Directed drift selected different movement strategy in fragmented habitats

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S1. APPENDIX

Mathematical Model

Let u and v be the high and low dispersers in $[0, L]$.

$$\begin{cases} u_t = d_1\Delta u - cu_x + u(m_i(x) - b_1 u - c_1 v), \\ v_t = d_2\Delta v - cv_x + v(m_i(x) - b_2 v - c_2 u), \\ d_1 u_x(0,T) - cu(0,T) = 0, d_2 v_x(0,T) - cv(0,T) = 0, \\ u_x(L,T) = v_x(L,T) = 0. \end{cases} \tag{Right Drift}$$

$$\begin{cases} u_t = d_1\Delta u + cu_x + u(m_i(x) - b_1 u - c_1 v), \\ v_t = d_2\Delta v + cv_x + v(m_i(x) - b_2 v - c_2 u), \\ u_x(0,T) = v_x(0,T) = 0, \\ d_1 u_x(L,T) - cu(L,T) = 0, d_2 v_x(L,T) - cv(L,T) = 0. \end{cases} \tag{Left Drift}$$

$$\begin{cases} u_t = d_1\Delta u + u(m_i(x) - b_1 u - c_1 v), \\ v_t = d_2\Delta v + v(m_i(x) - b_2 v - c_2 u), \\ u_x(0,T) = v_x(0,T) = 0, \\ u_x(L,T) = v_x(L,T) = 0, \end{cases} \tag{No Drift}$$

Theorem 1. *Consider the systems* (Right Drift)*,* (Left Drift)*,* (No Drift)*. These are well-posed. That is there exist classical solutions to these systems that are global in time.*

Proof. The analysis and the well-posedness of the model follow from the work of (Lou and Lutscher (2014); Lou et al. (2019)). Herein, our constructed $m_i(x)$ growth/resource functions that match the experimental setup can be devised using inverse hyperbolic type functions that are $C^\infty(\Omega)$. They are also chosen to be strictly positive on Ω, so any sort of degeneracy is avoided.

Also note although the methods of (Lou et al. (2019)) consider symmetric inter and intra-specific competition, a simple comparison argument such as

$$u(m_i(x) - u - v) \leq u(m_i(x) - b_1 u - c_1 v), \text{ for } b_1, c_1 > 1 \tag{1}$$

yields boundedness of solutions to our systems (Right Drift), (Left Drift), (No Drift). The results follow. □

Remark 1. *Note, Lou et al. (2019) considers the above when $b_1 = c_1 = b_2 = c_2$, and obtain competitive exclusion results. In our simulations, we see co-existence, but we have non-symmetric inter and intra-specific competition. This warrants further theoretical investigation. We conjecture coexistence can be proved under certain parametric assumptions on the competition coefficients.*

