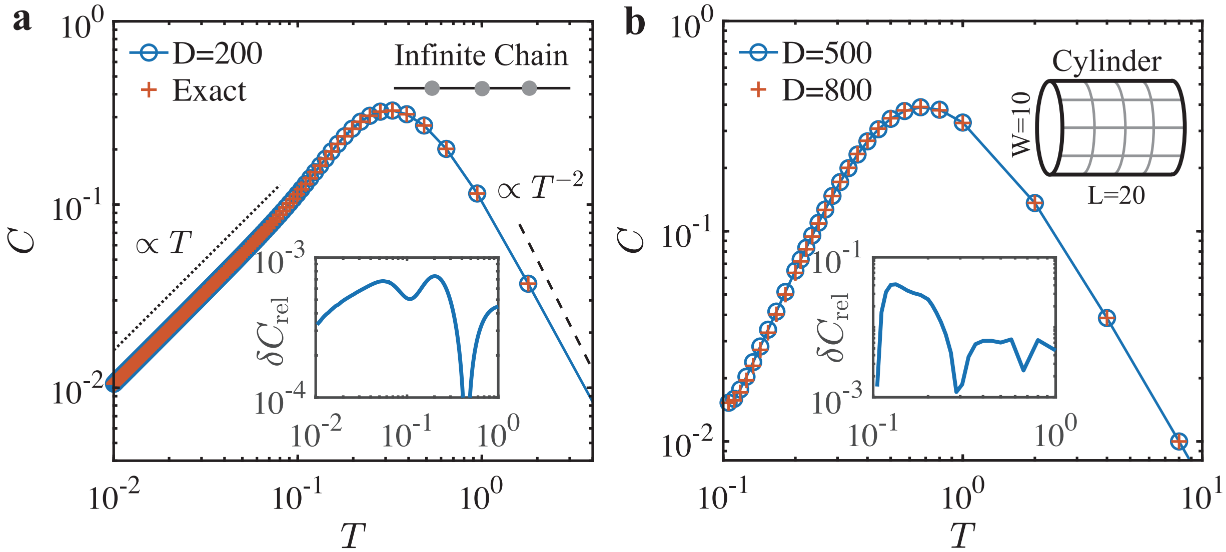
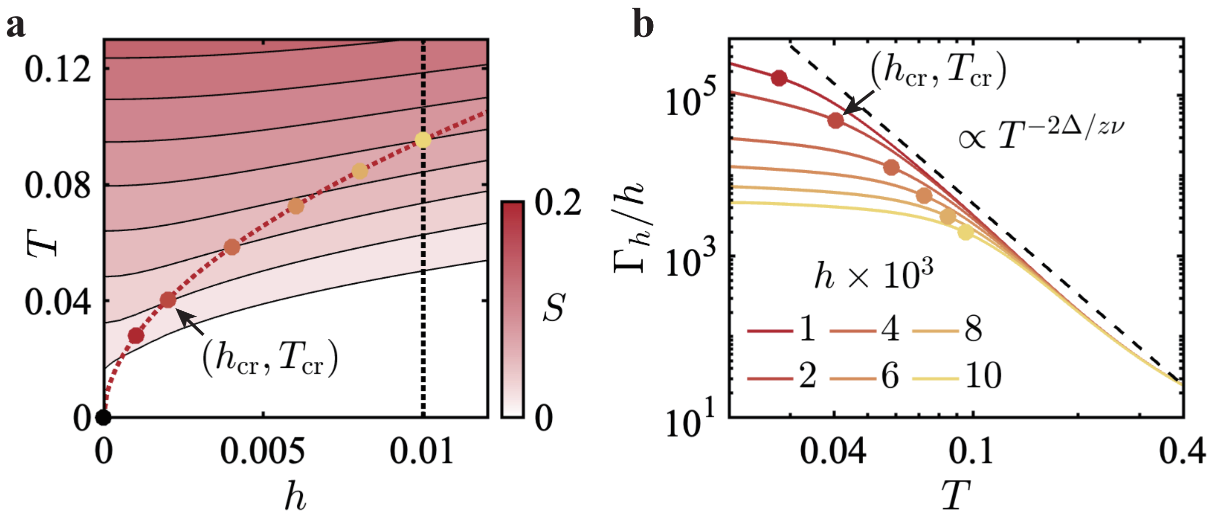
Supplementary Information for

