Supplementary Material for:
Observation of lump solitons in a photon fluid

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Supplementary Note 1. The integrable regime of the defocusing 2D NLS

The defocusing 2D NLS admits an integrable regime described by the KPI equation. Let us consider the NLS hydrodynamic solution written as a small perturbation over a flat background $\psi(x, y, z) = \sqrt{\rho_0 + \eta(x, y, z)}\, e^{i\phi(x,y,z)}$, with ρ_0 constant and $|\eta| \ll \rho_0$. The KPI regime is achieved by applying a multiple-scale expansion to the hydrodynamic equations obtained from the 2D NLS, and choosing different scales along x and y to pass from the isotropy of the 2D NLS to the anisotropy of the KPI. As a result of the expansion, the amplitude and phase perturbations (η and ϕ) are described by the KPI and the potential KPI equations respectively

$$\begin{aligned} &(-\eta_Z + 6\eta\eta_X + \eta_{XXX})_X - 3\eta_{YY} = 0; \\ &\phi_{XZ} + 6\frac{c_0}{\gamma}\phi_X\phi_{XX} - \phi_{XXXX} + 3\phi_{YY} = 0; \end{aligned} \tag{1}$$

where the KPI equations are defined in the moving reference frame $(X, Y, Z) = (x - c_0 z, y, z)$. In the KPI regime, the phase and amplitude of the NLS solution are bound by the relation $\phi_X = -\gamma/c_0\eta$ and are traveling along x with c_0 speed during the z-propagation. The choice of the coefficients in the KPI (Eqs.(1)) and in the 2D NLS (Equation (3) of the manuscript) implies a constraint between the strength of the nonlinearity γ and the plateau intensity ρ_0, which are bound by the relation $\rho_0 = \frac{3}{4|\gamma|}$. The NLS field speed is $c_0 = \pm\sqrt{-\gamma\rho_0}$ and assumes a constant value $c_0 = \sqrt{3/4}$ for all the possible choices of γ. As a consequence, the KPI regime is found in the NLS system when the background intensity ρ_0 is tuned in accordance to the nonlinear coefficient γ. This defines the integrability condition.

Supplementary Note 2. The lump soliton in NLS systems

Considering η as the lump solitonic solution of the KPI, the constraint $|\eta| \ll \rho_0$ becomes a constraint on the solution parameter ε. A small ε guarantees the perturbation to be small and the perturbative expansion to be more accurate. The field described by the KPI is defined on a moving reference frame $(x - c_0 z, y, z)$, thus the NLS lump soliton travels along x with ε-dependent velocity $c = c_0 - c_0/2\varepsilon$.

In Fig.1 numerical simulations of the 2D NLS are shown with the NLS lump as the initial condition. Simulations reveal that the observation of solitonic propagation of the lump is crucially determined by the relation $\rho_0 = 3/(4|\gamma|)$. The initial condition is defined with $\rho_0 = 1$, and the lump solitonic propagation is observed only for $\gamma = -0.75$. Different values of γ produce a lump distortion during the propagation.

In the experiment, the integrability condition links the plateau intensity I_0 to the experimental parameters V and t. We report in Fig.2 observations for different values of the experimental parameters V, t and I_0. The plateau intensity of the initial condition and the applied voltage are constant, but data are taken at different time instants. The soliton observation is prevented for values of t which do not satisfy the integrability condition.

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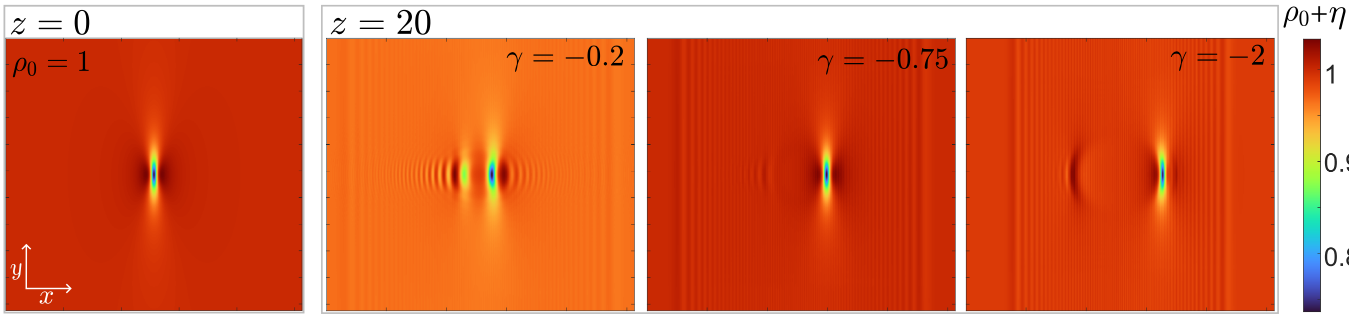
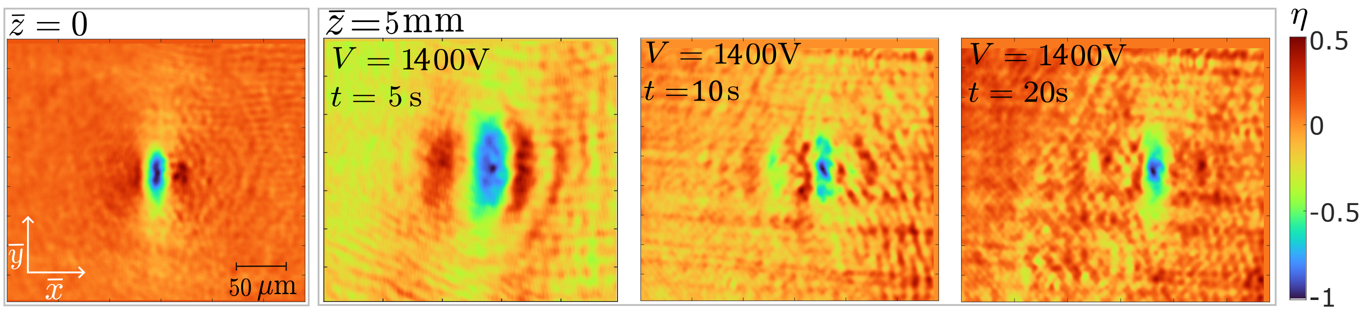