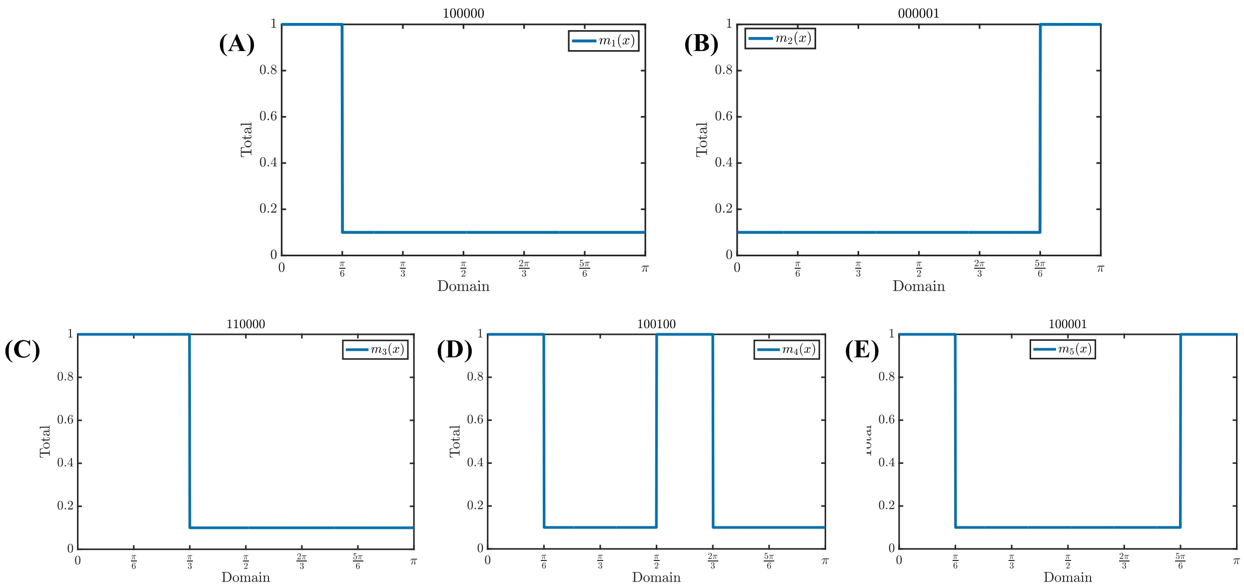

Figure S.1: Spatially explicit resource function for experimental environments

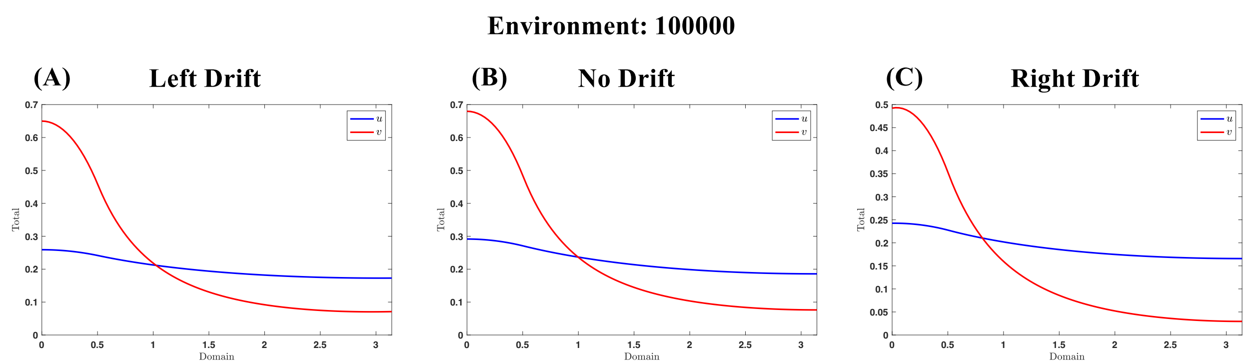

Figure S.2: Abundance in 1-column environment 100000. The initial conditions are $[u_0, v_0] = [0.5, 0.5]\mathbb{1}_\Omega$, where $\Omega = [0, \frac{\pi}{6}]$. $\mathbb{1}_{\Omega_1}$ denotes the indicator function, which gives 1 if in Ω and zero everywhere. The parameters are: $d_1 = 1, d_2 = 0.1, c = 0.01, b_1 = 1, b_2 = 1$ (A) $c_1 = 0.32$ and $c_2 = 0.5$ (B) $c_1 = 0.219$ and $c_2 = 0.38$ (C) $c_1 = 0.47$ and $c_2 = 1.05$.