# Quantum Supercritical Regime with Universal Magnetocaloric Scaling in Ising Magnets

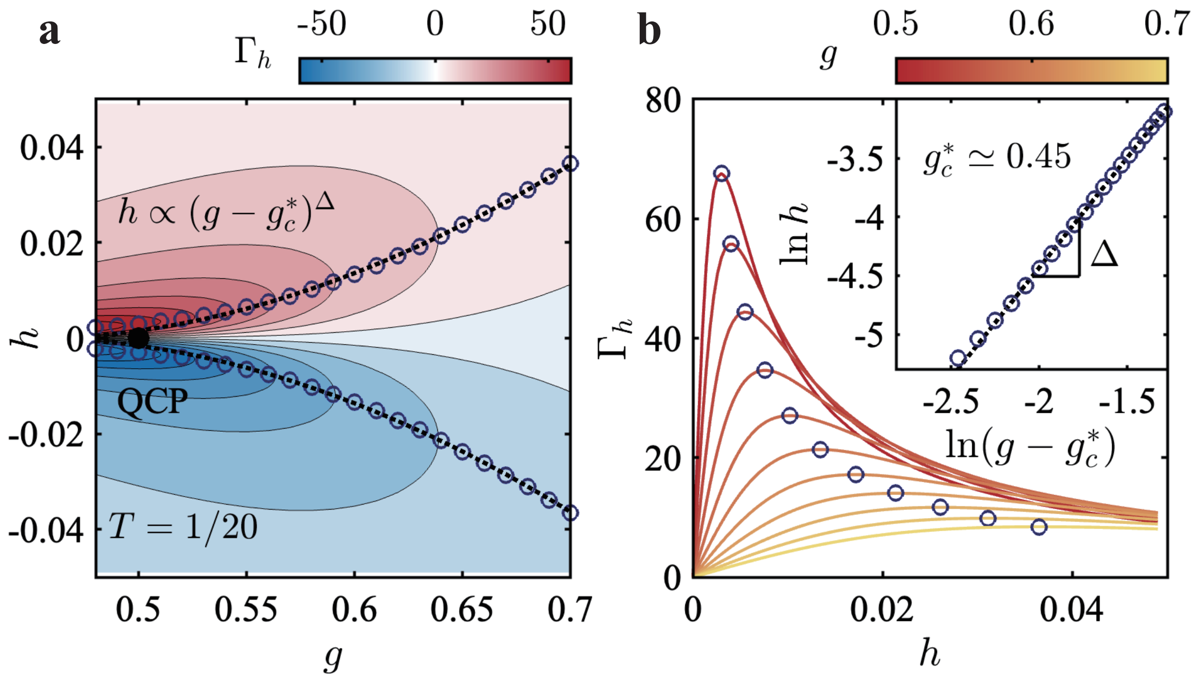
Lv *et al.*



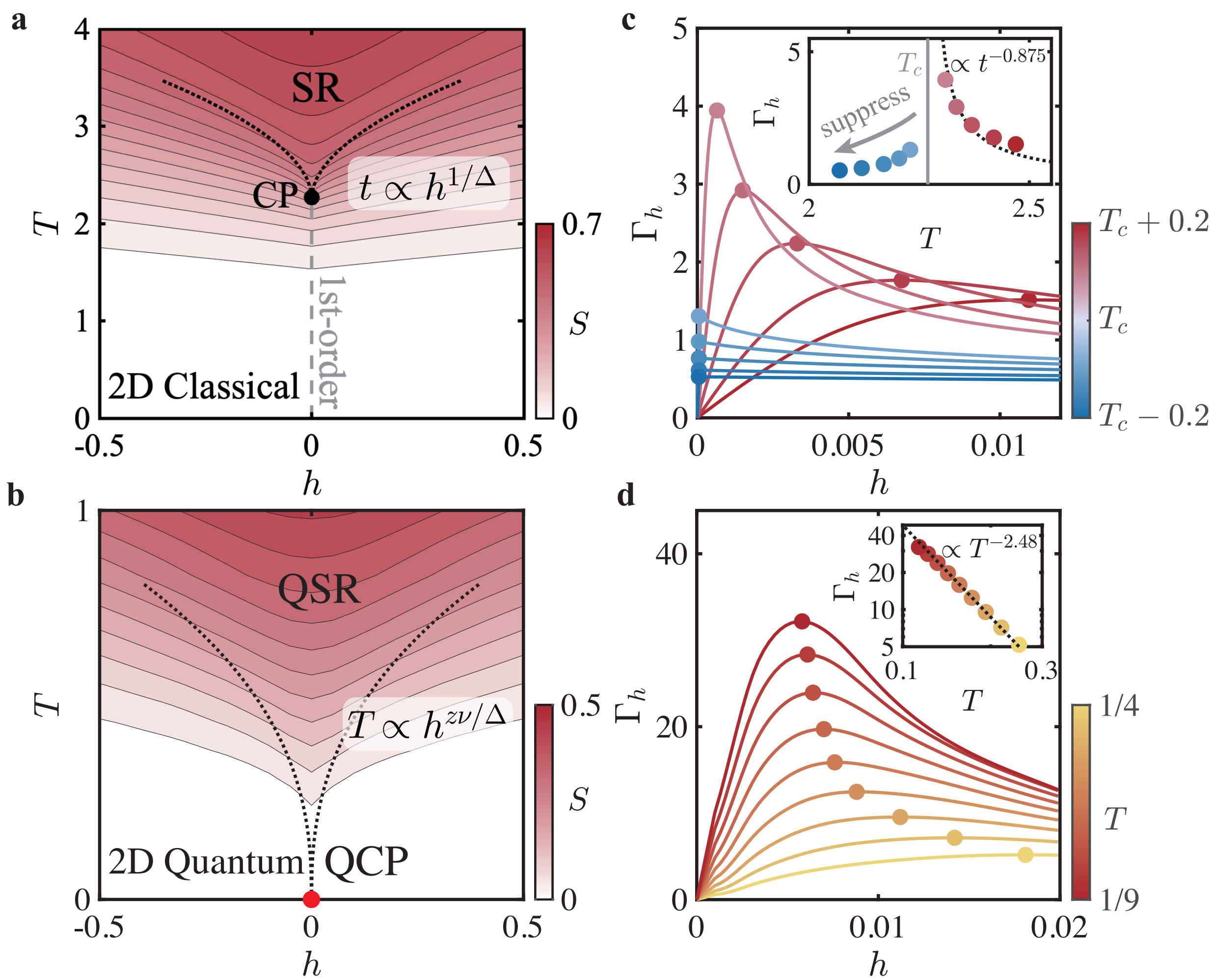

**Supplementary Figure 1. Convergence of the tensor-network calculations. a** The calculated specific heat $C$ of the 1D QIM at the critical field $g_c = 0.5$ (blue circles), which coincide with the exact results $C_{\mathrm{exact}}$ (red crosses). Inset shows the relative error $\delta C_{\mathrm{rel}} \equiv |C_{\mathrm{exact}} - C|/C_{\mathrm{exact}} < 10^{-3}$. **b** The specific heat of 2D QIM is simulated on the $10 \times 20$ cylinder (inset), at $g = 1.45 \approx g_c$. The results are well converged for bond dimension $D = 800$. Inset illustrates the relative difference between the $D = 500$ and 800 data.