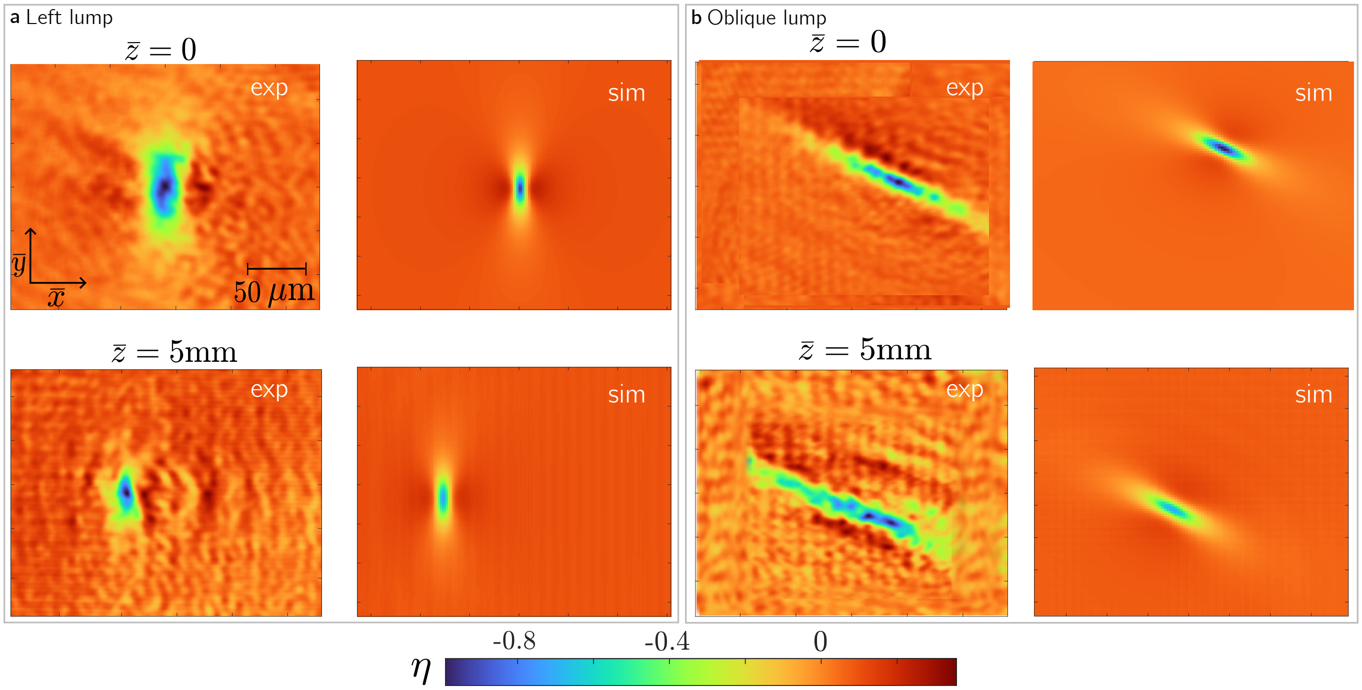
Supplementary Figure 1. **Integrability condition.** Simulations of the lump propagation with different nonlinear coefficients. The initial condition is prepared with a lump over a background $\rho_0 = 1$, and the propagation is studied by varying the nonlinearity γ. The solitonic propagation of the lump is observed only in the case with $\gamma = -0.75$ which satisfies the integrability condition $\rho_0 = 3/(4|\gamma|)$.



Supplementary Figure 2. **Non-integrable regime in the experiment.** The lump initial condition is prepared at the input of the crystal and the output is observed in time with constant applied voltage $V = 1400$V. The lump shape is preserved only when the observation time fits the integrability condition, namely when $t = 20$s. The other observations show a non-integrable regime.

Supplementary Note 3. Differently oriented lumps

In Figure 3, we report cases of rotated lumps that experience a displacement in the direction of the rotated x-axis. Figure 3(a) reports a left-shifting lump, obtained through a π-rotation, while the oblique-shifting lump in Fig.3(b) is obtained by rotating the lump of an angle of $-5\pi/4$ rad.



Supplementary Figure 3. **Rotated lumps.** (a) The left propagating lump is obtained through a rotation of π, undergoing a shift toward the negative x-axis. (b) The oblique lump is obtained through a rotation of $5\pi/4$ and reveals a shift in the bottom-left direction.