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## Research Article

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# Control of Superluminal Light Propagation in a Four-Level Atomic Medium via Laguerre-Gaussian Field Strength and Beam Waist Modulation

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## ABSTRACT

We used a theoretical approach to study and investigate that how light pulses move faster than light in a four level atomic medium. In this study Laguerre-Gaussian (LG) beam waist, represented by  $F$  and strength  $Z$  is controlled. We examine the localized absorption and dispersion characteristics inside the four level atomic medium, illustrating that altering the beam waist affects the group velocity. According to the findings that we obtained, the group velocity at lower beam waists is between  $-0.0004c$  to  $-0.0002c$ , and the group index is negative, ranging from  $-0.0015$  to  $-0.0005$  respectively. As the pulse waist expands the extent of localized absorption and dispersion widens, yet the superluminal performance stays unchanged. Potentially useful in fields such as gravitational wave detection, soliton radar systems, and high-speed optical computing, this study sheds fresh information on the adaptability of light propagation in dispersive surfaces.

**Keywords:** Laguerre-Gaussian (LG); Susceptibility; Group Velocity; Group Index.

## 1. Introduction

Superluminal propagation is the phenomena where group velocity of light in a medium exceeds the speed of light in a vacuum. Consequently, under some circumstances the envelope of the wave packet seems to move faster than the basic speed limit imposed by relativity [1, 2]. Light has been regulated to propagate superluminally in a number of different mediums. Scientists have used a variety of methods to control the group velocity allowing this phenomenon to take place [3-7]. This phenomenon has been thoroughly examined because of its significant potential in modern optical technology. A significant use is in optical switching, where regulating light speed may improve communication networks [8], additionally, it is essential for regulating and storing light [9, 10], which enhances data transfer and processing effectiveness. This phenomenon also contributes to the development of present-day optical technology through applications in quantum communication and nonlinear optics [11-13].

Changing the group velocity of light in a medium causes the subluminal (slow) and superluminal (fast) propagation of light possible. The medium refractive index, which tells us how light waves move through it, has a direct effect on this group velocity [14]. Electromagnetically induced transparency (EIT) which permits light to flow through a conventional opaque substances is caused by quantum perturbation in atomic systems. Destructive impacts of absorption routes cause EIT, which is characterized by a dramatic increase in transmission inside the absorption zone [15-17]. Concurrently, the group index of the medium rises as a result of normal dispersion, leading to subluminal propagation of light ( $v_g < c$ ), where light travels at a slower rate than light in a vacuum. It has been a great difficulty for researchers to achieve a change from subluminal to superluminal performance ( $v_g > c$ ) in the same atomic medium by changing particular parameters [18]. The key need for slow light propagation is the growing slope of dispersion gives rise to this dilemma. Researchers have tackled the difficulty of rapid light propagation which demands a decreasing slope of dispersion by employing a variety of gain-assisted mediums [19-24]. In subsequent work, this obstacle was removed, allowing for the use of electromagnetically induced transparency (EIT) to transition from subluminal to superluminal actions within the same atomic medium. Modifying several parameters, mostly the probe wave field strength, which allows this change to occur in same medium [25].

Research on the propagation of plane-wave light beams in dispersive material has been increased for the last several decades. In their interactions with various material media, researchers have shown that the group velocity of plane-wave beams may display propagation rates that are both subluminal [12, 26] and superluminal [20]. Slow and fast propagation has been reported in different kinds of quantum systems, such as quantum dots, grapheme, quantum circuits, atomic media, Fermi gases, and photonic crystals [27-32]. The optical interactions have recently received significant interest in scientific studies. Extensive research has been undertaken on Laguerre-Gaussian (LG) fields to investigate their distinctive features and uses. Recently, researches have investigated the group velocity of the Laguerre-Gaussian (LG) beam. Researchers seek to comprehend its behavior and possible uses in diverse optical systems [33].