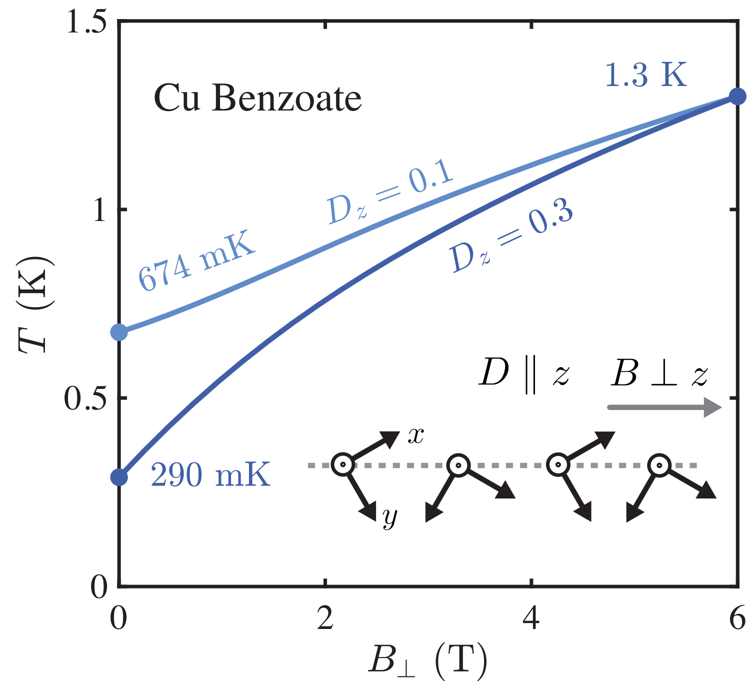

**Supplementary Figure 2. Quantum supercritical scaling of Grüneisen ratio $\Gamma_h$. a** The isentropes of 1D QIM in the $h$-$T$ plane. The red dashed line represents the QSR crossover line. **b** Results of the Grüneisen ratio $\Gamma_h$ with various fixed $h$. The black dashed line illustrates the scaling $\Gamma_h/h \propto T^{-2\Delta/z\nu}$. The red circles ($h_{\rm cr}$, $T_{\rm cr}$) are determined from Fig. 2**c** of the main text.




**Supplementary Figure 3. Crossover scaling and Grüneisen ratio $\Gamma_h$ in the $g$-$h$ plane. a** Contour of Grüneisen ratio $\Gamma_h$ of 1D QIM in the $g$-$h$ plane at a fixed temperature $T = 1/20$. The hollow circles indicate the peaks/dips of $\Gamma_h$ by scanning $h$ for various fixed $g$, with the curves of $\Gamma_h$ vs. $h$ in **b**. Inset illustrates the power-law scaling $h \propto (g - g_c^*)^\Delta$ of the peak locations, where the fitting yields $g_c^* \simeq 0.45$.




**Supplementary Figure** 4. **Grüneisen ratio of 2D classical and quantum Ising model. a** The isentropic lines of 2D classical Ising model, where the black circle marks the Curie point (CP). The black dotted lines are classical supercritical crossover lines with scaling $t \propto h^{1/\Delta}$ ($t \equiv T - T_c$), which correspond to the peaks/dips of $\Gamma_h$ in **c**, and the gray dashed line represents the first-order phase transition. **b** Isentropic lines of the 2D quantum Ising model at critical transverse field, where the red dot labels the QCP at $g_c \simeq 1.52$. The dashed lines represent the QSR crossover boundary. **c** For classical SR, the Grüneisen ratio peaks are indicated, whose peak values follows $\Gamma_h \propto t^{-(\Delta-1)} \propto t^{-0.875}$ and diverges at $T_c$; for $T < T_c$, $\Gamma_h$ get quickly suppressed in the ordered phase (see inset). **d** For the QSR, the peak values of the Grüneisen ratio are marked by red dots, the inset shows a divergent scaling behavior $\Gamma_h \propto T^{-2.48}$ in the zero-temperature limit.




**Supplementary Figure 5. Quantum supercritical cooling effect in Cu benzoate.** Simulated isentropic line of the representative Heisenberg-Dzyaloshinskii-Moriya (HDM) compound Cu benzoate ($J \simeq 18.2$ K, $D_z \simeq 0.1$) under a in-plane field $B_\perp$. The dark blue line represents the isentropic line for $D_z = 0.3$. The inset illustrates the effective transverse field $g$ and staggered longitudinal field $h_s$, originated from the external field $B_\perp$.