High-Throughput Discovery of Perturbation-Induced Topological Magnons: *Supplementary Materials*

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44.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 387

45 MSG $Pn\bar{3}m'$ (224.113) **388**
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46 MSG $Fd\bar{3}m'$ (227.131) **400**
46.1 WP: $16c$ . . . 400
46.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 400
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47 MSG $F_Sd\bar{3}c$ (228.139) **412**
47.1 WP: $32b$ . . . 412
47.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 412
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49 MSG $Ia\bar{3}d'$ (230.148) **428**
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50 MSG $Pn'n'n$ (48.260) **434**
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50.1.1 Topological bands in subgroup $P2'/c'$ (13.69) . . . 434
50.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 436

51 MSG $Pb'an'$ (50.282) **438**
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51.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 438
51.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 441
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51.2 WP: $4f$ . . . 445
51.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 445
51.2.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 447
51.2.3 Topological bands in subgroup $P2'/c'$ (13.69) . . . 448

52 MSG $Pn'n'a$ (52.310) **449**
52.1 WP: $4d$ . . . 450
52.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 450
52.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 451
52.1.3 Topological bands in subgroup $P2'/c'$ (13.69) . . . 453

53 MSG P_bnna (52.315) **454**
53.1 WP: $4c + 8f$ . . . 454
53.1.1 Topological bands in subgroup $P2'/c'$ (13.69) . . . 454
53.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 456
53.1.3 Topological bands in subgroup $P2'/c'$ (13.69) . . . 458
53.1.4 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 460
53.2 WP: $8f$ . . . 462
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53.2.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 465
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54 MSG P_Bnna (52.318) 474
54.1 WP: $4c+4c$ . . . 474
54.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 474
54.2 WP: $8f$ . . . 476
54.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 476

55 MSG P_Amna (53.333) 478
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55.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 480
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56.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 488
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57 MSG P_Imna (53.336) 493
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58 MSG P_Bcca (54.350) 499
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59 MSG P_Icca (54.352) 508
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59.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 508
59.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 510
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60 MSG $Pc'c'n$ (56.369) **516**
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61 MSG P_bccn (56.372) **520**
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62 MSG P_cccn (56.373) **523**
62.1 WP: $4a + 8c$ . . . 523
62.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 523
62.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 525
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63 MSG P_Accn (56.374) **531**
63.1 WP: $4a$ . . . 531
63.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 531
63.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 533
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63.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 537

64 MSG P_abcm (57.386) **538**
64.1 WP: $8d$ . . . 539
64.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 539
64.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 541

65 MSG P_Cbcm (57.391) **543**
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65.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 543
65.2 WP: $4c$ . . . 545
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66 MSG P_Bnnm (58.402) **549**
66.1 WP: $4a$ . . . 549
66.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 550
66.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 552
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66.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 555
66.1.6 Topological bands in subgroup $Pm'a'2$ (28.91) . . . 556

67 MSG $Pm'm'n$ (59.409) **557**
67.1 WP: $2b + 2b + 4c + 4d$ . . . 557
67.1.1 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 557
67.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 559

68 MSG $Pmm'n'$ (59.410) **561**
68.1 WP: $4c$ . . . 561
68.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 562
68.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 563
68.1.3 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 564

69 MSG P_Cbcn (60.431) **565**
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69.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 568
69.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 570
69.2 WP: $8e$ . . . 572
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69.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 574
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69.3.4 Topological bands in subgroup $P2'/m'$ (10.46) . . . 582
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70 MSG P_Ibcn (60.432) **585**
70.1 WP: $4a$ . . . 586
70.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 586
70.1.2 Topological bands in subgroup $P2'/m'$ (10.46) . . . 588
70.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 590
70.1.4 Topological bands in subgroup $P2'/c'$ (13.69) . . . 591
70.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 593
70.1.6 Topological bands in subgroup $Pm'a'2$ (28.91) . . . 594
70.2 WP: $4b + 8j$ . . . 595
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70.2.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 597
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70.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 601
70.3.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 603
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71 MSG $Pbca$ (61.433) **607**
71.1 WP: $4a$ . . . 607
71.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 607

72 MSG P_Cbca (61.439) **609**
72.1 WP: $4a + 8e$ . . . 609
72.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 609
72.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 611
72.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 613

73 MSG $Pnma$ (62.441) **615**
73.1 WP: $4b$ . . . 615
73.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 616
73.2 WP: $4a + 4c$ . . . 618
73.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 618
73.3 WP: $4b + 4c$ . . . 621
73.3.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 621
73.4 WP: $4a$ . . . 624
73.4.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 624

74 MSG $Pn'm'a$ (62.446) **626**
74.1 WP: $4b$ . . . 626
74.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 626
74.1.2 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 628
74.1.3 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 629
74.2 WP: $4c + 4c$ . . . 631
74.2.1 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 631
74.2.2 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 632
74.3 WP: $4b + 4b$ . . . 634
74.3.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 634
74.3.2 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 637
74.3.3 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 639
74.4 WP: $4c$ . . . 641
74.4.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 641
74.4.2 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 643
74.4.3 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 644
74.5 WP: $4a + 4c$ . . . 646
74.5.1 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 646
74.5.2 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 648
74.6 WP: $4b + 4c$ . . . 650
74.6.1 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 650
74.6.2 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 652
74.7 WP: $4a + 4a + 4c$ . . . 654
74.7.1 Topological bands in subgroup $P2'_1/m'$ (11.54) . . . 654
74.7.2 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 656

75 MSG $Pnm'a'$ (62.447) **658**
75.1 WP: $4a+4c$ . . . 658
75.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 658
75.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 660
75.2 WP: $4a$ . . . 662
75.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 662
75.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 664
75.2.3 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 665
75.3 WP: $4b+4c+4c$ . . . 667
75.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 667
75.3.2 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 669
75.4 WP: $4b+4c$ . . . 671
75.4.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 671
75.4.2 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 673
75.5 WP: $4b$ . . . 675
75.5.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 675
75.5.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 677
75.5.3 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 678
75.6 WP: $4c$ . . . 680
75.6.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 680
75.6.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 682
75.6.3 Topological bands in subgroup $P2_1'/m'$ (11.54) . . . 683
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75.7.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 684
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76 MSG $Pn'ma'$ (62.448) **688**
76.1 WP: $4b$ . . . 689
76.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 689
76.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 691
76.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 692
76.2 WP: $4a$ . . . 694
76.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 694
76.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 696
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76.3 WP: $4c$ . . . 698
76.3.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 698
76.3.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 700
76.3.3 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 701
76.4 WP: $4c+8d$ . . . 702
76.4.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 702
76.4.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 704
76.5 WP: $4b+4c$ . . . 706
76.5.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 706
76.5.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 708

76.6 WP: $4b + 4b$ . . . 710
76.6.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 710
76.6.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 713
76.6.3 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 715
76.7 WP: $8d$ . . . 717
76.7.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 717
76.7.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 719

77 MSG P_anma (62.450) **721**
77.1 WP: $4a + 4b$ . . . 721
77.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 721
77.2 WP: $4b$ . . . 723
77.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 723
77.3 WP: $4a$ . . . 725
77.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 725

78 MSG P_bnma (62.451) **726**
78.1 WP: $4d + 8h$ . . . 726
78.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 726
78.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 728
78.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 730
78.2 WP: $8g$ . . . 732
78.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 732
78.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 734
78.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 736
78.3 WP: $8h$ . . . 738
78.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 738
78.3.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 740
78.3.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 742

79 MSG P_cnma (62.452) **744**
79.1 WP: $16e$ . . . 744
79.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 744
79.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 746
79.2 WP: $8c$ . . . 748
79.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 748
79.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79) . . . 750

80 MSG C_cmcm (63.466) **752**
80.1 WP: $4b + 8e$ . . . 752
80.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 752
80.1.2 Topological bands in subgroup $P2'/m'$ (10.46) . . . 755
80.1.3 Topological bands in subgroup $C2'/m'$ (12.62) . . . 757
80.1.4 Topological bands in subgroup $C2'/c'$ (15.89) . . . 758
80.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 760

81 MSG C_Amcm (63.468) **761**
81.1 WP: $8d$ . . . 761
81.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 761
81.1.2 Topological bands in subgroup $P2'/c'$ (13.69) . . . 763
81.1.3 Topological bands in subgroup $C2'/m'$ (12.62) . . . 764
81.1.4 Topological bands in subgroup $C2'/c'$ (15.89) . . . 766
81.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 767
81.1.6 Topological bands in subgroup $Ab'm'2$ (39.199) . . . 768
81.1.7 Topological bands in subgroup $C2/m$ (12.58) . . . 769
81.1.8 Topological bands in subgroup $Cmm'a'$ (67.506) . . . 770
81.1.9 Topological bands in subgroup C_c2/m (12.63) . . . 771
81.2 WP: $8f$ . . . 772
81.2.1 Topological bands in subgroup $Ab'm'2$ (39.199) . . . 772

82 MSG C_Ammm (65.490) **773**
82.1 WP: $8f$ . . . 773
82.1.1 Topological bands in subgroup $Cc'c'2$ (37.183) . . . 773

83 MSG C_amma (67.509) **774**
83.1 WP: $4g$ . . . 775
83.1.1 Topological bands in subgroup $C2'/m'$ (12.62) . . . 775
83.2 WP: $4e$ . . . 776
83.2.1 Topological bands in subgroup $C2'/m'$ (12.62) . . . 776

84 MSG F_Smmm (69.526) **777**
84.1 WP: $8g$ . . . 777
84.1.1 Topological bands in subgroup $C2'/m'$ (12.62) . . . 777

85 MSG $Fd'd'd$ (70.530) **778**
85.1 WP: $16c + 8b$ . . . 778
85.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 778
85.1.2 Topological bands in subgroup $C2'/c'$ (15.89) . . . 780
85.1.3 Topological bands in subgroup $C2'/c'$ (15.89) . . . 782
85.2 WP: $16c$ . . . 783
85.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 783
85.2.2 Topological bands in subgroup $C2'/c'$ (15.89) . . . 785
85.2.3 Topological bands in subgroup $C2'/c'$ (15.89) . . . 786

86 MSG I_cbca (73.553) **787**
86.1 WP: $8g$ . . . 787
86.1.1 Topological bands in subgroup C_a2 (5.17) . . . 787

87 MSG $Im'm'a$ (74.558) **788**
87.1 WP: $4b$ . . . 789
87.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 789
87.1.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 790
87.1.3 Topological bands in subgroup $C2'/m'$ (12.62) . . . 791
87.2 WP: $4b + 4c$ . . . 792
87.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 792
87.2.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 793
87.2.3 Topological bands in subgroup $C2'/m'$ (12.62) . . . 795

88 MSG $Imm'a'$ (74.559) **796**
88.1 WP: $4c + 4c$ . . . 796
88.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 796
88.1.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 798
88.2 WP: $4b + 4c$ . . . 799
88.2.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 799
88.2.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 801
88.3 WP: $4a + 4c$ . . . 803
88.3.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 803
88.3.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 805
88.4 WP: $4b + 4c + 4e$ . . . 807
88.4.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 807
88.4.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 809
88.5 WP: $4d$ . . . 811
88.5.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 811
88.5.2 Topological bands in subgroup $C2'/m'$ (12.62) . . . 812

89 MSG $P4/n$ (85.59) **813**
89.1 WP: $2c + 4e$ . . . 813
89.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 814

90 MSG P_c4/n (85.64) **816**
90.1 WP: $4c + 4c$ . . . 816
90.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79) . . . 816
90.1.2 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 817

91 MSG P_C4_2/n (86.73) **819**
91.1 WP: $4c$ . . . 819
91.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 819
91.1.2 Topological bands in subgroup $P2'/m'$ (10.46) . . . 820
91.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 822

92 MSG $I4_1/a$ (88.81) **823**
92.1 WP: $4a + 8d$ . . . 823
92.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 823

93 MSG I_c4_1/a (88.86) 825
93.1 WP: $16c$ . . . 825
93.1.1 Topological bands in subgroup $P\bar{1}$ (2.4) . . . 825
93.1.2 Topological bands in subgroup $C2'/c'$ (15.89) . . . 828
93.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7) . . . 829

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94.4 Subgroup $C2/m$ (12.58) . . . 831
94.5 Subgroup $C2'/m'$ (12.62) . . . 831
94.6 Subgroup C_c2/m (12.63) . . . 831
94.7 Subgroup $I\bar{4}2'd'$ (122.337) . . . 831
94.8 Subgroup $P2'/c'$ (13.69) . . . 832
94.9 Subgroup $P2_1'/c'$ (14.79) . . . 832
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94.21 Subgroup $Ab'a'2$ (41.215) . . . 833
94.22 Subgroup C_a2 (5.17) . . . 833
94.23 Subgroup $Cmm'a'$ (67.506) . . . 833

1 Topological Magnon Materials

Supplementary Table 1: Summary of topological magnon materials.

	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
1 2	α-Fe_2O_3(955) $Ho(Co_{0.667}Ga_{0.333})_2$(31)	$C2'/c'$ (15.89) $8f$	B in generic direction $P\bar{1}$ (2.4)
3	$MnPt_{0.5}Pd_{0.5}$(866)	C_amma (67.509) $4g$	B $\perp$ [010] $C2'/m'$ (12.62)
4	$NiFe_2O_4$(858)	$I4_1/am'd'$ (141.557) $4a, 8d, 8d$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/c'$ (15.89) B $\perp$ [110] $C2'/c'$ (15.89) B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
5	$NdFeO_3$(760)		
6	$CeFeO_3$(720)		
7	$TbFeO_3$(681)		
8	$SmFeO_3$(670)		
9	$NaOsO_3$(410)		
10	$LaCrO_3$(310)		
11	$YCrO_3$(141)		
12	$LaMnO_3$(139.5)		
13	$ErCrO_3$(133)	$Pn'ma'$	B in generic direction $P\bar{1}$ (2.4)
14	$TmCrO_3$(124)	(62.448)	$\sigma \perp$ [001] $P2'_1/c'$ (14.79)
15	$YRuO_3$(97)	$4b$	$\sigma \perp$ [100] $P2'_1/c'$ (14.79)
16	$PrMnO_3$(91)		
17	$Pr_{0.95}K_{0.05}MnO_3$(85)		
18	$NdMnO_3$(78)		
19	$KMnF_3$(77)		
20	$NdMnO_3$(73)		
21	$(CH_3NH_3)(Co(COOH)_3$(15.9)		
22	$Ho_{0.2}Bi_{0.8}FeO_3$(-)		
23	$Ho_{0.15}Bi_{0.85}FeO_3$(-)		
24	$SrFeO_2F$(710)		
25	$Pb_{0.7}Bi_{0.3}Fe_{0.762}W_{0.231}O_3$(504)	I_c4/mcm	
26	$Pb_{0.8}Bi_{0.2}Fe_{0.728}W_{0.264}O_3$(435)	(140.550)	B $\perp$ [001] $C2'/m'$ (12.62)
27	$KNiF_3$(275)	$4a$	
28	$UNiGa_5$(86)		
29	$CeIr(In_{0.97}Cd_{0.03})_5$(3)		
30	$La_{0.5}Sr_{0.5}FeO_{2.5}F_{0.5}$(700)		
31	$La_{0.875}Ba_{0.125}Mn_{0.875}Ti_{0.125}O_3$(-)		
32	$La_{0.90}Ba_{0.10}Mn_{0.90}Ti_{0.10}O_3$(-)	$Pn'ma'$	B in generic direction $P\bar{1}$ (2.4)
33	$La_{0.95}Ba_{0.05}Mn_{0.95}Ti_{0.05}O_3$(-)	(62.448)	$\sigma \perp$ [001] $P2'_1/c'$ (14.79)
34	$La_{0.95}Ba_{0.05}MnO_3$(-)	$4a$	$\sigma \perp$ [100] $P2'_1/c'$ (14.79)
35	$La_{0.95}Ca_{0.05}MnO_3$(-)		
36	$LaMnO_3$(-)		
37	$YCr_{0.5}Mn_{0.5}O_3$(-)		
38	$Sr_2Cr_3As_2O_2$(590)	P_I4_2/mnm (136.506) $4d$	E $\parallel$ [001] + B $\parallel$ [001] + $\sigma \parallel$ [110] $Cc'c'2$ (37.183)
39	Mn_2As(573)		
40	Cr_2As(393)	P_anma	
41	Fe_2As(353)	(62.450)	B $\perp$ [010] $P2'_1/c'$ (14.79)
42	NdCoAsO(14)	$4a, 4b$	
43	$SrMnSb_2$(565)	$Pn'ma'$ (62.448) $4c$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
44	$SmFeO_3$(480)		
45	$SrRuO_3$(165)		
46	$TbCrO_3$(158)	$Pn'm'a$	B in generic direction $P\bar{1}$ (2.4)
47	$DyCrO_3$(146)	(62.446)	$\sigma \perp$ [010] $P2'_1/m'$ (11.54)
48	$TeNiO_3$(130)	$4b$	$\sigma \perp$ [100] $P2'_1/c'$ (14.79)
49	$Ca_2PrCr_2TaO_9$(130)		
50	$Ca_2PrCr_2NbO_9$(110)		
51	YVO_3(77)		

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
52	$ErMn_2Ge_2$(475)		
53	YMn_2Si_2(460)		
54	$CeCo_2P_2$(440)		
55	YMn_2Si_2(410)		
56	YMn_2Ge_2(395)		
57	$EuMn_2Si_2$(391)		B in generic direction $P\bar{1}$ (2.4)
58	$CeMn_2Si_2$(384)		B $\perp$ [110] $C2'/m'$ (12.62)
59	$NdMn_2Si_2$(382)		B $\perp$ [001] $P2'_1/m'$ (11.54)
60	$NdMn_2Si_2$(380)	P_I4/nnc	B $\perp$ [100] $P2'_1/m'$ (11.54)
61	$CeMn_2Si_2$(380)	(126.386)	σ in generic direction $P_S\bar{1}$ (2.7)
62	$CeMn_2Si_2$(379)	$4d$	E $\parallel$ [001] + σ $\parallel$ [110]
63	$PrMn_2Si_2$(368)		C_Acc2 (37.186)
64	$PrMn_2Si_2$(348)		E $\parallel$ [001] P_I4nc (104.210)
65	$CaCo_{1.86}As_2$(52)		
66	$ErMn_2Ge_2$(-)		
67	$ErMn_2Ge_2$(-)		
68	$ErMn_2Si_2$(-)		
69	$ErMn_2Si_2$(-)		
70	$TbMn_2Si_2$(-)		
71	Mn_3Pt(475)	P_c4_2/mcm (132.456) $4f$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/c'$ (15.89) B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [100] $P2'/m'$ (10.46) σ in generic direction $P_S\bar{1}$ (2.7)
72	$SrFeO_2$(473)	F_Smmm (69.526) $8g$	B $\perp$ [100] $C2'/m'$ (12.62)
73	$Ni_{1.64}Co_{0.36}Mn_{1.28}Ga_{0.72}$(450)	P_I4_2/mnm (136.506) $4d, 4d$	E $\parallel$ [001] + B $\parallel$ [001] + σ $\parallel$ [110] $Cc'c'2$ (37.183)
74	$Li_{0.5}FeCr_{1.5}O_4$(417)	$I4_1/am'd'$ (141.557) $4a, 8d$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/c'$ (15.89) B $\perp$ [110] $C2'/c'$ (15.89) B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)
75	$FeCr_2S_4$(172)		
76	Ce_5TeO_8(40)		
77	FeGe(410)	P_c6/mcc (192.252) $6f$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62) B $\perp$ [001] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
78	FeF_3(394)		
79	Ca_3LiRuO_6(117)		
80	$FeSO_4F$(100)	$C2'/c'$	
81	Sr_3LiRuO_6(90)	(15.89)	B in generic direction $P\bar{1}$ (2.4)
82	Sr_3NaRuO_6(70)	$4d$	
83	HoP(5.5)		
84	FeF_3(-)		

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
85	LiMnAs(393)		
86	LiMnAs(393)		
87	LiMnAs(393)		
88	CsMnP(295)		
89	CsMnP(295)		
90	CsMnBi(295)		B $\perp$ [110] $C2'/c'$ (15.89)
91	RbMnAs(295)	P_c4_2/ncm	B $\perp$ [001] $P2_1'/c'$ (14.79)
92	RbMnP(295)	(138.528)	B $\perp$ [100] $P2_1'/m'$ (11.54)
93	RbMnP(295)	$4b$	σ in generic direction $P_S\bar{1}$ (2.7)
94	KMnP(295)		E $\parallel$ [110] + B $\parallel$ [110]
95	KMnP(295)		$Ab'a'2$ (41.215)
96	KMnAs(293)		
97	KMnAs(293)		
98	RbMnAs(293)		
99	RbMnBi(-)		
100	LaCrAsO(-)		
			B in generic direction $P\bar{1}$ (2.4)
101	$LaSrFeO_4$(380)	P_C4_2/nnm	B $\perp$ [110] $C2'/m'$ (12.62)
102	Nd_2CuO_4(276)	(134.481)	B $\perp$ [001] $P2'/m'$ (10.46)
103	Pr_2CuO_4(270)	$4d$	B $\perp$ [100] $P2_1'/c'$ (14.79)
			σ in generic direction $P_S\bar{1}$ (2.7)
			B in generic direction $P\bar{1}$ (2.4)
104	$LaSrFeO_4$(380)	P_C4_2/ncm	B $\perp$ [110] $C2'/m'$ (12.62)
105	$LaCaFeO_4$(373)	(138.529)	B $\perp$ [001] $P2'/m'$ (10.46)
106	$LaBaFeO_4$(-)	$4d$	B $\perp$ [100] $P2'/c'$ (13.69)
			σ in generic direction $P_S\bar{1}$ (2.7)
107	$Mn_3Cu_{0.5}Ge_{0.5}N$(380)	$R\bar{3}m$	
108	Mn_3GaN(298)	(166.97)	B in generic direction $P\bar{1}$ (2.4)
109	Mn_3ZnN(183)	$9e$	
			B in generic direction $P\bar{1}$ (2.4)
			B $\perp$ [001] $P2'/m'$ (10.46)
		P_Ibcn	B $\perp$ [010] $P2_1'/c'$ (14.79)
110	$FeSn_2$(378)	(60.432)	B $\perp$ [100] $P2'/c'$ (13.69)
111	$FeGe_2$(315)	$4a$	σ in generic direction $P_S\bar{1}$ (2.7)
			E $\parallel$ [010] + B $\parallel$ [010]
			$Pm'a'2$ (28.91)
			B in generic direction $P\bar{1}$ (2.4)
		C_cmcm	B $\perp$ [001] $P2'/m'$ (10.46)
112	FeSn(373)	(63.466)	B $\perp$ [010] $C2'/m'$ (12.62)
		$4b, 8e$	B $\perp$ [100] $C2'/c'$ (15.89)
			σ in generic direction $P_S\bar{1}$ (2.7)
		P_Cbcm	
113	$LaSr_3Fe_3O_9$(350)	(57.391)	B $\perp$ [001] $P2_1'/c'$ (14.79)
		$4c, 8g$	
114	$La_{0.75}Bi_{0.25}Fe_{0.5}Cr_{0.5}O_3$(350)		
115	$LaCrO_3$(290)	$Pnma$	
116	$CeFeO_3$(220)	(62.441)	B in generic direction $P\bar{1}$ (2.4)
117	$InCrO_3$(93)	$4b$	
118	$TlCrO_3$(89)		
119	$ScCrO_3$(73)		
120	$FeBO_3$(348)	$C2'/c'$	
121	$FeOHSO_4$(125)	(15.89)	B in generic direction $P\bar{1}$ (2.4)
122	Ca_3LiOsO_6(117.1)	$4c$	

