Euclidean Framework for Recursive Feedback Dynamics and Multifractal Analysis

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1 Background and Framework

In the Euclidean context, we treat spacetime as a d-dimensional Euclidean space $\mathbb{R}^d$ rather than a Lorentzian manifold, focusing on temporal evolution within a mathematically simplified framework. This allows us to abstractly study the **recursive feedback dynamics** in **spatial** and **temporal** domains.

2 Generalized Influence Kernel

The **Generalized Influence Kernel** $\Phi(x, t)$ models how influences propagate through space and time, with feedback loops governed by recursive effects from past states. The kernel Φ represents a **temporal-spatial field** whose evolution is determined by both fractional time and space dynamics.

The proposed equation captures the recursive nature of the influence through fractional differential equations that govern **feedback networks**. These networks reflect how influences at a given space-time point depend not only on their immediate neighborhood but also on more distant or past states, forming a **non-local recursive structure**.

2.0.1 Fractional Diffusion Equation

The generalized fractional diffusion equation for $\Phi(x, t)$ is:

$$\mathscr{D}_t^p \Phi(x,t) = -\lambda(-\Delta)^{s/2}\Phi(x,t), \quad p \in (1,2),\ s > 0. \tag{1}$$

3 A. Fractional Time Derivative ($\mathscr{D}_t^p$)

The term $\mathscr{D}_t^p$ denotes the **Caputo fractional derivative** of order p, where $p \in (1, 2)$. This fractional derivative accounts for **memory effects** and **non-local temporal feedback**, meaning that the system's state at time t depends on a weighted combination of past states, with the weight determined by the fractional exponent p. It is defined as:

$$\mathscr{D}_t^p \Phi(x,t) = \frac{1}{\Gamma(2-p)} \int_0^t (t-\tau)^{1-p} \frac{d}{d\tau}\Phi(x,\tau)\, d\tau. \tag{2}$$

Here, the **Caputo derivative** ensures that the influence at time t integrates the history of the field, with more weight placed on **recent past states** for smaller p and more **distributed memory** for larger p.

4 B. Fractional Laplacian Operator ($(-\Delta)^{s/2}$)

The operator $(-\Delta)^{s/2}$ represents a **fractional Laplacian**, a non-local operator that governs spatial interactions in the system. The fractional Laplacian generalizes the conventional Laplacian operator Δ to capture **long-range interactions** and **spatial heterogeneity** in higher-order dimensions. It is defined in terms of a Fourier transform as:

$$(-\Delta)^{s/2}\Phi(x) = \mathcal{F}^{-1}\left(|k|^s \mathcal{F}[\Phi(x)]\right), \tag{3}$$

where $\mathcal{F}$ denotes the Fourier transform and k is the wavevector in $\mathbb{R}^d$.

This fractional spatial operator introduces **non-local spatial dynamics**, allowing the influence at any point in space to be affected not only by local changes but also by distant points, a critical feature for modeling **spatially distributed feedback** in recursive systems.

4.1 Asymptotic Behavior

The solution to this equation in Fourier-Laplace space is:

$$\tilde{\Phi}(k, u) = \frac{u^{p-1}}{u^p + \lambda|k|^s}, \tag{4}$$

where k is the wavevector and u the Laplace frequency. By applying Tauberian theorems, we find that for large time $t \gg 1$, the asymptotic decay of the influence kernel follows a power law:

