Supplementary Material (SM): A Low-Cost Policy for Reducing Methane Emissions in the Oil and Gas Industry

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1 Model Setup

We study an infinite-horizon production economy with $S \geq 2$ sovereign countries populated by four types of agents: oil & gas firms (upstream, section 1.1), refineries & transformation firms (midstream, section 1.2), consumers (downstream, section 1.3), and national governments (section 1.4). The economy is affected by climate change caused by anthropogenic greenhouse gas (GHG) emissions (section 1.5). In this section we present the setup of the model. Section 2 describes the main analytical results. Section 3 illustrates the identification of the structural parameters of the model.

1.1 Upstream: Oil&Gas Firms

In each country s there are K^s infinitely-living oil&gas profit-maximizing firms. Firm k in country s owns I^{ks} oil&gas fields denoted by $i \in \{1, 2, ..., I^{sk}\}$ and compete in a Cournot oligopoly fashion on the crude and natural gas markets.[1]

Production Technology. Let Oil_t^{iks} and Gas_t^{iks} be the Barrel of Oil Equivalent (BOE) amounts of crude and natural gas sold by field i in period t, respectively. Flare_t^{iks} denotes the BOE amount of natural gas flared and PInS_t^{iks} the amount purchased for in-situ use for electricity production or heating purposes. M_t^{iks} is the maintenance capital accumulated by the field and Z_t^{iks} the US dollar value of other net outputs, such as labor and electricity.[2] Lastly, ReInS_t^{iks} denotes the amount of extracted gas that is reused in-situ[3] and NRF_t^{iks} is the amount of flaring that field i cannot avoid to produce given the technology available in period t (minimum non-routine flaring). Following standard Microeconomic Theory, we assume that the oil&gas production technology is described by a field-specific real analytic transformation function $TF_t^{iks} : (-\infty, +\infty)^7 \to \mathbb{R}$, whose argument, the net output vector, writes $\left(\mathrm{Oil}_t^{iks}, \mathrm{Gas}_t^{iks}, \mathrm{Flare}_t^{iks}, \mathrm{PInS}_t^{iks}, \mathrm{M}_t^{iks}, \mathrm{Z}_t^{iks}; \mathrm{ReInS}_t^{iks}\right)$.

[1]However, oil fields usual produce modest quantities of gas, which make the effect of their productive choices on gas prices small or negligible. Because of that, we assume that oil firms are price-takers on the natural gas market.

[2]Note that the domain of Z_t^{iks} is $(-\infty, +\infty)$. However, it typically takes negative values because it includes the value of all productive inputs, such as labor, energy purchases, etc.

[3]Note that formally ReInS_t^{iks} is not a net output of the field transformation function, the quantity of gas reused in-situ affects the production technology by partially replacing other sources of energy required in the production process. Also note that the domain of each net input is the entire set of real number. However, when we setup the firm's problem, we restrict the range of feasible values by adding appropriate constraints (e.g., $\mathrm{Oil}_t^{iks} \geq 0$)

We assume that natural gas venting is a costless output and does not affect the production technology in any way other than, of course, through regulatory and fiscal costs, which we consider separately from the production technology in the next section. As a consequence of this assumption, we suppress Vent_t^i from the arguments of $TF_t^{iks}(\cdot)$. Thus, the production set is defined by the inequality:

$$TF_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{PInS}_t^{iks}, \text{M}_t^{iks}, \text{Z}_t^{i}; \text{ReInS}_t^{iks}\right) \leq 0 \ . \tag{1}$$

Note that $\text{PInS}_t^{iks}, \text{M}_t^{iks}, \text{Z}_t^{iks}; \text{ReInS}_t^{iks}$ are net input whose value is allowed to be positive or negative. The sign of these variables is determined endogenously and is shaped by the function TF_t^{iks}. For instance, the assumptions $\frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Oil}_t^i} \leq 0$ for all $\text{Oil}_t^{iks} \leq 0$ ensure that the oil production is always weakly positive. In particular, we restrict the attention to the class of weakly separable and twice differentiable convex transformation functions in the form:

$$\begin{aligned} TF_t^{iks} = \quad & F_t^{iks}\left(TF_{1t}^{iks}\left(\text{Oil}_t^i\right) + TF_{2t}^{iks}\left(\text{Gas}_t^i\right) + TF_{3t}^{iks}\left(\text{Flare}_t^i\right)\right. \\ & \left. + TF_{4t}^{iks}\left(\text{PInS}_t^{iks}; \text{ReInS}_t^{iks}\right) + TF_{5t}^{iks}\left(\text{M}_t^{iks}\right) + TF_{6t}^{iks}\left(\text{Z}_t^{iks}\right)\right) \end{aligned} \ , \tag{2}$$

where F_t^{iks} is strictly increasing and twice differentiable. Lastly, the production of field i in period t is bounded above by its capacity K_t^i, such that the aggregate production of hydrocarbons must satisfy the following inequality:

$$\text{Oil}_t^{iks} + \text{TotGas}_t^{iks} \leq \text{K}_t^{iks} \tag{3}$$

Where the variable TotGas_t^{iks} denotes the total amount of gas extracted from the field in period t. Note that the constraint in (3) is stated in terms of total hydrocarbons measured in BOE. Under the technological constraints we introduce in section 1.1.1, this modelling choice is equivalent to imposing a constraint on the oil production capacity. The management can sell, flare, vent, re-inject or use the extracted gas in-situ for electricity production, therefore its formula writes:

$$\text{TotGas}_t^{iks} = \text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{TotVent}_t^{iks} + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks} \ , \tag{4}$$

where TotVent_t^{iks} denotes the total amount of gas released in the atmosphere in period t.

Investment, Development, and Discoveries. The field faces investment costs in field development and new discoveries, as well as in field's capacity in the form, represented by a real analytic function

$$\begin{aligned} & InvCost_t^{iks}\left(\text{ID}_t^{iks}, \text{IM}_t^{iks}, \text{ReInj}_t^{iks}, \text{PInj}_t^{iks}, \text{OInj}_t^{iks}; PP_t^{iks,\text{Gas}}\right) = \\ & \text{ID}_t^{iks} + \text{IM}_t^{iks} + PP_t^{iks,\text{Gas}}\text{PInj}_t^{iks} + IC_t^{iks}\left(\text{ReInj}_t^{iks}, \text{PInj}_t^{iks}\right) + C_t^{iks}\text{OInj}_t^{iks} \end{aligned} \ , \tag{5}$$

where ID_t^{iks} is the US dollar amount of investment in field development and exploration, IM_t^{iks} is investment in field maintenance and $PP_t^{iks,\mathrm{Gas}}$ denotes the purchase price of natural gas from nearby fields.[4] The field capacity can be increased through the discovery and development of new reserves. Alternatively, the field's management can increase the pressure of the reservoir by injecting natural gas or other liquids and/or gases through injection wells. Injections ReInj_t^{iks} and PInj_t^{iks} denote the amounts injected natural gas produced by the field and purchased from nearby fields, respectively, whereas OInj_t^{iks} is the gas-equivalent amount of other types of injections, such as steam and chemicals. The increase in field capacity depends upon total injections, $\mathrm{TotInj}_{t-1}^{iks} = \mathrm{ReInj}_t^{iks} + \mathrm{PInj}_t^{iks} + \mathrm{OInj}_t^{iks}$. Lastly, the field's capacity declines with the amount of extracted hydrocarbons, capturing the fall in well pressure due to depletion. In detail, the capacity of oil field i in period $t+1$ solves the following inequality:

$$\mathrm{K}_{t+1}^{iks} \leq \mathrm{K}_t^{iks} + D_t^{iks}\left(\mathrm{ID}_t^{iks}, \mathrm{L}_{t-1}^{iks}\right) + B_t^i\left(\mathrm{TotInj}_{t-1}^{iks}\right) - \zeta\left[\mathrm{Oil}_t^{iks} + \mathrm{TotGas}_t^{iks}\right] \quad (6)$$

where D_t^{iks} and B_t^{iks} are real analytic functions capturing the effect on the field's capacity of investment in discoveries and injections, respectively[5], whereas L_{t-1}^{iks} denotes the cumulative investment in new discoveries up to period $t-1$ and follows a low of motion:

$$\mathrm{L}_t^{iks} = \mathrm{L}_{t-1}^{iks} + \mathrm{ID}_t^{iks}\ . \quad (7)$$

Before describing the firm's profit maximization problem we describe in detail the technological and regulatory constraint an oil&gas firm faces in the management of the natural gas produced by each field.

1.1.1 The Natural Gas Management Problem

Technological Constraints. Consider an oil&gas field i owned by firm k. In each period t, field i extracts Oil_t^{iks} and a quantity of natural gas, denoted by TotGas_t^{iks} and measured in BOE. Gas extraction may be either the outcome of a deliberate choice of the management or a byproduct of oil production. In both cases, we assume a constant field-specific gas-to-oil ratio $GOR^{iks} \in [0, +\infty)$, and

[4]Note that the purchase price of gas $PP_t^{iks,\mathrm{Gas}}$ faced by field i is allowed to differ from the gross sales price $P_t^{iks,\mathrm{Gas}}$. This capture cases in which oil firms purchase gas from nearby fields that are not connected to a gas pipeline at a cheaper-than-market price.

[5]The use of an inequality constraint captures the possibility that the firm chooses to disregard some of its productive capacity, for instance by postponing the start of productive activity of some newly tapped wells.

impose the constraint $\text{TotGas}_t^{iks} \geq GOR^{iks}\text{Oil}_t^{iks}$. This constraint captures the fact that a certain quantity of natural gas is extracted as a by-product of oil production and is trivially non-binding for gas-only fields. The total quantity of gas vented is divided into two macro categories. A quantity vented intentionally IVent_t^{iks} and a quantity vented unintentionally $\text{UVent}_t^{iks} = UVent_t^{iks}\left(\text{Oil}_t^{iks}, \text{M}_t^{iks}\right)$ (leaking, also known as unintentional venting),

$$\text{TotVent}_t^{iks} = \text{IVent}_t^{iks} + UVent_t^{iks}(\text{TotGas}_t^{iks}, \text{M}_t^{iks}) \tag{8}$$

The former is defined as the amount of gas vented as a direct and deterministic consequence of a deliberate action or omission by the firm's personnel which is not justified by true health and safety concerns. It is mostly due to the disposal of gas accumulated at the top of oil tanks. It therefore excludes leakages due to poor maintenance. The latter is for the most part caused by the cleaning, testing, and poor maintenance of the gas equipment and safety-related pressure releases.

The set of feasible net output vectors is determined by the production technology. Following standard Microeconomic Theory, we assume that the production technology of the oil field in period t is described a real analytic transformation function $TF_t^{iks}: (-\infty, +\infty)^6 \to \mathbb{R}$, whose argument – the net output vector – is described in detail in the next paragraphs. Note that TF_t^{iks} has a very large domain. However, we impose constraints to the firm's choice problem (e.g., $\text{Oil}_t^{iks} \geq 0$) in order to avoid unplausible outcomes. For the purpose of modeling the gas management problem, we impose three key restrictions on TF_t^{iks}. First, even if natural gas is often treated as a by-product of oil extraction by oil firms, its production for commercial purposes is costly for the firm; i.e., $\frac{\partial^2 TF_t^{iks}(\cdot)}{\partial \text{Gas}_t^{iks}} > 0$, because it requires energy to capture and compress the extracted gas prior to entering the market. Second, flaring is also (weakly) costly and such cost is weakly increasing and convex in the quantity of gas flared,

$$\frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Flare}_t^{iks}} \geq 0 \;\; \frac{\partial^2 TF_t^{iks}(\cdot)}{\partial \text{Flare}_t^{iks2}} > 0 \tag{9}$$

for all possible values of the argument of TF_t^{iks}. This convex cost structure is motivated by technological considerations. First, the high pressure gas contained in the heater-treater can be flared at a very small marginal production cost – virtually equal to zero[6]. However, the low pressure gas contained in the oil tank

[6]We assume that not all the gas, which goes into the flare-stack, is flared. In articular, we assume a 98% flaring rate within the flare stack, which corresponds to the best practice in the industry. In presence of strong wind and/or low tech combustors, the flaring efficiency could decline (as low as $\sim 91\%$) and a larger part of the gas in the heater-treater could be vented.

cannot be flared at a marginal cost equal to zero. It is necessary to use a small compressor, an air assisted blower, or a gas assist options to get the gas out of the tank in a pressurized form and then burn it, see Figure 1. This operation has a positive marginal cost, which is possibly increasing in the quantity of gas flared because the energy required to get the gas out of the tank increases at a more-than-proportional rate as the amount of residual gas in the tank decreases. The magnitude of $\frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Flare}_t^i}$ depends upon the specific configuration of every field. However, a rule-of-thumb estimation can be done multiplying the quantity of natural gas or electricity needed to re-pressurize the low pressure gas by the cost of electricity (for a detailed description see section VRU (lines 54-63) of the OPGEE 3.0 manual Brandt, Masnadi, Rutherford, and Englander (2020))[7]. Lastly, we assume that the amount of unintentional venting is a real analytic function of the amount of crude extracted Oil_t^{iks} and the stock of maintenance capital M_t^{iks} in the form:

$$UVent_t^{iks}\left(\text{TotGas}_t^{iks}, \text{M}_t^{iks}\right) = \vartheta^{iks}\text{TotGas}_t^{iks} - Maint_t^{iks}\left(\text{M}_t^{iks}\right) + \epsilon_t^{iks}\ , \tag{10}$$

where $Maint_t^{iks}(\cdot)$ is weakly increasing and concave, and ϵ_t^{iks} is an i.i.d. shock with $\mathbb{E}\left[\epsilon_t^{iks}\right] = 0$. For the sole purpose of studying the effect of a change in the leak detection technology, in section 2 of this document we assume the following functional form: $Maint_t^{iks}\left(\text{M}_t^{iks}\right) = LDT_t^{ks} MNT_t^{iks}\left(\text{M}_t^{iks}\right)$ for some increasing and concave function $MNT_t^{iks}(\cdot)$, where $LDT_t^{ks} \geq 0$ denotes the effectiveness of the leak-detection technology available to firm k in period t. Given the the assumption on the functional form of $UVent_t^{iks}$ stated above, the formula for TotGas_t^{iks} in (4) can be solved recursively to obtain:

$$\text{TotGas}_t^{iks} = \left(1 - \vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} - Maint_t^{iks}\left(\text{M}_t^{iks}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\right] \tag{11}$$

Lastly, the stock of maintenance capital follows a law of motion:

$$\text{M}_{t+1}^{iks} = \text{M}_t^{iks}\left(1 - \rho^i\right) + \text{IM}_t^{iks} \tag{12}$$

where IM_t^{iks} is the investment in the maintenance of field i made in period t.

Information. In order to correctly design the regulatory and fiscal framework in the next paragraphs, we first describe the information set of each player. Each oil&gas firm is assumed to possess full information at the moment in which production decisions are made. That is, a firm's information set Ω_t^{ks} includes the

[7]Note that in world with stringent regulation on methane emissions also the flaring of high pressure gas is costly because the flare stuck must be maintained to ensure that all the gas is combusted all the time.

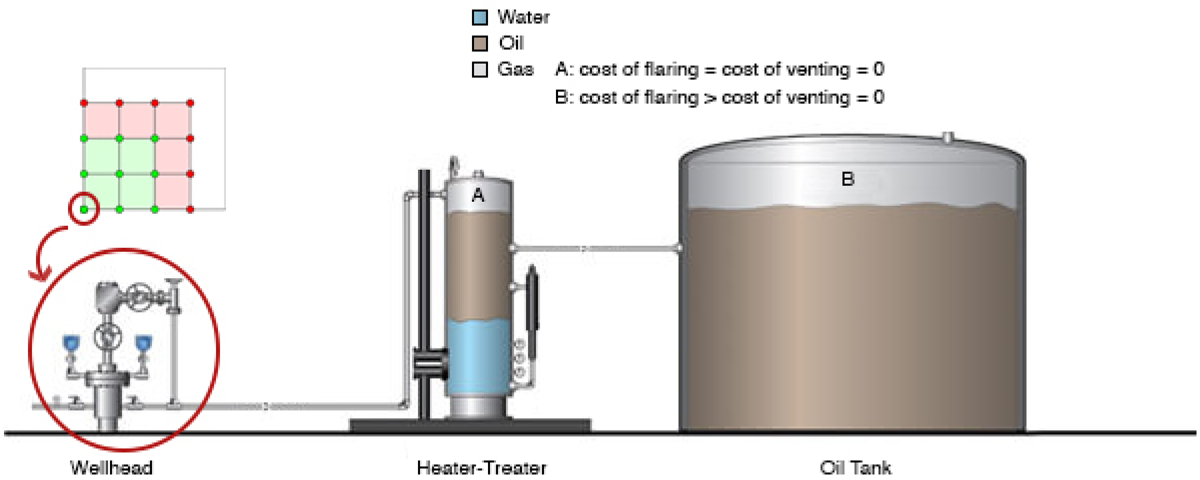

Figure 1: Flaring Marginal Costs according to the pressure of the co-extracted Gas.

full history of prices, own and other firm's costs, own and other firm's decisions and outcomes, tax rates and regulation in place. Moreover, each firm knows the future realizations of all fields' marginal costs and all other time-variant exogenous variables, and possess perfect foresight regarding all endogenous variables, such as prices and other firms' production choices. Because of this assumption, we can omit expectations in the firm's problem and treat it as an optimization in a deterministic environment. Note that under these assumptions, a the solution of the problem of a firm choosing all it production plans in period 1 for all periods $t = 1, 2...$ is identical to that of a firm choosing the production plan for each period $t = 1, 2, ...$ at the beginning of such period.

All the information –with one piece of information being a notable exception– is assumed to be public and contractible for the government, whose information set is denoted by Ω_t^{PUB}. For instance, oil and gas sales are well-documented in the firm's balance sheets. Moreover, they are relatively easy to measure and verify for the regulatory authority, implying that substantial misreporting for these variables is very unlikely. Regarding flaring, the regulator may not rely solely on self-reported quantities, which could be distorted using under/over billing tricks. In particular, the regulator can assess the volumes of disposed gas using quantity-monitoring technologies. They can supervise flaring activities using satellite, airplane and in-person tracking. All these methods tend to be accurate. Given these considerations, we assume thatOil_t^i, Gas_t^i, and Flare_t^i are fully observable and contractible by all agents. However, some information is not publicly available. In particular, gas venting is deemed hard to detect, measure and attribute to a specific emitter (Allen, Chen, & Dunn, 2021). In principle, the regulator can supervise venting using technologies similar to the ones adopted to monitor flaring. However, in the case of venting bottom-up as well as top- down measures tend to be inaccurate. A general lack in the understanding of the spatio-temporal heterogeneity of methane emissions renders these measures prone to commit measurement errors. Furthermore, most legislation regulate intentional venting, which is not easy to separate form unintentional venting. In other words, the regulator wants to supervise intentional venting but it is incapable to separate this quantity from unintentional venting and/or measurement difficulties. Moreover, even if some amount of venting is detected, it may be challenging for the regulatory authority to establish in legally binding terms that such venting occurred as the result of a voluntary action of the firm's personnel which was not justified by health and safety reasons. Thus, we assume that IVent_t^{iks} and UVent_t^{iks} are observable with probability equal to 1 by firm i only. The public only receives an imperfect contractible public signal $ivent_t^{iks} \in \{0,1\}$, where $ivent_t^{iks} = 1$ only if $\text{IVent}_t^{iks} > 0$, such that $Pr_t^{iks}\left(ivent_t^{iks} = 1 \mid \Omega_t^{PUB}, \text{IVent}_t^{iks} > 0\right) \in (0,1)$ and $Pr_t^{iks}\left(ivent_t^{iks} = 1 \mid \Omega_t^{PUB}, \text{IVent}_t^{iks} = 0\right) = 0$. This implies that even if TotGas_t^{iks}, Gas_t^{iks}, Flare_t^{iks}, ReInj_t^{iks}, ReInS_t^{iks} are public information, such that it is possible to obtain a reliable measure of total venting Vent_t^{i}, the intentional part IVent_t^{i} is only partially observable and contractible.

Flaring & Venting Regulation. firms in their production decisions are not solely shaped by technology. One must also account for the legal and fiscal restrictions that both flaring and venting face in most countries.

In the United States, flaring and venting are regulated at the federal and at the local level. Federal laws focus on offshore production, local requirements on onshore. Offshore fields must require flaring and venting permits to dispose of the extracted Gas. The permits are released by the Interior Bureau of Safety and Environmental Enforcement if at least one of the following criterion is met: 1) the national interest requires it (e.g. when a major hurricane could cause infrastructure damage), 2) the production from a completed well would likely be permanently lost, or 3) the short-term flaring or venting would likely yield a smaller volume of lost Gas than if the facility were to shut in and restart later (with flaring and venting necessary to restart the facility). Similarly, onshore fields must require flaring and venting permits. The latter are released by State Environmental agencies. Several states release unlimited flaring permits, while regulating/forbidding intentional venting (Alabama, Arizona, Colorado, Florida, Illinois, Indiana, Kentucky, Louisiana, Mississippi, Missouri, Nebraska, Nevada, Ohio, Oklahoma, Pennsylvania, South Dakota, Tennessee, West Virginia). Other states release a limited number of flaring permits, those capping the quantity of flaring, while regulating/-forbidding intentional venting (Alaska, California, Idaho, Kansas, Michigan, Montana, New Mexico, North Dakota, New York, Texas, Utah, Virginia, Wyoming). Finally, one state (Arkansas) taxes flaring, while regulating/forbidding intentional venting. Table 1 provides the legal sources of the current flaring and venting regulation in the United States.

Table 1: Sources of Legal Regulation

Region	Source
State Regulation	
Alabama	State Oil & Gas Board of Alabama Administrative Code, Rules 400-1 - 400-7. Alabama Statute, Title 09, Chapter 17, Section 9-17-11.
Alaska	Alaska Oil & Gas Conservation Act, Section 31.05.095
Arizona	Arizona Administrative Code Title 12; Chapter 7, Section R12-7-138
Arkansas	Arkansas Code, Title 15, Section 15-72-105 and Section 15-72-208
California	California Code of Regulations, Title 17, Division 3, Chapter 1, Subchapter 10 Climate Change, Article 4, Sub-article 13
Colorado	Colorado Code of Regulations, Rule 912, Page 183
Florida	Florida Statutes and Rules, Chapter 377, 62C-25 - 30
Idaho	Idaho Administrative Rule 20.07.02, Sections 413 and 430
Illinois	Illinois Oil and Gas Act (225 ILCS 725). Illinois Hydraulic Fracturing Regulatory Act, Section 245.900 and 245.910
Indiana	Indiana Code, Title 14, Article 37, Chapter 11, Subsection 14-37-11-1. Indiana Administrative Code, Title 312, Article 29, Subsection 312 IAC 29-3-3.
Kansas	Kansas Statute 55-102b. Kansas Administrative Regulations, Sections 82-3-208, 82-3-209, 82-3-314
Kentucky	Kentucky Revised Statutes Chapter 353, Section 353.160, 353.520
Louisiana	Louisiana Administrative Code, Title 33, Part III. Title 43, Part XIX
Michigan	Natural Resources and Environmental Protection Act, 1994, Public Act 451, Part 615
Mississippi	Mississippi Statewide Rules and Regulations, Rule 62
Missouri	Revised Statues of Missouri, Chapter 259.060
Montana	Administrative Rules of Montana (ARM) 17.8.1603, 17.8.1711, 36.22.1220
Nebraska	Revised Statutes of Nebraska, Chapter 57, Section 902 and 903
Nevada	Nevada Revised Statutes, Section 522.010 and 522.039. Nevada Administrative Codes, Section 522.3
New Mexico	New Mexico Administrative Code, Chapter 15, Title 19, Subsection 18
New York	New York Codes, Rules and Regulations, Title 6, Parts 550-559, Chapter V, Subchapter B
North Dakota	North Dakota Administrative Code 33.1-15-07-02, 33.1-15-03-03.1, 33.1-15-20. North Dakota Industrial Commission Order No. 24665
Ohio	Ohio Revised Code Title 15, Chapter 1509.20. Ohio Administrative Code, Chapter 1501:9-9
Oklahoma	Oklahoma Register, Chapter 10, Subsection 3-15. Oklahoma Statues 2-5-102, et seq.. Oklahoma Administrative Code, Title 252, Chapter 100
Pennsylvania	No specific Regulation
South Dakota	Administrative Rules of South Dakota, Article 74:12, Section 74:12:05:04
Tennessee	Tennessee Code, Chapter 1, Title 60, Section 60-1-101 and Section 60-1-102
Texas	Texas Air Quality State Implementation Plan, Regulation 30 TAC 115.720-115.729. Texas Administrative Code, Title 16, Part 1, Chapter 3, Rule 3.32
Utah	Utah Administrative Code, Rule R649-3
Virginia	Code of Virginia, Title 45.1, Chapter 22.1. Virginia Administrative Code, Title 4, Agency 25, Chapter 150
West Virginia	West Virginia Code, Chapter 22, Articles 5 and 18. West Virginia Legislative Rules, 45CSR, Series 6 and 13
Wyoming	
Federal Regulation	
US Federal Offshore	At the discretion of the Department of the Interior's Bureau of Safety and Environmental Enforcement (BSEE)

In order to capture these legal and fiscal restrictions, we assume that each oil&gas field also faces expected regulatory costs $RegCost_t^{iks}$, which depend upon the fines, taxes and emission permits the firm must pay to the government (if any). The flaring regulation is in the form of a threshold $\overline{\text{Flare}}_t^{iks}$[8], such that a violation occurs whenever the amount of gas flared exceed the threshold.[9] In line with the current regulation of all US states we assume that intentional venting is illegal, such that the firm faces a fine whenever $\text{IVent}_t^{iks} > 0$ is detected.

Let vf_t^{iks} and FF_t^{iks} denote the fines for violation of the venting and flaring regulation, respectively. Because flaring is assumed to be fully observable and contractible by the regulatory authority, any violation of the flaring regulation results in a fine with probability equal to 1. Conversely, because intentional venting is not perfectly observable and/or contractible, venting activity is detected with probability equal to the accuracy of the venting detection technology available to the regulator, denoted by $VDT_t^s \geq 0$. If detected, venting activity constitutes a violation of the venting regulation and results in a fine only when a signal $ivent_t^{iks} = 1$ is observed, which occurs with probability $Pr_t^{iks}\left(ivent_t^{iks} = 1 \mid \Omega_t^{ks}\right)$. For simplicity, we model the expected regulatory costs of venting as an increasing and strictly convex real analytic function of IVent_t^{iks} and Flare_t^{iks}; i.e., a, where the inclusion of Flare_t^{iks} in the arguments of PF_t^{iks} captures any possible substitutability between intentional venting and flaring for the oil&gas firm, whereas the second term in the square brackets PU_t^{iks} models the possibility that the level of maintenance affects the probability of detection of intentional venting, for instance because it may be hard for the regulatory authority to provide sufficient evidence that a given amount of venting performed by a poorly maintained field is the result of a deliberate action of the field management rather than an unintentional leakage due to damaged pipelines. Given these assumptions the formula for regulatory costs write:

$$\begin{aligned} RegCost_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}, \text{M}_t^{iks}\right) = \quad & VF_t^{iks}\ \left[PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right) + PU_t^{iks}\left(\text{M}_t^{iks}\right)\right] \\ & + FF_t^{iks} \times \mathbf{1}\left[\text{Flare}_t^{iks} > \overline{\text{Flare}}_t^{iks}\right] \end{aligned}, \tag{13}$$

where $VF_t^{iks} = vf_t^{iks} VDT_t^s$ denotes the expected value of the fine conditional on $ivent_t^{iks} = 1$. While these assumptions deliver an admittedly stylized picture of

[8]The *Zero Routine Flaring by 2030 Initiative* aims to set $\overline{\text{Flare}}_t^{iks} \cong 0$. We allow for a more general regulatory scenario with$\overline{\text{Flare}}_t^{iks} \geq 0$.

[9]Current United States regulation expresses $\overline{\text{Flare}}_t^{iks}$ as a ratio between the volume of Gas flared and the total volume of Gas extracted, $\overline{\text{Flare}}_t^{iks} = \text{Flare}_t^{iks}/\text{TotGas}_t^{iks}$. Other legislation (e.g. Canada) express $\overline{\text{Flare}}_t^{iks}$ as a ratio between the volume of gas flared and the volume of oil extracted, $\overline{\text{Flare}}_t^{iks} = \text{Flare}_t^{iks}/\text{Oil}_t^{iks}$.

the highly heterogeneous regulatory framework concerning flaring and venting in US States, we believe that it represents a useful simplification that incorporates all the key features for the purpose of this analysis.

Market Structure. We assume that the markets for crude oil and unrefined natural gas are imperfectly competitive. Specifically, oil firms compete on the crude market in a global Cournot-style oligopoly and are price-taker on the gas market. There is a unique global average crude price P_t^{Oil}, but individual fields face different prices $P_t^{iks,\text{Oil}} = P_t^{\text{Oil}} + \sigma^{iks}$, where σ^{iks} captures the time-invariant quality of crude from field i. Conversely, we assume that the quality of natural gas is identical across fields, and that gas firms compete in an oligopoly in quantities on the unrefined natural gas market, whose demand side consists of a number of midstream firms (gas-processing facilities), but we allow for gas produced in different countries to be imperfect substitutes. Thus, we allow for different gas prices $P_t^{s,\text{Gas}}$ across different countries. This assumption captures the geographically segmented nature of the natural gas market at international level and accommodates for the possibility of transport costs and bottlenecks affecting specific local segments of the market. Lastly, there is a market for each consumption good. Both midstream firms and consumers are assumed to be price-taker on all markets they participate in. The price of general consumption (numéraire) is normalized to 1, whereas the prices of oil&gas goods are listed in the vector $\mathbf{p}_t^s$ and they are allowed to differ across country. The collection of all prices in period t is $\mathbf{P}_t = \left\{P_t^{\text{Oil}}, \left\{P_t^{s,\text{Gas}}, \mathbf{p}_t^s\right\}_{s=1}^{S}\right\}$, whereas $\mathbf{P} = \{\mathbf{P}_t\}_{t=1}^{\infty}$ denotes the collection of all prices in all periods. All prices are assumed to be endogenously determined in a competitive equilibrium in each period t and all agents possess perfect foresight regarding future equilibrium prices. In particular, the price of crude oil must clear the global oil market, whereas each country possesses its own market for unrefined natural gas, resulting in a country-specific natural gas price that clears such market.

Fiscal Framework. Oil&gas firms in the US face a complex system of taxation, fees, and royalties collected at both Federal, State, and local level. For the sake of simplicity, we reduce this complex and heterogeneous framework to a stylized system of linear taxes. Moreover, the assumption on the information structure of the model implies that taxation can be imposed only on quantities that are observable and contractible by the government. As a result, the government can tax flaring, but taxation of intentional venting is not feasible. In principle, the regulator could impose a tax on all natural gas released into the atmosphere by the firm, irrespective of its intentional or unintentional nature. However, this regulatory approach could lead to unintended and undesirable safety consequences. By making it expensive for the firm to implement safety-related pressure relief measures, it could inadvertently raise the risk of explosions. Moreover, it would represent a monetary incentive to emission misreporting, as illustrated in section 5.2. Therefore, we propose an incentive scheme that avoids penalizing unintentional leaks. In detail, we assume a tax system on oil&gas upstream firms featuring: (i) a linear tax rate T_t^s on corporate income, defined as the firm's revenue minus total costs excluding the payment of fines; (ii) a vector of (possibly field-specific) specific linear taxes $\left(\tau_t^{iks,\text{Oil}}, \tau_t^{iks,\text{Gas}}\right)$ on oil and gas sales, which also includes any royalties on hydrocarbon extraction to be paid to the government; (iii) a (possibly field-specific) specific linear tax on flaring, which also includes the unitary cost of any flaring permits the firm may be required to purchase, denoted by $\tau_t^{iks,\text{Flare}}$ (it may be equal to zero). Lastly, firm k may be partially allowed to deduct from the taxable income generated by field i other production and/or investment cost that do not enter directly the firm's balance sheets, such as the value of extracted gas which is re-injected in the field or used in-situ for electricity production. Specifically, the firm's deductible amount is given by function $Deduct_t^{ikt}$, which has formula:

$$Deduct_t^{ikt}\left(\text{ReInj}_t^{iks}, \text{ReInS}_t^{iks}\right) = \delta_{0t}^{iks} + \delta_{1t}^{iks}\left(\text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\right) \ , \tag{14}$$

where δ_{0t}^{iks} denotes lump-sump deductions aiming to captures other off-balance firm-specific costs and δ_{1t}^{iks} denotes the deduction rate for gas re-injected or reused in-situ. These tax provisions are complemented by a system of taxes on other agents (midstream firms and consumers), which are described in detail in the corresponding sections of this Appendix. For a complete definition of a *tax scheme*, see Section 1.4.

Identification of Types of Upstream Fields for Tax Purposes. For tax purposes, oil&gas fields are classified in three mutually-exclusive categories (types): *oil fields*, *gas-only fields*, and *mixed oil&gas fields*. Let $MC_t^{iks,\text{Gas}} = \frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Gas}_t^{iks}} \Big/ \frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Z}_t^{iks}}$ and $MC_t^{iks,\text{Oil}} = \frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Oil}_t^{iks}} \Big/ \frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Z}_t^{iks}}$. We assume that $MC_t^{iks,\text{Gas}}$ is constant in all endogenous variables for all oil fields and impose specific restrictions that depends upon the type field considered, as described in the remainer of this paragraph. Firstly, we assume that all oil fields in each period t satisfy the following condition:

$$\begin{aligned} -\left[P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}}\right]\ \left(1+\vartheta^{iks}\right)^{-1} GOR^{iks} \\ > \left[P_t^{\text{Oil}} + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}(0)\right] \geq 0 \end{aligned} \tag{15}$$

in all periods $t = 1, 2, ...$ given the tax rates $\tau_t^{iks,\text{Gas}}, \tau_t^{iks,\text{Oil}}$ and equilibrium prices $P_t^{s,\text{Gas}}$, P_t^{Oil} that prevail under the existing tax scheme. Note that, because the tax reform we propose is such that for oil fields the change in $\tau_t^{iks,\text{Oil}}$, denoted by $\varDelta\tau_t^{iks,\text{Oil}}$, satisfies $\varDelta\tau_t^{iks,\text{Oil}} = -\varDelta\tau_t^{s,\text{Gas}} \left(1+\vartheta^{iks}\right)^{-1} GOR^{iks}$, the condition above is unaffected by the introduction of the tax reform at constant prices, meaning that the classification of a given field in the oil category does not change with the introduction of the tax reform as long as the reform does not affect equilibrium prices. Intuitively, condition (15) states that oil fields are those fields for which gas production is not profitable. Thus, natural gas production (if any) is a by-product of oil production for those fields. Secondly, we assume that all oil&gas fields satisfy the following condition:

$$\begin{aligned} P_t^{\text{Oil}} + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks}\right)\left(1+\vartheta^{iks}\right) \leq \\ P_t^{s,\text{Gas}}\left(1-\varsigma_t^{\text{Gas}}\right) - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}}\left(GOR^{iks}\text{Oil}_t^{iks}\right) \end{aligned} \tag{16}$$

for all $\text{Oil}_t^{iks} \geq 0$ in all periods $t = 1, 2, ...$ given the the tax rates $\tau_t^{iks,\text{Gas}}, \tau_t^{iks,\text{Oil}}$ and equilibrium prices $P_t^{s,\text{Gas}}$, P_t^{Oil} that prevail under the existing tax scheme. Note that if a field satisfies the condition above under the existing tax scheme, then it also satisfies the condition under the tax reform proposed in this paper at constant prices, meaning that the classification of a given field in the "other oil&gas" category does not change with the introduction of the tax reform as long as the reform does not affect equilibrium prices. Intuitively, this assumption (16) captures the fact that gas production is profitable for this type of fields. Lastly, gas-only fields are characterized by $MC_t^{iks,\text{Oil}}(0) = +\infty$ in all periods $t = 1, 2, ...$, implying that they never produce any positive quantity of crude. Given these assumptions, it is possible to show (see proof to Proposition 3 below) that a field is uniquely identified as an oil field in period t if its production choices satisfy $\text{Oil}_t^{iks} > 0$ and $\text{TotGas}_t^{iks} \leq \text{Oil}_t^{iks}\ GOR^{iks}$, as an other oil&gas field if $\text{Oil}_t^{iks} > 0$ and $\text{TotGas}_t^{iks} > \text{Oil}_t^{iks}\ GOR^{iks}$ and as a gas-only field if $\text{Oil}_t^{iks} = 0$. Moreover, we can show that the identification of a field's category is not affected by the introduction of the tax reform.

Technical Assumptions. Each field is endowed with an initial condition $\text{M}_0^{iks}, \text{K}_0^{iks}, \text{L}_{-1}^{iks}$. We impose a lower bound on ReInS_t^{iks} such that $\text{ReInS}_t^{iks} \geq RI_t^{iks}$ where for technical reasons we allow for RI_t^{iks} to be negative and arbitrarily large in magnitude. This restriction is mostly innocuous because we impose conditions on $TF_{4t}^{iks}\left(\text{PInS}_t^{iks}; \text{ReInS}_t^{iks}\right)$ that ensures that $\text{ReInS}_t^{iks} \geq 0$ at all optimal choices. Specifically, we impose that all oil fields and other oil&gas fields satisfy $\frac{\partial TF_t^{iks}(\cdot)}{\partial \text{ReInS}_t^{iks}} \Big/ \frac{\partial TF_t^{iks}(\cdot)}{\partial \text{Z}_t^{iks}} \geq \max\left\{P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}}, 0\right\}$ for all feasible $\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{PInS}_t^{iks}, \text{M}_t^{iks}, \text{Z}_t^{iks}$ at any value $\text{ReInS}_t^{iks} < 0$. For gas fields, we allow for occasional negative values for ReInS_t^{iks}, although negative values do not generally occur at the optimal choice.

This assumption is rather strong, but it is used solely for two specific purposes: (1) in section 1.1.3 to ensure that the firm's optimization problem satisfies the Slater's condition; and (2) in the proof to Proposition 3 Part (vi) to show that the tax scheme proposed increases consumer's welfare.

1.1.2 Oil&gas Firm's Problem

Each oil&gas field i owned by firm k generates revenues Rev_t^{iks} from selling net outputs $\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{PInS}_t^{iks}, \text{Z}_t^{iks}\right)$ at net prices $\mathbf{np}_t^{iks} = \left(P_t^{iks,\text{Oil}} - \tau_t^{iks,\text{Oil}}, P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}}, -\tau_t^{iks,\text{Flare}}, -PP_t^{iks,\text{Gas}}, 1\right)$ expressed in $\$/BOE$ for the first four arguments. The value of the last argument equals one because it corresponds to the normalized price of the numéraire. Under the assumptions stated in the previous paragraphs, the gross revenue from field i has formula:

$$\begin{aligned} Rev_t^{iks}(\cdot) = \; & \left(P_t^{iks,\text{Oil}} - \tau_t^{iks,\text{Oil}}\right)\text{Oil}_t^{iks} + \left(P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}}\right)\text{Gas}_t^{iks} \\ & -\tau_t^{iks,\text{Flare}}\text{Flare}_t^{iks} - PP_t^{iks,\text{Gas}}\text{PInS}_t^{iks} - \text{IM}_t^{iks} + \text{Z}_t^{iks} \end{aligned} \quad . \tag{17}$$

Such that the intra-temporal profits $\Pi_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInj}_t^{iks}, \text{PInj}_t^{iks}, \text{PInS}_t^{iks}, \text{Z}_t^{iks}, \text{ID}_t^{iks}, \text{M}_t^{iks}, \text{IM}_t^{iks}, \text{OInj}_t^{iks}, \text{IVent}_t^{iks}\right)$ generated by field i have formula:

$$\begin{aligned} \Pi_t^{iks}(\cdot) = \; & \left(1 - T_t^{ks}\right)\Big[Rev_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{PInj}_t^{iks}, \text{PInS}_t^{iks}, \text{Z}_t^{iks}\right) \\ & -InvCost_t^{iks}\left(\text{ID}_t^{iks}, \text{IM}_t^{iks}, \text{ReInj}_t^{iks}, \text{PInj}_t^{iks}, \text{OInj}_t^{iks}; PP_t^{iks,\text{Gas}}\right) \\ & +Deduct_t^{ikt}\left(\text{ReInj}_t^{iks}, \text{ReInS}_t^{iks}\right)\Big] - RegCost_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right) \end{aligned} \tag{18}$$

Therefore, after substituting (11) and (8) and the formula for $\text{TotInj}_{t-1}^{iks}$ into the inequalities (3), (6), and (7) and using such inequalities plus the inequality in (1) as constraints to the firm's choice, we can construct the firm k's profit maximization problem in period 1, which writes:

$$\max_{\substack{\{\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{IVent}_t^{iks}, \\ \text{ID}_t^{iks}, \text{ReInj}_t^{iks}, \text{OInj}_t^{iks} \text{ReInS}_t^{iks}, \text{PInj}_t^{iks}, \\ \text{PInS}_t^{iks}, \text{M}_{t+1}^{iks}, \text{IM}_t^{iks}, \text{L}_t^{iks}, \text{Z}_t^{iks}, \text{K}_{t+1}^{iks}\}_{i=1}^{I_k} \in X_u^s}} \sum_{t=1}^{\infty} \sum_{k=1}^{K^s} \sum_{i=1}^{I^{ks}} \beta^{t-1} \, \Pi_t^{iks}(\cdot)$$

$$s.t. \left\{ \begin{array}{c} TF_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInS}_t^{iks}, \text{PInS}_t^{iks}, \text{M}_t^{iks}, \text{Z}_t^{iks}\right) \leq 0 \\ \\ GOR^{iks}\text{Oil}_t^{iks} - \left(1-\vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} \right. \\ \left. -Maint_t^{iks}\left(\text{M}_t^{iks}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks} - \text{K}_t^{iks}\right] \leq 0 \\ \\ \text{Oil}_t^{iks} + \left(1-\vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} \right. \\ \left. -Maint_t^{iks}\left(\text{M}_t^{iks}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks} - \text{K}_t^{iks}\right] \leq 0 \\ \\ \text{K}_{t+1}^{iks} - D_t^{iks}\left(\text{ID}_t^{iks}, \text{L}_{t-1}^{iks}\right) - B_t^{iks}\left(\text{ReInj}_t^{iks} + \text{PInj}_t^{iks} + \text{OInj}_t^{iks}\right) \\ -\text{K}_t^{iks} + \zeta\left\{\text{Oil}_t^{iks} + \left(1-\vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} + \right.\right. \\ \left.\left. -Maint_t^{iks}\left(\text{M}_t^{iks}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\right]\right\} \leq 0 \\ \\ \text{Oil}_t^{iks} \geq 0, \text{Flare}_t^{iks} - \text{NRF}_t^{iks} \geq 0, \text{IVent}_t^{iks} \geq 0, \text{ReInj}_t^{iks} \geq 0, \\ \text{ReInS}_t^{iks} - RI_t^{iks} \geq 0, \text{M}_{t+1}^{iks} \geq 0, \text{L}_t^{iks} \geq 0, \text{K}_{t+1}^{iks} \geq 0 \\ \\ \text{L}_t^{iks} - \text{L}_{t-1}^{iks} - \text{ID}_t^{iks} = 0 \\ \\ \text{M}_{t+1}^{iks} - \text{M}_t^{iks}\left(1-\rho^{iks}\right) - \text{IM}_t^{iks} = 0 \end{array} \right\} \begin{array}{l} t = 1, 2, ..., \\ i = 1, ..., I_k \end{array} \tag{19}$$

where $X_u^s = \left\{\left\{X_{ut}^{iks}\right\}_{i=1}^{I^{ks}}\right\}_{t=1}^{\infty}$ with $X_{ut}^{iks} = (-\infty, +\infty)$[15]. That is, the firm k solves a constrained maximization problem with 14 inequality constraints and 2 linear equality constraints. Note that we are not constraining Gas_t^{iks}, ID_t^{iks} and IM_t^{iks} to be positive. This captures the fact that, in principle, oil field may purchase natural gas for injection or in-situ use. However, the extent of which Gas_t^{iks} can be negative is limited by the other constraints, in particular $GOR^{iks}\text{Oil}_t^{iks} - \text{TotGas}_t^{iks} \leq 0$ and $\text{Oil}_t^{iks} \geq 0$. Similarly, negative investment is allowed in our framework, but the extent of negative investment in maintenance and/or discoveries is bounded by the constraints on the values of L_t^{iks} and M_{t+1}^{iks}. Given these assumptions, the

Lagrangian of the firm's problem writes:

$$
\begin{aligned}
\mathcal{L}_u^{ks} = \sum_{t=1}^{\infty}\sum_{i=1}^{I^{ks}} \Big\{ & \big[\beta^{t-1}\Pi_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInj}_t^{iks},\right. \\
& \qquad \left.\text{PInj}_t^{iks}, \text{PInS}_t^{iks}, \text{Z}_t^{iks}, \text{ID}_t^{iks}, \text{M}_t^{iks}, \text{IM}_t^{iks}, \text{OInj}_t^{iks}, \text{IVent}_t^{iks}\right)\big] + \\
& - \phi_{1t}^{iks}\left[TF_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInS}_t^{iks}, \text{PInS}_t^{iks}, \text{M}_t^{iks}, \text{Z}_t^{iks}\right)\right] + \\
& - \phi_{2t}^{iks}\Big\{GOR^{iks}\text{Oil}_t^{iks} - \left(1-\vartheta^{iks}\right)^{-1}\big[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} + \\
& \qquad -Maint_t^{iks}\left(\text{M}_t^{i}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\big]\Big\} \\
& - \phi_{3t}^{iks}\Big\{\text{Oil}_t^{iks} + \left(1-\vartheta^{iks}\right)^{-1}\big[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} + \\
& \qquad -Maint_t^{iks}\left(\text{M}_t^{i}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\big] - \text{K}_t^{iks}\Big\} + \\
& - \phi_{4t}^{i}\Big\{\text{K}_{t+1}^{iks} - \text{K}_t^{iks} - D_t^{iks}\left(\text{ID}_t^{iks}, \text{L}_{t-1}^{iks}\right) - B_t^{iks}\left(\text{ReInj}_t^{iks} + \text{PInj}_t^{iks} + \text{OInj}_t^{iks}\right) + \\
& \qquad + \zeta\Big[\text{Oil}_t^{iks} + \left(1-\vartheta^{iks}\right)^{-1}\big[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks} + \\
& \qquad -Maint_t^{iks}\left(\text{M}_t^{i}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\big]\Big\} \\
& + \phi_{5t}^{iks}\text{Oil}_t^{iks} + \phi_{6t}^{iks}\left(\text{Flare}_t^{iks} - \text{NRF}_t^{iks}\right) + \phi_{7t}^{iks}\text{IVent}_t^{i} + \phi_{8t}^{iks}\text{ReInj}_t^{iks} \\
& + \phi_{9t}^{iks}\left(\text{ReInS}_t^{iks} - RI_t^{iks}\right) + \phi_{10t}^{iks}\text{M}_{t+1}^{iks} + \phi_{11t}^{iks}\text{L}_t^{i} + \phi_{12t}^{iks}\text{K}_{t+1}^{iks} \\
& - \lambda_{1t}^{iks}\left[\text{L}_t^{iks} - \text{L}_{t-1}^{iks} - \text{ID}_t^{iks}\right] - \lambda_{2t}^{iks}\left[\text{M}_{t+1}^{iks} - \text{M}_t^{iks}\left(1-\rho^{iks}\right) - \text{IM}_t^{iks}\right]\Big\}
\end{aligned}
$$

For ease of notation, we define the vector

$$
\mathbf{x}_t^{iks} = \left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInj}_t^{iks}, \text{PInj}_t^{iks}, \text{PInS}_t^{iks}, \text{Z}_t^{iks}, \text{ID}_t^{iks}, \text{M}_t^{iks}, \text{IM}_t^{iks}, \text{OInj}_t^{iks}, \text{IVent}_t^{iks}\right) \tag{20}
$$

with $\mathbf{x}_t^{iks} \in X_{ut}^{iks}$ and the corresponding profit function $\mathit{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks};\mathbf{T}\right)$, which we use in the proofs in section 1.1.3, where $\mathbf{T}$ denotes the tax scheme as defined in section 1.4, and the corresponding collection $\mathbf{x}^{iks} = \left\{\mathbf{x}_t^{iks}\right\}_{t=1}^{\infty}$. Given this newly defined notation, we restate the firm's problem in the following parsimonious form.

$$
\begin{aligned}
\max_{\mathbf{x}_t^{iks} \in X_u^s} \quad & \textstyle\sum_{t=1}^{\infty}\sum_{k=1}^{K^s}\sum_{i=1}^{I^{ks}} \beta^{t-1}\ \mathit{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks};\mathbf{T}\right) \\
s.t. \quad & \left\{\begin{array}{c} \left\{g_{w,ut}^{iks}\left(\mathbf{x}_t^{iks}\right) \leq 0\right\}_{w=1}^{14} \\ \\ \left\{e_{z,ut}^{iks}\left(\mathbf{x}_t^{iks}\right) \leq 0\right\}_{z=1}^{2} \end{array}\right\} \quad \begin{array}{l} t = 1, 2, ..., \\ i = 1, ..., I_k \end{array}
\end{aligned} \tag{21}
$$

where $g_{w,ut}^{iks}\left(\mathbf{x}_t^{iks}\right)$ corresponds to the wth inequality constraint and $e_{z,ut}^{iks}\left(\mathbf{x}_t^{iks}\right)$ to the zth equality constraint of the original firm's problem in (19).