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
123	MnCoGe(345)	$Pn'm'a$ (62.446) $4c, 4c$	$\sigma \perp$ [010] $P2'_1/m'$ (11.54) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
124 125 126	La_2NiO_4(330) Nd_2NiO_4(320) La_2CoO_4(275)	P_Cmna (53.335) $4a$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [010] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
127 128 129	La_2CuO_4(316) La_2CuO_4(50) Gd_2CuO_4(18)	P_Accn (56.374) $4a$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [010] $P2'/m'$ (10.46) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
130	CoO(290)	I_c4_1/acd (142.570) $16e$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/c'$ (15.89) B $\perp$ [001] $C2'/c'$ (15.89) B $\perp$ [100] $C2'/m'$ (12.62) σ in generic direction $P_S\bar{1}$ (2.7)
131 132	Sm_2CuO_4(280) Eu_2CuO_4(265)	P_C4_2/ncm (138.529) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [001] $P2'/m'$ (10.46) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
133 134 135	CrN(273) DyOCl(10) $ErFe_2Si_2$(2.6)	P_anma (62.450) $4b$	B $\perp$ [010] $P2'_1/c'$ (14.79)
136	$Bi_{0.85}Ca_{0.15}Fe_{0.55}Mn_{0.45}O_3$(268)	$Pn'm'a$ (62.446) $4b, 4b$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $P2'_1/m'$ (11.54) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
137 138	Mn_3Sn_2(262) Mn_3Sn_2(227)	$Pn'ma'$ (62.448) $4c, 8d$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
139 140	Nd_2CuO_4(245) UP(22.5)	P_C4_2/nnm (134.481) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [001] $P2'/m'$ (10.46) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
141 142	Sr_2IrO_4(240) Sr_2IrO_4(224)	P_Icca (54.352) $8c$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [010] $P2'_1/c'$ (14.79)
143	$Bi_{0.8}La_{0.2}Fe_{0.5}Mn_{0.5}O_3$(240)	$Imm'a'$ (74.559) $4c, 4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/m'$ (12.62)
144 145 146 147 148 149 150 151	$PrCrO_3$(239) $SmCrO_3$(192) $SmCrO_3$(191) $NdMn_{0.8}Fe_{0.2}O_3$(59) $Rb_2Fe_2O(AsO_4)_2$(25) $NdMnO_3$(15) $NdMn_{0.8}Fe_{0.2}O_3$(13) $NdMnO_3$(13)	$Pn'ma'$ (62.448) $4b, 4c$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
152 153	$Pr_{0.5}Sr_{0.5}CoO_3$(230) $Pr_{0.5}Sr_{0.4}Ba_{0.1}CoO_3$(226)	$Im'm'a$ (74.558) $4b$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/m'$ (12.62) $\sigma \perp$ [100] $C2'/m'$ (12.62)
154	$Cd_2Os_2O_7$(227)	$Fd\bar{3}m'$ (227.131) $16c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] + $\sigma \parallel$ [111] $C2'/m'$ (12.62) B $\parallel$ [110] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62)
155 156	BaCoSO(220) BaCoSO(-)	P_abcm (57.386) $8d$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79)
157 158 159	USb(213) NpBi(192.5) $Fe_{0.7}Mn_{0.3}$(-)	$Pn\bar{3}m'$ (224.113) $4b$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] + $\sigma \parallel$ [111] $C2'/m'$ (12.62) B $\parallel$ [110] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62)
160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177	CuO(213) Ba_2CoO_4(26) Ba_2CoO_4(25) Ba_2CoO_4(23) $LiFeGe_2O_6$(20) $Na_{0.5}Li_{0.5}FeGe_2O_6$(18) BiNiO($PO_4$)(17.5) $Li_{0.31}Na_{0.69}FeGe_2O_6$(15) BiCoO($PO_4$)(15) Li_2MnSiO_4(12) $BiMnTeO_6$(10) MnV_2O_6(4.7) ErNiGe(2.9) $BaNd_2O_4$(1.7) $BaNd_2O_4$(1.7) $GdPO_4$(0.8) $CoNb_2O_6$(-) Y_2BaCuO_5(-)	P_a2_1/c (14.80) $8e$	B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
178 179 180 181	USb_2(206) UP_2(203) UGeS(90) CeSbTe(2.3)	P_c4/ncc (130.432) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
182 183	$Sr_2CoO_2Ag_2Se_2$(190) $Ba_2CoO_2Ag_2Se_2$(180)	P_C4_2/n (86.73) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] $P2'/m'$ (10.46) σ in generic direction $P_S\bar{1}$ (2.7)
184 185 186	$FeCr_2S_4$(180) $FeCr_2S_4$(180) $NdCo_2$(100)	$I4_1/am'd'$ (141.557) $4b, 8c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/c'$ (15.89) B $\perp$ [110] $C2'/c'$ (15.89) B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)
187	$CaFe_4Al_8$(180)	$I4'/mmm'$ (139.535) $8f$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62)
188	$CoFePO_5$(175)	$Pnm'a'$ (62.447) $4a, 4c$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
189 190 191 192	Mn_3GaC(164) $KFe_3(OH)_6(SO_4)_2$(64) $NaFe_3(SO_4)_2(OH)_6$(50) $AgFe_3(SO_4)_2(OD)_6$(-)	$R_I\bar{3}c$ (167.108) $18e$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62) σ in generic direction $P_S\bar{1}$ (2.7)
193	$KCuMnS_2$(160)	C_Ammm (65.490) $8f$	E $\parallel$ [001] + B $\parallel$ [001] $Cc'c'2$ (37.183)
194	Ca_2MnGaO_5(160)	$Pnm'a'$ (62.447) $4a$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
195	$CaCu_3Fe_2Sb_2O_{12}$(160)	$Pn'n'n$ (48.260) $2b, 2c, 2d, 4f$	$\sigma \perp$ [010] $P2'/c'$ (13.69) $\sigma \perp$ [100] $P2'/c'$ (13.69)
196 197 198 199 200 201	$LiCoF_4$(150) $NaMnF_4$(13) $CuSb_2O_6$(8.7) $MnPb_4Sb_6S_{14}$(6) $FePb_4Sb_6S_{14}$(5) $Li_2Fe(SO_4)_2$(4)	P_a2_1/c (14.80) $4a$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
202	Mn_3CuN(143)	$P4/n$ (85.59) $2c, 4e$	B in generic direction $P\bar{1}$ (2.4)
203	Mn_3ZnN(140)	P_Ibcn (60.432) $4b, 8j$	B $\perp$ [010] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
204	$Pr_{0.5}Sr_{0.5}MnO_3$(140)	C_Amcm (63.468) $8d$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [001] $P2'/c'$ (13.69) B $\perp$ [010] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/c'$ (15.89) σ in generic direction $P_S\bar{1}$ (2.7) E $\parallel$ [100] + B $\parallel$ [100] $Ab'm'2$ (39.199) B $\parallel$ [100] + $\sigma \perp$ [100] $C2/m$ (12.58) B $\parallel$ [100] $Cmm'a'$ (67.506) $\sigma \perp$ [100] C_c2/m (12.63)
205 206	Nd_2NiO_4(130) $Nd_2NiO_{4.11}$(53)	$P4_2/nc'm'$ (138.525) $4d, 8i$	B $\parallel$ [100] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79)
207	$BiCrO_3$(114)	$C2/c$ (15.85) $4d, 4e$	B in generic direction $P\bar{1}$ (2.4)
208	GdMg(110)	$C2'/c'$ (15.89) $4a$	B in generic direction $P\bar{1}$ (2.4)
209	Ca_2RuO_4(110)	$Pbca$ (61.433) $4a$	B in generic direction $P\bar{1}$ (2.4)
210	Ca_2MnO_4(110)	$I4'_1/a'cd'$ (142.568) $8a$	B $\parallel$ [001] $I\bar{4}2'd'$ (122.337)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
211	Sr_2CoOsO_6(108)	$C2/c$ (15.85) $4c, 4d$	B in generic direction $P\bar{1}$ (2.4)
212 213 214	$(Tm_{0.7}Mn_{0.3})MnO_3$(104) $(Ho_{0.8}Mn_{0.2})MnO_3$(76) $(Ho_{0.8}Mn_{0.2})MnO_3$(40)	$Pnm'a'$ (62.447) $4b, 4c, 4c$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
215 216	Mn_5Si_3(100) Mn_5Si_3(99)	P_Cbcn (60.431) $8g$	B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
217	FeF_3(97)	$P6_3/m$ (176.143) $6g$	B in generic direction $P\bar{1}$ (2.4)
218	$Ho_2Ru_2O_7$(95)	$I4_1/am'd'$ (141.557) $8c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/c'$ (15.89) B $\perp$ [110] $C2'/c'$ (15.89) B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)
219	$FeSn_2$(93)	P_Cbcn (60.431) $8e$	B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
220	Fe_2MnBO_5(90)	P_bnma (62.451) $4d, 8h$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
221	$Er_2Ru_2O_7$(90)	$I4'_1/am'd$ (141.554) $8c, 8d$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)
222 223	Na_2NiFeF_7(88) Na_2NiFeF_7(88)	$Imm'a'$ (74.559) $4b, 4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/m'$ (12.62)
224 225	$MnTe_2$(86.5) NiS_2(39)	$Pa\bar{3}$ (205.33) $4a$	B in generic direction $P\bar{1}$ (2.4)
226	$TbCr_{0.5}Mn_{0.5}O_3$(84)	$Pn'ma'$ (62.448) $4b, 4b$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
227	UPd_2Ge_2(80)	P_c4/ncc (130.432) $4c, 4c$	B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
228	La_2NiO_4(80)	$Pc'c'n$ (56.369) $4a$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
229	$CsCo_2Se_2$(78)	C_Amcm (63.468) $8f$	E $\parallel$ [100] + B $\parallel$ [100] $Ab'm'2$ (39.199)
230 231	$Sr_2Fe_3S_2O_3$(76) Pr_2Pd_2In(5)	P_bnma (62.451) $8g$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
232	$NiCr_2O_4$(74)	$Fd'd'd$ (70.530) $16c, 8b$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/c'$ (15.89) $\sigma \perp$ [100] $C2'/c'$ (15.89)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
233 234	UGeTe(73) $NdScSiC_{0.5}H_{0.2}$(10)	P_I4/nnc (126.386) $4e$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [100] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
235	NdZn(70)	$P_In\bar{3}n$ (222.103) $8c$	B $\parallel$ [100] + $\sigma \parallel$ [110] $P2'_1/m'$ (11.54) B $\perp$ [100] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
236	Sr_2FeOsO_6(67)	P_c4/n (85.64) $4c, 4c$	B $\parallel$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
237 238 239	Cr_2ReO_6(67) $Co_3(PO_4)_2$(30) $CuFe_2(P_2O_7)_2$(15.5)	P_a2_1/c (14.80) $4a, 8e$	B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
240 241 242	$(Lu_{0.6}Mn_{0.4})MnO_3$(67) $CsMn_2F_6$(24.1) $Tb_{0.55}Sr_{0.45}MnO_3$(8)	$Pnm'a'$ (62.447) $4b, 4c$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
243 244	$Tb_{0.55}Sr_{0.45}MnO_3$(65) $Tb_{0.55}Sr_{0.45}MnO_3$(65)	$Pnm'a'$ (62.447) $4b$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
245	DyCu(64)	$Im\bar{3}m'$ (229.143) $8c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] + $\sigma \parallel$ [111] $C2'/m'$ (12.62) B $\parallel$ [110] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62)
246 247 248 249 250	TbPt(56) $TbPt_{0.8}Cu_{0.2}$(47) NdSi(46) DyPt(23) TmNi(7)	$Pn'm'a$ (62.446) $4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $P2'_1/m'$ (11.54) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
251	$La_2NiO_3F_2$(55)	P_Amna (53.333) $4e$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [001] $P2'/c'$ (13.69) B $\perp$ [010] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7) E $\parallel$ [100] + $\sigma \perp$ [100] P_a2 (3.4)
252 253 254 255 256 257	PrSi(54) $NdNi_{0.6}Cu_{0.4}$(37.5) HoNi(35.5) HoPt(16) ErPt(16) TmPt(6)	$Pnm'a'$ (62.447) $4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
258	MnV_2O_4(53)	$I4_1/a$ (88.81) $4a, 8d$	B in generic direction $P\bar{1}$ (2.4)
259	$Fe_2F_5(H_2O)_2$(48)	$Imm'a'$ (74.559) $4a, 4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/m'$ (12.62)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
260 261 262	Mn_2SiO_4(47.1) Mn_2GeO_4(47) $Ca_2Fe_2O_5$(-)	$Pn'm'a$ (62.446) $4a, 4c$	$\sigma \perp$ [010] $P2'_1/m'$ (11.54) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
263 264 265	La_3OsO_7(45) La_3OsO_7(45) $La_{2.8}Ca_{0.2}OsO_7$(33)	P_a2_1/m (11.55) $4a$	σ in generic direction $P_S\bar{1}$ (2.7)
266	$LaMn_3V_4O_{12}$(44)	$R_I\bar{3}$ (148.20) $18e$	B in generic direction $P\bar{1}$ (2.4) σ in generic direction $P_S\bar{1}$ (2.7)
267 268	Ho_3NiGe_2(43) Pr_3CoGe_2(28)	$Pnm'a'$ (62.447) $4c, 4c, 4c$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
269 270 271	NpTe(40) NpSe(38) NpS(23)	$F_Sd\bar{3}c$ (228.139) $32b$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] + $\sigma \parallel$ [111] $C2'/m'$ (12.62) B $\parallel$ [110] + $\sigma \parallel$ [111] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62) B $\parallel$ [100] + $\sigma \parallel$ [110] $C2'/c'$ (15.89) B $\perp$ [100] $C2'/c'$ (15.89) σ in generic direction $P_S\bar{1}$ (2.7)
272	Cr_2F_5(40)	$C2/c$ (15.85) $4a, 4b$	B in generic direction $P\bar{1}$ (2.4)
273	$Tb_{0.6}Y_{0.4}RhIn_5$(37.6)	C_amma (67.509) $4e$	B $\perp$ [010] $C2'/m'$ (12.62)
274 275 276 277	$NiSO_4$(37) $FeSO_4$(21) $CoSO_4$(15.5) PrCuSi(5.1)	P_Cbcn (60.431) $4a$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [010] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'/m'$ (10.46) σ in generic direction $P_S\bar{1}$ (2.7)
278 279 280	$SmCrO_3$(37) $SmCrO_3$(37) $TbFeO_3$(8.4)	$Pn'm'a$ (62.446) $4b, 4c$	$\sigma \perp$ [010] $P2'_1/m'$ (11.54) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
281 282 283 284	$NaFeSO_4F$(36) $NaCoSO_4F$(29) Sr_3ZnIrO_6(17) Ca_4IrO_6(12)	P_C2/c (13.74) $4a$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
285	Mn_3TeO_6(35.8)	P_a2_1/c (14.80) $4c, 8e$	B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
286	NdMg(35)	P_C4/nbm (125.373) $4f$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [110] $C2'/m'$ (12.62) B $\perp$ [001] $P2'/m'$ (10.46) B $\perp$ [100] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
287 288	Sc_2NiMnO_6(35) La_2CoPtO_6(28)	P_a2_1/c (14.80) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
289 290	$Cu_3Y(SeO_3)_2O_2Cl$(35) $Cu_3Bi(SeO_3)_2O_2Br$(27.4)	P_cccn (56.373) $4a, 8c$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [010] $P2'_1/m'$ (11.54) B $\perp$ [100] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
291 292	$(NH_2(CH_3)_2)(FeMn(HCOO)_6)$(35) $(NH_2(CH_3)_2)(FeCo(HCOO)_6)$(32)	$C2'/c'$ (15.89) $4a, 4e$	B in generic direction $P\bar{1}$ (2.4)
293	$PrMnSi_2$(34)	P_Bnna (52.318) $4c, 4c$	B $\perp$ [100] $P2'_1/c'$ (14.79)
294 295 296	$MnCO_3$(32) $NiCO_3$(30) $CoCO_3$(18)	$C2/c$ (15.85) $4c$	B in generic direction $P\bar{1}$ (2.4)
297	UO_2(30.8)	$Pn\bar{3}m'$ (224.113) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] + $\sigma \parallel$ [111] $C2'/m'$ (12.62) B $\parallel$ [110] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62)
298 299 300	Tb_2CoGa_8(28.5) Dy_2CoGa_8(15.2) Nd_2RhIn_8(10.63)	I_c4/mcm (140.550) $8g$	B $\perp$ [001] $C2'/m'$ (12.62)
301 302	U_2Rh_2Sn(28) U_2Ni_2Sn(25)	P_c4_2/mbc (135.492) $8g$	B $\perp$ [001] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
303	$CeRh_2Si_2$(27)	P_Bcca (54.350) $4g, 4g$	E $\parallel$ [010] + B $\parallel$ [010] + $\sigma \perp$ [010] $P2$ (3.1) E $\parallel$ [010] + B $\parallel$ [010] $Pm'm'2$ (25.60) E $\parallel$ [010] + $\sigma \perp$ [010] P_a2 (3.4)
304	$CeRu_2Al_{10}$(27)	P_Cbcm (57.391) $4c$	B $\perp$ [001] $P2'_1/c'$ (14.79)
305	$Mn_2SeO_3F_2$(26)	$Pn'ma'$ (62.448) $8d$	$\sigma \perp$ [001] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'_1/c'$ (14.79)
306	$Fe_2F_5(H_2O)_2$(26)	$C2'/c'$ (15.89) $4a, 4c$	B in generic direction $P\bar{1}$ (2.4)
307 308	$U_2Rh_3Si_5$(26) $Cu_3Ni_2SbO_6$(22.3)	P_C2/c (13.74) $8f$	B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
309	$CoNb_3S_6$(25)	$P_B2_12_12$ (18.22) $4a$	B $\parallel$ [001] $P22'2'_1$ (17.10)
310 311 312	$Tm_2Mn_2O_7$(25) Ho_2CrSbO_7(13) $Ho_2Ru_2O_7$(1.4)	$I4_1/am'd'$ (141.557) $8c, 8d$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/c'$ (15.89) B $\perp$ [110] $C2'/c'$ (15.89) B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
313 314	NdPt(23) $NaCrGe_2O_6$(6)	$C2'/c'$ (15.89) $4e$	B in generic direction $P\bar{1}$ (2.4)
315	$Tb_2Pd_{2.05}Sn_{0.95}$(20.8)	P_bnma (62.451) $8h$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
316	$ErFeCuGe_4O_{12}$(20)	P_c4/nnc (126.384) $4a, 8e$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
317	Bi_2RuMnO_7(20)	$Fd'd'd$ (70.530) $16c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/c'$ (15.89) $\sigma \perp$ [100] $C2'/c'$ (15.89)
318	$EuFe_2As_2$(19)	P_Cbca (61.439) $4a, 8e$	B $\perp$ [001] $P2'_1/c'$ (14.79) B $\perp$ [010] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
319	$CsMnF_4$(18.9)	$Pmm'n'$ (59.410) $4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'/c'$ (13.69) $\sigma \perp$ [010] $P2'_1/m'$ (11.54)
320	$NpNiGa_5$(18)	$Imm'a'$ (74.559) $4d$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/m'$ (12.62)
321	$U_2Pd_{2.35}Sn_{0.65}$(17)	P_c4/mnc (128.408) $8g$	B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
322 323 324	$CuSe_2O_5$(17) $NH_4FeCl_2(HCOO)$(6) $La_2NiO_3F_{1.93}$(-)	P_C2_1/c (14.84) $4c$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
325 326	U_2Ni_2In(15) U_2Ni_2In(14.0)	P_c4/mnc (128.408) $8h$	B $\perp$ [001] $P2'_1/m'$ (11.54) B $\perp$ [100] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
327 328	$FeSO_4$(14) $NaFePO_4$(13)	P_c2_1/c (14.82) $4c, 4d$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
329	$GdFeZnGe_4O_{12}$(13.8)	P_bnna (52.315) $4c, 8f$	B $\perp$ [001] $P2'/c'$ (13.69) B $\perp$ [010] $P2'/c'$ (13.69) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
330 331	$FeTa_2O_6$(13) $FeTa_2O_6$(8.5)	I_c4_1/a (88.86) $16c$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [100] $C2'/c'$ (15.89) σ in generic direction $P_S\bar{1}$ (2.7)
332 333	$GeNi_2O_4$(12.1) $Cu_3Mg(OD)_6Br_2$(5.4)	C_c2/m (12.63) $4b, 8e$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $C2'/c'$ (15.89) σ in generic direction $P_S\bar{1}$ (2.7)
334	$AgMnVO_4$(12.1)	P_a2_1/m (11.55) $4a, 4c$	σ in generic direction $P_S\bar{1}$ (2.7)
335	Ho_3Ge_4(12)	P_Bnna (52.318) $8f$	B $\perp$ [100] $P2'_1/c'$ (14.79)

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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
336 337	$CoSO_4$(12) $CoSO_4$(-)	$Pnma$ (62.441) $4a$	B in generic direction $P\bar{1}$ (2.4)
338	VPO_4(10.3)	P_cnma (62.452) $8c$	B $\perp$ [010] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'_1/c'$ (14.79)
339	$NdNiMg_{15}$(9)	P_Bcca (54.350) $4g$	E $\parallel$ [010] + B $\parallel$ [010] + $\sigma \perp$ [010] $P2$ (3.1) E $\parallel$ [010] + B $\parallel$ [010] $Pm'm'2$ (25.60) E $\parallel$ [010] + $\sigma \perp$ [010] P_a2 (3.4)
340	$[C(ND_2)_3]Mn(DCOO)_3$(8.73)	$Pn'n'a$ (52.310) $4d$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $P2'_1/c'$ (14.79) $\sigma \perp$ [100] $P2'/c'$ (13.69)
341	$ZrMn_2Ge_4O_{12}$(8)	$P4'/nbm'$ (125.367) $4f$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/m'$ (12.62) B $\perp$ [110] $C2'/m'$ (12.62)
342	Er_3Ge_4(7.3)	P_Cbcm (57.391) $4c, 8f$	B $\perp$ [001] $P2'_1/c'$ (14.79)
343	$Na_2BaFe(VO_4)_2$(7)	$C2'/c'$ (15.89) $4b$	B in generic direction $P\bar{1}$ (2.4)
344	$CeMnCoGe_4O_{12}$(6)	$Pb'an'$ (50.282) $4f, 4f$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'/c'$ (13.69) $\sigma \perp$ [100] $P2'/c'$ (13.69)
345 346	$Ce_2Ni_3Ge_5$(4.8) $Ce_2Ni_3Ge_5$(4.3)	P_Ibcn (60.432) $8j$	B $\perp$ [010] $P2'_1/c'$ (14.79) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
347	$CeCo_2Ge_4O_{12}$(4.5)	P_bnna (52.315) $8f$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [001] $P2'/c'$ (13.69) B $\perp$ [010] $P2'/c'$ (13.69) B $\perp$ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
348	$TlMnF_4$(4.2)	P_C2/c (13.74) $4b$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $P2'_1/c'$ (14.79) σ in generic direction $P_S\bar{1}$ (2.7)
349 350	Na_2NiCrF_7(4) Na_2NiCrF_7(4)	$Im'm'a$ (74.558) $4b, 4c$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [010] $C2'/m'$ (12.62) $\sigma \perp$ [100] $C2'/m'$ (12.62)
351 352	$CuCl(C_4H_4N_2)_2(BF_4)$(3.9) $CuBr(C_4H_4N_2)_2(BF_4)$(3.8)	P_bnna (52.315) $4b$	E $\parallel$ [100] + $\sigma \perp$ [100] P_a2 (3.4)
353	$Fe_4Si_2Sn_7O_{16}$(3.5)	C_c2/m (12.63) $8e$	B in generic direction $P\bar{1}$ (2.4) B $\perp$ [010] $C2'/c'$ (15.89) σ in generic direction $P_S\bar{1}$ (2.7)
354	$ZrCo_2Ge_4O_{12}$(3.5)	$Pb'an'$ (50.282) $4f$	B in generic direction $P\bar{1}$ (2.4) $\sigma \perp$ [001] $P2'/c'$ (13.69) $\sigma \perp$ [100] $P2'/c'$ (13.69)

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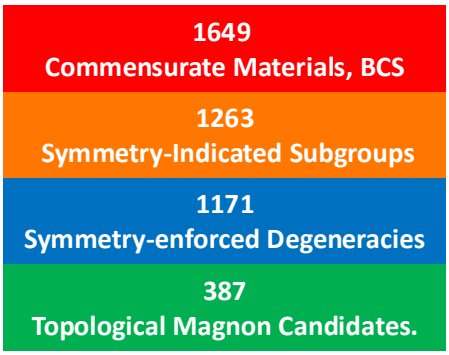
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	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
355	$CsCoCl_3(D_2O)_2$(3.4)	P_bccn (56.372) $8c$	B ⊥ [100] $P2'_1/c'$ (14.79)
356	HoRh(3.2)	$P_Ia\bar{3}$ (205.36) $8b$	B ∥ [110] $P2'/c'$ (13.69) B ⊥ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
357	HoRh(3.2)	P_C2_1/m (11.57) $4e$	B in generic direction $P\bar{1}$ (2.4) B ⊥ [010] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
358	$Yb_2Pd_2(In_{0.4}Sn_{0.6})$(3)	P_c4/mbm (127.396) $8h$	B ⊥ [001] $P2'_1/m'$ (11.54) σ in generic direction $P_S\bar{1}$ (2.7)
359	$CsNiF_3$(2.67)	P_Bnnm (58.402) $4a$	B in generic direction $P\bar{1}$ (2.4) B ⊥ [001] $P2'_1/c'$ (14.79) B ⊥ [010] $P2'_1/m'$ (11.54) B ⊥ [100] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7) E ∥ [001] + B ∥ [001] $Pm'a'2$ (28.91)
360 361 362 363 364 365 366 367	$Dy_3Al_5O_{12}$(2.49) $Tb_3Al_5O_{12}$(1.35) $Ho_3Al_5O_{12}$(0.85) $Er_3Ga_5O_{12}$(0.8) $Dy_3Ga_5O_{12}$(0.37) $Tb_3Ga_5O_{12}$(0.25) $Tb_3Ga_5O_{12}$(0.24) $Ho_3Ga_5O_{12}$(0.15)	$Ia\bar{3}d'$ (230.148) $24c$	B ∥ [100] + σ ∥ [111] $C2'/c'$ (15.89) B ∥ [110] $C2'/c'$ (15.89) B ⊥ [110] $C2'/c'$ (15.89)
368 369	$CdYb_2S_4$(1.92) $CdYb_2Se_4$(1.75)	$I4_1/amd$ (141.551) $8d$	B in generic direction $P\bar{1}$ (2.4)
370	$Na_3Co(CO_3)_2Cl$(1.5)	$Fd\bar{3}$ (203.26) $16c$	B in generic direction $P\bar{1}$ (2.4)
371	$NdPd_5Al_2$(1.3)	P_anma (62.450) $4a$	B ⊥ [010] $P2'_1/c'$ (14.79)
372	$Na_2BaMn(VO_4)_2$(1.3)	P_C2_1/c (14.84) $4d$	B in generic direction $P\bar{1}$ (2.4) B ⊥ [010] $P2'/c'$ (13.69) σ in generic direction $P_S\bar{1}$ (2.7)
373	$Er_2Ti_2O_7$(1.173)	$I4'_1/am'd$ (141.554) $8c$	B in generic direction $P\bar{1}$ (2.4) B ∥ [100] $C2'/m'$ (12.62) B ⊥ [100] $C2'/m'$ (12.62)
374 375 376	$Gd_2Sn_2O_7$(1) $Er_2Pt_2O_7$(0.38) $Er_2Sn_2O_7$(0.1)	$I4'_1/amd'$ (141.555) $8d$	B in generic direction $P\bar{1}$ (2.4) B ∥ [110] $C2'/c'$ (15.89) B ⊥ [110] $C2'/c'$ (15.89)
377	$Gd_2Ti_2O_7$(0.97)	$R_I\bar{3}m$ (166.102) $18e$	B in generic direction $P\bar{1}$ (2.4) B ⊥ [110] $C2'/c'$ (15.89) B ⊥ [100] $C2'/c'$ (15.89) σ in generic direction $P_S\bar{1}$ (2.7)

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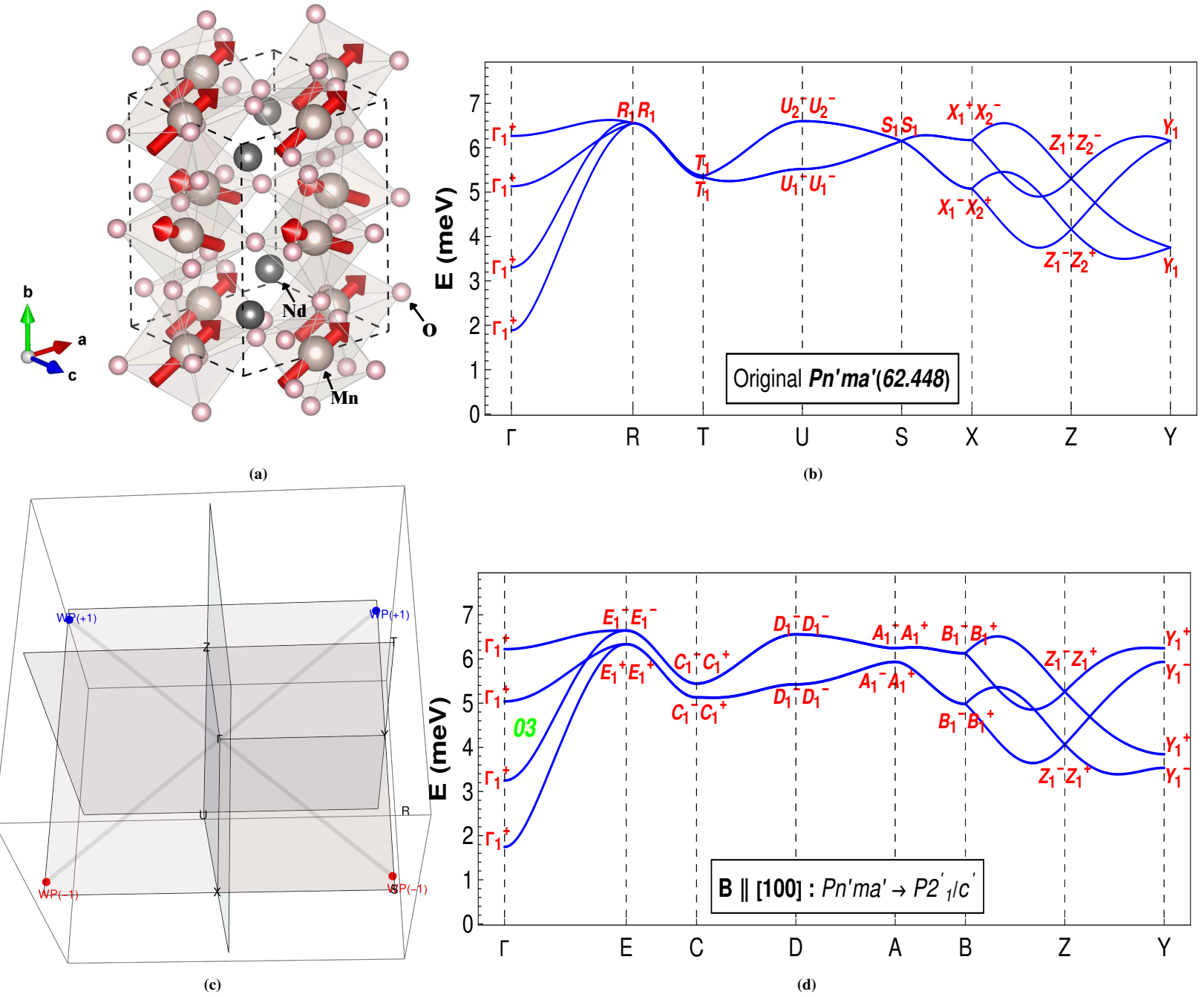
	Material(T_c[K])	MSG & WPs	Perturbation & Subgroup
378	$Nd_2Sn_2O_7$(0.91)		B in generic direction $P\bar{1}$ (2.4)
379	$Nd_2Hf_2O_7$(0.55)	$Fd\bar{3}m'$	B $\parallel$ [100] + σ $\parallel$ [111]
380	$Nd_2Zr_2O_7$(0.4)	(227.131)	$C2'/m'$ (12.62)
381	Nd_2ScNbO_7(0.371)	$16d$	B $\parallel$ [110] $C2'/m'$ (12.62)
382	$Sm_2Ti_2O_7$(0.35)		B $\perp$ [110] $C2'/m'$ (12.62)
383	$YbCo_2Si_2$(0.9)	I_cbca (73.553) $8g$	E $\parallel$ [001] + σ $\perp$ [001] C_a2 (5.17)
384	$Tb_2Sn_2O_7$(0.87)	$I4_1/am'd'$	B in generic direction $P\bar{1}$ (2.4) B $\parallel$ [110] $C2'/c'$ (15.89)
385	$Yb_2Ti_2O_7$(0.26)	(141.557)	B $\perp$ [110] $C2'/c'$ (15.89)
386	$Yb_2Sn_2O_7$(0.15)	$8d$	B $\parallel$ [100] $C2'/m'$ (12.62) B $\perp$ [100] $C2'/m'$ (12.62)
387	$Ca_2Fe_{0.875}Cr_{0.125}GaO_5$(-)	$Pn'm'a$ (62.446) $4a, 4a, 4c$	σ $\perp$ [010] $P2_1'/m'$ (11.54) σ $\perp$ [100] $P2_1'/c'$ (14.79)



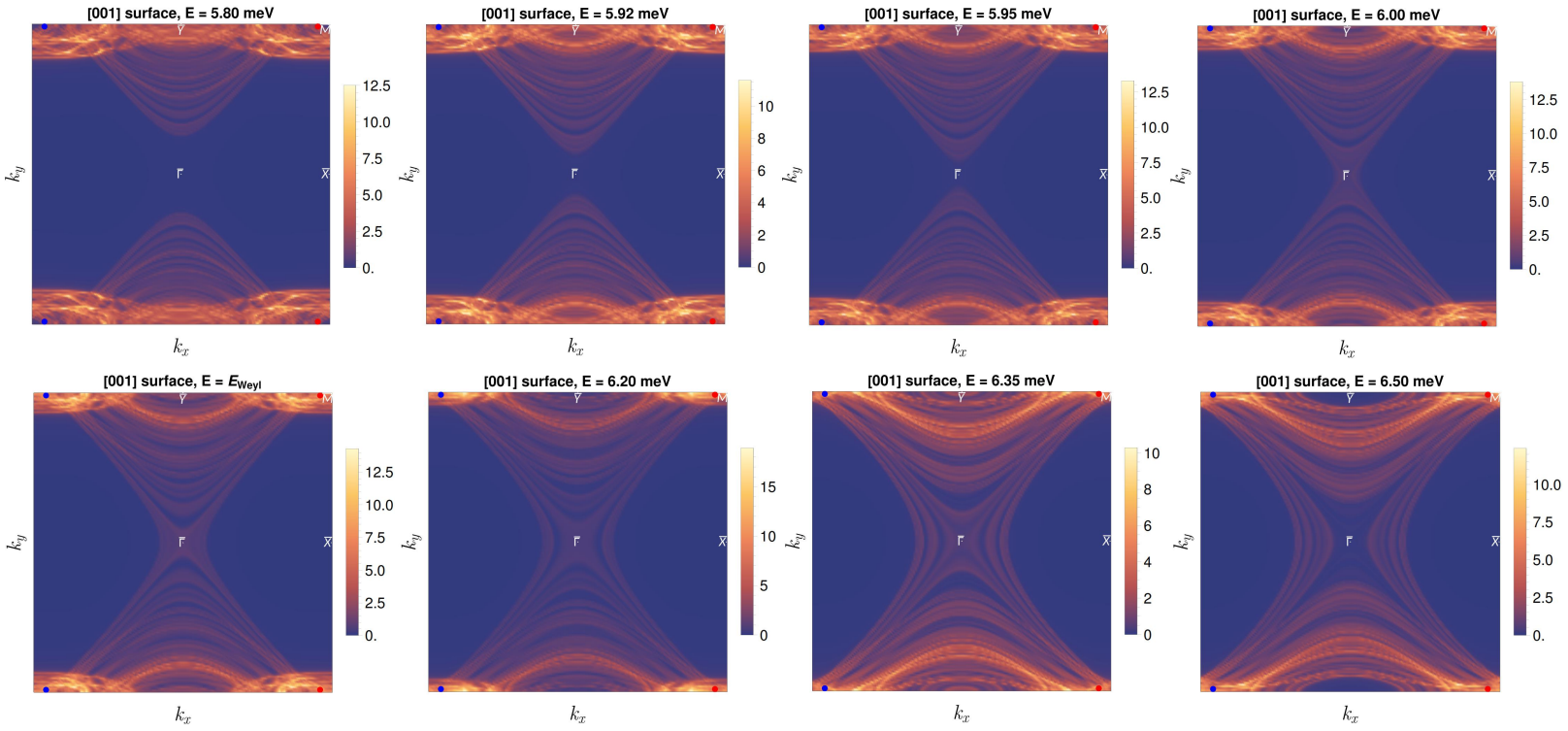
Supplementary Figure 1: Summary of the search program. Out of the 1649 commensurate magnetic materials in the BCS database, 387 admit perturbation-induced type-I topological magnons. By type-I[1], we refer to those which admit the symmetry indicated topological magnons regardless the form of the perturbation as long as it breaks the $\mathcal{MSG}$ into the relevant magnetic subgroup. A first filter excludes 386 materials that belong to $\mathcal{MSG}s$ without symmetry-indicated subgroups. A second filter excludes 92 materials that do not host symmetry-enforced degeneracies in their magnon spectrum as a band gap cannot be induced by perturbations at all high symmetry momenta. This results in 1171 materials that, after passing through our search algorithm (Fig.2 in main text), ended up with 387 successful candidates.

