

Numerical Methods.

We consider an up to three-component column of upwelling mantle, composed of typical mantle, mantle wallrock, and a vein of rising melt. In this 1-D column the solid mantle volume, upwelling rate, temperature, water content, and dry and wet solidus temperatures are dynamic functions of partial melting.

Thermal model. The temperature calculation is based on the conservation of energy:

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial z^2} \right) - u_z \frac{\partial T}{\partial z} + Q \quad (1)$$

where T denotes temperature, κ thermal diffusivity, z the vertical spatial coordinate and u_z the vertical velocity of upwelling mantle material (This simplification for uniform upward advective heat transport is an additional advantage of keeping the total upwelling flux of melt and solid constant within the column.). Here the heat source term Q is the latent heat that describes the energy consumed by the solid-to-melt phase change. The small temperature changes associated with the additional heat consumed by melting of wallrock due to its water equilibrium with adjacent rising melt are described below.

Mass conservation. The mantle's volume fraction will decrease due to the loss of material that becomes a fractional or dynamic increment of melt. As mass must be conserved in the 1D column:

$$\sum_{i=1}^{nc} Vol_i(z) \cdot u_z(z) + u_z(z) \cdot \int_{z_{bot}}^z \sum_{n=1}^{nc} Vol_i(z) \cdot \frac{dF_i}{dz}(z) dz = U_{bot} \quad (2)$$

where u_z is the velocity of the solid mantle, U_{bot} is the given velocity of the solid mantle at the initial depth of melting, $\frac{dF_i}{dz}$ is the i th mantle component's melt productivity, Vol_i is the i th mantle component's solid volume. (Here we also use the simplest approximation that the density of the mantle, wallrock and melt are the same.)

Fractional pressure-release melting. The dry solidus temperature for a mantle component is assumed to depend on its pressure and degree of depletion, which is the sum of three terms [Morgan, 2001]:

$$T_{sdry} = T_{s0} + \frac{dT_s}{dP} P + \frac{dT_s}{dF} F \quad (3)$$

where T_{s0} is solidus temperature of the rock at surface pressure, $\frac{dT_s}{dP}$ is its solidus-pressure dependence, $\frac{dT_s}{dF}$ is its solidus-depletion (depletion=degree of partial melt extraction) dependence,

P is its pressure and F is the degree of depletion of the lithologic component. Here we are using the assumption that all mantle has the same dry peridotitic solidus as that used by *Morgan* [2001].)

The wet solidus of the mantle component with water content X_{H_2O} is parameterized using the relation of *Katz et al.* [2003]

$$T_{swet} = T_{dry} - \Delta T(X_{H_2O}) \quad (4)$$

$$\Delta T(X_{H_2O}) = \Lambda * X_{H_2O}^\gamma \quad (5)$$

where X_{H_2O} is the water content in the ideal batch melt (wt.%), it can be calculated by dividing the water concentration in the solid (wt.%) by 0.01 (D value in the *Katz et al.* [2003]). Λ and γ are constant model parameters.

Melting happens when the temperature of a mantle component exceeds its solidus temperature. We determine melt productivity following *Morgan* [2001]. The melt productivity of component i , in thermal but not chemical equilibrium with other components j that each has a mass/volume fraction Vol_j is described by

$$-\frac{\partial F_i}{\partial P} = \frac{\frac{\partial T_i^s}{\partial P} - \frac{\alpha T}{\rho C_p} + \frac{T}{C_p} \sum_j Vol_j \Delta S_j \left(\frac{\partial T_i^s}{\partial P} - \frac{\partial T_j^s}{\partial P} \right)}{\frac{T}{C_p} [Vol_i \Delta S_i ((\frac{\partial T_i^s}{\partial F_i}) / (\frac{\partial T_j^s}{\partial F_j}))] + \frac{\partial T_i^s}{\partial F_i}} \quad (6)$$

where α is the thermal expansion coefficient, C_p is specific heat, ΔS is the change in entropy associated with the solid-to-melt phase change, T is temperature, T_i^s is the solidus temperature of component i . The melt productivity per unit mass of component i , as a function of depth, can be written as

$$\frac{\partial F_i}{\partial z} = -\rho g \frac{\partial F_i}{\partial P} \quad (7)$$

and the melt productivity of the mantle system (solid mantle + veins of melt) depends on the volume of solid component, see the equation 20 of *Morgan* [2001]

$$\frac{\partial F}{\partial z} = \sum_{i=1}^{nc} Vol_i \cdot \frac{\partial F_i}{\partial z} \quad (8)$$

To get the melt production rate (MPR) of the 1D column from the max depth z_{bot} of the model to the depth of z , we integrate the instantaneous melt productivity of the mantle system with respect to each depth increment dz

$$MPR(z) = \int_{z_{\text{bot}}}^z u_z \frac{\partial F}{\partial z} dz \quad (9)$$

where u_z is vertical upwelling rate of solid mantle, ρ is density, g is gravitational acceleration, and nc is the number of components (here 2).