1.1.3 Optimality Conditions

First, we establish that the firm's problem in (21) is a convex maximization problem. The objective function in (21) is a concave function given the assumptions on its functional form. The set X_u^s is convex. Moreover, each inequality constraint $g_{w,ut}^{iks}(x) \leq 0$ is such that $g_{w,ut}^{iks}\left(\mathbf{x}^{iks}\right)$ is a weakly convex function, and all equality constraints $e_{z,ut}^{iks}\left(\mathbf{x}^{iks}\right) = 0$ are linear, implying in turn that each set $CS_{ut}^{iks} \equiv \left\{\mathbf{x}^{iks} \in X_u^s \mid \mathbf{g}_{ut}^{iks}\left(\mathbf{x}^{iks}\right) \leq \mathbf{0}, \mathbf{e}_{ut}^{iks}\left(\mathbf{x}^{iks}\right) = \mathbf{0}\right\}$ for $i = 1, 2, ..., I^k$ and $t = 1, 2, ...$ is a convex set. Lastly, the set $\left[\bigcap_{i=1}^{I^k}\bigcap_{t=1}^{\infty} CS_{ut}^{iks}\right] \cap X_u^s$ is the intersection of convex sets, which is a convex set. Thus, the firm's problem is convex. Second, note that because the domain of the variables Slater's condition is satisfied. To prove this result we must prove that there exists a feasible choice vector that all the inequality constraints are satisfied with strict inequality. Consider the following choice vector. In each period t and for each field i, set $\mathrm{Oil}_t^{iks}, \mathrm{Gas}_t^{iks}, \mathrm{Flare}_t^{iks}, \mathrm{IVent}_t^{iks}, \mathrm{ReInj}_t^{iks}, \mathrm{M}_{t+1}^{iks}$ equal to an arbitrarily small strictly positive number ϵ_t^{iks}, $\mathrm{L}_t^{iks} = \mathrm{L}_{t-1}^{iks} > 0$, $\mathrm{K}_{t+1}^{iks} = \mathrm{K}_t^{iks} > 0$ $\mathrm{ReInS}_t^{iks} \leq 0$ and set Oil_t^{iks} equal to a strictly positive number that satisfies $\mathrm{Oil}_t^{iks} = \epsilon 2_t^{iks} < \frac{5\epsilon_t^{iks}}{\overline{G}}$, where $\overline{G}$ is the upper bound of GOR^{iks}. This ensures the proposed element satisfies seven constraints in (21), which correspond to constraints 2-7-8-9-10-11-12 in (19), with strict inequality. Then select $\mathrm{PInj}_t^{iks} = \mathrm{OInj}_t^{iks} = \epsilon_t^{iks}$, and $\mathrm{ReInS}_t^{iks} = -\max\left\{\epsilon 2_t^{iks} + 4\epsilon_t^{iks} + UVent_t^i\left(\epsilon 2_t^{iks}, \epsilon_t^{iks}\right), -D_t^i\left(\epsilon_t^{iks}, \mathrm{L}_0^{iks}\right)\right.$ $\left. -B_t^{iks}\left(\epsilon_t^{iks}\right) + \zeta\left[\epsilon 2_t^{iks} + 4\epsilon_t^{iks} + UVent_t^{iks}\left(\epsilon 2_t^{iks}, \epsilon_t^{iks}\right)\right]\right\} - \epsilon_t^{iks}$ (i.e., just sufficiently large in magnitude to ensure that constraints 3 and 4 are satisfied with strict inequality). Lastly, set Z_t^{iks} sufficiently small such that $TF_t^{iks}(\mathrm{Oil}_t^{iks}, \epsilon_t^{iks}, \epsilon_t^{iks}, \mathrm{ReInS}_t^{iks}, 0, \epsilon_t^{iks}, \mathrm{Z}_t^{iks}) < 0$, for which, given that TF_t^{iks} is weakly decreasing in ReInS_t^{iks} for negative values of ReInS_t^{iks} (see the technical assumptions section), it is sufficient that $TF_t^{iks}(\epsilon_t^{iks}, \epsilon_t^{iks}, \epsilon_t^{iks}, 0, 0, \epsilon_t^{iks}, \mathrm{Z}_t^{iks}) < 0$. Thus, because ϵ_t^{iks} is arbitrarily small and TF_t^{iks} has finite first derivatives and strictly positive first derivative w.r.t. Z_t^{iks}, there exists $\mathrm{Z}_t^{iks} < 0$ such that all the inequalities are satisfied with strict inequality. Thus, the Slater's condition is satisfied and the Karush–Kuhn–Tucker theorem implies the global maximizer of the constrained optimization problem (if it exists) has to satisfy the KKT conditions (First-order necessary conditions - FOCs). Let $\varsigma_t^{\mathrm{Oil}}$ ($\varsigma_t^{s,\mathrm{Gas}}$) denote the elasticity of oil (natural gas) demand, and $MS_t^{ks,\mathrm{Oil}}$ ($MS_t^{ks,\mathrm{Gas}}$) be the share of the oil (natural gas) market that is controlled by firm k. We derive the FOCs for a global maximum, which for each $t = 1, 2, ...$ and each $i = 1, 2, .., I^k$ write:

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{Oil}_t^{iks}} = \left[P_t^{\text{Oil}}\left(1-\varsigma_t^{\text{Oil}}MS_t^{ks,\text{Oil}}\right)+\sigma_1^{iks}-\tau_t^{iks,\text{Oil}}\right](1-T_t^s)-\phi_{1t}^{iks}\frac{\partial TF_{1t}^{iks}(.)}{\partial \text{Oil}_t^{iks}}$$

$$-\phi_{2t}^{iks}GOR^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta+\phi_{5t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{Gas}_t^{iks}} = \left[P_t^{s,\text{Gas}}\left(1-\varsigma_t^{s,\text{Gas}}MS_t^{ks,\text{Gas}}\mathbf{1}\left[\text{Oil}_t^{isk}=0\right]\right)-\tau_t^{isk,\text{Gas}}\right](1-T_t^s)$$

$$-\phi_{1t}^{iks}\frac{\partial TF_{1t}^{iks}(.)}{\partial \text{Gas}_t^{iks}}+\left(\phi_{2t}^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{Flare}_t^{iks}} = -\tau_t^{iks,\text{Flare}}(1-T_t^s)-VF_t^{iks}\frac{PF_t^{iks}(\cdot)}{\partial \text{Flare}_t^{iks}}-\phi_{1t}^{iks}\frac{\partial TF_{3t}^{iks}(.)}{\partial \text{Flare}_t^{iks}}$$

$$+\left(\phi_{2t}^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}+\phi_{6t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{IVent}_t^{iks}} = -VF_t^{iks}\frac{PF_t^{iks}(\cdot)}{\partial \text{IVent}_t^{i}}+\left(\phi_{2t}^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}+\phi_{9t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{PInj}_t^{iks}} = -\frac{IC_t^{iks}\left(\text{ReInj}_t^{iks},\text{PInj}_t^{iks}\right)}{\partial \text{PInj}_t^{iks}}(1-T_t^s)-PP_t^{iks,\text{Gas}}(1-T_t^s)+\phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial \text{PInj}_t^{iks}}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{ReInj}_t^{iks}} = \left[-\frac{IC_t^{iks}\left(\text{ReInj}_t^{iks},\text{PInj}_t^{iks}\right)}{\partial \text{ReInj}_t^{iks}}+\delta_{1t}^{iks}\right](1-T_t^s)$$

$$+\left(\phi_{2t}^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}+\phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial \text{ReInj}_t^{iks}}+\phi_{8t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{OInj}_t^{iks}} = -C_t^{iks}(1-T_t^s)+\phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial \text{OInj}_t^{iks}}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{ReInS}_t^{iks}} = -\phi_{1t}^{iks}\frac{\partial TF_{4t}^{iks}(.)}{\partial \text{ReInS}_t^{iks}}+\delta_{1t}^{iks}+\left(\phi_{2t}^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}+\phi_{9t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{PInS}_t^{iks}} = -PP_t^{iks,\text{Gas}}(1-T_t^s)+\phi_{1t}^{iks}\frac{\partial TF_{4t}^{iks}(.)}{\partial \text{PInS}_t^{iks}}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{Z}_t^{iks}} = (1-T_t^s)-\phi_{1t}^{iks}\frac{\partial TF_t^{iks}(.)}{\partial \text{Z}_t^{iks}}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{K}_{t+1}^{iks}} = -\phi_{4t}^{iks}+\beta\phi_{3t+1}^{iks}+\beta\phi_{4t+1}^{iks}+\phi_{11t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{ID}_t^{iks}} = -(1-T_t^s)+\phi_{4t}^{iks}\frac{\partial D_t^{iks}\left(\text{ID}_t^{iks},\text{L}_{t-1}^{iks}\right)}{\partial \text{ID}_t^{i}}-\lambda_{1t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{L}_t^{iks}} = \lambda_{1t}^{iks}+\beta\phi_{4t+1}^{iks}\frac{\partial D_{t+1}^{iks}\left(\text{ID}_{t+1}^{iks},\text{L}_t^{iks}\right)}{\partial \text{L}_t^{iks}}-\beta\lambda_{1t+1}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{IM}_t^{iks}} = -(1-T_t^s)+\lambda_{2t}^{iks}=0$$

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{M}_{t+1}^{iks}} = -\beta\phi_{1t+1}^{iks}\frac{\partial TF_{5t+1}^{iks}(.)}{\partial \text{M}_{t+1}^{iks}}-\lambda_{2t}^{iks}+\beta\lambda_{2t+1}^{iks}\left(1-\rho^{iks}\right)+\phi_{10t}^{iks}-\beta VF_t^{iks}\frac{PF_{t+1}^{iks}(\cdot)}{\partial \text{M}_{t+1}^{iks}}$$

$$-\beta\left(1-\vartheta^{iks}\right)^{-1}\left[\phi_{2t+1}^{iks}-\phi_{3t+1}^{iks}-\phi_{4t+1}^{iks}\zeta\right]\frac{\partial Maint_{t+1}^{iks}(.)}{\partial \text{M}_{t+1}^{iks}}=0$$

plus the standard primal feasibility conditions $\left\{\left\{\left\{g^{iks}_{w,ut}(\cdot) \leq 0\right\}_{w=1}^{12}, \left\{e^{iks}_{z,ut}(\cdot) = 0\right\}_{z=1}^{2}\right\}_{i=1}^{I^k}\right\}_{t=1}^{\infty}$, the dual feasibility conditions $\left\{\left\{\left\{\phi^{iks}_{wt} \geq 0\right\}_{w=1}^{12}\right\}_{i=1}^{I^k}\right\}_{t=1}^{\infty}$, and the complimentary slackness conditions $\left\{\left\{\left\{\phi^{iks}_{wt} g^{iks}_{w,ut}(\cdot) = 0\right\}_{w=1}^{12}\right\}_{i=1}^{I^k}\right\}_{t=1}^{\infty}$.

1.2 Midstream: Refinery&Transformation Firms

We assume there are J^s price-taker and infinitely-living midstream firms in each country s and each period t. This category includes oil refineries, gas-processing facilities, and transformation firms which use crude O_t^{js} and natural gas G_t^{js} (measured in BOE) and other commodities with US dollar value MZ_t^{js} as inputs (negative net outputs) and produce a B-dimensional vector of oil&gas products $\mathbf{y}_t^{js}$ (positive net outputs) and a weakly positive amount of flaring F_t^{js}. Transformation firms include all type of firms that use crude oil and/or unrefined natural gas as inputs, such as power plants and petrolchemical firms. Oil&gas products include, among other, gasoline, heavy fuels, LPG, natural gas, and plastic materials. Lastly, midstream firms may recover some of the natural gas produced as a byproduct of crude processing and use it for electricity production in-situ in quantity MInS_t^{js}.

Technological Constraints. The firm's technology is represented by the real analytic transformation function $MTF_t^{js} : X_t^m \to \mathbb{R}$ where $X_t^m = (-\infty, +\infty)^{B+5}$. As a consequence, the production set of the R&F firm is described by the inequality:

$$MTF_t^j\left(\mathbf{y}_t^j, \mathrm{O}_t^j, \mathrm{G}_t^j, \mathrm{F}_t^j, \mathrm{MInS}_t^j, \mathrm{MZ}_t^j\right) \leq 0 \tag{22}$$

For tractability, we assume that MTF_t^{js} possess the additively separable form:

$$\begin{aligned} MTF_t^{js}\left(\mathbf{y}_t^{js}, \mathrm{O}_t^{js}, \mathrm{G}_t^{js}, \mathrm{F}_t^{js}, \mathrm{MInS}_t^{js}, \mathrm{MZ}_t^{js}\right) = \quad & MTF_{1t}^j\left(\mathbf{y}_t^j\right) + MTF_{2t}^j\left(\mathrm{O}_t^j\right) \\ & + MTF_{3t}^j\left(\mathrm{G}_t^j\right) + MTF_{4t}^j\left(\mathrm{F}_t^j\right) \\ & + MTF_{5t}^j\left(\mathrm{MInS}_t^j\right) + MTF_{6t}^j\left(\mathrm{MZ}_t^j\right) \end{aligned} \tag{23}$$

The technology of any midstream firm satisfies the constraints $\mathrm{O}_t^{js} \leq 0$ and $\mathrm{G}_t^{js} \leq 0$, $\mathrm{F}_t^{js} \geq RF_t^{js}$; i.e., crude and unrefined natural gas are net inputs for midstream firms, and some of them –specifically, oil refineries and gas processing facilities– must produce a certain amount of flaring (minimum non-routine flaring). First, oil refineries are defined as midstream firms such that $\frac{\partial MTF_2^{js}\left(\mathrm{O}_t^{js}\right)}{\partial \mathrm{O}_t^{js}} > 0$ for all O_t^j, $\frac{\partial MTF_4^{js}\left(\mathrm{F}_t^{js}\right)}{\partial \mathrm{F}_t^{js}} \neq +\infty$, and $\frac{\partial MTF_3^{js}\left(\mathrm{G}_t^{js}\right)}{\partial \mathrm{G}_t^{j}} = 0$ for all G_t^j. Under these assumptions, the constraint $\mathrm{G}_t^{js} \leq 0$ is always binding, implying that oil refineries optimally only use crude oil and MZ_t^j as inputs and they may produce positive flaring and natural gas as by-products. Second, in a similar way, a gas processing facility features $\frac{\partial MTF_2^{js}(\mathrm{G}_t^{js})}{\partial \mathrm{G}_t^{js}} > 0$ for all G_t^{js}, $\frac{\partial MTF_4^{js}(\mathrm{F}_t^{js})}{\partial \mathrm{F}_t^{js}} \neq +\infty$, and $\frac{\partial MTF_3^{js}(\mathrm{O}_t^{js})}{\partial \mathrm{O}_t^{js}} = 0$ for all O_t^{js}. Third, midstream firms other than oil refineries and gas processing facilities ("transformation firms") feature $\frac{\partial MTF_4^{js}(\mathrm{F}_t^{js})}{\partial \mathrm{F}_t^{js}} = +\infty$ for all $\mathrm{F}_t^{js} \geq 0$ and $RF_t^{js} = 0$; i.e., they do not perform any gas flaring. Fourth, we do not restrict the elements of vector $\mathbf{y}_t^{js}$ to be weakly positive in order to capture the fact that some goods that are net output for some midstream firms may be net inputs for other firms in the same class. For instance, refined natural gas is a net output for gas processing facilities and a net input for gas-operated steel factories. Lastly, all midstream firms face the following constraint:

$$-\mathrm{MInS}_t^{js} - GOR^{js}\mathrm{O}_t^{js} - \mathrm{F}_t^{js} \leq 0$$

where GOR^{js} represents the amount of natural gas that is produced as byproduct of crude processing per unit of crude. This constraint captures the fact that such byproduct gas can be either flared or used in-situ for the production of electricity. Note that this constraint is not binding for gas processing facilities that feature $\mathrm{O}_t^{js} = 0$ at the optimal choice, and for other midstream firms that are characterized by $GOR^{js} = 0$; i.e., they do not produce any natural gas as byproduct.

Information. In terms of information structure, we assume that midstream firms operate under full information. Differently from upstream firms, however, the information set of midstream firms corresponds to the public one; i.e., $\Omega_t^j = \Omega_t^{PUB}$ for all j. This assumption captures the fact that midstream firms do not engage in venting of natural gas of any type because of safety concerns, therefore all their endogenous choices are observable and contractible for the government.

Flaring & Venting Regulation. Midstream firms (including oil refineries and gas processing facilities) cannot vent any amount of natural gas because of safety concerns. For the same reason, they are also typically allowed to flare gas resulting from their operations. Therefore, assume no venting and/or flaring regulation applies to such firms.

Fiscal Framework. We assume a tax system on midstream firms featuring: (i) a linear tax rate T_t^s on corporate income, defined as the firm's revenue minus total costs;(ii) a (possibly firm-specific) specific linear tax on flaring, which also includes the unitary cost of any flaring permits the firm may be required to purchase, denoted by $\tau_t^{js,\mathrm{F}}$ (it may be equal to zero). (iii) a vector of (possibly field-specific)

specific linear sales taxes $\mathbf{a}_t^{js}$ on oil&gas products; (iv) a (possibly field-specific) specific linear sales tax b_t^{js} on natural gas. Note that taxes (iii) and (iv) are defined on net supplies. Thus, if the corresponding net outputs are negatives, their values should be interpreted as subsidy rates rather than tax rates.

1.2.1 Midstream Firm's Problem

We define $X_m = \{X_{mt}\}_{t=1}^{\infty}$. The within-period profits of a midstream firm write:

$$\Pi_t^{js}(\cdot) = (1 - T^s)\left[\left(\mathbf{p}_t^s - \mathbf{a}_t^{js}\right)' \mathbf{y}_t^{js} + P_t^{\text{Oil}}\text{O}_t^{js} + \left(P_t^{s,\text{Gas}} - b_t^{js}\right)\text{G}_t^{js} + \text{MZ}_t^{js} - \text{F}_t^{js}\tau_t^{js,\text{F}}\right] \tag{24}$$

Given the assumptions stated in the previous section, the problem of a midstream firm writes:

$$\begin{array}{cc} \max\limits_{\substack{\{\mathbf{y}_t^{js},\text{O}_t^{js},\text{G}_t^{js},\text{F}_t^{js},\\ \text{MInS}_t^{js},\text{MZ}_t^{js}\}_{t=1}^{\infty}\in X_m}} & \sum_{t=1}^{\infty}\beta^{t-1}\Pi_t^{js}\left(\mathbf{y}_t^{js},\text{O}_t^{js},\text{G}_t^{js},\text{F}_t^{js},\text{MInS}_t^{js},\text{MZ}_t^{js}\right) \\ & MTF_t^{js}\left(\mathbf{y}_t^{js},\text{O}_t^{js},\text{G}_t^{js},\text{F}_t^{js},\text{MInS}_t^{js},\text{MZ}_t^{js}\right) \leq 0 \\ s.t. & -\text{MInS}_t^{js} - GOR^{js}\text{O}_t^{js} - \text{F}_t^{js} \leq 0 \\ & \text{O}_t^{js} \leq 0,\ \text{G}_t^{js} \leq 0,\ \text{F}_t^{js} - RF_t^{js} \geq 0 \end{array} \tag{25}$$

The Lagrangian of this problem writes:

$$\begin{aligned} \mathcal{L}_m^{js} = \ & \sum_{t=1}^{\infty}\beta^{t-1}\Big\{(1 - T_t^s)\left[\left(\mathbf{p}_t^s - \mathbf{a}_t^{js}\right)'\mathbf{y}_t^{js} + P_t^{\text{Oil}}\text{O}_t^{js} + \left(P_t^{s,\text{Gas}} - b_t^{js}\right)\text{G}_t^{js} - \text{F}_t^{js}\tau_t^{js,\text{F}} + \text{MZ}_t^{js}\right] \\ & -\psi_{1t}^{js}\left[MTF_t^{js}\left(\mathbf{y}_t^{js},\text{O}_t^{js},\text{G}_t^{js},\text{F}_t^{js},\text{MInS}_t^{js},\text{MZ}_t^{js}\right)\right] \\ & -\psi_{2t}^{js}\left[-\text{MInS}_t^{js} - GOR^{js}\text{O}_t^{js} - \text{F}_t^{js}\right] \\ & -\psi_{3t}^{js}\text{O}_t^{js} - \psi_{4t}^{j}\text{G}_t^{js} + \psi_{5t}^{j}\left[\text{F}_t^{js} - RF_t^{js}\right] \end{aligned} \tag{26}$$

For ease of notation, we define the vector

$$\mathbf{z}_t^{js} = \left(\text{y}_{1t}^{js},\text{y}_{2t}^{js},...,\text{y}_{Bt}^{js},\text{O}_t^{js},\text{G}_t^{js},\text{F}_t^{js},\text{MInS}_t^{js},\text{MZ}_t^{js}\right) \tag{27}$$

with $\mathbf{z}_t^{js} \in X_{mt}$ and the corresponding profit function $\mathit{\Pi}_t^{js}\left(\mathbf{z}_t^{js};\mathbf{T}\right)$, which we use in the proofs in section 1.1.3, where $\mathbf{T}$ denotes the tax scheme as defined in section 1.4, and the corresponding collection $\mathbf{z}^{js} = \left\{\mathbf{z}_t^{js}\right\}_{t=1}^{\infty}$. Given this newly defined notation, we restate the firm's problem in the following parsimonious form.

$$\max_{\mathbf{z}_t^{js} \in X_m} \quad \sum_{t=1}^{\infty} \beta^{t-1} \; \Pi_t^{js}\left(\mathbf{z}_t^{js}; \mathbf{T}\right)$$
$$s.t. \left\{ \; \left\{ g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right) \leq 0 \right\}_{w=1}^{5} \; \right\}_{t=1,2,...,} \tag{28}$$

where $g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right)$ corresponds to the wth inequality constraint of the original firm's problem in (25).

1.2.2 Optimality Conditions

First, we establish that the firm's problem in (28) is a convex maximization problem. The objective function in (28) is a concave function because it is linear. The set X_m is convex. Moreover, each inequality constraint $g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right) \leq 0$ is such that $g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right)$ is a weakly convex function, implying in turn that each set $CS_{w,mt}^{js} \equiv \left\{\mathbf{z}_t^{js} \in X_{mt} \mid g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right) \leq 0\right\}$ for $w = 1, 2, 3, 4, 5$ and $t = 1, 2, ...$ is a convex set. Lastly, the set $\left[\bigcap_{w=1}^{5} \bigcap_{t=1}^{\infty} CS_{w,mt}^{js}\right] \cap X^m$ is the intersection of convex sets, which is a convex set. Thus, the firm's problem is convex. Second, note that the Slater's condition is satisfied. To prove this result, it is sufficient to choose an arbitrarily small strictly positive ε_t^{js} and set $\mathbf{y}_t^{js} = \mathbf{0}$, $\mathrm{O}_t^{js} = -\varepsilon_t^{js}$, $\mathrm{G}_t^{js} \leq -\varepsilon_t^{js}$, $\mathrm{F}_t^{js} = RF_t^{js} + \varepsilon_t^{js}$, $\mathrm{MInS}_t^{js} = GOR^{js}\varepsilon_t^{js} - RF_t^{js}$ and set MZ_t^{js} to a value small enough to ensure that the first constraint is satisfied with strict inequality, where such value exists given that $\frac{\partial MTF_{6t}^{js}\left(\mathrm{MZ}_t^{js}\right)}{\mathrm{MZ}_t^{js}} > 0$ for all values of MZ_t^{js}. Thus, the Slater's condition is satisfied and the Karush–Kuhn–Tucker theorem implies the global maximizer of the constrained optimization problem (if it exists) has to satisfy the KKT conditions (First-order necessary conditions - FOCs). Second, we derive the First-order Necessary Conditions for a global maximum, which write:

$$
\begin{aligned}
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{O}_t^{js}} &= P_t^{\mathrm{Oil}}\left(1-T_t^s\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{O}_t^{js}}+\psi_{2t}^{js}GOR^{js}-\psi_{3t}^{js}=0 \\
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{G}_t^{js}} &= \left(P_t^{s,\mathrm{Gas}}-b_t^{js}\right)\left(1-T_t^s\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{G}_t^{js}}-\psi_{4t}^{js}=0 \\
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{F}_t^{js}} &= -\tau_t^{js,\mathrm{F}}\left(1-T_t^s\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{F}_t^{js}}+\psi_{2t}^{js}-\psi_{5t}^{js}=0 \\
\frac{\partial\mathcal{L}_m^{j}}{\partial \mathrm{MInS}_t^{js}} &= -\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{MInS}_t^{js}}+\psi_{2t}^{js}=0 \\
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{MZ}_t^{js}} &= \left(1-T_t^{js}\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{MZ}_t^{js}}=0 \qquad\qquad \forall t=1,2,... \\
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{y}_{1t}^{j}} &= \left(p_{bt}^s-a_{bt}^{js}\right)\left(1-T_t^s\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{y}_{bt}^{js}}=0 \\
&\vdots \\
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{y}_{bt}^{js}} &= \left(p_{bt}^s-a_{bt}^{js}\right)\left(1-T_t^s\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{y}_{bt}^{js}}=0 \\
&\vdots \\
\frac{\partial\mathcal{L}_m^{js}}{\partial \mathrm{y}_{Bt}^{js}} &= \left(p_{bt}^s-a_{bt}^{js}\right)\left(1-T_t^s\right)-\psi_{1t}^{js}\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{y}_{Bt}^{js}}=0
\end{aligned}
\tag{29}
$$

plus the standard primal feasibility $\left\{\left\{g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right)\leq 0\right\}_{w=1}^5\right\}_{t=1}^\infty$, dual feasibility $\left\{\left\{\psi_{wt}^{js}\geq 0\right\}_{w=1}^5\right\}_{t=1}^\infty$, and complimentary slackness conditions $\left\{\left\{\psi_{wt}^{js}g_{w,mt}^{js}\left(\mathbf{z}_t^{js}\right)=0\right\}_{w=1}^5\right\}_{t=1}^\infty$.

1.3 Downstream: Consumers

We assume a single infinitely-living and price-taker consumer in each country $s=1,2,...,S$, who consumes the numéraire good C_t^s (other consumption) and a B-dimensional vector of oil&gas products (gasoline, natural gas, electricity, plastic materials, etc.), with typical element $\mathbf{c}_t^s$. Each element of vector $\mathbf{c}_t^s$ is expressed in an appropriate unit (e.g., gallons, KWh, Mt, etc.), whereas other consumption C_t^s is expressed in USD.

Preferences. In each period t the consumer has preferences over C_t^s, $\mathbf{c}_t^s$, and the concentration (stock) of greenhouse gases in the atmosphere expressed in CO_2-equivalent amounts. In detail, their within-period preferences of a consumer in country s are represented by the utility function:

$$U\left(C_t^s,\mathbf{c}_t^s,ExcTCO2e_t\right)=C_t^s+u^s\left(\mathbf{c}_t^s\right)-Ext\times ExcTCO2e_t \tag{30}$$

where u^s is continuous and strictly concave. Ext represents the marginal social cost of one CO_2-equivalent unit of greenhouse gases in the atmosphere (i.e., the price of CO_2) and $ExtTCO2e_t=TCO2e_t-\overline{TCO2e_t}$ is the difference between the actual concentration of GHG in the atmosphere in period t and a target value $\overline{TCO2e_t}$ for such variable (e.g., the concentration observed before the industrial revolution, $\sim$ 1750 AD). Thus, the term $Ext \times ExcTCO2e_t$ in formula (30) represents the disutility from excess GHG concentration in the atmosphere. Provided that the average global temperature is increasing in $TCO2e_t$, this term can be interpreted as a measure of the utility cost of Global Warming faced by the consumer in a given period t. The concentration of GHGs in the atmosphere is taken as given by the consumer, such that the amount of individual emissions which contribute to increase the value of $TCO2e_t$ (see equation (50) in the climate change section) is excluded from the consumer's problem. This assumption is equivalent to that of an economy with a large number of identical consumers, such that the consumption choices of each consumer have a negligible impact on global emissions.

Gross Income. In each period t the consumer in country s earns exogenous income e_t^s (e.g., labor income) and capital income. Regarding the former, we assume that it is large enough to ensure positive consumption. Regarding the latter, we assume that all firms in each country are owned by domestic consumers. This assumption implies that firms' net profits from each oil&gas firm $\sum_{k=1}^{K^s} \Pi_t^{iks}$ and each midstream firm $\sum_{j=1}^{J^s} \Pi_t^{js}$ are entering the consumer's income. Consumers take firms' profits as given because they do not manage the firms' production choices and are price-takers.

Fiscal Framework. In each period t the consumer in country s pays a net lump-sum income tax $ITax_t^s$. As a result, their disposable income in period t –denoted by Y_t^s– has formula:

$$Y_t^s = e_t^s + \left[\sum_{k=1}^{K^s} \Pi_t^{iks} + \sum_{j=1}^{J^s} \Pi_t^{js} \right] - ITax_t^s \tag{31}$$

However, the consumer does not exert control on neither firm nor government's decisions, meaning that Y_t^s is treated by the consumer as exogenous. Moreover, the consumer also faces a vector $\mathbf{v}_t^s$ of specific linear consumption taxes on oil&gas products. Thus, in each period t the consumer in country s faces a budget constraint:

$$C_t^s + (\mathbf{p}_t + \mathbf{v}_t^s)' \, \mathbf{c}_t^s - Y_t^s \leq 0 \tag{32}$$

Because consumer's disposable income Y_t^s is treated as given, the consumer's choice set reduces to $X_c = \{X_{ct}\}_{t=1}^{\infty}$ where $X_{ct} = \left\{ \left(C_t^S, \mathbf{c}_t^s\right) \mid \left(C_t^S, \mathbf{c}_t^s\right) \in (0, +\infty) \times (0, +\infty)^B \right\}$.

As a result, the consumer in country s solves the following problem:

$$\begin{array}{ll} \max\limits_{\left\{\left(C_t^S,\mathbf{c}_t^s\right)\right\}_{t=1}^{\infty}\in X_c} & \sum_{t=1}^{\infty}\beta^{t-1}\left[C_t^s+u^s\left(\mathbf{c}_t^s\right)-Ext\times ExcTCO2e_t\right] \\ & s.t.\ \left\{C_t^s+\left(\mathbf{p}_t^s+\mathbf{v}_t^s\right)'\mathbf{c}_t^s-e_t^s-Y_t^s\leq 0,\right. \\ & \left.\left\{c_{bt}^s\geq 0\right\}_{b=1}^{B}\right\}_{t=1}^{\infty} \end{array} \tag{33}$$

The Lagrangian of consumer s's problem writes:

$$\begin{array}{rl} \mathcal{L}_c^s = & \sum_{t=1}^{\infty}\beta^{t-1}\left\{C_t^s+u\left(\mathbf{c}_t^s\right)-Ext\times ExcTCO2e_t\right. \\ & -\theta_{0t}^s\left[C_t^s+\left(\mathbf{p}_t^s+\mathbf{v}_t^s\right)'\mathbf{c}_t^s-Y_t^s\right] \\ & \left.\sum_{b=1}^{B}\theta_{bt}^s c_{bt}^s\right\} \end{array} \tag{34}$$

Note that we are not constraining C_t^s to be weakly positive. However, the assumption that e_t^s is large ensures positive consumption in each period t.

For ease of notation, we define the vector

$$\mathbf{b}_t^s=\left(\mathrm{C}_t^s,\mathrm{c}_{1t}^s,\mathrm{c}_{2t}^{js},...,\mathrm{c}_{Dt}^s\right) \tag{35}$$

with $\mathbf{b}_t^{js}\in X_{ct}$ and the corresponding utility function $\mathbb{U}_t^s\left(\mathbf{b}_t^s;\mathbf{T},ExcTCO2e_t\right)$, which we use in the proofs in section 1.1.3, where $\mathbf{T}$ denotes the tax scheme as defined in section 1.4, and the corresponding collection $\mathbf{b}^s=\{\mathbf{b}_t^s\}_{t=1}^{\infty}$. Given this newly defined notation, we restate the firm's problem in the following parsimonious form.

$$\begin{array}{ll} \max\limits_{\mathbf{b}_t^s\in X_c} & \sum_{t=1}^{\infty}\beta^{t-1}\,\mathbb{U}_t^s\left(\mathbf{b}_t^s;\mathbf{T},ExcTCO2e_t\right) \\ & s.t.\ \left\{\ \left\{g_{w,ct}^s\left(\mathbf{b}_t^{js}\right)\leq 0\right\}_{w=1}^{B+1}\ \right\}_{t=1,2,...,} \end{array} \tag{36}$$

where $g_{w,ct}^{js}\left(\mathbf{b}_t^{js}\right)$ corresponds to the wth inequality constraint of the original firm's problem in (33).

1.3.1 Optimality Conditions

First, we establish that the consumer's problem in (36) is a convex maximization problem. The objective function in (36) is a concave given the assumption that u^s is strictly concave. The set X_c is convex. Moreover, each inequality constraint $g_{w,ct}^s\left(\mathbf{b}_t^s\right)\leq 0$ is such that $g_{w,ct}^s\left(\mathbf{b}_t^s\right)$ is a weakly convex function, implying in turn that each set $CS_{ct}^{s} \equiv \left\{ \mathbf{b}_t^s \in X_{ct} \mid g_{w,ct}^{s}\left(\mathbf{b}_t^s\right) \leq 0 \right\}$ for $w = 1, 2, ..., B+1$ and $t = 1, 2, ...$ is a convex set. Lastly, the set $[\bigcap_{t=1}^{\infty} CS_{ct}^{s}] \cap X_c$ is the intersection of convex sets, which is a convex set. Thus, the firm's problem is convex. Moreover, it is easy to show that Slater's condition is satisfied. To prove that result, note that it is sufficient to set all c_{bt}^{s} equal to an arbitrarily small and strictly positive value, and such bundle satisfies all the constraints with strict inequality given that e_t^s is assumed to be large. This implies that a global maximizer exists and solves the First-order Conditions. Second, we derive the First-order Necessary Conditions for a global maximum, which write:

$$\begin{array}{rcl} \frac{\partial \mathcal{L}_c^s}{\partial \mathrm{C}_t^s} & = & 1 - \theta_{0t}^s = 0 \\ \frac{\partial \mathcal{L}_c^s}{\partial \mathrm{c}_{bt}^s} & = & \frac{\partial u^s(\mathbf{c}_t^s)}{\partial \mathrm{c}_{bt}^s} - \theta_{bt}^s \left(p_{kt} + v_{kt}\right) + \theta_{bt}^s = 0 \end{array} \quad \forall t = 1, 2, ... \tag{37}$$

plus the standard primal feasibility $\left\{ \left\{ g_{w,ct}^{s}\left(\mathbf{b}_t^s\right) \leq 0 \right\}_{w=1}^{B+1} \right\}_{t=1}^{\infty}$, dual feasibility $\left\{ \left\{ \theta_{wt}^s \geq 0 \right\}_{w=1}^{B+1} \right\}_{t=1}^{\infty}$, and complimentary slackness conditions $\left\{ \left\{ \theta_{wt}^s g_{w,ct}^{s}\left(\mathbf{b}_t^s\right) = 0 \right\}_{w=1}^{B+1} \right\}_{t=1}^{\infty}$.

1.4 Government

We assume a government in each country s which spend an exogenous amount G_t^s in each period t. Public spending is a mere cost for the society[10] and is financed through solely through tax revenues (there is no sovereign debt). Tax revenues include those generated by: (i) specific taxes on the production of oil, gas, and other commodities, (ii) specific taxes on the consumption of goods, (iii) personal income taxes $ITax_t^s$, (iv) corporate income taxes, (v) flaring tax (if any), and (vi) the payments of any fine due by firms because of the violation of flaring and/or venting regulation (if any). The government is a passive player solely defined by its budget constraint, which is assumed to be balanced in every period and has the following functional form:

$$\begin{gathered} G_t^s - \sum_{i \in K^s} \tau_t^{iks,\mathrm{Oil}} \mathrm{Oil}_t^{iks} + \tau_t^{iks,\mathrm{Gas}} \mathrm{Gas}_t^{iks} + \tau_t^{iks,\mathrm{Flare}} \mathrm{Flare}_t^{i} \\ + VF_t^{iks} \times \mathbf{1}\left[\mathrm{ivent}_t^{iks} = 1\right] + FF_t^{iks} \times \mathbf{1}\left[\mathrm{Flare}_t^{iks} > \bar{F}^i\right] + \left(\mathbf{a}_t^{js}\right)' \mathbf{y}_t^{js} \\ + \tau_t^{js,\mathrm{F}} \mathrm{F}_t^{js} + \left(\mathbf{v}_t\right)' \mathbf{c}_t^s + ITax_t^s + \left[\left(\sum_{k=1}^{K^s} \sum_{i=1}^{I^k} T_t^{ks} G\Pi_t^{iks} \right) + \left(\sum_{j=1}^{J^s} T_t^{js} G\Pi_t^{js} \right) \right] = 0 \end{gathered} \tag{38}$$

[10]This assumption can be easily relaxed. For instance, one could impose the alternative assumption that G_t^s enters the utility function of the consumer in country s, with no consequences for the present analysis.

where

$$G\Pi_t^{iks} = \begin{array}{c} Rev_t^{iks}\left(\mathrm{Oil}_t^{iks}, \mathrm{Gas}_t^{iks}, \mathrm{Flare}_t^{iks}, \mathrm{PInj}_t^{iks}, \mathrm{PInS}_t^{iks}, \mathrm{Z}_t^{iks}\right) \\ -InvCost_t^{iks}\left(\mathrm{ID}_t^{iks}, \mathrm{IM}_t^{iks}, \mathrm{ReInj}_t^{iks}, \mathrm{PInj}_t^{iks}, \mathrm{OInj}_t^{iks}; PP_t^{iks,\mathrm{Gas}}\right) \end{array} \tag{39}$$

and

$$G\Pi_t^{js} = \left[\left(\mathbf{p}_t^s - \mathbf{a}_t^{js}\right)' \mathbf{y}_t^{js} + P_t^{\mathrm{Oil}}\mathrm{O}_t^{js} + P_t^{s,\mathrm{Gas}}\mathrm{G}_t^{js} + \mathrm{MZ}_t^{js} - \mathrm{F}_t^{js}\tau_t^{js,\mathrm{F}}\right] \tag{40}$$

are the gross profits (excluding fines) generated by field i owned by upstream firm k and by midstream firm j, respectively. The rates of each tax $\tau_t^{iks,\mathrm{Oil}}, \tau_t^{iks,\mathrm{Gas}}, \tau_t^{iks,\mathrm{Flare}}, T_t^{ks}, \tau_t^{js,\mathrm{F}}\mathbf{a}_t^{js}, T_t^{js}, \mathbf{v}_t^s$, are the (exogenous) policy variables that we seek to set at a desirable level. A tax scheme in country s is the collection $\mathbf{T}^s$ of all taxes imposes on all firms and consumers in each period t; i.e.,

$$\mathbf{T}^s = \left\{\left\{\tau_t^{iks,\mathrm{Oil}}, \tau_t^{iks,\mathrm{Gas}}, \tau_t^{iks,\mathrm{Flare}}, T_t^{ks}\right\}_{k=1}^{K^s}, \left\{\tau_t^{js,\mathrm{F}}\mathbf{a}_t^{js}, T_t^{js}\right\}_{j=1}^{J^s}, \mathbf{v}_t^s, ITax_t^s\right\}_{t=1}^{\infty} \tag{41}$$

Lastly, we denote with $\mathbf{T} = \{\mathbf{T}^s\}_{s=1}^{S}$ the collection of all the tax schemes adopted by each country $s = 1, 2, ..., S$.

1.4.1 Tax Reform

A tax reform is defined as a change in the value of some of the tax rates in $\mathbf{T}^s$ relative to their values under the existing tax scheme, that delivers a new tax scheme $\check{\mathbf{T}}^s$. We use the symbol Δ to denote a change in a given variable; for instance:

$$\Delta\tau_t^{iks,\mathrm{Oil}} = \check{\tau}_t^{iks,\mathrm{Oil}} - \tau_t^{iks,\mathrm{Oil}} \tag{42}$$

where $\check{\tau}_t^{iks,\mathrm{Oil}}$ is an element of $\check{\mathbf{T}}^s$. Let $type_t^{iks} \in \{oil, gas, mixed\}$ denote the type of oil&gas field, as defined in section 1.1.1. The reform proposed in this paper consists in the following adjustments:

1. A change in the tax rate on unrefined natural gas sales:

$$\Delta\tau_t^{iks,\mathrm{Gas}} = \begin{cases} \min_{i\in\{\{1,2,...,I_k\}\}_{k=1}^{K}}\left\{P_t^{s,\mathrm{Gas}} - \tau_t^{iks,\mathrm{Gas}} - MC_t^{iks,\mathrm{Gas}}\right\} & if \quad type_t^{iks} = oil \\ \frac{P_t^{s,\mathrm{Gas}}\left(1-\varsigma_t^{\mathrm{Gas}}MS_t^{ks,\mathrm{Gas}}\right)}{\eta_{\mathrm{G},P}^{s,GO}}\frac{\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\Delta\mathrm{Gas}_t^{iks}\mathbf{1}\left[\mathrm{Oil}_t^{iks}>0\right]}{\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\mathrm{Gas}_t^{iks}\mathbf{1}\left[\mathrm{Oil}_t^{iks}=0\right]} & if \quad type_t^{iks} = gas \\ 0 & otherwise \end{cases} \tag{43}$$

2. A change in the tax rate on crude oil sales:

$$\Delta\tau_t^{iks,\mathrm{Oil}} = \begin{cases} -\Delta\tau_t^{iks,\mathrm{Gas}} GOR^{iks}\left(1-\vartheta^{iks}\right) & if \quad type_t^{iks} = oil \\ 0 & otherwise \end{cases} \tag{44}$$

3. A change in the rate of deduction of non-commercial gas use and unavoidable gas losses:

$$\Delta\delta_{1t}^{iks} = \begin{cases} -\Delta\tau_t^{iks,\mathrm{Gas}} & if \quad type_t^{iks} = oil \\ 0 & otherwise \end{cases} \tag{45}$$

4. A change in the lump-sum deduction amount:

$$\Delta\delta_{0t}^{iks} = \begin{cases} -\Delta\tau_t^{iks,\mathrm{Gas}} \widehat{Gas}_t^{iks} & if \quad type_t^{iks} = gas \\ -\Delta\tau_t^{iks,\mathrm{Gas}} \left(\mathrm{NRF}_t^{iks} - \widehat{Maint}_t^{iks}\right) & if \quad type_t^{iks} = oil \\ 0 & otherwise \end{cases} \tag{46}$$

where

$$\widehat{Gas}_t^{iks} = \frac{\sum_{l\neq k}\sum_{j=1}^{I^l} \mathrm{Gas}_t^{jls}\mathbf{1}\left[type_t^{jls} = Gas\right]}{(K^s-1)\left(\sum_{j=1}^{I^k}\mathbf{1}\left[type_t^{jls} = Gas\right]\right)} \tag{47}$$

is the average natural gas production from gas-only fields owned by firms other than k, and

$$\widehat{Maint}_t^{iks} = \frac{\sum_{l\neq k}\sum_{j=1}^{I^l} \mathrm{ResGas}_t^{jls}\mathbf{1}\left[type_t^{jls} = Oil\right]}{(K^s-1)\sum_{j=1}^{I^k}\mathbf{1}\left[type_t^{jks} = Oil\right]} \tag{48}$$

where residual gas ResGas_t^{jls} has formula:

$$\mathrm{ResGas}_t^{jls} = \left(1-\vartheta^{iks}\right)^{-1}\mathrm{TotGas}_t^{jls} - \mathrm{ReInj}_t^{jls} - \mathrm{ReInS}_t^{jls} - RF_t^{jls} - \mathrm{Gas}_t^{jls} \tag{49}$$

and represents the average natural gas extracted from oil fields owned by firms other than k that cannot be attributed to observable variables; i.e., it equals the sum over fields j owned by firms other than k of $\mathrm{IVent}_t^{jls} + \mathrm{Flare}_t^{jls} - RF_t^{jls} - Maint_t^{jls}\left(\mathrm{M}_t^{jls}\right)$. Note that both $\widehat{Gas}_t^{iks}$ and $\widehat{Maint}_t^{iks}$ are independent of changes in choice variables controlled by firm k.

A further optional tax adjustment consists in the introduction of a tax rate on flaring for midstream firms, whose description we postpone to section 2.2. Lastly, we assume that the current (pre-reform) tax scheme features no flaring fee (i.e., $FF_t^{iks} = 0$) and the same tax rate on natural gas production for all firms; i.e., $\tau_t^{iks,\mathrm{Gas}} = \tau_t^{s,\mathrm{Gas}}$ for all fields i in country s.

1.5 Climate Change

We assume that the excess concentration of greenhouse gases in the atmosphere follows a simple law of motion:

$$-\Lambda Emissions_t + ExcTCO2e_{t+1} - (1-\Gamma)\, ExcTCO2e_t = 0 \tag{50}$$

where $Emissions_t$ represents the global emissions of greenhouse gases in period t, Λ is a parameter that captures the effect of new GHG emissions in period t on the concentration of GHG in the atmosphere, whereas Γ represents the annual rate of natural decline of the excess concentration of GHG in the atmosphere. While admittedly stylized, these assumptions capture the key consequence of GHG emissions for the purpose of this analysis: the long-lasting effect on the global climate, which represent a cost for individuals both in the current period and in the future (see section 1.3). Note that all climate change variables have no superscript: this notation captures the fact that they are global-level variables.

2 Main Analytical Results

This section contains the main analytical results and their proofs.

2.1 Upstream

We present the main theoretical findings regarding oil and gas firms. In detail:

1. Proposition 1 shows that, under mild restrictions, an increase in the specific tax on flaring and/or a tightening in the flaring ceiling both result in increased intentional venting and higher GHG emissions.

2. Proposition 2A shows that, under mild restrictions, an increase in the fine for illegal venting and/or the regulator's ability to detect and punish illegal venting both result in reduced investment in maintenance and increased leaking.

3. Proposition 2B shows that an increase in the firm's ability to detect leaks results in larger investment in equipment maintenance if and only if natural gas sales are profitable. In this case, it also unambiguously reduces methane leaking.

4. Proposition 3 shows that the reform presented in section 1.4.1 produces all the desirable outcomes outlined in the main manuscript, including the elimination of both intentional venting and routine flaring, no effect on equilibrium oil and natural gas prices, and weakly larger corporate profits, tax revenue, and consumer's welfare.