2 Linear Spin Wave Calculations

Here we give more details about the LSW calculations whose results were presented in the main text, and include related Supplementary Figures [2-6].



Supplementary Figure 2: Topological magnon band structure of $NdMnO_3$. a) **Magnetic unit cell of $NdMnO_3$**. Light pink spheres indicate the Oxygen sites which form corner-sharing networks around the Mn rose spheres while black are Nd sites. Size of Nd spheres is reduced relative to those of Mn for a better visualization of the magnetic order. Red arrows indicate the magnetic moment direction of each of the four Mn magnetic sublattices. Spins mostly align along the a-axis with a small canting towards the b-axis. Dashed lines indicate the boundaries of the magnetic unit cell with the inset highlighting a,b,c axes. b) **Unperturbed magnon spectrum for $NdMnO_3$**. Red labels are the magnon band representations at high symmetry points ($\mathcal{HSP}s$). Band degeneracies are consistent with the dimensionality of the induced band representation discussed in the text which enforces four-fold degeneracy at $\mathcal{R}$,$\mathcal{S}$ and two-fold at all other $\mathcal{HSP}s$ except for the Γ point that has four separate bands (i.e. four 1D *irreps*). c) **Diagnosed Weyl magnons in perturbed $NdMnO_3$** with blue and red labels indicate locations of Weyl points (labeled WP) of positive and negative charge respectively. Each Weyl point on the graph is connected to its inversion symmetric partner as indicated by the dark lines. Due to the symmetries discussed in the main text, for each Weyl point $(k_x, k_y, 0)$ there is a Weyl point of the same charge located at $(k_x, -k_y, 0)$, in addition to an opposite charge located at $(-k_x, -k_y, 0)$ related by inversion, giving rise to a pair of magnon arcs between projections of Weyl points as shown in Supplementary Figure 3. Solid green and black lines in the picture connect inversion-related weyl partners. d) **Perturbed magnon spectrum for $NdMnO_3$.** By breaking the $\mathcal{MSG}$ into the $P2_1'/c'$ subgroup through applying a magnetic field parallel to the a axis, the four-fold degeneracies at R, S points break into two two-folds, and therefore a band gap is opened at all of $\mathcal{HSP}s$. Calculated value of the band gap $\mathcal{SI}$ is labeled in green. Red labels are the magnon representations at the $\mathcal{HSP}s$.



Supplementary Figure 3: Formation of Magnon Arcs in $NdMnO_3$. Evolution of the surface spectral density on the (001) surface across different isoenergy cuts near the Weyl node energy. The intensity maps show the momentum-resolved surface density of states in the $k_x - k_y$ plane at various energies from 5.80 meV to 6.50 meV. The Weyl node energy corresponds to $E \simeq 6.07$ meV (bottom left). At this energy, distinct arc-like surface states emerge, connecting projected Weyl nodes (marked in red, with -1 chirality, and blue, with $+1$ chirality) close to the BZ corners. The two magnon arcs intersect to form an X-shaped connectivity pattern, which disappears when the iso-energy cut is taken away from the Weyl node energy, confirming their topological origin.

Spin wave spectrum of $NdMnO_3$

The crystal structure of $NdMnO_3$ belongs to the space group *Pnma* (62.448) which has four Mn sublattices at the $\mathcal{WP}$ 4b:

$$\begin{aligned} r_A &= (0, 0, c/2) \\ r_B &= (a/2, 0, 0) \\ r_C &= (0, b/2, c/2) \\ r_D &= (a/2, b/2, 0) \end{aligned} \tag{1}$$

where Cartesian coordinates are used. In the magnetic phase below $T_N \sim 78$ K, the Mn atoms become magnetized giving rise to the magnetic order[2]:

$$\begin{aligned} \langle \boldsymbol{S}_{A,B} \rangle &\propto (0.933, -0.359, 0) \\ \langle \boldsymbol{S}_{C,D} \rangle &\propto (-0.933, -0.359, 0). \end{aligned} \tag{2}$$

We build our spin model based on the interaction bonds defined between pairs of sites as following:

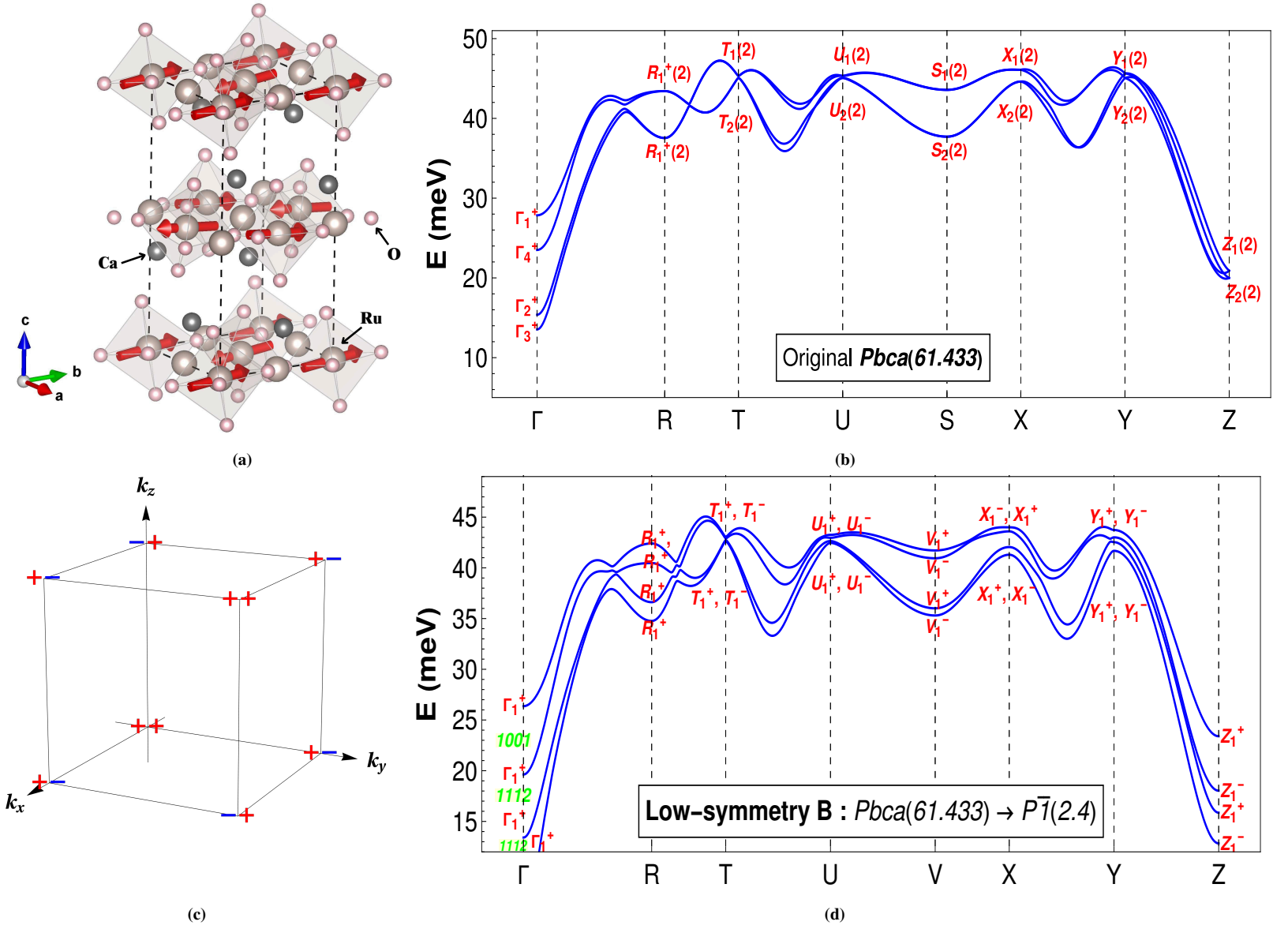
$$\begin{aligned} \mathbf{J_1} : r_B &\iff r_D \\ \mathbf{J_2} : r_B &\iff r_A \\ \mathbf{J_3} : r_B &\iff r_B + (0, 0, c) \\ \mathbf{J_4} : r_B &\iff r_B + (a/2, b/2, c/2) \\ \mathbf{J_5} : r_B &\iff r_B + (a, 0, 0) \end{aligned} \tag{3}$$

where $\mathbf{J_1}$ is constrained by $\{m_{010}|0, \frac{1}{2}, 0\}$ and therefore have the anti-symmetric form:

$$\mathbf{J_1} = \begin{pmatrix} J_1^{xx} & D_1^z & \Gamma_1^{xz} \\ -D_1^z & J_1^{yy} & D_1^x \\ \Gamma_1^{xz} & -D_1^x & J_1^{zz} \end{pmatrix} \tag{4}$$

while $\mathbf{J_2}$ is not symmetry-constrained and therefore take the general form:

$$\mathbf{J_2} = \begin{pmatrix} J_2^{xx} & D_2^z + \Gamma_2^{xy} & -D_2^y + \Gamma_2^{xz} \\ -D_2^z + \Gamma_2^{xy} & J_2^{yy} & D_2^x + \Gamma_2^{yz} \\ D_2^y + \Gamma_2^{xz} & -D_2^x + \Gamma_2^{yz} & J_2^{zz} \end{pmatrix} \tag{5}$$



Supplementary Figure 4: Topological magnon band structure of Ca_2RuO_4. a) **Magnetic unit cell of Ca_2RuO_4.** Light pink spheres indicate the Oxygen sites which form corner-sharing networks around the Ru rose spheres while black are Calcium sites. Size of Ca spheres is reduced relative to those of Ru for a better vision of the magnetic order. Red arrows indicate the magnetic moment direction of each of the four Ru magnetic sublattices. Spins mostly align along the b-axis with a small canting towards the c-axis. Dashed lines indicate the boundaries of the magnetic unit cell with the a,b,c axes are highlighted in the inset. b) **Unperturbed magnon spectrum for Ca_2RuO_4.** Red labels are the magnon band representations at $\mathcal{HSP}s$. Band degeneracies are consistent with the dimensionality of the induced band representation of this magnetic structure. c) **Inversion $\mathcal{I}$ eigenvalues** of the upper two bands at the eight $\mathcal{HSP}s$ of the Brillouin zone. d) **Perturbed magnon spectrum for Ca_2RuO_4.** By breaking the $\mathcal{MSG}$ into the $P\bar{1}$ subgroup through applying a magnetic field in a low-symmetry direction, the two-fold degeneracies at all $\mathcal{HSP}s$ break, and therefore three band gaps are opened at all $\mathcal{HSP}s$. Calculated values of the band gaps $\mathcal{SI}$ are labeled in green. Red labels are the magnon representations at the $\mathcal{HSP}s$. Since at some points the band gap is small, we write the band *irreps* of two bands with a (,) in between to be understood that the first label is for the lower energy band.

$\mathbf{J_3}$ is constrained by inversion $\{-1|0\}$ and therefore take the general symmetric form:

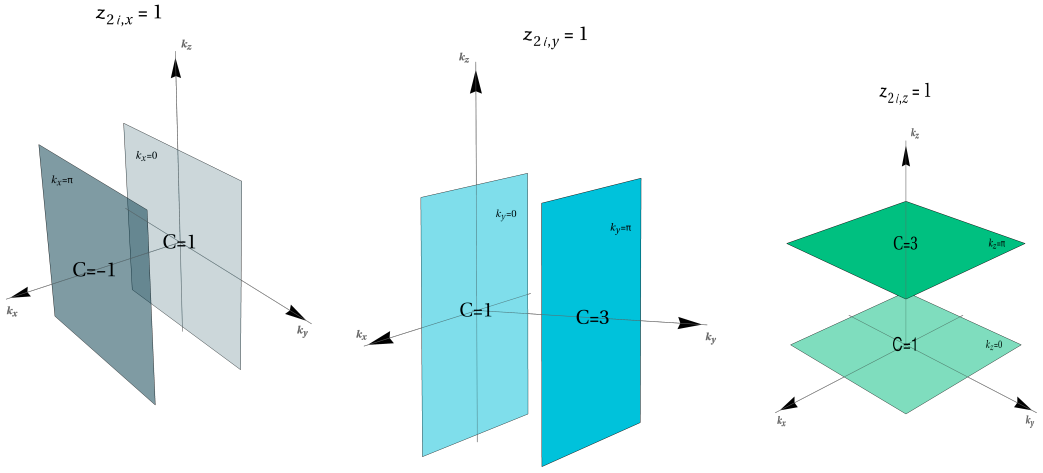
$$\mathbf{J_3} = \begin{pmatrix} J_3^{xx} & \Gamma_3^{xy} & \Gamma_3^{xz} \\ \Gamma_3^{xy} & J_3^{yy} & \Gamma_3^{yz} \\ \Gamma_3^{xz} & \Gamma_3^{yz} & J_3^{zz} \end{pmatrix} \tag{6}$$

and $\mathbf{J_4}$ is not symmetry-constrained, so it takes the general form:

$$\mathbf{J_4} = \begin{pmatrix} J_4^{xx} & D_4^z + \Gamma_4^{xy} & -D_4^y + \Gamma_4^{xz} \\ -D_4^z + \Gamma_4^{xy} & J_4^{yy} & D_4^x + \Gamma_4^{yz} \\ D_4^y + \Gamma_4^{xz} & -D_4^x + \Gamma_4^{yz} & J_4^{zz} \end{pmatrix} \tag{7}$$

and $\mathbf{J}_5$ is constrained by inversion $\{-1|0\}$ and therefore take the general symmetric form:

$$\mathbf{J_5} = \begin{pmatrix} J_5^{xx} & \Gamma_5^{xy} & \Gamma_5^{xz} \\ \Gamma_5^{xy} & J_5^{yy} & \Gamma_5^{yz} \\ \Gamma_5^{xz} & \Gamma_5^{yz} & J_5^{zz} \end{pmatrix} \tag{8}$$



(a) **(b)** **(c)**

Supplementary Figure 5: Chern numbers on high-symmetry planes in perturbed Ca_2RuO_4. a) Inversion $\mathcal{I}$ eigenvalues of the upper two bands at the eight $\mathcal{HSP}s$ of the Brillouin zone. b-d) We check consistency between the $\mathcal{SI}$ value of the second magnon band gap enforced by symmetries and the calculated Chern numbers over the $k_i = 0, \pi$ planes of the BZ using our spin model. The $\mathcal{Z}_4 = 2$ value reflects the difference between the Chern number over the $k_z = 0$ and $k_z = \pi$ planes while the $\mathcal{Z}_{2i,x_i} = 1$ $\mathcal{SI}$ value reflects the Chern number (*modulo 2*) over the $k_{x_i} = 0, \pi$ planes.

and we include an extra single-ion anisotropy term of the form $-\sum_i A^{xx}(S_i^x)^2$ that helps in stabilizing the ground state along the experimentally-reported directions. We perform the the linear spinwave expansion as in Eq. 15 in main text using the Heisenberg exchange values $J_1 = 1$ meV, $J_2 = -0.3$ meV, $J_3 = -0.1$ meV, $J_4 = 0.05$ meV, $J_5 = -0.3$ meV and supplement them with a Dzyaloshinskii–Moriya interaction (DMI) on the first nearest neighbor (NN) bond $D_1^z = 1.1$ meV and an anisotropy $A^{xx} = 0.4$ meV. we reach the magnon Hamiltonian $\mathbf{R}(\boldsymbol{k})$ in Eq. 15 in main text which we use to diagonalize $\mathbf{M} \cdot \mathbf{R}(\boldsymbol{k})$ and find the magnon spectrum as shown in Fig. 2b and the magnon wavefunctions which we use to calculate the *irreps* of magnon bands.

Topology-Related Details of $NdMnO_3$

To understand the field-induced topology discussed in main text, we need to look at the decomposition of irreps at the $\mathcal{HSP}s$. To see this we firstly note that the standard axes and origin of the magnetic subgroup are related to those of the original $\mathcal{MSG}$ (unmarked) through the transformation:

$$\begin{aligned} a &\to a' = -b \\ b &\to b' = -c \\ c &\to c' = a \\ o &\to o' = o \end{aligned} \tag{9}$$

Then, the *irreps* decomposition proceeds as following. At $A\left(\frac{1}{2}, 0, \frac{1}{2}\right)$, equivalent to $S\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ in the original $\mathcal{MSG}$,

$$S_1 S_1(4) \to 2A_1^- A_1^+(2) \tag{10}$$

At $B\left(0, 0, \frac{1}{2}\right)$, equivalent to $X\left(\frac{1}{2}, 0, 0\right)$ in the original $\mathcal{MSG}$,

$$\begin{aligned} X_1^+ X_2^-(2) &\to B_1^- B_1^+(2) \\ X_1^- X_2^+(2) &\to B_1^- B_1^+(2) \end{aligned} \tag{11}$$

At $C\left(\frac{1}{2}, \frac{1}{2}, 0\right)$, equivalent to $T\left(0, \frac{1}{2}, \frac{1}{2}\right)$ in the original $\mathcal{MSG}$,

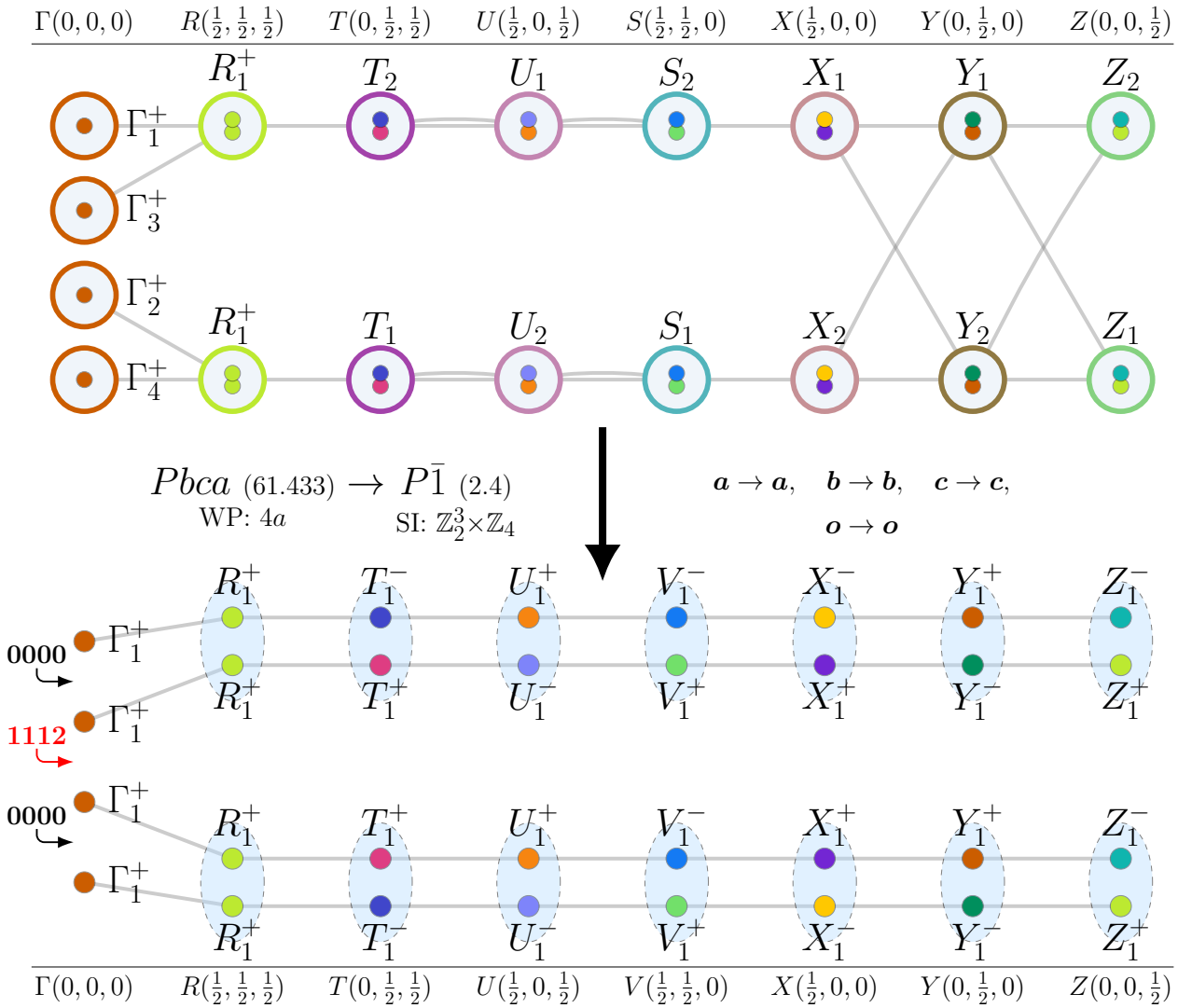
$$T_1(2) \to C_1^- C_1^+(2) \tag{12}$$

At $D\left(0, \frac{1}{2}, \frac{1}{2}\right)$, equivalent to $U\left(\frac{1}{2}, 0, \frac{1}{2}\right)$ in the original $\mathcal{MSG}$,

$$\begin{aligned} U_1^+ U_1^+(2) &\to D_1^+ D_1^+(2) \\ U_1^- U_1^-(2) &\to D_1^- D_1^-(2) \\ U_2^+ U_2^+(2) &\to D_1^+ D_1^+(2) \\ U_2^- U_2^-(2) &\to D_1^- D_1^-(2) \end{aligned} \tag{13}$$

At $E\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, equivalent to $R\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ in the original $\mathcal{MSG}$,

$$R_1 R_1(4) \to E_1^+ E_1^+(2) \oplus E_1^- E_1^-(2) \tag{14}$$



Supplementary Figure 6: Schematic representation of the magnon band structure in Ca_2RuO_4. Upon the symmetry breaking discussed in the main text, the resulting order of the *irreps* at the $\mathcal{HSP}s$ is not unique. This is one possibility the band structure might fall into. Unlike the constructed spin model that produced a 1112 and 1001 $\mathcal{SI}s$ for the first and third gaps, a different spin model can give rise to a different possibility like the one presented here in which the $\mathcal{SI}$ is 0000. Regardless of the spin Hamiltonian details, the $\mathcal{SI}$ of the middle band gap belongs to the same topological class.

At $\Gamma(0,0,0)$,

$$\begin{aligned}\Gamma_1^+(1) &\to \Gamma_1^+(1)\\ \Gamma_1^-(1) &\to \Gamma_1^-(1)\\ \Gamma_2^+(1) &\to \Gamma_1^+(1)\\ \Gamma_2^-(1) &\to \Gamma_1^-(1)\end{aligned} \tag{15}$$

At $Y\left(\frac{1}{2},0,0\right)$, equivalent to $Y\left(0,\frac{1}{2},0\right)$ in the original $\mathcal{MSG}$,

$$Y_1(2) \to Y_1^+(1) \oplus Y_1^-(1) \tag{16}$$

Finally, at $Z\left(0,\frac{1}{2},0\right)$, equivalent to $Z\left(0,0,\frac{1}{2}\right)$ in the original $\mathcal{MSG}$,

$$\begin{aligned}Z_1^+Z_2^-(2) &\to Z_1^-Z_1^+(2)\\ Z_1^-Z_2^+(2) &\to Z_1^-Z_1^+(2)\end{aligned} \tag{17}$$

Spinwave spectrum of Ca_2RuO_4

The crystal structure of Ca_2RuO_4 belongs to the space group *Pbca* (61.433) which has four Ru sublattices at the 4a $\mathcal{WP}$:

$$\begin{aligned} r_A &= (0,0,0) \\ r_B &= (a/2, b/2, 0) \\ r_C &= (0, b/2, c/2) \\ r_D &= (a/2, 0, c/2) \end{aligned} \tag{18}$$

where Cartesian coordinates are used. In the magnetic phase below $T_N \sim 110$ K, the Ru atoms become magnetized giving rise to the magnetic order[3]:

$$\begin{aligned} \langle \boldsymbol{S}_A \rangle &\propto (0, 1, 0.1) \\ \langle \boldsymbol{S}_B \rangle &\propto (0, -1, -0.1). \\ \langle \boldsymbol{S}_C \rangle &\propto (0, 1, -0.1). \\ \langle \boldsymbol{S}_D \rangle &\propto (0, -1, 0.1). \end{aligned} \tag{19}$$

We build our spin model based on the interaction bonds defined between pairs of sites as following:

$$\begin{aligned} \mathbf{J_1} &: r_A \iff r_B \\ \mathbf{J_2} &: r_A \iff r_A + (a,0,0) \\ \mathbf{J_3} &: r_A \iff r_A + (0,b,0) \\ \mathbf{J_4} &: r_A \iff r_B + (a,0,0) \\ \mathbf{J_5} &: r_A \iff r_A + (a,b,0) \\ \mathbf{J_6} &: r_A \iff r_D \\ \mathbf{J_7} &: r_A \iff r_A + (0, b/2, c/2) \\ \mathbf{J_8} &: r_A \iff r_A + (a/2, b, c/2) \\ \mathbf{J_9} &: r_A \iff r_A + (0,0,C) \\ \mathbf{J_{10}} &: r_A \iff r_A + (a, b/2, c/2) \\ \mathbf{J_{11}} &: r_A \iff r_A + (a,b,0) \end{aligned} \tag{20}$$

where Inversion $\{-1|0\}$ constrain $\mathbf{J}_i$ where $i = 2, 3, 5, 9$ and therefore they take the general symmetric form:

$$\mathbf{J_i} = \begin{pmatrix} J_i^{xx} & \Gamma_i^{xy} & \Gamma_i^{xz} \\ \Gamma_i^{xy} & J_i^{yy} & \Gamma_i^{yz} \\ \Gamma_i^{xz} & \Gamma_i^{yz} & J_i^{zz} \end{pmatrix} \tag{21}$$

and the rest of bonds are not symmetry-constrained and therefore take the generic form:

$$\mathbf{J_k} = \begin{pmatrix} J_k^{xx} & D_k^z + \Gamma_k^{xy} & -D_k^y + \Gamma_k^{xz} \\ -D_k^z + \Gamma_k^{xy} & J_k^{yy} & D_k^x + \Gamma_k^{yz} \\ D_k^y + \Gamma_k^{xz} & -D_k^x + \Gamma_k^{yz} & J_k^{zz} \end{pmatrix} \tag{22}$$

with $k = 1, 4, 5, 6, 7, 10$, and we include an extra single-ion anisotropy term of the form $-\sum_i A^{yy}(S_i^y)^2$ that helps in stabilizing the ground state along the experimentally-reported directions[4]. We perform the linear spinwave calculation using the interaction parameters of the Heisenberg type to the first three NNs[4] $J_1 = 8$ meV, $J_2 = 0.7$ meV, $J_3 = 0.7$ meV and we set $S = 0.67$ similar to[5 4]. For further couplings, we allow for off-diagonal exchange interactions to remove accidental degeneracies in the magnon spectrum and their used values are $J_4^{xx} = 0.2$ meV, $J_4^{xz} = 0.2$ meV, $J_4^{yz} = 0.09$ meV, $J_4^{zx} = 0.2$ meV, $J_4^{zy} = 0.06$ meV, $J_4^{yy} = 0.0002$ meV, $J_5^{xx} = J_5^{yy} = J_5^{zz} = -0.1$ meV, $J_5^{xz} = -0.6$ meV, $J_5^{yz} = 0.8$ meV, $J_6^{xx} = J_6^{yy} = J_6^{zz} = 0.03$ meV, $J_7^{xx} = J_7^{zz} = -0.25$ meV, $J_7^{yy} = -0.5$ meV, $J_8^{xx} = 0.01$ meV, $J_8^{xy} = 0.5$ meV, $J_8^{yy} = J_8^{zz} = 0.1$ meV, $J_8^{zy} = 0.2$ meV, $J_9^{xx} = J_9^{yy} = J_9^{zz} = -0.02$ meV, $J_9^{yz} = -0.5$ meV, $J_{10}^{xx} = J_{10}^{yy} = -0.02$ meV, $J_{10}^{zz} = -0.2$ meV, $J_{10}^{xy} = 0.5$ meV, $J_{10}^{yz} = -0.02$ meV and anisotropy $A^{yy} = 0.735$ meV. Based on this spin model, the magnon Hamiltonian $\mathbf{R}(\boldsymbol{k})$ is calculated and the magnon spectrum and band *irreps* are shown in Supplementary Figure 4b.