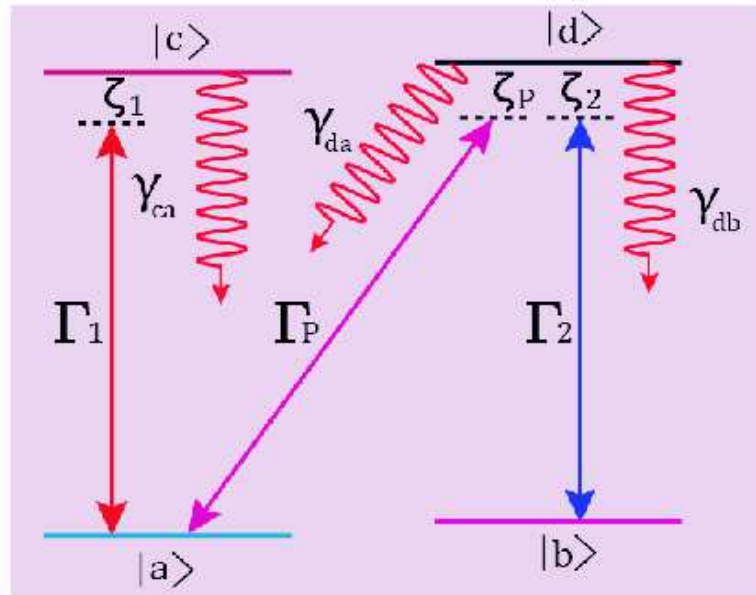
In a comparable investigation, researchers identified a link between the group velocity of the Laguerre-Gaussian field and propagation of the medium. This connection is essential for comprehending light propagation in complex optical systems [34].

The propagation of light in atomic media has been an important field of research in optics and quantum physics for years, and the ability to manipulate light velocity particularly the switch between subluminal and superluminal propagation has extraordinary relevance in terms of fundamental science and technologies. It has been demonstrated before in the literature that group velocity can be manipulated through different media and methods, however, the growing field of LG based studies still presents an ongoing challenge in detail related to group velocity manipulation in this context.

We investigate in this manuscript the superluminal propagation of light beams in a four-level atomic system under the influence of Laguerre-Gaussian (LG) beam waist and strength. The goal of this research is to use structured optical fields to find new methods of adjusting light speed in dispersive medium. This work advances our knowledge of localized absorption and dispersion properties and may find use in cutting-edge domains including quantum computation, high-speed optical computation, soliton radar systems, and gravitational wave detection. This research addresses the deficiency in current literature about LG field-induced modulation of group velocity and hence, enhancing our fundamental understanding of light-matter interactions and facilitating developments in photonic and quantum technology.

## 2. Atomic Model of Cesium-137 Atom

A four-level atomic system is considered as displayed in Fig. 1. This system is characterized by a probe wave field of Rabi frequency  $\Gamma_p$  is connected between state  $|a\rangle$  and  $|d\rangle$ . Rabi frequencies of control fields can be written as;



**Figure1:** Four-level atomic system

$$\Gamma_{1,2}(\Gamma_{1,2}^*) = |\Gamma_{1,2}| e^{i(\Phi_{1,2})} |\Gamma_{1,2}^*| e^{-i(\Phi_{1,2})} \quad (1)$$

These Rabi frequencies are applied to the system. State  $|a\rangle$  and  $|d\rangle$  is coupled by Rabi frequency  $\Gamma_p$  having probe field frequency detuning  $\zeta_p$ . Level  $|a\rangle$  and  $|c\rangle$  is characterized by Rabi frequency  $\Gamma_1$  with detuning  $\zeta_1$ . Similarly state  $|b\rangle$  and  $|d\rangle$  is connected by Rabi frequency  $\Gamma_2$  with detuning  $\zeta_2$ . To explain the dynamic and optical properties of the proposed atomic system, the Hamiltonian interaction as;

$$H_I = -\frac{\hbar}{2} [\Gamma_1 e^{-i\zeta_1 t} |a\rangle\langle c| + \Gamma_2 e^{-i\zeta_2 t} |b\rangle\langle d| + \Gamma_p e^{-i\zeta_p t} |a\rangle\langle d|] + H.C \quad (2)$$

Here;

$$\zeta_1 = \omega_{ac} - \omega_a, \quad \zeta_2 = \omega_{bd} - \omega_b, \quad \zeta_p = \omega_{ad} - \omega_p$$