Proposition 1. *If (i)* $\frac{\partial^2 PF_t^{iks}\left(IVent_t^{iks},Flare_t^{iks}\right)}{\partial IVent_t^{iks}\partial Flare_t^{iks}} \geq 0$ *then intentional venting* $IVent_t^{iks}$ *by field i is weakly increasing in the flaring tax rate* $\tau_t^{iks,Flare}$ *and weakly decreasing in the flaring ceiling* $\overline{Flare_t}^{iks}$*. Moreover, if (ii)* $\frac{\partial^2 PF_t^{iks}\left(IVent_t^{iks},Flare_t^{iks}\right)}{\partial IVent_t^{iks}\partial Flare_t^{iks}} \left[\frac{\partial^2 PF_t^{i}\left(IVent_t^{iks},Flare_t^{iks}\right)}{\partial\left(IVent_t^{iks}\right)^2}\right]^{-1} > \frac{CI^{Flare}}{CI^{Vent}}$ *then the total GHG emissions* $CO2e_t^{iks}$ *by field i is weakly increasing in the flaring tax rate* $\tau_t^{iks,Flare}$ *and weakly decreasing in the flaring ceiling* $\overline{Flare_t}^{iks}$*.*

Proof. Part (i): effect of an increase in $\tau_t^{iks,\text{Flare}}$ for field i. First, note that the first constraint is always binding, implying $\phi_{1t}^{iks} > 0$ at the optimal solution, because otherwise $\frac{\partial TF_{6t}^{i}(.)}{\partial \text{Z}_t^{i}} > 0$ implies that it would be possible to obtain strictly larger profits in period t with no effect on the profit gained in any other period through a marginal increase in Z_t^{iks}. Secondly, under the assumption that $MC_t^{iks,\text{Gas}} \equiv \frac{\partial TF_{2t}^{iks}(.)}{\partial \text{Gas}_t^{iks}} \Big/ \frac{\partial TF_{6t}^{iks}(.)}{\partial \text{Z}_t^{iks}}$ is constant and given that the domain of the variable Gas_t^{iks} is unbounded and we know that the solution must satisfy the FOCs, the FOC w.r.t. Gas_t^{iks} implies that $-\phi_{2t}^{iks} + \phi_{3t}^{iks} + \phi_{4t}^{iks}\zeta$ is constant in $\tau_t^{iks,\text{Flare}}$. Moreover, the assumptions that (1) $MC_t^{iks,\text{Flare}}\left(\text{Flare}_t^{iks}\right)$ is constant in Z_t^{iks} and that (2) M_t^{iks} enters the formula for $Pr_t^i\left(ivent_t^{iks} = 1 \mid \Omega_t^{ks}\right)$ in an additively separable fashion imply that all variables except possibly Flare_t^{iks},IVent_t^{iks}, and Gas_t^{iks} are all constant in $\tau_t^{iks,\text{Flare}}$. In order to study the effect of a marginal change in $\tau_t^{iks,\text{Flare}}$ on these three variables we must distinguish two cases. Case 1. If the FOC w.r.t. ϕ_{6t}^{iks} is binding, then the optimal level of Flare_t^{iks} is a corner solution $\text{Flare}_t^{iks} = \text{NRF}_t^{iks}$ at the baseline value of $\tau_t^{iks,\text{Flare}}$ (and in turn $\text{Flare}_t^{iks} = 0$) , then $\frac{d\text{Flare}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = 0$ and trivially the FOCs imply $\frac{d\text{IVent}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = \frac{d\text{Flare}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = \frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = 0$ and therefore $\frac{d\text{CO2e}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = 0$. Case 2. If the FOC w.r.t. at the baseline value of $\tau_t^{iks,\text{Flare}}$ Flare_t^{iks} are binding (and in turn $\text{Flare}_t^{iks} \geq 0$), then we must make use of the FOCs w.r.t. Flare_t^{iks} and IVent_t^{iks}. We define the marginal cost of flaring as follows: $MC_t^{iks,\text{Flare}}\left(\text{Flare}_t^{iks}\right) \equiv \frac{\partial TF_{3t}^{iks}(.)}{\partial \text{Flare}_t^{iks}} \Big/ \frac{\partial TF_{6t}^{iks}(.)}{\partial \text{Z}_t^{iks}}$. Then under the condition:

$$\begin{aligned}(1 - T_t^s)\, MC_t^{iks,\text{Flare}}\left(\text{Flare}_t^{iks}\right) + VF_t^{iks}\, \frac{\partial PF_t^{iks}\left(\text{IVent}_t^{iks},\text{Flare}_t^{iks}\right)}{\partial \text{Flare}_t^{iks}}\Bigg|_{\text{IVent}_t^{iks}=0} \\ \geq VF_t^{iks}\, \frac{\partial PF_t^{iks}\left(\text{IVent}_t^{iks},\text{Flare}_t^{iks}\right)}{\partial \text{IVent}_t^{iks}}\Bigg|_{\text{IVent}_t^{iks}=0}\end{aligned} \tag{51}$$

for all values of Flare_t^{iks}, it must be true that if $\text{Flare}_t^{iks} > 0$ at the baseline value of $\tau_t^{iks,\text{Flare}} \geq 0$ then the primal feasibility condition $\text{IVent}_t^{iks} \geq 0$ is also not binding. We prove this result by contradiction. Suppose it is. $\text{Flare}_t^{iks} > \text{NRF}_t^{iks}$ implies $\phi_{6t}^{iks} = 0$, and in turn at the optimal solution:

$$\begin{aligned} -\tau_t^{iks,\text{Flare}}\left(1 - T_t^{ks}\right) - VF_t^{iks} \times \left.\frac{\partial PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{Flare}_t^{iks}}\right|_{\text{IVent}_t^{iks}=0} \\ -\phi_{1t}^{iks}\frac{\partial TF_{3t}^{iks}(.)}{\partial \text{Flare}_t^{iks}} + \left(\phi_{2t}^{iks} - \phi_{3t}^{iks} - \phi_{4t}^{iks}\zeta\right)\left(1 - \vartheta^{iks}\right)^{-1} = 0 \end{aligned} \tag{52}$$

If the constraint $\text{IVent}_t^{iks} \geq 0$ is binding, then $\phi_{7t}^{iks} > 0$ and therefore at the optimal solution:

$$-VF_t^{iks} \times \left.\frac{\partial PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{IVent}_t^{iks}}\right|_{\text{IVent}_t^{iks}=0} + \left(\phi_{2t}^{iks} - \phi_{3t}^{iks} - \phi_{4t}^{iks}\zeta\right)\left(1 - \vartheta^{iks}\right)^{-1} < 0 \tag{53}$$

Combining these two inequalities, we obtain

$$\begin{aligned} \left(1 - T_t^i\right) MC_t^{iks,\text{Flare}}\left(\text{Flare}_t^{iks}\right) + VF_t^{iks} \times \left.\frac{\partial PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{Flare}_t^{iks}}\right|_{\text{IVent}_t^{iks}=0} \\ +\tau_t^{iks,\text{Flare}}\left(1 - T_t^{ks}\right) - VF_t^{iks} \times \left.\frac{\partial PF_t^{i}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{IVent}_t^{iks}}\right|_{\text{IVent}_t^{iks}=0} < 0 \end{aligned}, \tag{54}$$

which leads to a contradiction. Thus, the solution for IVent_t^{iks} is not a corner solution with respect to its natural boundary. Moreover, the firm's objective function is globally concave and (partially) strictly concave in IVent_t^{iks} and Flare_t^{iks}. Thus we can obtain the marginal effects on $\tau_t^{iks,\text{Flare}}$ on IVent_t^{iks} and Flare_t^{iks} by differentiating the FOC w.r.t. IVent_t^{iks} and Flare_t^{iks} evaluated at $\phi_{6t}^{iks} = 0$, $\phi_{7t}^{iks} = 0$, and solving for the derivatives of interest, to get:

$$\frac{d\text{IVent}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = \frac{\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \text{IVent}_t^i \partial \text{Flare}_t^i}\left(1 - T_t^s\right)}{\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{IVent}_t^{iks}\right)^2}\left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{Flare}_t^{iks}\right)^2} + \frac{1}{VF_t^{iks}}\frac{\partial^2 MC_t^{iks,\text{Flare}}(\cdot)}{\partial\left(\text{Flare}_t^{iks}\right)^2}\right) - \left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}}\right)^2} \tag{55}$$

which is positive if $\frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}} \geq 0$, and

$$\frac{d\text{Flare}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = -\frac{\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{IVent}_t^{iks}\right)^2}\left(1 - T_t^s\right)}{\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{IVent}_t^{iks}\right)^2}\left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{Flare}_t^{iks}\right)^2} + \frac{1}{VF_t^{iks}}\frac{\partial^2 MC_t^{iks,\text{Flare}}(\cdot)}{\partial\left(\text{Flare}_t^{iks}\right)^2}\right) - \left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}}\right)^2} \tag{56}$$

and the residual effect on Gas_t^{iks}, which is obtained using $\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = -\left(\frac{d\text{IVent}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} + \frac{d\text{Flare}_t^{iks}}{d\tau_t^{iks,\text{Flare}}}\right)$, writes:

$$\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = \frac{\left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{IVent}_t^{iks}\right)^2} - \frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\text{IVent}_t^{iks}\partial\text{Flare}_t^{iks}}\right)\left(1 - T_t^{ks}\right)}{\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{IVent}_t^{iks}\right)^2}\left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\left(\text{Flare}_t^{iks}\right)^2} + \frac{1}{VF_t^{iks}}\frac{\partial^2 MC_t^{iks,\text{Flare}}(\cdot)}{\partial\left(\text{Flare}_t^{iks}\right)^2}\right) - \left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial\text{IVent}_t^{iks}\partial\text{Flare}_t^{iks}}\right)^2} \tag{57}$$

However, if the change in the tax rate extends to all the oil&gas fields, then the change in the gas supply may affect the equilibrium price of natural gas, with effects on the levels of flaring and venting optimally chosen by each firm. The overall effect writes:

$$\begin{aligned}\frac{d\text{CO2e}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} = \ & \left(\frac{d\text{IVent}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} + \frac{d\text{IVent}_t^{i}}{dP_t^{s,\text{Gas}}}\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}}\right) CI^{\text{Vent}} + \left(\frac{d\text{Flare}_t^{iks}}{d\tau_t^{i,\text{Flare}}} + \frac{d\text{Flare}_t^{iks}}{dP_t^{s,\text{Gas}}}\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}}\right) CI^{\text{Flare}} \\ & + \left(\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} + \frac{d\text{Gas}_t^{iks}}{dP_t^{s,\text{Gas}}}\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}}\right) CI^{iks,\text{Gas}}\end{aligned} \tag{58}$$

Because flaring and gas production are gross substitutes, we can show that $\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} + \frac{d\text{Gas}_t^{iks}}{dP_t^{s,\text{Gas}}}\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}} \geq 0$, $\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}} \leq 0$, $\frac{d\text{Flare}_t^{iks}}{dP_t^{s,\text{Gas}}} \leq 0$ and $\frac{d\text{IVent}_t^{iks}}{dP_t^{s,\text{Gas}}} \leq 0$. To see how, first note that (i) $\frac{d\text{Gas}_t^{iks}}{dP_t^{s,\text{Gas}}} \geq 0$ by the law of supply. We prove that (ii) $\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}} \leq 0$ by contradiction. Suppose $\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}} > 0$. Then, because the demand side of the economy is not directly affected by $\tau_t^{iks,\text{Flare}}$, the aggregate gas supply at national level must be lower after the rise in $\tau_t^{iks,\text{Flare}}$, implying that at least one field must reduce its gas supply following a rise in $\tau_t^{iks,\text{Flare}}$. But combining the FOCs of the oil&gas firm w.r.t. Gas_t^{iks} and Flare_t^{iks}, which are both binding in this case, Gas_t^{iks} is weakly increasing in $P_t^{s,\text{Gas}}$ at constant $\tau_t^{iks,\text{Flare}}$ and in $\tau_t^{iks,\text{Flare}}$ at constant $P_t^{s,\text{Gas}}$. This implies in turn that a marginal increase in both variables translates to an increase in Gas_t^{iks} in all fields, leading to a contradiction. Second, we prove that (iii) $\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} + \frac{d\text{Gas}_t^{iks}}{dP_t^{s,\text{Gas}}}\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}} \geq 0$ by contradiction. Suppose $\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} + \frac{d\text{Gas}_t^{iks}}{dP_t^{s,\text{Gas}}}\frac{dP_t^{s,\text{Gas}}}{d\tau_t^{iks,\text{Flare}}} < 0$. Because the second term is positive (see point (i) and (ii)), it must be true that $\frac{d\text{Gas}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} < 0$. But because the denominator of (57) is positive by convexity of the objective function, this is true only if $\frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks},\text{Flare}_t^{iks}\right)}{\partial\left(\text{IVent}_t^{iks}\right)^2} < \frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks},\text{Flare}_t^{iks}\right)}{\partial\text{IVent}_t^{iks}\partial\text{Flare}_t^{iks}}$, which contradicts the assumption that the function PF_t^{iks} is convex, because that is true only if $\frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks},\text{Flare}_t^{iks}\right)}{\partial\left(\text{IVent}_t^{iks}\right)^2} \geq \frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks},\text{Flare}_t^{iks}\right)}{\partial\text{IVent}_t^{iks}\partial\text{Flare}_t^{iks}}$. Lastly, $\frac{d\text{Flare}_t^{iks}}{dP_t^{s,\text{Gas}}} \leq$ and $\frac{d\text{IVent}_t^{iks}}{dP_t^{s,\text{Gas}}} \leq 0$ can be proved by substituting the FOC w.r.t. Gas_t^{iks} into the FOCs w.r.t. (a)

Flare_t^{iks} and (b) IVent_t^{iks} and totally differentiating each of the two resulting equation w.r.t. Flare_t^{iks} and IVent_t^{iks}, respectively. Thus, it is possible to calculate a lower bound on the effect of interest because the following inequality holds:

$$\frac{d\text{CO2e}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} \geq \frac{d\text{IVent}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} CI^{\text{Vent}} + \frac{d\text{Flare}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} CI^{\text{Flare}} \tag{59}$$

where the RHS of equation 59 corresponds to the effect of a marginal increase in $\tau_t^{iks,\text{Flare}}$ at constant gas prices. Substituting the formulas in 55 and 56 into 59 we obtain:

$$\frac{d\text{CO2e}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} > \frac{\left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}} CI^{\text{Vent}} - \frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \left(\text{IVent}_t^{iks}\right)^2} CI^{\text{Flare}}\right)(1 - T_t^s)}{\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \left(\text{IVent}_t^{iks}\right)^2}\left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \left(\text{Flare}_t^{iks}\right)^2} + \frac{1}{VF_t^{iks}}\frac{\partial^2 MC_t^{iks,\text{Flare}}(\cdot)}{\partial \left(\text{Flare}_t^{iks}\right)^2}\right) - \left(\frac{\partial^2 PF_t^{iks}(\cdot)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}}\right)^2} \geq 0 \tag{60}$$

Thus, it is sufficient to derive a condition for the second inequality above to be weakly satisfied to obtain a sufficient condition for $\frac{d\text{CO2e}_t^{iks}}{d\tau_t^{iks,\text{Flare}}} > 0$. Because the firm's objective function is (partially) strictly concave in Flare_t^{iks} and IVent_t^{iks}, the denominator of the RHS of 60 is positive. Thus, a sufficient condition for the inequality in 60 to hold true writes:

$$\frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}} \left[\frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \left(\text{IVent}_t^{iks}\right)^2}\right]^{-1} > \frac{CI^{\text{Flare}}}{CI^{\text{Vent}}}, \tag{61}$$

which corresponds to the condition stated in Proposition 1.

Part (ii): effect of a marginal decrease in $\overline{\text{Flare}}_t^{iks}$. Note that if the flaring ceiling is not binding for field i then the effect of a marginal change in the value of $\overline{\text{Flare}}_t^{iks}$ is trivially zero. If the ceiling is binding, then the problem is equivalent at the margin to that in 19 but with three extra conditions in each period t and each field $i = 1, 2, ..., I^k$: a constraint $\text{Flare}_t^{iks} - \overline{\text{Flare}}_t^{iks} \leq 0$ with associated KT multiplier ϕ_{13t}^i, a dual feasibility condition $\phi_{13t}^{iks} \geq 0$ and a complementary slackness condition $\phi_{13t}^{iks}\left(\text{Flare}_t^{iks} - \overline{\text{Flare}}_t^{iks}\right) = 0$. Then whenever this new constraint is binding ($\phi_{13t}^{iks} > 0$) it can be interpreted as the shadow price of relaxing the constraint. This implies that the effect of a marginal decrease in $\overline{\text{Flare}}_t^{iks}$ is equivalent to that of a marginal increase in the flaring tax multiplied by $\phi_{13t}^{iks}/\left(1 - T_t^{ks}\right) > 0$. Thus, the sign of the effect of a marginal decrease in $\overline{\text{Flare}}_t^{iks}$ is the same as that of a marginal increase in the flaring tax $\tau_t^{iks,\text{Flare}}$, which is stated in part (i) of Proposition 1. Q.E.D. Note that $\frac{\partial^2 PF_t^{iks}\left(\text{IVent}_t^{iks}, \text{Flare}_t^{iks}\right)}{\partial \text{IVent}_t^{iks}\partial \text{Flare}_t^{iks}} \geq 0$ is a sufficient condition for total emission being increasing in the flaring tax rate, but that may be true even if such condition is not satisfied. Secondly, note that the RHS of inequality (61) is a very small number (< 0.076, calculated using the values $CI^{\text{Flare}} = 0.3018$ $TCO2e/BOE$ and $CI^{\text{Vent}} = 3.9583$ $TCO2e/BOE$ from Brandt, Masnadi, Englander, Koomey, and Gordon (2018)) implying that the condition is satisfied even for modest degrees of substitutability between flaring and intentional venting. In the empirical section, we show that this condition is satisfied in the data, and we quantify the detrimental effect of an increase in flaring taxation. Note that under the parametric specification for PF_t^{iks} adopted in the empirical section of this paper, the condition in (61) reduces to $\frac{\kappa_2^{iks}}{\kappa_2^{iks}} \geq \frac{CI^{\text{Flare}}}{CI^{\text{Vent}}}$, where the threshold $\frac{\kappa_2^{iks}}{\kappa_1^{iks}}$ represents the marginal effect of reducing flaring by one unit on the amount of intentional venting and is identified by the model, allowing for estimation of the effect of interest.

Proposition 2A. *If $\frac{\partial PU_t^{iks}(\cdot)}{\partial M_t^{iks}} \geq 0$, then an increase in either the venting fine vf_t^{iks} or the regulator's venting detection accuracy VDT_t^s (or both) for an oil field i translates into (i) weakly lower maintenance M_t^{iks} and (ii) weakly larger unintentional venting $UVent_t^{iks}$ for any given quantity of extracted gas $TotGas_t^{iks}$.*

Proof. Recall that $VF_{t+1}^{iks} = vf_{t+1}^{iks} VDT_{t+1}^s$ with $vf_{t+1}^{iks} \geq 0$ and $VDT_{t+1}^s \geq 0$. Thus, it is sufficient to show that results (i) and (ii) hold true for an increase in VF_{t+1}^{iks}. Part (i). Case (1): the seventh constraint is binding (and therefore $\phi_{7t}^{iks} > 0$ and $\text{IVent}_t^{iks} = 0$). Then given the assumption $Pr_t^{iks}\left(ivent_t^{iks} = 1 \mid \Omega_t^{PUB}, \text{IVent}_t^{iks} = 0\right) = 0$ the value of VF_{t+1}^{iks} does not enter any binding optimality condition, trivially implying $\frac{d\text{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}} = 0$ and $\frac{dUVent_{t+1}^{iks}}{dVF_{t+1}^{iks}} = 0$. Case (2): the the ninth constraint is not binding (and therefore $\phi_{7t}^{iks} = 0$ and $\text{IVent}_t^{iks} \geq 0$). Let us consider the FOCs of the firm's problem w.r.t. M_{t+1}^{iks} and IM_t^{iks}:

$$\begin{aligned}\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{M}_{t+1}^{iks}} = \quad & -\beta\phi_{1t+1}^{iks}\frac{\partial TF_{5t+1}^{iks}\left(\text{M}_{t+1}^{iks}\right)}{\partial \text{M}_{t+1}^{iks}} - \lambda_{2t}^{iks} + \phi_{10t}^{iks} + \beta\lambda_{2t+1}^{iks}\left(1-\rho^{iks}\right) \\ & -\beta VF_{t+1}^{iks}\frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial \text{M}_{t+1}^{i}} - \beta\left[\phi_{2t+1}^{iks} - \phi_{3t+1}^{iks} - \phi_{4t+1}^{iks}\zeta\right]\left(1-\vartheta^{iks}\right)^{-1}\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \text{M}_{t+1}^{iks}} = 0\end{aligned} \tag{62}$$

and

$$\frac{\partial \mathcal{L}_u^{ks}}{\partial \text{IM}_t^{iks}} = -\lambda_{2t}^{iks} + (1 - T_t^s) = 0 \tag{63}$$

where the second condition is always binding given that IM_t^{iks} possesses unbounded support. Second, the sixth constraint is an equality constraint and, as such, it is always binding. Thus, we can substitute $\lambda_{2t}^{i} = \left(1 - T_t^{ks}\right)$ into the FOC w.r.t. M_{t+1}^{iks} to obtain the condition:

$$\beta\left(1-T_{t+1}^{s}\right)MPE_{t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)-\left[\left(1-T_{t}^{s}\right)-\beta\left(1-\rho^{i}\right)\left(1-T_{t+1}^{s}\right)\right]+\phi_{10t}^{iks}$$
$$-\beta\left[VF_{t+1}^{iks}\frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i}}-\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}\right]=0 \tag{64}$$

where the above follows the fact that $\mathrm{IVent}_{t+1}^{iks}$ is constant in M_{t+1}^{iks} and $MPE_{t+1}^{i}\left(\mathrm{M}_{t+1}^{iks}\right)\equiv\frac{\partial TF_{5t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)}{\partial \mathrm{M}_{t+1}^{iks}}\Big/\frac{\partial TF_{6t+1}^{iks}\left(\mathrm{Z}_{t+1}^{iks}\right)}{\partial \mathrm{Z}_{t+1}^{iks}}$. If the tenth constraint is binding ($\phi_{10t}^{i}>0$), then $\mathrm{M}_{t+1}^{iks}=0$ and the derivative of interest is simply $\frac{d\mathrm{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}}=0$. If the the tenth constraint is non-binding ($\phi_{10t}^{i}=0$), then we can totally differentiate the F.O.C. w.r.t. VF_{t+1}^{iks} to obtain:

$$-\beta\frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i}}+\beta\left\{\left(1-T_{t+1}^{s}\right)\frac{\partial MPE_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}-VF_{t+1}^{iks}\frac{\partial^{2}PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\;2}}\right.$$
$$\left.+\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}\frac{\partial^{2}Maint_{t+1}^{i}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\;2}}\right\}\frac{d\mathrm{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}}=0 \tag{65}$$

Note that because the optimization problem is convex, the Second-order Necessary Conditions for a global maximum must be satisfied; i.e., the bordered Hessian matrix must be negative semi-definite. This condition implies that the second derivative of the Lagrangian w.r.t. M_{t+1}^{i} must satisfy:

$$\frac{\partial^{2}\mathcal{L}_{u}^{ks}}{\partial \mathrm{M}_{t+1}^{iks\;2}}=\beta\left\{\left(1-T_{t+1}^{s}\right)\frac{\partial MPE_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}-VF_{t+1}^{iks}\frac{\partial^{2}PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\;2}}\right.$$
$$\left.+\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}\frac{\partial^{2}Maint_{t+1}^{i}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\;2}}\right\}\leq 0 \tag{66}$$

at all possible values of the choice variables in X^{u}. Even if the optimization problem is not convex over the entire range of values of the choice variables, the condition in (66) must be satisfied at least locally for the solution to be a maximum. In particular, evaluating condition (66) at the candidate solution values that solve the FOCs, the SONCs are satisfied only if:

$$\beta\left\{\left(1-T_{t+1}^{s}\right)\frac{\partial MPE_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}-VF_{t+1}^{iks}\frac{\partial^{2}PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\;2}}\right.$$
$$\left.+\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}\frac{\partial^{2}Maint_{t+1}^{i}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\;2}}\right\}\leq 0 \tag{67}$$

Lastly, note that if the weak inequality (67) is satisfied with strict equality, then the FOC in (62) does not have a unique solution for M_{t+1}^{iks}, implying that the optimal value of M_{t+1}^{iks} is pinned down by the constraint $\mathrm{M}_{t+1}^{iks}-\mathrm{M}_{t}^{iks}\left(1-\rho^{iks}\right)-\mathrm{IM}_{t}^{iks}\leq 0$ and it is therefore constant in VF_{t+1}^{iks}. Conversely, if the inequality (67) is satisfied with strict inequality, then we can solve (65) with respect to $\frac{d\mathrm{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}}$ and obtain:

$$\frac{d\mathrm{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}} = \frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i}} \Bigg\{ \left(1 - T_{t+1}^{s}\right) \frac{\partial MPE_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}} - VF_{t+1}^{iks} \frac{\partial^2 PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\,2}} + \left(1 - T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}} - \tau_{t+1}^{iks,\mathrm{Gas}} - MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1 - \vartheta^{iks}\right)^{-1} \frac{\partial^2 Maint_{t+1}^{i}(\cdot)}{\partial \mathrm{M}_{t+1}^{i\,2}} \Bigg\}\Bigg]^{-1}, \tag{68}$$

where the sign of the second part at the RHS of (68) is strictly negative. Thus, this implies that either $\frac{d\mathrm{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}} = 0$ or the sign of $\frac{d\mathrm{M}_{t+1}^{iks}}{dVF_{t+1}^{iks}}$ is the same as that of:

$$-\frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}} \tag{69}$$

which is strictly negative as long as $PU_{t+1}^{iks}(\cdot)$ is increasing in M_{t+1}^{iks}, Q.E.D.

Part (ii). Recall that $UVent_t^i(\cdot) = \vartheta^{iks}\mathrm{TotGas}_{t+1}^{iks} - Maint_t^{iks}\left(\mathrm{M}_t^{iks}\right) + \epsilon_t^{iks}$. Thus, a marginal increase in VF_t^{iks} may affect the optimal choice of TotGas_t^{iks}, but at any given level of gas extracted $\mathrm{TotGas}_t^{iks} = \overline{\mathrm{TotGas}}_t^{iks}$, we obtain:

$$\frac{\partial UVent_t^{iks}\left(\overline{\mathrm{TotGas}}_t^{iks}, \mathrm{M}_t^{iks}\right)}{\partial VF_t^{iks}} = -\frac{\partial Maint_t^{iks}\left(\mathrm{M}_t^{iks}\right)}{\partial \mathrm{M}_t^{iks}} \frac{d\mathrm{M}_t^{iks}}{dVF_t^{iks}} \geq 0\,, \tag{70}$$

which is weakly positive because $Maint_t^{iks}$ is increasing in M_t^{iks} and because if $\frac{\partial PU_t^{iks}(\cdot)}{\partial \mathrm{M}_t^{iks}} \geq 0$, then we have $\frac{d\mathrm{M}_t^{iks}}{dVF_t^{iks}} \leq 0$ from part (i) of this proof. Q.E.D.

Proposition 2B. *If natural gas sales are not profitable for an oil field i; i.e.,* $P_t^{s,Gas} - \tau_t^{iks,Gas} - MC_t^{iks,Gas} < 0$, *then an increase in the firm's leakage detection accuracy* LDT_t^{ks} *translates into (i) weakly lower maintenance* M_t^{iks} *for any given quantity of extracted gas* $TotGas_t^{iks}$*. If natural gas sales are profitable for an oil field i; i.e.,* $P_t^{s,Gas} - \tau_t^{iks,Gas} - MC_t^{iks,Gas} \geq 0$, *then an increase in the firm's leakage detection accuracy* LDT_t^{ks} *translates into (ii) weakly larger maintenance* M_t^{iks} *and (iii) strictly lower unintentional venting* $UVent_t^{iks}$ *for any given quantity of extracted gas* $TotGas_t^{iks}$*.*

Proof. Part (i) and (ii). Consider the F.O.C. of the oil and gas firm's problem w.r.t. M_{t+1}^{iks}. From the proof to Proposition 2A, using $Maint_{t+1}^{iks}(\cdot) = LDT_t^{ks}MNT_{t+1}^{iks}(\cdot)$, we know that such condition can be rewritten as follows:

$$\begin{gathered}\beta\left(1-T_{t+1}^{s}\right)MPE_{t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)-\left[\left(1-T_{t}^{s}\right)-\beta\left(1-\rho^{i}\right)\left(1-T_{t+1}^{s}\right)\right]+\phi_{10t}^{iks}\\-\beta\left[VF_{t+1}^{iks}\frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks}}\right.\\\left.-\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}LDT_{t+1}^{ks}\frac{\partial MNT_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks}}\right]=0\end{gathered}\tag{71}$$

If the tenth constraint is binding ($\phi_{10t}^{i}>0$), then $\mathrm{M}_{t+1}^{iks}=0$ and the derivative of interest is simply $\frac{d\mathrm{M}_{t+1}^{iks}}{dLDT_{t+1}^{ks}}=0$. If the the tenth constraint is non-binding ($\phi_{10t}^{i}=0$), then we can totally differentiate the F.O.C. in (71) w.r.t. LDT_{t+1}^{ks} to obtain:

$$\begin{gathered}\beta\left(1-T_{t+1}^{s}\right)\frac{\partial^{2}MPE_{t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)}{\partial\mathrm{M}_{t+1}^{iks\,2}}\frac{d\mathrm{M}_{t+1}^{iks}}{dLDT_{t+1}^{ks}}\\-\beta\left[VF_{t+1}^{iks}\frac{\partial^{2}PU_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks\,2}}\frac{d\mathrm{M}_{t+1}^{iks}}{dLDT_{t+1}^{ks}}-\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\right.\\\left.\left(1-\vartheta^{iks}\right)^{-1}\left(LDT_{t+1}^{ks}\frac{\partial^{2}MNT_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks\,2}}\frac{d\mathrm{M}_{t+1}^{iks}}{dLDT_{t+1}^{ks}}+\frac{\partial MNT_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks}}\right)\right]=0\end{gathered}\tag{72}$$

Solving the equation (72) w.r.t. $\frac{d\mathrm{M}_{t+1}^{iks}}{dLDT_{t+1}^{ks}}$ we obtain:

$$\frac{d\mathrm{M}_{t+1}^{iks}}{dLDT_{t+1}^{ks}}=\begin{gathered}-\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}\frac{\partial MNT_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks}}\\\left[\left(1-T_{t+1}^{s}\right)\frac{\partial^{2}MPE_{t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)}{\partial\mathrm{M}_{t+1}^{iks\,2}}-VF_{t+1}^{iks}\frac{\partial^{2}PU_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks\,2}}\right.\\\left.+\left(1-T_{t+1}^{s}\right)\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-\vartheta^{iks}\right)^{-1}LDT_{t+1}^{ks}\frac{\partial^{2}MNT_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks\,2}}\right]^{-1}\end{gathered}\tag{73}$$

Note that the term in square brackets in formula (73) must be strictly negative whenever the LHS is non-zero, as shown in the proof to Proposition 2A. Lastly, $\frac{\partial MNT_{t+1}^{iks}(\cdot)}{\partial\mathrm{M}_{t+1}^{iks}}\geq 0$ by assumption, therefore by evaluating the same condition at time t we obtain:

$$\begin{matrix}\frac{d\mathrm{M}_{t}^{iks}}{dLDT_{t}^{ks}}\geq 0 & if\quad P_{t}^{s,\mathrm{Gas}}-\tau_{t}^{iks,\mathrm{Gas}}-MC_{t}^{iks,\mathrm{Gas}}\geq 0\\\frac{d\mathrm{M}_{t}^{iks}}{dLDT_{t}^{ks}}<0 & otherwise\end{matrix},\tag{74}$$

that is, $\mathrm{M}_{t}^{iks}$is increasing in LDT_{t}^{ks} if and only if $P_{t}^{s,\mathrm{Gas}}-\tau_{t}^{iks,\mathrm{Gas}}-MC_{t}^{iks,\mathrm{Gas}}\geq 0$. Q.E.D.

Part (iii). Lastly, note that for any given constant value of $\mathrm{TotGas}_{t}^{iks}=\overline{\mathrm{TotGas}_{t}}^{iks}$, we get:

$$\frac{dUVent_{t}^{iks}\left(\overline{\mathrm{TotGas}_{t}}^{iks},\mathrm{M}_{t}^{iks}\right)}{dLDT_{t}^{ks}}=-\left[Maint_{t}^{iks}\left(\mathrm{M}_{t}^{iks}\right)+LDT_{t}^{ks}\frac{\partial Maint_{t}^{iks}(\cdot)}{\partial\mathrm{M}_{t}^{iks}}\frac{d\mathrm{M}_{t}^{iks}}{dLDT_{t}^{ks}}\right],\tag{75}$$

which given $\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}} \geq 0$ is weakly positive whenever $\frac{d\mathrm{M}_t^{iks}}{dLDT_t^{ks}} \geq 0$. Because this occurs if $P_t^{s,\mathrm{Gas}} - \tau_t^{iks,\mathrm{Gas}} - MC_t^{iks,\mathrm{Gas}} \geq 0$, this condition is sufficient to ensure $\frac{dUVent_t^{iks}\left(\overline{\mathrm{TotGas}}_t^{iks}, \mathrm{M}_t^{iks}\right)}{dLDT_t^{ks}} \geq 0$. Q.E.D.

Proposition 3. *In each period t the tax reform $\check{\mathbf{T}}^s$ translates into (i) zero intentional venting; (ii) zero routine flaring; (iii) weakly lower unintentional venting for all oil&gas firms; (iv) no effect on the equilibrium level of all prices; (v) no effect on the equilibrium demand of intermediate goods O_t^{js}, G_t^{js}, and consumption goods $\mathbf{c}_t^s$; (v) strictly lower GHG emissions;, (vi) weakly larger aggregate present-discounted corporate profits, tax revenue, and net consumer income, and (vii) strictly larger social welfare.*

Proof. We begin with proving result (iv). Then we use result (iv) to prove that the results in parts (i), (ii), (iii), (v), (vi), and (vii).

Part (iv). First, recall that the first constraint is always binding (see proof to Proposition 1). Thus, $\phi_{1t}^{iks} > 0$. Second, because the optimization problem is convex and satisfy the Slater's condition, the optimal solution satisfies the FOCs. Third, consider the constraint 2, which writes:

$$GOR^{iks}\mathrm{Oil}_t^{iks} - \left(1 - \vartheta^{iks}\right)^{-1} \left[\mathrm{Gas}_t^{iks} + \mathrm{Flare}_t^{iks} + \mathrm{IVent}_t^{iks} - Maint_t^{iks}\left(\mathrm{M}_t^{iks}\right) + \mathrm{ReInj}_t^{iks} + \mathrm{ReInS}_t^{iks}\right] \leq 0 \tag{76}$$

we aim to show that this constraint is always binding for all oil fields and never binding for all gas fields.

Step 1. We prove that for an oil field constraint 2 is always binding at the optimal solution. This result ensures that an oil fields remains classified as such even after the tax reform is implemented. Proof. By assumption, a field classified as an oil field in period t satisfies

$$\begin{aligned} 0 < \left[P_t^{\mathrm{Oil}}\left(1 - \varsigma_t^{\mathrm{Oil}} MS_t^{ks,\mathrm{Oil}}\right) + \sigma_1^{iks} - \tau_t^{iks,\mathrm{Oil}} - MC_t^{iks,\mathrm{Oil}}\left(\mathrm{Oil}_t^{iks}\right)\right] < \\ - \left[P_t^{s,\mathrm{Gas}} - \tau_t^{iks,\mathrm{Gas}} - MC_t^{iks,\mathrm{Gas}}\right] \left(1 - \vartheta^{iks}\right) GOR^{iks} \end{aligned} \tag{77}$$

for all $\mathrm{Oil}_t^{iks} \geq 0$. Note that this condition is unaffected by the introduction of the tax reform at constant prices because $\Delta\tau_t^{iks,\mathrm{Oil}} = -\Delta\tau_t^{s,\mathrm{Gas}}\left(1 - \vartheta^{iks}\right) GOR^{iks}$ implies that the condition is identical under $\check{\mathbf{T}}^s$ and $\mathbf{T}^s$ for any feasible value of

Oil_t^{iks}. Suppose the constraint (76) is not binding in period t, therefore $\phi_{2t}^{iks} = 0$. In such case, the field is classified as an "other oil&gas field" if $\text{Oil}_t^{iks} > 0$ and faces the original tax scheme regardless of the implementation of the reform, or as a gas-only field if $\text{Oil}_t^{iks} = 0$. Suppose the optimal choice features $\text{Oil}_t^{iks} > 0$. Combining the FOCs w.r.t. Oil_t^{iks} and Gas_t^{iks} and setting $\phi_{2t}^{iks} = 0$ the optimality condition for oil production writes:

$$\begin{aligned}&\left(1 - T_t^{ks}\right)\left[P_t^{\text{Oil}}\left(1 - \varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right) + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{i*}\right)\right] \\ &\quad - \left[P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}}\right]\left(1 - T_t^{s}\right)\left(1 - \vartheta^{iks}\right)^{-1} + \phi_{5t}^{iks} = 0\end{aligned} \tag{78}$$

Using condition (77) into the optimality condition (78) and noticing that it implies $P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}} \leq 0$, we obtain:

$$\left[P_t^{\text{Oil}}\left(1 - \varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right) - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks*}\right)\right]\left[1 + \left(GOR^{iks}\right)^{-1}\right](1 - T_t^{s}) + \phi_{5t}^{iks} < 0 \tag{79}$$

Thus, because $\phi_{5t}^{iks*} \geq 0$, the inequality is satisfied only if

$$\left[P_t^{\text{Oil}}\left(1 - \varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right) + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks*}\right)\right](1 - T_t^{s}) < 0 \tag{80}$$

However, note that the FOC w.r.t. Oil_t^{iks} at the optimal vector writes:

$$\begin{aligned}&\left[P_t^{\text{Oil}}\left(1 - \varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right) + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks*}\right)\right](1 - T_t^{s}) \\ &\qquad - \left(\phi_{3t}^{iks} + \phi_{4t}^{iks}\zeta\right) + \phi_{5t}^{iks} = 0\end{aligned} \quad , \tag{81}$$

which implies that either $\phi_{5t}^{iks*} > 0$ and therefore $\text{Oil}_t^{iks*} = 0$, implying that the field would not be classified as an oil field for the purposes of the tax scheme, or if $\phi_{5t}^{iks*} = 0$, and therefore given that the K-T multipliers must satisfy $\phi_{3t}^{iks} + \phi_{4t}^{iks}\zeta \geq 0$ we obtain:

$$\left[P_t^{\text{Oil}}\left(1 - \varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right) + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks*}\right)\right](1 - T_t^{s}) \geq 0 \ . \tag{82}$$

Comparing inequalities (80) and (82) we find that for Oil_t^{iks*} to be optimal the left hand side of the inequality (80) should be both strictly negative and weakly positive. This leads to a contradiction. Lastly, suppose $\text{Oil}_t^{iks*} = 0$ and $\text{Gas}_t^{iks*} > 0$. $P_t^{\text{Gas}} + \sigma_2^{iks} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}} < 0$ implies that the firm is making negative profits in period t. Moreover, $\text{Oil}_t^{iks*} = 0$ implies that the second constraint is not binding. Thus, there exists an alternative feasible choice $\tilde{\mathbf{x}}^{ks} \in X^u$ such that (i) $\tilde{\text{Gas}}_t^{iks} = 0$ and (ii) all other choice variables are unchanged, which delivers strictly larger profit in period t and the same profit as $\mathbf{x}^{ks*}$ in all other periods $r \neq t$. Thus, the choice $\mathbf{x}^{ks*}$ cannot be optimal. This leads to a contradiction. Q.E.D.

Step 2a. For a "other oil&gas field" constraint 2 is always not binding at the optimal solution. Suppose it is binding, which implies $\phi_{2t}^{iks*} > 0$. Then combining the FOCs w.r.t. Gas_t^{iks} and Oil_t^{iks} we obtain:

$$\begin{gathered} -\left(1-T_t^s\right)\left[P_t^{\text{Oil}}\left(1-\varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right)+\sigma_1^{iks}-\tau_t^{iks,\text{Oil}}-MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks*}\right)\right]\left(1-\vartheta^{iks}\right) \\ +\left[P_t^{s,\text{Gas}}\left(1-\varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\right)+\sigma_2^{iks}-\tau_t^{iks,\text{Gas}}-MC_t^{iks,\text{Gas}}\left(GOR^{iks}\text{Oil}_t^{iks}\right)\right]\left(1-T_t^s\right)= \\ -\phi_{2t}^{iks}\left(1+GOR^{iks}\right)\left(1-\vartheta^{iks}\right)+\phi_{5t}^{iks}\left(1-\vartheta^{iks}\right) \end{gathered} \tag{83}$$

But by assumption all "other oil&gas field" satisfy the following condition:

$$\begin{gathered} \left[P_t^{\text{Oil}}+\sigma_1^{iks}-\tau_t^{iks,\text{Oil}}-MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks}\right)\right]\left(1-\vartheta^{iks}\right) \leq \\ P_t^{s,\text{Gas}}\left(1-\varsigma_t^{\text{Gas}}\right)+\sigma_2^{iks}-\tau_t^{iks,\text{Gas}}-MC_t^{iks,\text{Gas}}\left(GOR^{iks}\text{Oil}_t^{iks}\right) \end{gathered} \tag{84}$$

for all $\text{Oil}_t^{iks} \geq 0$ implies:

$$-\phi_{2t}^{iks}\left(1+GOR^{iks}\right)\left(1-\vartheta^{iks}\right)+\phi_{5t}^{iks}\left(1-\vartheta^{iks}\right) \geq 0 \tag{85}$$

Thus, either (i) $\phi_{5t}^{iks*} > 0$ with $\text{Oil}_t^{iks*} = 0$ and given that we have stated that the constraint is binding, $\text{Gas}_t^{iks*} = 0$; i.e., the field is inactive, or (ii) $\phi_{5t}^{iks*} = 0$ with $\text{Oil}_t^{iks*} > 0$, then the inequality above is not satisfied. This leads to a contradiction.

Step 2b. For a gas-only field constraint 2 is always not binding at the optimal solution. Note that the assumption $MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks}\right) = +\infty$ ensures that $\text{Oil}_t^{iks*} = 0$ for all gas-only fields. Thus, the constraint 2 is always trivially non-binding for any active gas field; i.e., such that $\text{Gas}_t^{iks*} > 0$.

Step 3. The constraint 4 is always binding and therefore $\phi_{4t}^{iks} > 0$. Suppose the constraint 4 is not binding. Then, because of the complimentary slackness condition at the optimal choice we must have:

$$\begin{gathered} \text{K}_{t+1}^{iks}-\text{K}_t^{iks}-D_t^{iks}\left(\text{ID}_t^{iks},\text{L}_{t-1}^{iks}\right)-B_t^{iks}\left(\text{ReInj}_t^{iks}+\text{PInj}_t^{iks}+\text{OInj}_t^{iks}\right) \\ +\zeta\left\{\text{Oil}_t^{iks}+\left(1-\vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks}+\text{Flare}_t^{iks}\right.\right. \\ \left.\left.+\text{IVent}_t^{iks}-Maint_t^{iks}\left(\text{M}_t^{iks}\right)+\text{ReInj}_t^{iks}+\text{ReInS}_t^{iks}\right]\right\}<0 \end{gathered} \tag{86}$$

Because B_t^{iks} is strictly increasing, this implies that a choice vector $\tilde{\mathbf{x}}^{ks}$ identical to the optimal one $\mathbf{x}^{ks*}$ except for $\widetilde{\text{OInj}}_t^i = \text{OInj}_t^{i*} - a$ for arbitrarily small strictly positive a is feasible. This alternative choice vector is feasible and delivers exactly the same profit from field i in all periods $s \neq t$ and strictly larger profit in period t, and it does not affect the profit generated by fields other than i. This implies that the total present discounted value of profits from choosing vector $\tilde{\mathbf{x}}^{ks}$ are strictly larger that those from $\mathbf{x}^{ks*}$. Thus, the original choice vector $\mathbf{x}^{ks*}$ cannot be optimal. This leads to a contradiction.

Step 4. For an oil field the problem can be split into two independent optimization problems. Recall from Step 1 that the second constraint is always binding for oil fields, implying that the corresponding KT multiplier satisfies $\phi_{2t}^{iks} > 0$. Moreover, from Step 4 we know that the fourth K-T multiplier satisfies $\phi_{4t}^{iks} > 0$. Using these result and combining together the FOCs w.r.t. Gas_t^{iks}, and OInj_t^{iks}, Z_t^{iks}, we obtain the following conditions:

$$
\begin{gathered}
GOR^{iks}\text{Oil}_t^{iks} - \left(1 - \vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks} + \text{Flare}_t^{iks} + \text{IVent}_t^{iks}\right. \\
\left. - Maint_t^{iks}\left(\text{M}_t^{iks}\right) + \text{ReInj}_t^{iks} + \text{ReInS}_t^{iks}\right] = 0 \\
\left[P_t^{s,\text{Gas}} - \tau_t^{s,\text{Gas}} - MC_t^{iks,\text{Gas}}\right]\left(1 - T_t^s\right) + \left(\phi_{2t}^{iks} - \phi_{3t}^{iks} - \phi_{4t}^{iks}\zeta\right)\left(1 - \vartheta^{iks}\right)^{-1} = 0 \\
C_t^{iks}\left(1 - T_t^s\right) = \phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial \text{OInj}_t^i} \\
\left(1 - T_t^s\right) = \phi_{1t}^{iks}\frac{\partial TF_t^{iks}(.)}{\partial \text{Z}_t^{iks}}
\end{gathered}
\tag{87}
$$

Substitute these four conditions into the FOCs. We obtain the following equilibrium conditions, divided into three subsets, which correspond to three *reduced problems.*

Reduced Problem 1. Define the collections of endogenous variables $y1_t^{iks} = \{\text{Flare}_t^{iks}, \text{IVent}_t^{iks}, \text{ReInj}_t^{iks}, \text{PI}$ $\phi 1_t^{iks} = \{\phi_{8t}^{iks}, \phi_{9t}^{iks}, \phi_{10t}^{iks}, \phi_{11t}^{iks}, \phi_{12t}^{iks}\}$, and $\lambda 1_t^{iks} = \lambda_{2t}^{iks}$. Also define the sets $Y1^{ks}$ with typical element $y1^{ks} = \left\{\left\{y1_t^{iks}\right\}_{t=1}^{\infty}\right\}_{i=1}^{I^k}$, $\Psi 1^{ks}$ with typical element $\psi 1^{ks} = \left\{\left\{\psi 1_t^{iks}\right\}_{t=1}^{\infty}\right\}_{i=1}^{I^k}$, and $\Lambda 1^{ks}$ with typical element $\lambda 1^{ks} = \left\{\left\{\lambda 1_t^{iks}\right\}_{t=1}^{\infty}\right\}_{i=1}^{I^k}$. Note that $\mathbf{x}^{ks} \in X^{ks} \to y1^{ks} \in Y1^{ks}$. Also note that the inequality constraints 8, 9, 10, 11, 12 can be redefined as functions $g1_{jt}^{iks}$ such that $g1_{jt}^{iks}\left(y1_t^{iks}\right) \geq 0$ if and only if $g_{wt}^{j,m}\left(\mathbf{x}^{ks}\right) \leq 0$ given an appropriate choice of the index j. Consider the following subset of $24 \times I^k \times T$ conditions:

$$F1_{1t}^{iks} = -VF_t^{iks} \times \frac{PF_t^{iks}(\cdot)}{\partial \mathrm{Flare}_t^{iks}} - (1-T_t^s)\Big[P_t^{s,\mathrm{Gas}} - \tau_t^{iks,\mathrm{Gas}}$$

$$-MC_t^{iks,\mathrm{Gas}} + \tau_t^{iks,\mathrm{Flare}} + MC_t^{iks,\mathrm{Flare}}\left(\mathrm{Flare}_t^{iks}\right)\Big] + \phi_{8t}^{iks} = 0$$

$$F1_{2t}^{iks} = -VF_t^{iks} \times \frac{PF_t^{iks}(\cdot)}{\partial \mathrm{IVent}_t^{iks}} - (1-T_t^s)\left(P_t^{s,\mathrm{Gas}} - \tau_t^{s,\mathrm{Gas}} - MC_t^{iks,\mathrm{Gas}}\right) + \phi_{9t}^{iks} = 0$$

$$F1_{3t}^{iks} = -\left[\frac{IC_t^{iks}\left(\mathrm{ReInj}_t^{iks}, \mathrm{PInj}_t^{iks}\right)}{\partial \mathrm{PInj}_t^{iks}} + PP_t^{iks,\mathrm{Gas}}\right](1-T_t^s) + C_t^{iks}(1-T_t^s) = 0$$

$$F1_{4t}^{iks} = \left[-\frac{IC_t^{iks}\left(\mathrm{ReInj}_t^{iks}, \mathrm{PInj}_t^{iks}\right)}{\partial \mathrm{ReInj}_t^{iks}} + \delta_{1t}^{iks}\right](1-T_t^s)$$

$$-(1-T_t^s)\left(P_t^{s,\mathrm{Gas}} - \tau_t^{s,\mathrm{Gas}} - MC_t^{iks,\mathrm{Gas}} - C_t^{iks}\right) + \phi_{10t}^{iks} = 0$$

$$F1_{5t}^{iks} = -(1-T_t^s)\left(P_t^{s,\mathrm{Gas}} - \tau_t^{s,\mathrm{Gas}} - MC_t^{iks,\mathrm{Gas}} - MP_t^{iks,\mathrm{ReIns}}\left(\mathrm{ReInS}_t^{iks}, \mathrm{PInS}_t^{iks}\right) - \delta_{1t}^{iks}\right) + \phi_{11t}^{iks} = 0$$

$$F1_{6t}^{iks} = (1-T_t^s)\left[MP_t^{iks,\mathrm{PIns}}\left(\mathrm{ReInS}_t^{iks}, \mathrm{PInS}_t^{iks}\right) - PP_t^{iks,\mathrm{Gas}}\right] = 0$$

$$F1_{7t}^{iks} = -(1-T_t^s) + \lambda_{2t}^{iks} = 0$$

$$F1_{8t}^{iks} = -(1-T_t^s)\beta MP_{t+1}^{iks,\mathrm{M}}\left(\mathrm{M}_{t+1}^{iks}\right) - \lambda_{2t}^{iks} + \beta\lambda_{2t+1}^{iks}\left(1-\rho^{iks}\right)$$

$$-\beta\left[P_{t+1}^{s,\mathrm{Gas}} - \tau_{t+1}^{s,\mathrm{Gas}} - MC_{t+1}^{iks,\mathrm{Gas}}\right]\frac{\partial Maint_{t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)}{\partial \mathrm{M}_{t+1}^{iks}} + \beta\phi_{12t}^{iks} = 0$$

$$E1_{1t}^{iks} = \mathrm{M}_{t+1}^{iks} - \mathrm{M}_t^{iks}\left(1-\rho^{iks}\right) - \mathrm{IM}_t^{iks} = 0$$

$$I1_{1t}^{iks} = \mathrm{Flare}_t^{iks} - \mathrm{NRF}_t^{iks} \geq 0$$

$$I1_{2t}^{iks} = \mathrm{IVent}_t^{iks} \geq 0$$

$$I1_{3t}^{iks} = \mathrm{ReInj}_t^{iks} \geq 0$$

$$I1_{4t}^{iks} = \mathrm{ReInS}_t^{iks} \geq 0$$

$$I1_{5t}^{iks} = \mathrm{M}_{t+1}^{iks} \geq 0$$

$$M1_{jt}^{iks} = \phi 1_{jt}^{iks} \geq 0, \;\; j = 1,2,3,4,5$$

$$S1_{jt}^{iks} = \phi 1_{jt}^{iks} g1_{jt}^{iks}\left(y1_t^{iks}\right) \geq 0 \;\; j = 1,2,3,4,5$$

In the system above, we have 24 conditions per period t per field i. However, making use of the complimentary slackness conditions $S1_{jt}^{iks}$ for $j = 1,2,3,4,5$ we are left with 14 binding conditions per period t per field i. We have 14 unknown variables per period t and per field i: 8 in collection $y1_t^{iks}$, 5 in $\psi 1_t^{iks}$, plus $\lambda 1_t^{iks}$, all independent of the realizations of the variables outside of $Y1^{ks} \times \Psi 1^{ks} \times \Lambda 1^{ks}$. Thus the problem can be solved independently of the optimality of the other variables in the full problem. As we have shown that a solution to the full optimization problem exists, then a solution to the reduced problem must also exist. Thus, it must be the collection $\left\{y1^{ks*}, \psi 1^{ks}, \lambda 1^{ks}\right\}$ that solve this system.