(k_x, k_y, k_z)	Topological Charge
(1.62561, -0.46326, 3.05048)	+1
(-1.60916, 0.45319, 2.99134)	−1
(-0.80420, -0.22994, 2.59234)	−1
(-0.34714, -0.09112, 2.35056)	+1
(1.78258, -0.36203, 2.32929)	−1
(2.10685, -1.06362, 2.15571)	+1
(2.31297, 1.63148, 2.13981)	+1
(0.09406, -0.89653, 2.00765)	−1
(0.08200, -0.91530, 2.00562)	−1
(-0.01273, -3.07374, 1.99766)	−1
(-0.00477, 1.79253, 1.99405)	−1
(-2.58046, 0.10317, 1.82548)	−1
(-2.18417, 2.94025, 1.21087)	+1
(-2.02496, 3.03443, 0.93751)	+1
(2.02496, -3.03443, -0.93751)	−1
(2.18417, -2.94025, -1.21087)	−1
(2.58046, -0.10317, -1.82548)	+1
(0.00477, -1.79253, -1.99405)	+1
(0.01273, 3.07374, -1.99766)	+1
(-0.08200, 0.91530, -2.00562)	+1
(-0.09406, 0.89653, -2.00765)	+1
(-2.31297, -1.63148, -2.13981)	−1
(-2.10685, 1.06362, -2.15571)	−1
(-1.78258, 0.36203, -2.32929)	+1
(0.34714, 0.09112, -2.35056)	−1
(0.80420, 0.22994, -2.59234)	+1
(1.60916, -0.45319, -2.99134)	+1
(-1.62561, 0.46326, -3.05048)	−1

Supplementary Table 2: Explicit locations of Weyl Points between bands 2 and 3 in the perturbed LSW spectrum of Ca_2RuO_4 and their charges. Calculation is based on the spin model given above in the previous subsection 2.

3 MSG P_a2_1/m (11.55)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2_1/m$ (11.50).

Trivial-SI Subgroups: Pc' (7.26), $P2_1'$ (4.9), P_S1 (1.3), Pm (6.18), P_am (6.21), $P2_1$ (4.7), P_a2_1 (4.10).

3.1 WP: $4a$

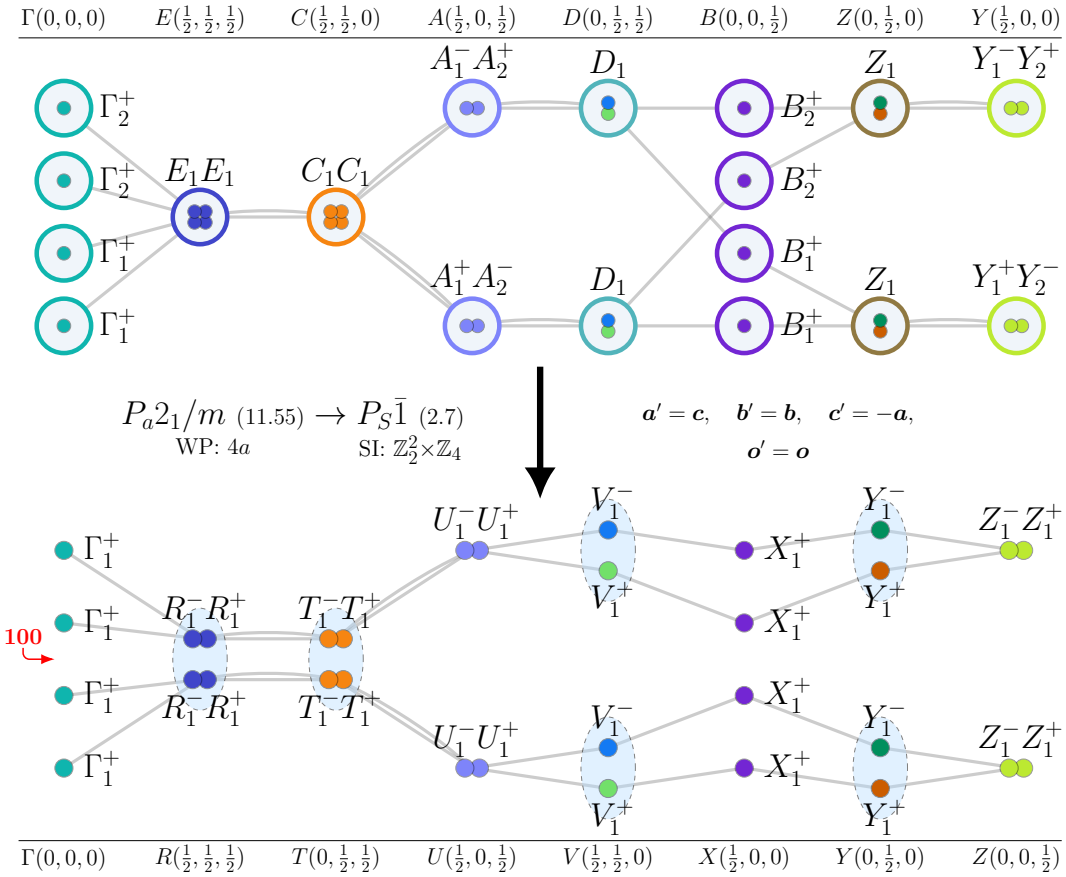
BCS Materials: La_3OsO_7 (45 K)[1], La_3OsO_7 (45 K)[2], $La_{2.8}Ca_{0.2}OsO_7$ (33 K)[3].

3.1.1 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.

[1]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.571
[2]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.570
[3]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.572



Supplementary Figure 7: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_a2_1/m (11.55).

Supplementary Table 3: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/m (11.55), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

Supplementary Table 4: Possible SI values in each gap. Supergroup: P_a2_1/m (11.55), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

3.2 WP: $4a + 4c$

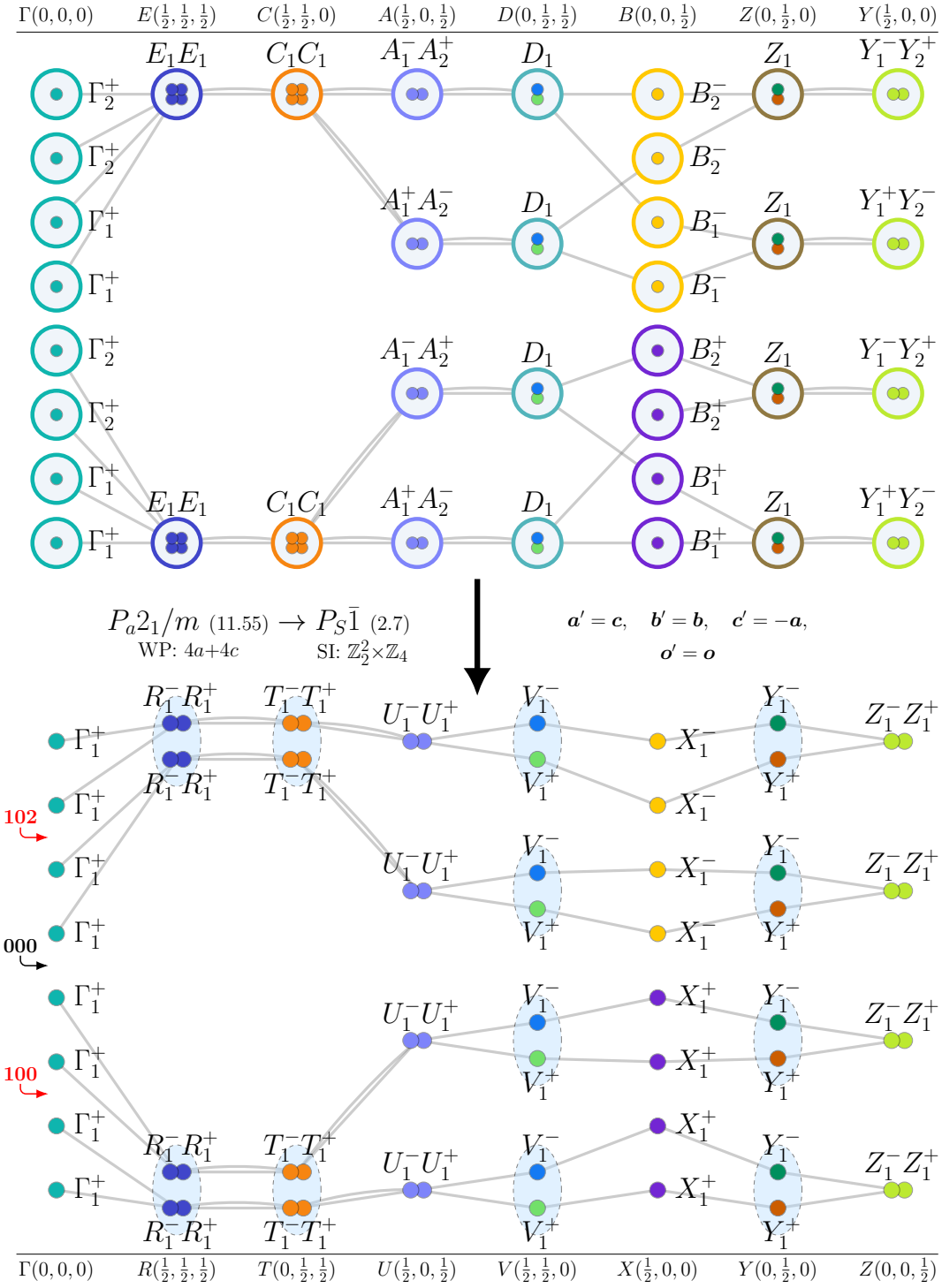
BCS Materials: $AgMnVO_4$ (12.1 K)[4].

[4]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.116`

3.2.1 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 8: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4a + 4c$ of supergroup P_a2_1/m (11.55).

Supplementary Table 5: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/m (11.55), WP: $4a$+$4c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1

continued on next page...

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SI	Possible #Gaps
011	0, 1
013	0, 1
100	0, 1, 2
102	0, 1, 2
111	0, 1
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 6: Possible SI values in each gap. Supergroup: P_a2_1/m (11.55), WP: $4a$+$4c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	100, 102, 111
3	G
4	000, 002, 011, 013
5	G
6	100, 102, 113
7	G
8	000

4 MSG P_C2_1/m (11.57)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2_1/m$ (11.50).

Trivial-SI Subgroups: Pc' (7.26), $P2'$ (3.3), P_S1 (1.3), Pm (6.18), P_Cm (6.23), $P2_1$ (4.7), P_C2_1 (4.12).

4.1 WP: $4e$

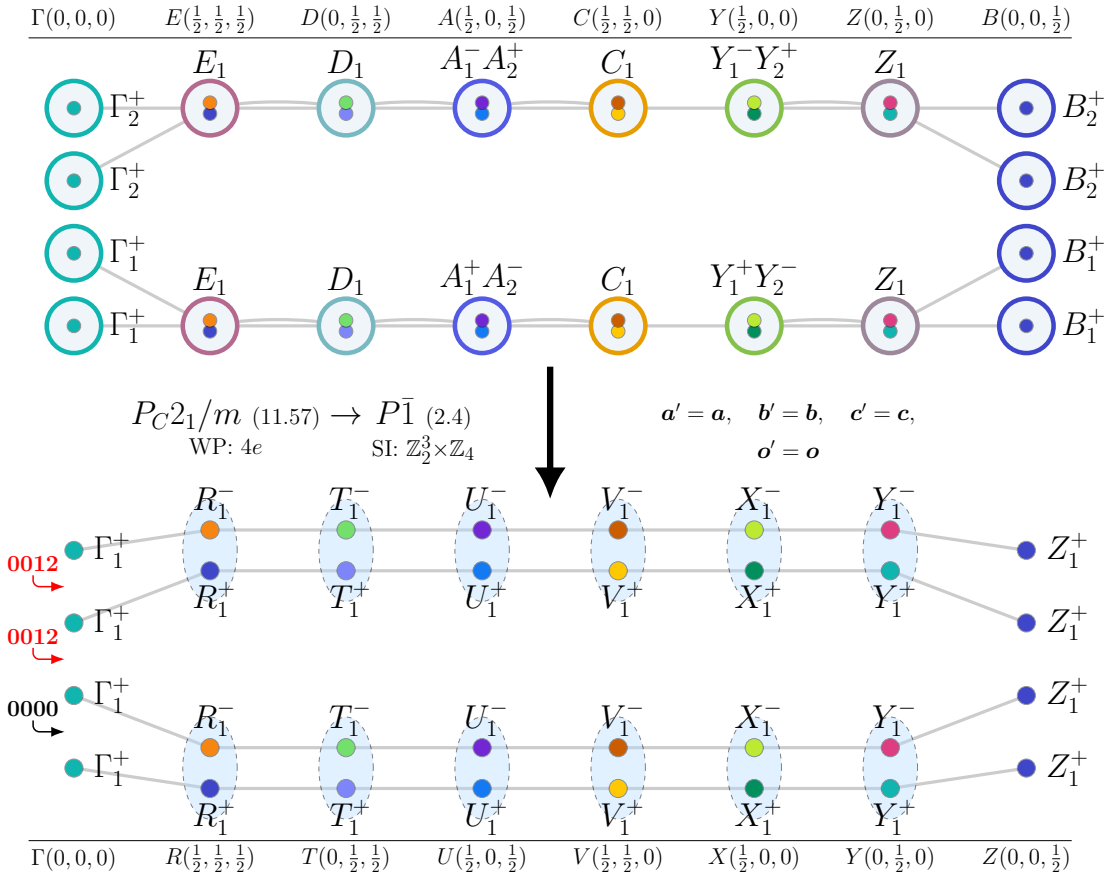
BCS Materials: HoRh (3.2 K)[5].

4.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[5]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.71`



Supplementary Figure 9: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4e$ of supergroup P_C2_1/m (11.57).

Supplementary Table 7: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/m (11.57), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0011	0, 1, 2
0012	1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1
1102	0, 1, 2
1103	0, 1

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SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

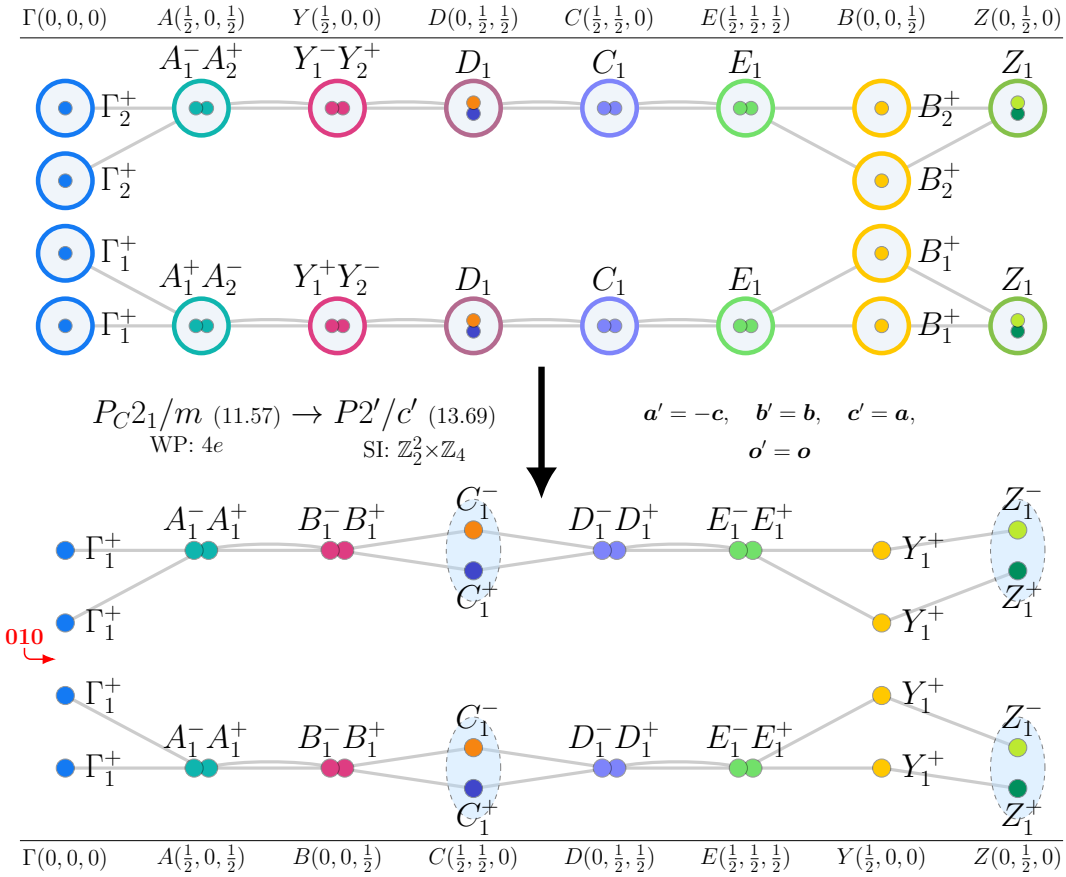
Supplementary Table 8: Possible SI values in each gap. Supergroup: $P_C 2_1/m$ (11.57), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1112, 1113
2	0012
3	0000, 0001, 0003, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1111, 1112
4	0000

4.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 10: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4e$ of supergroup P_C2_1/m (11.57).

Supplementary Table 9: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/m (11.57), WP: $4e$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
010	1
gapless	2
nontrivial	1

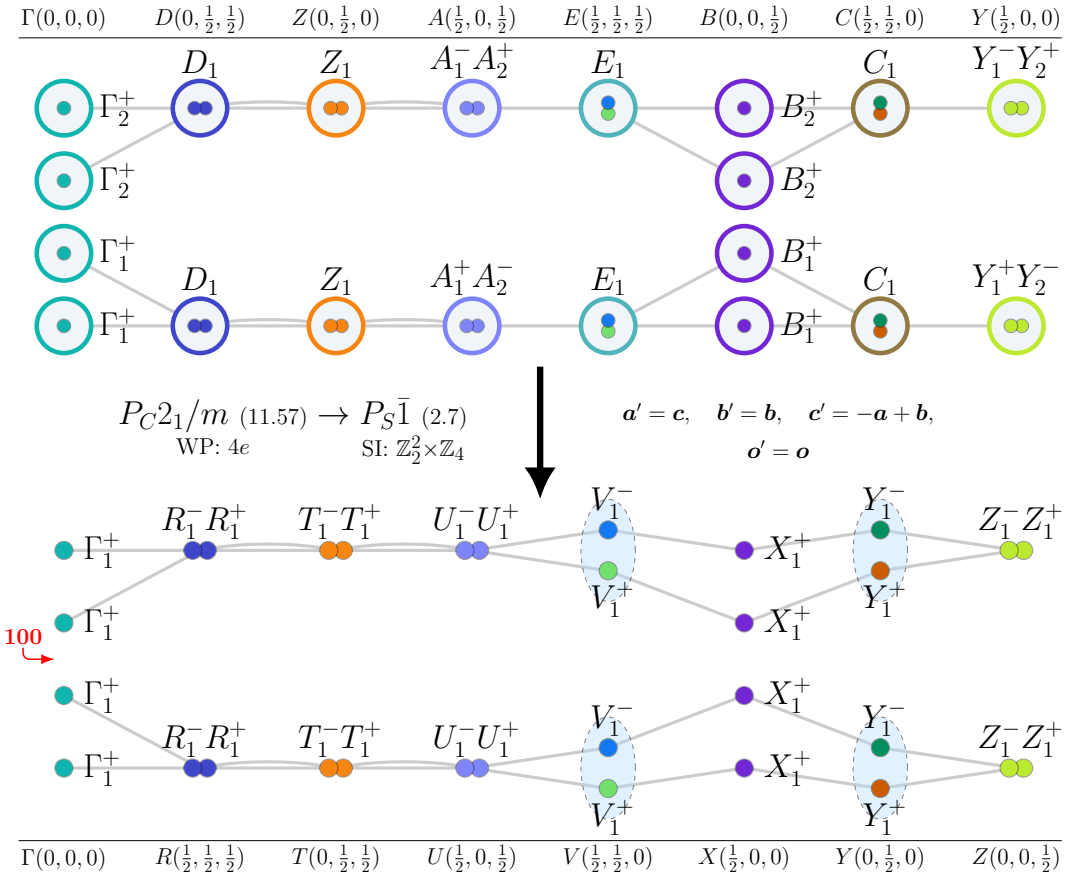
Supplementary Table 10: Possible SI values in each gap. Supergroup: P_C2_1/m (11.57), WP: $4e$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	010
3	G
4	000

4.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 11: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4e$ of supergroup P_C2_1/m (11.57).

Supplementary Table 11: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/m (11.57), WP: $4e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

Supplementary Table 12: Possible SI values in each gap. Supergroup: P_C2_1/m (11.57), WP: $4e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

5 MSG $P4'/nbm'$ (125.367)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P2$ (3.1), $Cm'm'2$ (35.168), $P2/c$ (13.65), $Cm'm'a$ (67.505), $P2$ (3.1), $P2/c$ (13.65), $Pban$ (50.277).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), $C2'$ (5.15), Pc (7.24), $Abm'2'$ (39.198), Pc (7.24), $Pba2$ (32.135), $P4'bm'$ (100.174), $Pnc2$ (30.111).

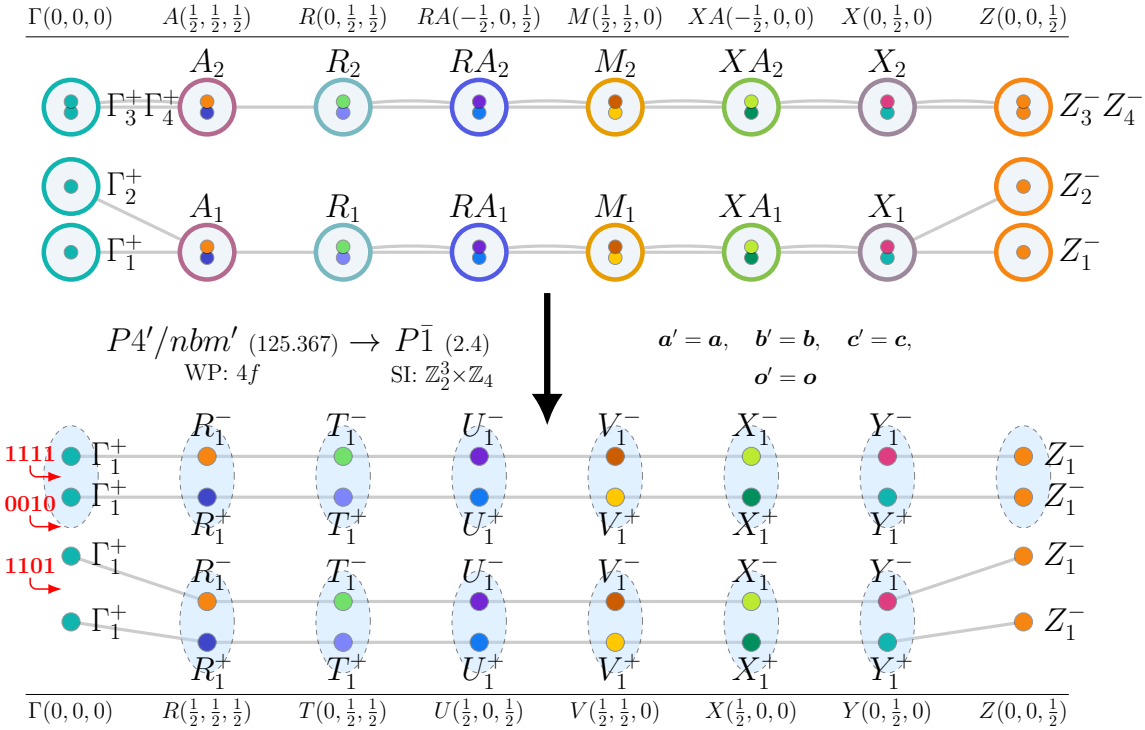
5.1 WP: $4f$

BCS Materials: $ZrMn_2Ge_4O_{12}$ (8 K)[6].

5.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [001] and strain ⊥ [100],
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].



Supplementary Figure 12: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4f$ of supergroup $P4'/nbm'$ (125.367).

[6]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.315`

Supplementary Table 13: Number of times a given SI can appear in the band structure. Supergroup: $P4'/nbm'$ (125.367), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	1, 2, 3
0011	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1
1102	0, 1, 2
1103	0, 1
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

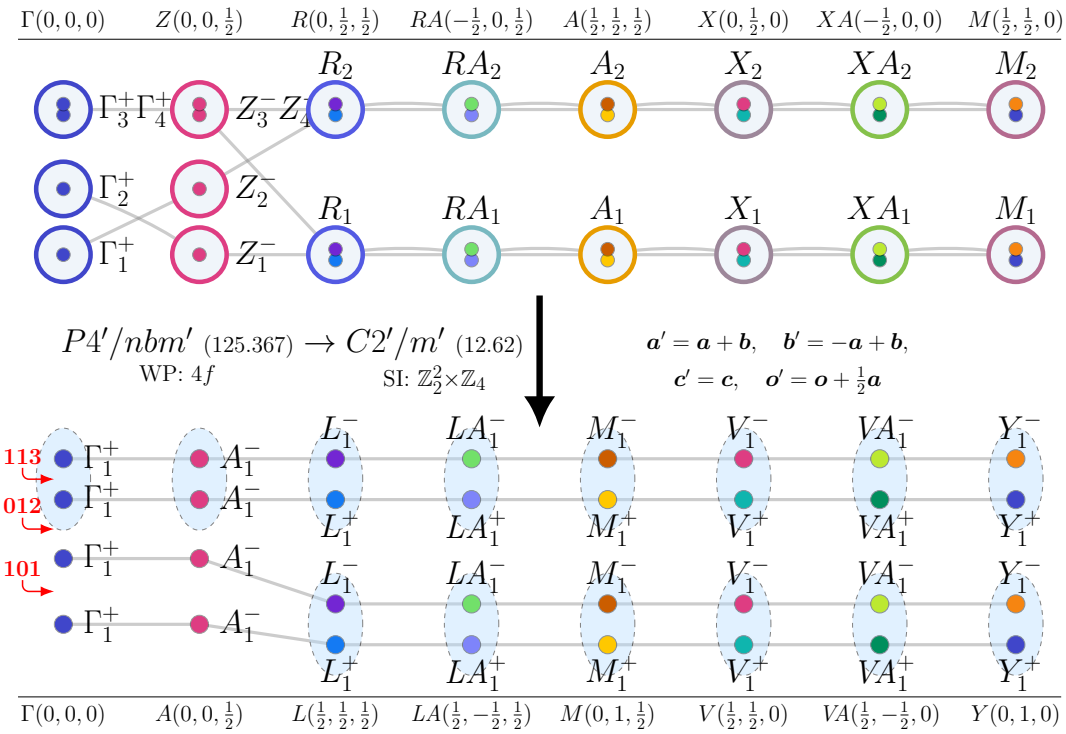
Supplementary Table 14: Possible SI values in each gap. Supergroup: $P4'/nbm'$ (125.367), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1112, 1113
2	0010
3	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1111, 1112
4	0000

5.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [110].



Supplementary Figure 13: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4f$ of supergroup $P4'/nbm'$ (125.367).

Supplementary Table 15: Number of times a given SI can appear in the band structure. Supergroup: $P4'/nbm'$ (125.367), WP: $4f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	1, 2, 3
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 16: Possible SI values in each gap. Supergroup: $P4'/nbm'$ (125.367), WP: $4f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	012

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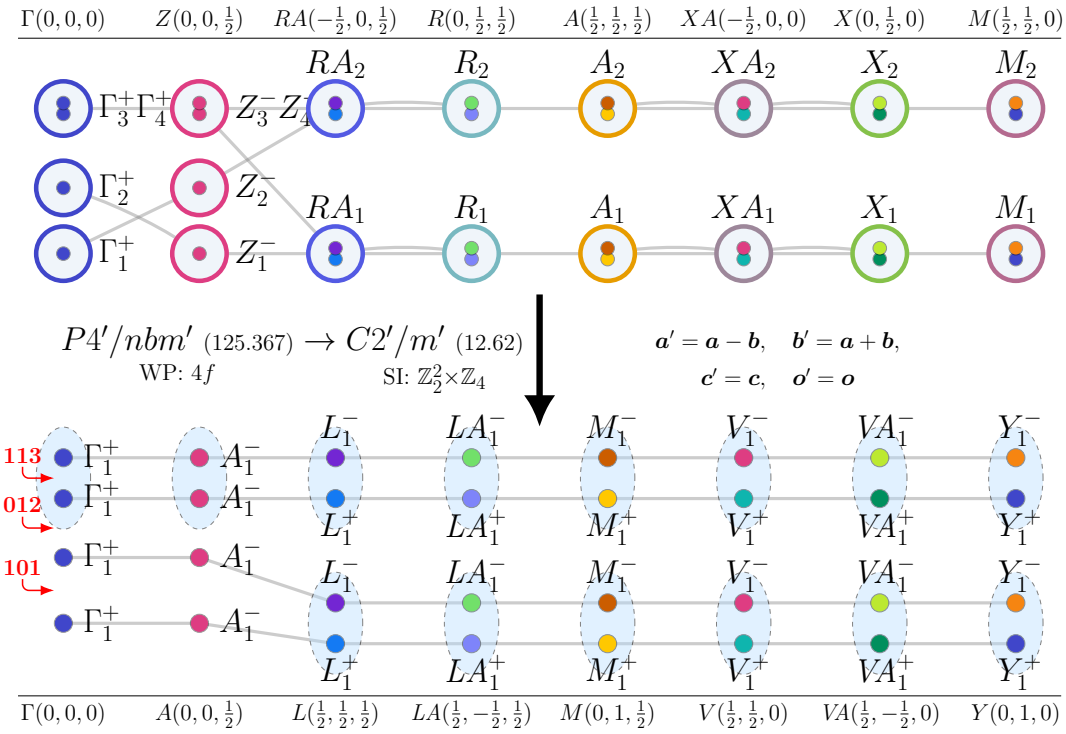
...continued

Gap#	Possible SI Values
3	000, 003, 012, 013, 100 103, 112, 113
4	000

5.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 14: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4f$ of supergroup $P4'/nbm'$ (125.367).