Where  $H.C$  represents Hermitian conjugate, the system detuning parameters is related to the difference of resonant frequency of the atomic system and frequencies of control and probe wave field  $\omega_a, \omega_b, \omega_p$ . These frequencies represent frequencies of probe and control field, while  $\omega_{ac}, \omega_{bd}, \omega_{ad}$  represents resonant frequencies between the two states. The master density matrix equation as follow;

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} (H\rho - \rho H) - \frac{1}{2} \sum \gamma_{ij} (\beta\beta^\dagger\rho + \rho\beta\beta^\dagger - 2\beta^\dagger\beta\rho) \quad (3)$$

Equation (3) has two parts, first part show the interaction of system and second part is phenomenological one represents the atomic decay rate along with rising and lowering operators  $\beta^\dagger, \beta$ . According to our proposed four-level atomic system four coupled rates equations are calculated as given bellow.

$$\dot{\tilde{\rho}}_{ad} = \tilde{\rho}_{ad} \left[ i\zeta_p - \frac{1}{b}(\gamma_{da} + \gamma_{ca}) \right] + \frac{i}{b} \Gamma_1 \tilde{\rho}_{cd} + \frac{i}{b} \Gamma_p (\tilde{\rho}_{dd} - \tilde{\rho}_{aa}) - \frac{i}{b} \Gamma_2 \tilde{\rho}_{ab} \quad (4)$$

$$\dot{\tilde{\rho}}_{cd} = \tilde{\rho}_{cd} (i\zeta_p - i\zeta_1) + \frac{i}{b} \Gamma_1^* \tilde{\rho}_{ad} - \frac{i}{b} \Gamma_p \tilde{\rho}_{ca} - \frac{i}{b} \Gamma_b \tilde{\rho}_{cb} \quad (5)$$

$$\dot{\tilde{\rho}}_{ab} = \tilde{\rho}_{ab} \left[ i\zeta_p - i\zeta_b - \frac{1}{b}(\gamma_{da} + \gamma_{db} + \gamma_{ca}) \right] + \frac{i}{b} \Gamma_1 \tilde{\rho}_{cb} + \frac{i}{b} \Gamma_p \tilde{\rho}_{db} - \frac{i}{b} \Gamma_b^* \tilde{\rho}_{ad} \quad (6)$$

$$\dot{\tilde{\rho}}_{cb} = \tilde{\rho}_{cb} \left[ (i\zeta_p - i\zeta_1 - i\zeta_b) - \frac{1}{b} \gamma_{ab} \right] + \frac{i}{b} \Gamma_1^* \tilde{\rho}_{ab} - \frac{i}{b} \Gamma_b^* \tilde{\rho}_{cd} \quad (7)$$

Equations (4-7) is solved analytically by using initial population condition and first order perturbation approximation by the following equation:

$$P(t) = Y^{-1}N \quad (8)$$

Where  $P(t)$  and  $N$  are column matrices, while  $Y$  is a  $4 \times 4$  matrix.

Coherence between state  $|a\rangle$  and  $|d\rangle$  as follow:

$$\tilde{\rho}_{ad} = -2i\Gamma_p \left( \frac{4G_2G_3G_4 + G_2\Gamma_1\Gamma_1^* + G_3\Gamma_2\Gamma_2^*}{N + G_2G_4\Gamma_2 + \Gamma_2(-\Gamma_1 + \Gamma_2)\Gamma_2^*} \right) \quad (9)$$

In equation (9),  $G_1, G_2, G_3, G_4$  and  $N$  are analytical constants.

Where;

$$G_1 = i\zeta_p - \frac{1}{2}(\gamma_{da} + \gamma_{ca}) \quad (10)$$

$$G_2 = i(\zeta_p - \zeta_1) \quad (11)$$

$$G_3 = i(\zeta_p - \zeta_2) - \frac{1}{2}(\gamma_{da} + \gamma_{db} + \gamma_{ca}) \quad (12)$$

$$G_4 = i(\zeta_p - \zeta_1 - \zeta_2) - \frac{1}{2}\gamma_{db} \quad (13)$$

$$N = \Gamma_1(\zeta_1 - \zeta_2)(\Gamma_1^*)^2 + 4G_1G_3(4G_2G_4 + \Gamma_2\Gamma_2^*) + \Gamma_1^*(4(G_1G_2\Gamma_1 + G_3G_4\Gamma_1)) \quad (14)$$