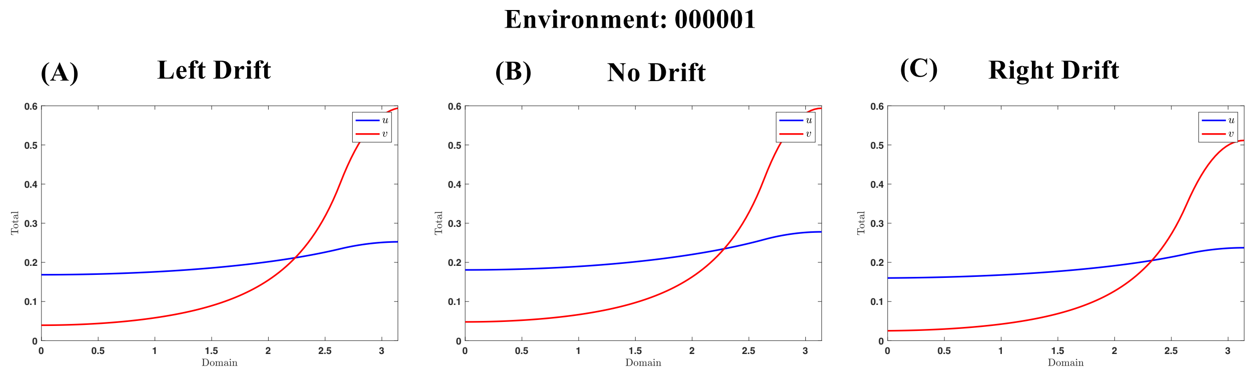

Figure S.3: Abundance in 1-column environment 000001. The initial conditions are $[u_0, v_0] = [0.5, 0.5]\mathbb{1}_{\Omega}$, where $\Omega = [0, \frac{\pi}{6}]$. $\mathbb{1}_{\Omega_1}$ denotes the indicator function, which gives 1 if in Ω and zero everywhere. The parameters are: $d_1 = 1, d_2 = 0.1, c = 0.01, b_1 = 1, b_2 = 1$ (A) $c_1 = 0.42$ and $c_2 = 0.88$ (B) $c_1 = 0.3$ and $c_2 = 0.67$ (C) $c_1 = 0.5$ and $c_2 = 1.05$.

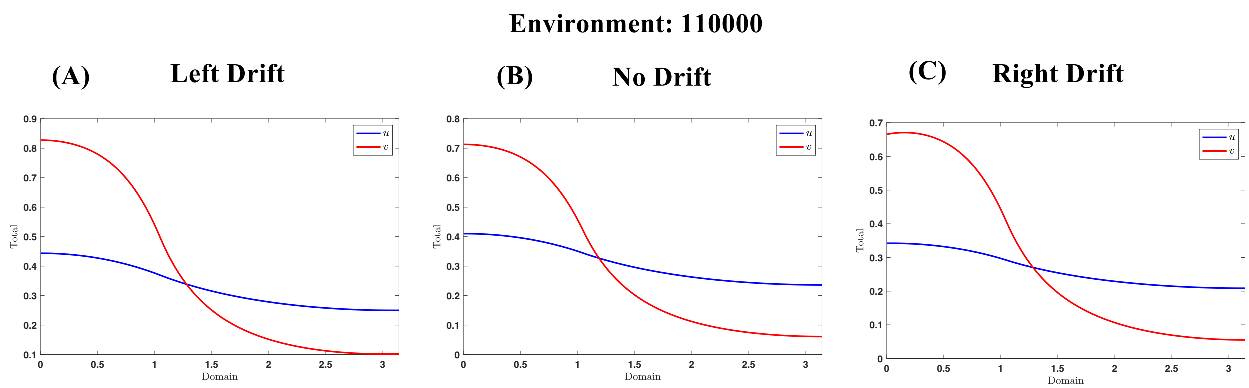

Figure S.4: Abundance in 110000 environment. The initial conditions are $[u_0, v_0] = [0.5, 0.5]\mathbb{1}_{\Omega}$, where $\Omega = [0, \frac{\pi}{6}]$. $\mathbb{1}_{\Omega_1}$ denotes the indicator function, which gives 1 if in Ω and zero everywhere. The parameters are: $d_1 = 1, d_2 = 0.1, c = 0.01, b_1 = 1, b_2 = 1$ (A) $c_1 = 0.32$ and $c_2 = 0.3$ (B) $c_1 = 0.44$ and $c_2 = 0.6$ (C) $c_1 = 0.58$ and $c_2 = 0.78$.