Supplementary Table 17: Number of times a given SI can appear in the band structure. Supergroup: $P4'/nbm'$ (125.367), WP: $4f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	1, 2, 3
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0

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...continued

SI	Possible #Gaps
nontrivial	1, 2, 3

Supplementary Table 18: Possible SI values in each gap. Supergroup: $P4'/nbm'$ (125.367), WP: $4f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	012
3	000, 003, 012, 013, 100 103, 112, 113
4	000

6 MSG P_C4/nbm (125.373)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2'/m'$ (10.46), $P2'_1/m'$ (11.54), $P_S\bar{1}$ (2.7), C_a2 (5.17), $C2/m$ (12.58), $Cmm'm'$ (65.486), C_a2/m (12.64), $P2$ (3.1), $Cm'm'2$ (35.168), $Pm'm'2$ (25.60), P_a2 (3.4), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pm'm'n$ (59.409), P_c2/c (13.72), $P2$ (3.1), $Pm'm'2$ (25.60), $P2/c$ (13.65), $Pm'm'a$ (51.294), P_A2/c (13.73), P_Cban (50.287), $P4m'm'$ (99.167), $P4/nm'm'$ (129.417).

Trivial-SI Subgroups: Cm' (8.34), Pm' (6.20), Pm' (6.20), $C2'$ (5.15), $P2'$ (3.3), $P2'_1$ (4.9), P_S1 (1.3), Cm (8.32), $Cm'm2'$ (35.167), C_am (8.36), Pc (7.24), $Abm'2'$ (39.198), $Pm'n2'_1$ (31.125), P_cc (7.28), Pc (7.24), $Pm'a2'$ (28.89), P_Ac (7.31), $C2$ (5.13), $Am'm'2$ (38.191), A_bbm2 (39.201), C_amm2 (35.170), P_Cba2 (32.141), C_amma (67.509), P_C2 (3.6), P_Anc2 (30.119), P_C4bm (100.177).

6.1 WP: $4f$

BCS Materials: NdMg (35 K)[7].

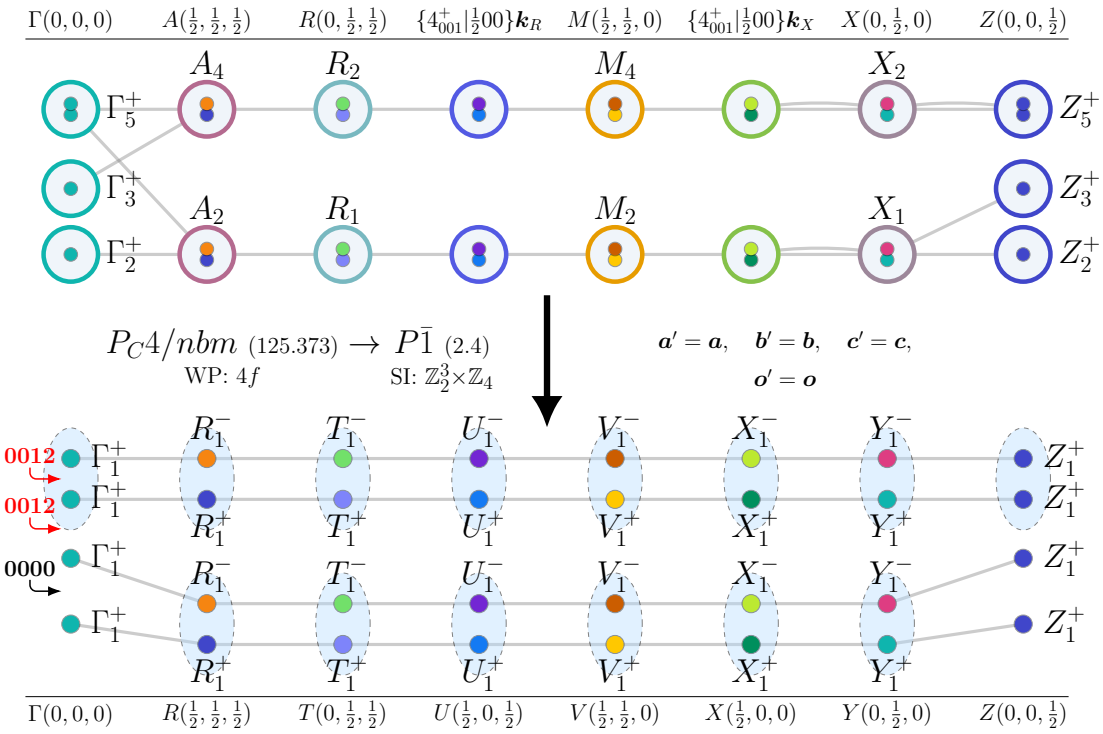
6.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,

[7]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.14`

- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 15: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4f$ of supergroup P_C4/nbm (125.373).

Supplementary Table 19: Number of times a given SI can appear in the band structure. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0011	0, 1, 2
0012	1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2

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SI	Possible #Gaps
1100	0, 1, 2
1101	0, 1
1102	0, 1, 2
1103	0, 1
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

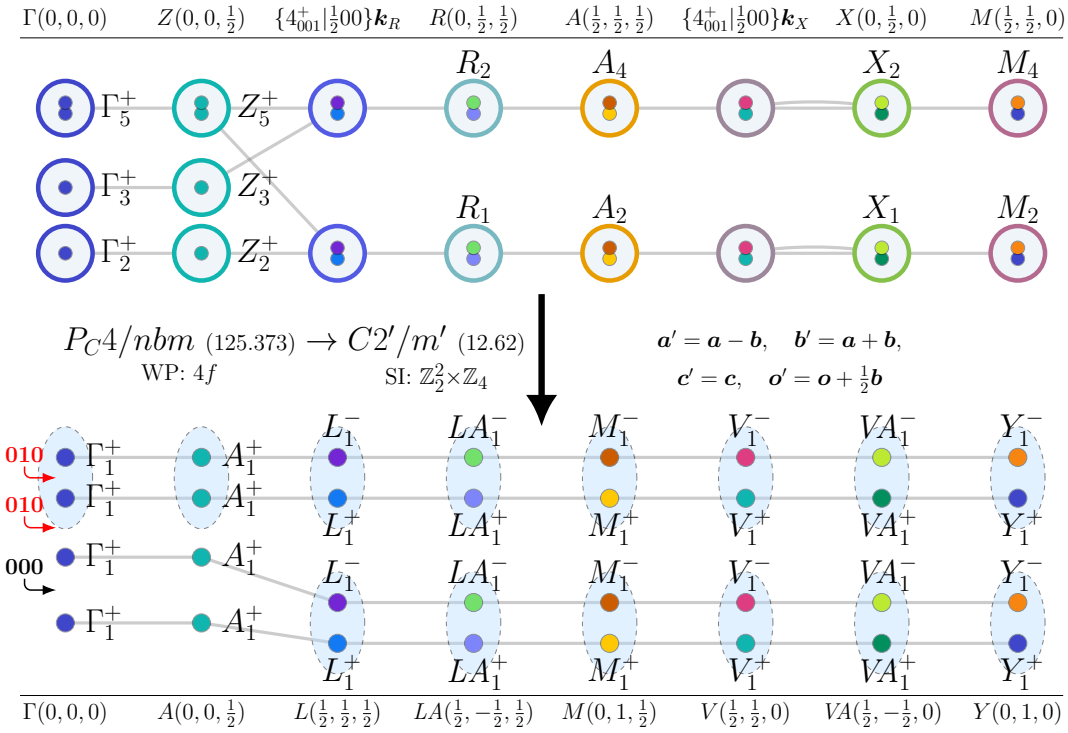
Supplementary Table 20: Possible SI values in each gap. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1112, 1113
2	0012
3	0000, 0001, 0003, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1111, 1112
4	0000

6.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 16: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4f$ of supergroup P_C4/nbm (125.373).

Supplementary Table 21: Number of times a given SI can appear in the band structure. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
010	1, 2, 3
011	0, 1
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
110	0, 1, 2
111	0, 1
113	0, 1
gapless	0
nontrivial	1, 2, 3

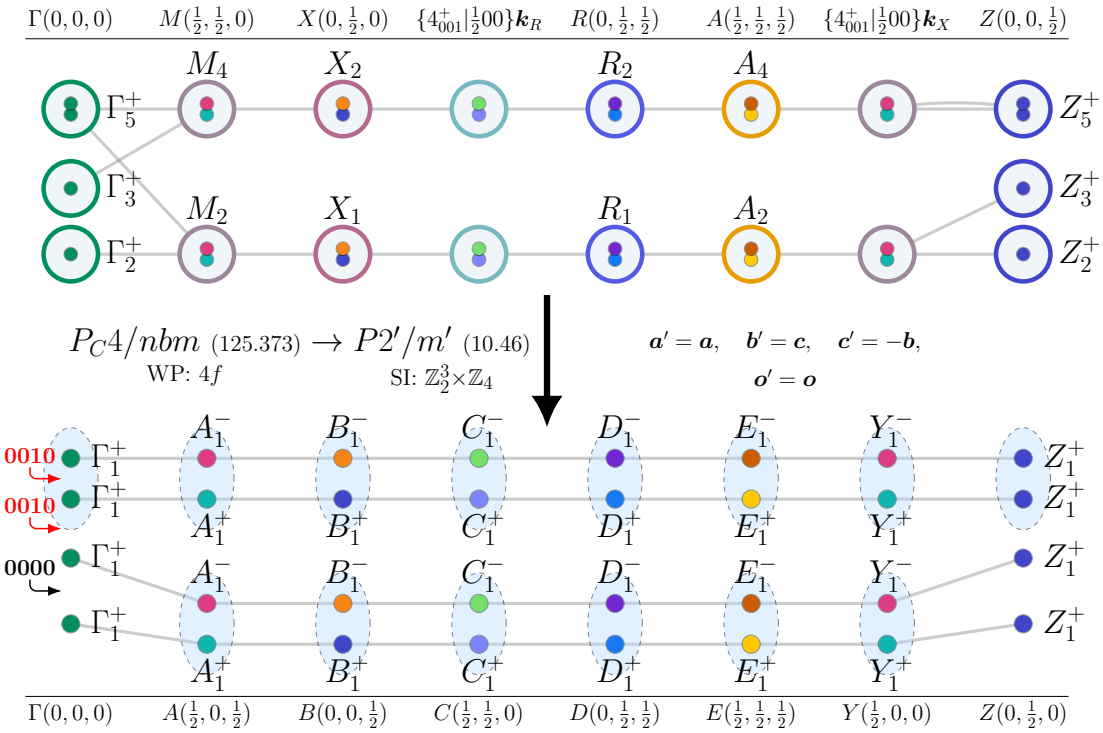
Supplementary Table 22: Possible SI values in each gap. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 003, 010, 011, 100 103, 110, 111
2	010
3	000, 001, 010, 013, 100 101, 110, 113
4	000

6.1.3 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 17: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4f$ of supergroup P_C4/nbm (125.373).

Supplementary Table 23: Number of times a given SI can appear in the band structure. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	1, 2, 3
0011	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2

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SI	Possible #Gaps
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1
1102	0, 1, 2
1103	0, 1
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

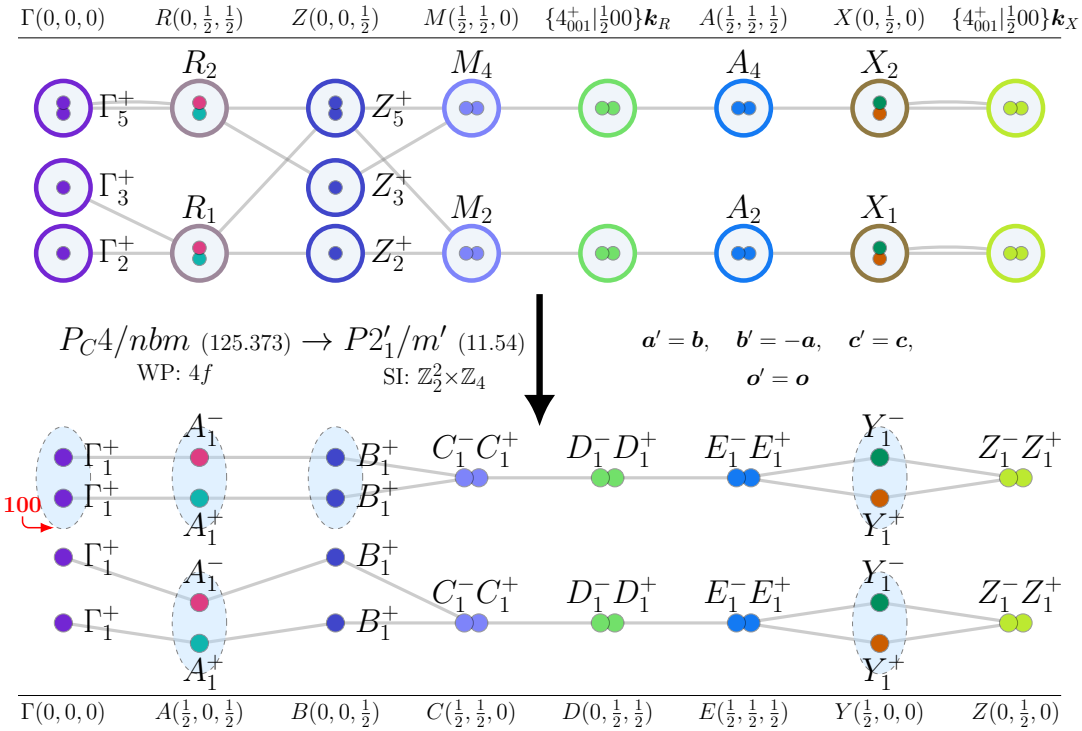
Supplementary Table 24: Possible SI values in each gap. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1111, 1112
2	0010
3	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1112, 1113
4	0000

6.1.4 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 18: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4f$ of supergroup P_C4/nbm (125.373).

Supplementary Table 25: Number of times a given SI can appear in the band structure. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

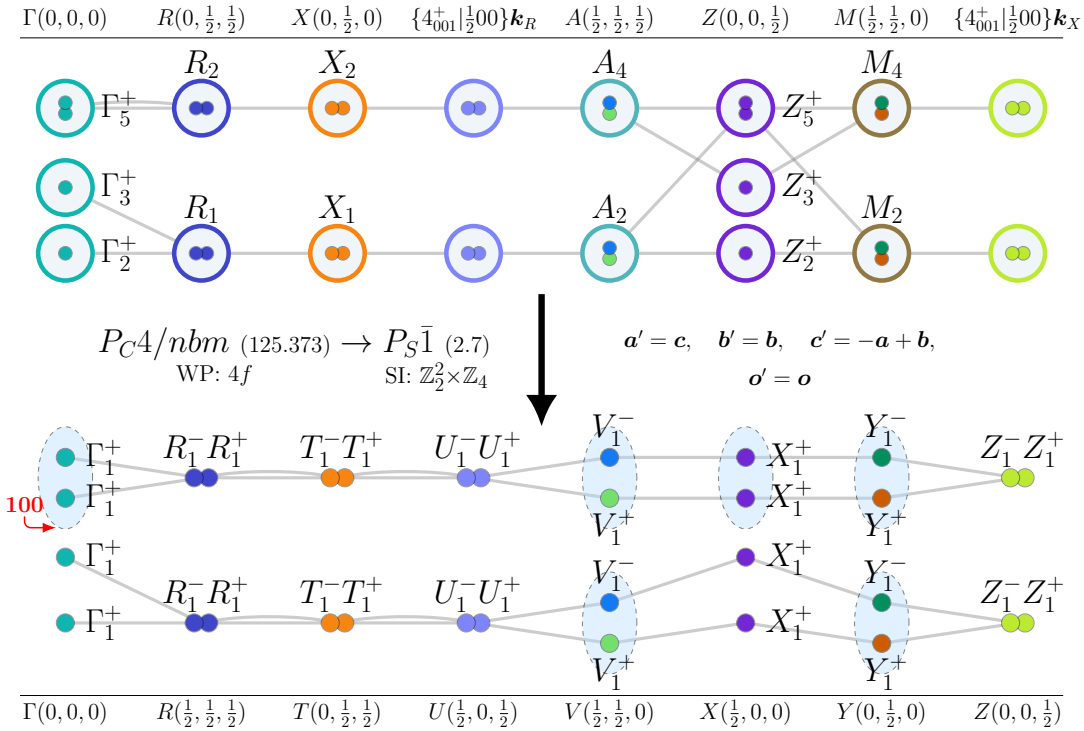
Supplementary Table 26: Possible SI values in each gap. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

6.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 19: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4f$ of supergroup P_C4/nbm (125.373).

Supplementary Table 27: Number of times a given SI can appear in the band structure. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

Supplementary Table 28: Possible SI values in each gap. Supergroup: P_C4/nbm (125.373), WP: $4f$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

7 MSG P_c4/nnc (126.384)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2_1'/c'$ (14.79), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $Ab'm'2$ (39.199), $C2/c$ (15.85), $Cm'ca'$ (64.476), C_c2/c (15.90), $P2$ (3.1), $Cm'm'2$ (35.168), $Pb'a'2$ (32.138), P_b2 (3.5), C_ccc2 (37.184), P_cnn2 (34.161), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pb'a'n$ (50.281), P_b2/c (13.71), C_ccca (68.518), $P2$ (3.1), P_a2 (3.4), $P2/c$ (13.65), $Pnn'a'$ (52.311), P_a2/c (13.70), P_cnnn (48.262), $P4b'm'$ (100.175), P_c4nc (104.208), $P4/nb'm'$ (125.369).

Trivial-SI Subgroups: Cm' (8.34), Pc' (7.26), Pc' (7.26), $C2'$ (5.15), $P2_1'$ (4.9), $P2'$ (3.3), P_S1 (1.3), Cc (9.37), $Cm'c2_1'$ (36.174), C_cc (9.40), Pc (7.24), $Abm'2'$ (39.198), $Pnc'2'$ (30.114), P_bc (7.29), Pc (7.24), $Pna'2_1'$ (33.147), P_ac (7.27), $C2$ (5.13), C_c2 (5.16),

$A_a ba2$ (41.216), $Pn'c'2$ (30.115), $P_a nn2$ (34.160).

7.1 WP: $4a + 8e$

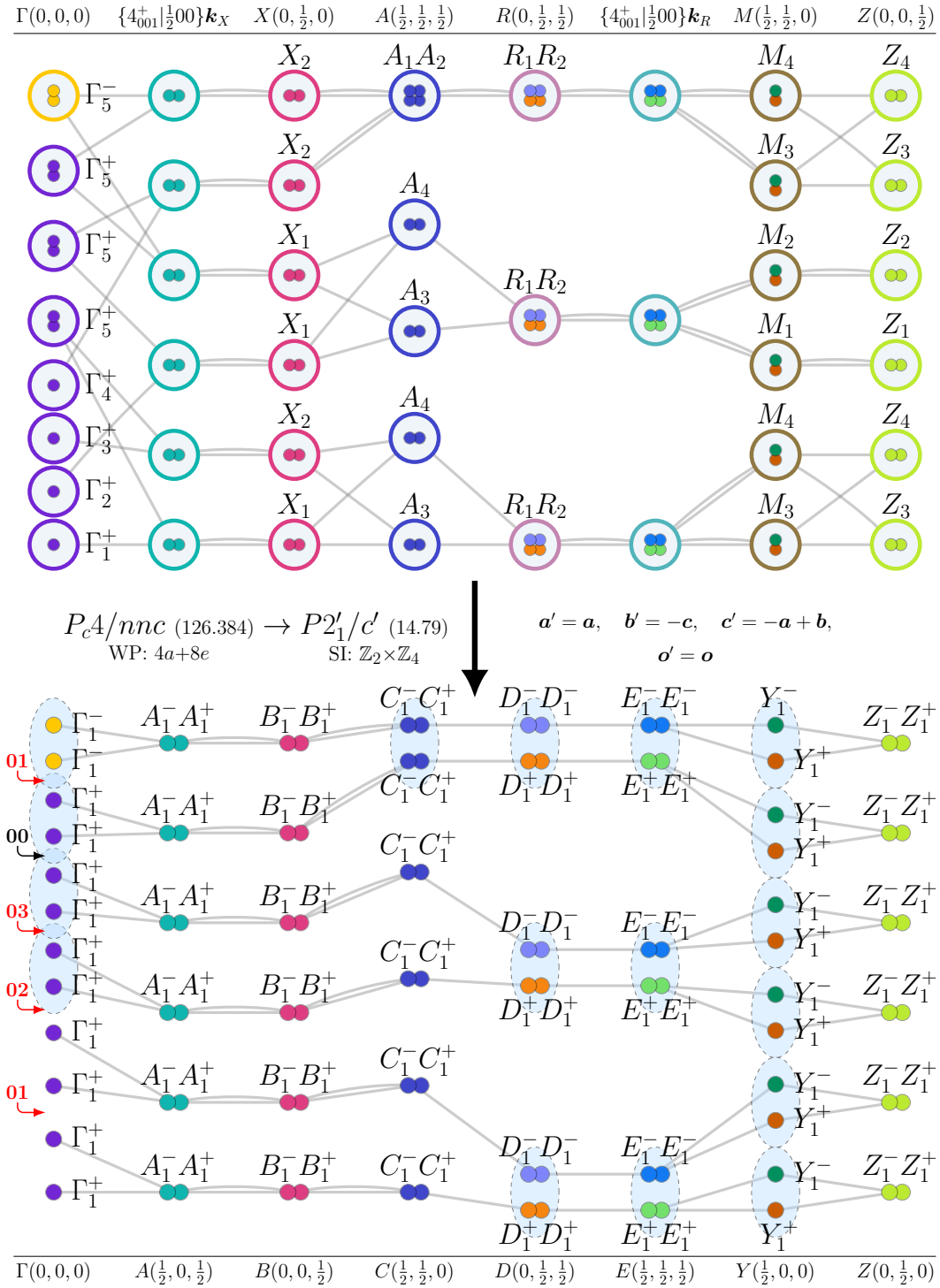
BCS Materials: $ErFeCuGe_4O_{12}$ (20 K)[8].

7.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].

[8]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.236`



Supplementary Figure 20: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4a+8e$ of supergroup P_c4/nnc (126.384).

Supplementary Table 29: Number of times a given SI can appear in the band structure. Supergroup: P_c4/nnc (126.384), WP: $4a$+$8e$, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2, 3
02	1, 2
03	0, 1, 2, 3
10	0, 1
11	0, 1

continued on next page...

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SI	Possible #Gaps
12	0, 1
13	0, 1
gapless	6
nontrivial	4, 5

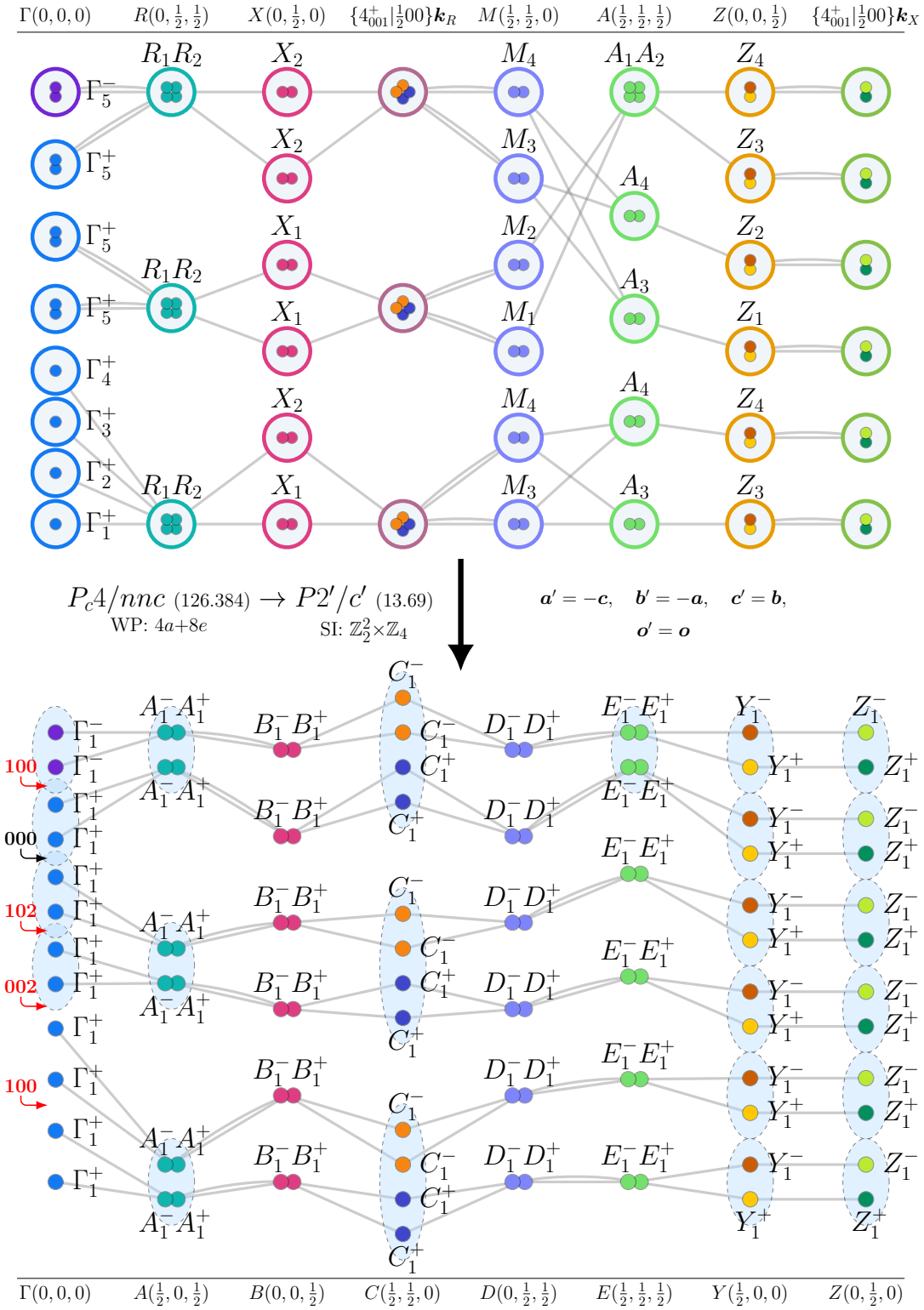
Supplementary Table 30: Possible SI values in each gap. Supergroup: P_c4/nnc (126.384), WP: $4a$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 13
9	G
10	01, 03, 10, 12
11	G
12	00

7.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 21: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4a+8e$ of supergroup P_c4/nnc (126.384).

Supplementary Table 31: Number of times a given SI can appear in the band structure. Supergroup: P_c4/nnc (126.384), WP: $4a$+$8e$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
011	0, 1
013	0, 1

continued on next page...

...continued

SI	Possible #Gaps
100	0, 1, 2, 3
102	0, 1, 2, 3
111	0, 1, 2, 3
113	0, 1, 2, 3
gapless	6
nontrivial	3, 4, 5

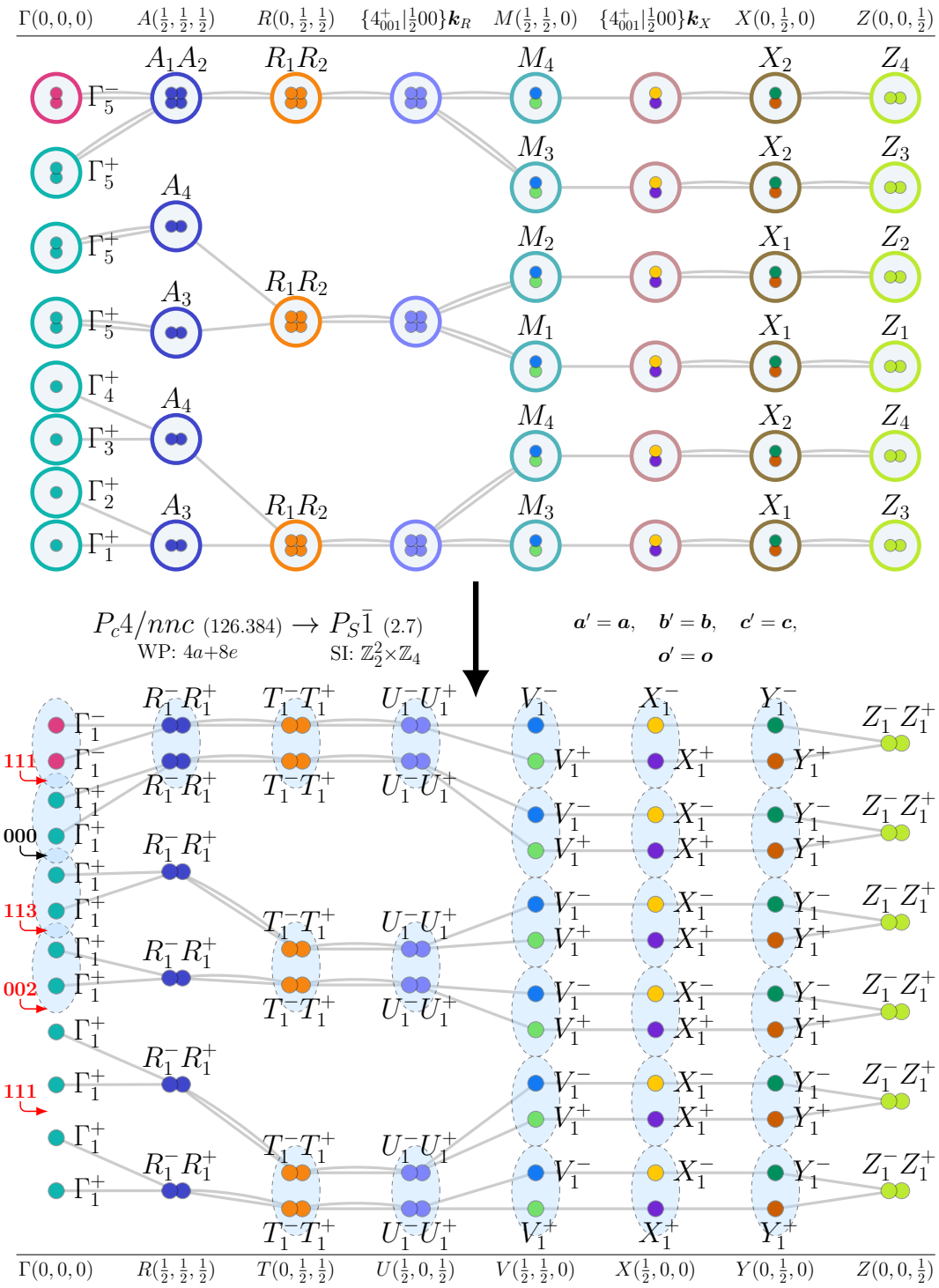
Supplementary Table 32: Possible SI values in each gap. Supergroup: P_c4/nnc (126.384), WP: $4a$+$8e$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	000, 011, 013, 100, 102 111, 113
3	G
4	000, 002, 111
5	G
6	002, 011, 013, 100, 102 111, 113
7	G
8	000, 002, 113
9	G
10	000, 011, 013, 100, 102 111, 113
11	G
12	000

7.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 22: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4a+8e$ of supergroup P_c4/nnc (126.384).