The response of a quantum system or optical medium to the external wave field is determined by optical susceptibility denoted by  $\chi$  which has two parts real and imaginary part. Real part shows dispersion while imaginary part shows absorption spectrum. To calculate the optical susceptibility for the proposed atomic system the electrical susceptibility in term of electric polarization as follow:

$$\chi_e = \frac{P}{\epsilon_0 E} \quad (15)$$

Where;

$$P = \sigma_{ad}\rho_{ad} \quad (16)$$

$$E = \frac{\Gamma_p \hbar}{\sigma_{ad}} \quad (17)$$

Using equation 16 and 17 in equation 15 we get:

$$\chi_e = \frac{2n\sigma_{ad}^2}{\epsilon_0 \hbar \Gamma_p} \tilde{\rho}_{ad} \quad (18)$$

Where  $n$  represents number of atoms,  $\sigma_{ad}$  is the dipole moment, and  $\epsilon_0$  is the free space permittivity.

Modified Control Field Rabi Frequencies ( $\Gamma_1, \Gamma_2$ ) with Laguerre Coordinates:

$$\Gamma_1 = Z1 \text{Exp} \left[ -i \text{ArcTan} \left[ \frac{y}{x} \right] \frac{\sqrt{x^2 + y^2}}{F} \right] \text{Exp} \left[ -\frac{x^2 + y^2}{F^2} \right] \quad (19)$$

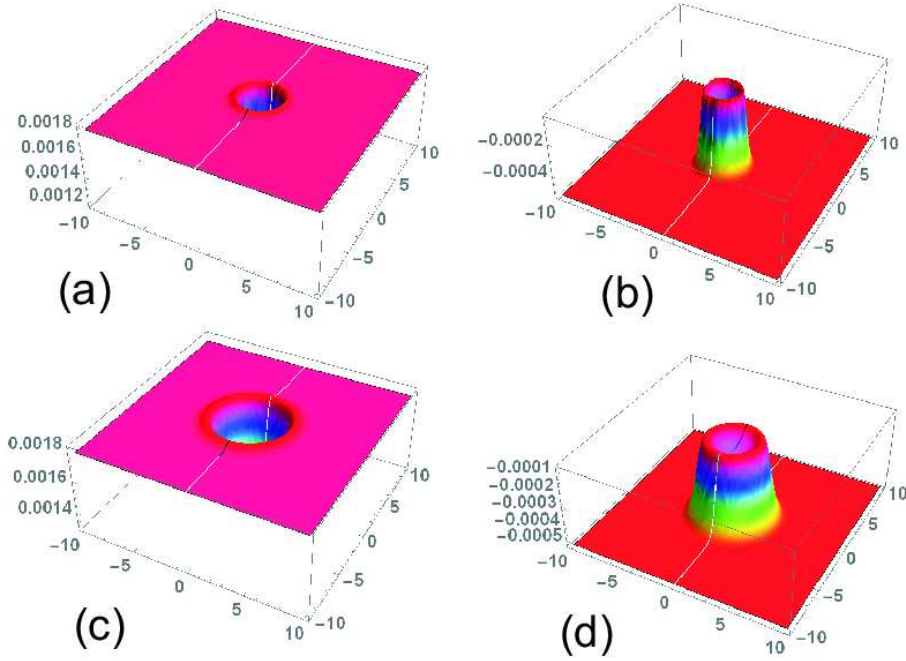
$$\Gamma_2 = Z_2 \text{Exp} \left[ -i \text{ArcTan} \left[ \frac{y}{x} \right] \frac{\sqrt{x^2+y^2}}{F} \right] \text{Exp} \left[ -\frac{x^2+y^2}{F^2} \right] \quad (20)$$

$Z_1$  and  $Z_2$  are space independent parts of Rabi frequencies,  $X$  and  $Y$  are Laguerre coordinates and  $F$  is the beam waist.

