Digital Twin based Sorting Optimization for Parcel Distribution Centers in Logistics Networks

SUPPLEMENTARY NOTE I: PORTION OF PARCELS TARGETED TO EACH DESTINATION OVER CONSECUTIVE DAYS

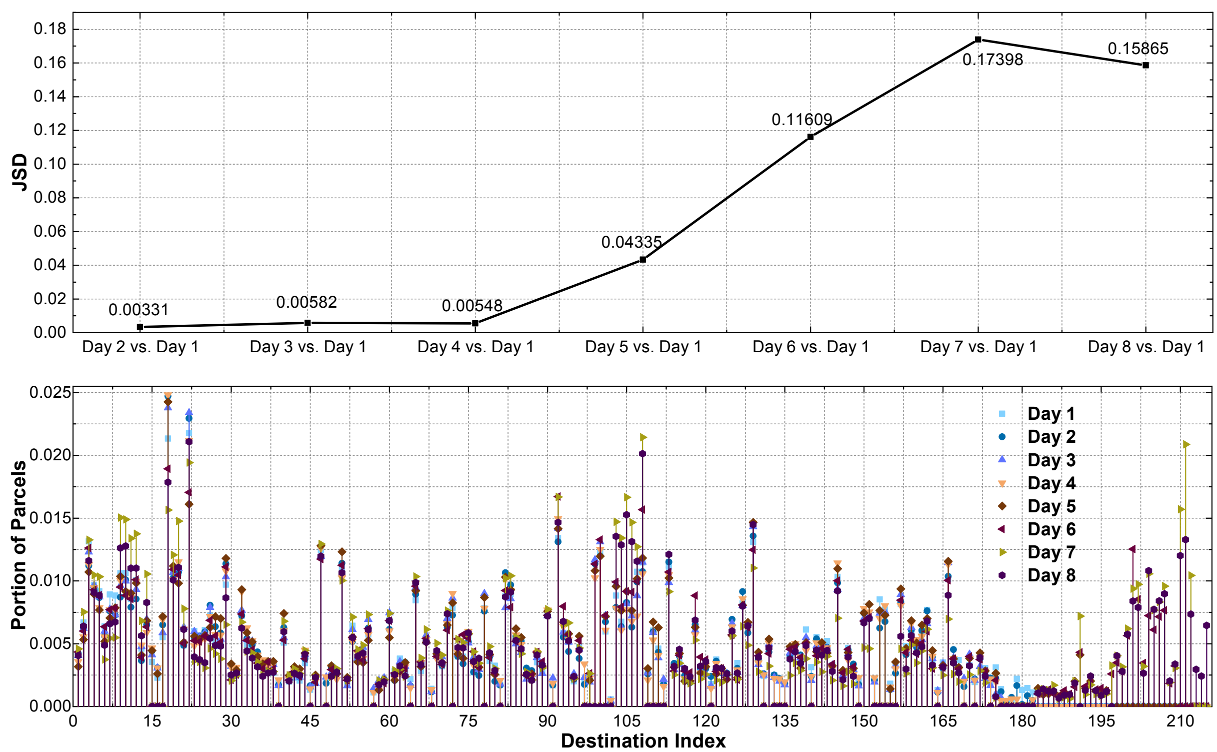

Fig. 1. **Jensen-Shannon divergence (JSD) values quantifying the similarity in the portion of parcels targeted to each destination over consecutive days relative to Day 1.** The first plot shows the JSD values comparing each subsequent day to Day 1. The second plot details the portion of parcels targeted to each destination over 8 consecutive days.

To quantify the similarity of the portion of parcels targeted to each destination across sorting shifts over consecutive days, we employ the Jensen-Shannon divergence (JSD) as the metric. Specifically, we denote the portion of parcels targeted to each destination on two different days by $\mathcal{P} = \{P_d\}_{d=1}^D$ and $\mathcal{P}' = \{P'_d\}_{d=1}^D$, where for each $d \in \mathcal{D}$, $P_d \geq 0$, $P'_d \geq 0$, and $\sum_{d=1}^D P_d = \sum_{d=1}^D P'_d = 1$. The JSD between $\mathcal{P}$ and $\mathcal{P}'$ is given as