Supplementary Table 33: Number of times a given SI can appear in the band structure. Supergroup: P_c4/nnc (126.384), WP: $4a$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
111	1, 2, 3
113	1, 2, 3
gapless	6
nontrivial	3, 4, 5

Supplementary Table 34: Possible SI values in each gap. Supergroup: P_c4/nnc (126.384), WP: $4a$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 111, 113
3	G
4	000, 002, 111
5	G
6	002, 111, 113
7	G
8	000, 002, 113
9	G
10	000, 111, 113
11	G
12	000

8 MSG P_I4/nnc (126.386)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2_1'/m'$ (11.54), $P2_1'/m'$ (11.54), $P_S\bar{1}$ (2.7), $C2/c$ (15.85), $Cm'cm'$ (63.464), C_c2/c (15.90), $P2$ (3.1), $Cm'm'2$ (35.168), $Pm'm'2$ (25.60), C_Acc2 (37.186), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pm'm'n$ (59.409), P_A2/c (13.73), C_Acca (68.520), $P2$ (3.1), $Pm'm'2$ (25.60), $P2/c$ (13.65), $Pm'm'n$ (59.409), P_A2/c (13.73), P_Innn (48.264), $P4m'm'$ (99.167), P_I4nc (104.210), $P4/nm'm'$ (129.417).

Trivial-SI Subgroups: Cm' (8.34), Pm' (6.20), Pm' (6.20), $C2'$ (5.15), $P2_1'$ (4.9), $P2_1'$ (4.9), P_S1 (1.3), Cc (9.37), $Cm'c2_1'$ (36.174), C_cc (9.40), Pc (7.24), $Abm'2'$ (39.198), $Pm'n2_1'$ (31.125), P_Ac (7.31), Pc (7.24), $Pm'n2_1'$ (31.125), P_Ac (7.31), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_Bba2 (41.218), P_C2 (3.6), P_Inn2 (34.164), P_C2 (3.6), P_Inn2 (34.164).

8.1 WP: $4d$

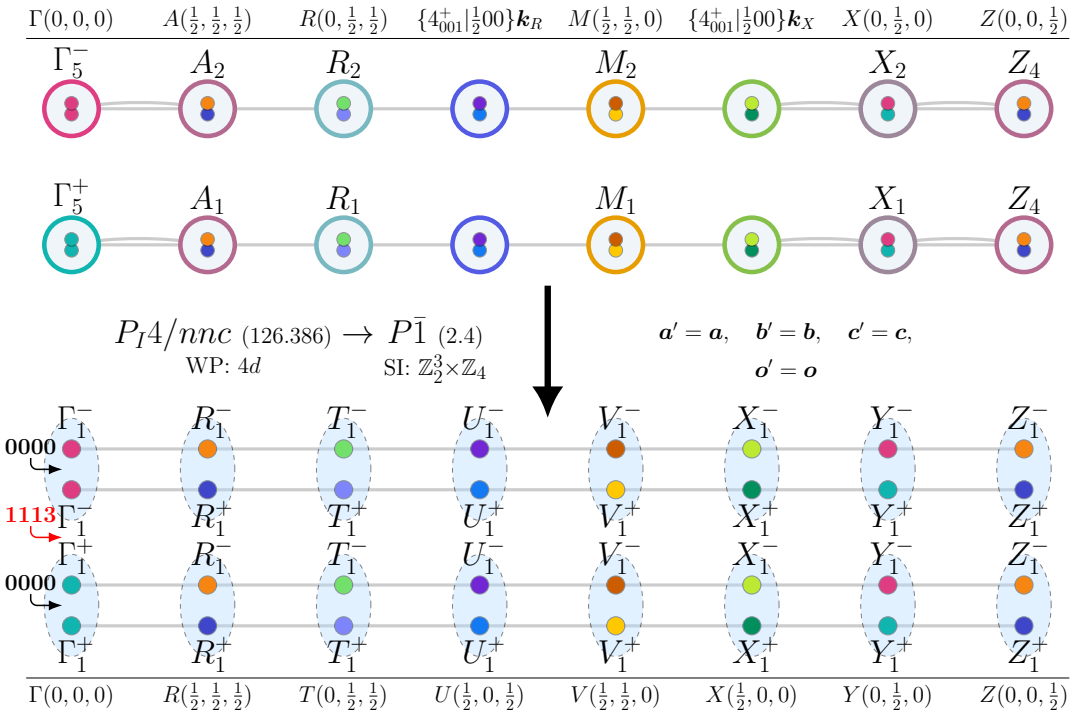
BCS Materials: $ErMn_2Ge_2$ (475 K)[9], YMn_2Si_2 (460 K)[10], $CeCo_2P_2$ (440 K)[11], YMn_2Si_2 (410 K)[12], YMn_2Ge_2 (395 K)[13], $EuMn_2Si_2$ (391 K)[14], $CeMn_2Si_2$ (384 K)[15], $NdMn_2Si_2$ (382 K)[16], $NdMn_2Si_2$ (380 K)[17], $CeMn_2Si_2$ (380 K)[18], $CeMn_2Si_2$ (379 K)[19], $PrMn_2Si_2$ (368 K)[20], $PrMn_2Si_2$ (348 K)[21], $CaCo_{1.86}As_2$ (52 K)[22], $ErMn_2Ge_2$[23], $ErMn_2Ge_2$[24], $ErMn_2Si_2$[25], $ErMn_2Si_2$[26], $TbMn_2Si_2$[27].

[9]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.640
[10]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.495
[11]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.253
[12]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.469
[13]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.496
[14]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.453
[15]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.489
[16]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.493
[17]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.494
[18]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.490
[19]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.488
[20]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.492
[21]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.491
[22]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.691
[23]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.639
[24]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.638
[25]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.637
[26]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.636
[27]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.468

8.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 23: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 35: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1, 2, 3
1112	0, 1, 2
1113	0, 1, 2, 3
gapless	0
nontrivial	1, 2, 3

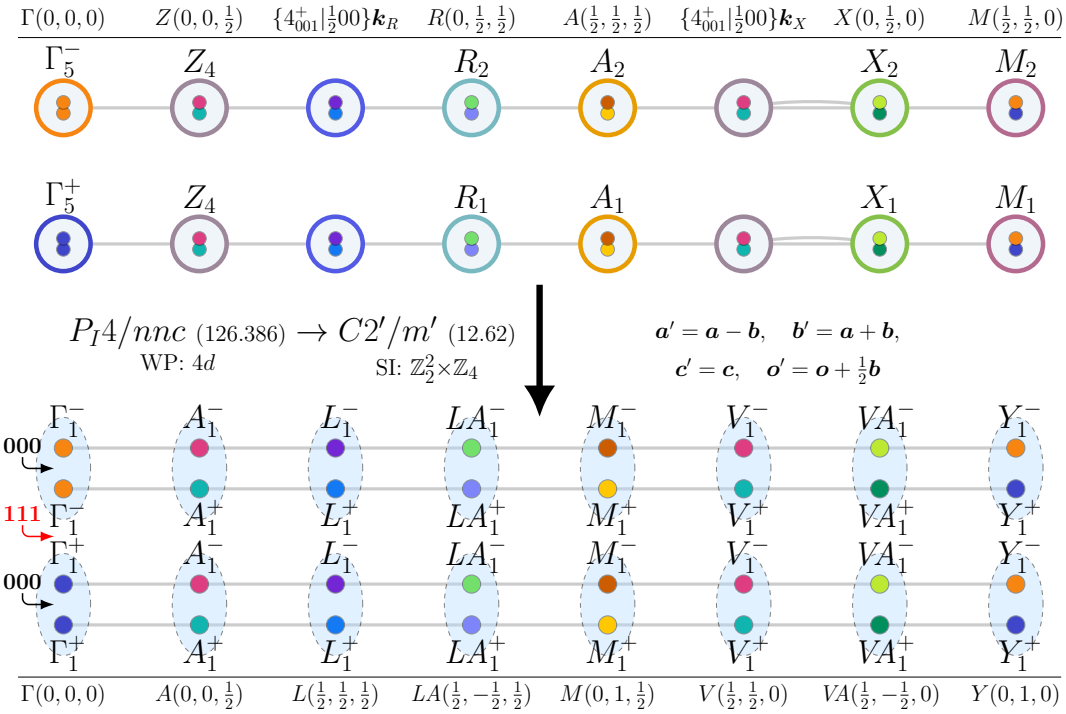
Supplementary Table 36: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	1111, 1113
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

8.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [001] and strain ⊥ [110],
- B ⊥ [110].



Supplementary Figure 24: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 37: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
011	0, 1, 2
012	0, 1, 2
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1, 2, 3
112	0, 1, 2
113	0, 1, 2, 3
gapless	0
nontrivial	1, 2, 3

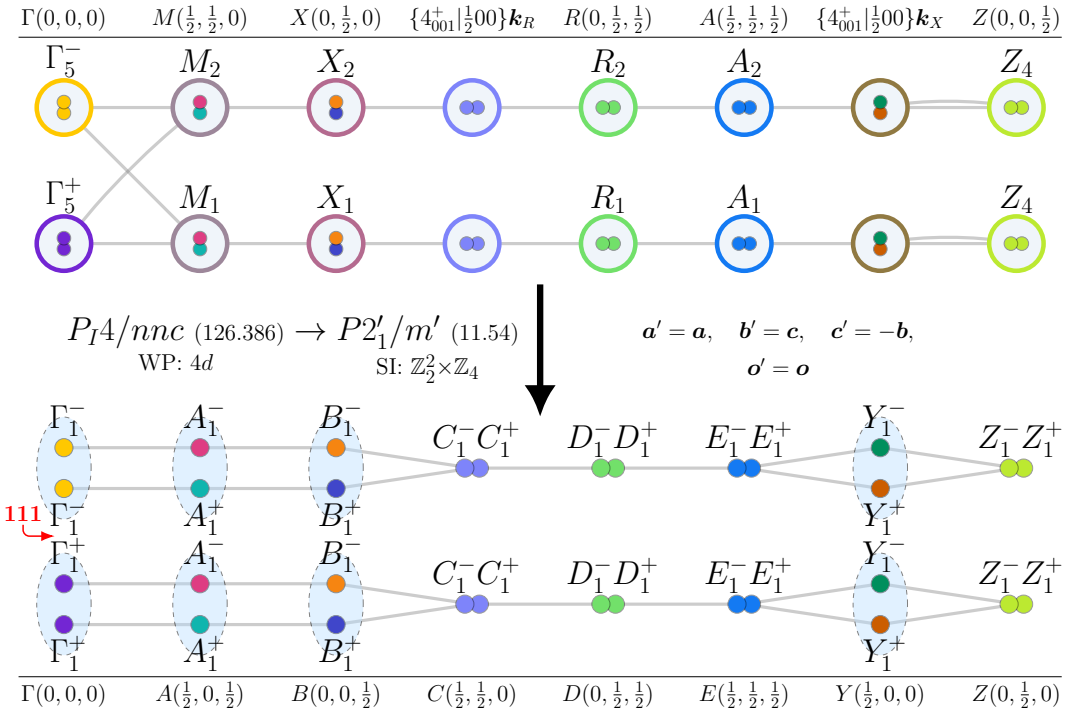
Supplementary Table 38: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113
2	111, 113
3	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113
4	000

8.1.3 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 25: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 39: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0
111	0, 1

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...continued

SI	Possible #Gaps
113	0, 1
gapless	2
nontrivial	1

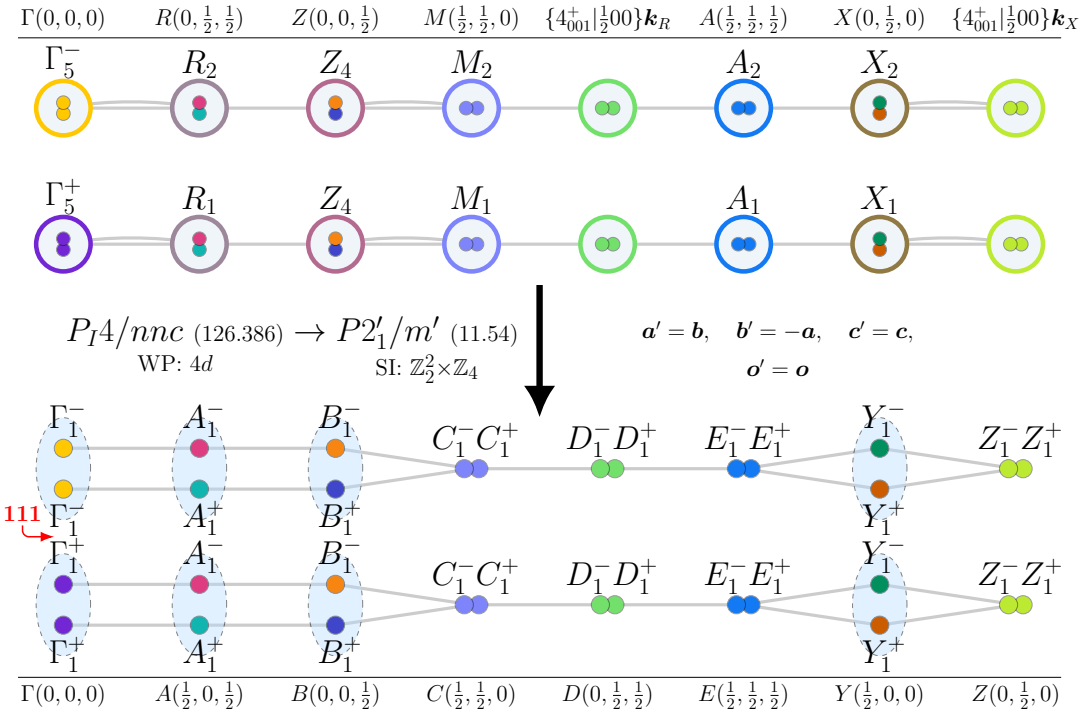
Supplementary Table 40: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000

8.1.4 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 26: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 41: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0

continued on next page...

...continued

SI	Possible #Gaps
111	0, 1
113	0, 1
gapless	2
nontrivial	1

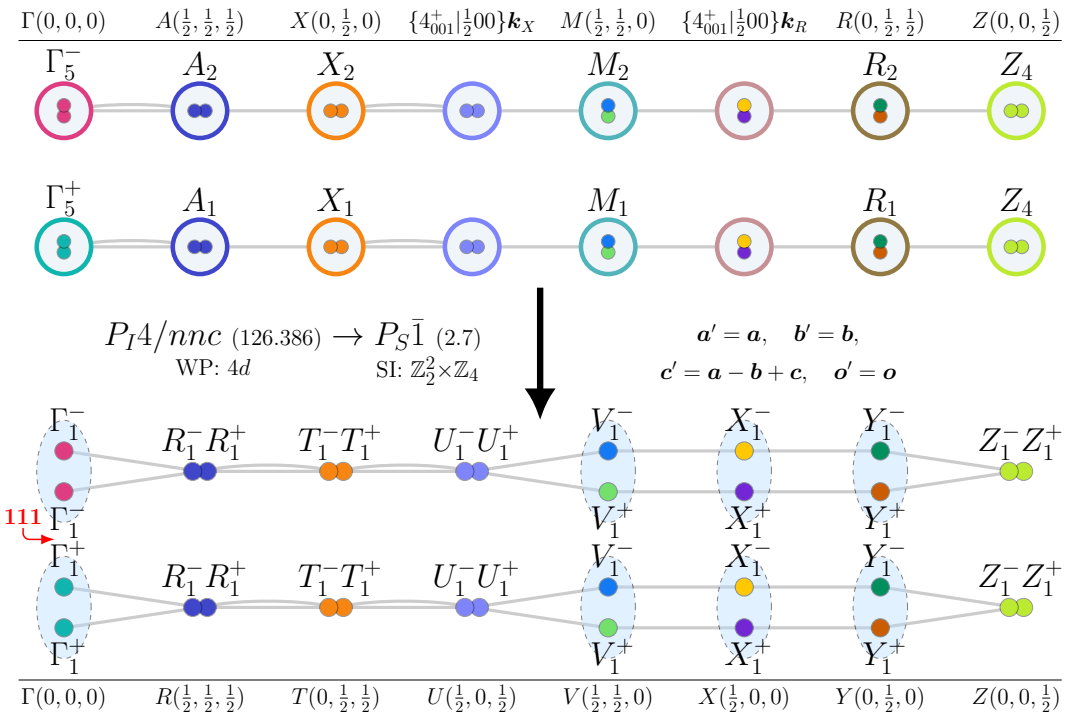
Supplementary Table 42: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000

8.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 27: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 43: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
111	0, 1
113	0, 1

continued on next page...

...continued

SI	Possible #Gaps
gapless	2
nontrivial	1

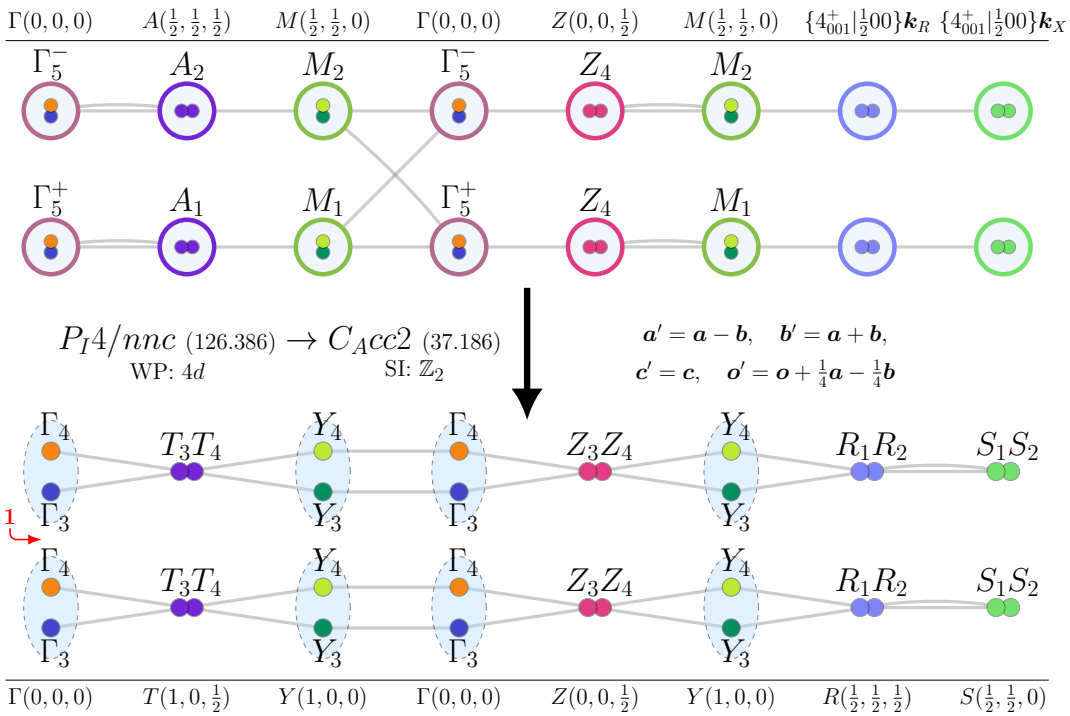
Supplementary Table 44: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000

8.1.6 Topological bands in subgroup C_Acc2 (37.186)

Perturbation:

- E ∥ [001] and strain ∥ [110].



Supplementary Figure 28: Topological magnon bands in subgroup C_Acc2 (37.186) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 45: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: C_Acc2 (37.186).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

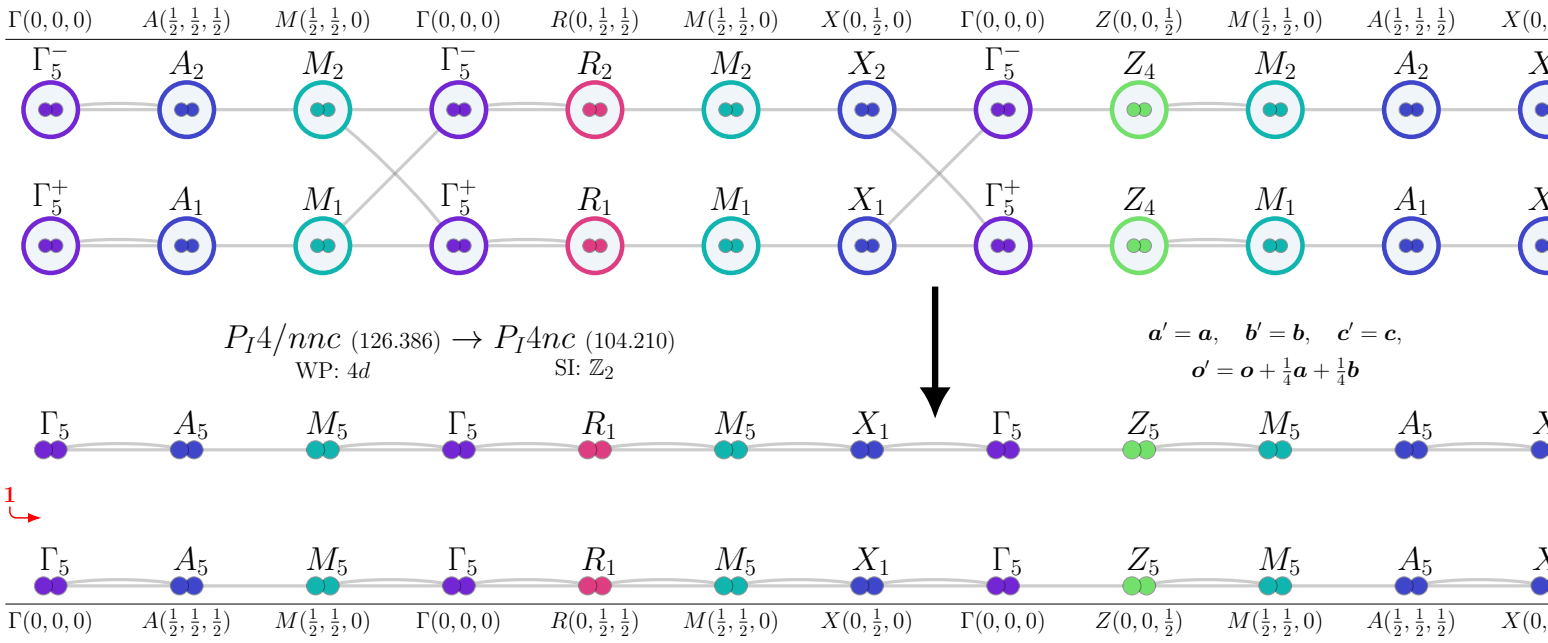
Supplementary Table 46: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: C_Acc2 (37.186).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

8.1.7 Topological bands in subgroup P_I4nc (104.210)

Perturbation:

- E ∥ [001].



Supplementary Figure 29: Topological magnon bands in subgroup P_I4nc (104.210) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4/nnc (126.386).

Supplementary Table 47: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: P_I4nc (104.210).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 48: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4d$, Subgroup: P_I4nc (104.210).

Gap#	Possible SI Values
1	G
2	1

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Gap#	Possible SI Values
3	G
4	0

8.2 WP: $4e$

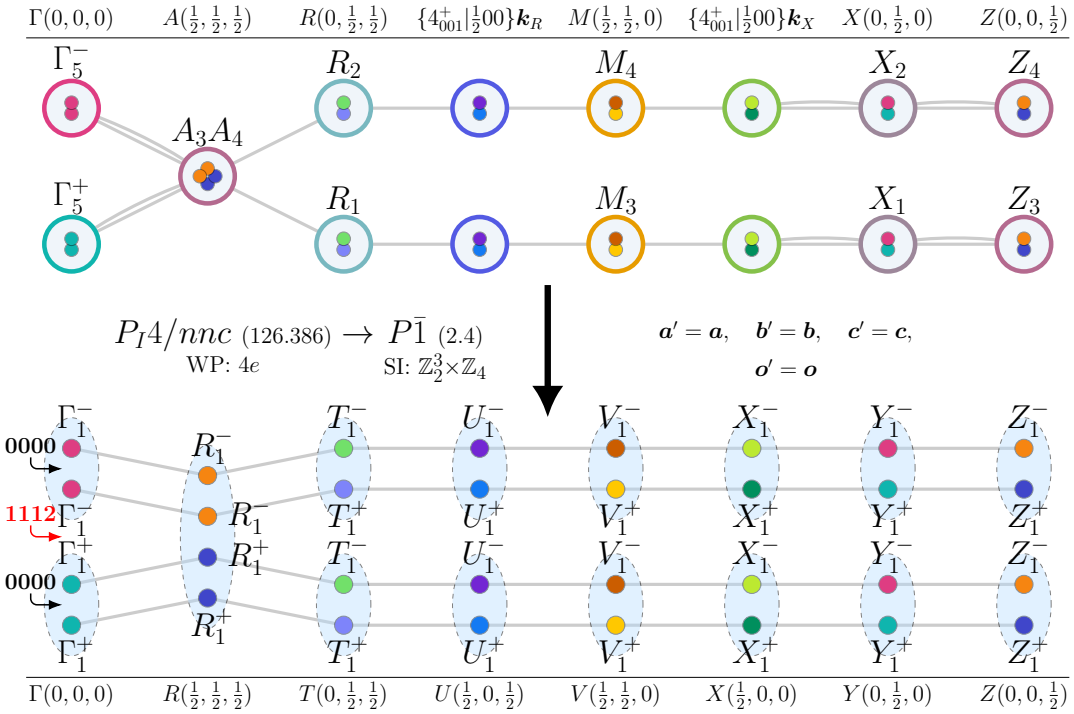
BCS Materials: UGeTe (73 K)[28], $NdScSiC_{0.5}H_{0.2}$ (10 K)[29].

8.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B $\parallel$ [001] and strain in generic direction,
- B $\parallel$ [100] and strain $\perp$ [110],
- B $\parallel$ [100] and strain in generic direction,
- B $\parallel$ [110] and strain $\perp$ [100],
- B $\parallel$ [110] and strain in generic direction,
- B $\perp$ [001] and strain $\perp$ [100],
- B $\perp$ [001] and strain $\perp$ [110],
- B $\perp$ [001] and strain in generic direction,
- B $\perp$ [100] and strain $\parallel$ [110],
- B $\perp$ [100] and strain $\perp$ [001],
- B $\perp$ [100] and strain $\perp$ [110],
- B $\perp$ [100] and strain in generic direction,
- B $\perp$ [110] and strain $\parallel$ [100],
- B $\perp$ [110] and strain $\perp$ [001],
- B $\perp$ [110] and strain $\perp$ [100],
- B $\perp$ [110] and strain in generic direction,
- B in generic direction.

[28] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.425`
[29] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.503`



Supplementary Figure 30: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4e$ of supergroup P_I4/nnc (126.386).

Supplementary Table 49: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

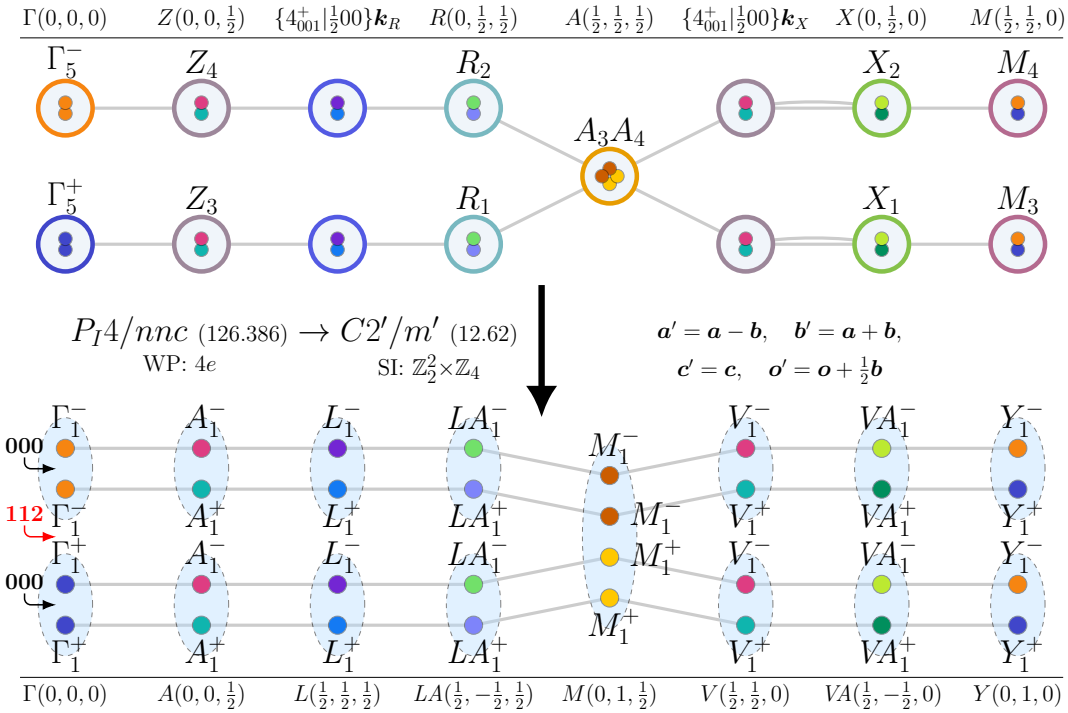
Supplementary Table 50: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	1110, 1111, 1112, 1113
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

8.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 31: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4e$ of supergroup P_I4/nnc (126.386).