$$\Phi(x, t) \sim t^{-p}. \tag{5}$$

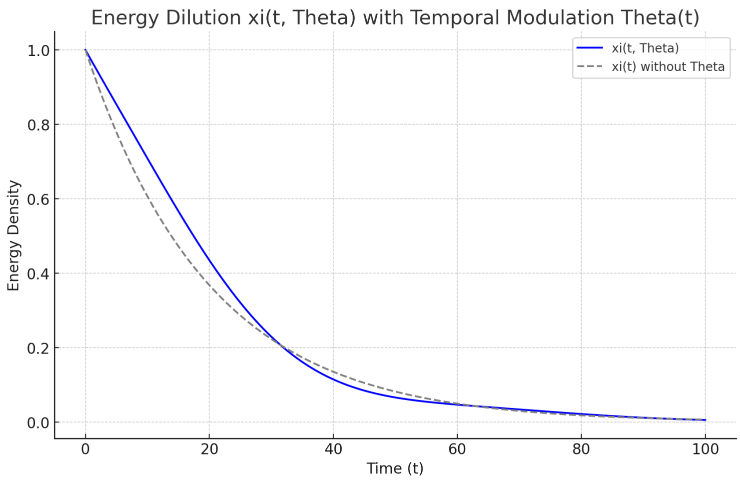

Figure 1: Log-log plot of $\Phi(t)$ showing t^{-p} decay for $p = 1.5$, validating fractional dynamics.

5 Fractal Structure and Multiscale Feedback

5.1 Fractal Dynamics in Euclidean Space

Recursive dynamics naturally lead to fractal structures in spacetime. The fractal dimension D of the influence kernel $\Phi(x,t)$ can be computed using a multifractal analysis:

$$\langle |I(t+\Delta) - I(t)|^q \rangle \sim \Delta^{\tau(q)}, \quad \tau(q) = q\alpha - \frac{\phi}{2}q^2, \tag{6}$$

where α is the scaling exponent and ϕ is the fractal parameter that governs the multifractal spectrum. The singularity spectrum, denoted $f(\alpha)$, is derived using the Legendre transform:

$$f(\alpha) = \phi\left(\frac{\alpha}{\phi} - \frac{1}{2}\right)^2. \tag{7}$$

5.2 Scaling Behavior and Universal Critical Exponent

The system exhibits universal scaling behavior near critical points. The critical threshold T_c corresponds to the point of bifurcation in the recursive dynamics:

$$T_c = \frac{\alpha\beta k}{4}, \tag{8}$$

where α and β are parameters that control feedback strength and recursive interaction strength.

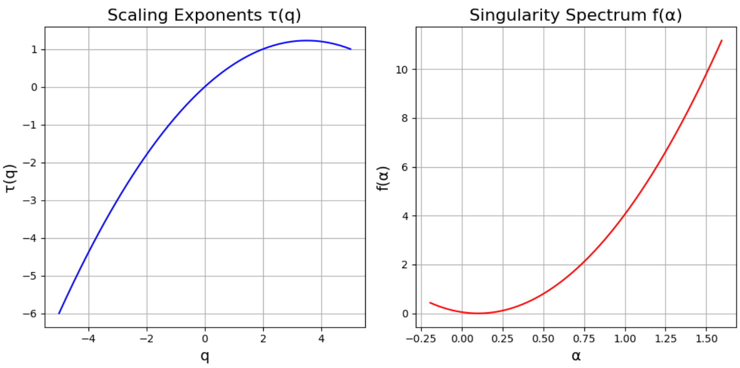

Figure 2: Multifractal spectrum of the coupled field $I(t)$ showing the scaling exponents $\tau(q)$ and singularity spectrum $f(\alpha)$.

6 Coupled Feedback Networks and Hierarchical Renormalization

6.1 Renormalization Group Approach: Momentum-Shell Integration

The Renormalization Group (RG) framework offers a systematic method for integrating out high-momentum modes within recursive dynamics, enabling us to study the flow of coupling constants at various scales. The RG flow equations for the fields ϕ_d and π_d are given by:

$$\frac{d\phi_d}{dl} = (d - 2 + \eta)\phi_d - C_d\pi_d^2, \quad \frac{d\pi_d}{dl} = (2d - 4 + \eta)\pi_d, \tag{9}$$

where l is the scale parameter, d is the spatial dimension, η is the anomalous dimension arising from vertex corrections, and C_d is a constant that depends on the specific form of the interaction. The fields ϕ_d and π_d represent the scalar and momentum fields, respectively, within the system.

The RG flow captures how these fields evolve under changes in scale, ultimately leading to critical scaling at the fixed points, where the system undergoes phase transitions or other emergent behaviors.

6.2 Critical Scaling and Emergent Phenomena

The fixed points of the RG flow represent the critical points where the system exhibits a change in behavior, such as a transition from ordered to disordered or chaotic states. These fixed points are characterized by universal scaling relations and can be classified as either stable or unstable, depending on their nature.