### 3. Results and Discussions

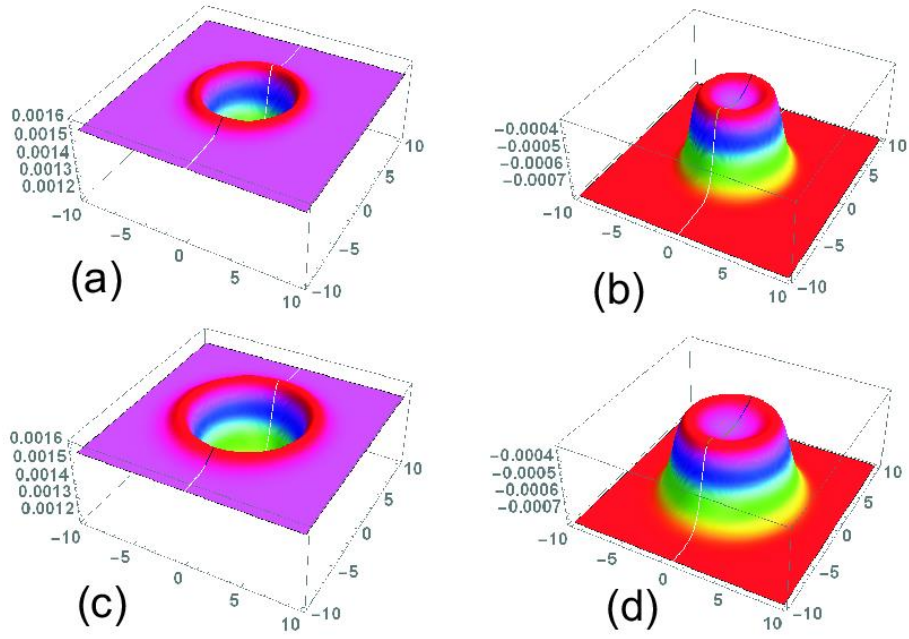
Superluminal propagation of probe fields has been investigated in a four-level cesium atomic model by modulating the various properties of the probe (absorption, dispersion, group index, and group velocity) by Laguerre-Gaussian forms of the control driving field. Careful tuning of the control field intensities and waists enables precise speed control of light in the medium. In addition, this investigation provides insights into optical control in quantum systems and will enable optical applications in. This work is in atomic units. Cesium atomic decay rate is taken  $\gamma = 1\text{MHz}$ ,  $\lambda = \frac{2\pi c}{\omega}$ ,  $\omega = 1000\gamma$  and all others parameters are taken in  $\gamma$  units.

In Fig.2. The figures show absorption and dispersion vs Laguerre coordinates  $x/\lambda$  and  $y/\lambda$ , adjusted to the wavelength of the pulse ( $\lambda$ ). Around the origin, there is a localized absorption spectrum (dark center area) is reported. The control driving wave fields pulse waists is taken to  $F = 0.5\lambda$ . Typical absorption depth is examined at and around the  $x$  and  $y$ -plane origins is displayed in Fig. 2.a. This area of absorption has an unusual dispersion spectrum is displayed in Fig. 2.b. A normal dispersion zone is related with slow group velocity, whereas anomalous dispersion is associated with rapid or superluminal group velocity in the medium. Altering the pulse waist from  $1.5$  to  $2.5 \lambda$  improves the localized absorption depth at the origin of the  $x, y$  plane is displayed in Fig. 2.c. In this absorption depth area, the associated dispersion likewise exhibits a concentrated peak and broadens as the pulse width increases is displayed in Fig. 2.d.



**Figure 2:** Absorption and dispersion processes are analyzed with the help of Laguerre coordinates using normalized spatial coordinates  $x/\lambda$  and  $y/\lambda$ ,  $\gamma = 1$ ,  $\gamma_{da} = 1\gamma$ ,  $\gamma_{db} = 1.2\gamma$ ,  $\gamma_{ca} = 1\gamma$ ,  $\zeta_p = 0.3\gamma$ ,  $\zeta_1 = 0.1\gamma$ ,  $\zeta_2 = 0.09\gamma$ ,  $Z_{1,2} = 2\gamma$  (a, b)  $F = 1.5\lambda$  (c, d),  $F = 2.5\lambda$