Reduced Problem 2. Define the variable total gas injection as follows: $\text{TotInj}_t^{iks} = \text{ReInj}_t^{iks} + \text{PInj}_t^{iks} + \text{OInj}_t^{iks}$. Then define the collections of endogenous variables $y2_t^{iks} = \left\{\text{Oil}_t^{iks}, \text{ID}_t^{iks}, \text{TotInj}_t^{iks}, \text{L}_t^{iks}, \text{K}_{t+1}^{iks}\right\}$, $\phi 2_t^{iks} = \left\{\phi_{3t}^{iks}, \phi_{4t}^{iks}, \phi_{7t}^{iks}, \phi_{13t}^{iks}, \phi_{14t}^{iks}\right\}$, and $\lambda 2_t^{iks} = \lambda_{1t}^{iks}$.. Also define the sets $Y2^{ks}$ with typical element $y2^{ks} = \left\{\left\{y1_t^{iks}\right\}_{t=1}^{\infty}\right\}_{i=1}^{I^k}$, $\Psi 2^{ks}$ with typical element $\phi 2^{ks} = \left\{\left\{\psi 1_t^{iks}\right\}_{t=1}^{\infty}\right\}_{i=1}^{I^k}$, and $\Lambda 2^{ks}$ with typical element $\lambda 2^{ks} = \left\{\left\{\lambda 2_t^{iks}\right\}_{t=1}^{\infty}\right\}_{i=1}^{I^k}$. Note that $\mathbf{x}^{ks} \in X^{ks} \rightarrow y2^{ks} \in Y2^{ks}$. Also note that the inequality constraints 3, 4, 7, 13, 14 can be redefined as functions $g2_{jt}^{iks}$ such that $g2_{jt}^{iks}\left(y2_t^{iks}\right) \geq 0$ if and only if $g_{wt}^{j,m}\left(\mathbf{x}^k\right) \leq 0$ given an appropriate choice of the index j. Consider the following subset of $21 \times I^k \times T$ conditions:

$$
\begin{aligned}
F2_{1t}^{iks} &= \left[P_t^{\text{Oil}}\left(1 - \varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right) + \sigma_1^{iks} - \tau_t^{iks,\text{Oil}} - MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^{iks}\right)\right](1 - T_t^s) \\
&\quad + \left[P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}}\right](1 - T_t^s)\ GOR^{iks}\left(1 - \vartheta^{iks}\right) \\
&\quad - \left(\phi_{3t}^{iks} + \phi_{4t}^{iks}\zeta\right)\left[1 + GOR^{iks}\right] + \phi_{5t}^{iks} = 0 \\
F2_{2t}^{iks} &= -\phi_{4t}^{iks} + \beta\phi_{3t+1}^{iks} + \beta\phi_{4t+1}^{iks} + \phi_{13t}^{iks} = 0 \\
F2_{3t}^{iks} &= -(1 - T_t^s) + \phi_{4t}^{iks}\frac{\partial D_t^{iks}\left(\text{ID}_t^{iks}, \text{L}_{t-1}^{iks}\right)}{\partial \text{ID}_t^i} - \lambda_{1t}^{iks} = 0 \\
F2_{4t}^{iks} &= \lambda_{1t}^{iks} + \beta\phi_{4t+1}^{iks}\frac{\partial D_{t+1}^{iks}\left(\text{ID}_{t+1}^{iks}, \text{L}_t^{iks}\right)}{\partial \text{L}_t^i} - \beta\lambda_{1t+1}^{iks} = 0 \\
F2_{5t}^{iks} &= C_t^{iks}(1 - T_t^s) - \phi_{4t}^{iks}\frac{\partial B_t^{iks}\left(\text{TotInj}_t^{iks}\right)}{\partial \text{TotInj}_t^{iks}} = 0 \\
E2_{1t}^{iks} &= \text{L}_t^{iks} - \text{L}_{t-1}^{iks} - \text{ID}_t^{iks} = 0 \\
I2_{1t}^{iks} &= \text{Oil}_t^{iks}\left(1 + GOR^{iks}\right) - \text{K}_t^{iks} \leq 0 \\
I2_{2t}^{iks} &= \text{K}_{t+1}^{iks} - \text{K}_t^{iks} - D_t^{iks}\left(\text{ID}_t^{iks}, \text{L}_{t-1}^{iks}\right) - B_t^{iks}\left(\text{TotInj}_t^{iks}\right) + \zeta\text{Oil}_t^{iks}\left(1 + GOR^{iks}\right) \leq 0 \\
I2_{3t}^{iks} &= \text{Oil}_t^{iks} \geq 0 \\
I2_{4t}^{i,k} &= \text{L}_t^{iks} \geq 0 \\
I2_{5t}^{iks} &= \text{K}_{t+1}^{iks} \geq 0 \\
M2_{jt}^{iks} &= \phi 2_{jt}^{iks} \geq 0,\ j = 1,2,3,4,5 \\
S2_{jt}^{iks} &= \phi 2_{jt}^{iks} g2_{jt}^{iks}\left(y2_t^{iks}\right) \geq 0\ j = 1,2,3,4,5
\end{aligned}
$$

In the system above, we have 21 unknown variables per period t and per field i. However, making use of the complimentary slackness conditions $S2_{jt}^{iks}$ for $j = 1,2,3,4,5$ we are left with 11 binding conditions per period t per field i. We have 11 unknown variables per period t and per field i: 5 in collection $y2_t^{ks}$, 5 in $\psi 2_t^{ks}$, plus $\lambda 2_t^{ks}$, all independent of the realizations of the variables outside of $Y2^{ks}\times\Psi 2^{ks}\times\Lambda 2^{ks}$. Thus the problem can be solved independently of the optimality of the other variables in the full problem. As we have shown that a solution to the full optimization problem exists, then a solution to the reduced problem must also exist. Thus, it must be the collection $\left\{y2^{ks*}, \psi 2^{ks*}, \lambda 2^{ks*}\right\}$ that solve this system. In particular, note that given that the change in the tax rate on oil production satisfies:

$$\Delta\tau_t^{iks,\text{Oil}} = -GOR^{iks}\left(1-\vartheta^{iks}\right)\Delta\tau_t^{s,\text{Gas}} \tag{88}$$

then all the equilibrium conditions of reduced problem 2 are unaffected by the tax reform. Therefore, the optimal levels of all endogenous variables in $y2_t^{ks}$ $\psi 2_t^{ks}$, and $\lambda 2_t^{ks}$ for all $t = 1, 2, ...$ – including Oil_t^{iks}– are unaffected by the policy change from $\mathbf{T}^s$ to $\check{\mathbf{T}}^s$.

Reduced problem 3. Lastly, the optimal values of Z_t^{iks}, Gas_t^{iks}, ϕ_{1t}^{iks}, ϕ_{2t}^{iks} can be pinned down using the optimal values of the other endogenous variables from $y1^{ks*}, \phi 1^{ks*}$ and $y2^{ks*}, \phi 2^{ks*}$, plus the four conditions:

$$\begin{aligned}
F3_{1t}^{iks} = \quad & \left[P_t^{s,\text{Gas}} - \tau_t^{s,\text{Gas}} - MC_t^{iks,\text{Gas}}\right](1-T_t^s) + \left(\phi_{2t}^{iks} - \phi_{3t}^{iks*} - \phi_{4t}^{iks*}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1} = 0 \\
F3_{2t}^{iks} = \quad & = (1-T_t^s) - \phi_{1t}^{iks}\frac{\partial TF_t^{iks}(.)}{\partial \text{Z}_t^{iks}} = 0 \\
I3_{1t}^{iks} = \quad & TF_t^{iks}(\text{Oil}_t^{iks*}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks*}, \text{ReInS}_t^{iks*}, \text{PInS}_t^{iks*}, \text{M}_t^{iks*}, \text{Z}_t^{iks}) \leq 0 \\
I3_{2t}^{iks} = \quad & GOR^{iks}\text{Oil}_t^{iks*} - \left(1-\vartheta^{iks}\right)^{-1}\left[\text{Gas}_t^{iks} + \text{Flare}_t^{iks*} + \text{IVent}_t^{iks*}\right. \\
& \left. - Maint_t^{iks}\left(GOR^{iks}\text{Oil}_t^{iks*}, \text{M}_t^{iks*}\right) + \text{ReInj}_t^{iks*} + \text{ReInS}_t^{iks*}\right] \leq 0
\end{aligned},$$

which are all binding. Specifically, we have shown that $I3_{2t}^{iks}$ is binding in Step 1 of this proof, and that $I3_{1t}^{iks}$ is binding in the proof to Proposition 1. Thus, after substituting the optimal values of $y1^{ks*}, \phi 1^{ks*}$ and $y2^{ks*}, \phi 2^{ks*}$ from the corresponding reduced problems into conditions $F3_{1t}^{iks}$, $F3_{2t}^{iks}$, $I3_{1t}^{iks}$ and $I3_{2t}^{iks}$ and set the conditions to hold with strict equality, we can solve for Z_t^{iks} and Gas_t^{iks} from conditions $I3_{1t}^{iks}$ and $F3_{2t}^{iks}$, and for ϕ_{2t}^{iks} from condition $F3_{1t}^{iks}$, for all fields $i = 1, 2, ..., I^k$ and all periods $t = 1, 2, ...$

Step 5. The oil firms' field-level and total optimal oil supply is unaffected by the tax reform at constant global oil price P_t^{Oil},constant gas price $P_t^{s,\text{Gas}}$, and constant aggregate oil supply from firms other than k. Suppose the aggregate oil supply from firms other than k; i.e., $\sum_{l\neq k}\sum_{i=1}^{I^l}\text{Oil}_t^{ils}$, is unaffected by the tax reform. Then recall that the formula for the tax reform implies $\Delta\tau_t^{iks,\text{Oil}}GOR^{iks}\left(1-\vartheta^{iks}\right) = -\Delta\tau_t^{s,\text{Gas}}$. Thus, the formula for the optimal positive oil production for field i in period t, which writes:

$$\left[P_t^{\text{Oil}}\left(1-\varsigma_t^{\text{Oil}} MS_t^{k,\text{Oil}}\right)+\sigma_1^{iks}-\tau_t^{iks,\text{Oil}}-MC_t^{iks,\text{Oil}}\left(\text{Oil}_t^i\right)\right]\left(1-T_t^s\right)$$
$$+\left[P_t^{s,\text{Gas}}-\tau_t^{s,\text{Gas}}-MC_t^{iks,\text{Gas}}\right]\left(1-T_t^s\right)\ GOR^{iks}\left(1-\vartheta^{iks}\right)$$
$$-\left(\phi_{3t}^{iks}+\phi_{4t}^{iks}\zeta\right)\left[1+GOR^{iks}\right]+\phi_{5t}^{iks}=0$$

is unaffected by the reform. The tax reform does not enter any other condition of the reduced problem 2, implying in turn that the optimal values of $\text{Oil}_t^{iks}, \text{ID}_t^{iks}, \text{TotInj}_t^{iks}, \text{L}_t^{iks}, \text{K}_{t+1}^{iks}, \phi_{3t}^{iks}, \phi$ are unchanged for all $i = 1, 2, .., I^k$ and all $t = 1, 2, ...$ As a result, the aggregate oil supply from all the fields owned by firm k in period t; i.e., $\sum_{i=1}^{I^k} \text{Oil}_t^{iks}$ is also unchanged, as well as $MS_t^{ks,\text{Oil}}$, which is a function of $\sum_{i=1}^{I^k} \text{Oil}_t^{iks}$, of the aggregate oil supply from firms other than k and of the oil price P_t^{Oil}. Note that the restrictions imposed on oil fields imply that oil production is always profitable, i.e., the constraint $\text{Oil}_t^{iks} \leq 0$ is never binding. That is, an oil field hit the zero production point only if $\text{K}_t^{iks} = 0$; i.e., the field does not have any residual capacity because the net return to investments in discoveries and/or injections have become weakly negative.

Step 5/b. First, because each oil firm's optimal oil supply is unaffected by the tax reform at constant global oil price P_t^{Oil},constant gas price $P_t^{s,\text{Gas}}$, and constant aggregate oil supply from firms other than k, then the global oil supply is also unchanged at any given global oil price P_t^{Oil}. Second, the new tax on gas fields ensures that the aggregate gas supply of country s is unchanged, ensuring that the national and the global natural gas supply are also unchanged at any given national gas prices $\left\{P_t^{s,\text{Gas}}\right\}_{s=1}^{S}$. Third, because the demand for all goods is unaffected by the tax reform then the equilibrium prices must be unchanged too. Under the assumptions imposed on B^{iks}, if $\text{PInS}_t^{iks} = 0$ we get for gas only fields $MP_t^{iks,\text{ReInS}}\left(\text{ReInS}_t^{iks}\right) = MP_t^{iks,\text{ReInS}} > 0$. Let us consider fields such that $\text{ReInS}_t^{iks} > 0$, we obtain that the production of gas-only fields solves:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{Gas}_t^{iks}} &= \left[P_t^{s,\mathrm{Gas}}\left(1-\varsigma_t^{\mathrm{Gas}} MS_t^{ks,\mathrm{Gas}}\left(\mathrm{Gas}_t^{iks}\right)\right)-\tau_t^{iks,\mathrm{Gas}}\right]\left(1-T_t^s\right) \\
&\quad -\phi_{1t}^{iks}\frac{\partial TF_{1t}^{iks}(.)}{\partial \mathrm{Gas}_t^i}-\left(\phi_{3t}^{iks}+\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{ReInS}_t^{iks}} &= \phi_{1t}^{iks}\frac{\partial TF_{4t}^{iks}(.)}{\partial \mathrm{ReInS}_t^{iks}}-\left(\phi_{3t}^{iks}+\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}+\phi_{11t}^{iks}=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{Z}_t^{iks}} &= \left(1-T_t^s\right)-\phi_{1t}^{iks}\frac{\partial TF_t^{iks}(.)}{\partial \mathrm{Z}_t^{iks}}=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{K}_{t+1}^{iks}} &= -\phi_{4t}^i+\beta\phi_{3t+1}^i+\beta\phi_{4t+1}^i+\phi_{14t}^i=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{ID}_t^i} &= -\left(1-T_t^s\right)+\phi_{4t}^{iks}\frac{\partial D_t^{iks}\left(\mathrm{ID}_t^{iks},\mathrm{L}_{t-1}^{iks}\right)}{\partial \mathrm{ID}_t^i}-\lambda_{1t}^{iks}=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{L}_t^i} &= \lambda_{1t}^{iks}+\beta\phi_{4t+1}^{iks}\frac{\partial D_{t+1}^{iks}\left(\mathrm{ID}_{t+1}^{iks},\mathrm{L}_t^{iks}\right)}{\partial \mathrm{L}_t^i}-\beta\lambda_{1t+1}^{iks}+\phi_{13t}^{iks}=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{IM}_t^{iks}} &= -\left(1-T_t^{iks}\right)+\lambda_{2t}^{iks}=0 \\
\frac{\partial \mathcal{L}_u^{ks}}{\partial \mathrm{M}_{t+1}^{iks}} &= -\phi_{1t+1}^{iks}\frac{\partial TF_{5t+1}^{iks}(.)}{\partial \mathrm{M}_{t+1}^{iks}}-\lambda_{2t}^{iks}+\beta\lambda_{2t+1}^{iks}\left(1-\rho^{iks}\right)+\phi_{12t}^{iks} \\
&\quad -\beta\left[\phi_{3t+1}^{iks}+\phi_{4t+1}^{iks}\zeta+\delta_{1t+1}^{iks}\right]\frac{\partial UVent_{t+1}^{iks}(.)}{\partial \mathrm{M}_{t+1}^{iks}}=0 \\
EG_{1t}^{iks} &= \mathrm{L}_t^{iks}-\mathrm{L}_{t-1}^{iks}-\mathrm{ID}_t^{iks}=0 \\
EG_{2t}^{iks} &= \mathrm{M}_{t+1}^{iks}-\mathrm{M}_t^{iks}\left(1-\rho^{iks}\right)-\mathrm{IM}_t^{iks}=0 \\
IG_{1t}^{iks} &= TF_t^{iks}(0,\mathrm{Gas}_t^{iks},\mathrm{NRF}_t^{iks},\mathrm{ReInS}_t^{iks},0,\mathrm{M}_t^{iks},\mathrm{Z}_t^{iks})\le 0 \\
IG_{2t}^{iks} &= \left(1-\vartheta^{iks}\right)^{-1}\left[\mathrm{Gas}_t^{iks}+\mathrm{Flare}_t^{iks}+\mathrm{IVent}_t^{iks}-Maint_t^{iks}\left(\mathrm{M}_t^{iks}\right)+\mathrm{ReInS}_t^{iks}\right]-\mathrm{K}_t^{iks}\le 0 \\
IG_{3t}^{iks} &= \mathrm{K}_{t+1}^{iks}-\mathrm{K}_t^{iks}-D_t^{iks}\left(\mathrm{ID}_t^{iks},\mathrm{L}_{t-1}^{iks}\right) \\
&\quad +\zeta\left(1-\vartheta^{iks}\right)^{-1}\left[\mathrm{Gas}_t^{iks}+\mathrm{Flare}_t^{iks}+\mathrm{IVent}_t^{iks}-Maint_t^{iks}\left(\mathrm{M}_t^{iks}\right)+\mathrm{ReInS}_t^{iks}\right]\le 0
\end{aligned}
$$

Plus $\phi_{1t}^{iks}\ge 0,\phi_{3t}^{iks}\ge 0,\phi_{4t}^{iks}\ge 0,\phi_{9t}^{iks}\ge 0,\phi_{10t}^{iks}\ge 0,\phi_{11t}^{iks}\ge 0,\phi_{12t}^{iks}\ge 0$ and the usual complementary slackness conditions. Using these FOCs, the optimality condition for gas production writes:

$$
\left[P_t^{s,\mathrm{Gas}}\left(1-\varsigma_t^{\mathrm{Gas}}MS_t^{ks,\mathrm{Gas}}\left(\mathrm{Gas}_t^{iks}\right)\right)-\tau_t^{iks,\mathrm{Gas}}-MC_t^{iks,\mathrm{Gas}}\left(\mathrm{Gas}_t^{iks}\right)-MP_t^{iks,\mathrm{ReInS}}\right]\left(1-T_t^s\right)=0 \tag{89}
$$

which can be differentiated to obtain the marginal effect of an increase in the linear tax rate on gas production:

$$\begin{aligned}\frac{\partial \text{Gas}_t^{iks}}{\partial \tau_t^{s,\text{Gas}}} &= -\left[P_t^{s,\text{Gas}} \frac{\partial \varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\left(\text{Gas}_t^{iks}\right)}{\partial \text{Gas}_t^{iks}} + \frac{\partial MC_t^{iks,\text{Gas}}\left(\text{Gas}_t^{iks}\right)}{\partial \text{Gas}_t^{iks}}\right]^{-1} \\ &= -\frac{\partial \text{Gas}_t^{iks}}{\partial P_t^{s,\text{Gas}}}\left(1-\varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\right)^{-1}\end{aligned} \tag{90}$$

Thus, the effect of a change in the linear tax rate on gas production on the gas supply of gas-only filed i writes:

$$\varDelta \text{Gas}_t^{iks} \simeq -\min\left\{\frac{\partial \text{Gas}_t^{iks}}{\partial P_t^{s,\text{Gas}}}\left(1-\varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\right)^{-1} \varDelta\tau_t^{iks,\text{Gas}}, \text{Gas}_t^{iks}\right\} \tag{91}$$

Because the tax adjustment is small, we can approximate the result by assuming that for all active gas-only fields the the corrective tax affects production only at the intensive margin. Thus, after setting $\varDelta\tau_t^{iks,\text{Gas}} = \varDelta\tau_t^{s,\text{Gas}}\left(1-\varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\right)$ we obtain a formula for the effect of the corrective tax, which writes:

$$\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} \varDelta \text{Gas}_t^{iks} \mathbf{1}\left[\text{Oil}_t^{iks}=0\right] \simeq -\eta_{\text{G},P}^{s,GO}\left(\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} \text{Gas}_t^{iks} \mathbf{1}\left[\text{Oil}_t^{iks}=0\right]\right)\frac{\varDelta\tau_t^{s,\text{Gas}}}{P_t^{s,\text{Gas}}} \tag{92}$$

where $\eta_{\text{G},P}^{s,GO}$ denotes the aggregate own-price elasticity of gas supply by gas-only fields. Also note that the optimal values of maintenance is unaffected, because the equilibrium condition for M_{t+1}^{iks} imply that either $\phi_{12t}^{i} > 0$ and M_{t+1}^{iks}, such that the optimal level of M_{t+1}^{iks} is unaffected by marginal changes in $\tau_t^{iks,\text{Gas}}$, or $\phi_{12t}^{iks} = 0$ and the optimality condition writes:

$$\left(1-T_t^s\right)\left[MP_{t+1}^{iks,\text{M}}\left(\text{M}_{t+1}^{iks}\right) - 1 + \beta\left(1-\rho^{iks}\right)\right] + \beta\left(1-T_t^i\right)\frac{MP_t^{i,\text{ReInS}}}{1+\vartheta^i}\frac{\partial Maint_{t+1}^{iks}(.)}{\partial \text{M}_{t+1}^{iks}} = 0$$

which is constant in $\tau_t^{iks,\text{Gas}}$. Lastly, we need to ensure that the tax is set to a level such that the reduced gas supply from gas-only fields exactly offsets the increased supply by gas fields. Thus, the optimal corrective tax solves:

$$\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} \varDelta \text{Gas}_t^{iks} \mathbf{1}\left[\text{Oil}_t^{iks}=0\right] = -\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} \varDelta \text{Gas}_t^{iks} \mathbf{1}\left[\text{Oil}_t^{iks}>0\right] \tag{93}$$

where $\varDelta \text{Gas}_t^{iks}$ is the change in gas supply by field i. The above equation solves for:

$$\frac{\varDelta\tau_t^{iks,\text{Gas}}}{P_t^{s,\text{Gas}}} = \frac{1-\varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}}{\eta_{\text{G},P}^{s,GO}} \frac{\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} \Delta \text{Gas}_t^{iks} \mathbf{1}\left[\text{Oil}_t^{iks}>0\right]}{\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} \text{Gas}_t^{iks} \mathbf{1}\left[\text{Oil}_t^{iks}=0\right]} \tag{94}$$

which represents the optimal adjustment tax,; i.e., one that ensures zero change in the aggregate country-level and global natural gas supply.

$$AS_t^{r,\text{Gas}}(\mathbf{P};\mathbf{T}) = \sum_{k=1}^{K^r}\sum_{i=1}^{I^k}\text{Gas}_t^{ikr*}(\mathbf{P};\mathbf{T}) \tag{95}$$

Thus, given that the tax reform is such that $\Delta\tau_t^{s,\text{Gas}}$ solves equation (94), the aggregate supply of gas $AS_t^{r,\text{Gas}}(\mathbf{P};\mathbf{T}^r)$ by each country $r = 1, 2, ..., S$ – including $r = s$ – is unaffected by the tax scheme at any given price collection $\mathbf{P} = \{\mathbf{P}_t\}_{t=1}^{\infty}$; i.e.,

$$AS_t^{r,\text{Gas}}(\mathbf{P};\mathbf{T}) = AS_t^{r,\text{Gas}}(\mathbf{P};\check{\mathbf{T}})$$

Step 6. Let $\text{Gas}_t^{ikr*}(\mathbf{P};\mathbf{T}^r)$ and $\text{Oil}_t^{ikr*}(\mathbf{P};\mathbf{T}^r)$ denote the supply of natural gas and oil by field i in period t, respectively. Under the proposed tax scheme, we get that in each period t the following results hold true:

1. The aggregate supply of gas $AS_t^{r,\text{Gas}}(\mathbf{P};\mathbf{T}^r)$ by each country $r = 1, 2, ..., S$ – including $r = s$ – is unaffected by the tax scheme at any given price collection $\mathbf{P} = \{\mathbf{P}_t\}_{t=1}^{\infty}$ by Step 5 bis; i.e.,

   $$AS_t^{r,\text{Gas}}(\mathbf{P};\mathbf{T}) = AS_t^{r,\text{Gas}}(\mathbf{P};\check{\mathbf{T}})$$

   for all $r = 1, 2, ..., S$ and all $t = 1, 2, ...$

2. The supply of oil by each field i in country s is unaffected by the tax scheme at any given price collection$\mathbf{P} = \{\mathbf{P}_t\}_{t=1}^{\infty}$ by Step 5. Moreover, the supply of oil by each field i in country $r \neq s$ is also unaffected by the tax scheme implemented by country s, except possibly through price effects. Thus, at any given price collection $\mathbf{P}$. Thus, the supply of oil by each field i in country $r \neq s$ is unaffected by the tax scheme at any given price collection $\mathbf{P}$. As a result, the aggregate global supply function of crude by each country $r = 1, 2, ..., S$ – including $r = s$ – is unaffected by the tax scheme at any given price collection $\mathbf{P}$; i.e.,

   $$AS_t^{\text{Oil}}(\mathbf{P};\mathbf{T}^r) = AS_t^{\text{Oil}}(\mathbf{P};\check{\mathbf{T}})$$

   for all $t = 1, 2, ...$

3. The aggregate supply of all consumption goods y_{bt}^{js} for $b = 1, 2, ..., B$ by each country $r = 1, 2, ..., S$ – including $r = s$ – is unaffected by the tax scheme at any given price collection $\mathbf{P}$. Thus, the aggregate supply function of a given

consumption good from country r is unchanged in each country $r = 1, 2, ..., S$; i.e.,

$$AS^r_{bt}\left(\mathbf{P};\mathbf{T}\right) = AS^r_{bt}\left(\mathbf{P};\check{\mathbf{T}}\right)$$

for all $r = 1, 2, ..., S$, all $b = 1, 2, ..., B$, and all $t = 1, 2, ...$

4. The aggregate demand function in each country r of gas $AD^{r,\text{Gas}}_t\left(\mathbf{P};\mathbf{T}\right)$ and all consumption goods $AD^r_{bt}\left(\mathbf{P};\mathbf{T}\right)$, as well as the global demand for crude oil $AD^{\text{Oil}}_t\left(\mathbf{P};\mathbf{T}\right)$ are all unaffected by the tax collection $\check{\mathbf{T}}$ relative to $\mathbf{T}$ because no tax change affects midstream and downstream markets. Because all the aggregate demand and all the aggregate supply functions are unaffected by the tax reform, the market equilibrium in each period $t = 1, 2, ...$ under scheme $\check{\mathbf{T}}$ solves the same system of $[1 + s \times (B + 1)]$ equations per period that delivers the equilibrium price vector under scheme $\mathbf{T}$. Thus, the equilibrium price collection $\mathbf{P}$ that clears all the markets in each period t under scheme $\mathbf{T}$, also clears all the markets in each period t under the tax scheme $\check{\mathbf{T}}$. Thus, we have shown that the proposed tax reform has no effect on the equilibrium prices of oil, gas, and of all consumption goods. Q.E.D.

Part (i). Recall that the FOCs of the oil&gas firm's problem w.r.t. IVent^{iks}_t, Gas^{iks}_t, and Z^{iks}_t write:

$$-VF^{iks}_t \times \frac{PF^{iks}_t\left(\cdot\right)}{\partial \text{IVent}^{iks}_t} + \left(\phi^{iks}_{2t} - \phi^{iks}_{3t} - \phi^{iks}_{4t}\zeta\right)\left(1 - \vartheta^{iks}_t\right)^{-1} + \phi^{iks}_{9t} = 0\ , \tag{96}$$

$$\begin{array}{l}\left[P^{s,\text{Gas}}_t\left(1 - \varsigma^{\text{Gas}}_t MS^{ks,\text{Gas}}_t\right) - \tau^{s,\text{Gas}}_t\right]\left(1 - T^s_t\right) \\ -\phi^{iks}_{1t}\frac{\partial TF^{iks}_{2t}(.)}{\partial \text{Gas}^{iks}_t} + \left(\phi^{iks}_{2t} - \phi^{iks}_{3t} - \phi^{iks}_{4t}\zeta\right)\left(1 - \vartheta^{iks}_t\right)^{-1}\end{array} = 0\ , \tag{97}$$

and

$$\left(1 - T^i_t\right) + \phi^{iks}_{1t}\frac{\partial TF^{iks}_{6t}(.)}{\partial \text{Z}^{iks}_t} = 0\ , \tag{98}$$

respectively. Recall that IVent^{iks}_t is equal to zero if the FOC in 96 is satisfied at $\text{IVent}^{iks}_t = 0$. Combining the three conditions in (96), (97), and (98) and using $\phi^{iks}_{9t} \geq 0$, one gets that IVent^{iks}_t is equal to zero if:

$$\left[P^{s,\text{Gas}}_t\left(1 - \varsigma^{\text{Gas}}_t MS^{ks,\text{Gas}}_t\right) - \tau^{iks,\text{Gas}}_t - MC^{iks,\text{Gas}}_t\right]\left(1 - T^s_t\right) \geq -VF^{iks}_t\frac{\partial PF^{iks}_{t+1}\left(\cdot\right)}{\partial \text{IVent}^{iks}_t} \tag{99}$$

at $\text{IVent}_t^{iks} = 0$ and for all possible values of Flare_t^{iks}. Thus, a sufficient condition for (99) to hold true for all oil&gas fields is to set a uniform tax rebate $\Delta\tau_t^{iks,\text{Gas}} = \Delta\tau_t^{s,\text{Gas}}$ for all i with formula:

$$\Delta\tau_t^{s,\text{Gas}} = \min_{i \in \left\{\left\{1,2,\dots,I^k\right\}\right\}_{k=1}^{K^s}} \left\{P_t^{s,\text{Gas}} - \tau_t^{iks,\text{Gas}} - MC_t^{iks,\text{Gas}}\right\} \tag{100}$$

where $\tau_t^{iks,\text{Gas}}$ denotes the marginal tax rate on gas sales faced by any field i given the current tax framework. Note that $\Delta\tau_t^{s,\text{Gas}}$ is typically negative.

Part (ii). Recall that the FOCs of the oil&gas firm's problem w.r.t. Flare_t^{iks} writes:

$$\begin{array}{c} -\tau_t^{iks,\text{Flare}}\left(1 - T_t^s\right) - VF_t^{iks} \times \frac{PF_t^{iks}(\cdot)}{\partial \text{Flare}_t^{iks}} - \phi_{1t}^{iks} \frac{\partial TF_{3t}^{iks}(.)}{\partial \text{Flare}_t^{iks}} \\ + \left(\phi_{2t}^{iks} - \phi_{3t}^{iks} - \phi_{4t}^{iks}\zeta\right)\left(1 - \vartheta_t^{iks}\right)^{-1} + \phi_{8t}^{iks} = 0 \end{array} , \tag{101}$$

with $\phi_{8t}^{iks} \geq 0$. Combining the condition in (101) with those in (97) and (98), one gets that Flare_t^{iks} is equal to its lower bound NRF_t^{iks} if:

$$\begin{array}{c} \left[P_t^{s,\text{Gas}}\left(1 - \varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\right) - \tau_t^{s,\text{Gas}} - MC_t^{iks,\text{Gas}}\right]\left(1 - T_t^s\right) \geq \\ \left[-MC_t^{iks,\text{Flare}}(0) - \tau_t^{iks,\text{Flare}}\right]\left(1 - T_t^s\right) - VF_t^{iks} \times \frac{PF_t^{iks}(\cdot)}{\partial \text{Flare}_t^{iks}} \end{array} \tag{102}$$

for all possible values of IVent_t^{iks}. Thus, a sufficient condition for (102) to hold true for all oil&gas fields i is:

$$\left[P_t^{s,\text{Gas}}\left(1 - \varsigma_t^{\text{Gas}} MS_t^{ks,\text{Gas}}\right) - \tau_t^{s,\text{Gas}} - MC_t^{iks,\text{Gas}}\right] \geq \tau_t^{iks,\text{Flare}} \tag{103}$$

Note that formula (100) implies:

$$\Delta\tau_t^{s,\text{Gas}} \leq P_t^{s,\text{Gas}} - \tau_t^{s,\text{Gas}} - MC_t^{iks,\text{Gas}} \tag{104}$$

Using the definition of the tax rebate from part (i), $\Delta\tau_t^{s,\text{Gas}} = \breve{\tau}_t^{iks,\text{Gas}} - \tau_t^{iks,\text{Gas}}$, and substituting (104) into the sufficient condition for zero routine flaring in (103), we obtain that – given $\Delta\tau_t^{s,\text{Gas}}$ from (100) for all $i = 1, 2, ...I^k$ and all $k = 1, 2, ...K^s$, a sufficient condition for zero routine flaring by each foil&gas firm $k = 1, 2, ...K^s$ is:

$$\tau_t^{iks,\text{Flare}} \leq 0 \ \forall i = 1, 2, ..., I^k \ , \tag{105}$$

which implies that, as long as no flaring tax is introduced; i.e., $\tau_t^{iks,\text{Flare}} = 0$ for all $i = 1, 2, ..., I^k$, the tax rebate $\Delta\tau_t^{s,\text{Gas}}$ ensures zero routine flaring by all oil&gas firms.

Part (iii). Recall that the optimality condition for M_{t+1}^{iks} for an oil&gas firm with respect to oil field i in period $t+1$ writes:

$$\begin{aligned} &\beta\left(1-T_{t+1}^{s}\right) MPE_{t+1}^{iks}\left(\mathrm{M}_{t+1}^{iks}\right)-\left[\left(1-T_{t}^{s}\right)-\beta\left(1-\rho^{iks}\right)\left(1-T_{t+1}^{s}\right)\right]+\phi_{12t}^{iks} \\ &+\beta\left[VF_{t+1}^{iks}\frac{\partial PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}+\left(P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right)\left(1-T_{t+1}^{s}\right)\right]\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}=0 \end{aligned} \tag{106}$$

Recall that $\Delta\delta_{1t+1}^{iks}=-\Delta\tau_{t+1}^{s,\mathrm{Gas}}$ for oil fields and $\Delta\delta_{1t+1}^{iks}=0$ for gas fields. Because the optimality condition for M_{t+1}^{iks} in (106) is unaffected by changes in tax rates other than $\tau_{t+1}^{iks,\mathrm{Gas}}$, it is sufficient to show that $\frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{iks,\mathrm{Gas}}}\leq 0$. If the twelfth constraint is non-binding ($\phi_{12t+1}^{iks}>0$), then $\mathrm{M}_{t+1}^{iks}=0$ and the derivative of interest is simply $\frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{iks,\mathrm{Gas}}}=0$. If the the twelfth constraint is binding ($\phi_{12t+1}^{iks}=0$), then given the convexity of the firm's maximization problem (see proof to Proposition 2), we can totally differentiate the F.O.C. w.r.t. $\tau_{t+1}^{iks,\mathrm{Gas}}$ to obtain:

$$\begin{aligned} &-\beta\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}\left(1-T_{t+1}^{s}\right)+\beta\left\{VF_{t+1}^{iks}\frac{\partial^2 PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t}^{iks\ 2}}\right. \\ &+\left(1-T_{t+1}^{s}\right)\left[P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right]\frac{\partial^2 Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks\ 2}} \\ &\left.+\left(1-T_{t+1}^{s}\right)\frac{\partial MPE_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}\right\}\frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{iks,\mathrm{Gas}}}=0 \end{aligned} \tag{107}$$

Where the second term in equation (107) is weakly negative, as shown in the proof to Proposition 2. In particular, if such term is equal to zero at the optimal level of M_{t+1}^{iks}, then the FOC in (106) does not have a unique solution for M_{t+1}^{iks}, implying that the optimal value of M_{t+1}^{iks} is pinned down by the constraint $\mathrm{M}_{t+1}^{iks}-\mathrm{M}_{t}^{iks}\left(1-\rho^{iks}\right)-\mathrm{IM}_{t}^{iks}\leq 0$ and it is therefore constant in $\tau_{t+1}^{iks,\mathrm{Gas}}$. Instead, if the second term in equation (107) is strictly negative, then we can solve (107) with respect to $\frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{iks,\mathrm{Gas}}}$ and obtain::

$$\begin{aligned} \frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{iks,\mathrm{Gas}}}= &\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}\left\{\frac{VF_{t+1}^{iks}}{1-T_{t+1}^{s}}\frac{\partial^2 PU_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t}^{iks\ 2}}\right. \\ &\left.+\left[P_{t+1}^{s,\mathrm{Gas}}-\tau_{t+1}^{iks,\mathrm{Gas}}-MC_{t+1}^{iks,\mathrm{Gas}}\right]\frac{\partial^2 Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks\ 2}}-\frac{\partial MPE_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}\right\}^{-1}\leq 0 \end{aligned}, \tag{108}$$

where the first term $\frac{\partial Maint_{t+1}^{iks}(\cdot)}{\partial \mathrm{M}_{t+1}^{iks}}$ is strictly positive by assumption and the second term is strictly negative, implying that $\frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{iks,\mathrm{Gas}}}\leq 0$.

Because the optimality condition for M_{t+1}^{iks} in (106) is unaffected by changes in tax rates other than $\tau_{t+1}^{iks,\mathrm{Gas}}$ and δ_{1t+1}^{iks}, it is sufficient to consider the effect of the change in these two tax variables. If the twelfth constraint is binding ($\phi_{12t+1}^{iks}>0$), then $\mathrm{M}_{t+1}^{iks}=0$ and the derivative of interest is simply $\frac{d\mathrm{M}_{t+1}^{iks}}{d\tau_{t+1}^{s,\mathrm{Gas}}}=\frac{d\mathrm{M}_{t+1}^{iks}}{d\delta_{1t+1}^{iks}}=0$. If the the twelfth constraint is not binding ($\phi_{12t+1}^{iks}=0$), then the optimal level of M_{t+1}^{iks} solves equation (106) evaluated at $\phi_{12t+1}^{iks}=0$. In this case, $\varDelta\delta_{1t+1}^{iks}=-\varDelta\tau_{t+1}^{s,\mathrm{Gas}}$ implies that the equation (106) is unaffected by the introduction of the reform, therefore the solution to the equation is also unchanged.

The optimality conditions for ReInj_t^{iks} , PInj_t^{iks} and OInj_t^{iks} for an oil&gas firm with respect to oil field i in period t write:

$$\begin{gathered}\left[-\frac{IC_t^{iks}\left(\mathrm{ReInj}_t^{iks},\mathrm{PInj}_t^{iks}\right)}{\partial\mathrm{ReInj}_t^{iks}}+\delta_{1t}^{iks}-\left(P_t^{s,\mathrm{Gas}}-\tau_t^{isk,\mathrm{Gas}}-MC_t^{iks,\mathrm{Gas}}\right)\right](1-T_t^s)\\ +\phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial\mathrm{ReInj}_t^{iks}}+\phi_{8t}^{iks}=0\\ \left[-\frac{IC_t^{iks}\left(\mathrm{ReInj}_t^{iks},\mathrm{PInj}_t^{iks}\right)}{\partial\mathrm{PInj}_t^{iks}}-PP_t^{iks,\mathrm{Gas}}\right](1-T_t^s)+\phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial\mathrm{PInj}_t^{iks}}=0\\ -C_t^{iks}\left(1-T_t^s\right)+\phi_{4t}^{iks}\frac{\partial B_t^{iks}(\cdot)}{\partial\mathrm{OInj}_t^{iks}}=0\end{gathered}\tag{109}$$

plus $\phi_{8t}^{iks}\geq 0$ and $\mathrm{ReInj}_t^{iks}\geq 0$. First, note that $\varDelta\delta_{1t}^{iks}=-\varDelta\tau_t^{s,\mathrm{Gas}}$ implies that the FOC w.r.t. ReInj_t^{iks} is unaffected by the tax reform. Because the third condition is always binding (see Step 3 of the proof to part (iv)), then it can be substituted into the other two conditions, implying that the FOC w.r.t. PInj_t^{iks} is also unaffected by the tax reform. Lastly, we know that ϕ_{4t}^{iks} is unaffected by the tax reform from Step 4 of part (iv) of this proof. Thus, the optimal values of ReInj_t^{iks}, PInj_t^{iks}, OInj_t^{iks}, and ϕ_{8t}^{iks} given the optimal value of ϕ_{4t}^{iks}, which is unaffected by the tax reform, solve a system of four equations corresponding to four binding conditions (the three equations in (109) plus either $\phi_{8t}^{iks}=0$ or $\mathrm{ReInj}_t^{iks}=0$) with four unknown. Because all the equations are unaffected by the tax reform, the solution to the system must be the same. In the same way, it is possible to show that the optimal value of ReInS_t^{iks} and PInS_t^{iks} are both unaffected by the tax reform. Q.E.D.

Part (v). Proof. All prices are unaffected by the tax reform by part (iv), and the tax reform does not directly affect the optimality conditions of any midstream firm and any consumer in each period t. Thus, any individual demand function from either midstream firms (demand for G_t^{js} and O_t^{js}) or consumers (demand for y_{bt}^{js} for $b=1,2,...,B$) evaluated at the equilibrium price collection $\mathbf{P}=\{\mathbf{P}_t\}_{t=1}^{\infty}$ must deliver the same equilibrium demand levels under both scheme $\mathbf{T}^s$ and $\check{\mathbf{T}}^s$. As a consequence, each corresponding aggregate demand must also be unchanged under scheme $\check{\mathbf{T}}^s$ relative to scheme $\mathbf{T}^s$.

Part (v). Proof. Note that all midstream and downstream GHG emissions are unchanged because the optimal choice of each midstream firm and each consumer is unaffected by the tax reform. In the upstream level, however, the emissions in each period t change as follows:

$$\begin{aligned}\mathit{\Delta Emissions}_t^s = \; & \textstyle\sum_{k=1}^{K^s}\sum_{i=1}^{I^K} CI^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks} + CI^{\text{Flare}}\Delta\text{Flare}_t^{iks} \\ & +CI^{\text{Vent}}\left(\Delta\text{IVent}_t^{iks} + \Delta\text{UVent}_t^{iks}\right) \\ & +CI^{iks,\text{ReInj}}\Delta\text{ReInj}_t^{i} + CI^{iks,\text{ReIns}}\Delta\text{ReIns}_t^{i}\end{aligned} \tag{110}$$

Note that $\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\Delta\text{Gas}_t^{iks} = 0$. Thus, as long as the average carbon intensity of gas extraction is the same, then $\sum_{k=1}^{K^s}\sum_{i=1}^{I^k} CI^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks} = 0$. All the other terms: $\Delta\text{Flare}_t^{iks}$, $\Delta\text{IVent}_t^{iks}$, $\Delta\text{UVent}_t^{iks}$, $\Delta\text{ReInj}_t^{iks}$, $\Delta\text{ReIns}_t^{iks}$ are weakly negative, with $\Delta\text{Flare}_t^{iks}$ strictly negative for at least one field i. Thus, $\mathit{\Delta Emissions}_t^s < 0$. On the other hands, all the productive choices made by firms in countries other than s are unaffected by the tax reform, implying that the emissions in such countries are also unchanged; i.e., $\mathit{\Delta Emissions}_t^r = 0$ for all $r \neq s$. Lastly, these two results together imply $\mathit{\Delta Emissions}_t = \sum_{r=1}^{S}\mathit{\Delta Emissions}_t^s < 0$. Q.E.D.

Part (vi). Proof. First we show that firm's profits are weakly larger unchanged under $\check{\mathbf{T}}^s$ relative to $\mathbf{T}^s$. Using the finding that the first constraint $TF_t^{iks}(\cdot) \leq 0$ is always binding (see proof to part (iii)) and that Π_t^{iks} is linear in Z_t^{iks}, we can define the function at $Z_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInS}_t^{iks}, \text{PInS}_t^{iks}, \text{M}_t^{iks}\right)$ such that $TF_t^{iks}(\cdot) = 0$ at $\text{Z}_t^{iks} = Z_t^{iks}(\cdot)$, and use it to re-define the profit function as:

$$\begin{aligned}\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks};\mathbf{T}^s\right) = \; & \Pi_t^{iks}\left(\text{Oil}_t^{iks}, \text{Gas}_t^{iks}, \text{Flare}_t^{iks}, \text{ReInj}_t^{iks}, \text{PInj}_t^{iks}, \text{PInS}_t^{iks},\right. \\ & \left. Z_t^{iks}(\cdot), \text{ID}_t^{iks}, \text{M}_t^{iks}, \text{IM}_t^{iks}, \text{OInj}_t^{iks}, \text{IVent}_t^{i};\mathbf{T}^s\right)\end{aligned} \tag{111}$$

and note that for each $\text{x}_{jt}^{iks} \neq \text{Z}_t^{iks}$ we get:

$$\frac{\partial Z_t^{iks}(\cdot)}{\partial \text{x}_{jt}^{iks}} = -\left.\frac{\partial TF_t^{iks}(.)}{\partial \text{x}_{jt}^{iks}}\right/\frac{\partial TF_t^{iks}(.)}{\partial \text{Z}_t^{iks}} \tag{112}$$

This implies that the derivative of $\tilde{\Pi}_t^{iks}$ with respect to x_{jt}^{iks} writes:

$$\frac{\partial \tilde{\Pi}_t^{iks}(\cdot)}{\partial \text{x}_{jt}^{iks}} = \frac{\partial \Pi_t^{iks}(\cdot)}{\partial \text{x}_{jt}^{iks}} - \left(1 - T_t^s\right)\left.\frac{\partial TF_t^{iks}(.)}{\partial \text{x}_{jt}^{iks}}\right/\frac{\partial TF_t^{iks}(.)}{\partial \text{Z}_t^{iks}} \tag{113}$$

for each $\text{x}_{jt}^{iks} \neq \text{Z}_t^{iks}$. Recall that the change in the tax deduction has formula:

$$\mathit{\Delta}\delta_{1t}^{iks} = -\mathit{\Delta}\tau_t^{s,\text{Gas}}\mathbf{1}\left[type_t^{iks} = Oil\right] \tag{114}$$

for all $i = 1, 2, ..., I^k$, and

$$\mathit{\Delta}\delta_{0t}^{iks} = -\mathit{\Delta}\tau_t^{s,\text{Gas}}\left\{\widehat{Gas}_t^{iks}\mathbf{1}\left[type_t^{iks} = Gas\right] + \left(\text{NRF}_t^{iks} - \widehat{Maint}_t^{iks}\right)\mathbf{1}\left[type_t^{iks} = Oil\right]\right\} \tag{115}$$

where the variable $\widehat{Gas}_t^{iks}$ is defined as follows:

$$\widehat{Gas}_t^{iks} = \frac{\sum_{l\neq k}\sum_{j=1}^{I^l} \check{\text{Gas}}_t^{jls}\mathbf{1}\left[type_t^{jls} = Gas\right]}{(K^s-1)\left(\sum_{j=1}^{I^k}\mathbf{1}\left[type_t^{jls} = Gas\right]\right)} \tag{116}$$

and $\widehat{Maint}_t^{iks}$ has formula:

$$\widehat{Maint}_t^{iks} = \frac{\sum_{l\neq k}\sum_{j=1}^{I^l} \text{Res}\check{\text{Gas}}_t^{jls}\mathbf{1}\left[type_t^{jls} = Oil\right]}{(K^s-1)\sum_{j=1}^{I^k}\mathbf{1}\left[type_t^{jks} = Oil\right]} \tag{117}$$

Thus, we can calculate the profit generated by such a bundle. First, note that the linearity of the profit function in the tax rates on oil and gas implies:

$$\begin{aligned}\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks};\check{\mathbf{T}}^s\right) = \quad & \tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks};\mathbf{T}^s\right) - \Delta\tau_t^{s,\text{Oil}}\tilde{\text{Oil}}_t^{iks} \\ & -\Delta\tau_t^{s,\text{Gas}}\tilde{\text{Gas}}_t^{iks} + \Delta\delta_{0t}^{iks} + \Delta\delta_{1t}^{iks}\left(\text{Re}\tilde{\text{Inj}}_t^{iks} + \text{Re}\tilde{\text{InS}}_t^{iks}\right)\end{aligned} \tag{118}$$

where $\Delta\delta_{0t}^{iks} = -\Delta\tau_t^{s,\text{Gas}}\left(\text{NRF}_t^{iks} - \widehat{Maint}_t^{iks}\right)$ for oil fields and $\delta_{0t}^{iks} = -\Delta\tau_t^{s,\text{Gas}}\widehat{Gas}_t^{iks}$ for gas-only fields.