The change in temperature during melting over a decompression interval dP in fractional melting is discussed in *Morgan* [2001]:

$$\frac{dT}{dP} = \frac{\partial T_i^s}{\partial P} + \frac{dT_i^s}{dF_i} \frac{dF_i}{dP} \quad (10)$$

Once mantle components melt, water will be strongly partitioned from solid into melt. We use a fractional melting equation to calculate the current water concentration left in a solid mantle component during its partial melting:

$$X^s = X^{s0} * (1 - dF)^{\left[\frac{1}{D_{H_2O}} - 1\right]} \quad (11)$$

where X_C^{s0} is the initial water concentration in the solid component at the current step, dF is its small degree change of melt depletion at this step, and D_{H_2O} is the partition coefficient of water between this component solid and its melt.

The effect of water depletion on the solidus temperature in the $\frac{\partial T_i^s}{\partial F_i}$ term of the equation (6) is treated by a water “depletion” related part $\frac{\partial T_C^s}{\partial F_C^{H_2O}}$ (modified after *Hasenclever* [2010]), which is added to the $\frac{dT_i^s}{dF_i}$ term of equations (6) and (10) when water is extracted during the melting of mantle component i :

$$\frac{\partial T_C^s}{\partial F_C^{H_2O}} = \frac{\partial T_C^s}{\partial X_C^s} \cdot \frac{\partial X_C^s}{\partial F_C^{H_2O}} \quad (12)$$

$$\frac{\partial T_C^s}{\partial X_C^s} = \gamma \cdot \Lambda \cdot X_C^{\gamma-1} \quad (13)$$

$$\frac{\partial X_C}{\partial F_C^{H_2O}} = -X^{s0} * \left(\frac{1}{D_{H_2O}} - 1\right) * (1 - F_C^{H_2O})^{\left(\frac{1}{D_{H_2O}} - 2\right)} \quad (14)$$

$$F_C^{H_2O} = 1 - \left(\frac{X_C^s}{X^{s0}}\right)^{\frac{D_{H_2O}}{1-D_{H_2O}}} \quad (15)$$

where $\frac{\partial T_C^s}{\partial X_C}$ is the derivative of equation (5) with respect to X_C , $\frac{\partial X_C}{\partial F_C}$ is the derivative of fractional melting equation with respect to current water depletion degree F_C , X^{s0} is the initial water concentration in solid, X_C^s is the current water concentration in solid. D_{H_2O} is the water partition coefficient. Water's partition coefficient is assumed to be the same partition coefficient as that for the incompatible element Cerium, i.e. ~ 0.01 .

Water exchange between wallrock and melt rising in adjacent veins. In order to simulate the exchange of water between vein melt and adjacent wallrock, we assume there exists a limiting vein fraction within the mantle. As long as the cumulative volume of melt generated by a certain depth does not exceed this threshold, the melt rising within veins will be assumed to be equal to the summed melt production beneath this depth. Otherwise, the melt volume within veins is capped at the limiting vein fraction (with melts rising more rapidly within these veins than their surrounding mantle).

During decompression melting, water in the wallrock is assumed to be in equilibrium with a fraction of the melt rising in its adjacent vein — i.e. the water content of the wallrock will reflect a diffusive water equilibrium between the wallrock and a fraction of melt in its neighboring vein, due to their close proximity on a length scale where water diffusive transport is significant. The bulk of the rising mantle continues to melt by fractional or dynamic melting. Assuming water conservation and chemical equilibrium, the water concentration in the wallrock and vein melt is given

by:

$$X_{vein}^{eq} = \frac{Vol_{vein}X_{vein} + Vol_{WR}X_{WR}}{Vol_{vein} + Vol_{WR} * D_{H_2O}} \quad (16)$$

$$X_{WR}^{eq} = X_{vein}^{eq} * D_{H_2O} \quad (17)$$

where X_{vein}^{eq} and X_{WR}^{eq} is the water concentration of the melt in the vein and the wallrock after their water concentration reaches equilibrium; Vol_{vein} and Vol_{WR} is the mass/volume fraction of the melt in the vein and wallrock; X_{vein} and X_{WR} is the water concentration of the melt in the vein and wallrock after fractional melting process but before water equilibrium.