$$\mathbf{JS}(\mathcal{P}\|\mathcal{P}') = \frac{1}{2}\,\mathbf{KL}(\mathcal{P}\|\mathcal{M}) + \frac{1}{2}\,\mathbf{KL}(\mathcal{P}'\|\mathcal{M}), \tag{1}$$

where $\mathcal{M} = \{M_d\}_{d=1}^D$ is the element-wise average portion of parcels targeted to each destination, with $M_d = \frac{P_d + P'_d}{2}, \forall d \in \mathcal{D}$, and $\mathbf{KL}(\cdot\|\cdot)$ denotes the Kullback-Leibler (KL) divergence, i.e.,

$$\begin{aligned} \mathbf{KL}(\mathcal{P}\|\mathcal{M}) &= \sum_{d=1}^D P_d \log \frac{P_d}{M_d}, \\ \mathbf{KL}(\mathcal{P}'\|\mathcal{M}) &= \sum_{d=1}^D P'_d \log \frac{P'_d}{M_d}. \end{aligned} \tag{2}$$

The resulting JSD value, ranging from 0 (indicating identical distributions) to $\ln 2 \approx 0.693$ (indicating maximum divergence), provides a quantitative measure of the similarity in the portion of parcels targeted to each destination across sorting shifts over consecutive days.

By analyzing the real-world parcel data from SF Express, as detailed in Fig. 1, we observe notably low JSD values during the initial consecutive days (Day 2 vs. Day 1: 0.00331, Day 3 vs. Day 1: 0.00582, Day 4 vs. Day 1: 0.00548), indicating minimal variation. However, the divergence progressively increases in subsequent days (Day 5 vs. Day 1: 0.04335, Day 6 vs. Day 1: 0.11609, Day 7 vs. Day 1: 0.17398, Day 8 vs. Day 1: 0.15865).

These results indicate that the portion of parcels targeted to each destination remains relatively similar over consecutive days, particularly within the initial days. Consequently, updating sorting plans on a three-day basis provides a practical balance between operational efficiency and deployment cost.

Supplementary Methods II: Fidelity of the PSDTS

The fidelity of the proposed PSDTS is defined as the weighted average accuracy between the simulated and actual recorded number of sorted parcels over fixed-length time intervals throughout the sorting shift. Specifically, we first divide the sorting time of each sorting shift into Z fixed-length time intervals. In the o-th interval, we record the number of sorted parcels in the PSDTS as A^o and in the real-world as B^o, and calculate the parcel-count weight which corresponds to the fraction of simulated sorted parcels in interval o relative to the total simulated sorted parcels, ensuring that time intervals with more sorted parcels contribute to the overall fidelity. The fidelity F is then defined as

$$F = \sum_{o=1}^{Z} \left(1 - \frac{\left| B^o - A^o \right|}{\max\{B^o, A^o\}} \right) \frac{A^o}{A^{\text{all}}},$$

where A^{all} denotes the total simulated sorted parcels in the sorting shift.

Supplementary Methods III: Solution space analyses

A. *Solution space for problem (**P1**)*

Given the number of destinations D and the number of grids X, the total number of solutions of problem **(P1)** is D^X. Considering each destination has to be assigned at least one grid, the number of valid solutions can be reduced to

$$D^X - \binom{D}{1}(D-1)^X + \binom{D}{2}(D-2)^X - \cdots + (-1)^{D-1}\binom{D}{D-1},$$

by using the inclusion-exclusion principle, which can be simplified as

$$D^X + \sum_{k=1}^{D-1} (-1)^k \binom{D}{k} (D-k)^X.$$

B. *Solution space for problem (**P2**)*

The bounded assignment constraint in (8) requires $\alpha_d \geq 1, \forall d \in \mathcal{D}$. We introduce a change of variables $y_d = \alpha_d - 1$, so that $y_d \geq 0$. Substituting into the practical constraint

$$\alpha_1 + \alpha_2 + \cdots + \alpha_D = X,$$

yields the equivalent form

$$y_1 + y_2 + \cdots + y_D = X - D.$$

Therefore, the number of non-negative integer solutions to this equation is $\binom{X-1}{D-1} = \frac{(X-1)!}{(D-1)!(X-D)!}$.