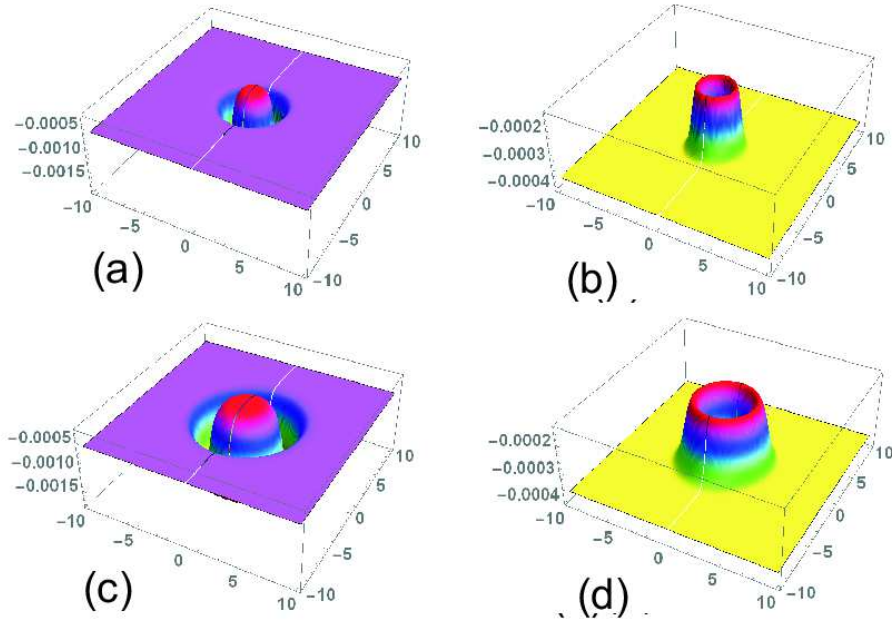
In Fig.3. The figures show absorption and dispersion against Laguerre coordinates  $x/\lambda$  and  $y/\lambda$ , adjusted to the wavelength of the pulse ( $\lambda$ ). In this scenario, Laguerre driving fields' pulse waist is first taken to  $F = 3.5\lambda$  then to  $F = 4.5\lambda$ . A confined absorption spectrum is again detected, with a large zone surrounding the point of origin. The usual wide absorption depth has been observed at and around the x- and y-plane origins is illustrated in Fig. 3.a. additionally, there is localized unusual dispersion in the dispersion spectrum, which is typically wide see Fig. 3.b. By altering the pulse waist from  $F = 3.5\lambda$  to  $4.5\lambda$  the localized absorption spectra depth become wider around the x and y plane is see Fig. 3.c. As the pulse waist increases, the associated dispersion widens, but it still retains a localized peak in the absorption depth zone is shown in Fig. 3.d



**Figure 3:** Absorption and dispersion processes are analyzed with the help of Laguerre coordinates using normalized spatial coordinates  $x/\lambda$  and  $y/\lambda$ ,  $\gamma = 1$ ,  $\gamma_{da} = 1\gamma$ ,  $\gamma_{db} = 1\gamma$ ,  $\gamma_{ca} = 1\gamma$ ,  $\zeta_p = 0.3\gamma$ ,  $\zeta_1 = 0.1\gamma$ ,  $\zeta_2 = 0.3\gamma$ ,  $Z_{1,2} = 2\gamma$  (a, b)  $F = 3.5\lambda$  (c, d)  $F = 4.5\lambda$

In Figure 4. The group velocity ( $v_g$ ) and group index ( $n_g$ ) of the probe pulse in the four-level atomic medium are shown in relation to the Laguerre coordinates  $x/\lambda$  and  $y/\lambda$ . In this case, the Laguerre driving wave fields' pulse waist is extended to  $1.5\lambda$  and  $2.5\lambda$ . A negative group index for the atomic medium indicates that superluminal propagation is taking place. A negative group index causes the wave to appear to be traveling in the opposite direction. There is a negative change in both the delay time (the amount of time it takes for the wave to move) and the group velocity (the speed of the pulse in the material). So, the wave seems to leave the material before it completely enters, but it's actually simply a particular wave effect; it doesn't actually exceed the speed of light. The delay time is the time it takes for a pulse to travel a certain distance in a vacuum minus the time it takes for the pulse to travel the same distance via a medium. A negative delay time indicates that the pulse travels the same distance through the medium with less time spent there than in a vacuum. When the group index is negative, the delay time and group velocity are both negative. Although it appears as though the pulse is traveling faster in the medium than in a vacuum, this is actually only a wave effect and not true transit at the speed of light. A phenomenon known as superluminal propagation occurs when the group velocity is negative, making the wave appear to travel faster than the speed of light in a vacuum (c). Nevertheless, this is only a consequence of the wave's behavior in the atomic medium and does not indicate that real information travels rapidly than light. In order to see more concentrated wave characteristics, narrow and localized patches of group index and group