(a) Profits - Oil Fields. Note that the introduction of the tax reform $\check{\mathbf{T}}^s$ causes all prices to be unchanged relative to $\mathbf{T}^s$. Consider the following choice vector: $\tilde{\mathbf{x}}_t^{iks}$, where $\tilde{\text{x}}_{jt}^{iks} = \text{x}_{jt}^{iks}$ for all j except the following. Set $\tilde{\text{Gas}}_t^{iks} = \text{Gas}_t^{iks} + \text{Flare}_t^{iks} - \text{NRF}_t^{iks} + \text{IVent}_t^{iks} - Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right) + Maint_t^{iks}\left(\text{M}_t^{iks}\right)$, $\tilde{\text{Flare}}_t^{iks} = \text{NRF}_t^{iks}$, and $\tilde{\text{IVent}}_t^{iks} = 0$ for all fields and leave all other choice variables (other than Z_t^{iks}) unchanged; i.e., $\text{Re}\tilde{\text{Inj}}_t^{iks} = \text{ReInj}_t^{iks}$, $\text{Re}\tilde{\text{InS}}_t^{iks} = \text{ReInS}_t^{iks}$, etc. Note that choice vector $\tilde{\mathbf{x}}_t^{iks}$ lies within the domain of the function $\tilde{\Pi}_t^{iks}$ and it is feasible given the assumptions of upstream fields. Lastly, note that the FOCs of the firm's problem imply: $\frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{Gas}_t^{iks}} = \frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{Flare}_t^{iks}} = \frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{IVent}_t^{iks}}$ whenever the constraints $\text{Flare}_t^{iks} - \text{NRF}_t^{iks} \geq 0$, $\text{IVent}_t^{iks} \geq 0$ are not binding. If any of the these two constraints is binding, then the change in the value of the corresponding variable is equal to zero at the margin. Using these two result and the differentiablity of $\tilde{\Pi}_t^{iks}$, we can obtain a linear approximation of $\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)$ for oil fields as follows:

$$\begin{aligned}\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks},\mathbf{T}^s\right) \simeq \quad & \left.\frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{Gas}_t^{i}}\right|_{\mathbf{x}_t^{iks},\mathbf{T}^s}\left(\tilde{\text{Gas}}_t^{iks} - \text{Gas}_t^{iks}\right) - \left.\frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{Flare}_t^{iks}}\right|_{\mathbf{x}_t^{iks},\mathbf{T}^s}\left(\text{Flare}_t^{iks} - \text{NRF}_t^{iks}\right) \\ & - \left.\frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{IVent}_t^{iks}}\right|_{\mathbf{x}_t^{iks},\mathbf{T}^s}\text{IVent}_t^{iks} = \\ = \quad & \left.\frac{\partial\tilde{\Pi}_t^{iks}(\cdot)}{\partial\text{Gas}_t^{i}}\right|_{\mathbf{x}_t^{iks},\mathbf{T}^s}\left[\left(\tilde{\text{Gas}}_t^{iks} - \text{Gas}_t^{iks}\right) - \left(\text{Flare}_t^{iks} - \text{NRF}_t^{iks} + \text{IVent}_t^{iks}\right)\right] \\ \simeq \tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right) &\end{aligned} \tag{119}$$

Thus, we found that for oil fields, the following linear approximation holds true:

$$\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \mathbf{T}^s\right) \simeq \tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks}, \mathbf{T}^s\right) \tag{120}$$

Using result (120) into (118) we obtain an approximation of $\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right)$, which writes:

$$\begin{aligned} &\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks}, \mathbf{T}^s\right) - \Delta\tau_t^{s,\mathrm{Oil}}\check{\mathrm{Oil}}_t^{iks} - \Delta\tau_t^{s,\mathrm{Gas}}\tilde{\check{\mathrm{Gas}}}_t^{iks} \\ &+\Delta\delta_{0t}^{iks} + \Delta\delta_{1t}^{iks}\left(\tilde{\mathrm{ReInj}}_t^{iks} + \tilde{\mathrm{ReInS}}_t^{iks}\right) \end{aligned} \tag{121}$$

which for oil fields, given $\Delta\tau_t^{s,\mathrm{Gas}} = -\Delta\delta_{1t}^{iks}$ and $\Delta\delta_{0t}^{iks} = -\Delta\tau_t^{s,\mathrm{Gas}}\left(\mathrm{NRF}_t^{iks} - \widehat{Maint}_t^{iks}\right)$, the formula above is equivalent to:

$$\begin{aligned} \tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right) \simeq \quad & \tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks}, \mathbf{T}^s\right) - \Delta\tau_t^{s,\mathrm{Oil}}\check{\mathrm{Oil}}_t^{iks} \\ & -\Delta\tau_t^{s,\mathrm{Gas}}\left[\tilde{\check{\mathrm{Gas}}}_t^{iks} - \widehat{Maint}_t^{iks} + \tilde{\mathrm{ReInj}}_t^{iks} + \tilde{\mathrm{ReInS}}_t^{iks} + \mathrm{NRF}_t^{iks}\right] \end{aligned} \tag{122}$$

which using the fact that for oil fields $GOR^{iks}\mathrm{Oil}_t^{iks} = \mathrm{Tot}\check{\mathrm{Gas}}_t^{iks}$ and $\Delta\tau_t^{iks,\mathrm{Oil}} = -\Delta\tau_t^{s,\mathrm{Gas}}GOR^{iks}\left(1-\vartheta^{iks}\right)$, the formula above rewrites as follows:

$$\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right) \simeq \tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks}, \mathbf{T}^s\right) + \Delta\tau_t^{s,\mathrm{Gas}}\left[\widehat{Maint}_t^{iks} - Maint_t^{iks}\left(\check{\mathrm{M}}_t^{iks}\right)\right] \tag{123}$$

(b) Profits - Gas-only fields. For gas-only fields we set $\tilde{\check{\mathrm{Gas}}}_t^{iks} = \check{\mathrm{Gas}}_t^{iks}$ and $\tilde{\mathrm{ReInS}}_t^{iks} = \mathrm{ReInS}_t^{iks} - \tilde{\check{\mathrm{Gas}}}_t^{iks} + \mathrm{Gas}_t^{iks}$. Then, using the same method as in the previous paragraph, it is possible to show that $\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \mathbf{T}^s\right) \simeq \tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks}, \mathbf{T}^s\right)$ holds true for gas-only fields. Second, for gas-only fields we have $\check{\mathrm{Oil}}_t^{iks} = 0$ and $\Delta\delta_{0t}^{iks} = \Delta\tau_t^{s,\mathrm{Gas}}\widehat{Gas}_t^{iks}$. Thus, the approximate value of $\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right)$ writes:

$$\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right) \simeq \tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks}, \mathbf{T}^s\right) - \Delta\tau_t^{s,\mathrm{Gas}}\left[\check{\mathrm{Gas}}_t^{iks} - \widehat{Gas}_t^{iks}\right]\left(1 - T_t^s\right) \tag{124}$$

where the formula for $\widehat{Gas}_t^{iks}$ is provided in the previous paragraph.

(c) *Profits - Aggregate.* By optimality, because all prices are unchanged by the introduction of the tax reform and the bundle $\left\{\left\{\tilde{\mathbf{x}}_t^{iks}\right\}_{i=1}^{I^k}\right\}_{t=1}^{\infty}$ is feasible for firm k but it is not chosen, the present discounted value of all firms profits generated by $\left\{\left\{\tilde{\mathbf{x}}_t^{iks}\right\}_{i=1}^{I^k}\right\}_{t=1}^{\infty}$ cannot fall short that obtained by the optimal bundle $\left\{\left\{\check{\mathbf{x}}_t^{iks}\right\}_{i=1}^{I^k}\right\}_{t=1}^{\infty}$ given tax policy $\check{\mathbf{T}}^s$. Therefore,

$$\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\tilde{\Pi}_t^{iks}\left(\check{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right) \geq \sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks}, \check{\mathbf{T}}^s\right) \tag{125}$$

Subtracting $\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)$ from both side of the inequality (125) we obtain:

$$\begin{gathered}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\left[\tilde{\Pi}_t^{iks}\left(\check{\mathbf{x}}_t^{iks},\check{\mathbf{T}}^s\right)-\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)\right]\geq\\ \sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\left[\tilde{\Pi}_t^{iks}\left(\tilde{\mathbf{x}}_t^{iks},\check{\mathbf{T}}^s\right)-\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)\right]\end{gathered}\tag{126}$$

Lastly, substituting (123) and (124) into (126), and denoting $\Delta\tilde{\Pi}_t^{iks}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)=\tilde{\Pi}_t^{iks}\left(\check{\mathbf{x}}_t^{iks},\check{\mathbf{T}}^s\right)-\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)$, we obtain:

$$\begin{gathered}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\Delta\tilde{\Pi}_t^{iks}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)\geq\\ \sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\Delta\tau_t^{s,\text{Gas}}\left\{\left[\widehat{Maint}_t^{iks}-Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)\right]\mathbf{1}\left[type_t^{iks}=oil\right]\right.\\ \left.+\left[\widehat{Gas}_t^{iks}-\check{\text{Gas}}_t^{iks}\right]\mathbf{1}\left[type_t^{iks}=gas\right]\right\}\left(1-T_t^s\right)\end{gathered}\tag{127}$$

That is, the change in the present discounted value of profits generated by oil fields profits is weakly larger than $\sum_{t=1}^{\infty}\beta^{t-1}\Delta\tau_t^{s,\text{Gas}}\left[\widehat{Maint}_t^{iks}-Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)\right](1-T_t^s)$, whereas the change in those generated by gas-only firms it is weakly larger than $\sum_{t=1}^{\infty}\beta^{t-1}\Delta\tau_t^{s,\text{Gas}}\left[\widehat{Gas}_t^{iks}-\check{\text{Gas}}_t^{iks}\right](1-T_t^s)$. Because the $\widehat{Maint}_t^{iks}-Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)$ – which can be either positive or negative – is typically very small in magnitude, we can conclude that the effect of the tax reform on oil fields' profits is negligible. This finding rules out concerns regarding possible effects of the tax reform on the extensive margin of oil production. Lastly, note that the profits of other oil&gas fields (i.e., neither oil fields not gas-only fields) are unaffected by the tax reform, we can sum up the the formula for oil&gas firm's profits over all firms, to obtain that the aggregate value of present-discounted profits for oil&gas firms, which write:

$$\begin{gathered}\sum_{k=1}^{K^s}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\left[\tilde{\Pi}_t^{iks}\left(\check{\mathbf{x}}_t^{iks},\check{\mathbf{T}}^s\right)-\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)\right]\geq\\ \sum_{k=1}^{K^s}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\Delta\tau_t^{s,\text{Gas}}\left\{\left[\widehat{Maint}_t^{iks}-Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)\right]\mathbf{1}\left[type_t^{iks}=oil\right]\right.\\ \left.+\left[\widehat{Gas}_t^{iks}-\check{\text{Gas}}_t^{iks}\right]\mathbf{1}\left[type_t^{iks}=gas\right]\right\}\left(1-T_t^s\right)\end{gathered}\tag{128}$$

Using the fact that the tax reform implies $\check{\text{IVent}}_t^{iks}=0$ and $\check{\text{Flare}}_t^{iks}=\text{NRF}_t^{iks}$, the formula for $\widehat{Maint}_t^{iks}$ implies:

$$\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\widehat{Maint}_t^{iks}=\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)\tag{129}$$

and that the formula for $\widehat{Gas}_t^{iks}$ implies:

$$\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\widehat{Gas}_t^{iks}=\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\check{\text{Gas}}_t^{iks}\tag{130}$$

we can use the results (129) and (130) into (128) to obtain:

$$\sum_{k=1}^{K^s}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\left[\tilde{\Pi}_t^{iks}\left(\check{\mathbf{x}}_t^{iks},\check{\mathbf{T}}^s\right)-\tilde{\Pi}_t^{iks}\left(\mathbf{x}_t^{iks},\mathbf{T}^s\right)\right]\geq 0\ , \tag{131}$$

that is, the aggregate present discounted value of profits from oil&gas firms is weakly larger under the tax reform $\check{\mathbf{T}}^s$ relative to $\mathbf{T}^s$.

(d) Tax Revenue - Oil Fields. Note that for oil fields:

$$\text{Tot}\check{\text{Gas}}_t^{iks}=\left(1-\vartheta^{iks}\right)^{-1}\left[\check{\text{Gas}}_t^{iks}-Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)+\text{Re}\check{\text{I}}\text{nj}_t^{iks}+\text{ReInS}_t^{iks}+\text{NRF}_t^{iks}\right] \tag{132}$$

First, note that because the aggregate present discounted value of profits is larger under the tax reform $\check{\mathbf{T}}^s$ relative to $\mathbf{T}^s$, then aggregate present discounted value of tax revenue is also larger. Thus, for oil fields the change in tax revenue for the government writes:

$$\begin{aligned}\varDelta TRev^{s,\text{Oil}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)\geq\quad & \textstyle\sum_{t=1}^{\infty}\beta^{t-1}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\Big\{\varDelta\tau_t^{s,\text{Gas}} \\ & \left[\text{Tot}\check{\text{Gas}}_t^{iks}-\widehat{Maint}_t^{iks}+Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)\right]+ \\ & \tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}+\varDelta\tau_t^{i,\text{Oil}}\check{\text{O}}\text{il}_t^{iks}\Big\}\left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Oil\right]\end{aligned} \tag{133}$$

which using $\varDelta\tau_t^{iks,\text{Oil}}\check{\text{O}}\text{il}_t^{iks}=\varDelta\tau_t^{s,\text{Gas}}\text{Tot}\check{\text{Gas}}_t^{iks}$ reduces to:

$$\begin{aligned}\varDelta TRev^{s,\text{Oil}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)=\quad & \textstyle\sum_{t=1}^{\infty}\beta^{t-1}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\Big\{\tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}\left(1-T_t^s\right) \\ & +\varDelta\tau_t^{s,\text{Gas}}\left[Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right)-\widehat{Maint}_t^{iks}\right]\Big\} \\ & \left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Oil\right]\end{aligned} \tag{134}$$

Using the fact that the tax reform implies $\text{IV}\check{\text{e}}\text{nt}_t^{iks}=0$ and $\text{Fl}\check{\text{a}}\text{re}_t^{iks}=\text{NRF}_t^{iks}$, the formula for $\widehat{Maint}_t^{iks}$ implies:

$$\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\widehat{Maint}_t^{iks}\quad=\quad\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}Maint_t^{iks}\left(\check{\text{M}}_t^{iks}\right) \tag{135}$$

Using (135) into (134) the formula reduces to:

$$\varDelta TRev^{s,\text{Oil}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)=\sum_{t=1}^{\infty}\beta^{t-1}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}\left(1-T_t^s\right)\left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Oil\right] \tag{136}$$

(e) Tax Revenue - Gas-only Fields. Note that for gas-only fields the change in tax revenue for the government writes:

$$\begin{aligned} \Delta TRev^{s,\text{Gas}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right) = & \sum_{t=1}^{\infty}\beta^{t-1}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\left[\Delta\tau_t^{s,\text{Gas}}\left(\check{\text{Gas}}_t^{iks}-\widehat{Gas}_t^{iks}\right)\right. \\ & \left.+\tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}\right]\left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Gas\right] \end{aligned} \tag{137}$$

The formula for $\widehat{Gas}_t^{iks}$ implies:

$$\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\widehat{Gas}_t^{iks} = \sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\check{\text{Gas}}_t^{iks} \tag{138}$$

Using (138) into (137) the formula for $\Delta Rev_t^{s,\text{Gas}}$ reduces to:

$$\Delta TRev^{s,\text{Gas}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right) = \sum_{t=1}^{\infty}\beta^{t-1}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}\left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Gas\right] \tag{139}$$

(f) Tax Revenue - Aggregate. Lastly, we can calculate the change in aggregate revenue:

$$\begin{aligned} \Delta TRev^s\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right) = & \quad \Delta TRev^{s,\text{Oil}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)+\Delta TRev^{s,\text{Gas}}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right) \\ & = \sum_{t=1}^{\infty}\beta^{t-1}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}\left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Oil\right] \\ & \quad +\tau_t^{iks,\text{Gas}}\Delta\text{Gas}_t^{iks}\left(1-T_t^s\right)\mathbf{1}\left[type_t^{iks}=Oil\right] \\ & = \sum_{t=1}^{\infty}\beta^{t-1}\tau_t^{s,\text{Gas}}\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\Delta\text{Gas}_t^{iks}\left(1-T_t^s\right)=0 \end{aligned} \tag{140}$$

where the last equality follows from the fact that the aggregate natural gas supply is unaffected by the policy, and therefore $\sum_{k=1}^{K^s}\sum_{i=1}^{I^k}\Delta\text{Gas}_t^{iks}=0$, and that we have assumed that prior to the reform all natural gas is taxed at the same rate $\tau_t^{s,\text{Gas}}$.

(g) Net Lifetime Income. The balanced budget assumption implies $\Delta ITax_t^s = -\Delta TRev_t^s$. The change in the aggregate lifetime income ΔLY^s of the consumer in country s is given by:

$$\Delta LY^s\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right) = \sum_{k=1}^{K^s}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\left[\Delta\tilde{\Pi}_t^{iks}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)+\Delta ITax_t^s\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)\right] \tag{141}$$

Substituting the value of $\sum_{k=1}^{K^s}\sum_{t=1}^{\infty}\sum_{i=1}^{I^k}\beta^{t-1}\Delta\tilde{\Pi}_t^{iks}\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)\geq 0$ from (127) and of $\sum_{t=1}^{\infty}\beta^{t-1}\Delta ITax_t^s\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)=-\Delta Rev^s\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)$ from (140) into (141) we obtain:

$$\Delta LY^s\left(\mathbf{T}^s,\check{\mathbf{T}}^s\right)\geq 0 \tag{142}$$

i.e., the consumer in country s enjoys weakly larger net lifetime income under tax scheme $\check{\mathbf{T}}^s$ relative to tax scheme $\mathbf{T}^s$ in each period t. Q.E.D.

Part (vii). *Proof.* In order to calculate the change in utility we must calculate the change in consumer's consumption (see part (vi)) and in the utility loss due to climate change. Note that because by part (v) of Proposition 3, global GHG emissions are lower under the tax scheme $\check{\mathbf{T}}^s$ relative to tax scheme $\mathbf{T}^s$ in every period $t = 1, 2, ...$; i.e., $Emis\check{s}ions_t < Emissions_t$, the law of motion of the excess GHGs in the atmosphere:

$$-\Lambda Emissions_t + ExcTCO_2e_{t+1} - (1 - \Gamma)\, ExcTCO_2e_t = 0 \tag{143}$$

implies that $Exc\check{T}CO_2e_t < ExcTCO_2e_t$ in each period $t = 1, 2, ...$ Moreover, we have shown in part (iv) that $\check{\mathbf{c}}_t^s = \mathbf{c}_t^s$ and that all prices and consumption taxes are unaffected by the tax reform. Therefore, the consumer's budget constraint $C_t^s + (\mathbf{p}_t + \mathbf{v}_t^s)' \mathbf{c}_t^s - Y_t^s \leq 0$, which must be binding at the optimal consumer's choice, implies

$$\sum_{t=1}^{\infty} \beta^{t-1} \left(\check{C}_t^s - C_t^s\right) = \sum_{t=1}^{\infty} \beta^{t-1} \left(\check{Y}_t^s - Y_t^s\right) = \mathit{\Delta} LY^s \left(\mathbf{T}^s, \check{\mathbf{T}}^s\right) \geq 0 \tag{144}$$

Thus, we have shown that the present discounted value of general consumption C_t^s, weakly larger under tax scheme $\check{\mathbf{T}}^s$ relative to tax scheme $\mathbf{T}^s$. Lastly, because all prices are unaffected by tax scheme $\check{\mathbf{T}}^s$ relative to tax scheme $\mathbf{T}^s$ in every period $t = 1, 2, ...,$ then $\mathbf{c}_t^r$ and C_t^r are unaffected by the change in tax policy in country r for all $r \neq s$. Thus,

$$\check{C}_t^s + u^s\left(\check{\mathbf{c}}_t^s\right) - Ext \times Exc\check{T}CO_2e_t \geq C_t^s + u^s\left(\mathbf{c}_t^s\right) - Ext \times \mathit{\Delta} TCO_2e\ ; \tag{145}$$

for all $s = 1, 2, ..., S$ in each period $t = 1, 2, ...,$ and therefore:

$$\sum_{t=1}^{\infty} \beta^{t-1} \left[\check{C}_t^s + u^s\left(\check{\mathbf{c}}_t^s\right) - Ext \times Exc\check{T}CO_2e_t\right] \geq \sum_{t=1}^{\infty} \beta^{t-1} \left[C_t^s + u^s\left(\mathbf{c}_t^s\right) - Ext \times \mathit{\Delta} TCO_2e\right]\ ; \tag{146}$$

for all $s = 1, 2, ..., S$; i.e., the lifetime utility of a consumer in each country s is strictly larger under tax scheme $\check{\mathbf{T}}^s$ relative to tax scheme $\mathbf{T}^s$. Q.E.D.

2.2 Midstream

In this subsection we present the main theoretical result regarding refineries, which is in sharp contrast with our findings concerning oil and gas firms (see Proposition

1). In detail, Proposition 4 shows that a specific tax on flaring performed by midstream firms (refineries) can effectively eliminate routine flaring in this part of the industry without producing significant side effects.

Let $MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^j\right) \equiv \frac{\partial MTF_4^{js}\left(\mathrm{F}_t^j\right)}{\partial \mathrm{F}_t^{js}} \Big/ \frac{\partial MTF_6^{js}\left(\mathrm{MZ}_t^j\right)}{\partial \mathrm{MZ}_t^{js}}$ be the marginal cost of flaring and $\check{MC}_t^{js,\mathrm{F}}$ represent the maximum value of $MC_t^{js,\mathrm{F}}$ for firm j (see section 1.2).

Proposition 4. *A specific linear tax on flaring at rate $\tau_t^{js,F} \geq \check{MC}_t^{js,F}$ implies $F_t^{js} = 0$ and no effect on O_t^{js}, G_t^{js}, $\mathbf{y}_t^{js}$ for all midstream firms $j = 1, 2, ..., J$.*

Proof. First, note that by assumption all midstream firms other than oil refineries feature $MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{js}\right) = +\infty$, implying that for such firms the constraint $\mathrm{F}_t^{js*} \geq RF_t^{js}$ and in turn, $\mathrm{F}_t^{js*} = RF_t^{js}$. Second, refineries are defined as all midstream firms such that $\frac{\partial MTF_2^{js}\left(\mathrm{O}_t^{js}\right)}{\partial \mathrm{O}_t^{js}} > 0$, $MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{js}\right) \neq +\infty$, and $\frac{\partial MTF_3^{js}\left(\mathrm{G}_t^{js}\right)}{\partial \mathrm{G}_t^{js}} = 0$ for $\mathrm{G}_t^{js} < 0$. Thus, using the FOCs of the midstream firms w.r.t. F_t^{js} and MZ_t^{js}, where the latter is always binding because $\mathrm{MZ}_t^{js} \in (-\infty, +\infty)$, we get:

$$-\tau_t^{js,\mathrm{F}}\left(1 - T_t^s\right) - \left(1 - T_t^s\right) \frac{\partial MTF_4^{js}\left(\mathrm{F}_t^{js}\right)}{\partial \mathrm{F}_t^{js}} \Bigg/ \frac{\partial MTF_6^{js}\left(\mathrm{MZ}_t^{js}\right)}{\partial \mathrm{MZ}_t^{js}} + \psi_{2t}^{js} \leq 0 \quad (147)$$

whereas using the FOCs of the midstream firms w.r.t. O_t^{js} and MZ_t^{js} we get:

$$P_t^{\mathrm{Oil}}\left(1 - T_t^s\right) - \left(1 - T_t^s\right) \frac{\partial MTF_2^{js}\left(\mathrm{F}_t^{js}\right)}{\partial \mathrm{O}_t^{js}} \Bigg/ \frac{\partial MTF_6^{js}\left(\mathrm{MZ}_t^{js}\right)}{\partial \mathrm{MZ}_t^{js}} + \psi_{2t}^{js} GOR^{js} \leq 0 \quad (148)$$

Let us define the marginal product of oil $MP_t^{js,\mathrm{O}}\left(\mathrm{O}_t^{js}\right) \equiv \frac{\partial MTF_2^{js}\left(\mathrm{O}_t^{js}\right)}{\partial \mathrm{O}_t^{js}} \Big/ \frac{\partial MTF_6^{js}\left(\mathrm{MZ}_t^{js}\right)}{\partial \mathrm{MZ}_t^{js}}$. Considering oil refineries which are active; i.e., they feature $\mathrm{O}_t^{js} > 0$, the condition (148) rewrites:

$$\left(1 - T_t^s\right)\left[MP_t^{js,\mathrm{O}}\left(\mathrm{O}_t^{js}\right) - P_t^{\mathrm{Oil}}\right] = \psi_{2t}^{js} GOR^{js} \quad (149)$$

and the condition (147) rewrites:

$$-\left[\tau_t^{js,\mathrm{F}} + MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{js}\right)\right]\left(1 - T_t^s\right) + \psi_{2t}^{js} \leq 0 \quad (150)$$

Combining conditions (149) and (150) we obtain:

$$-\tau_t^{js,\mathrm{F}} - MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{js}\right) - \frac{P_t^{\mathrm{Oil}} - MP_t^{js,\mathrm{O}}\left(\mathrm{O}_t^{js}\right)}{GOR^{js}} \leq 0 \quad (151)$$

Thus, zero flaring is ensured if:

$$\tau_t^{js,\mathrm{F}} \geq \frac{MP_t^{js,\mathrm{O}}\left(\mathrm{O}_t^{js*}\right) - P_t^{\mathrm{Oil}}}{GOR^{js}} - MC_t^{js,\mathrm{F}}\left(0\right) \tag{152}$$

whereas with zero flaring tax the FOCs imply:

$$0 \geq \frac{MP_t^{js,\mathrm{O}}\left(\mathrm{O}_t^{js**}\right) - P_t^{\mathrm{Oil}}}{GOR^{js}} - MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{js**}\right) \tag{153}$$

However, the optimal quantity of oil with positive flare tax in (152) may differ from the quantity with no flaring tax in (153). In order to tackle this issue, let us define $MC_t^{js,\mathrm{MInS}} \equiv \frac{\partial MTF_5^{js}\left(\mathrm{MInS}_t^j\right)}{\partial \mathrm{MInS}_t^{js}} \Big/ \frac{\partial MTF_6^{js}\left(\mathrm{MZ}_t^{js}\right)}{\partial \mathrm{MZ}_t^{js}}$. Assuming that $MC_t^{js,\mathrm{MInS}}$ is constant, the FOC w.r.t. MInS_t^{js} writes:

$$\left(1 - T_t^s\right) MC_t^{js,\mathrm{MInS}} = \psi_{2t}^{js} \tag{154}$$

Substituting (154) into (149) we obtain:

$$MP_t^{js,\mathrm{O}}\left(\mathrm{O}_t^j\right) = P_t^{\mathrm{Oil}} + MC_t^{js,\mathrm{MInS}} GOR^{js} \tag{155}$$

which implies that O_t^{js*} is independent of $MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{j**}\right)$ at given P_t^{Oil}. Thus, we get $\mathrm{O}_t^{js*} = \mathrm{O}_t^{js**}$. As a consequence, combining inequalities (153) and (152), a sufficient condition for inequality to hold is:

$$\tau_t^{js,\mathrm{F}} \geq MC_t^{js,\mathrm{F}}\left(\mathrm{F}_t^{js**}\right) - MC_t^{js,\mathrm{F}}\left(0\right) \tag{156}$$

Lastly, because the marginal cost of flaring is weakly positive and increasing, a sufficient condition for the inequality (156) to hold true is:

$$\tau_t^{js,\mathrm{F}} \geq \check{M}C_t^{js,\mathrm{F}} \ , \tag{157}$$

which implies that if the flaring tax satisfies inequality (157) the optimal amount of flaring chosen by each refinery is $\mathrm{F}_t^{js*} = 0$. Lastly, note that the optimality conditions for all other endogenous variables are unaffected by the introduction of , implying that all optimal choices –except possibly MInS_t^{js} and MZ_t^{js} – are unaffected by the policy reform, implying that aggregate net production choices $\mathrm{O}_t^{js}, \mathrm{G}_t^{js}, \mathbf{y}_t^{js}$ are all unaffected. Q.E.D.

3 Identification

In order to quantify the aggregate reduction in GHG emissions caused by the implementation of the tax reform proposed in the previous section we need to propose a method to identify and estimate the amount of (unobservable) intentional venting produced by of each oil&gas firm. Note that because all the empirical analysis makes use of data from a single country (the US), for ease of notation in this section we suppress the country superscript s.

3.1 Assumptions

For the purpose of identification, we need to impose additional restrictions to our model. Specifically, we assume that

1. The expected regulatory cost of venting possesses a linear-quadratic function of IVent_t^i and Flare_t^i; i.e.,

$$VF_t^{ik} \times Pr_t^{ik}\left(ivent_t^{ik} = 1 \mid \Omega_t^{ks}\right) = \begin{array}{r} \kappa_0^{ik}\text{IVent}_t^{ik} + \frac{\kappa_1}{2}\text{IVent}_t^{ik\ 2} \\ +\kappa_2\text{IVent}_t^{ik}\ \left(\text{Flare}_t^{ik} - \text{NRF}_t^{ik}\right) + \tilde{\kappa}_3\text{M}_t^{ik} \end{array}. \tag{158}$$

2. The marginal cost (including the specific tax on gas sales) of gas production for oil fields $MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}}$ can be decomposed as follows:

$$MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} = \overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} + \nu_t^{ik}$$

   where ν_t^{ik} is independent across fields normally distributed with zero mean and standard deviation σ_ν, potentially serially correlated, and such that $E\left[\nu_t^{ik}\right] = 0$.

3. We impose the following parametric restrictions regarding the effect of maintenance on capital on the level of unintentional venting and on firm's costs: $\frac{\partial Maint_t^{ik}(\cdot)}{\partial \text{M}_t^{ik}} = \kappa_4$ and $MPE_{t+1}^{ik}\left(\text{M}_{t+1}^{ik}\right) = \kappa_5 + \kappa_6\text{M}_t^{ik}$. Moreover, we assume that the level of maintenance capital is strictly positive for all fields in our sample. Given these assumptions, the formula for the optimal level of maintenance capital writes:

$$\begin{array}{r} \text{M}_t^{ik} = -\frac{\kappa_5}{\kappa_6} + \frac{1-T_{t-1}^s}{\beta\kappa_6(1-T_t)} - \left(1-\rho^{ik}\right) + \frac{\kappa_4}{\kappa_6}\frac{VF_t^{ik}\kappa_3}{1-T_t} \\ +\frac{\kappa_4}{\kappa_6}\left[P_t^{\text{Gas}} - \tau_t^{ik,\text{Gas}} - MC_t^{ik,\text{Gas}}\right] \end{array} \tag{159}$$

4. We assume zero flaring fines; i.e., $FF_t^{ik} = 0$, and time-invariant $T_t^s = T^s$, $\tau_t^{ik,\text{Gas}} = \tau^{ik,\text{Gas}}$ and $VF_t^{ik} = VF^{ik}$. As a result, the formula for regulatory costs writes:

$$RegCost_t^{ik}\left(\text{IVent}_t^{ik}, \text{Flare}_t^{ik}, \text{M}_t^{ik}\right) = \begin{array}{r} const^{ik} + \kappa_0^{ik}\text{IVent}_t^{ik} + \frac{\kappa_1}{2}\text{IVent}_t^{ik\ 2} \\ +\kappa_2\text{IVent}_t^{ik}\ \left(\text{Flare}_t^{ik} - \text{NRF}_t^{ik}\right) + \kappa_3 P_t^{\text{Gas}} + \upsilon_t^{ik} \end{array}. \tag{160}$$

where $\kappa_3 = \tilde{\kappa}_3 \frac{\kappa_4}{\kappa_6}\, const^{ik} = -\kappa_3 + \frac{\tilde{\kappa}_3}{\beta\kappa_6} - \left(1 - \rho^{ik}\right) + \kappa_3 \frac{VF^{ik}\kappa_3}{1-T} - \kappa_3 \left[\tau^{ik,\text{Gas}} + MC^{ik,\text{Gas}}\right]$ and $\upsilon_t^{ik} = -\kappa_3 \nu_t^{ik}$

5. The marginal cost of flaring $MC_t^{ik,\text{Flare}}$ has linear formula:

$$MC_t^{ik,\text{Flare}} (1 - T_t) = \pi_0^{ik} + \pi_1 \left(\text{Flare}_t^{ik} - \text{NRF}_t^{ik}\right) \tag{161}$$

6. We assume $\kappa_0^{ik} \simeq \pi_0^{ik}$. That is, the marginal expected cost of flaring and intentional venting both evaluated at $\text{Flare}_t^{ik} = 0$ and $\text{IVent}_t^{ik} = 0$ are approximately equal to each other. For instance, it is plausible to assume that for very low levels of flaring and intentional venting, the marginal expected cost of both flaring and intentional venting is very close to zero. Moreover, we assume $\pi_1 > 0$, $\kappa_1 > 0$, $\kappa_1 > \kappa_2$ and $\pi_1 > \kappa_2$ to ensures that the costs are convex and, in turn, that the supply substitution matrix is positive definite. Note that the last two inequalities also imply that the own-price effects on the supply of Flare_t^{ik} and IVent_t^{ik} are greater in magnitude than the cross-price effects.

7. We assume a field-specific level of minimum routine flaring NRF_t^{ik}, which is allowed to depend upon the realization of the field's marginal cost of gas production; i.e., $\text{NRF}_t^{ik} = \zeta_0^{ik} + \zeta_1 \left(MC_t^{ik,\text{Gas}} + \tau_t^{ik}\right)$. Given this assumption, the product of parameters $\pi_1\zeta_1$ captures the extent of the co-movement between the marginal cost of gas production and the marginal cost of flaring due, for instance, to changes in electricity prices that affect both type of costs.

3.2 Identification: Structural Equations

We define the following variable:

$$\text{OtherGas}_t^{ik} = \text{TotGas}_t^{ik} - \text{Gas}_t^{ik} - \text{Flare}_t^{ik}\ , \tag{162}$$

which is observable by the econometrician because both TotGas_t^{ik}, Flare_t^{ik} and Gas_t^{ik} are. Using the formula for TotGas_t^{ik}, the definition above implies:

$$\text{OtherGas}_t^{ik} = \text{UVent}_t^{ik} + \text{IVent}_t^{ik} + \text{ReInj}_t^{ik} + \text{ReInS}_t^{ik} \tag{163}$$

In order to identify the key parameters of the structural model from equation (163), one faces several issues, which are summarized below.

1. The variable IVent_t^{ik} is not observed by the econometrician separately from UVent_t^{ik} and is censored at $\text{IVent}_t^{ik} = 0$, because a field cannot run negative intentional venting. Thus, one cannot neither directly observe whether $\text{IVent}_t^{i} > 0$ or not.

2. The realizations of $\frac{\nu_t^{ik}}{\kappa_1}$ which enter the formula for IVent_t^{ik} are not observed separately from any shock that enters UVent_t^{ik}. Thus, one cannot directly identify IVent_t^{ik} separately from UVent_t^{ik} .

3. Note that the model implies a link between $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$ and $\text{IVent}_t^{ik} > 0$. Thus, one could, in principle, use the value of Flare_t^{ik} as a selection tool to separate IVent_t^{ik} from UVent_t^{ik}. However, even if one could separate IVent_t^{ik} from UVent_t^{ik} using Flare_t^{ik}, other identification issues may occur. In particular, note that the formulas for IVent_t^{ik} and Flare_t^{ik} imply $E\left[\frac{\nu_t^{ik}}{\kappa_1} \mid \text{Flare}_t^{ik} > \text{NRF}_t^{ik}\right] \neq 0$. Because $\frac{\nu_t^{ik}}{\kappa_1}$ is part of IVent_t^{ik}, any attempt to use Flare_t^{ik} to quantify IVent_t^{ik} must account for the fact that the stochastic component of IVent_t^{ik} is not independent of Flare_t^{ik}.

4. We do not observes the values of ReInj_t^{i} and ReInS_t^{i}, UVent_t^{ik}. We only observe (estimates of) $\text{GasInj}_t^{i} = \text{ReInj}_t^{i} + \text{PInj}_t^{i}$ and $\text{GasInS}_t^{i} = \text{ReInS}_t^{i} + \text{PInS}_t^{i}$.

In the next sections we propose a method to solve all these issues. Specifically, sections 3.3 and 3.4 describe a two-step procedure to identify IVent_t^{ik} separately from UVent_t^{ik}. In section 3.5 and 3.6 we illustrate the methodology adopted to quantify ReInj_t^{i} and ReInS_t^{i}, respectively. Section 3.7 present the final empirical equation derived in sections 3.3. 3.4, 3.5 and 3.6. Lastly section 3.8 describes a procedure to identify the effect of a flaring tax, and section 3.9 provides a discussion of the main assumptions underpinning identification.

3.3 Identification: Intentional Venting

Using the parametric restriction from section 3.1 and the optimality conditions from section 2, we obtain the following formulas for the optimal levels of flaring and intentional venting:

$$\text{IVent}_t^{ik} = \max\left\{-\frac{\kappa_0^{ik} + \kappa_2^{ik}\left(\text{Flare}_t^{ik} - \text{NRF}_t^{ik}\right)}{\kappa_1} + \frac{1}{\kappa_1}\left[\overline{MC}^{ik,\text{Gas}} + \tau^{ik} - P_t^{\text{Gas}} + \nu_t^{ik}\right], 0\right\} \tag{164}$$

and

$$\text{Flare}_t^{ik} = \max\left\{-\frac{\pi_0^{ik} + \tau_t^{ik,\text{Flare}} + \kappa_2^{ik}\text{IVent}_t^{ik}}{\pi_1} + RF_t^{ik} + \frac{1}{\pi_1}\left[\overline{MC}^{ik,\text{Gas}} + \tau^{ik} - P_t^{\text{Gas}} + \nu_t^{ik}\right], \text{NRF}_t^{ik}\right\}, \tag{165}$$

respectively. This section makes use of the assumption $\kappa_0^{ik} \simeq \pi_0^{ik}$ to solve the first identification problem stated in section 3.2. Specifically, this assumption implies that if flaring is strictly positive, intentional venting is also strictly positive – although the two quantities may obviously differ from each other. In other words, observing a value for flaring $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$ is sufficient to identify $\text{IVent}_t^{ik} > 0$. This result is summarized by the following proposition.

Proposition 5. *If $\kappa_0^{ik} = \pi_0^{ik}$ and $\tau_t^{ik,Flare} = 0$ then $Flare_t^{ik} > NRF_t^{ik}$ if and only if $IVent_t^{ik} > 0$.*

Proof. Step 1. Suppose $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$ and $\text{IVent}_t^{ik} = 0$. The formula for Flare_t^{ik}:

$$\text{Flare}_t^{ik} = \max\left\{-\frac{\pi_0^{ik} + \tau_t^{ik,\text{Flare}} + \kappa_2^{ik}\text{IVent}_t^{ik}}{\pi_1} + \text{NRF}_t^{ik} + \frac{1}{\pi_1}\left[MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}}\right], \text{NRF}_t^{ik}\right\} \tag{166}$$

implies that $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$ at $\text{IVent}_t^{ik} = 0$ only if:

$$MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}} - \pi_0^{ik} - \tau_t^{ik,\text{Flare}} > 0 \tag{167}$$

From the formula of IVent_t^{ik}, at $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$ we get that $\text{IVent}_t^{ik} = 0$ only if the latent variable IVent_t^{ik*} is weakly negative, i.e.,

$$\text{IVent}_t^{ik*} = \frac{\pi_1\left(\pi_0^{ik} + \tau_t^{ik,\text{Flare}} - \kappa_0^{ik}\right)}{\kappa_1\pi_1 - \left(\kappa_2^{ik}\right)^2} + \frac{\pi_1 - \kappa_2^{ik}}{\kappa_1\pi_1 - \left(\kappa_2^{ik}\right)^2}\left[MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}} - \pi_0^{ik} - \tau_t^{ik,\text{Flare}}\right] \leq 0 \tag{168}$$

Because the second term of (168) is strictly positive by result (167), $\text{IVent}_t^{ik} = 0$ is true only if:

$$\tau_t^{ik,\text{Flare}} - \kappa_0^{ik} + \pi_0^{ik} < 0 \tag{169}$$

which for $\tau_t^{ik,\text{Flare}} = 0$ implies

$$\pi_0^{ik} < \kappa_0^{ik} \tag{170}$$

which leads to a contradiction of the assumption $\kappa_0^{ik} = \pi_0^{ik}$. Thus, if $\kappa_0^{ik} = \pi_0^{ik}$, then $\text{Flare}_t^{ik} > 0 \longrightarrow \text{IVent}_t^{ik} > 0$.

Step 2. Suppose $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$ and $\text{IVent}_t^{ik} > 0$. From the formula of IVent_t^{ik}, at $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$ we get:

$$\text{IVent}_t^{ik*} = \frac{1}{\kappa_1}\left[MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}} - \kappa_0^{ik}\right] > 0 \tag{171}$$

where the term is square brackets must be strictly positive; i.e.,

$$MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}} - \kappa_0^{ik} > 0 \tag{172}$$

The formula for Flare_t^{ik} implies that $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$ only if:

$$\text{Flare}_t^{ik*} = -\frac{\pi_0^{ik} + \tau_t^{ik,\text{Flare}} + \kappa_2^{ik}\text{IVent}_t^{ik}}{\pi_1} + \text{NRF}_t^{ik} + \frac{1}{\pi_1}\left[MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}}\right] \leq \text{NRF}_t^{ik} \tag{173}$$

Substituting the formula for IVent_t^{ik} and $\pi_0^{ik} = \kappa_0^{ik}$ we obtain

$$-\kappa_1\frac{\pi_0^{ik} - \kappa_0^{ik} + \tau_t^{ik,\text{Flare}}}{\pi_1\kappa_1 - \left(\kappa_2^{ik}\right)^2} + \frac{\kappa_1 - \kappa_2^{ik}}{\pi_1\kappa_1 - \left(\kappa_2^{ik}\right)^2}\left[MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - P_t^{\text{Gas}} - \kappa_0^{ik}\right] \leq 0 \tag{174}$$

because the second term of (174) is strictly positive by result (172), $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$ is true only if:

$$\pi_0^{ik} - \kappa_0^{ik} + \tau_t^{ik,\text{Flare}} > 0 \tag{175}$$

which for $\tau_t^{ik,\text{Flare}} = 0$ implies

$$\pi_0^{ik} > \kappa_0^{ik} \tag{176}$$

which leads to a contradiction of the assumption $\kappa_0^{ik} = \pi_0^{ik}$. Thus, if $\kappa_0^{ik} = \pi_0^{ik}$, then $\text{Flare}_t^{ik} > \text{NRF}_t^{ik} \longleftrightarrow \text{IVent}_t^{ik} > 0$. Q.E.D.

3.3.1 Step 1: Selection Equation

Proposition 5 allows us to establish whether $\text{IVent}_t^{ik} > 0$ or not simply using the value of the observable variable Flare_t^{ik}, providing in turn an empirical tool to tackle the identification problem number 1 in section 3.2. Moreover, this finding also provides a route to tackle identification problem 2. Specifically, we can introduce in the structural equation (163) a dummy $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$ to separate the effect of a given variable on IVent_t^{ik} from the effect of the same variable on UVent_t^{ik}. However, in order to quantify IVent_t^{ik} we still need to tackle identification problem 3. We solve this problem using a two-step approach. Using the result in Proposition 5 we know that $\text{Flare}_t^{ik} > \text{NRF}_t^{ik} \longleftrightarrow \text{IVent}_t^{ik} > 0$. Thus, we can substitute the formula for IVent_t^{ik} into the formula for Flare_t^{ik} to obtain:

$$\text{Flare}_t^{ik} = \alpha_0^{ik} + \alpha_1 P_t^{\text{Gas}} + \eta_t^{ik} \tag{177}$$

where using $\text{NRF}_t^{ik} = \zeta_0^{ik} + \zeta_1 \left(MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} \right)$ we obtain the parameters α_0^{ik}, α_1 and the stochastic component η_t^{ik}, which have formulas:

$$\begin{array}{ll} \alpha_0^{ik} = & \frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \left(\overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} - \kappa_0^{ik} \right) + \zeta_0^{ik} + \zeta_1 \left(\overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} \right) \\ \alpha_1 = & -\frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \\ \eta_t^{ik} = & \left(\frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} + \zeta_1 \right) \nu_t^{ik} \end{array} \tag{178}$$

respectively. Note that the equation in (177) resembles that of a standard *censored regression*. Thus, it can be estimated using standard tools for censored data. In particular, we use a version of the Tobit type I model that allows for serial correlation on the η_t^{ik}'s. Having estimated the parameters α_0^{ik}, α_1^{ik} using a consistent estimator, we can calculate the residuals for each observation such that $\text{Flare}_t^{ik} > 0$ as follows:

$$\widehat{res}_t^{ik} = \text{Flare}_t^{ik} - \hat{\alpha}_0^{ik} - \hat{\alpha}_1 P_t^{\text{Gas}}$$

where $\widehat{res}_t^{ik}$ represents an estimate of $\eta_t^{ik} = \left(\frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} + \zeta_1 \right) \nu_t^{ik}$ for the cases in which Flare_t^{ik}. Conversely, when $\text{Flare}_t^{ik} = 0$ we do not possess a point estimate for η_t^{ik}. However, we can construct the expected value of η_t^{ik} conditional on $\text{Flare}_t^{ik} = 0$ using the normality assumption. Specifically, the conditional expectation of η_t^{ik} has formula:

$$E \left[\eta_t^{ik} \mid \eta_t^{ik} \leq -\alpha_0^{ik} - \alpha_1 P_t^{\text{Gas}} \right] = -\hat{\sigma}_\eta \frac{\phi \left(\alpha_0^{ik} + \alpha_1 P_t^{\text{Gas}} / \sigma_\eta \right)}{\Phi \left(\alpha_0^{ik} + \alpha_1 P_t^{\text{Gas}} / \sigma_\eta \right)} \tag{179}$$

Substituting α_0^{ik}, α_1, and σ_η with their estimators obtained from the censored regression in (177) into the formula above, we can construct the estimator for η_t^{ik} as follows:

$$\hat{\eta}_t^{ik} = \begin{cases} \widehat{res}_t^{ik} & if \ \text{Flare}_t^{ik} > \text{NRF}_t^{ik}. \\ -\hat{\sigma}_\eta \frac{\phi \left(-\widehat{\text{Flare}}_t^{ik} / \hat{\sigma}_\eta \right)}{\Phi \left(-\widehat{\text{Flare}}_t^{ik} / \hat{\sigma}_\eta \right)} & otherwise \end{cases} \tag{180}$$

Thus, we can use this estimate in the main structural equation (163) in order to quantify IVent_t^{ik} accounting for the fact that $E \left[\nu_t^{ik} \mid \text{Flare}_t^{ik} > \text{NRF}_t^{ik} \right] \neq 0$. This procedure is illustrated in the next subsection.

3.3.2 Step 2: Formula for IVent_t^{ik} Conditional on Flare_t^{ik}

Using the result in Proposition 5, the formula for IVent_t^{ik} conditional on $\text{Flare}_t^{ik} > 0$ writes:

$$\text{IVent}_t^{ik} = -\frac{\kappa_2 \tau_t^{ik,\text{Flare}}}{\pi_1\kappa_1 - {\kappa_2}^2} + \frac{\pi_1 - \kappa_2}{\pi_1\kappa_1 - {\kappa_2}^2}\left(\overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} - \kappa_0^{ik}\right) - \frac{\pi_1 - \kappa_2}{\pi_1\kappa_1 - \kappa_2^2} P_t^{\text{Gas}} + \frac{\pi_1 - \kappa_2}{\kappa_1\pi_1 - {\kappa_2}^2}\nu_t^{ik} \tag{181}$$

Thus, we can rewrite the formula for IVent_t^{ik} as follows:

$$\text{IVent}_t^{ik} = \delta_1^{ik} D_t^{ik} + \delta_2 P_t^{\text{Gas}} D_t^{ik} + \delta_3 \eta_t^{ik} D_t^{ik} \tag{182}$$

where D_t^{ik} is a dummy variable defined as follows:

$$D_t^{ik} = \begin{cases} 1 & if\ \text{Flare}_t^{ik} > \text{NRF}_t^{ik} \\ 0 & otherwise \end{cases} \tag{183}$$

and where the parameters δ_1^{ik}, δ_2, δ_3 have formulas:

$$\begin{aligned} \delta_1^{ik} = &\ \tfrac{\pi_1-\kappa_2}{\pi_1\kappa_1-{\kappa_2}^2}\left(\overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} - \kappa_0^{ik}\right) - \tfrac{\kappa_2\tau_t^{ik,\text{Flare}}}{\pi_1\kappa_1-{\kappa_2}^2} \\ \delta_2 = &\ -\tfrac{\pi_1-\kappa_2}{\pi_1\kappa_1-\kappa_2^2} \\ \delta_3 = &\ \tfrac{\pi_1-\kappa_2}{\kappa_1\pi_1-{\kappa_2}^2}\left(\zeta_1 + \tfrac{\kappa_1-\kappa_2}{\kappa_1\pi_1-{\kappa_2}^2}\right)^{-1} \end{aligned} \tag{184}$$

Lastly, note that η_t^{ik} is not observable by the econometrician. However, from section 3.3 we know that we can construct an estimate of η_t^{ik}, with formula (180), which we can use in equation (182). Note that this method allows us to tackle the identification issue number 2 in section 3.2.

3.4 Identification: Unintentional Venting

Using the binding constraint for oil fields $\text{TotGas}_t^{ik} = GOR^{ik}\text{Oil}_t^{ik}$ and the optimality condition for M_t^{ik} in (159) we get the formula for UVent_t^{ik}:

$$\begin{aligned} \text{UVent}_t^{ik} = &\ \tfrac{\kappa_4\kappa_5}{\kappa_6} + \kappa_4\left(1-\rho^{ik}\right) - \tfrac{(\kappa_4)^2}{\kappa_6}\left(\tfrac{VF^{ik}\kappa_3}{1-T} - \overline{MC}^{ik,\text{Gas}} - \overline{\tau}^{ik,\text{Gas}}\right) \\ &- \tfrac{\kappa_4}{\beta\kappa_6} + \vartheta^{ik} GOR^{ik}\text{Oil}_t^{ik} - \tfrac{(\kappa_4)^2}{\kappa_6} P_t^{\text{Gas}} + \tfrac{(\kappa_4)^2}{\kappa_6}\nu_t^{ik} + \epsilon_t^{ik} \end{aligned} \tag{185}$$

where $E\left[\epsilon_t^{ik}\right] = 0$. Note that ν_t^{ik} is not observable by the econometrician. However, from section 3.3 we know that we can construct an estimate of $\eta_t^{ik} = \left(\frac{\kappa_1-\kappa_2}{\kappa_1\pi_1-{\kappa_2}^2} + \zeta_1\right)\nu_t^{ik}$, with formula (180), which we can use in equation (185) by applying simple change of variable $\nu_t^{ik} = \left(\frac{\kappa_1-\kappa_2}{\kappa_1\pi_1-\kappa_2{}^2}+\zeta_1\right)^{-1}\eta_t^{ik}$. Thus, the formula for UVent_t^{ik} can be written as follows:

$$\text{UVent}_t^{ik} = \delta_0^{ik} + \delta_4^{ik}\text{Oil}_t^{ik} + \delta_5 P_t^{\text{Gas}} + \delta_6\eta_t^{ik} + \epsilon_t^{ik} \tag{186}$$

where the parameters possess the following formulas:

$$\begin{aligned} \delta_0^{ik} &= \frac{\kappa_4\kappa_5}{\kappa_6} - \frac{\kappa_4}{\beta\kappa_6} + \kappa_4\left(1-\rho^{ik}\right) - \frac{(\kappa_4)^2}{\kappa_6}\left(\frac{VF^{ik}\kappa_3}{1-T} - \overline{MC}^{ik,\text{Gas}} - \overline{\tau}^{ik,\text{Gas}}\right) \\ \delta_4^{ik} &= \vartheta^{ik}GOR^{ik} \\ \delta_5 &= -\frac{(\kappa_4)^2}{\kappa_6} \\ \delta_6 &= \frac{(\kappa_4)^2}{\kappa_6}\left(\frac{\kappa_1-\kappa_2}{\kappa_1\pi_1-\kappa_2{}^2}+\zeta_1\right)^{-1} \end{aligned} \tag{187}$$

The formula in (186) represents the second object of the structural equation we aim to estimate.