The critical temperature, denoted as T_c, represents the boundary between these phases. At T_c, the system undergoes a phase transition, and the behavior near this point is governed by scaling laws that describe the emergence of large-scale phenomena from the recursive dynamics.

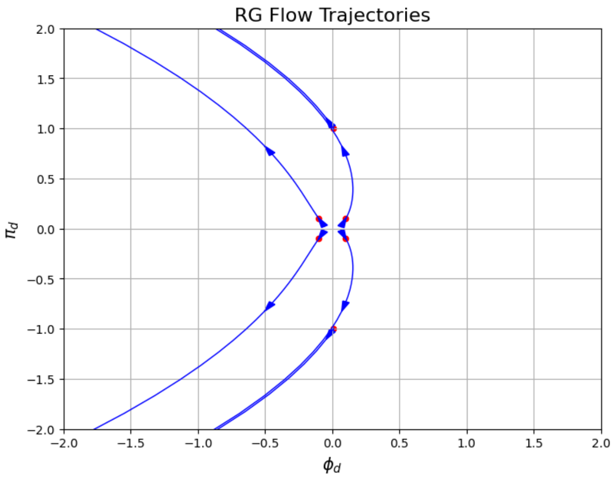

Figure 3: Renormalization Group (RG) flow diagram showing trajectories in the space of coupling constants. The separatrix (dashed line) corresponds to the critical threshold T_c, marking the boundary between ordered and disordered phases.

7 Applications in Physical Systems and Network Theory

7.1 Feedback Networks in Social Systems: Viral Propagation

In the context of social networks, the propagation of content or influence can be modeled using a master equation that describes the flow of probabilities across nodes in the network. The equation governing the evolution of the probability distribution $P(m, t)$ for the number of infected nodes is given by:

$$\frac{dP}{dt} = \gamma \sum_{m'=0}^{m} \left[w(m' \to m)P(m') - w(m \to m')P(m) \right], \tag{10}$$

where $P(m,t)$ represents the probability of having m infected nodes at time t, and the transition rates $w(m \to m')$ characterize the likelihood of an infection propagating from one node to another. The rates $w(m \to m+1)$ depend on the network structure and the influence kernel, which determines how content spreads through the network. The feedback dynamics are recursive, reflecting how the state of the network at each time step is influenced by the previous states.

7.2 Gravitational Wave Echoes: Recursive Feedback in Spacetime

In the study of gravitational waves (GW) and their echoes, recursive feedback processes govern the signal evolution, with time delays associated with the gravitational memory effect. The echo signal can be expressed as:

$$h_n(t) = A_n h_0(t - n\Delta t), \tag{11}$$

where $h_n(t)$ represents the n-th echo at time t, and A_n is the amplitude of the n-th echo. The time delay between consecutive echoes is given by:

$$\Delta t = \phi \frac{2GM}{c^3}, \tag{12}$$

where Δt is the time interval between echoes, ϕ is the golden ratio governing the recursive dynamics of the system, G is the gravitational constant, M is the mass of the source, and c is the speed of light. The golden ratio ϕ plays a crucial role in shaping the recursive feedback behavior, influencing the temporal spacing and amplitude modulation of the GW echoes. This model connects the recursive feedback dynamics seen in social networks with the propagation of influences in spacetime, particularly in the context of gravitational waves.

8 Conclusion

We have traced the mathematical lineage from simple trigonometric parameterizations, through classical roulettes, to a fully normalized, higher-dimensional geometric framework with dynamical feedback. The process involves:

1. Starting with basic circles and extending to tori and classical roulette curves (cycloids, trochoids, epicycloids, etc.).

2. Normalizing parameters to achieve dimensionless scaling.

3. Embedding into higher dimensions and introducing recursive influence, gravitational feedback, temporal scaling, energy decay, curvature modulation, and quantum corrections.

This evolution leads to the Cykloid Geometry: a stable, dynamic, and physically interpretable structure that respects classical geometric intuition with advanced concepts needed to describe non-local quantum phenomena and higher-dimensional gravitational effects.

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