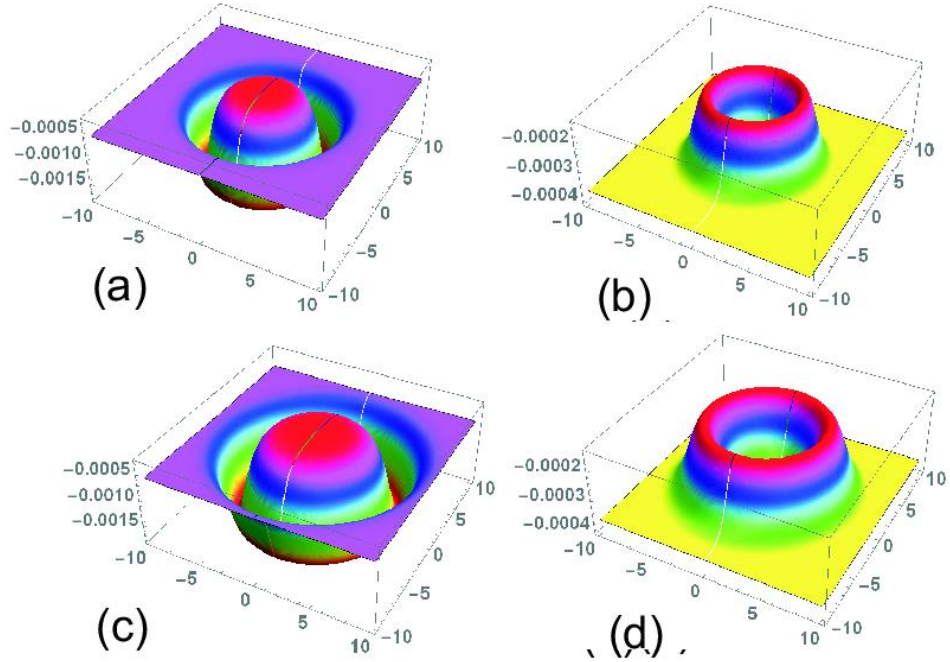
velocity are seen around the origin when the pulse waist is adjusted to  $1.5\lambda$ . In the confined absorption and dispersion zone around the origin of  $x/\lambda$  and  $y/\lambda$ , the group index differs from -0.0015 to -0.0005, as seen in Fig. 4.a. This suggests minute changes in wave behavior in that particular place. As shown in Fig. 4.b, the group velocity ranges from -0.0004 to -0.0002 c in the localized absorption and dispersion zone close to the origin of  $x/\lambda$  and  $y/\lambda$ . This suggests minor variations in wave speed inside the given region. When the beam waist rises from  $1.5\lambda$  to  $2.5\lambda$ , the group index and velocity stay unchanged. However, the area with a negative group index and velocity grows, as seen in Fig. 4.c, d.



**Figure 4:** Absorption and dispersion processes are analyzed with the help of Laguerre coordinates using normalized spatial coordinates  $x/\lambda$  and  $y/\lambda$ ,  $\gamma = 1$ ,  $\gamma_{da} = 0.234\gamma$ ,  $\gamma_{db} = 0.1\gamma$ ,  $\gamma_{ca} = 0.01\gamma$ ,  $\zeta_p = 0.02\gamma$ ,  $\zeta_1 = 0.1\gamma$ ,  $\zeta_2 = -2\gamma$ ,  $Z_{1,2} = 2\gamma$ ,  $F = 1.5\lambda$  (b)  $\gamma = 1$ ,  $\gamma_{da,db,ca} = 1\gamma$ ,  $\zeta_p = 0.2\gamma$ ,  $\zeta_1 = 0.1\gamma$ ,  $\zeta_2 = 2\gamma$ ,  $Z_1 = 2\gamma$ ,  $Z_2 = 6\gamma$ ,  $F = 1.5\lambda$  (c, d)  $F = 2.5\lambda$