3.5 Gas Injections

We assume that the cost of performing gas injections for firm i has the following functional form:

$$IC_t^{ik}\left(\text{ReInj}_t^{ik},\text{PInj}_t^{ik}\right) = a_1^{ik}\text{ReInj}_t^{ik} + a_2^{ik}\text{PInj}_t^{ik} + A^{ik}\left(\text{ReInj}_t^{ik},\text{PInj}_t^{ik}\right) \tag{188}$$

where $A^{ik}: (-\infty,+\infty)^2 \to \mathbb{R}$ is a strictly convex, homogeneous of degree $\varrho^{ik} > 0$ real analytic function that satisfies $A_1^{ik}\left(0,\text{PInj}_t^{ik}\right) + a_1^{ik} \leq 0$ for all $\text{PInj}_t^{ik} > 0$ and $A_2^{ik}\left(\text{ReInj}_t^{ik},0\right) + a_2^{ik} + PP_t^{ik,\text{Gas}} \leq 0$ for all $\text{ReInj}_t^{ik} > 0$, where A_j^{ik} denotes the first derivative of A^{ik} with respect to its jth argument. Using formula (188) into the FOCs of the oil firm problem w.r.t. $\text{ReInj}_t^{ik}, \text{PInj}_t^{ik}$, and OInj_t^{ik} we get:

$$\begin{aligned} \frac{\partial\mathcal{L}_u^k}{\partial\text{ReInj}_t^{ik}} &= -\left[A_1^{ik}\left(\text{ReInj}_t^{ik},\text{PInj}_t^{ik}\right) + a_1^{ik}\right](1-T_t) \\ &\quad + \left(-\phi_{2t}^{ik} + \phi_{3t}^{ik} + \phi_{4t}^{ik}\zeta\right)\left(1-\vartheta^{ik}\right)^{-1} + \phi_{4t}^{ik}\frac{\partial B_t^{ik}\left(\text{TotInj}_t^{ik}\right)}{\partial\text{ReInj}_t^{ik}} + \phi_{8t}^{ik} = 0 \\ \frac{\partial\mathcal{L}_u^k}{\partial\text{PInj}_t^{ik}} &= -\left[A_2^{ik}\left(\text{ReInj}_t^{ik},\text{PInj}_t^{ik}\right) + a_2^{ik} + PP_t^{ik,\text{Gas}}\right](1-T_t) + \phi_{4t}^{i}\frac{\partial B_t^{ik}\left(\text{TotInj}_t^{ik}\right)}{\partial\text{PInj}_t^{ik}} = 0 \\ \frac{\partial\mathcal{L}_u^k}{\partial\text{OInj}_t^{ik}} &= -C_{3t}^{ik}(1-T_t) + \phi_{4t}^{i}\frac{\partial B_t^{i}\left(\text{TotInj}_t^{ik}\right)}{\partial\text{OInj}_t^{i}} = 0 \end{aligned} \tag{189}$$

Note that the assumptions on A^{ik} implies that $\text{ReInj}_t^{ik} > 0$ if $\text{PInj}_t^{ik} > 0$, implying in turn that the observable variable total gas injection $\text{GasInj}_t^{ik} > 0$ is sufficient to establish that both $\text{ReInj}_t^{ik} > 0$ and $\text{PInj}_t^{ik} > 0$ must hold true, and therefore $\phi_{8t}^{ik} = 0$. Using the optimality condition of the oil firm's problem w.r.t. Gas_t^{ik}:

$$\left[P_t^{\text{Gas}} - \tau_t^{ik,\text{Gas}} - MC_t^{ik,\text{Gas}}\right](1-T_t) + \left(-\phi_{2t}^{ik} + \phi_{3t}^{ik} + \phi_{4t}^{ik}\zeta\right)\left(1-\vartheta^{ik}\right)^{-1} = 0 \tag{190}$$

and recalling that the third condition is always binding (see proof to Proposition 3), the optimality conditions for any field such that $\text{GasInj}_t^{ik} > 0$ become:

$$-\left[A_1^{ik}\left(\text{ReInj}_t^{ik}, \text{PInj}_t^{ik}\right) + a_1^{ik}\right] - \left(P_t^{\text{Gas}} - \tau_t^{ik,\text{Gas}} - MC_t^{ik,\text{Gas}}\right) - C_{3t}^{ik} = 0 \quad (191)$$

and

$$-\left[A_2^{ik}\left(\text{ReInj}_t^{ik}, \text{PInj}_t^{ik}\right) + a_2^{ik} + PP_t^{ik,\text{Gas}}\right] - C_{3t}^{ik} = 0 \quad (192)$$

Combining the two conditions and using the homogeneity of A^{ik}, we get:

$$h^{ik\ -1}\left(\frac{\text{PInj}_t^{ik}}{\text{ReInj}_t^{ik}}\right) = \frac{C_{3t}^{ik} - PP_t^{ik,\text{Gas}} - a_2^{ik}}{C_{3t}^{ik} - P_t^{i,Gas} + MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - a_1^{ik}} \quad (193)$$

for some real analytic function h^{ik} with formula:

$$h^{ik\ -1}\left(\frac{\text{PInj}_t^{ik}}{\text{ReInj}_t^{ik}}\right) = \frac{A_2^{ik}\left(\text{ReInj}_t^{ik}, \text{PInj}_t^{ik}\right)}{A_1^{ik}\left(\text{ReInj}_t^{ik}, \text{PInj}_t^{ik}\right)}\ , \quad (194)$$

where $h^{ik\ -1}$ is a function of $\frac{\text{PInj}_t^{ik}}{\text{ReInj}_t^{ik}}$ because of the homogeneity of A^{ik}, and solves for:

$$\text{PInj}_t^{ik} = h^{ik}\left(\frac{C_t^{ik} - PP_t^{ik,\text{Gas}} - a_2^{ik}}{C_t^{ik} - P_t^{\text{Gas}} + MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - a_1^{ik}}\right)\text{ReInj}_t^{ik} \quad (195)$$

Using the formula for gas injections:

$$\text{GasInj}_t^{ik} = \text{ReInj}_t^{ik} + \text{PInj}_t^{ik} \quad (196)$$

we obtain

$$\text{GasInj}_t^{i} = \left[1 + h^{ik}\left(\frac{C_t^{ik} - PP_t^{ik,\text{Gas}} - a_2^{ik}}{C_t^{ik} - P_t^{\text{Gas}} + MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} - a_1^{ik}}\right)\right]\text{ReInj}_t^{i} \quad (197)$$

Lastly we use the formulas for $PP_t^{ik,\text{Gas}}$:

$$PP_t^{ik,\text{Gas}} = P_t^{\text{Gas}} - \sigma_2^{ik}, \quad (198)$$

where σ_2^{ik} captures the segmentation of the local natural gas market for injections, and for $MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}}$:

$$MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} = \overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} + \nu_t^{ik} \quad (199)$$

into equation (197) to obtain:

$$\text{ReInj}_t^{ik} = SR^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)\text{GasInj}_t^{ik} \quad (200)$$

where $SR^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)$ is a real analytic function with formula:

$$SR^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right) = \left[1 + h^{ik}\left(\frac{\sigma_2^{ik} - a_2^{ik} - P_t^{\text{Gas}}}{\overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} - a_1^{ik} + \nu_t^{ik} - P_t^{\text{Gas}}}\right)\right]^{-1} \tag{201}$$

The formula in (200) show that, whenever $\text{GasInj}_t^{ik} > 0$, the use of own gas and purchased gas for gas injection is regulated by a share SR^{ik} which is a function of the gas price, the realized shock on the firm marginal cost and other non time-variant variables. Because h^{ik} is a real analytic function, $SR_t^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)$ also is also real analytic. Thus, it is twice differentiable. This property allows us to define all the derivatives of SR^{ik} evaluated at $P_t^{\text{Gas}} = \overline{P}_t^{\text{Gas}}, \nu_t^{ik} = \overline{\nu}_t^{ik}$ as follows:

$$SR_{jl}^{ik} = \left.\frac{\partial^{j+k} SR^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)}{\partial\left(P_t^{\text{Gas}}\right)^j \partial\left(\nu_t^i\right)^l}\right|_{P_t^{\text{Gas}} = \overline{P}_t^{\text{Gas}}, \nu_t^{ik} = \overline{\nu}_t^{ik}} \tag{202}$$

Moreover, because SR^{ik} is a real analytic function, the formula for ReInj_t^{ik} can be written as a Taylor Series:

$$\text{ReInj}_t^{ik} = \left[\sum_{j=0}^{\infty}\sum_{j=0}^{\infty}\frac{SR_{jl}^{ik}}{j!l!}\left(P_t^{\text{Gas}} - \overline{P}_t^{\text{Gas}}\right)^j\left(\nu_t^{ik} - \overline{\nu}_t^{ik}\right)^l\right]\text{GasInj}_t^{ik} \tag{203}$$

However, the formula above is not suitable for empirical purposes. Therefore, we rely on an approximate a formula for ReInj_t^{ik} using a Jth-order Taylor approximation:

$$\text{ReInj}_t^{ik} \simeq \left[\sum_{j=0}^{J}\sum_{l=0}^{J}\frac{SR_{jl}^{ik}}{j!l!}\left(P_t^{\text{Gas}} - \overline{P}_t^{\text{Gas}}\right)^j\left(\nu_t^{ik} - \overline{\nu}_t^{ik}\right)^l\right]\text{GasInj}_t^{ik} \tag{204}$$

Specifically, for practical purposes in our empirical analysis we use a first-order (i.e., linear) Taylor approximation:

$$\text{ReInj}_t^{ik} \simeq \left(SR_{00}^{ik} - \overline{P}_t^{\text{Gas}} - \overline{\nu}_t^{ik}\right)\text{GasInj}_t^{ik} + SR_{10}^{ik}P_t^{\text{Gas}}\text{GasInj}_t^{ik} + SR_{01}^{ik}\nu_t^{ik}\text{GasInj}_t^{ik} \tag{205}$$

Lastly, note that ν_t^{ik} is not observable by the econometrician. However, from section 3.3 we know that we can construct an estimate of $\eta_t^{ik} = \left(\frac{\kappa_1 - \kappa_2}{\kappa_1\pi_1 - {\kappa_2}^2} + \zeta_1\right)\nu_t^{ik}$, with formula (180), which we can use in equation (205) by applying simple change of variable $\nu_t^{ik} = \left(\frac{\kappa_1 - \kappa_2}{\kappa_1\pi_1 - {\kappa_2}^2} + \zeta_1\right)^{-1}\eta_t^{ik}$. Thus, the approximate formula for ReInj_t^{ik} can be written as follows:

$$\text{ReInj}_t^{ik} = \delta_7^{ik}\text{GasInj}_t^{i} + \delta_8^{ik}P_t^{Gas} \times \text{GasInj}_t^{ik} + \delta_9^{ik}\eta_t^{ik} \times \text{GasInj}_t^{ik} \tag{206}$$

where the coefficients $\delta_7^{ik}, \delta_8^{ik}, \delta_9^{ik}$ are field-specific coefficients and in particular $\delta_9^{ik} = SR_{01}^{ik}\left(\frac{\kappa_1 - \kappa_2}{\kappa_1\pi_1 - {\kappa_2}^2} + \zeta_1\right)^{-1}$. Note that the coefficient in front of GasInj_t^{ik} always implies that ReInj_t^{ik} is a share $\in (0,1)$ of GasInj_t^{ik} as long as $\text{GasInj}_t^{ik} > 0$, and $\text{ReInj}_t^{ik} = 0$ whenever $\text{GasInj}_t^{ik} = 0$ then , as expected.

3.6 In-Situ Use

We assume that the transformation function of field i in period t satisfies the following conditions:

$$\frac{\partial TF_{4t}^{ik}\left(\mathrm{PInS}_t^{ik};\mathrm{ReInS}_t^{ik}\right)}{\partial \mathrm{ReInS}_t^{ik}} \Bigg/ \frac{\partial TF_{6t}^{ik}\left(\mathrm{Z}_t^{ik}\right)}{\partial \mathrm{Z}_t^{ik}} = \left[b_1^{ik} + B_1^{ik}\left(\mathrm{ReInS}_t^{i},\mathrm{PInS}_t^{i}\right)\right](1-T_t)$$

and

$$\frac{\partial TF_{4t}^{ik}\left(\mathrm{PInS}_t^{ik};\mathrm{ReInS}_t^{ik}\right)}{\partial \mathrm{PInS}_t^{ik}} \Bigg/ \frac{\partial TF_{6t}^{ik}\left(\mathrm{Z}_t^{ik}\right)}{\partial \mathrm{Z}_t^{ik}} = \left[b_2^{ik} + B_2^{ik}\left(\mathrm{ReInS}_t^{i},\mathrm{PInS}_t^{i}\right)\right](1-T_t)$$

where $B^{ik}:\ (-\infty,+\infty)^2 \to \mathbb{R}$ is a strictly convex, homogeneous of degree $\varphi^{ik}>0$ real analytic function that satisfies $B_1^{ik}\left(0,\mathrm{PInS}_t^{ik}\right)+b_1^{ik}\leq 0$ for all $\mathrm{PInS}_t^{ik}>0$ and $B_2^{ik}\left(\mathrm{ReInS}_t^{ik},0\right)+b_2^{ik}+PP_t^{ik,\mathrm{Gas}}\leq 0$ for all $\mathrm{ReInS}_t^{ik}>0$, where B_j^{ik} denotes the first derivative of B^{ik} with respect to its jth argument. Note that the assumptions on B^{ik} implies that $\mathrm{ReInS}_t^{ik}>0$ if $\mathrm{PInS}_t^{ik}>0$, implying in turn that the observable variable total gas injection $\mathrm{GasInS}_t^{ik}>0$ is sufficient to establish that both $\mathrm{ReInj}_t^{ik}>0$ and $\mathrm{PInj}_t^{ik}>0$ must hold true, and therefore $\phi_{9t}^{ik}=0$. Using formula (188) into the FOCs of the oil firm problem w.r.t. ReInS_t^{ik} and PInS_t^{ik}, we get:

$$\begin{aligned}\frac{\partial \mathcal{L}_u^k}{\partial \mathrm{ReInS}_t^{ik}} &= -\left[B_1^{ik}\left(\mathrm{ReInS}_t^{ik},\mathrm{PInS}_t^{ik}\right)+b_1^{ik}\right](1-T_t)\\ &\quad +\left(-\phi_{2t}^{ik}+\phi_{3t}^{ik}+\phi_{4t}^{ik}\zeta\right)\left(1-\vartheta^{ik}\right)^{-1}+\phi_{9t}^{iks}=0\\ \frac{\partial \mathcal{L}_u^k}{\partial \mathrm{PInS}_t^{ik}} &= -\left[B_2^{ik}\left(\mathrm{ReInS}_t^{ik},\mathrm{PInS}_t^{ik}\right)+b_2^{ik}+PP_t^{ik,\mathrm{Gas}}\right](1-T_t)=0\end{aligned} \tag{207}$$

Combining the conditions above with the optimality condition of the firm's problem w.r.t. Gas_t^{ik},

$$\frac{\partial \mathcal{L}_u^k}{\partial \mathrm{Gas}_t^{ik}} = \left[P_t^{\mathrm{Gas}}-\tau_t^{ik,\mathrm{Gas}}+MC_t^{ik,\mathrm{Gas}}\right](1-T_t)+\left(-\phi_{2t}^{ik}+\phi_{3t}^{ik}+\phi_{4t}^{ik}\zeta\right)\left(1-\vartheta^{ik}\right)^{-1}=0 \tag{208}$$

the optimality conditions for any field such that $\mathrm{GasInS}_t^{ik}>0$ become:

$$\begin{aligned}\frac{\partial \mathcal{L}_u^k}{\partial \mathrm{ReInS}_t^{ik}} &= -\left[B_1^{ik}\left(\mathrm{ReInS}_t^{ik},\mathrm{PInS}_t^{ik}\right)+b_1^{ik}\right](1-T_t)\\ &\quad -\left[P_t^{\mathrm{Gas}}-\tau_t^{ik,\mathrm{Gas}}+MC_t^{ik,\mathrm{Gas}}\right](1-T_t)+\phi_{9t}^{iks}=0\\ \frac{\partial \mathcal{L}_u^k}{\partial \mathrm{PInS}_t^{ik}} &= -\left[B_2^{ik}\left(\mathrm{ReInS}_t^{ik},\mathrm{PInS}_t^{ik}\right)+b_2^{ik}+PP_t^{ik,\mathrm{Gas}}\right](1-T_t)=0\end{aligned} \tag{209}$$

Using the formula for gas in-situ use:

$$\mathrm{GasInS}_t^{ik}=\mathrm{ReInS}_t^{ik}+\mathrm{PInS}_t^{ik} \tag{210}$$

and following the same procedure used in the previous section for gas injections, we obtain the following results:

$$\text{ReInS}_t^i = \left[1 + g^{ik}\left(\frac{b_2^{ik} + PP_t^{ik,\text{Gas}}}{b_1^{ik} - MC_t^{ik,\text{Gas}} - \tau_t^{ik,\text{Gas}} + P_t^{\text{Gas}}}\right)\right] \text{GasInS}_t^i \tag{211}$$

or some real analytic function g^{ik}. Lastly we use the formulas for $PP_t^{ik,\text{Gas}}$:

$$PP_t^{ik,\text{Gas}} = P_t^{\text{Gas}} - \sigma_2^{ik} \tag{212}$$

and for $MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}}$:

$$MC_t^{ik,\text{Gas}} + \tau_t^{ik,\text{Gas}} = \overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} + \nu_t^{ik} \tag{213}$$

into equation (211) to obtain:

$$\text{ReInj}_t^i = SI^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right) \text{GasInj}_t^i \tag{214}$$

where

$$SI^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right) = \left[1 + g^{ik}\left(\frac{-b_2^{ik} - P_t^{\text{Gas}} + \sigma_2^{ik}}{-b_1^{ik} + \overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik,\text{Gas}} + \nu_t^{ik} - P_t^{\text{Gas}}}\right)\right]^{-1} \tag{215}$$

The formula in (215) show that, whenever $\text{GasInS}_t^{ik} > 0$, the use of own gas and purchased gas for in-situ use is regulated by a share SI^{ik} which is a function of the gas price, the realized shock on the firm marginal cost and other non time-variant variables. Because g^{ik} is a real analytic function, $SI_t^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)$ also is also real analytic. Thus, it is twice differentiable. This property allows us to define all the derivatives of SI^{ik} evaluated at $P_t^{\text{Gas}} = \overline{P}_t^{\text{Gas}}, \nu_t^{ik} = \overline{\nu}_t^{ik}$ as follows:

$$SI_{jl}^{ik} = \left.\frac{\partial^{j+k} SI^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)}{\partial\left(P_t^{\text{Gas}}\right)^j \partial\left(\nu_t^i\right)^l}\right|_{P_t^{\text{Gas}} = \overline{P}_t^{\text{Gas}}, \nu_t^{ik} = \overline{\nu}_t^{ik}} \tag{216}$$

Moreover, because SI^{ik} is a real analytic function, the formula for ReInS_t^{ik} can be written as a Taylor Series:

$$\text{ReInS}_t^{ik} = \left[\sum_{j=0}^{\infty}\sum_{j=0}^{\infty} \frac{SI_{jl}^{ik}}{j!l!}\left(P_t^{\text{Gas}} - \overline{P}_t^{\text{Gas}}\right)^j \left(\nu_t^{ik} - \overline{\nu}_t^{ik}\right)^l\right] \text{GasInS}_t^{ik} \tag{217}$$

However, the formula above is not suitable for empirical purposes. Therefore, we rely on an approximate a formula for ReInS_t^{ik} using a Jth-order Taylor approximation:

$$\text{ReInS}_t^{ik} \simeq \left[\sum_{j=0}^{J}\sum_{l=0}^{J} \frac{SI_{jl}^{ik}}{j!l!}\left(P_t^{\text{Gas}} - \overline{P}_t^{\text{Gas}}\right)^j \left(\nu_t^{ik} - \overline{\nu}_t^{ik}\right)^l\right] \text{GasInS}_t^{ik} \tag{218}$$

Specifically, for practical purposes in our empirical analysis we use a first-order (i.e., linear) Taylor approximation:

$$\text{ReInS}_t^{ik} \simeq \left(SI_{00}^{ik} - \overline{P}_t^{\text{Gas}} - \overline{\nu}_t^{ik}\right)\text{GasInS}_t^{ik} + SI_{10}^{ik}P_t^{\text{Gas}}\text{GasInS}_t^{ik} + SI_{01}^{ik}\nu_t^{ik}\text{GasInS}_t^{ik} \tag{219}$$

Lastly, note that ν_t^{ik} is not observable by the econometrician. However, from section 3.3 we know that we can construct an estimate of $\eta_t^{ik} = \left(\frac{\kappa_1-\kappa_2}{\kappa_1\pi_1-{\kappa_2}^2} + \zeta_1\right)\nu_t^{ik}$, with formula (180), which we can use in equation (219) by applying simple change of variable $\nu_t^{ik} = \left(\frac{\kappa_1-\kappa_2}{\kappa_1\pi_1-{\kappa_2}^2} + \zeta_1\right)^{-1}\eta_t^{ik}$. Thus, the approximate formula for ReInS_t^{ik} can be written as follows:

$$\text{ReInS}_t^{ik} = \delta_{10}^{ik}\text{GasInS}_t^{i} + \delta_{11}^{ik}P_t^{Gas}\times\text{GasInS}_t^{ik} + \delta_{12}^{ik}\eta_t^{ik}\times\text{GasInS}_t^{ik} \tag{220}$$

where the coefficients $\delta_{10}^{ik}, \delta_{11}^{ik}, \delta_{12}^{ik}$ are field-specific coefficients and in particular $\delta_{12}^{ik} = SI_{01}^{ik}\left(\frac{\kappa_1-\kappa_2}{\kappa_1\pi_1-{\kappa_2}^2} + \zeta_1\right)^{-1}$. Note that the coefficient in front of GasInS_t^{ik} always implies that ReInS_t^{ik} is a share $\in (0,1)$ of GasInS_t^{ik} as long as $\text{GasInS}_t^{ik} > 0$, and $\text{ReInS}_t^{ik} = 0$ whenever $\text{GasInS}_t^{ik} = 0$ then , as expected.

3.7 Identification: Main Equation

The formulas for $\text{IVent}_t^{ik}, \text{UVent}_t^{ik}, \text{ReInj}_t^{ik}, \text{ReInS}_t^{ik}$ derived in the previous sections write:

$$\begin{aligned}
\text{IVent}_t^{ik} &= \delta_1^{ik}D_t^{ik} + \delta_2 P_t^{\text{Gas}}D_t^{ik} + \delta_3\eta_t^{ik}D_t^{ik}\\
\text{UVent}_t^{ik} &= \delta_0^{ik} + \delta_4^{ik}\text{Oil}_t^{ik} + \delta_5 P_t^{\text{Gas}} + \delta_6\eta_t^{ik} + \epsilon_t^{ik}\\
\text{ReInj}_t^{ik} &= \delta_7^{ik}\text{GasInj}_t^{i} + \delta_8^{ik}P_t^{\text{Gas}}\times\text{GasInj}_t^{ik} + \delta_9^{ik}\eta_t^{ik}\times\text{GasInj}_t^{ik}\\
\text{ReInS}_t^{ik} &= \delta_{10}^{ik}\text{GasInS}_t^{i} + \delta_{11}^{ik}P_t^{\text{Gas}}\times\text{GasInS}_t^{ik} + \delta_{12}^{ik}\eta_t^{ik}\times\text{GasInS}_t^{ik}
\end{aligned}$$

Substitute the formulas above into the OtherGas_t^{ik} equation:

$$\text{OtherGas}_t^{ik} = \text{IVent}_t^{ik} + \text{UVent}_t^{ik} + \text{ReInj}_t^{ik} + \text{ReInS}_t^{ik} \tag{221}$$

to obtain the empirical structural equation:

$$\begin{aligned}
\text{OtherGas}_t^{ik} = \;& \delta_0^{ik} + \delta_1^{ik}D_t^{ik} + \delta_2 P_t^{\text{Gas}}D_t^{ik} + \delta_3\eta_t^{ik}D_t^{ik}\\
&+\delta_4^{ik}\text{Oil}_t^{ik} + \delta_5 P_t^{\text{Gas}} + \delta_6\eta_t^{ik} + \delta_7^{ik}\text{GasInj}_t^{i}\\
&+\delta_8^{ik}P_t^{\text{Gas}}\times\text{GasInj}_t^{ik} + \delta_9^{ik}\eta_t^{ik}\times\text{GasInj}_t^{ik} + \delta_{10}^{ik}\text{GasInS}_t^{i}\\
&+\delta_{11}^{ik}P_t^{\text{Gas}}\times\text{GasInS}_t^{ik} + \delta_{12}^{ik}\eta_t^{ik}\times\text{GasInS}_t^{ik} + \epsilon_t^{ik}
\end{aligned} \tag{222}$$

which can be estimated using the estimates of $\hat{\eta}_t^{ik}$ obtained from the residuals of the flaring equation (first stage) as a new explanatory variable. Then, one can use the estimated parameters $\hat{\delta}_0^{ik}, \hat{\delta}_1^{ik}, \hat{\delta}_2, \hat{\delta}_3, \hat{\delta}_4^{ik}, \hat{\delta}_5, \hat{\delta}_6, \hat{\delta}_7^{ik}, \hat{\delta}_8^{ik}, \hat{\delta}_9^{ik}, \hat{\delta}_{10}^{ik}, \hat{\delta}_{11}^{ik}, \hat{\delta}_{12}^{ik}$ together with the regression residuals $\widehat{res2}_t^{ik}$ to construct estimates of the quantities of interest, specifically:

$$\widehat{\text{IVent}}_t^{ik} = \hat{\delta}_1^{ik} D_t^{ik} + \hat{\delta}_2 P_t^{\text{Gas}} D_t^{ik} + \hat{\delta}_3 \hat{\eta}_t^{ik} D_t^{ik} \tag{223}$$

and

$$\widehat{\text{UVent}}_t^{ik} = \hat{\delta}_0^{ik} + \hat{\delta}_4^{ik} \text{Oil}_t^{ik} + \hat{\delta}_5 P_t^{\text{Gas}} + \hat{\delta}_6 \hat{\eta}_t^{ik} + \widehat{res2}_t^{ik} \tag{224}$$

3.8 Identifications of Bounds on the Effect of a Flaring Tax

Exploiting the relationship between the structural parameters and the empirical equation in (222), which is illustrated below:

$$\begin{array}{ll}
\delta_1^{ik} = & \frac{\pi_1 - \kappa_2}{\pi_1 \kappa_1 - {\kappa_2}^2} \left(\overline{MC}^{ik,\text{Gas}} + \tau^{ik} - \kappa_0^{ik} \right) + \zeta_0^{ik} + \zeta_1 \left(\overline{MC}^{ik,\text{Gas}} + \overline{\tau}^{ik} \right) \\
\delta_2 = & -\frac{\pi_1 - \kappa_2}{\pi_1 \kappa_1 - \kappa_2^2} \\
\delta_3 = & \frac{\pi_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \left(\zeta_1 + \frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \right)^{-1} \\
\delta_4^{ik} = & \vartheta^{ik} GOR^{ik} \\
\delta_5 = & -\frac{(\kappa_4)^2}{\kappa_6} \\
\delta_6 = & \frac{(\kappa_4)^2}{\kappa_6} \left(\frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} + \zeta_1 \right)^{-1} \\
\alpha_0^{ik} = & \frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \left(\overline{MC}^{ik} + \tau^{ik} - \kappa_0^{ik} \right) + \zeta_0^{ik} + \zeta_1 \overline{MC}^{ik,\text{Gas}} \\
\alpha_1 = & -\frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \\
\eta_t^{ik} = & \left(\frac{\kappa_1 - \kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} + \zeta_1 \right) \nu_t^{ik}
\end{array}, \tag{225}$$

we can use the estimated parameters to identify bounds on the structural objects of interest. Moreover, for the purpose of this section we impose the following additional assumptions: (i) $\delta_3 \leq 0$, which can be easily tested by checking the sign of the estimated parameter $\hat{\delta}_3$; and $\frac{\partial \text{Flare}_t^{ik*}}{\partial \tau_t^{ik,\text{Flare}}} \leq \frac{\partial \text{NRF}_t^{ik}}{\partial MC_t^{ik,\text{Gas}}}$; i.e., the effect of a marginal increase in the marginal cost of natural gas production is either positive or negative but not too large in magnitude relative to the (negative) effect of a marginal increase in the flaring tax on the amount of flaring performed by each field i. Note that the assumption stated in the previous sections also imply: $\alpha_1 \leq 0$ and $\delta_2 \leq 0$, which can also be tested by checking the sign of the corresponding estimated parameters. Then we can use the parameters α_1, δ_1, and δ_3 to identify bounds on the derivative of interest, as illustrated in the remainder of this section.

3.8.1 Lower bound on $\frac{\partial \textbf{IVent}_t^{ik}}{\partial \tau_t^{ik,\textbf{Flare}}}$

From the structural equation for IVent_t^{ik} we know that the derivative of interest has formula:

$$\frac{\partial \text{IVent}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} = \frac{\kappa_2}{\pi_1\kappa_1 - \kappa_2^2}\mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{226}$$

Consider the ratio of parameters δ_2 and δ_3. Using their structural equations in (225) we obtain:

$$\frac{\delta_2}{\delta_3} = -\left(\zeta_1 + \frac{\kappa_1 - \kappa_2}{\kappa_1\pi_1 - {\kappa_2}^2}\right) \tag{227}$$

which solves for

$$\frac{\kappa_2}{\pi_1\kappa_1 - \kappa_2^2} = \frac{\delta_2}{\delta_3} + \left(\zeta_1 + \frac{\kappa_1}{\kappa_1\pi_1 - {\kappa_2}^2}\right) \tag{228}$$

Under the assumption $\frac{\partial \text{Flare}_t^{ik*}}{\partial \tau_t^{ik,\text{Flare}}} \leq \frac{\partial \text{NRF}_t^{ik}}{\partial MC_t^{ik,\text{Gas}}}$ it must be true that $\zeta_1 + \frac{\kappa_1}{\kappa_1\pi_1 - {\kappa_2}^2} \geq 0$, which used in (228) implies:

$$\frac{\kappa_2}{\pi_1\kappa_1 - \kappa_2^2} \geq \frac{\delta_2}{\delta_3} \tag{229}$$

which can be combined with (226) to obtain a lower bound for $\frac{\partial \text{IVent}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}}$, which writes:

$$\frac{\partial \text{IVent}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq \frac{\delta_2}{\delta_3}\mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{230}$$

and note that the inequality in (230) implies that if $\frac{\delta_2}{\delta_3} \geq 0$, then $\frac{\partial \text{IVent}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq 0$, i.e., a marginal increase in the flaring tax translate in a weakly larger amount of intentional venting by field i.

In a similar way, we can use the model to derive another lower bound on the value of the derivative of interest as follows. Note that the formulas for the structural parameters in (187) imply:

$$\frac{\delta_5}{\delta_6} = -\left(\zeta_1 + \frac{\kappa_1 - \kappa_2}{\kappa_1\pi_1 - {\kappa_2}^2}\right) \tag{231}$$

Following the same argument illustrated in the previous paragraph and using the previously stated assumptions on ζ_1, equation (231) implies:

$$\frac{\kappa_2}{\pi_1\kappa_1 - \kappa_2^2} \geq \frac{\delta_5}{\delta_6}\mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{232}$$

Because both bound must be satisfied, we can combine them to obtain a single tighter lower bound, which has formula:

$$\frac{\partial \text{IVent}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq \max\left\{\frac{\delta_2}{\delta_3}, \frac{\delta_5}{\delta_6}\right\}\mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{233}$$

3.8.2 Lower bound on $\frac{\partial \mathbf{Flare}_t^{ik}}{\partial \tau_t^{ik,\mathbf{Flare}}}$

From the structural equation for Flare_t^{ik} we know that the derivative of interest has formula:

$$\frac{\partial \text{Flare}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} = -\frac{\kappa_1}{\pi_1 \kappa_1 - \kappa_2^2} \mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{234}$$

Consider the parameter α_1. Using its structural equation in (225) we obtain:

$$\frac{\kappa_1}{\kappa_1 \pi_1 - {\kappa_2}^2} = \frac{\kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} - \alpha_1. \tag{235}$$

Given that $\frac{\kappa_2}{\kappa_1 \pi_1 - {\kappa_2}^2} \geq 0$, the equation above implies:

$$\frac{\kappa_1}{\kappa_1 \pi_1 - {\kappa_2}^2} \leq -\alpha_1 \tag{236}$$

which can be combined with (234) to obtain a lower bound for $\frac{\partial \text{Flare}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}}$, which writes:

$$\frac{\partial \text{Flare}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq \alpha_1 \mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{237}$$

and note that the inequality in (237) implies that if $\alpha_1 \leq 0$, then $\frac{\partial \text{Flare}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \leq 0$, i.e., a marginal increase in the flaring tax translate in a weakly lower amount of flaring by field i, as expected.

3.8.3 Lower bound on $\frac{\partial \mathbf{CO2e}_t^{ik}}{\partial \tau_t^{ik,\mathbf{Flare}}}$

The formula for a lower bound on overall effect of a marginal increase in the flaring tax on field i's CO2-equivalent GHG emissions is given by equation (59) of the theory section of this appendix, and writes:

$$\frac{\partial \text{CO2e}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq \frac{\partial \text{IVent}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} CI^{\text{Vent}} + \frac{\partial \text{Flare}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} CI^{\text{Flare}} \tag{238}$$

Using results (237) and (230) into (238) we obtain a lower bound for the overall effect of a marginal increase in the flaring tax CO2-equivalent GHG emissions, which writes:

$$\frac{\partial \text{CO2e}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq \left(\max\left\{ \frac{\delta_2}{\delta_3}, \frac{\delta_5}{\delta_6} \right\} CI^{\text{Vent}} + \alpha_1 CI^{\text{Flare}} \right) \mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{239}$$

which can be calculated using the estimates for α_1, δ_2, δ_3, δ_5, and δ_6 obtained using the method illustrated in section 3.7, and standard values for the carbon intensity of flaring and venting from the literature. In particular, the formula above implies that $\frac{\partial \text{CO2e}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq 0$ if the following inequality holds true:

$$-\frac{1}{\alpha_1} \max\left\{\frac{\delta_2}{\delta_3}, \frac{\delta_5}{\delta_6}\right\} \geq \frac{CI^{\text{Flare}}}{CI^{\text{Vent}}} \tag{240}$$

For instance, if we use the standard conversion values from Brandt et al. (2018), namely $CI^{\text{Flare}} = 0.3018\ TCO2e/BOE$ and $CI^{\text{Vent}} = 3.9583\ TCO2e/BOE$, such that $\frac{CI^{\text{Flare}}}{CI^{\text{Vent}}} \simeq 0.07624$, we get that the condition in (239) rewrites:

$$\frac{\partial \text{CO2e}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq \left(3.9583 \max\left\{\frac{\delta_2}{\delta_3}, \frac{\delta_5}{\delta_6}\right\} + 0.3018\alpha_1\right) \mathbf{1}\left[\text{Flare}_t^{ik} > 0\right] \tag{241}$$

and therefore that $\frac{\partial \text{CO2e}_t^{ik}}{\partial \tau_t^{ik,\text{Flare}}} \geq 0$ if

$$-\frac{1}{\alpha_1} \max\left\{\frac{\delta_2}{\delta_3}, \frac{\delta_5}{\delta_6}\right\} \geq 0.07624 \tag{242}$$

which can be tested using the estimates for α_1, δ_2, δ_3, δ_5, and δ_6 obtained using the method illustrated in section 3.7. Using our estimates for these parameters, which we present in section 4, we find that the empirical value of the left-hand side of inequality 242 is 2.05687, meaning that the inequality is satisfied. In turn, this finding represents compelling evidence that the introduction of a flaring tax would increase rather than decrease the overall greenhouse gas emissions produced by the oil&gas fields in our sample.

3.9 Identification: Discussion of Identifying Assumptions

The methodology for the identification of the key structural parameters of the model illustrated in this section is based on three key assumptions. In this section we describe these assumptions in depth and discuss the possible consequences of relaxing each of them.

- Key Assumption 1: $\pi_0^{ik} \simeq \kappa_0^{ik}$. This assumption states that the marginal expected costs of flaring and intentional venting are approximately the same at $\text{IVent}_t^{ik} = 0$ and $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$. This assumption is crucial for Proposition 5 to hold true, which states that $\text{IVent}_t^{ik} > 0$ if and only if $\text{Flare}_t^{ik} > \text{NRF}_t^{ik}$. In turn, it ensures that we can use the dummy variable D_t^{ik} as a tool to separate the effect of the natural gas price and the cost shock ν_t^{ik} on IVent_t^{ik}

relative to the effect of the same variables on UVent_t^{ik}. We conjecture that the expected marginal cost of intentional venting at $\text{IVent}_t^{ik} = 0$ should be extremely close to zero, because the regulatory authority would not even start an investigation for a possible voluntary leak of an extremely small amount of natural gas, which means that the probability of a fine being issued remains arbitrarily close to zero for low levels of IVent_t^{ik}. However, one may conjecture that the marginal cost of flaring is non-zero even at $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$, because the cost of the routine maintenance of the flare stack may be roughly proportional to the amount of flaring performed. If the expected marginal cost of intentional venting evaluated at $\text{IVent}_t^{ik} = 0$ and $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$ is lower than the marginal cost of flaring at $\text{IVent}_t^{ik} = 0$ and $\text{Flare}_t^{ik} = 0$; i.e., $\pi_0^{ik} \geq \kappa_0^{ik}$, then we may have cases in which $\text{IVent}_t^{ik} > 0$ and $\text{Flare}_t^{ik} = \text{NRF}_t^{ik}$. In such a case, we conjecture that this amount of intentional venting would be incorrectly accounted as part of UVent_t^{ik}, causing the latter to be overestimated and IVent_t^{ik} to be underestimated. However, because the marginal cost of flaring is deemed to be extremely small, we conjecture that such estimation bias, if it occurs at all in our sample, is likely to be very small in magnitude. Lastly, because our proposed policy reform primarily targets intentional venting, this potential bias should result in an underestimation of the policy emission-reduction effects, which is consistent with our goal of obtaining prudent estimates.

- Key Assumptions 2 and 3: $A^{ik}\left(\text{ReInj}_t^{ik}, \text{PInj}_t^{ik}\right)$ and $B^{ik}\left(\text{ReInS}_t^{ik}, \text{PInS}_t^{ik}\right)$ are homogeneous functions. These two assumptions ensure that the formulas for ReInj_t^{ik} and ReInS_t^{ik} write $\text{ReInj}_t^{ik} = SR^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)\text{GasInj}_t^{ik}$ and $\text{ReInS}_t^{ik} = SI^{ik}\left(P_t^{\text{Gas}}, \nu_t^{ik}\right)\text{GasInS}_t^{ik}$, respectively. This is crucial for identification because it implies that ReInj_t^{ik} and ReInS_t^{ik} equal zero whenever $\text{GasInj}_t^{ik} = 0$ and $\text{GasInS}_t^{ik} = 0$, respectively. This ensures that the field fixed effect δ_0^{ik} in the structural equation can be entirely attributed to UVent_t^{ik}, which is therefore separately identified from ReInj_t^{ik} and ReInS_t^{ik}. Moreover, it also implies that ReInj_t^{ik} and ReInS_t^{ik} may be functions of P_t^{Gas} and ν_t^{ik}, but only with a functional form that is non-additive in GasInj_t^{ik} and GasInS_t^{ik}, respectively. As a consequence, this assumption allows one to identify the effect of P_t^{Gas} and ν_t^{ik} on ReInj_t^{ik} and ReInS_t^{ik} separately from the effect of the same variables on UVent_t^{ik}. We cannot exclude that the second implication may fail to hold true in our analysis. We conjecture that relaxing either of these assumptions (or both), may cause part of the effect of P_t^{Gas} and ν_t^{ik} to be attributed to GasInj_t^{ik} and GasInS_t^{ik} instead of UVent_t^{ik}. However, given that the effect of P_t^{Gas} on all those variables should be negative given the assumptions of our model and is empirically larger in magnitude relative to the effect of ν_t^{ik}, we also conjecture that a potential misspecification of the function $A^{ik}$and B^{ik} may lead to an underestimation of UVent_t^{ik} and, in turn, of the emission-reduction effects of our proposd tax reform. This is, once again, consistent with our goal of obtaining prudent estimates.

Thus, the key take-home from this analysis of the key identifying assumption of this empirical model is that the estimates of IVent_t^{ik} and UVent_t^{ik} obtained under these assumptions are likely to represent lower bounds for the quantities of interest in the case in which some of these assumptions do not hold true. Similarly, the estimated emission-reduction effects of the proposed tax reform should be interpreted as a lower bound.

4 Economic & Environmental Consequences

To compute the economic and environmental effects of the tax package, we use the Rystadt Shale Well Database Rystad Energy (n.d.). The Rystadt Shale Well Database assigns to every well a unique ID. The latter identifies one, and only one, onshore oilfield. Aggregating them, we obtain information about 1,464 oil & gas fields over a sixteen years interval (2005-2020). The combination of the cross-sectional and of the temporal dimension creates a micro-panel (cross-sectional dimension $\gg$ time dimension; 1,464 $\gg$ 16), made out of 18,909 data points[11].

Out of 1,464 oil & gas fields, Rystadt classifies 1,091 as oilfields and 373 as gas fields. Among the oilfields, 325 (i.e. 29.79% of the sample) extract "conventional" oil by recovering high viscosity liquids from permeable rocks (Light & Medium). The remaining oilfields (i.e. 51.6% of the sample) produce "unconventional" oil either extracting low viscosity liquids from permeable rocks (Heavy & Extra Heavy, 32 oilfields) or extracting high viscosity liquids from impermeable rocks (Shale & Tight, 531 oilfields). Finally, 209 oilfields (i.e. 19.16% of the sample) are hard to classify since Rystad does not directly label these formations and we do not have information about the API gravity and/or the lithology of the rocks. Therefore, we generate a fourth category (Other), which incorporates oilfields with little or no information about the API gravity of the oil and/or the lithology of the rocks containing it. Out of 373 fields, which predominantly produce gas, 330 (i.e. 88.47%

[11]The panel in unbalanced. A balanced panel would have had $1{,}464 \cdot 16 = 23{,}424$ data points. The unbalanced nature of the sample emerges because 692 fields are observed in every period, while the rest (784) either star or end their production during the studied period.

of the sample) extract raw methane from low permeability rocks (Shale & Tight gas fields) and 43 (i.e. 11.94% of the sample) from coal beds (Coalbed Methane).

Among oilfields, the output composition differs across categories. Shale & Tight formations are the main oil producers. They extract circa half of the oil (43.2%, yearly average 2.55 million barrels per day BBL/Day) and one fourth of the total natural gas (26.40%, yearly average 1.45 million barrels of oil equivalent BOE/-Day). Light & Medium formations are responsible for a comparable quantity of production (38.10%, yearly average 1.29 million BBL/Day). Other formations are the third most important oil producers. They extract circa 10% of the total production (9.94%, yearly average 0.38 million BBL/Day) and almost 20% of the natural gas production (19.50%, yearly average 0.93 million BOE/Day). Finally, Heavy & Extra Heavy formations are the least important oil producers. They extract less than one tenth of the oil (8.78%, yearly average 0.31 million BBL/Day) and they virtually do not extract natural gas (0.48%, yearly average 0.02 million BOE/Day). Table 2 presents the summary statistics for the different types of oil producers.

Table 2: Oil & Gas Production

	Oil (BBL/Day)					Total Natural Gas (BOE/Day)				
	Mean	Median	SD	Min	Max	Mean	Median	SD	Min	Max
Light & Medium	4371	394	18077	0	371643	8342	509	70088	0	1301162
Heavy & Extra Heavy	9798	5117	14188	0	90354	669	176	1395	0	10286
Shale & Tight	6283	1349	13800	0	162382	3569	850	7622	0	93413
Other Oil	2389	46	7213	0	96023	5785	339	15797	0	123664

Most of the output is concentrated in few regions, which contain only one type of oil. The largest fraction of the oil is extracted from the Permian basin (located between Texas and New Mexico), and the Bakken basin (located in west North Dakota). Both basins contain Shale & Tight deposits. Figure 2 shows the 2019 production[12].

[12]Alaska's production is not displayed in the map but available upon request.

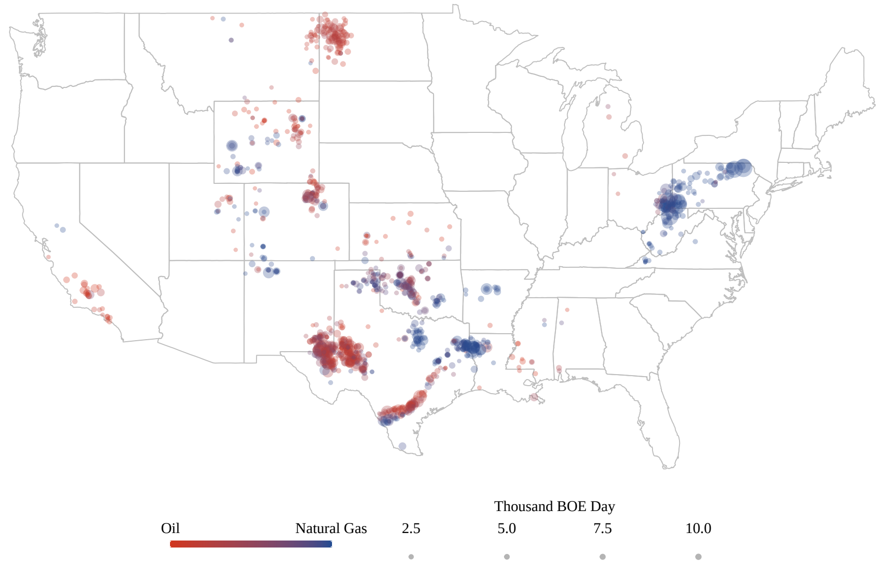

Figure 2: The 2019 Oil & Gas production in the United States. The colors of the dots reflect the composition of the outcome (Oil •, Gas •). The size of the dots reflects the aggregate volume of production.

4.1 First Step Estimation

The first step of the estimation process involves running a panel Tobit model,

$$\text{Flare}_t^{ik} = \begin{cases} \alpha_0^{ik} + \alpha_1 P_t^{\text{Gas}} + \eta_t^{ik} & \text{if } \ \eta_t^{ik} > -\alpha_0^{ik} - \alpha_1 P_t^{\text{Gas}} + \text{NRF}_t^{ik} \\ 0 & \text{otherwise} \end{cases}, \tag{243}$$

where the dependent variable, subject to left-censoring, represents flaring. The censoring occurs every time routine flaring is bigger than zero. Routine flaring denotes the proportion of total flaring that can be mitigated through managerial adjustments. In other words, it is the share of flaring not attributed to safety or maintenance concerns, which is instead categorized as non-routine flaring.

In order to divide routine from non-routine flaring, we study the flaring behaviour of producers who have the incentive to minimize their flaring behaviour, namely natural gas fields. These formations have as their primary source of income revenues obtained from selling natural gas. Therefore, they can be used to construct a benchmark in terms of minimum amount of flaring, which cannot be avoided. Table 3 presents the summary statistics for different types of natural gas producers.

Table 3: Flaring among Natural Gas Producers (BOE/Day)

	Mean	Median	SD	Min	Max
Coalbed Methane	11.28	1.61	27.77	0.00	202.66
Shale & Tight	56.10	5.20	169.88	0.00	1495.70

The median natural gas field has a flaring ratio, $\text{Flare}_t^{ik}/\text{TotGas}_t^{ik}$, of 0.05%, see Figure 3[13]. We use this quantity to identify the minimum amount of flaring necessary to guarantee the safety and the orderly maintenance of production. In other words, we assume that it is not possible for oilfields to flare less than 0.05% of all the extracted natural gas without incurring into technical problems[14]. Therefore, we define non-routine flaring,

$$\text{NRF}_t^{ik} = \begin{cases} \text{mdn}\left(\frac{\text{Flare}_t^{ik}}{\text{TotGas}_t^{ik}}\right) \cdot \text{TotGas}_t^{ik} & \text{if } \text{mdn}\left(\frac{\text{Flare}_t^{ik}}{\text{TotGas}_t^{ik}}\right) \cdot \text{TotGas}_t^{ik} \leq \text{Flare}_t^{ik} \\ \text{Flare}_t^{ik} & \text{otherwise .} \end{cases}$$

Combining this definition with a selection procedure on the original dataset, we can construct the selection rule of equation (243). First, we drop all the natural gas fields reducing the dataset from 18,909 to 14,267 data points. Then, we check that all the remaining oilfields are consistent with the definition of an oilfield given by the Energy Information Administration (EIA). In other words, we check that each oilfield contained in the dataset has a gas-oil ratio smaller than 100,000 standardized cubic feet of natural gas per BBL. This second step reduces the dataset to 11,993 data points because there are 287 fields (mostly classified as Other Oil), which Rystad labels as oilfields, that do not respect the EIA definition. Then, we drop all the observations for which Flare_t^{ik} is an NA. This third step reduces the size of the dataset from 11,993 to 5,539 data points (i.e. 38.82% of the original sample). After this last step, we can construct the dependent variable of equation

[13]Note that the average flaring ratio is bigger (0.44%) due to few outliers among Shale & Tight gas fields.

[14]While this assumption allows us to construct the reference case, its impact on the economic and environmental consequences of the policy are negligible. We study what happens with increasingly bigger thresholds and the results are virtually unchanged. All the results are available upon request.

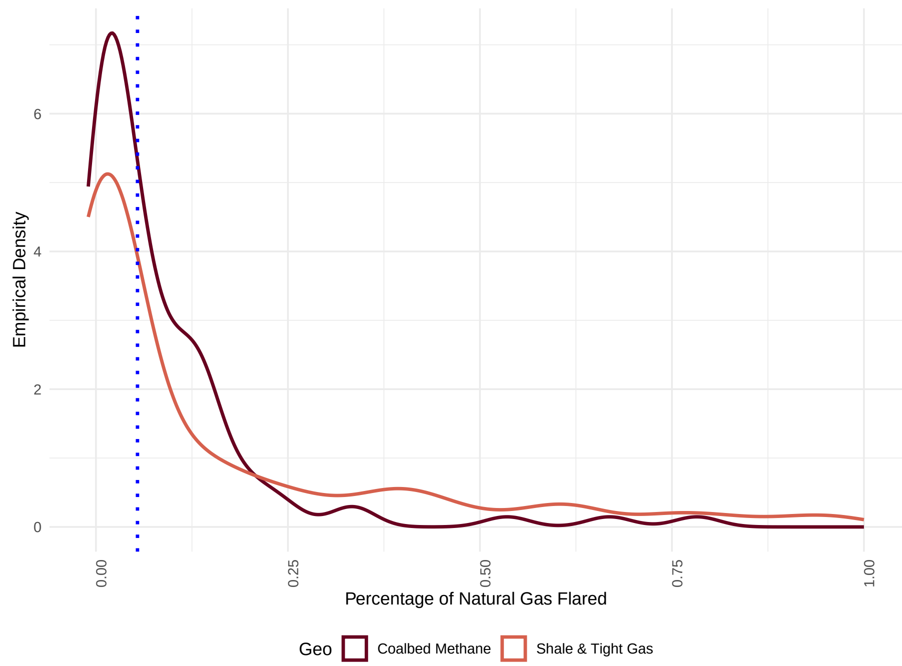

Figure 3: The empirical density function of the flaring rate for natural gas fields. The colors define the two types of formation (Coalbed Methane, Shale & Tight Gas). The blue dotted line indicates the median of their flaring rate (i.e. the fraction of natural gas that they flare divided by the total quantity of natural gas that they extract.). What is left-hand side of the vertical line is considered non-routine flaring.