Mantle wallrock will further melt after water is diffusively added from an adjacent melt-filled vein. This additional water has diffusively migrated from the water-rich vein melt into its adjacent relatively water-poor wallrock, which will cause the solidus of the wallrock to decrease.

In this case, mantle wallrock will experience enhanced partial melting relative to the bulk of its surrounding mantle that cannot diffusively equilibrate with a nearby vein of ascending more water-rich melt. This will also lead to a local increase in the absorption of latent heat. The wallrock will lose some of this water during its additional increment of melt production.

Conservation of energy also describes this process:

$$Vol_{WR} dF_{WR}^n L = C_p \Delta T \quad (18)$$

where Vol_{WR} is the mass/volume fraction of wallrock; dF_{WR}^n is the new change in depletion for the wallrock due to the addition of water from the water-riched melt, which is unknown and needed to calculate; L is the latent heat; C_p is specific heat; ΔT is the change of the system temperature:

$$\Delta T = T_0 - T_n \quad (19)$$

where T_0 is the system temperature before melting, and T_n is the new system temperature after an increment of wallrock melting. Here ΔT also equals the change in wet solidus temperature of the wallrock:

$$\Delta T = T_{S_{WR}^{wet}{}^0} - T_{S_{WR}^{wet}{}^n} \quad (20)$$

where $T_{S_{WR}^{wet}{}^0}$ is the wet solidus temperature of the wallrock before melting but after the addition of water, and $T_{S_{WR}^{wet}{}^n}$ is the new wet solidus temperature of the wallrock after its remelting. $T_{S_{wall}^{wet}{}^n}$ is equal to T_n because the temperature of the system is equal to the solidus temperature of the wallrock at the end of the melting:

$$T_n = T_{S_{WR}^{wet}{}^n} \quad (21)$$

Combining the above equations, dF_{wall}^n can be solved by finding the root of the implicit equation below

$$Vol_{WR} dF_{WR}^n L = C_p (T_0 - T_{S_{WR}^{wet}{}^n}(dF_{WR}^n)) \quad (22)$$

where the change in depletion of the wallrock is dF_{WR}^n , and the new wet solidus temperature for the wallrock $T_{S_{WR}^{wet}{}^n}$ is determined by substituting the new depletion $F_n = F + dF_{WR}^n$ and current water concentration X_C^S into the equations (3),(4),(5). And the new system's temperature T_n is equal to the new wet solidus temperature for the wallrock $T_{S_{WR}^{wet}{}^n}$ according to equation (21).

The parameter values used here are summarized in Table 1.

Table 1. List of used parameters

Symbols	Meaning	Values	Units
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T_{s0}	solidus temperature of rock at surface pressure	1151	°C
$\frac{dT_s}{dP}$	the solidus-pressure dependence	71	°C/GPa
$\frac{dT_s}{dF}$	the solidus-depletion dependence	150	°C
Λ	Λ is constant for wet solidus	43	°C wt% ^{-γ}
γ	γ is constant for wet solidus	0.75	1
κ	Thermal diffusivity	10 ⁻⁶	m ² /s
E_a	Activation energy	400	kJ/mol
V_a	Activation volume	0	m ⁻³ /mol
ρ	Mantle density	3300	kg/m ³
g	Gravitational acceleration	-9.81	m/s ²
R	Ideal gas constant	8.314	J mol ⁻¹ K ⁻¹
α	Thermal expansion coefficient	3 × 10 ⁻⁵	°C ⁻¹
β	Coefficient for Fe-depletion buoyancy scale	3 × 10 ⁻²	1
L/C_p	Latent heat divided by specific heat	600	K/kg

References

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- Katz, R. F., M. Spiegelman, and C. H. Langmuir (2003), A new parameterization of hydrous mantle melting, *Geochem. Geophys. Geosyst.*, 4(9), n/a-n/a, doi:Artn 1073 10.1029/2002gc000433.
- Morgan, J. P. (2001), Thermodynamics of pressure release melting of a veined plum pudding mantle, *Geochem. Geophys. Geosyst.*, 2(4).