In Figure 5, Laguerre coordinates  $x/\lambda$  and  $y/\lambda$  are used to display the group index and group velocity of the probe wave pulse in the atomic medium in the plots. Here, the Laguerre driving fields' beam waist is first set to  $F = 3.5\lambda$  and subsequently raised to  $F = 4.5\lambda$ . The atomic medium exhibits superluminal performance where  $v_g > c$  at the identical length ( $L$ ) since both its group index and group velocity are negative. The areas of group velocity and group index widen when the pulse waist is adjusted to  $F = 3.5\lambda$ . The propagation of the pulse in the atomic medium is impacted by the wave characteristics becoming more dispersed around the origin. Within the confined absorption and dispersion zone close to the origin of  $x/\lambda$  and  $y/\lambda$ , the group index varies between -0.0015 and -0.0005. As seen in Fig. 5 .a, this point to minute

changes in wave behavior that impact the way atomic medium absorbs and distributes the probe wave pulse. Figure 5.b shows that in the localized absorption and dispersion zones near the origin of  $x/\lambda$  and  $y/\lambda$ , the group velocity value varies between  $0.0004c$  and  $0.0002c$ . Upon increasing the beam waist from  $F = 3.5\lambda$  to  $F = 4.5\lambda$ , the group velocity and group index values do not change. On the other hand, as Figs. 5.c and 5.d demonstrate, the areas with a negative group index and negative group velocity grow, impacting a wider area. Experimental observations have verified the existence of subluminal (slower than light) and superluminal (faster than light) propagation. Several studies showing similar impacts in various media have been carried out by several researchers.



**Figure 5:** Absorption and dispersion processes are analyzed with the help of Laguerre coordinates using normalized spatial coordinates  $x/\lambda$  and  $y/\lambda$ ,  $\gamma = 1$ ,  $\gamma_{da} = 0.234\gamma$ ,  $\gamma_{db} = 0.1\gamma$ ,  $\gamma_{ca} = 0.01\gamma$ ,  $\zeta_p = 0.02\gamma$ ,  $\zeta_1 = 0.1\gamma$ ,  $\zeta_2 = -2\gamma$ ,  $Z_{1,2} = 2\gamma$ ,  $F = 1.5\lambda$  (b)  $\gamma = 1$ ,  $\gamma_{da,db,ca} = 1\gamma$ ,  $\zeta_p = 0.2\gamma$ ,  $\zeta_1 = 0.1\gamma$ ,  $\zeta_2 = 2\gamma$ ,  $Z_1 = 2\gamma$ ,  $Z_2 = 6\gamma$ ,  $F = 3.5\lambda$  (c, d)  $F = 4.5\lambda$

#### 4. Conclusion

Here we have reported a systematic study of superluminal propagation of light in four-level cesium-137 atomic medium driven by Laguerre-Gaussian (LG) fields. The work showed that absorption and dispersion characteristics can be controlled through the strength and beam waist of control driving field. One of the main results is that increasing the beam waist leads to broader regions of localized absorption and dispersion, but that negative group index and group

velocity associated with superluminal behavior persists. Such insights have important implications for using beam waist parameters to tune the crossover between subluminal and superluminal light propagations. This phenomenon is of great importance for various technological applications such as gravitational wave detection, soliton radar systems, high-speed computing, and time-cloaking technologies. The findings add to the knowledge on light-matter interactions and could help in advanced optical communication and signal processing-based systems.

**Ethics declaration**

Ethics, Consent to Participate, and Consent to Publish declarations: Not applicable.

**Clinical trial registration**

Not applicable

**Author Contributions**

All authors have contributed equally: Methodology, Formal analysis, writing original draft preparation, writing review & editing.

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**Conflict of Interests**

This work does not have any potential conflicts of interest.

**Data Availability Statement**

The associated data is available upon request from the corresponding author.

**Declaration Statement of Generative AI**

The author declare they have not used Artificial Intelligence (AI) tools in the creation of this article

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