(243),

$$\text{RoutineFlare}_t^{ik} = \begin{cases} \text{Flare}_t^{ik} - \text{NRF}_t^{ik} & \text{if Flare}_t^{ik} - \text{NRF}_t^{ik} > 0 \\ 0 & \text{otherwise ,} \end{cases}$$

for all the oilfields for which we have information about flaring. Given the two previous definitions, we can compare non-routine with routine flaring. According to our calculation, on average a Light & Medium deposit non-routinely flares 2.36 BOE/Day and routinely flares 24.70 BOE/Day. In other words, routine flaring is more than ten times bigger than non-routine one. The same is true if the medians are compare. Both of which are significantly smaller due to the presence of few outliers, which flare up to 827.6 BOE/Day, see Figure 4. All these proportion are similar for Heavy & Extra Heavy and Other Oil, with Other Oil having a particularly fat right tail, see Table 4.

Having defined routine and non-routine flaring, we can run model (243) using

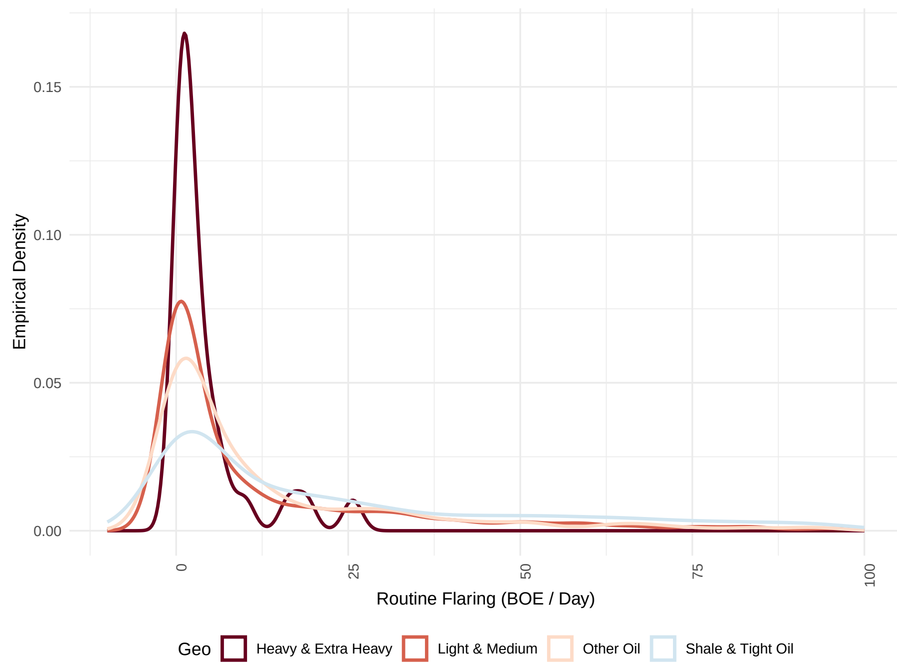

Figure 4: The empirical density function of the routine flaring for oilfields. The colors define the four types of formation (Heavy & Extra Heavy, Light & Medium, Other Oil, and Shale & Tight). The shape of the density is the one of a left censored variable.

Table 4: Summary Statistics Non-Routine vs Routine Flaring

	Non-Routine Flaring (BOE/Day)					Routine Flaring (BOE/Day)				
	Mean	Median	SD	Min	Max	Mean	Median	SD	Min	Max
Light & Medium	2.36	0.53	8.99	0.00	129.50	24.70	3.80	64.87	0.00	698.10
Heavy & Extra Heavy	0.20	0.09	0.24	0.00	0.66	4.30	1.75	6.24	0.00	25.65
Shale & Tight	2.38	0.81	4.22	0.00	40.95	237.00	44.00	558.67	0.00	9656.00
Other Oil	3.23	0.37	7.97	0.00	50.81	73.60	8.30	214.50	0.00	2433.90

the Henry Hub spot price P_t^{Gas}, measured in (United States) Dollars/BOE. The model requires the estimation of a field-specific coefficient α_0^{ik}, which measures the unobserved intrinsic characteristics that lead a field to flare, and a population coefficient α_1, which measures the sensitivity of flaring to a change in the price of natural gas. To estimate these two parameters, along with the residuals, we cannot use the within transformation since the fixed-effects panel estimation is affected by the incidental parameters problem[15] Lancaster (2000); Neyman and Scott (1948).

[15] Note that, even if the magnitude of the coefficients could be estimated consistently with T

Therefore, we assume that the individual effects are independent from the Henry Hub spot price and estimate the parameters consistently using a random effect model. In particular, we assume $\alpha_0^{ik} \overset{iid}{\sim} \mathcal{N}(0,1)$, and maximize the likelihood,

$$\mathcal{L}^{ik} = \int_{-\infty}^{\infty} \left\{ \prod_{t=1}^{T^{ik}} \left[\Phi\left(\frac{-\alpha_0^{ik} - \alpha_1 P_t^{\text{Gas}}}{\sigma_\eta} \right) \right]^{D_t^{ik}} \left[\frac{1}{\sigma_\eta} \phi\left(\frac{\text{Flare}_t^{ik} - \alpha_0^{ik} - \alpha_1 P_t^{\text{Gas}}}{\sigma_\eta} \right) \right]^{1-D_t^{ik}} \right\} \phi\left(\frac{\alpha_0^{ik}}{\sigma_{\alpha_0^{ik}}} \right) d\alpha_0^{ik} ,$$

to estimates the five parameters of interest $(\alpha_0^{ik}, \alpha_1, \sigma_{\alpha_0^{ik}}, \sigma_\eta)$, where D_t^{ik} is a dummy variable, which takes value equal to one if the field is doing routine flaring and zero otherwise, and σ_η is the standard deviation of the error term η. We set as initial values of the optimization $(\alpha_0^{ik} = 0, \alpha_1 = 0)$ and $\sigma_{\alpha_0^{ik}}$ as the standard deviation of the (column) mean of Flare_t^{ik} (i.e. the standard deviation of the mean of flare across oilfields), and $\sigma_\eta = \text{sd}(\text{Flare}_t^{ik})$. Using the `nlminb` package, we obtain an unconstrained optimization using a quasi-Newton method optimizer running the FORTRAN PORT library. The likelihood converges after 18 iterations (value of 35,141). The resulting $\hat{\eta}$ has the empirical density function shown in Figure 5.

The empirical density of η_t^{ik} is in line with what the theoretical model predicts. η_t^{ik} represents the non-visible part of the marginal costs of producing natural gas. If routine flaring is bigger than zero, it means that the oilfield faces 'high' marginal costs of gas production. Therefore, the distribution of $\hat{\eta}_t^{ik}$ must be centered around positive numbers. Table 5 breaks the results for the different types of formations.

Table 5: Summary Statistics $\hat{\eta}_t^{ik}$

	Routine Flaring ¿ 0 (BOE/Day)					Routine Flaring = 0 (BOE/Day)				
	Mean	Median	SD	Min	Max	Mean	Median	SD	Min	Max
Light & Medium	63.70	45.30	82.87	-262.20	526.40	-254.00	-280.00	90.04	-568.00	-80.00
Heavy & Extra Heavy	17.10	6.65	20.84	-0.32	76.64	NA	NA	NA	NA	NA
Shale & Tight	57.00	7.00	431.66	-1785.00	7746.00	-326.00	-322.00	146.77	-1894.00	-107.00
Other Oil	58.10	14.50	107.06	-94.10	451.00	-264.00	-265.00	98.13	-460.00	-119.00

According to our estimates, when routine flaring is positive, $\hat{\eta}_t^{ik}$ is negative for the first three deciles of its distribution, but then shifts to positive with a median expected value of 15 BOE/Day and an average of 58 BOE/Day. When we break down these results across different types of oil formations, they remain consistent with the theoretical model. All four formation types show a positive mean, with Light & Medium, Shale & Tight, and Other Oil formations displaying similar values

small (in our case 16) using special maximization routines as the ones described in Greene (2001) and Webel (2011). Their variance would still be inconsistent Henningsen (2010).

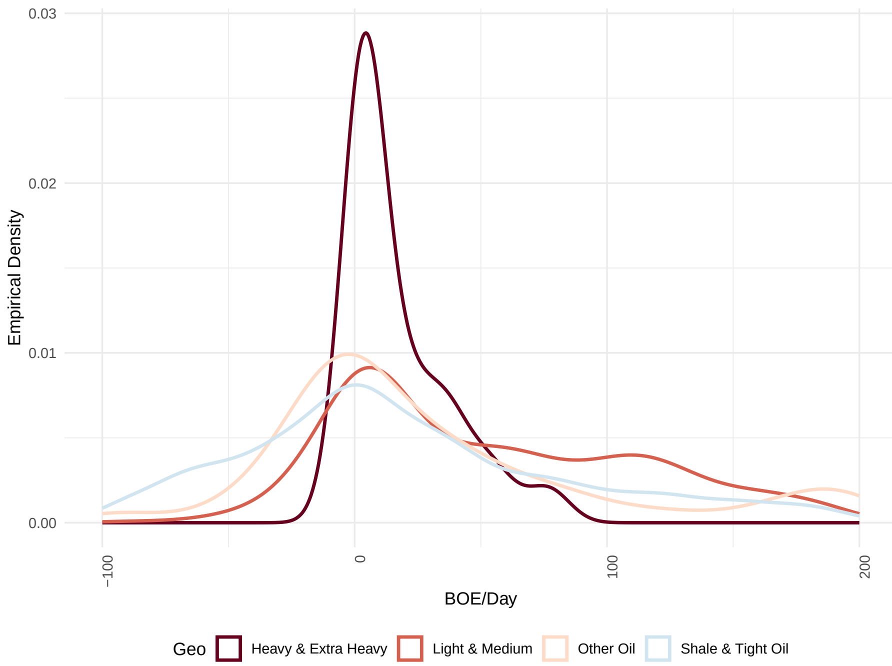

Figure 5: The empirical density function of the estimated unobserved part of natural gas marginal production costs for oilfields with positive routine flaring. For all four types of formation the expected value of $\hat{\eta}_t^{ik}$ is positive. The maxima are on average four times bigger than the minima highlighting a positive skewness of the distribution as suggested by the theoretical model.

around 60 BOE/Day, while Heavy & Extra Heavy formations show a significantly lower mean of 17.10 BOE/Day. This is not surprising since the natural gas content of heavy deposits is minimal. Similarly, all medians are positive, and the spread between the minimum and maximum values suggests a right-skewed distribution. In contrast, the magnitude of $\hat{\eta}_t^{ik}$ when $\text{RoutineFlare}_t^{ik} = 0$ is negative by construction, amounting to $-391.50\frac{\phi(-\widehat{\text{Flare}t}^{ik}/391.50)}{\Phi(-\widehat{\text{Flare}t}^{ik}/391.50)}$. Lastly, the fixed coefficient $\hat{\alpha}_1$ is estimated at -1.74, indicating that for every 1 \$ increase in the price of natural gas (in Dollar/BOE), flaring at the oilfield declines by 1.74 BOE/Day.

4.2 Second Step Estimation

The second step of the estimation process involves running a panel linear model,

$$\begin{aligned}\text{OtherGas}_t^{ik} =&\delta_0^{ik} + \delta_1 D_t^{ik} + \delta_2 P_t^{\text{Gas}} D_t^{ik} + \delta_3 \hat{\eta}_t^{ik} D_t^{ik} + \delta_4 \text{Oil}_t^{ik} + \delta_5 P_t^{\text{Gas}} + \delta_6 \hat{\eta}_t^{ik} + \\ &+ \delta_7 \text{GasInj}_t^{ik} + \delta_8 P_t^{\text{Gas}} \text{GasInj}_t^{ik} + \delta_9 \hat{\eta}_t^{ik} \text{GasInj}_t^{ik} + \\ &+ \delta_{10} \text{GasInS}_t^{i} + \delta_{11} P_t^{\text{Gas}} \text{GasInS}_t^{ik} + \delta_{12} \hat{\eta}_t^{ik} \text{GasInS}_t^{ik} + \epsilon_t^{ik} \ , \qquad (244)\end{aligned}$$

where the dependent variable represents the quantity of extracted gas that is not sold or flared, as defined in equation (162), measured in BOE/Day. In other words, the dependent variable equals the quantity of natural gas vented intentionally and unintentionally, plus the quantity (re-)injection, plus the quantity used onsite to generate heat or electricity, as described in equation (163). D_t^{ik} is a dummy variable, which takes value equal to one if the field is doing routine flaring and zero otherwise, P_t^{Gas} is the price of natural gas, as defined in section 4.1, while $\hat{\eta}_t^{ik}$ are the residuals obtained in the first step regression. Finally, all the other terms control for the volumes of natural gas injected or used in situ. δ_0^{ik} is an unobserved field specific effect, which might correlate with the other parameters as well as with the other explanatory variables. All the other are fixed coefficients. Finally, ϵ_t^{ik} is an error term normally distributed with mean zero and finite variance.

Table 6: Other Gas among Oil Producers (BOE/Day)

	Mean	Median	SD	Min	Max
Light & Medium	238.00	30.00	851.25	-60.00	8555.00
Heavy & Extra Heavy	1.29	0.40	1.53	0.00	4.30
Shale & Tight	204.00	38.00	473.17	-1.00	5925.00
Other Oil	171.20	33.40	276.55	0.00	1269.50

The dependent variable has an expected value of 210 BOE/Day and a median of 34 BOE/Day. The mean is significantly larger than the median due to a few outliers, particularly in the Light & Medium and Shale & Tight formations, which produce up to 8,555 and 5,925 BOE/Day, respectively, that are neither sold nor flared. Additionally, the standard deviation is notably higher for Light & Medium formations compared to other types of oil. According to a covariate-augmented Dickey-Fuller test with one and two lags (p-value = 0.01), the dependent variable is stationary. This allows us to run equation (244) in levels without encountering issues related to non-stationarity. We run three standard linear panel regressions: pooled ordinary least squares (OLS), a random effects model assuming that δ_0^{ik} is normally distributed with homoskedastic variance $\delta_0^{ik} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_{\delta_0})$ and independent of the regressors, and a fixed effects model. To check for autocorrelation among the estimated residuals $\hat{\epsilon}_t^{ik}$, we perform three Breusch-Godfrey tests for panel models, all of which indicate the presence of autocorrelation. Additionally, an F-test for cross-sectional and time effects suggeste the presence of fixed effects in the model. Finally, we run a Hausman test to differentiate between the random and fixed effects models, and the results rejected the random effects model. These findings collectively suggest that: 1) the error terms may be autocorrelated, 2) the variance of the residuals is heteroskedastic across the cross-sectional dimension, 3) the panel structure of the dataset matters, and 4) the unobserved individual effects are likely correlated with the explanatory variables.

Therefore, we run equation (244) using a feasible generalized least squared model, which included field-level fixed effects. This method uses a two-step estimation process. In the first step an ordinary least square estimation is done on equation (244) using the fixed effect option. Then, the resulting residuals $\hat{\epsilon}_t^{ik}$ are used to estimate an error covariance matrix to be used in a feasible generalized least square analysis. In this way, the error covariance structure inside each oilfield is fully unrestricted and is therefore robust against any type of intra-group heteroskedasticity and serial correlation[16]. Table 7 reports the results of the within estimation of the feasible generalized least squared model[17].

Variables	Estimate	Std. Error	z-Value	Pr(> $\|z\|$)
Dummy	**103.63*** **	7.41	14.00	0.00
Spot Price · Dummy	**-2.37*** **	0.10	-22.60	0.00
First Stage Residual · Dummy	**0.19*** **	0.02	11.10	0.00
Oil	**0.01*** **	0.00	49.90	0.00
Lag Future Price	**-0.68*** **	0.05	-12.50	0.00
First Stage Residual	**-0.19*** **	0.02	-11.00	0.00
Residual standard error:	$\sqrt{MSE} = 0.32$ on 4770 df			
Adjusted R-Squared:	0.68			

Table 7: GLS Regression Results to quantify Venting in Flaring Oilfields

[16]Note that this method requires the estimation of $T(T+1)/2$ variance parameters. Therefore, for an individual fixed effect efficiency requires $N >> T$. This requirement is respected since the dataset has a cross-sectional dimension of 556 oilfields and a time dimension of 16 years.

[17]Only the coefficients relevant for intentional and unintentional venting are reported. The remaining ones, which are second-order Taylor approximations of the (re-)injection and in situ functions do not have a direct interpretation. Their sign and magnitude is only relevant to net out these two options and do not over-estimate the impact of the policy reform.

The first three coefficients, denoted as $(\delta_1, \delta_2, \delta_3)$, measure the extent of intentional venting in flaring oilfields. The coefficient $\hat{\delta}_1$ is positive, with an estimated value of 103.66 BOE/Day, indicating the maximum amount of gas a flaring oilfield is willing to intentionally vent when the expected natural gas price is zero. This value represents an upper limit for intentional venting, assuming the oilfield is flaring and faces marginal costs for selling natural gas that align with its unconditional expectations. From this upper bound, intentional venting decreases by 2.37 BOE/Day for each one Dollar/BOE increase in the natural gas price, as reflected by the coefficient $\hat{\delta}_2 = -2.37$ BOE^2/Dollar, conditional on flaring being greater than zero. Additionally, $\hat{\delta}_3$, estimated at 0.19, suggests that for every one BOE increase in flaring at constant prices, intentional venting rises by 0.19 BOE/Day, again conditional on flaring being greater than zero.

The second set of three coefficients, denoted as $(\delta_4, \delta_5, \delta_6)$, characterizes the magnitude of unintentional venting in both flaring and non-flaring oilfields. The coefficient $\hat{\delta}_4$ is positive, with an estimated value of 0.01, suggesting that, all else being equal, higher oil production leads to an increase in unintentional venting. The coefficient $\hat{\delta}_5$ is negative, with an estimated value of 0.68 BOE^2/Dollar, indicating that the level of maintenance of natural gas equipment increases as the price of natural gas rises. It is interesting to notice that the magnitude of this effect is smaller of the ones obtained by an increase in natural gas prices for flaring (-1.75 BOE/Day), and for intentional venting (-2.37 BOE/Day), suggesting that maintenance plays a smaller indirect role in shaping the responsiveness of GHG emissions to a change in natural gas prices. Finally, $\hat{\delta}_6$ is negative, signifying that lower expected future gas production costs incentivize increased maintenance today in preparation for more efficient future operations. The statistical significance of this last coefficient indirectly demonstrates that a joint taxation of flaring and venting would be ineffective, as oilfields under this framework would reduce maintenance activities.

Table 8: Summary Statistics of Estimated Venting

	IVent_t^{ik} (BOE/Day)					UVent_t^{ik} (BOE/Day)				
	Mean	Median	SD	Min	Max	Mean	Median	SD	Min	Max
Light & Medium	43.00	52.00	35.48	0.00	150.00	170.00	0.00	646.75	0.00	4500.00
Heavy & Extra Heavy	64.00	67.00	8.40	46.00	78.00	0.00	0.00	0.00	0.00	0.00
Shale & Tight	69.00	62.00	76.18	0.00	1500.00	36.00	0.00	274.55	0.00	1900.00
Other Oil	44.00	51.00	37.36	0.00	150.00	36.00	0.00	90.72	0.00	370.00

4.3 Policy Outcome

The resulting intentional venting and unintentional venting estimates offer a clear contrast in the scale and distribution of gas venting across different types of oil formations. For intentional venting, Shale & Tight formations exhibit the highest mean venting rate at 69 BOE/Day, with significant variability, as indicated by a high standard deviation of 76.18 BOE/Day. This suggests that intentional venting practices in these formations are more inconsistent, with some fields experiencing substantially higher venting rates. In contrast, Heavy & Extra Heavy oil formations show more uniform behavior, with a mean of 64 BOE/Day and a relatively low standard deviation of 8.40 BOE/Day. Light & Medium and Other Oil formations have similar mean venting rates, around 43-44 BOE/Day, but Light & Medium fields demonstrate a slightly higher spread in values, suggesting more variation in venting practices. For unintentional venting, the results are more varied. Light & Medium formations exhibit the highest mean UVent rate at 170 BOE/Day, driven by a few extreme outliers, as reflected in the large standard deviation of 646.75 BOE/Day. Shale & Tight formations also show notable unintentional venting, with a mean of 36 BOE/Day and a significant standard deviation, again suggesting variability across fields. Notably, Heavy & Extra Heavy formations show no unintentional venting, possibly due to the inherent characteristics of the formation (i.e. extremely low gas-oil ratio). Similarly, Other Oil formations have low UVent values, with a mean of 36 BOE/Day and less variation compared to other types. Overall, our estimates reveal that intentional venting tends to be higher and more consistent in Shale & Tight formations, while unintentional venting is more pronounced in Light & Medium formations, likely driven by equipment degradation and operational practices.

Economic Outcome By averaging across the time dimension to create a cross-sectional dataset, we can evaluate the average economic performance of the 556 observed oilfields over the study period. This approach allows us to compare the energy wasted vs the energy recovered by the policy. Summing the energy lost through non-routine flaring, routine flaring, intentional venting, and unintentional venting, and then dividing this total by the sum of oil and natural gas extracted, we find that, on average, 2.78% of all energy is wasted (4.44% of oil energy and 7.09% of natural gas energy). This equates to an aggregate average waste of 0.19 million BOE/Day, with an average waste of 350 BOE/Day and a median waste of 140 BOE/Day. As in previous analyses, the mean is significantly skewed by a small number of outliers. If 100% of routine flaring and intentional venting would have been saved by implementing the reform in it most ambitious version, on average 1.09% of all the extracted energy would have been wasted (1.74% of oil energy and 2.92% of natural gas energy). This equates to an aggregate average waste of 0.08 million BOE/Day, with an average waste of 139 BOE/Day and a median one of 18 BOE/Day. In other words, for an average natural gas price of 21.07 Dollars/BOE the average waste would decline from 7,374 Dollars/Day to 2,929 Dollars/Day (the median waste would shift from 2,950 Dollars/Day to 379.3 Dollars/Day). The savings are not equally divided from routine flaring and intentional venting. The former account for a total of 0.0860 million BOE/Day and 0.0334 million BOE/Day of intentional venting. The part of unrecovered waste is in minimal part due to non-routine flaring, which amounts to a total of 0.0012 million BOE/Day and 0.0759 million of BOE/Day of unintentional venting.

Table 9: Delta in Oil and Natural Gas Tax for different Methane Savings Scenarios

	Oil Tax ($/BBL)					Natural Gas Tax ($/BOE)
	Mean	Median	SD	Min	Max	
25%	6.75	2.50	12.56	0.00	84.29	-5.23
50%	13.45	4.98	25.03	0.00	168.01	-10.43
75%	20.92	7.75	38.95	0.00	261.40	-16.23
100%	101.30	37.50	188.70	0.00	1266.10	-78.63

The summary statistics presented in Table 9 illustrate the required changes in oil and natural gas taxes under different methane-saving scenarios. Table 10 shows that, as the percentage of methane emissions saved increases, so does the magnitude of energy savings. For flaring, the mean savings range from 36.70 BOE/Day at a 25% savings scenario to 80.30 BOE/Day at 100%, with median values displaying a similar upward trend. The substantial standard deviations, especially in the 100% savings scenario (91.53 BOE/Day), indicate significant variability across oilfields. This is likely due to differences in the existing levels of waste. The same pattern is observed for venting savings, where mean values rise from 15.10 BOE/-Day at 25% savings to 60.00 BOE/Day at 100%. However, the greater divergence between the mean and median values in the 100% savings scenario underscores the uneven distribution of venting reduction across oilfields. The high maximum venting savings of 239.90 BOE/Day in the 100% scenario further emphasize the potential for significant reductions in emissions if the most ambitious methane-saving policies are implemented.

From an economic standpoint, the potential financial impact of these energy savings is substantial. With every saved BOE valued at 21.07 Dollars, achieving a 100% methane savings rate could result in daily savings of approximately 1,691

Table 10: Summary Statistics of Energy Savings for Different Methane Savings Scenarios

	Flaring Savings (BOE/Day)					Venting Savings (BOE/Day)					Total Savings (BOE/Day)
	Mean	Median	SD	Min	Max	Mean	Median	SD	Min	Max	Total Savings
25%	36.70	23.70	48.26	0.00	366.90	15.10	16.00	3.55	0.00	16.00	28806
50%	41.40	28.70	50.67	0.00	375.90	29.90	31.80	7.33	0.00	31.80	39637
75%	46.30	34.80	53.66	0.00	386.00	45.10	49.50	11.89	0.00	49.50	50812
100%	80.30	41.00	91.53	0.00	494.50	60.00	62.70	25.63	0.00	239.90	78013

Dollars from reduced flaring and 1,264 Dollars from reduced venting, on average. These savings represent a meaningful offset to the costs associated with higher taxes on oil production under methane-reduction policies. At the 100% savings level, oil taxes rise sharply, with an average increase to 101.30 Dollars/BBL. However, the natural gas tax simultaneously decreases to -78.63 Dollars/BBL, signaling a balanced approach to promoting environmental sustainability while maintaining economic viability for oilfields. The significant reduction in natural gas taxes mitigates the cost burden of achieving higher methane savings, encouraging firms to adopt technologies and practices that reduce flaring and venting. At the same time, the rising oil tax ensures that the policy retains a level of economic rigor, making it costly for firms to ignore potential savings opportunities. Furthermore, the variability in the savings across oilfields suggests that some fields, particularly those with higher venting and flaring rates, stand to benefit more substantially from the reforms, potentially driving industry-wide adoption of more sustainable practices.

Environmental Outcome This energy waste translates to aggregate emissions of 0.46 million tons of CO_2e/Day, of which 0.16 million tons are currently in play due to routine flaring and intentional venting. Eliminating routine flaring entirely would reduce emissions by 0.03 million tons of CO_2e/Day, while completely stopping intentional venting would save an additional 0.16 million tons of CO_2e/Day. Consequently, the emissions from intentional venting are approximately five times those from routine flaring. Of the emissions not in play, 99.87% are attributable to unintentional venting, amounting to 0.30 million tons of CO_2e/Day. The average recoverable emissions account for 35.31% of the total, with routine flaring representing 98.63% of all observable flaring-related CO_2e emissions and intentional venting accounting for 30.05% of all methane emissions.

The Table 11 presents the emissions savings in terms of tonnes for the same

Table 11: Summary Statistics of Emissions Savings for Different Methane Savings Scenarios

	Flaring Savings (TCO_2e/Day)					Venting Savings (TCO_2e/Day)					Total Savings (TCO_2e/Day)
	Mean	Median	SD	Min	Max	Mean	Median	SD	Min	Max	Total Savings
25%	13.76	8.87	18.09	0.00	137.55	59.80	63.20	14.07	0.00	63.20	40883
50%	15.50	10.77	19.00	0.00	140.94	118.00	126.00	29.01	0.00	126.00	74401
75%	17.35	13.05	20.12	0.00	144.72	179.00	196.00	47.07	0.00	196.00	108956
100%	30.10	15.39	34.42	0.00	185.39	238.00	248.00	101.40	0.00	950.00	148857

methane savings, offering an environmental perspective on the energy savings highlighted in the previous table. For flaring the mean savings range from 13.76 TCO_2e/Day at 25% savings to 30.10 TCO_2e/Day at 100%. The median values follow a similar pattern, increasing from 8.87 TCO_2e/Day to 15.39 TCO_2e/Day across the scenarios. While the standard deviations are large, particularly in the 100% scenario, they highlight the variation in potential emissions savings across oilfields. This variability suggests that some oilfields are contributing disproportionately to flaring emissions and would benefit more from aggressive methane reduction policies. The maximum flaring savings at 100%, reaching 185.39 TCO_2e/Day, indicate that substantial reductions are possible under ideal conditions.

The mean venting savings range from 59.80 TCO_2e/Day at 25% savings to 238.00 TCO_2e/Day at 100%. This significant increase across scenarios, especially at the upper end, highlights the environmental importance of reducing methane emissions. The maximum venting savings at 100% (950 TCO_2e/Day) are notably high. When these emissions savings are compared to the energy savings discussed earlier, it is clear that focusing on methane reduction (through venting savings) can lead to far greater environmental gains relative to flaring reductions. While the previous table indicated that venting savings could yield significant energy conservation, the current table shows that these reductions have an even more pronounced impact in terms of mitigating climate change.

5 Discussion & Further Policy Proposals

5.1 Efficiency of the Proposed Solution

It is easy to verify that the allocation generated by our proposed tax scheme is generally not Pareto-efficient. The reader may wonder why this scheme is preferable to Piguouvian taxation or other traditional approaches based on the Polluter-Pays principle, such as emission markets, which are well-known for inducing the First-best allocation in some circumstances.

The answer to this question lies in two key assumptions of our model, which closely mimic two core features of oil & gas markets and cause the First-best allocation to be unattainable in this economy. The first assumption is that intentional venting of natural gas is not perfectly observable and/or not contractible by the regulator. This assumption is not only justified by the fact that methane emissions are not easy to quantify and monitor. Perhaps more importantly, it is extremely difficult for the regulator to prove whether a certain amount of methane emission is "deliberate" or not in a legally binding way, because oil firms often claim that venting is justified by safety concerns (e.g., fire or explosion risk) or independent of their control to avoid punishment. Given this issue, one may wonder why the regulator does not tax all methane emissions, regardless of their (intentional or unintentional) nature. To see why, note that the existence of safety concerns implies that a regulator committing to punish venting even if they cannot prove it to be "avoidable" may induce firms to adopt a risky behavior with respect to fire and explosion hazard. Moreover, a tax on unintentional methane emissions my encourage emission misreporting, as we argue later in this section. The second assumption is that the level of maintenance of an oilfield is also not observable and/or contractible. This assumption follows the fact that the regulator may perhaps observe the firm's monetary investment in maintenance, but cannot easily assess whether such investment targets leakages reduction and/or detection in an effective way. As a result, if leakage-reducing maintenance becomes unprofitable, firms can either waste their maintenance investment in ineffective activities of divert some of it towards targets other than leakage reduction, in a way that is hard for the regulator to detect. These two assumptions together have dramatic consequences for the effectiveness of traditional pricing schemes.

First, any scheme that increases the cost of flaring relative to that of venting causes unwanted substitution between these two practices. This means that a flaring tax typically results in an increase in intentional venting, as illustrated in Proposition 1. For instance, our empirical results suggests that during the period 2005-2020 the introduction of a flaring tax of 1\$ per BOE of flared natural gas would have caused an average increase in intentional venting by the firms included in our sample equal to at least $\hat{\delta}_5/\hat{\delta}_6 \mathbf{1}[\text{Flare}_t^{ik} > 0] = 3.58$ BOE/Day.

Second, unless the production for commercial purposes of the co-extracted gas (or the alternative uses such as re-injection and in-situ use) is so profitable for a given firm that routine flaring and intentional venting are not a concern, any scheme that increases the cost of natural gas disposal (either flaring or intentional venting, or both) reduces the incentives for the firm to carry out effective maintenance aiming at reducing and detecting leakages. This implies that the introduction of a flaring tax or an increase in either the fines for intentional venting or the effectiveness of its detection by the regulator (or both) typically result in lower maintenance and increased natural gas leaking (a.k.a. "unintentional venting"), as illustrated in Proposition 2. For instance, our empirical results suggests that during the period 2005-2020 the introduction of a flaring tax of 1$ per BOE of flared natural gas would have caused an average increase in unintentional venting by the firms included in our sample equal to at least $\hat{\delta}_5 = 0.68$ BOE/Day.

These two pieces of evidence illustrate how in our setup - characterized by asymmetric information and limited enforcement - the Polluter-Pays principle does not work. Thus, a price scheme can achieve the elimination of both routine flaring and intentional venting without causing either substitution between the two practices or an increase in non-voluntary natural gas leakages only if it does not increase the overall marginal cost of gas disposal. Our proposed scheme possesses this feature. As a consequence, it eliminates both routine flaring and intentional venting while simultaneously reducing leaking (unintentional venting).

5.2 Political Economy & Implementation

One of the major shortcomings of most traditional pricing policies based on the Polluter-Pays principle, such as carbon taxes and emission markets, is that they typically result in lower output and higher equilibrium consumer prices for the goods affected directly or indirectly by the pricing scheme. This has important consequences that often undermine both their effectiveness and the political support they enjoy. First, lower output and higher consumer prices typically result in lower corporate profits and reduced consumption of certain goods. These undesirable outcomes incentivize firms to lobby against the implementation of such policies and consumers to support political parties that oppose them. Moreover, if these policies are introduced in a single country (or in a limited group of countries), they tend to reduce the competitiveness of domestically produced products on global markets, as they become more expensive relative to similar goods produced abroad. In turn, this may result in lower output and unemployment. Moreover, it may cause *carbon leakage*: the production of emission-intense goods may move from countries that apply a pricing scheme to those that do not, causing free-riding and resulting in limited or no effect of those policies on global GHG emissions. Our proposed reform is immune to these side effects because it has zero impact on all equilibrium prices and (approximately) no effect on firms' profitability, consumers' purchasing power, and government revenue. Figure 6 show the Strengths, Weaknesses, Opportunities, and Threats of the proposed reform versus two standard alternatives: the introduction of a flaring tax and the combination of a flaring tax with venting regulation and/or taxation.

The intuition underpinning these desirable outcomes is simple. Our reform requires oil producing firms to pay an extra tax which is proportional to the total natural gas extracted from the oilfield net of the quantity of natural gas that is re-injected or used in-situ for electricity and heating generation or that cannot be recovered using currently available technology and the industry's best practices (i.e., non-routine flaring and the amount of unintentional venting that cannot be avoided even with adequate maintenance). The resulting extra oil tax amount is proportional to the maximum avoidable methane footprint that the firm's oil extraction activity generates; i.e., the pricing scheme makes the firm *internalize* the potential social cost of their methane emissions, in the same way as a traditional Pigouvian tax would. However, the price scheme also provides a tax rebate which is proportional to the amount of natural gas that is sold by the firm. This second component of the reform serves two purposes. Firstly, it makes natural gas production relatively more profitable than both flaring and venting, inducing oil & gas firms to capture and sell on the market all the co-extracted natural gas that they would have flared and/or vented otherwise. Secondly, it exactly offsets the additional marginal cost of oil production caused by the increase in the oil tax. As a result, the overall marginal cost of oil production - including the cost of managing the co-extracted natural gas - in unchanged after the reform is implemented. This implies in turn that the oil production choices of all firms are unaffected at given market prices. Moreover, as long as the tax rebate on natural gas production is sufficiently large to eliminate both routine flaring and intentional venting, the rebate also compensates each firm such that its profits are approximately unaffected by the reform.

The reform also prescribes a small increase in the marginal tax rate on natural gas produced by gas-only firms and a small increase in deductions for gas-only fields. The former ensures that the additional gas supply generated by the elimination of routine flaring and intentional venting is exactly compensated by a fall in the supply of natural gas from gas-only fields. The latter compensates gas-only firms for the small profit loss they face because of the extra tax and should help in preventing lobbying by this type of firms against the implementation of the reform. As a result of these corrective taxes, the aggregate supply of both oil and natural

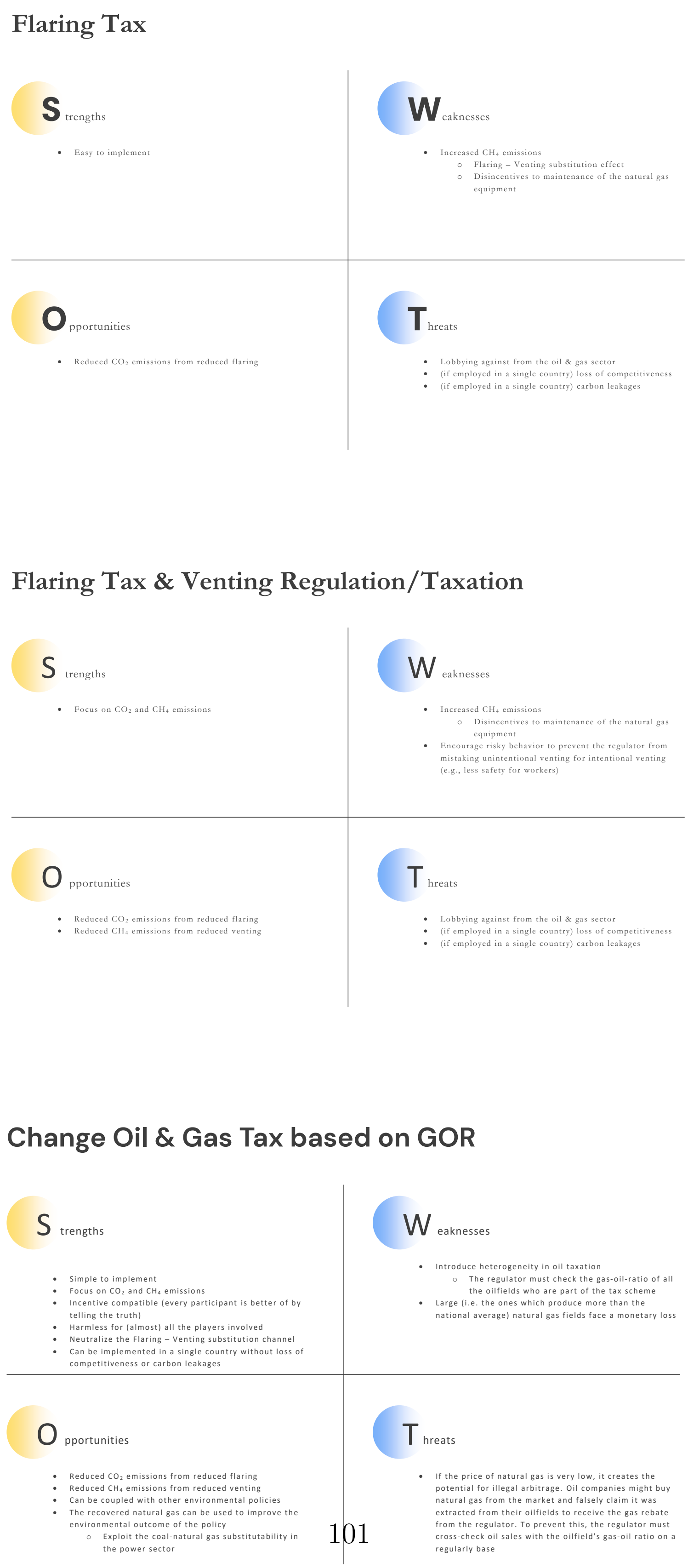

Figure 6: Differences in Strengths, Weaknesses, Opportunities, and Threats of the proposed policy versus two standard alternatives.

gas and, in turn, their equilibrium market prices are unaffected by the reform. Moreover, all oil & gas firms' profits are approximately unchanged, meaning that the oil & gas industry has little or no incentive to lobby against the implementation of the reform. For the same reason, our proposed reform is immune to loss of competitiveness and carbon leakage. Its introduction in a single country does not changes the firms' incentive to produce domestically and/or relocate production abroad. Similarly, the fact that the reform has no effect on all equilibrium prices of consumption goods and a weakly positive effect on government revenue implies that consumers and taxpayers have no incentive to oppose it though voting and/or collective action.

From a mechanism design perspective, note that - as long as the gas-oil ratio, the sales of oil and natural gas, and the quantity of natural gas that are either flared, re-injected or used in-situ for electricity production are fully observable by the regulator - each firm's total methane emission can be easily and accurately calculated using a simple formula, meaning that any attempt of cheating would be immediately detected. One may object that the quantity of natural gas re-injected and used in-situ may not be easily observable by the regulator, who may in fact have to rely on self-reported measures. While this is a valid concern, there are strong arguments suggesting that it is not a major one. First of all, it is relatively simple and cheap for the regulator to monitor ex-post the quantity of natural gas re-injected or used in-situ by a firm to detect substantial misreporting. For instance, natural gas injections typically affect the field's gas-oil ratio, whereas in-situ use to produce electricity is driven by the firm's electricity needs net of its purchases from the power grid. Any inconsistency between these measures and the reported quantities of co-extracted natural gas re-injected and used in-situ by the firm would constitute a strong signal of a likely attempt of cheating. There is an even more compelling theoretical argument that should reassure the reader regarding this potential issue. If the tax rates are set equal to their recommended level stated in section 1.4.1, then by Proposition 3 the quantity of co-extracted natural gas, which is intentionally vented by each oil & gas firm tends to zero. Recall that under our proposed tax regime any firm, which does not perform illegal venting, face no fines or fees for its unintentional methane leakages. Thus, because no methane is released intentionally, the firm's management has no strict incentive to manipulate the self-reported values of natural gas emissions, because at the firm's optimal choice the expected cost due to venting regulation is equal to zero and cannot be reduced any further. In fact, the introduction of a small fine for detected misreporting is sufficient to make it *strictly* unprofitable in expectation. In other words, truth-telling is *incentive compatible*. Even in the prudent empirical scenarios illustrated in section 5.2, the majority of the oil & gas firms is shown to react to the reform by either eliminating or drastically reducing intentional venting, meaning that the extent of misreporting is likely to be either null or extremely limited. Together with the safety concerns mentioned in the main body of the paper, this result represents a further theoretical reason to recommend no taxation on unintentional venting (leaking).

From a theoretical perspective, the main potential weakness of our proposed reform is that, if the tax rebate on natural gas sales from oilfields required to eliminate routine flaring and (intentional) venting is substantially larger than the market price of natural gas, then the scheme may promote illegal arbitrage on the natural gas market. That is, oil & gas firms may have an incentive to purchase natural gas from non-monitored sources (e.g., black market) and pretend it has been extracted from an oilfield to obtain the rebate and earn a positive profit. Because of that, our results explore four different venting reduction targets (10%, 50%, 90% and 100%), each corresponding to different values for the tax rebate. Our empirical estimates show that during the 2005-2020 time period the complete elimination of intentional venting would have required a tax rebate rate which is more than three times larger than the average market price of natural gas over the period of interest and therefore potentially strongly prone to promote illegal arbitrage. This finding is a direct consequence of the presence in our sample of a very small number of oilfields that feature an extremely large gas-oil ratio. However, all the other venting reduction targets (10%, 50%, 90%) could have been achieved with a tax rebate rate which is lower than the average market price of natural gas during the time span of interest. Thus, for the sake of prudence in the main body of the article we recommend the adoption of a venting reduction target up to 90% target, which delivers the largest emission reductions without generating excessively sizable incentives to perform illegal arbitrage.

5.3 Link to the Current Tax Structure

For the sake of simplicity and ease of interpretation the baseline model presented in section 1 of this appendix assumes that oil and gas taxation is levied through *specific taxes* on oil and natural gas sales. However, the proposed setup does not represent an accurate description of the US tax system, which is based mostly on *ad valorem* taxes, with a few exceptions. In this section, we show how the results presented in section 2 and 3 hold true even if a more realistic tax system is assumed. In detail, we borrow the setup in Kunce, Gerking, Morgan, and Maddux (2003), which provides a stylized but sufficiently realistic model of the US tax system with respect to oil and gas firms. First, we assume a tax system featuring two linear corporate tax rates on firm's profits: one at Federal level and one at State level, denoted by $T_{t,US}^{ks}$ and $T_{t,S}^{ks}$, respectively. After noticing that, relative to the baseline model, $\left(1-T_{t,US}^{ks}\right)\left(1-T_{t,S}^{ks}\right)$ simply replaces $1-T_t^{ks}$ in each firm k's objective function and that $T_{t,S}^{ks}T_{t,US}^{ks}$ replaces T_t^{ks} in the formula for total tax revenues, it is easy to show that this first change in the tax system is fully innocuous for our predictions. Secondly, we introduce taxes on oil and gas production that mimic those that are imposed by most US States and the Federal Government. Specifically, let $\tau_R^{ks,\text{Oil}}$ and $\tau_R^{ks,\text{Gas}}$ the royalty rates on production of oil and gas from public (state and federal) land. Moreover, we denote with $\tau_P^{ks,\text{Oil}}$ and $\tau_P^{ks,\text{Gas}}$ the production (severance) tax rate on production of oil and gas, respectively. Lastly, δ_{US} denotes the federal percentage depletion allowance weighted by the percentage of production attributable to eligible producers (non-integrated independents). Given these assumptions, the formula for firm k's revenue from oil and gas production in field i (before corporate taxes) writes:

$$
\begin{aligned}
&P_t^{iks,\text{Oil}}\left[(1-\tau_R^{ks,\text{Oil}})(1-\tau_P^{ks,\text{Oil}})+\frac{T_{t,US}^{ks}(1-\tau_R^{ks,\text{Oil}})}{(1-T_{t,US}^{ks})(1-T_{t,S}^{ks})}\delta_{US}\right]\text{Oil}_t^{iks}+\\
&+P_t^{s,\text{Gas}}\left[(1-\tau_R^{ks,\text{Gas}})(1-\tau_P^{ks,\text{Gas}})+\frac{T_{t,US}^{ks}(1-\tau_R^{ks,\text{Gas}})}{(1-T_{t,US}^{ks})(1-T_{t,S}^{ks})}\delta_{US}\right]\text{Gas}_t^{iks}+\\
&-\tau_t^{iks,\text{Flare}}\text{Flare}_t^{iks}-PP_t^{iks,\text{Gas}}\text{PInS}_t^{iks}-\text{IM}_t^{iks}+\text{Z}_t^{iks}\ .
\end{aligned}
$$

Replacing $1-T_t^{ks}$ with $(1-T_{t,US}^{ks})(1-T_{t,S}^{ks})$ in (18), we find that the F.O.C.s w.r.t. Oil_t^{iks} and Gas_t^{iks} in (19) become:

$$
\begin{aligned}
\frac{\partial\mathcal{L}_u^{ks}}{\partial\text{Oil}_t^{iks}}= &\quad P_t^{\text{Oil}}\left(1-\varsigma_t^{\text{Oil}}MS_t^{ks,\text{Oil}}+\frac{\sigma^{iks}}{P_t^{\text{Oil}}}\right)\\
&\left[\left(1-\tau_R^{ks,\text{Oil}}\right)\left(1-\tau_P^{ks,\text{Oil}}\right)\left(1-T_{t,US}^{ks}\right)\left(1-T_{t,S}^{ks}\right)+T_{t,US}^{ks}\left(1-\tau_R^{ks,\text{Oil}}\right)\delta_{US}\right]\\
&-\phi_{1t}^{iks}\frac{\partial TF_{1t}^{iks}(.)}{\partial\text{Oil}_t^{iks}}-\phi_{2t}^{iks}GOR^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta+\phi_{5t}^{iks}=0\\
\frac{\partial\mathcal{L}_u^{ks}}{\partial\text{Gas}_t^{iks}}= &\quad P_t^{s,\text{Gas}}\left(1-\varsigma_t^{s,\text{Gas}}MS_t^{ks,\text{Gas}}\mathbf{1}\left[\text{Oil}_t^{isk}=0\right]\right)\\
&\left[\left(1-\tau_R^{ks,\text{Gas}}\right)\left(1-\tau_{t,P}^{ks,\text{Gas}}\right)\left(1-T_{t,US}^{ks}\right)\left(1-T_{t,S}^{ks}\right)+T_{t,US}^{ks}\left(1-\tau_R^{ks,\text{Gas}}\right)\delta_{US}\right]\\
&-\phi_{1t}^{iks}\frac{\partial TF_{1t}^{iks}(.)}{\partial\text{Gas}_t^{iks}}+\left(\phi_{2t}^{iks}-\phi_{3t}^{iks}-\phi_{4t}^{iks}\zeta\right)\left(1-\vartheta^{iks}\right)^{-1}=0
\end{aligned}
\tag{245}
$$

Moreover, we find that all the other F.O.C.s of firm k in (19) are unchanged, except for featuring the term $\left(1-T_{t,US}^{ks}\right)\left(1-T_{t,S}^{ks}\right)$ instead of $\left(1-T_t^{ks}\right)$. Following the same steps as those presented in section 2, we find that the incentives generated by $\Delta\tau_t^{jks,\text{Oil}}$, $\Delta\tau_t^{s,\text{Gas}}$ for oil fields in the baseline model are replicated in the alternative model if the following equations hold true:

$$\begin{aligned} \mathit{\Delta}\tau_{t,P}^{s,\text{Gas}} = & \quad \min_{i\in\{\{1,2,\ldots,I_k\}\}_{k=1}^{K}} \left\{1 - \tau_{t,P}^{ks,\text{Gas}} + T_{t,US}^{ks}\delta_{US} - \frac{MC_t^{iks,\text{Gas}}}{P_t^{s,\text{Gas}}\left(1-\tau_R^{ks,\text{Gas}}\right)}\right\} \\ \mathit{\Delta}\tau_{t,P}^{iks,\text{Oil}} = & \quad -\mathit{\Delta}\tau_{t,P}^{s,\text{Gas}} GOR^{iks}\left(1-\vartheta^{iks}\right) \frac{P_t^{s,\text{Gas}}}{P_t^{\text{Oil}}\left(1-\varsigma_t^{\text{Oil}} MS_t^{ks,\text{Oil}}\right)+\sigma^{iks}} \end{aligned} \tag{246}$$

That is, it is sufficient to adjust the severance tax on oil and gas production at Federal level to obtain the same optimal choices obtained in the baseline setup under tax adjustments (43) and (44). In a similar way, it is possible to derive the formulas for the adjustment in the severance tax rates of gas-only fields, as well as in the rate of deduction of non-commercial gas use and unavoidable gas losses and in the lump-sum deduction amount for oil fields, in a way that generates the same incentives produced by the changes in the specific tax rates listed in (45) and (46). This ensures that and all the equilibrium outcomes are the same as those generated by the baseline model under the tax scheme outlined in (43), (44), (45) and (46). The intuition underpinning the formulas in (246) is unchanged with respect to those in (43) and (44). Namely, the reduction in the tax rate on natural gas production must be exactly compensated in terms of marginal profits for firm k by an increase in the tax rate on oil production, which is proportional to the gas/oil ratio. However, because severance taxes are *ad valorem* rather than specific taxes, their marginal effect on firm's profits is a function of oil and gas prices. Thus, in order to exactly compensate the firm at the margin, the optimal tax rate on oil production must also be multiplied by a term that is a function of the prices of natural gas and oil and that also adjusts for the market power of firm k on the crude market.