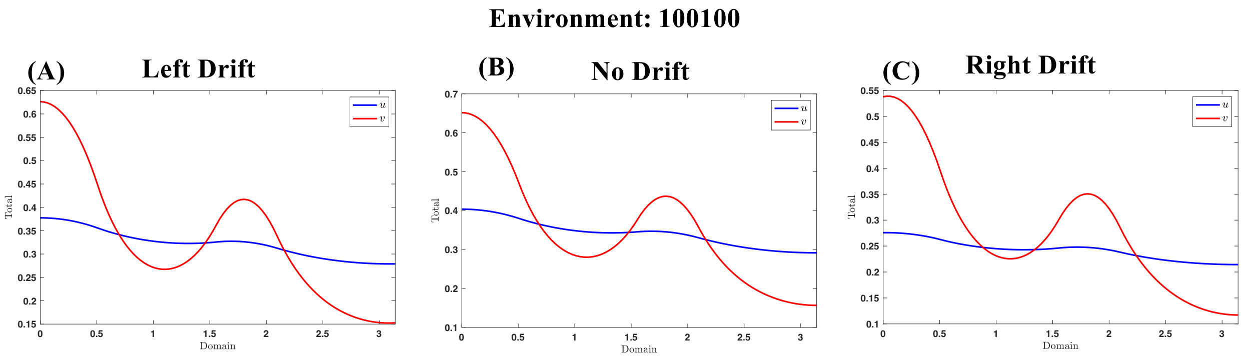

Figure S.5: Abundance in 100100 environment. The initial conditions are $[u_0, v_0] = [0.5, 0.5]\mathbb{1}_{\Omega}$, where $\Omega = [0, \frac{\pi}{6}]$. $\mathbb{1}_{\Omega_1}$ denotes the indicator function, which gives 1 if in Ω and zero everywhere. The parameters are: $d_1 = 1, d_2 = 0.1, c = 0.01, b_1 = 1, b_2 = 1$ (A) $c_1 = 0.29$ and $c_2 = 0.44$ (B) $c_1 = 0.219$ and $c_2 = 0.37$ (C) $c_1 = 0.596$ and $c_2 = 0.825$.

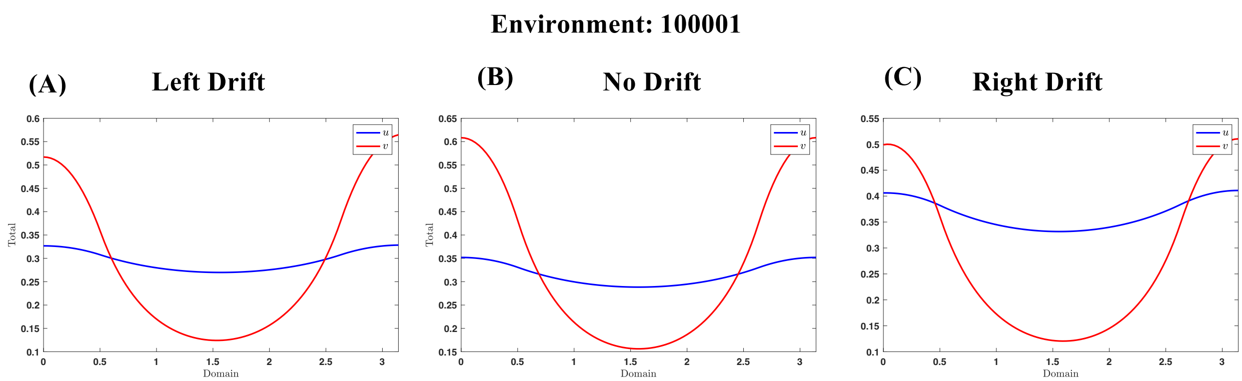

Figure S.6: Abundance in 100001 environment. The initial conditions are $[u_0, v_0] = [0.5, 0.5]\mathbb{1}_{\Omega}$, where $\Omega = [0, \frac{\pi}{6}]$. $\mathbb{1}_{\Omega_1}$ denotes the indicator function, which gives 1 if in Ω and zero everywhere. The parameters are: $d_1 = 1, d_2 = 0.1, c = 0.01, b_1 = 1, b_2 = 1$ (A) $c_1 = 0.42$ and $c_2 = 0.76$ (B) $c_1 = 0.29$ and $c_2 = 0.5$ (C) $c_1 = 0.17$ and $c_2 = 0.62$.

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