

Model	Reference	Group		Duration (kyr)	Ra_{τ}	Ra_c	AD/NAD_{median}	f_{dip}	E_m/E_k	Rm	Ro	f_{ohm}	$S_{ 30 median} (^{\circ})$	$S_{ 30 iqr} (^{\circ})$	AD/NAD_{TAF}	ZND/ND_{TAF}	
THOM1-1.5	This study	THOM1	$q^*=0$	Thermal $E = 3 \times 10^{-4}$ $Pr = 1$ $Pm = 3$	3426	150	0	42.8	0.84	2.54	107	0.011	0.37	5.9	0.4	289.1	0.99
THOM1-2	Me21				5688	200	0	26.4	0.75	3.42	121	0.012	0.43	7.4	0.3	178.7	1.00
THOM1-3	Me21				5110	300	0	22.4	0.70	2.98	148	0.015	0.43	7.9	0.4	147.8	0.99
THOM1-4	Me21				4345	400	0	22.5	0.69	2.27	176	0.018	0.41	7.9	0.5	130.1	1.00
THOM1-8	Me21				4007	800	0	16.7	0.63	1.16	262	0.026	0.35	9.3	0.6	113.6	0.96
THOM1-12	Me21				3156	1200	0	12.1	0.57	0.75	333	0.033	0.31	11.4	0.6	136.4	0.97
THOM1-17.5	This study				765	1750	0	0.2	0.15	0.17	724	0.072	0.19	53.8	2.5	36.0	0.92
THET1-1.2	This study	THET1	$q^*=2.3$		709	120	0	40.2	0.79	4.60	94	0.009	0.43	6.0	0.5	94.3	0.67
THET1-1.5	This study				2393	150	0	29.1	0.77	4.64	104	0.010	0.45	6.9	0.6	80.0	0.85
THET1-2	This study				2673	200	0	16.6	0.66	4.52	115	0.012	0.46	8.0	0.6	45.4	0.73
THET1-2.5	This study				708	250	0	14.1	0.64	3.84	148	0.015	0.45	8.4	0.9	45.2	0.83
THET1-3.5	This study				1230	350	0	13.8	0.63	2.61	156	0.016	0.40	9.1	0.7	40.1	0.67
THET1-5	This study				1718	500	0	13.7	0.61	1.76	205	0.021	0.36	9.3	1.1	40.6	0.62
THET1-7.5	This study				795	750	0	10.1	0.56	0.91	288	0.029	0.29	11.2	1.6	26.8	0.45
THET1-8	This study				182	800	0	9.4	0.53	0.91	274	0.027	0.34	11.9	2.9	28.0	0.44
THET1-12	This study				795	1200	0	7.9	0.50	0.60	340	0.034	0.27	13.9	3.6	24.2	0.32
THET1-17.25	This study				836	1725	0	5.7	0.46	0.46	389	0.039	0.26	16.8	5.7	19.2	0.29
THET1-24.15	This study				804	2415	0	3.5	0.38	0.34	496	0.050	0.24	20.7	7.0	11.6	0.29
TCHET1-1.5	This study	TCHET1	$q^*=2.3$	Thermochemical $E = 3 \times 10^{-4}$ $Pr = 1$ $Pm = 3$	771	150	7500	53.2	0.82	1.73	125	0.013	0.31	5.4	0.6	224.5	0.34
TCHET1-2	This study				870	200	10000	42.2	0.78	1.56	146	0.015	0.31	5.9	0.8	155.6	0.36
TCHET1-3	This study				983	300	15000	27.8	0.72	1.13	185	0.019	0.29	7.1	1.0	118.9	0.46
TCHET1-5	This study				497	500	25000	16.7	0.64	0.79	244	0.024	0.27	9.6	1.6	82.2	0.50
TCHET1-7.5	This study				432	750	37350	10.8	0.57	0.56	306	0.031	0.25	12.2	3.6	35.5	0.41
TCHET1-10	This study				480	1000	49800	5.3	0.48	0.45	357	0.036	0.24	15.5	5.5	12.7	0.38
TCHET1-12	This study				385	1200	60000	3.8	0.41	0.37	398	0.040	0.23	17.4	6.9	9.3	0.35
THOM2-20	MD23	THOM2	$q^*=0$	Thermal $E = 2 \times 10^{-5}$ $Pr = 0.2$ $Pm = 1$	121	2000	0	22.6	0.71	2.17	889	0.018	0.67	8.0	0.8	1008.3	0.76
THOM2-60	MD23	THOM2			127	6000	0	0.5	0.19	0.41	1848	0.037	0.52	46.0	10.3	52.6	0.90
THET2-20	MD23	THET2			116	2000	0	18.5	0.64	2.69	851	0.017	0.68	9.2	0.7	109.6	0.48
THET2-60	MD23	THET2			131	6000	0	25.0	0.66	0.85	1608	0.032	0.59	9.4	1.2	173.8	0.47
THET3-20	MD23	THET3			53	2000	0	8.5	0.54	3.54	830	0.017	0.69	12.0	2.5	21.4	0.36
THET3-60	MD23	THET3			91	6000	0	17.5	0.63	1.41	1483	0.030	0.61	9.6	1.8	68.8	0.63

Table S1: Summary of studied dynamo simulations. References Me21 and MD23 refer to refs 57 and 8 respectively. Group refers to definitions given in the Methods. Calculated durations assume a magnetic diffusion time of 200 kyr. AD/NAD_{median} is the median of the time series of time-instantaneous ratios of AD/NAD calculated at Earth's surface (ref 33; see Methods); f_{dip} is time-average dipolar fraction of the field at the core-mantle boundary (ref 60). E_m/E_k is the ratio of total magnetic and kinetic energies. Rm is the magnetic Reynolds number and Ro the Rossby Number. All other parameters are defined in the text.