5.4 Alternative Solutions

The core of our tax reform proposal consists in the adjustment of two tax rates: the tax rate on crude sales and on gas sales by oil fields. The other tax provisions, such as the change in the tax rate faced by gas-only fields, are not crucial. They are meant to offset the excess supply of natural gas caused by the reduction of gas waste (i.e., the elimination of flaring and intentional venting and the reduction of unintentional venting) and avoid in turn any possible effect of the policy reform on equilibrium prices. However, the tax on gas production imposed on gas-only fields is not the only possible way to offset such excess natural gas supply. One could obtain a similar result through increasing the tax rate on the purchase of goods that are gross substitutes to natural gas in some midstream industry. An example is given by the rise of a specific tax on coal use in electricity production. If the cross-price elasticity of the demand for gas by power plants with respect to the price of coal is positive (i.e., coal and natural gas are gross substitutes in the production of electricity) and sufficiently large in magnitude, then there exists a specific tax rate on coal purchases by power plants which exactly offsets the excess natural gas supply mentioned above, ensuring that the natural gas price is unchanged by the introduction of the tax reform. However, note that there is no guarantee that such a policy would deliver the same level of power plants' profits that prevail under the original tax scheme. Thus, this solution only preserves some of the results stated in Proposition 3. An alternative approach is that of eliminating the excess natural gas supply via direct government purchases or via subsidies to alternative uses, such as the production of blue hydrogen. This approach would avoid losses for all firms and ensures weakly larger consumption of consumption goods $\mathbf{c}_t^s$, but would drain government revenue, implying that the policy may not be revenue-neutral and cause a fall in other consumption C_t^s. Thus, this solution also preserves some but not all the results stated in Proposition 3.

5.5 Tax on Coal and Gas Purchases

Let us consider an alternative tax scheme that allows for the excess gas supply due to the reduction of flaring and venting performed by oil firms to be offset by the demand from the power sector, with no effect on electricity output and price. In particular, the alternative scheme is identical to the baseline reform with respect to the taxation of oil firms, but does not prescribe any change in the taxation of gas firms. Conversely, the alternative scheme introduces linear taxes on the purchase of natural gas and thermal coal by firms operating in the power sector. Let the l-th net output of midstream firms y_{lt}^{js} be electricity, and the k-th net output y_{kt}^{js} be thermal coal. The excess gas supply due to the effect of the reform on the oil extraction sector at constant oil and gas prices is

$$\text{ExcessGas}_t^s = -\sum_{k=1}^{K^s} \sum_{i=1}^{I^{ks}} \left(\varDelta\text{Flare}_t^{iks} + \varDelta\text{Vent}_t^{iks} \right) \ , \tag{247}$$

where $\varDelta\text{Flare}_t^{iks}$ and $\varDelta\text{Vent}_t^{iks}$ represent the reduction in flaring and venting by oil&gas firm i due to the introduction of the reform at constant prices. Let $E^s \subseteq \{1, 2, ..., J^s\}$ be the set of midstream firms operating in the power sector of country s and $\eta_{xy}^{s,E}$ denote the cross-price elasticity of the net supply of commodity x with respect to the price of commodity y within the power sector of country s. For instance,

$$\eta_{Gk}^{s,E} = \frac{\partial \sum_{j \in E^s} \mathrm{G}_t^{js}}{\partial p_{kt}^s} \frac{p_{kt}^s}{\sum_{j \in E^s} \mathrm{G}_t^{js}} \tag{248}$$

is the elasticity of the net supply of natural gas from the power sector with respect to the price of thermal coal. Firstly, we assume that thermal coal is supplied to firms in country s at a global market price $p_{kt}^s = p_{kt}^s \left(\mathrm{Coal}_t\right)$, where Coal_t represents the global supply of thermal coal. Secondly, we assume that thermal coal is used solely by power firms in this economy. As a consequence, the cross-price elasticities of the net supplies of midstream firms, other than coal-fueled power plants with respect to the price of thermal coal, equal zero. This assumption implies that, for instance, the equilibrium net output choices of oil refineries is unaffected by changes in the price of coal as long as the prices of crude and refined oil products are unchanged. Moreover, we assume that the cross-price elasticities of the net supply of other inputs used by the power sector with respect to p_{kt}^s and $P_t^{s,\mathrm{Gas}}$ are equal to zero. This is equivalent to assume that power plants other than fossil fuel operated ones (e.g., nuclear power plants) cannot use either gas nor coal as inputs. Under these assumptions, the net output supply of all firms other than natural gas- and coal-powered power plants is unaffected by the tax scheme. However, the scheme may, in principle, affect the market price of coal because it implies a fall in the demand for coal by power plants. In turn, because the own-price elasticity of the supply of electricity from coal-fueled power plant is typically different from zero, this implies that a change in the specific tax on coal consumption from coal-fueled power plant may affect the equilibrium price of coal. Specifically, the total effect of a change in the specific tax on coal purchases on the price of coal equals

$$\frac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S} - \eta_{kk}^{s,E}} , \tag{249}$$

where η_{kk}^{sS} denotes the own-price supply elasticity of coal in country s. The alternative scheme aims to achieve two targets: (1) eliminating the excess natural gas supply due to the taxation on oil firms and (2) delivering zero effect on the prices of consumer goods, such that the consumption of energy-related goods is unaffected by the policy change. Firstly, in order to offset the excess supply of gas, the scheme must solve:

$$\mathrm{ExcessGas}_t^s = \sum_{j \in E^s} \frac{\partial \mathrm{G}_t^{js}}{\partial P_t^{s,\mathrm{Gas}}} \Delta b_t^{js} + \frac{\partial \mathrm{G}_t^{js}}{\partial p_{kt}^s} \frac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S} - \eta_{kk}^{s,D}} \Delta a_{kt}^{js} \tag{250}$$

where a_{kt}^{js} and b_t^{js} are linear specific taxes on coal and gas transactions made by power sector firms, respectively. Given that natural gas and coal are net outputs that typically have negative values for power sector firms (i.e., they are net inputs), the values of a_{kt}^{js} and b_t^{js} should be interpreted as (possibly negative-valued) subsidies. Thus, an increase in a_{kt}^{js} and b_t^{js} corresponds to a reduction in the tax rate on coal (natural gas) purchased by power firms. Secondly, we want the scheme to ensure unchanged electricity price for consumers. For small price changes, a zero effect of the tax scheme on electricity prices is obtained if the following equation is satisfied:

$$\sum_{j \in E^s} \frac{\partial \mathrm{y}_{lt}^{js}}{\partial P_t^{s,\mathrm{Gas}}} \Delta b_t^{js} + \frac{\partial \mathrm{y}_{lt}^{js}}{\partial p_{kt}^{s}} \frac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S} - \eta_{kk}^{s,D}} \Delta a_{kt}^{js} = 0 \tag{251}$$

Using the definitions of the price elasticities, the equations (250) and (251) can be rewritten as:

$$\mathrm{ExcessGas}_t^s = \eta_{GG}^E \frac{\sum_{s\in E} \mathrm{G}_t^{js}}{P_t^{s,\mathrm{Gas}}} \Delta b_t^s + \eta_{Gk}^E \frac{\sum_{s\in E} \mathrm{G}_t^{js}}{p_{kt}^s} \frac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S} - \eta_{kk}^{s,D}} \Delta a_{kt}^s \tag{252}$$

and

$$\eta_{lG}^E \frac{\sum_{s\in E} \mathrm{y}_{lt}^{js}}{P_t^{\mathrm{Gas}}} \Delta b_t^s + \eta_{lk}^E \frac{\sum_{s\in E} \mathrm{y}_{lt}^{js}}{p_{kt}^{js}} \frac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S} - \eta_{kk}^{s,D}} \Delta a_t^s = 0 \ . \tag{253}$$

The scheme solving the system of equations (252) and (253) ensures that the electricity output y_{lt}^{js} is unchanged at constant natural gas market price $P_t^{s,\mathrm{Gas}}$, implying in turn that the equilibrium price of electricity is also unaffected by the policy. We assume that the technology of a fossil fuel-operated power plant is captured by the transformation function

$$MTF_t^{js}\left(\mathbf{y}_t^{js}, \mathrm{O}_t^{js}, \mathrm{G}_t^{js}, \mathrm{F}_t^{js}, \mathrm{MInS}_t^{js}, \mathrm{MZ}_t^{js}\right) \tag{254}$$

satisfies $\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{x}_{ht}^{js}} \to 0$ for $\mathrm{x}_{ht}^{js} \leq 0$ and $\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{x}_{ht}^{js}} \to +\infty$ $\mathrm{x}_{ht}^{js} > 0$ for any argument x_{ht}^{js} other than y_{lt}^{js}, y_{kt}^{js}, and G_t^{js}. This assumption ensures that the optimal choice of any net output x_{ht}^{js} other than y_{lt}^{js}, y_{kt}^{js}, and G_t^{js} equals zero. Setting $MTF_t^{js}(\cdot)$ equal to zero and differentiating it with respect to the price of a commodity, e.g., P_t^{Gas} and rearranging the resulting equation, we obtain:

$$\frac{\partial \mathrm{y}_{lt}^{js}}{\partial P_t^{\mathrm{Gas}}} + \left(\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{y}_{kt}^{js}} \Big/ \frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{y}_{lt}^{js}} \right) \frac{\partial \mathrm{y}_{kt}^{js}}{\partial P_t^{\mathrm{Gas}}} + \left(\frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{G}_t^{js}} \Big/ \frac{\partial MTF_t^{js}(\cdot)}{\partial \mathrm{y}_{lt}^{js}} \right) \frac{\partial \mathrm{G}_t^{js}}{\partial P_t^{\mathrm{Gas}}} = 0 \ . \tag{255}$$

The equilibrium change in the output of electricity is equal to the equilibrium change in the amount of natural gas and coal used in electricity production multiplied by the marginal rate of transformation between natural gas and electricity and coal and electricity, respectively. Note that the F.O.C.s of the firm's maximization problem imply that at the optimal choice the marginal rate of transformation between natural gas (thermal coal) and electricity is the same for all the firms that consume a positive amount of natural gas (thermal coal) as long as all such firms face the same marginal tax rate on natural gas and thermal coal consumption, and electricity production. Let $\zeta^{s,\mathrm{Gas}}$ ($\zeta^{s,k}$) denote the equilibrium industry-level time-invariant marginal rate of transformation between natural gas (thermal coal) and electricity. Under these assumptions, at the optimal choice for all firms the formula for the marginal effect of a change in natural gas price on the aggregate electricity production writes:

$$\frac{\partial \sum_{j\in E^s} \mathrm{y}_{lt}^{js}}{\partial P_t^{\mathrm{Gas}}} = -\left(\zeta^{s,\mathrm{Gas}}\frac{\partial \sum_{j\in E^s} \mathrm{G}_t^{js}}{\partial P_t^{\mathrm{Gas}}} + \zeta^{s,k}\frac{\partial \sum_{j\in E^s} \mathrm{y}_{kt}^{js}}{\partial P_t^{\mathrm{Gas}}}\right) \tag{256}$$

Similarly, with respect to the price of coal we get:

$$\frac{\partial \sum_{j\in E^s} \mathrm{y}_{lt}^{js}}{\partial p_{kt}^{js}} = -\left(\zeta^{s,\mathrm{Gas}}\frac{\partial \sum_{j\in E^s} \mathrm{G}_t^{js}}{\partial p_{kt}^{js}} + \zeta^{s,k}\frac{\partial \sum_{j\in E^s} \mathrm{y}_{kt}^{js}}{\partial p_{kt}^{js}}\right) \tag{257}$$

Using the two formulas above, we can obtain the formulas for η_{lG}^E,

$$\eta_{lG}^E = -\left(\zeta^{s,\mathrm{Gas}}\eta_{GG}^E\frac{\sum_{j\in E^s} \mathrm{G}_t^{js}}{P_t^{s,\mathrm{Gas}}} + \zeta^{s,k}\eta_{kG}^E\frac{\sum_{j\in E^s} \mathrm{y}_{kt}^{js}}{P_t^{s,\mathrm{Gas}}}\right)\frac{\partial P_t^{\mathrm{Gas}}}{\sum_{j\in E^s} \mathrm{y}_{lt}^{js}}\ , \tag{258}$$

and η_{lk}^E,

$$\eta_{lk}^E = -\left(\zeta^{s,\mathrm{Gas}}\eta_{Gk}^E\frac{\sum_{j\in E^s} \mathrm{G}_t^{js}}{p_{kt}^{js}} + \zeta^{s,k}\eta_{kk}^E\frac{\sum_{j\in E^s} \mathrm{y}_{kt}^{js}}{p_{kt}^{js}}\right)\frac{p_{kt}^{js}}{\sum_{s\in E} \mathrm{y}_{lt}^{js}}\ , \tag{259}$$

We substitute formulas (258) and (259) into equation (253) to get:

$$\begin{aligned} -\left[\zeta^{s,\mathrm{Gas}}\eta_{GG}^E\left(\textstyle\sum_{j\in E} \mathrm{G}_t^{js}\right) + \zeta^{s,k}\eta_{kG}^E\left(\textstyle\sum_{j\in E} \mathrm{y}_{kt}^{js}\right)\right]\tfrac{\Delta b_t^s}{P_t^{s,\mathrm{Gas}}} \\ -\left[\zeta^{s,\mathrm{Gas}}\eta_{Gk}^E\left(\textstyle\sum_{j\in E} \mathrm{G}_t^{js}\right) + \zeta^{s,k}\eta_{kk}^E\left(\textstyle\sum_{j\in E} \mathrm{y}_{kt}^{js}\right)\right]\tfrac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S}-\eta_{kk}^{s,D}}\tfrac{\Delta a_t^s}{p_{kt}^{js}} = 0 \end{aligned} \tag{260}$$

Solve the system of equations (252) and (260) for $\frac{\Delta a_t^s}{p_{kt}^s}$ and $\frac{\Delta b_t^s}{P_t^{s,\mathrm{Gas}}}$ to by how much taxation on coal and natural gas purchases should change

$$\begin{cases} \frac{\Delta a_t^s}{p_{kt}^s} & = -\frac{\eta_{kk}^{s,S}-\eta_{kk}^{s,E}}{\eta_{kk}^{s,S}\left(\eta_{kk}^E\eta_{GG}^E-\eta_{kG}^E\eta_{Gk}^E\right)}\left(\eta_{kG}^E + \frac{\zeta^{s,\mathrm{Gas}}\sum_{j\in E^s}\mathrm{G}_t^{js}}{\zeta^{s,k}\sum_{j\in E^s}\mathrm{y}_{kt}^{js}}\eta_{GG}^E\right)\frac{\mathrm{ExcessGas}_t^s}{\sum_{j\in E}\mathrm{G}_t^{js}} \\ \frac{\Delta b_t^s}{P_t^{s,\mathrm{Gas}}} & = \frac{1}{\eta_{kk}^E\eta_{GG}^E-\eta_{kG}^E\eta_{Gk}^E}\left(\eta_{kk}^E + \eta_{Gk}^E\frac{\zeta^{s,\mathrm{Gas}}\sum_{j\in E^s}\mathrm{G}_t^{js}}{\zeta^{s,k}\sum_{j\in E^s}\mathrm{y}_{kt}^{js}}\right)\frac{\mathrm{ExcessGas}_t^s}{\sum_{s\in E}\mathrm{G}_t^{js}} \end{cases} \tag{261}$$

5.5.1 Estimation of Net Supply Elasticities

In order to estimate values for the net supply elasticities of interest, $\eta^E_{kk}, \eta^E_{GG}, \eta^E_{kG}, \eta^E_{Gk}$ we start from the elasticities of conditional factor demands. Let w^s_j denote the cost share of input j in non-renewable electricity production,

$$w^s_{jt} = \frac{p_{jt}q_{jt}}{\sum_{h=1}^{3} p_{ht}q_{ht}} \tag{262}$$

where q_{jt} denotes the aggregate quantity of input $j \in \{1, 2, 3\}$ used by the power sector in country s. Specifically, subscript 1 corresponds to natural gas, 2 to thermal coal, and 3 to nuclear fuel (uranium). For instance, q_{1t} denotes the amount of natural gas demanded by the power sector of country s in period t, and must satisfy $q_{1t} = -\sum_{j\in E^s} \mathrm{G}^{js}_t$ at any given price vector (i.e., at given prices, the values of the factor demands must be equal to the negative of the values of the net supply functions). Following Considine (1989) and EIA (2012), we assume that w^s_{jt} has the functional form

$$w^s_{jt} = \frac{\exp\left\{\chi^s_j + \sum_{z=1}^{3} \psi^s_{jz} \ln p^s_{zt} + \phi^s_j \ln e^s_t\right\}}{\sum_{h=1}^{3} \exp\left\{\chi^s_h + \sum_{z=1}^{3} \psi^s_{hz} \ln p^s_{zt} + \phi^s_h \ln e^s_t\right\}} \tag{263}$$

for each $j \in \{1, 2, 3\}$, where $e^s_t = \sum_{j\in E^s}^{J^s} y_{lt}$ is the aggregate supply of electricity from non-renewable sources of country s. We differentiate (263) w.r.t. p_{kt} to obtain the elasticity of cost share w^s_{jt} with respect to input price p_{kt}, denoted by ϑ^s_{jk},

$$\vartheta^s_{jk} = \frac{\partial w^s_{jt}}{\partial p_{kt}} \frac{p_{kt}}{w^s_{jt}} = \psi^s_{jk} - \sum_{h=1}^{3} \psi^s_{hk} w^s_{ht} \tag{264}$$

In a similar way, we can differentiate (262) w.r.t. p_{kt} to obtain another formula for ϑ^s_{jk},

$$\vartheta^s_{jk} = \epsilon^{s,E}_{jk} - w^s_{kt} - \sum_{h=1}^{3} w^s_{ht}\epsilon^{s,E}_{hz} + \mathbf{1}\left[k = j\right] \ . \tag{265}$$

The homogeneity of the conditional demand functions implies:

$$\sum_{j=1}^{3} \epsilon^{s,E}_{kj} = \sum_{j=1}^{3} \frac{\partial q^s_{kt}}{\partial p_{jt}} \frac{p_{jt}}{q^s_{kt}} = 0 \ \forall k \tag{266}$$

Secondly, the symmetry of the substitution matrix implies

$$\frac{\partial q^s_{kt}}{\partial p_{jt}} = \frac{\partial q^s_{jt}}{\partial p_{kt}} \ \forall j, k \tag{267}$$

using condition (267) into (266) we obtain:

$$\sum_{j=1}^{3} \epsilon_{jk}^{s,E} w_{jt}^{s} = 0 \ \forall k \tag{268}$$

Using this result in (265) and imposing the normalization $\sum_{h=1}^{3} \psi_{hk}^{s} w_{ht}^{s} = 0$, we can equate the RHS of (264) with the RHS of (265) and solve for $\epsilon_{jk}^{s,E}$ to get:

$$\epsilon_{jk}^{s,E} = \left(\Psi_{jk}^{s} + 1\right) w_{kt}^{s} - \mathbf{1}\left[k = j\right] \tag{269}$$

where $\Psi_{jk}^{s} = \psi_{jk}^{s}/w_{kt}^{s}$. Note that the formula for the cost share (263) implies:

$$\ln w_{jt}^{s} = \chi_{1}^{s} + \psi_{j1t}^{s} \ln p_{1t}^{s} + \psi_{j2t}^{s} \ln p_{2t}^{s} + \psi_{j3t}^{s} \ln p_{3t}^{s} + \phi_{j}^{s} \ln e_{t}^{s} - \ln C\left(e_{t}^{s}, \mathbf{p}_{t}^{s}\right) \tag{270}$$

for $j = \{1, 2, 3\}$ where $C\left(e_{t}^{s}, \mathbf{p}_{t}^{s}\right) = \sum_{h=1}^{3} p_{ht} q_{ht}^{*}$ is the cost function. Moreover, the homogeneity condition (266) is satisfied if and only if:

$$\psi_{jj}^{s} = -\sum_{k \neq j} \psi_{jk}^{s} \ \forall j \tag{271}$$

Furthermore, the symmetry condition (267) implies:

$$\frac{\psi_{kj}^{s}}{w_{jt}^{s}} = \frac{\psi_{jk}^{s}}{w_{kt}^{s}} \ \forall j, k \tag{272}$$

Lastly, the fact that the cost share must add up to one for any possible value of e_{t}^{s} implies:

$$\phi_{3}^{s} = -\phi_{1}^{s} \frac{w_{1t}^{s}}{w_{3t}^{s}} - \phi_{2}^{s} \frac{w_{2t}^{s}}{w_{3t}^{s}} \tag{273}$$

Define $\Phi_{j}^{s} = \frac{\phi_{j}^{s}}{w_{3t}^{s}}$ for $j = \{1, 2, 3\}$. Using conditions (271) and (272) into (270) for $j = 1, 2, 3$ and combining the three resulting equations we obtain the following system of equations

$$\begin{cases} \ln\left(\frac{w_{1t}^{s}}{w_{3t}^{s}}\right) = & \left(\chi_{1t}^{s} - \chi_{3t}^{s}\right) - \left[\Psi_{12}^{s} w_{2t}^{s} + \Psi_{13}^{s}\left(w_{1t}^{s} + w_{3t}^{s}\right)\right] \ln \frac{p_{1t}^{s}}{p_{3t}^{s}} + \left(\Psi_{12}^{s} w_{2t}^{s} - \Psi_{23}^{s} w_{3t}^{s}\right) \ln \frac{p_{2t}^{s}}{p_{3t}^{s}} + \\ & + \left[\Phi_{1}^{s}\left(w_{1t}^{s} + w_{3t}^{s}\right) + \Phi_{2}^{s} w_{2t}^{s}\right] \ln e_{t}^{s} \\ \ln\left(\frac{w_{2t}^{s}}{w_{3t}^{s}}\right) = & \left(\chi_{2t}^{s} - \chi_{3t}^{s}\right) + \left(\Psi_{12}^{s} w_{1t}^{s} - \Psi_{13t}^{s} w_{1t}^{s}\right) \ln \frac{p_{1t}^{s}}{p_{2t}^{s}} - \left[\Psi_{23}^{s}\left(w_{2t}^{s} + w_{3t}^{s}\right) + \Psi_{13t}^{s} w_{1t}^{s}\right] \ln \frac{p_{2t}^{s}}{p_{3t}^{s}} + \\ & + \left[\Phi_{2}^{s}\left(w_{2t}^{s} + w_{3t}^{s}\right) + \Phi_{1}^{s} w_{1t}^{s}\right] \ln e_{t}^{s} \end{cases} \tag{274}$$

where the third equation for $\ln\left(\frac{w_{1t}^{s}}{w_{2t}^{s}}\right)$ is omitted because costs share must add up to one, implying that the inclusion of the third equation would result in over-identification. Lastly, we define ω_{1t}^{s} and ω_{2t}^{s} as

$$\begin{aligned} \omega_{1t}^{s} = & \ \chi_{1t}^{s} - \chi_{3t}^{s} - \chi_{1}^{s} + \chi_{3}^{s} + \alpha_{14}^{s} \ln \frac{w_{1t-1}^{s}}{w_{3t-1}^{s}} \\ \omega_{2t}^{s} = & \ \chi_{2t}^{s} - \chi_{3t}^{s} - \chi_{2}^{s} + \chi_{3}^{s} + \alpha_{24}^{s} \ln \frac{w_{2t-1}^{s}}{w_{3t-1}^{s}} \end{aligned} \tag{275}$$

and we assume they are i.i.d. shocks. Under these assumptions, we can write the empirical equations:

$$\begin{cases} \ln\left(\frac{w_{1t}^s}{w_{3t}^s}\right) = & \alpha_{10}^s + \alpha_{11}^s \ln\frac{p_{1t}^s}{p_{3t}^s} + \alpha_{12}^s \ln\frac{p_{2t}^s}{p_{3t}^s} + \alpha_{13}^s e_t^s + \alpha_{14}^s \ln\frac{w_{1t-1}^s}{w_{3t-1}^s} + \omega_{1t}^s \\ \ln\left(\frac{w_{2t}^s}{w_{3t}^s}\right) = & \alpha_{20}^s + \alpha_{21}^s \ln\frac{p_{1t}^s}{p_{3t}^s} + \alpha_{22}^s \ln\frac{p_{2t}^s}{p_{3t}^s} + \alpha_{23}^s e_t^s + \alpha_{24}^s \ln\frac{w_{2t-1}^s}{w_{3t-1}^s} + \omega_{2t}^s \end{cases} \tag{276}$$

where the parameters $\theta^s = \left\{\alpha_{1j}^s, \alpha_{2j}^s\right\}_{j=0}^4$ in (276) map into the structural parameters in (274), delivering a linear system of ten equations and ten unknowns that can be solved to obtain formulas for all the structural parameters of interest as functions of $\left\{\alpha_{1j}^s, \alpha_{2j}^s\right\}_{j=0}^4$ and, in turn, for the elasticities of interest $\{\{\epsilon_{jk}^{s,E}\}_{j=1}^3\}_{k=1}^3$.

In order to fit the system of equations (276), we collect power plant level data on fuel consumption, electricity generation, fuel costs, fuel quantities received, and indicators of the quality of the fuel received using the forms-923 and -860 of the EIA database for the years 2005-2020 (EIA, 2024a, 2024b)[18].

In order to compute cost shares, we need information about fuel consumption and fuel costs. Fuel consumption data are available in volumetric units and MMBTU in the form-923. We take into account only the volumes of fuel consumed, measured in MMBTU, to generate electricity. As some Combined Heat and Power (CHP) plants generate not only electricity but also district heating, we break the raw data into total power consumption and fuel consumption for electricity generation. Once the power plant level quantities are collected, we aggregate them across time (from month to year), energy source (from energy source to fuel type[19]), power plants, and regions (from NERC regions to the entire US).

Prices for delivered fuels, measured in \$/MMBTU, are derived from the monthly receipts of received fuel quantities by the power plants and plant-specific monthly fuel costs including transportation again using form-923 (EIA, 2012). This approach takes into account long transportation routes to the power plant, which increase the variable costs of the plants and thus the cost of electricity generation (Hughes & Lange, 2018). Furthermore, high fuel consumption gets higher weight, which results from the use of lower-quality fuels with low heat content. In the absence of raw data on specific monthly fuel costs, these were imputed with the

[18]We filter out power plants located in Alaska and Hawaii, as these two states tend to be independent in terms of their electricity generation and have little connection to the rest of the US.

[19]We decide to aggregate between the specific energy source instead of the higher-level fuel type because of the qualitative differences between coal types such as lignite or anthracite, which can result in high deviations in the fuel costs.

costs of the nearest power plants using the same energy source measured by the Haversine distance using the following algorithm:

Algorithm 1 Imputation of Missing Fuel Costs for Power Plants

```
function imputeFuelCostsTimeBased(df)
    df ← Group df by plantId, blockNumber, and energySource
    for each entry of grouped df do
        if fuelCost_t == -9999 and (fuelCost_{t−1} and fuelCost_{t+1}) > 0 then
            fuelCost ← 1/2 × (fuelCost_{t−1} + fuelCost_{t+1})
        end if
    end for
    return df
end function

function imputeFuelCostsGeoBased(df)
    missingIndices ← Which entries in df have fuelCost == −9999
    validIndices ← Which entries in df have fuelCost > 0
    if missingIndices and validIndices! = ∅ then
        distances ← Haversine distances on longitude and latitude with R-function geosphere::distm(missing entries, valid entries)
        distances[distances > 200 km] ← NA
        for each entry in missingIndices do
            closest ← Which index with minimum distance[distance != NA]
            imputedFuelCost ← fuelCost [closest]
        end for
    end if
    return df
end function

df ← imputeFuelCostsTimeBased(df)
df ← imputeFuelCostsGeoBased(df)
```

The monthly fuel costs per specific energy source is obtained using a volume-weighted aggregate, which aggregates across time, energy source, power plants, and regions,

$$p_{j,es,t,m,n} = \frac{\sum_{b \in B} \left(p_{j,es,t,m,n,b} \times o_{j,es,t,m,n,b} \times \rho_{j,es,t,m,n,b}\right)}{\sum_{b \in B} \left(o_{j,es,t,m,n,b} \times \rho_{j,es,t,m,n,b}\right)} \tag{277}$$

$$p_{jt} = \frac{1}{N} \times \sum_{n=1}^{N} \left(\frac{p_{j,es,t,m,n}}{100}\right) \tag{278}$$

where o is the volumes of fuel delivered to power plant n, located in block b, in month m, year t, of fuel type j belonging to energy source es, and ρ is the heat content of the fuel, measured in MMBTU/Unit of Fuel.

Fuel prices for uranium used in nuclear power plants in $/MMBTU are derived from the public wholesale prices provided by the EIA (EIA, 2024d). The volume-weighted mean total purchase price in $/pound of U_3O_8 equivalent per year is used as the basis for calculation, as it includes all potential countries of origin for the uranium, and it integrates short-term, medium-term, and long-term purchase contracts. Electricity generation costs in $/kWh were subsequently derived from the regression and additional cost information provided by the WNA (EIA, 2024c) and converted into $/MMBTU.

Putting together all the previous information, it is possible to fit equation (276) as a seemingly unrelated regression (SUR) in a system of equations that is estimated simultaneously comparable to EIA (2012). Correlation of the error terms between equations is explicitly taken into account for the estimation of coefficients, what extends the SUR model from the assumptions of conventional OLS. In this way, the complex mutual and partially time-shifted dependencies between prices and demand for different fuels for electricity generation can be considered (Considine & Mount, 1984; Jones, 1995). The system of equations is implemented in using the `R` package *systemfit*. The results of the regression are shown in 13.

5.5.2 From Conditional Factor Demands to Net Supplies

In order to obtain values for the elasticities of the aggregate net supply functions of coal and natural gas from the power sector, we use the assumption that fossil fuel-operated power plants only use natural gas and/or thermal coal as inputs and only produce electricity as outputs. Given these assumptions, the following equations must hold true:

$$\frac{\partial}{\partial p_{ht}} \left[\sum_{j \in E^s} \mathrm{G}_t^{js}\right] = -\frac{\partial q_{1t}^s}{\partial p_{ht}} - \frac{\partial q_{1t}^s}{\partial e_t^s} \frac{\partial}{\partial p_{ht}} \left[\sum_{j \in E^s} y_{lt}^{js}\right] \tag{279}$$

Year	Coal	Natural Gas	Nuclear
2005	1.872	8.754	0.439
2006	1.755	7.566	0.459
2007	1.852	7.480	0.525
2008	2.306	9.339	0.587
2009	2.615	5.067	0.587
2010	2.413	5.336	0.603
2011	2.524	5.012	0.633
2012	2.609	3.669	0.630
2013	2.630	4.569	0.615
2014	2.522	5.532	0.588
2015	2.387	3.740	0.579
2016	2.318	3.250	0.571
2017	2.256	3.749	0.554
2018	2.310	3.947	0.554
2019	2.209	3.113	0.539
2020	2.378	2.757	0.528

Table 12: Weighted average fuel prices [US-Dollar/MMBTU]

and

$$\frac{\partial}{\partial p_{ht}}\left[\sum_{j\in E^s} y_{kt}^{js}\right] = -\frac{\partial q_{2t}^s}{\partial p_{ht}} - \frac{\partial q_{2t}^s}{\partial e_t^s}\frac{\partial}{\partial p_{ht}}\left[\sum_{j\in E^s} y_{lt}^{js}\right] \tag{280}$$

for each $h = 1, 2, 3$. Note that our assumptions imply $\eta_{zh}^{s,E} = 0$ for all h other than $h = z$ and $h = l$ for nuclear fuels. Using the formulas in (256) and (257) and the definition of net supply elasticity, the equations above rewrite as follows.

$$\eta_{Gh}^{s,E} = -\epsilon_{1h}^s + \frac{\epsilon_{1e}^s}{e_t^s}\left[\zeta^{s,\text{Gas}}\eta_{Gh}^{s,E}q_{1t}^s + \zeta^{s,\text{k}}\eta_{kh}^{s,E}q_{2t}^s\right] \tag{281}$$

$$\eta_{kh}^{s,E} = -\epsilon_{2h}^s + \frac{\epsilon_{2e}^s}{e_t^s}\left[\zeta^{s,\text{Gas}}\eta_{Gh}^{s,E}q_{1t}^s + \zeta^{s,\text{k}}\eta_{kh}^{s,E}q_{2t}^s\right] \tag{282}$$

Solving the system of equations (281) and (282), we obtain the formulas for the net supply elasticities of interest, which write:

$$\begin{aligned}
\eta_{GG}^{s,E} &= -\epsilon_{11}^s - \epsilon_{1e}^s\frac{\zeta^{s,\text{k}}\epsilon_{21}^s q_{2t}^s + \zeta^{s,\text{Gas}}\epsilon_{11}^s q_{1t}^s}{e_t^s - \zeta^{s,\text{k}}\epsilon_{2e}^s q_{2t}^s - \zeta^{s,\text{Gas}}\epsilon_{1e}^s q_{1t}^s}\\
\eta_{Gk}^{s,E} &= -\epsilon_{12}^s - \epsilon_{1e}^s\frac{\zeta^{s,\text{k}}\epsilon_{22}^s q_{2t}^s + \zeta^{s,\text{Gas}}\epsilon_{12}^s q_{1t}^s}{e_t^s - \zeta^{s,\text{k}}\epsilon_{2e}^s q_{2t}^s - \zeta^{s,\text{Gas}}\epsilon_{1e}^s q_{1t}^s}\\
\eta_{kG}^{s,E} &= -\epsilon_{21}^s - \epsilon_{2e}^s\frac{\zeta^{s,\text{k}}\epsilon_{21}^s q_{2t}^s + \zeta^{s,\text{Gas}}\epsilon_{11}^s q_{1t}^s}{e_t^s - \zeta^{s,\text{k}}\epsilon_{2e}^s q_{2t}^s - \zeta^{s,\text{Gas}}\epsilon_{1e}^s q_{1t}^s}\\
\eta_{kk}^{s,E} &= -\epsilon_{22}^s - \epsilon_{2e}^s\frac{\zeta^{s,\text{k}}\epsilon_{22}^s q_{2t}^s + \zeta^{s,\text{Gas}}\epsilon_{12}^s q_{1t}^s}{e_t^s - \zeta^{s,\text{k}}\epsilon_{2e}^s q_{2t}^s - \zeta^{s,\text{Gas}}\epsilon_{1e}^s q_{1t}^s}
\end{aligned} \tag{283}$$

Equation 1	Estimate	Std. Error	t-Value	Pr(> |t|)
(Intercept)	15.53	18.66	0.83	0.42
Log price ratio gas/nuclear	**0.70*** **	(0.08)	8.86	0.00
Log price ratio coal/nuclear	-0.52	(0.32)	-1.58	0.14
Log total electricity generation	-0.64	(0.84)	-0.76	0.46
Log lagged consumption gas/nuclear	0.34	(0.17)	2.07	0.06
Residual standard error:	0.06 on 10 df (15 observations for 5 parameters)			
MSE:	0.003			
Adjusted R-Squared:	0.93			
Equation 2	Estimate	Std. Error	t-Value	Pr(> |t|)
(Intercept)	-23.34	24.58	-0.95	0.36
Log price ratio gas/nuclear	0.14	(0.08)	1.74	0.11
Log price ratio coal/nuclear	**1.31** **	(0.38)	3.48	0.01
Log total electricity generation	1.03	(1.11)	0.93	0.38
Log lagged consumption coal/nuclear	**0.96*** **	(0.17)	5.59	0.00
Residual standard error:	0.063 on 10 df (15 observations for 5 parameters)			
MSE:	0.004			
Adjusted R-Squared:	0.95			

Table 13: SUR model results

Lastly, substituting the formulas for the conditional demand elasticities from (269) into the formulas in (283) we obtain the formulas for the net supply elasticities of interest as functions of estimated parameters and known quantities, namely:

$$\begin{aligned}
\eta_{GG}^{s,E} &= \Psi_{12}^s w_{2t}^s + \Psi_{13}^s w_{3t}^s - w_{1t}^s + 1 - \Phi_1^s w_{3t}^s \frac{\zeta^{s,\mathrm{k}}\left(\Psi_{12}^s w_{1t}^s + w_{1t}^s\right) q_{2t}^s - \zeta^{s,\mathrm{Gas}}\left(\Psi_{12}^s w_{2t}^s + \Psi_{13}^s w_{3t}^s - w_{1t}^s + 1\right) q_{1t}^s}{e_t^s - \zeta^{s,\mathrm{k}} \Phi_2^s w_{3t}^s q_{2t}^s - \zeta^{s,\mathrm{Gas}} \Phi_1^s w_{3t}^s q_{1t}^s} \\
\eta_{Gk}^{s,E} &= -\left(\Psi_{12}^s w_{2t}^s + w_{2t}^s\right) + \Phi_1^s w_{3t}^s \frac{\zeta^{s,\mathrm{k}}\left(\Psi_{12}^s w_{1t}^s + \Psi_{23}^s w_{3t}^s - w_{2t}^s + 1\right) q_{2t}^s - \zeta^{s,\mathrm{Gas}}\left(\Psi_{12}^s w_{2t}^s + w_{2t}^s\right) q_{1t}^s}{e_t^s - \zeta^{s,\mathrm{k}} \Phi_2^s w_{3t}^s q_{2t}^s - \zeta^{s,\mathrm{Gas}} \Phi_1^s w_{3t}^s q_{1t}^s} \\
\eta_{kG}^{s,E} &= -\left(\Psi_{12}^s w_{1t}^s + w_{1t}^s\right) - \Phi_2^s w_{3t}^s \frac{\zeta^{s,\mathrm{k}}\left(\Psi_{12}^s w_{1t}^s + w_{1t}^s\right) q_{2t}^s - \zeta^{s,\mathrm{Gas}}\left(\Psi_{12}^s w_{2t}^s + \Psi_{13}^s w_{3t}^s - w_{1t}^s + 1\right) q_{1t}^s}{e_t^s - \zeta^{s,\mathrm{k}} \Phi_2^s w_{3t}^s q_{2t}^s - \zeta^{s,\mathrm{Gas}} \Phi_1^s w_{3t}^s q_{1t}^s} \\
\eta_{kk}^{s,E} &= \Psi_{12}^s w_{1t}^s + \Psi_{23}^s w_{3t}^s - w_{2t}^s + 1 + \Phi_2^s w_{3t}^s \frac{\zeta^{s,\mathrm{k}}\left(\Psi_{12}^s w_{1t}^s + \Psi_{23}^s w_{3t}^s - w_{2t}^s + 1\right) q_{2t}^s - \zeta^{s,\mathrm{Gas}}\left(\Psi_{12}^s w_{2t}^s + w_{2t}^s\right) q_{1t}^s}{e_t^s - \zeta^{s,\mathrm{k}} \Phi_2^s w_{3t}^s q_{2t}^s - \zeta^{s,\mathrm{Gas}} \Phi_1^s w_{3t}^s q_{1t}^s}
\end{aligned} \tag{284}$$

which we can use in (261) to calculate the adjustment in the tax rates that offsets any effect of the tax reform on natural gas and electricity prices.

Using the regression coefficients, we can calculate the net-supply elasticities for natural gas and coal[20]. That means we are measuring to what extent the supply of

[20]The only elasticity not calculated is $\eta_{kk}^{s,S}$, which we assume equal to 0.89 following Dahl

fuels (taking into account imports and exports) reacts relatively to a change in the fuel price of one percentage point. Since the data is aggregated to an annual resolution and collected over a period of 16 years, the elasticities can be considered as long-term stable. Regulatory changes and the reduction or expansion of production capacities (through the exploitation of additional gas fields or the opening of mines) are explicitly reflected in the elasticity. The results for the own-price elasticities (η_{GG} and η_{CC}) as well as the cross-price elasticities (η_{GC} and η_{CG}) are shown in Table 14. The positive net-supply own-price elasticity confirm the expectation that power plants will react to a rise in fuel prices by reducing production volumes in the long term. That is, the (negative) net supply of fuel by power plants becomes smaller in magnitude, resulting in turn in a fall in the firm's electricity output. Regarding the strength of the elasticity, we find that for natural gas, overall, the suppliers' reaction is inelastic. That is in line with results from the literature (Mason & Roberts, 2018; Ponce & Neumann, 2014). The own-price elasticity of coal suggests that coal mine operators react roughly proportionally to relative price changes. Again this result is in line with previous findings (Coglianese, Gerarden, & Stock, 2020; Dahl, 2009). The cross-price elasticities illustrate that e.g., coal mine operators supply less coal to power plants in the long term and presumably also reduce supply capacity when the price of natural gas rises. The magnitude of both elasticities is roughly the same and lower than one, which means that the price-induced reactions of supply to the electricity sector are more or less similar and inelastic.

η_{GG}	η_{GC}	η_{CG}	η_{CC}
0.910	-0.826	-0.895	1.033

Table 14: Net-supply elasticities for coal and natural gas

Substituting the long-term net-supply price elasticities into the system of equations (261), we can calculate the change in the coal and natural gas tax, which would make sure that all the excess natural gas supply due to the elimination of routine flaring and venting is absorbed and that the prices of consumer goods is unchanged. The results per year and long-term average can be seen in Table 15.

In the case of the tax change for natural gas, we see that the values from 2005 to 2019 are consistently negative but decreasing in magnitude. This ensures that the additional natural gas is purchased by the electricity sector, thanks to the reduced net average gas price. Only in 2020 the positive sign for the natural gas

(2009).

	Δ tax coal	Δ tax natural gas
2005	0.000273	-0.0115
2006	0.000483	-0.0166
2007	0.000523	-0.0170
2008	0.00124	-0.0408
2009	0.00196	-0.0310
2010	0.00262	-0.0480
2011	0.00663	-0.114
2012	0.00734	-0.106
2013	0.0138	-0.230
2014	0.0178	-0.362
2015	0.00859	-0.165
2016	0.00368	-0.0856
2017	0.00525	-0.129
2018	0.000790	-0.129
2019	-0.0122	-0.0108
2020	-0.00764	0.0271
∅	**0.00243**	**-0.0724**

Table 15: Yearly tax rate changes on coal and natural gas purchases by power firms

tax indicates that suppliers would need to pay higher taxes for the delivery of the excess gas. The annual change in the tax rate for coal purchases is positive up to and including 2018, which means that coal-operated power plants pay an extra amount in \$/MMBTU of coal. However, from 2019 to 2020 the sign is reversed, so that theoretically coal-operated power plants would have faced a lower tax rate for the delivery of coal.

5.5.3 Environmental Effects

We can calculate the change in emissions relative to the baseline policy reform using the formula

$$\text{ExcessGas}_t^s CI^{\text{Gas}} - \left[\frac{\partial \sum_{j \in E^s} \mathrm{y}_{kt}^{js}}{\partial p_{kt}^s} \frac{\eta_{kk}^{s,S}}{\eta_{kk}^{s,S} - \eta_{kk}^{s,E}} \Delta a_t^s + \frac{\partial \sum_{j \in E^s} \mathrm{y}_{kt}^{js}}{\partial P_t^{s,\text{Gas}}} \Delta b_t^s \right] CI^{\text{k}} \ . \quad (285)$$

Substituting (261) allows us to rewrite (285) as

$$\text{ExcessGas}_t^s \left(CI^{\text{Gas}} - \frac{\zeta^{s,\text{Gas}}}{\zeta^{s,k}} CI^{\text{k}} \right) \,, \tag{286}$$

where $\zeta^{s,\text{Gas}}/\zeta^{s,k}$ represents the marginal rate of technical substitution between natural gas and coal of the power sector in country s[21]. In order to compute equation 286, we need to calculate the carbon intensity of the conversion parameters ζ. To obtain these numbers, we collect data for fuel consumption, electricity generation, and emission rates for each region were the extra gas could have been produced combining information from the EIA with the ones recorded by the eGRID Program (i.e., the program responsible for collecting the emissions of every NERC region for every fuel type used in the power sector) (EPA, 2020). We compute the CI^{Gas} multiplying the output emission rates by the efficiency of the different gas power plant. The output emission rates are calculated as total annual adjusted emissions divided by annual net generation

$$\text{Output Emission Rate} = \frac{\text{Total annual adjusted emissions [TonCO}_2\text{]}}{\text{Annual net electricity generation [MWh]}} \tag{287}$$

as in EPA (2020). Since, the emission data are not fully available from for the entire analysed period, we use the 2020 emission rates to obtain a conservative estimate (EPA, 2020). The efficiency of the gas power plants, expressed as the amount of electrical energy produced (megawatt-hours, MWh) per unit of thermal energy input (MMBTU)

$$\text{Gas Power Plant Efficiency} = \frac{\text{Annual net electricity generation [MWh]}}{\text{Units of thermal energy input [MMBTU]}} \tag{288}$$

measure how efficient different power plants in converting thermal energy into electrical energy. Both output emission rates and gas power plant efficiency are calculated for individual market areas using specific emission rates[22]. Then, we average them for the period under consideration to obtain the time-invariant carbon intensity

$$CI^{\text{Gas}} = \text{Gas Output Emission Rate} \cdot \text{Gas Power Plant Efficiency}. \tag{289}$$

[21] As mention in section 5.4, the alternative scheme presented in section 5.5 may not be revenue- and profit-enhancing. This scheme only ensures unchanged consumption of energy-related goods in country s, but cannot ensure that the policy is welfare-improving. Moreover, if this alternative reform is implemented solely in country s, the change in the thermal coal price, even if it is likely to be modest in magnitude, may cause carbon leakages towards countries other than s. Thus, equation 286 measures the extra emissions savings in country s. However, it does not account for possible carbon leakages.

[22] Efficiencies lower than 0 or higher than 0.7 have been filtered out due to physical and technical limitations of the power plants (Fu, Anantharaman, Jordal, & Gundersen, 2015).

In a similar way, we can compute the carbon intensity multiplied by the ratio of the substitution coefficients calculating the emissions avoided by generating electricity from natural gas instead of coal. The joint product $\zeta^{s,\text{Gas}}/\zeta^{s,\text{Coal}} CI^{\text{Coal}}$ is determined by the amount of electricity generated from natural gas and the emission factor for coal-based power generation under the assumption that the amount of generated electricity remains constant,

$$\frac{\zeta^{\text{Gas}}}{\zeta^{\text{Coal}}} CI^{\text{Coal}} = \text{Coal Output Emission Rate} \cdot \text{Gas Power Plant Efficiency}. \quad (290)$$

The additional emission savings resulting from the conversion of the natural gas into electricity and substituting coal in the electricity sector are -135 million metric Ton of Carbon Dioxide Equivalent (MM TCO_2e) for the entire period under investigation from 2005 to 2020, -8,4 MM TCO_2e on average per year and 0.023 MM TCO_2e on average per day. The largest emission savings can be achieved in the TRE NERC region, as the amount of surplus natural gas and thus the substitution of coal is highest there.

NERC Region	**Power Sector Emissions Savings 2005-2020 (t)**	**Power Sector Emissions Savings per year (t/year)**	**Power Sector Emissions Savings per Day (t/year/day)**
MRO	-32,775,224	-2,048,452	-5,612
TRE	-92,185,563	-5,761,598	-15,785
WECC	-9,546,946	-596,684	-1,635
$\sum$	-134,507,733	-8,406,734	-23,032

Table 16: Delta Emissions Results

5.5.4 Model's Limitations

There are some limitations in our approach, which should be mentioned.

First, considering nuclear power as qualitatively similar to a thermal power plant type in the analysis of net supply substitution elasticities in the power sector is not common in the literature. Nuclear power plants are unlikely to be switched in the dispatch decision of the power plant portfolio by the utilities based on fuel cost changes, preferring coal or natural gas combustion over nuclear power. Moreover, differently from some papers in the literature (EIA, 2012; Jones, 1995), our analysis does not allow for electricity generation using oil or oil derivatives such as petroleum coke. This simplifying assumption may potentially distort our estimates for the net supply elasticities in the power sector. However, given the extremely small share of power generation that the excluded inputs accounts for in the US during the last two decades, we believe the size of the aforementioned distortion should be negligible.

Second, we assume that the price of natural gas only depends on (national or regional) aggregate demand and supply, and does not vary with the quantity purchased by a given plant. This assumption could be interpreted as a stylized representation of an economic environment in which there is sufficient transport capacity and infrastructure for any additional quantity of natural gas delivered from the oilfields to the power plants, as otherwise additional and increasing cost surcharges for the transport of natural gas would have to be added to the fuel prices as natural gas sales by oil & gas firms increase.

Third, the Seemingly Unrelated Regression (SUR) model we employ assumes linearity, which may not accurately capture the potentially non-linear relationships in the energy market. Furthermore, we use yearly averages to compute the net supply elasticities, which do not account for seasonal monthly variations in energy consumption and production. This simplification may have important consequences because dispatch decisions in electricity markets are based on hourly (but mostly daily) commodity prices. Thus, even if at a yearly granularity, gas fired generation is cheaper (or more expensive) than coal fired generation, during significant parts of the year the reality could be vice versa.

Fourth, the calculation of emissions does not take into account upstream emissions, which are generated for the exploration and development of oil fields or gas wells and any upstream technology used. However, this limitation only affects the estimation of the emission levels, but not the quantification of the economic and environmental effects of our proposed policy, because the reform does not affect oil & gas firm's incentives to invest in exploration and development. As a result, those types of emissions should not be affected by the policy change and be largely irrelevant for the purpose of our policy evaluation exercise.

These limitations indicate areas for future research to refine the methodology and ideas to mitigate flaring and venting in the oil industry.

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