Supplementary Table 51: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
011	0, 1, 2
012	0, 1, 2
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3
113	0, 1, 2, 3
gapless	0
nontrivial	1, 2, 3

Supplementary Table 52: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113
2	110, 111, 112, 113
3	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113

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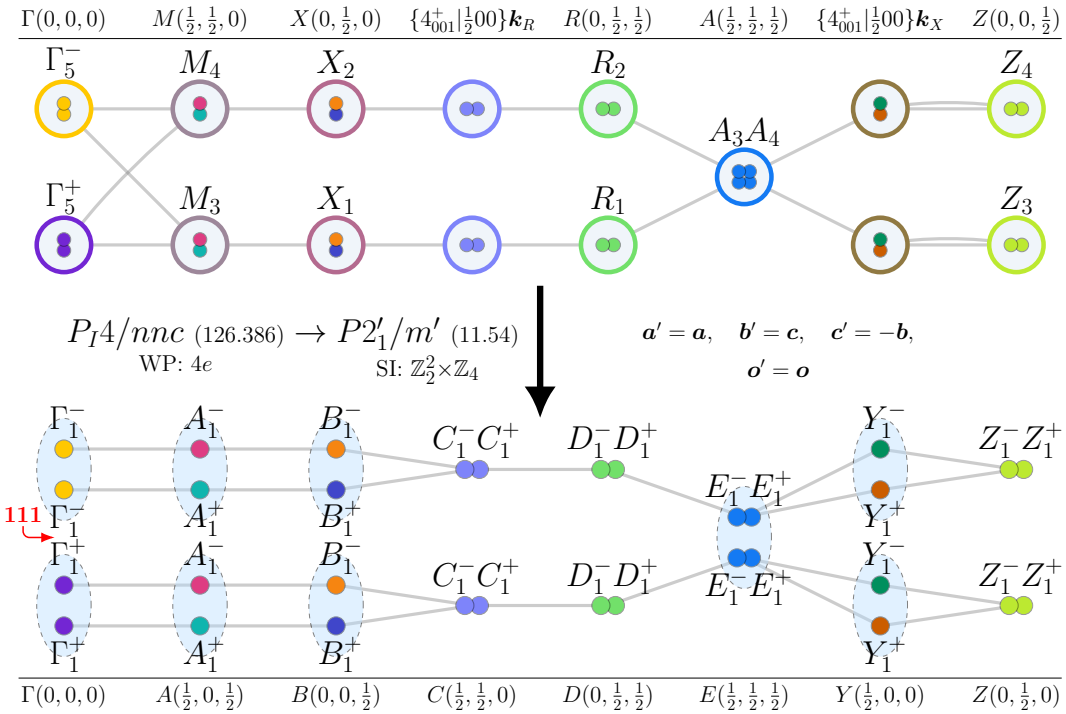
...continued

Gap#	Possible SI Values
4	000

8.2.3 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 32: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $4e$ of supergroup P_I4/nnc (126.386).

Supplementary Table 53: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0
111	0, 1
113	0, 1
gapless	2
nontrivial	1

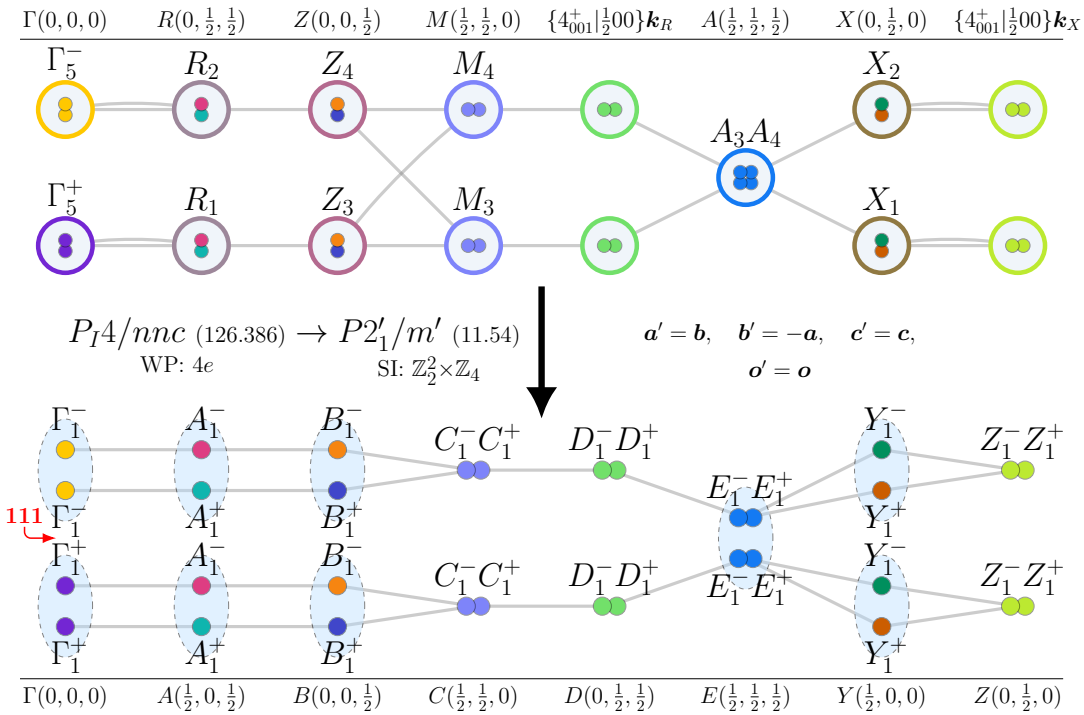
Supplementary Table 54: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000

8.2.4 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 33: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $4e$ of supergroup P_I4/nnc (126.386).

Supplementary Table 55: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0
111	0, 1
113	0, 1
gapless	2
nontrivial	1

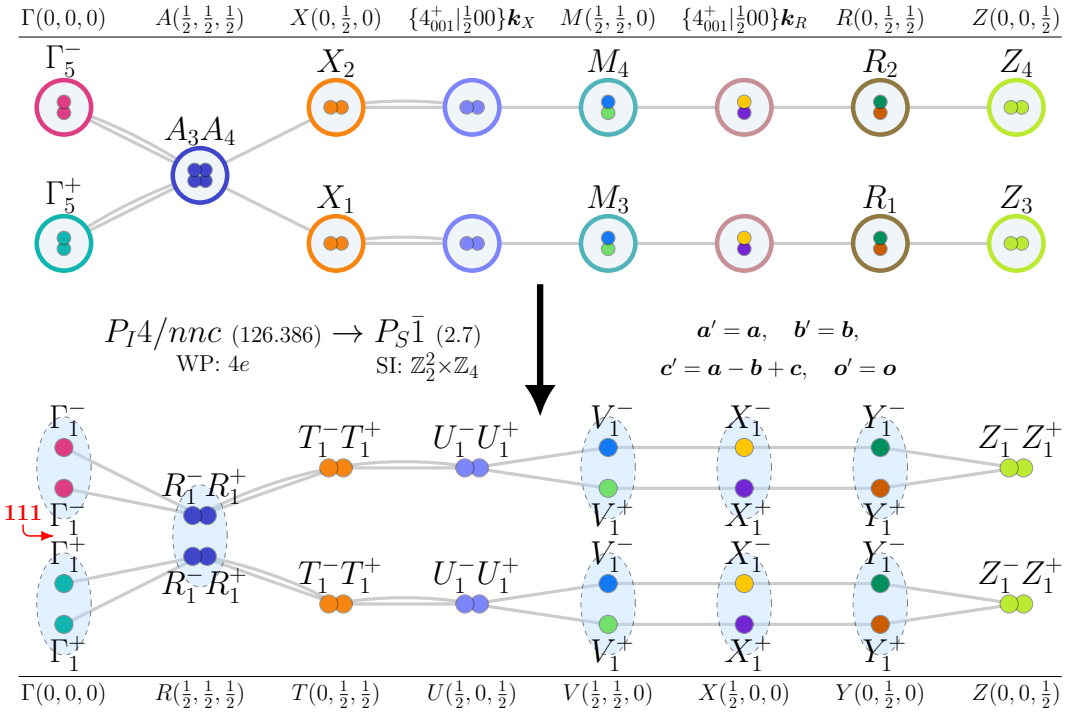
Supplementary Table 56: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000

8.2.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 34: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4e$ of supergroup P_I4/nnc (126.386).

Supplementary Table 57: Number of times a given SI can appear in the band structure. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
111	0, 1
113	0, 1
gapless	2
nontrivial	1

Supplementary Table 58: Possible SI values in each gap. Supergroup: P_I4/nnc (126.386), WP: $4e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G

continued on next page...

...continued

Gap#	Possible SI Values
2	111, 113
3	G
4	000

9 MSG C_c2/m (12.63)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), $C2/m$ (12.58).

Trivial-SI Subgroups: Cc' (9.39), $C2'$ (5.15), P_S1 (1.3), Cm (8.32), C_cm (8.35), $C2$ (5.13), C_c2 (5.16).

9.1 WP: $4b + 8e$

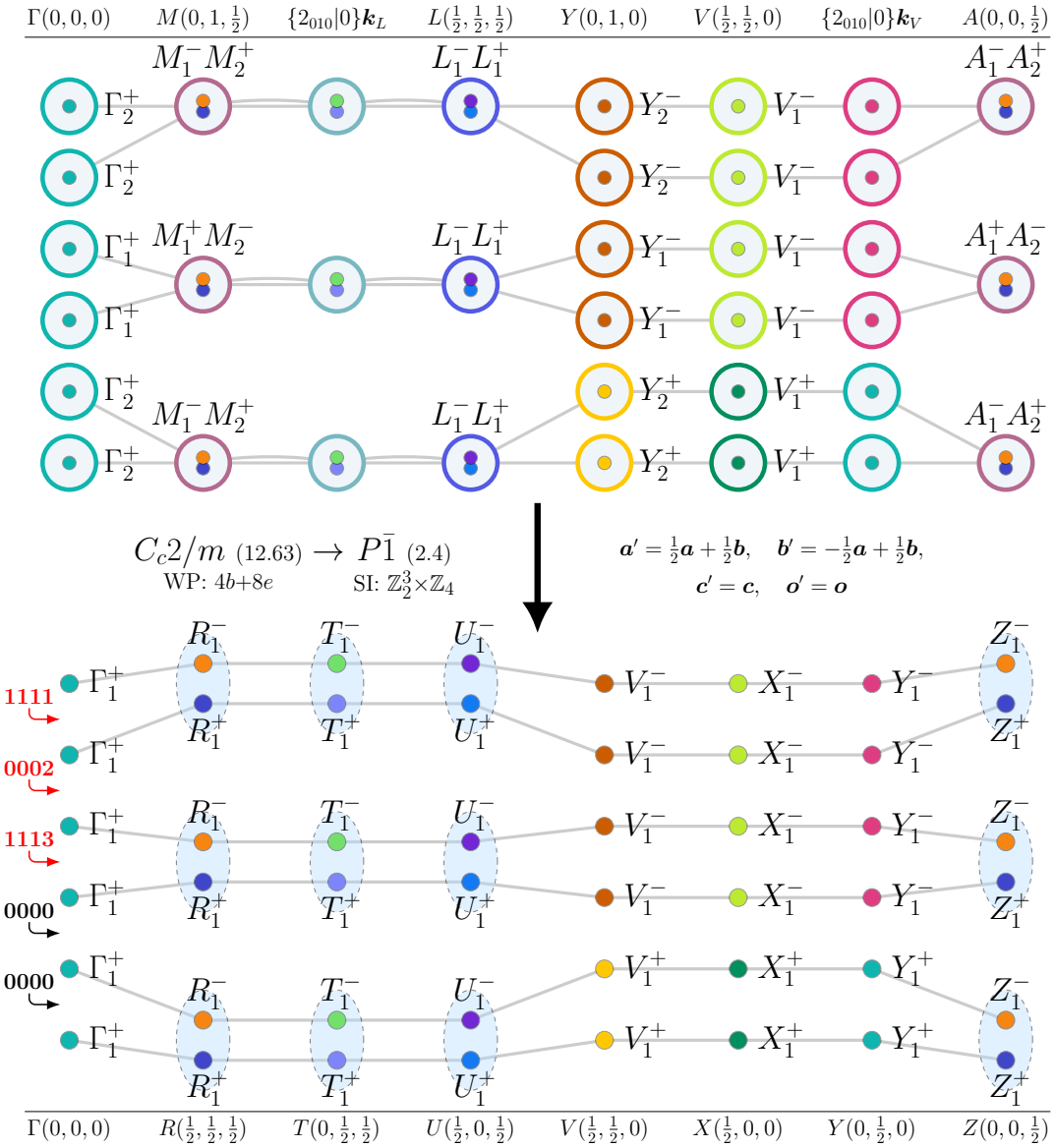
BCS Materials: $GeNi_2O_4$ (12.1 K)[30], $Cu_3Mg(OD)_6Br_2$ (5.4 K)[31].

9.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[30] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.560`
[31] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.397`



Supplementary Figure 35: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b+8e$ of supergroup C_c2/m (12.63).

Supplementary Table 59: Number of times a given SI can appear in the band structure. Supergroup: C_c2/m (12.63), WP: $4b$+$8e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0100	0, 1, 2, 3
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3

continued on next page...

...continued

SI	Possible #Gaps
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

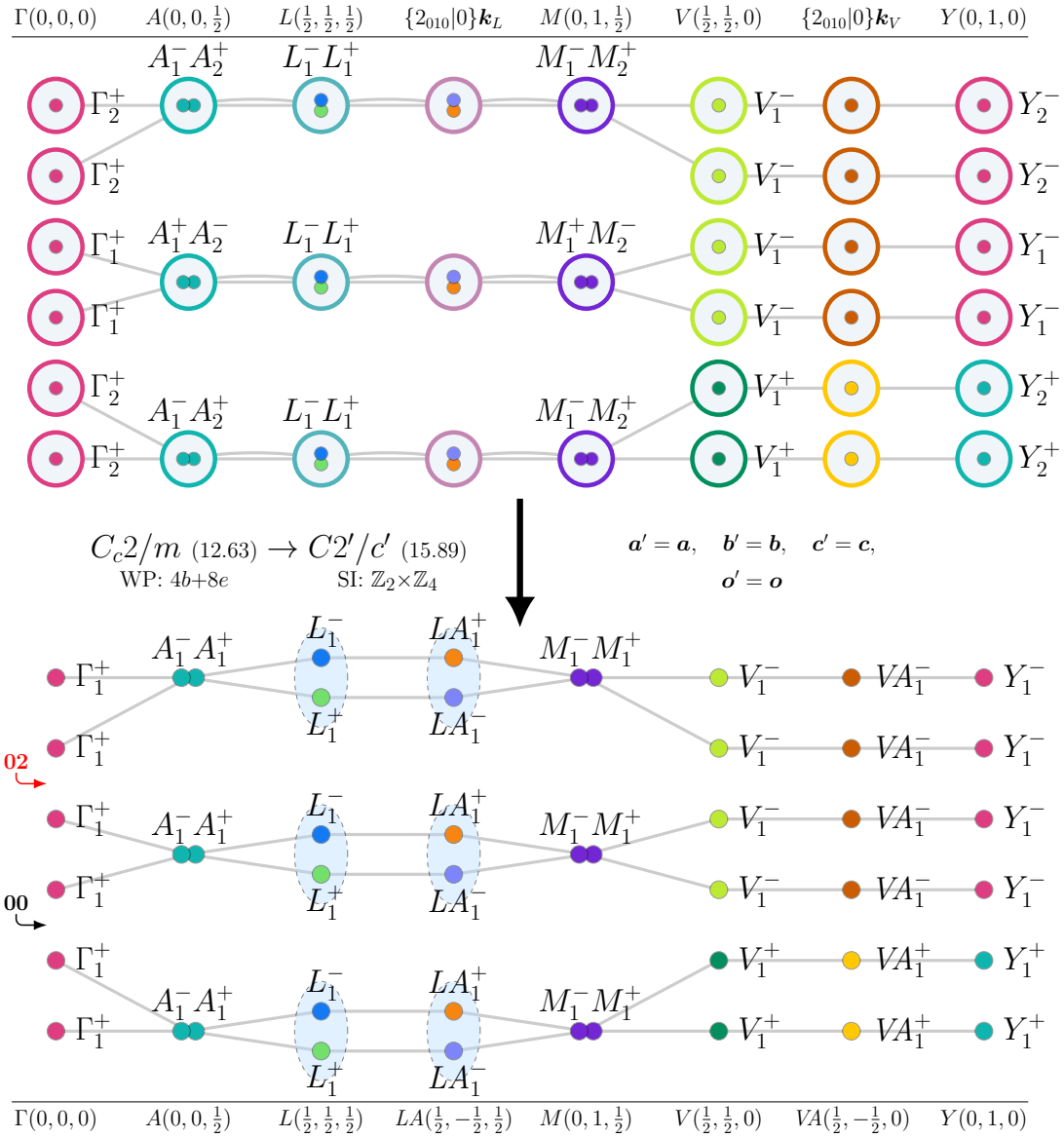
Supplementary Table 60: Possible SI values in each gap. Supergroup: C_c2/m (12.63), WP: $4b$+$8e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1112, 1113
2	0000, 0002, 0011, 1100, 1102 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1103, 1110, 1111, 1112, 1113
4	0000, 0002, 0013, 1100, 1102 1111
5	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1111, 1112
6	0000

9.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 36: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4b+8e$ of supergroup C_c2/m (12.63).

Supplementary Table 61: Number of times a given SI can appear in the band structure. Supergroup: C_c2/m (12.63), WP: $4b$+$8e$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
01	0, 1
02	0, 1, 2
03	0, 1
10	0, 1, 2
11	0, 1
12	0, 1
13	0, 1
gapless	3
nontrivial	1, 2

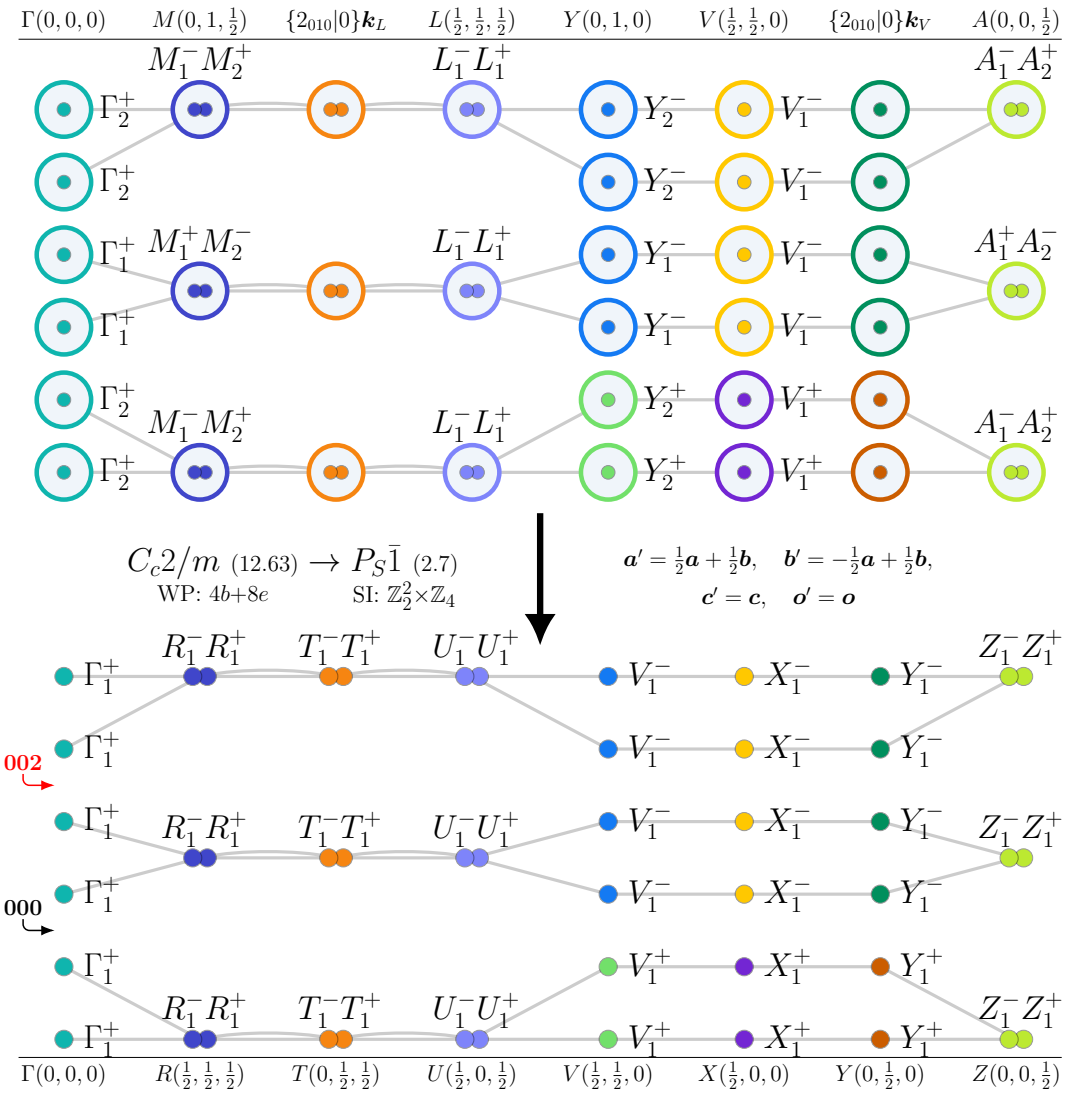
Supplementary Table 62: Possible SI values in each gap. Supergroup: C_c2/m (12.63), WP: $4b$+$8e$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 01, 02, 10, 12 13
3	G
4	00, 02, 03, 10, 11 12
5	G
6	00

9.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 37: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4b + 8e$ of supergroup C_c2/m (12.63).

Supplementary Table 63: Number of times a given SI can appear in the band structure. Supergroup: C_c2/m (12.63), WP: $4b$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1
002	0, 1, 2
003	0, 1
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	3
nontrivial	1, 2

Supplementary Table 64: Possible SI values in each gap. Supergroup: C_c2/m (12.63), WP: $4b$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 002, 003, 110, 111 112
3	G
4	000, 001, 002, 110, 112 113
5	G
6	000

9.2 WP: $8e$

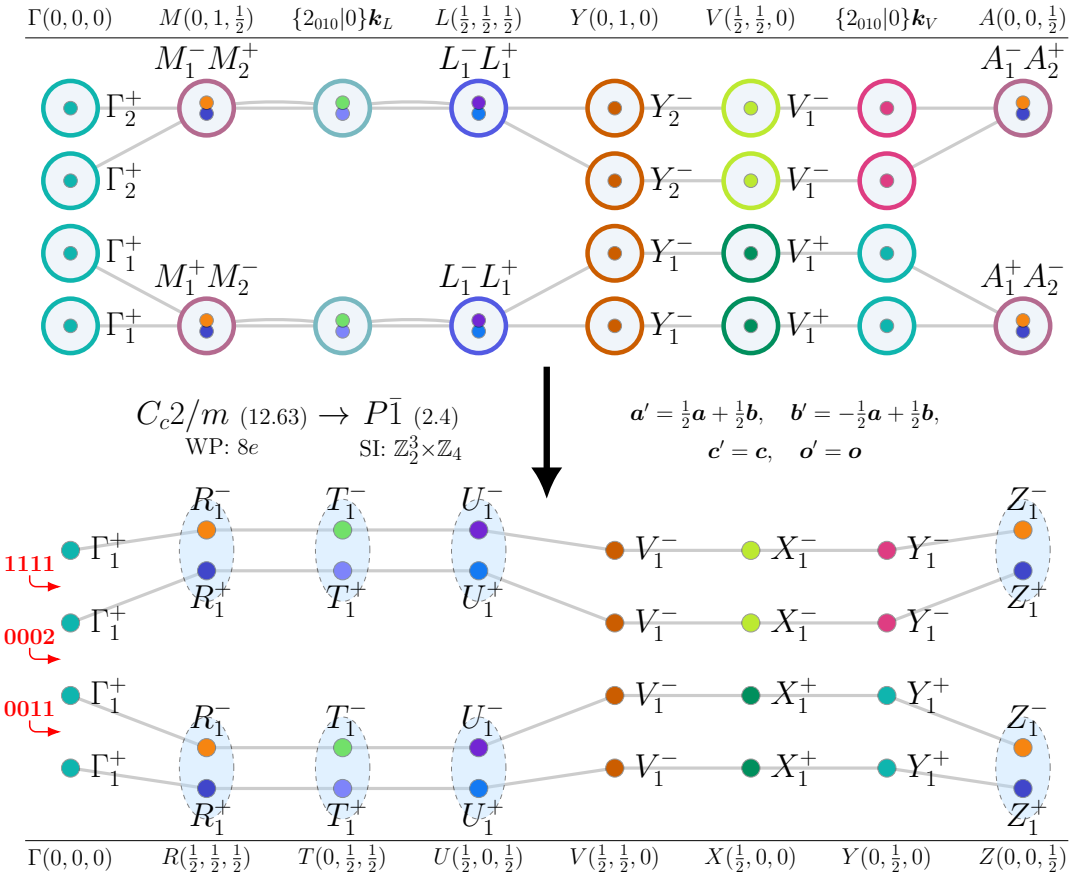
BCS Materials: $Fe_4Si_2Sn_7O_{16}$ (3.5 K)[32].

9.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[32]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.197`



Supplementary Figure 38: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8e$ of supergroup C_c2/m (12.63).

Supplementary Table 65: Number of times a given SI can appear in the band structure. Supergroup: C_c2/m (12.63), WP: $8e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0002	0, 1
0010	0, 1, 2
0011	0, 1
0012	0, 1, 2
0013	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	3

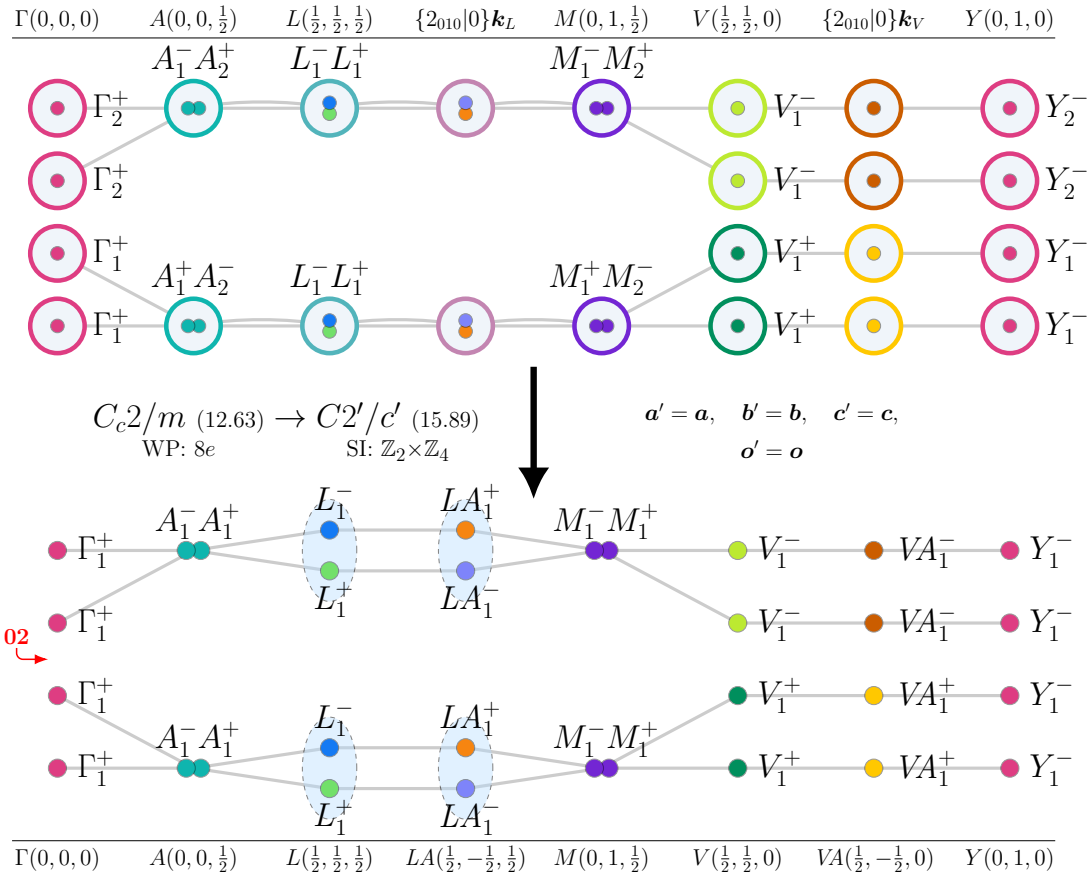
Supplementary Table 66: Possible SI values in each gap. Supergroup: C_c2/m (12.63), WP: $8e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0110, 0111 0112, 0113, 1010, 1011, 1012 1013, 1110, 1112, 1113
2	0002, 1100
3	0010, 0012, 0013, 0110, 0111 0112, 0113, 1010, 1011, 1012 1013, 1110, 1111, 1112
4	0000

9.2.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 39: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8e$ of supergroup C_c2/m (12.63).

Supplementary Table 67: Number of times a given SI can appear in the band structure. Supergroup: C_c2/m (12.63), WP: $8e$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	0, 1
10	0, 1
gapless	2
nontrivial	1

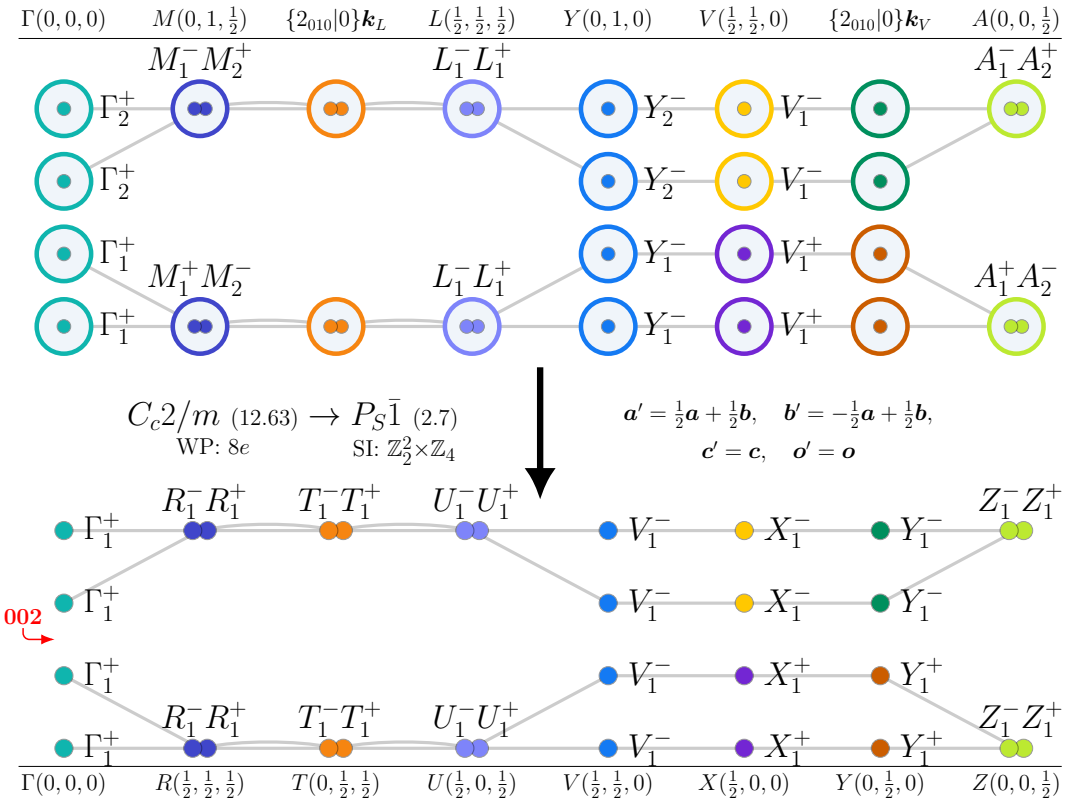
Supplementary Table 68: Possible SI values in each gap. Supergroup: C_c2/m (12.63), WP: $8e$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	02, 10
3	G
4	00

9.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 40: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8e$ of supergroup C_c2/m (12.63).

Supplementary Table 69: Number of times a given SI can appear in the band structure. Supergroup: C_c2/m (12.63), WP: $8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	0, 1
110	0, 1
gapless	2
nontrivial	1

Supplementary Table 70: Possible SI values in each gap. Supergroup: C_c2/m (12.63), WP: $8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	002, 110
3	G
4	000

10 MSG P_c4/mbm (127.396)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $P2'_1/m'$ (11.54), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $C2/m$ (12.58), $Cmc'm'$ (63.463), C_c2/m (12.63), $P2$ (3.1), $Cc'c'2$ (37.183), P_b2 (3.5), P_cba2 (32.139), $P2/m$ (10.42), $Cc'c'm$ (66.495), $Pn'n'm$ (58.397), P_b2/m (10.48), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_a2_1/c (14.80), $P4n'c'$ (104.207), $P4/mn'c'$ (128.405).

Trivial-SI Subgroups: Cc' (9.39), Pm' (6.20), Pc' (7.26), $C2'$ (5.15), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Cm (8.32), $Cmc'2'_1$ (36.175), C_cm (8.35), Pm (6.18), $Ama'2'$ (40.206), $Pmn'2'_1$ (31.126), P_bm (6.22), Pc (7.24), $Pn'a2'_1$ (33.146), P_ac (7.27), $C2$ (5.13), $Am'a'2$ (40.207), C_c2 (5.16), A_amm2 (38.192), $Pn'n'2$ (34.159), C_cmm2 (35.169), C_cmmm (65.488), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_amc2_1 (26.71), P_cbam (55.361), P_c4bm (100.176).

10.1 WP: $8h$

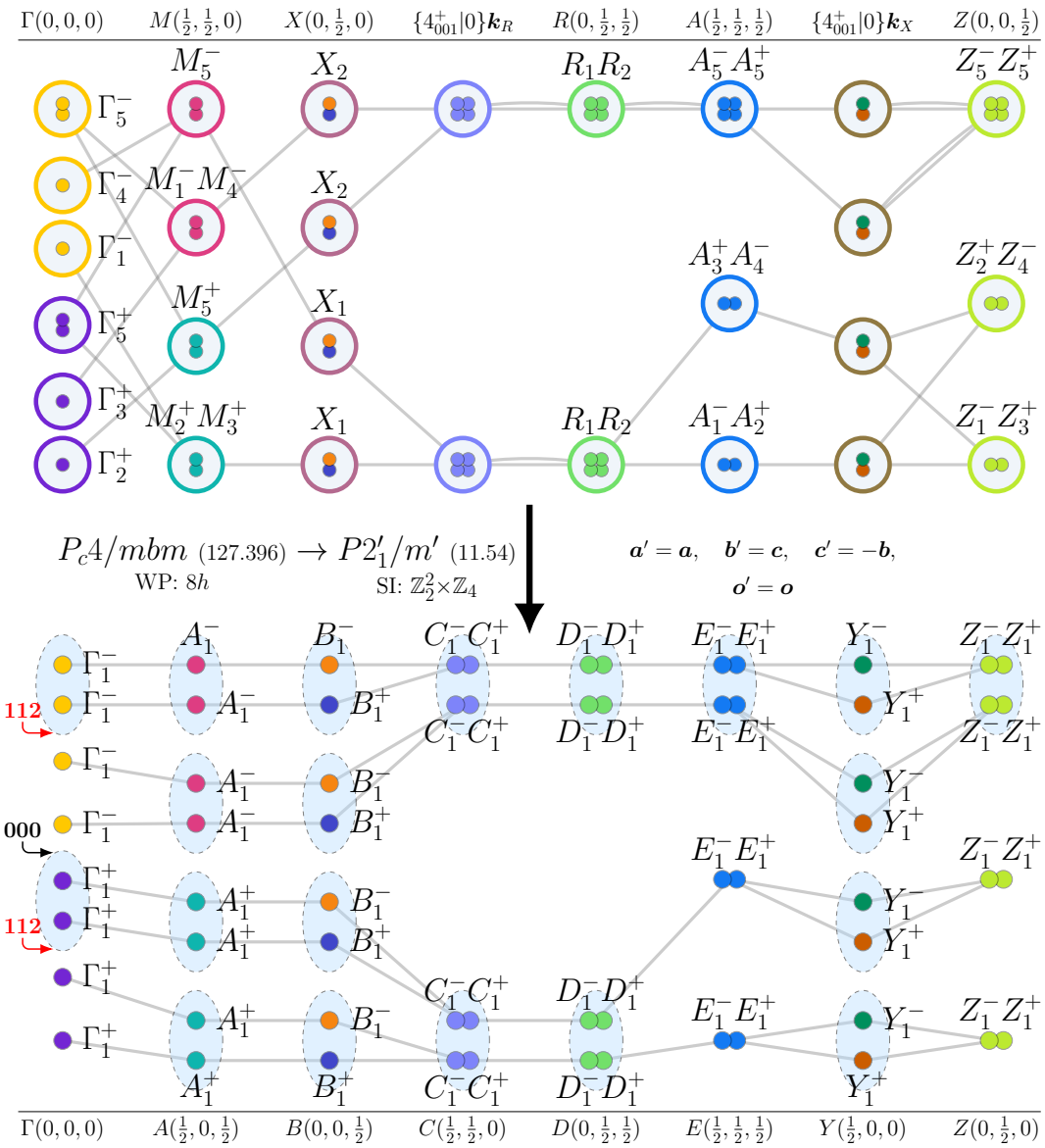
BCS Materials: $Yb_2Pd_2(In_{0.4}Sn_{0.6})$ (3 K)[33].

10.1.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].

[33]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.333`



Supplementary Figure 41: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $8h$ of supergroup P_c4/mbm (127.396).

Supplementary Table 71: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mbm (127.396), WP: $8h$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

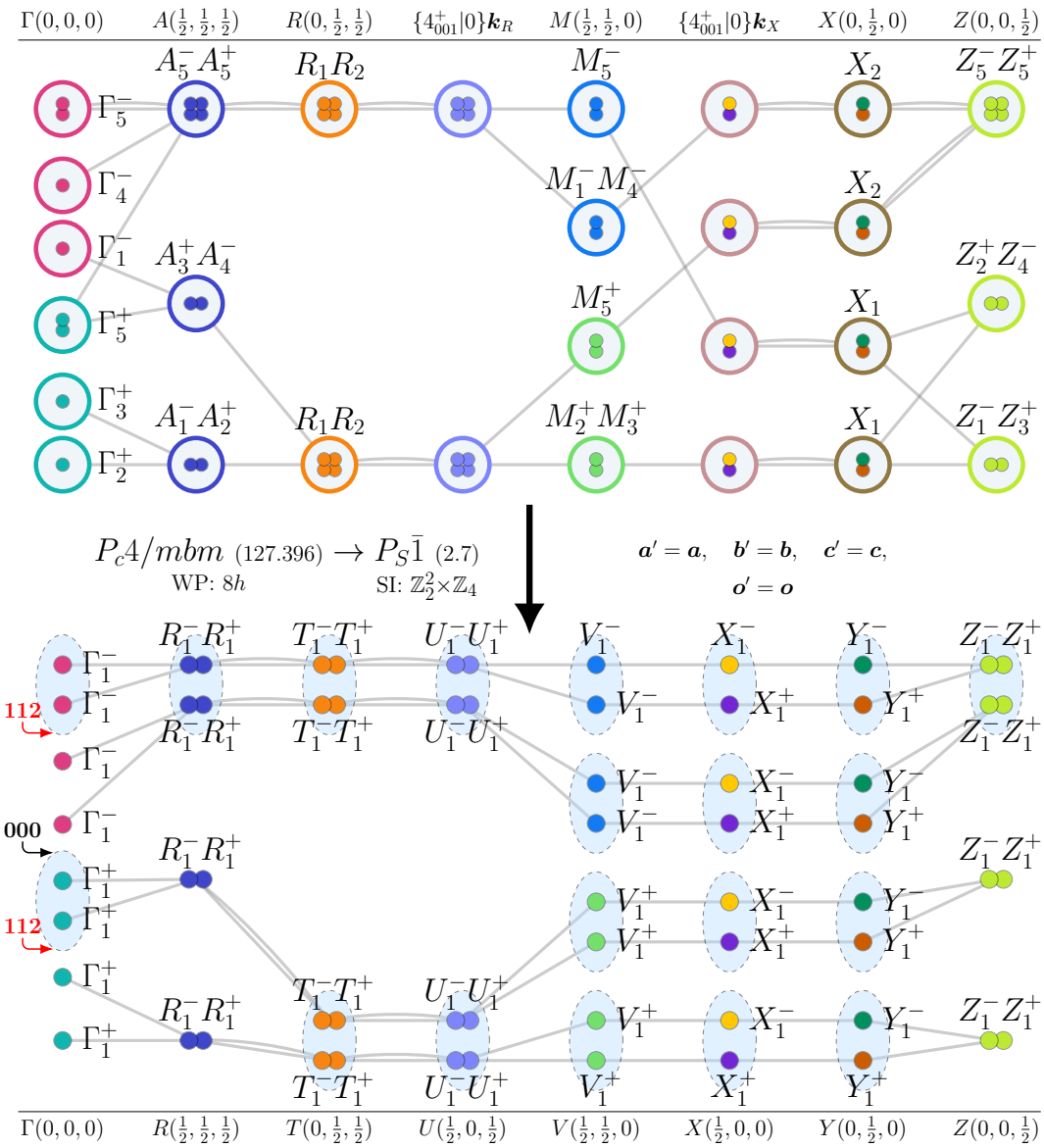
Supplementary Table 72: Possible SI values in each gap. Supergroup: P_c4/mbm (127.396), WP: $8h$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

10.1.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 42: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8h$ of supergroup P_c4/mbm (127.396).

Supplementary Table 73: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mbm (127.396), WP: $8h$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 74: Possible SI values in each gap. Supergroup: P_c4/mbm (127.396), WP: $8h$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

11 MSG P_c4/mnc (128.408)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2'_1/m'$ (11.54), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $C2/c$ (15.85), $Cm'cm'$ (63.464), C_c2/c (15.90), $P2$ (3.1), $Cm'm'2$ (35.168), $Pb'a'2$ (32.138), P_b2 (3.5), C_ccc2 (37.184), P_cnn2 (34.161), $P2/m$ (10.42), $Cm'm'm$ (65.485), $Pb'a'm$ (55.357), P_b2/m (10.48), C_ccm (66.498), $P2_1/c$ (14.75), $Pnm'a'$ (62.447), P_a2_1/c (14.80), P_cnnm (58.401), $P4b'm'$ (100.175), P_c4nc (104.208), $P4/mb'm'$ (127.393).

Trivial-SI Subgroups: Cm' (8.34), Pm' (6.20), Pc' (7.26), $C2'$ (5.15), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Cc (9.37), $Cm'c2'_1$ (36.174), C_cc (9.40), Pm (6.18), $Amm'2'$ (38.190), $Pmc'2'_1$ (26.69), P_bm (6.22), Pc (7.24), $Pna'2'_1$ (33.147), P_ac (7.27), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_ama2 (40.208), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_a2_1 (4.10), P_amn2_1 (31.128).

11.1 WP: $8g$

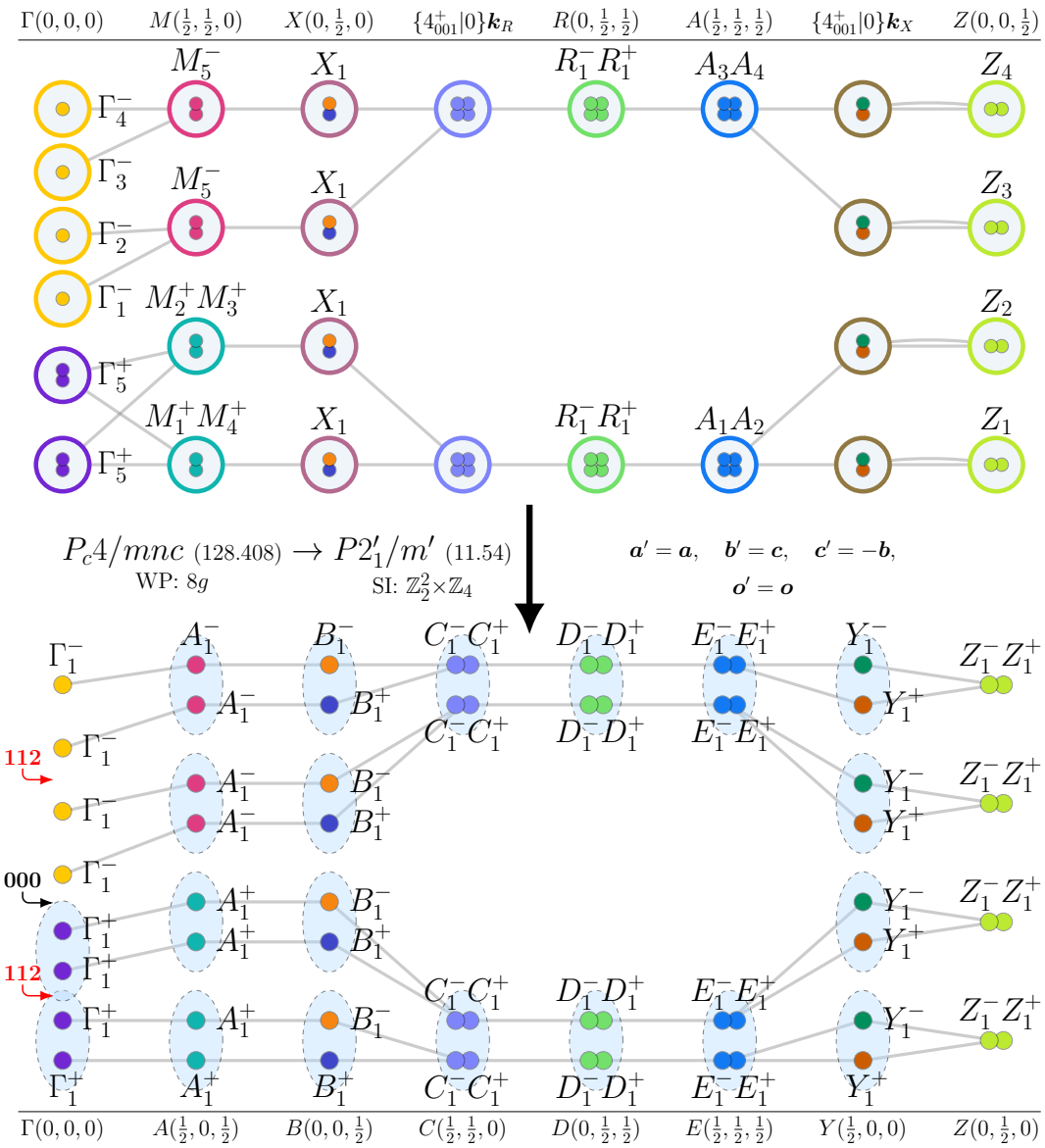
BCS Materials: $U_2Pd_{2.35}Sn_{0.65}$ (17 K)[34].

11.1.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].

[34]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.337`



Supplementary Figure 43: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $8g$ of supergroup P_c4/mnc (128.408).

Supplementary Table 75: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mnc (128.408), WP: $8g$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

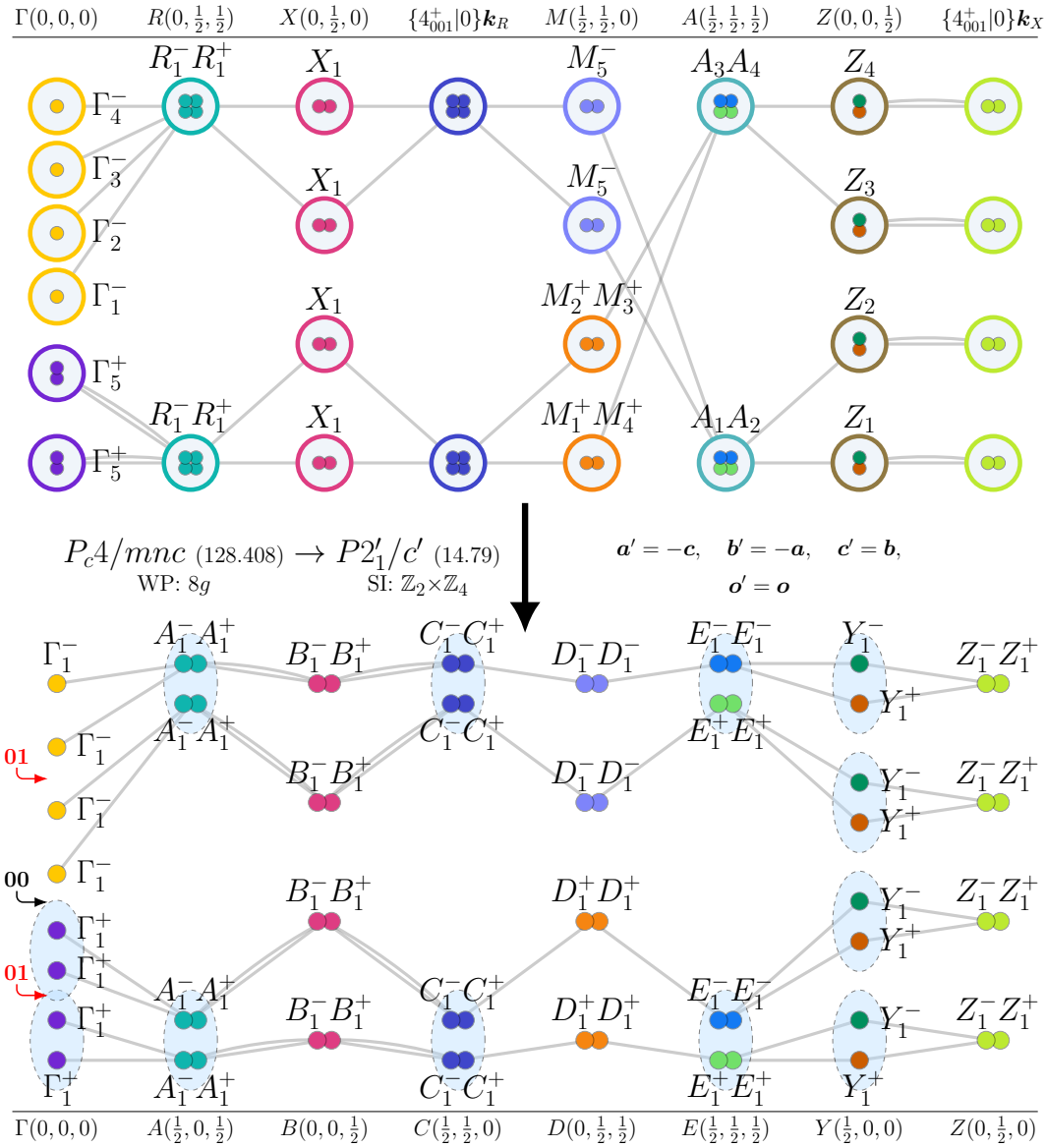
Supplementary Table 76: Possible SI values in each gap. Supergroup: P_c4/mnc (128.408), WP: $8g$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

11.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 44: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff position $8g$ of supergroup P_c4/mnc (128.408).

Supplementary Table 77: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mnc (128.408), WP: $8g$, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

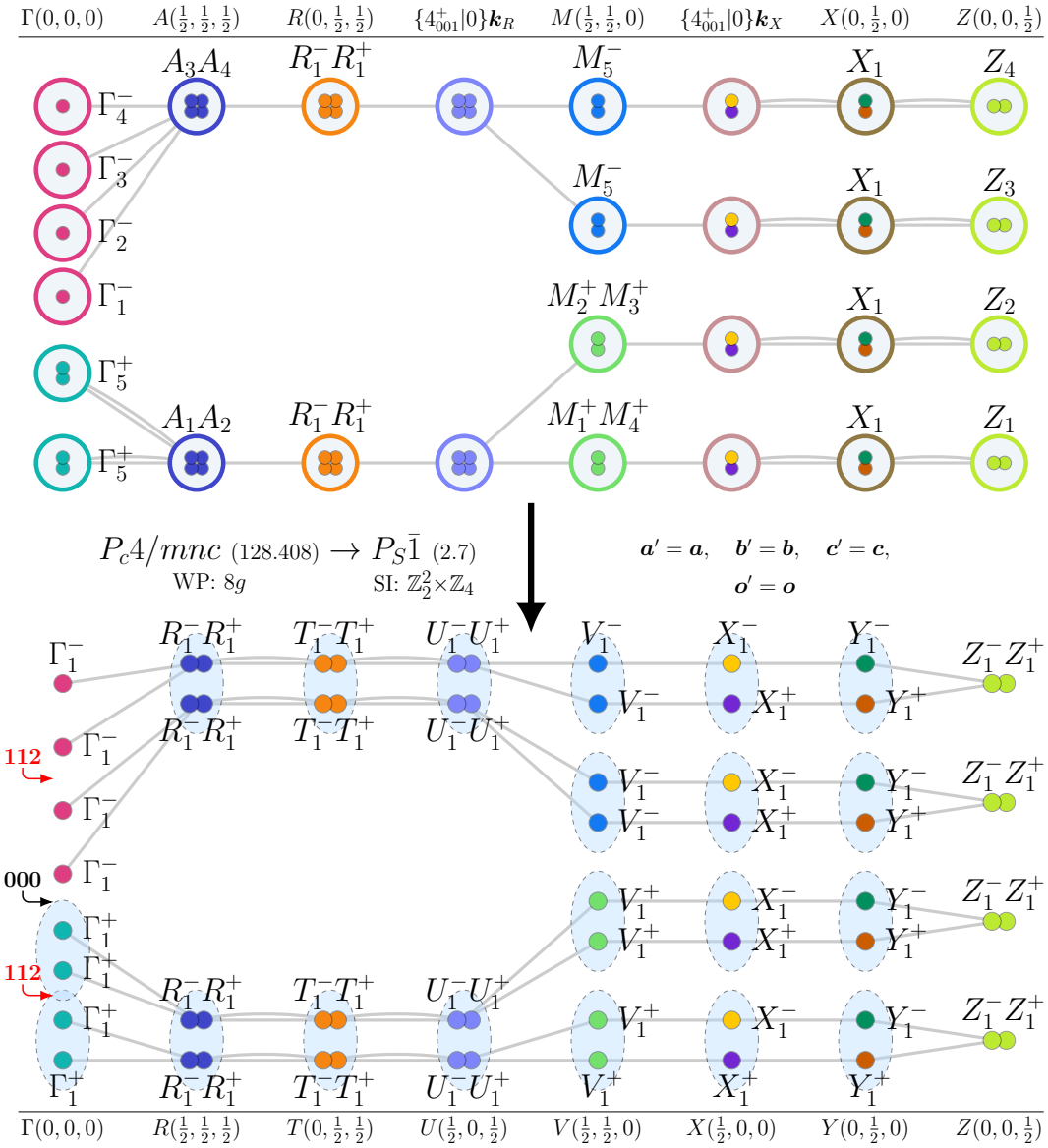
Supplementary Table 78: Possible SI values in each gap. Supergroup: P_c4/mnc (128.408), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

11.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 45: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8g$ of supergroup P_c4/mnc (128.408).

Supplementary Table 79: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mnc (128.408), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 80: Possible SI values in each gap. Supergroup: P_c4/mnc (128.408), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

11.2 WP: $8h$

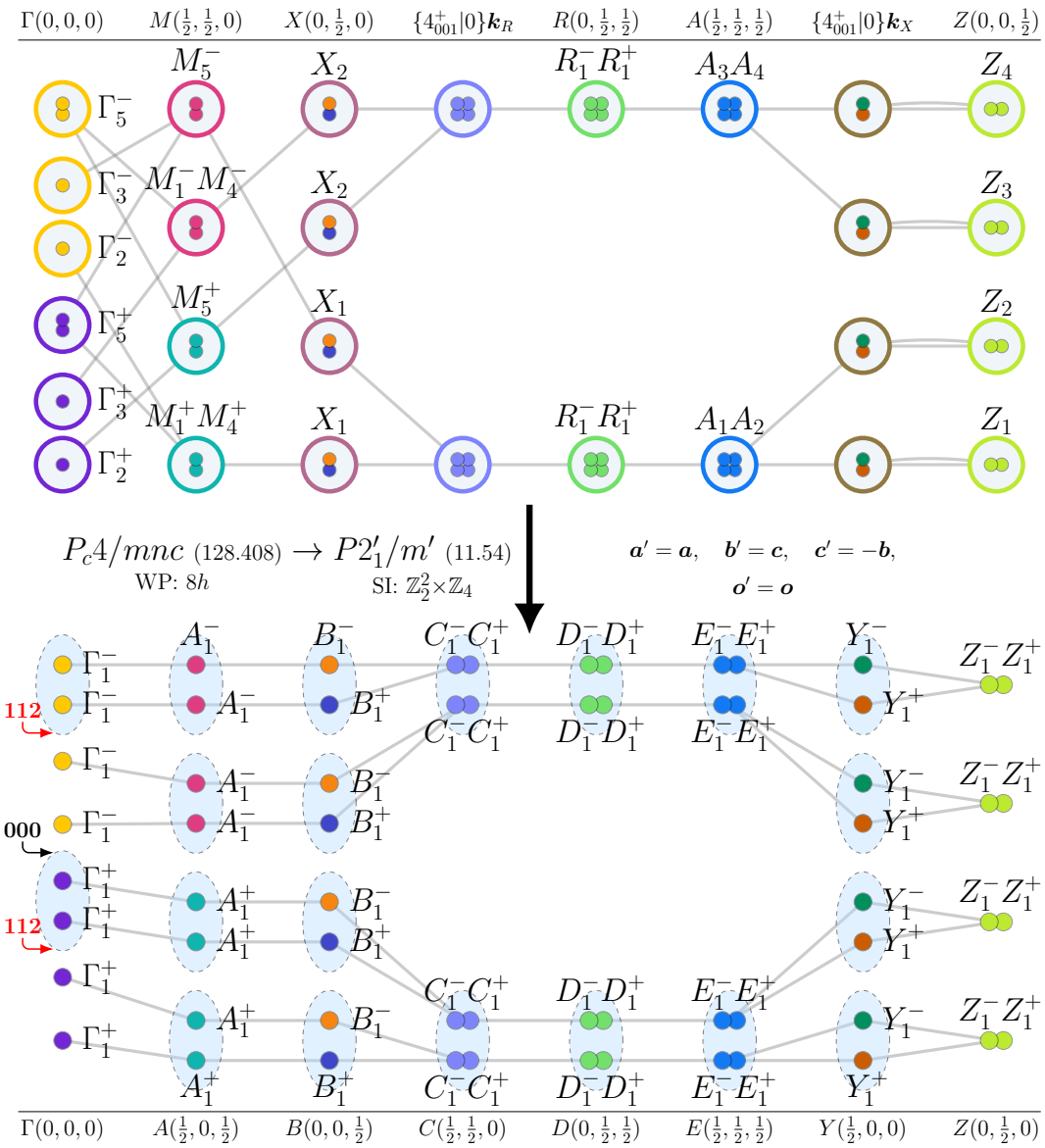
BCS Materials: U_2Ni_2In (15 K)[35], U_2Ni_2In (14.0 K)[36].

11.2.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].

[35]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.338`
[36]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.549`



Supplementary Figure 46: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $8h$ of supergroup P_c4/mnc (128.408).

Supplementary Table 81: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mnc (128.408), WP: $8h$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

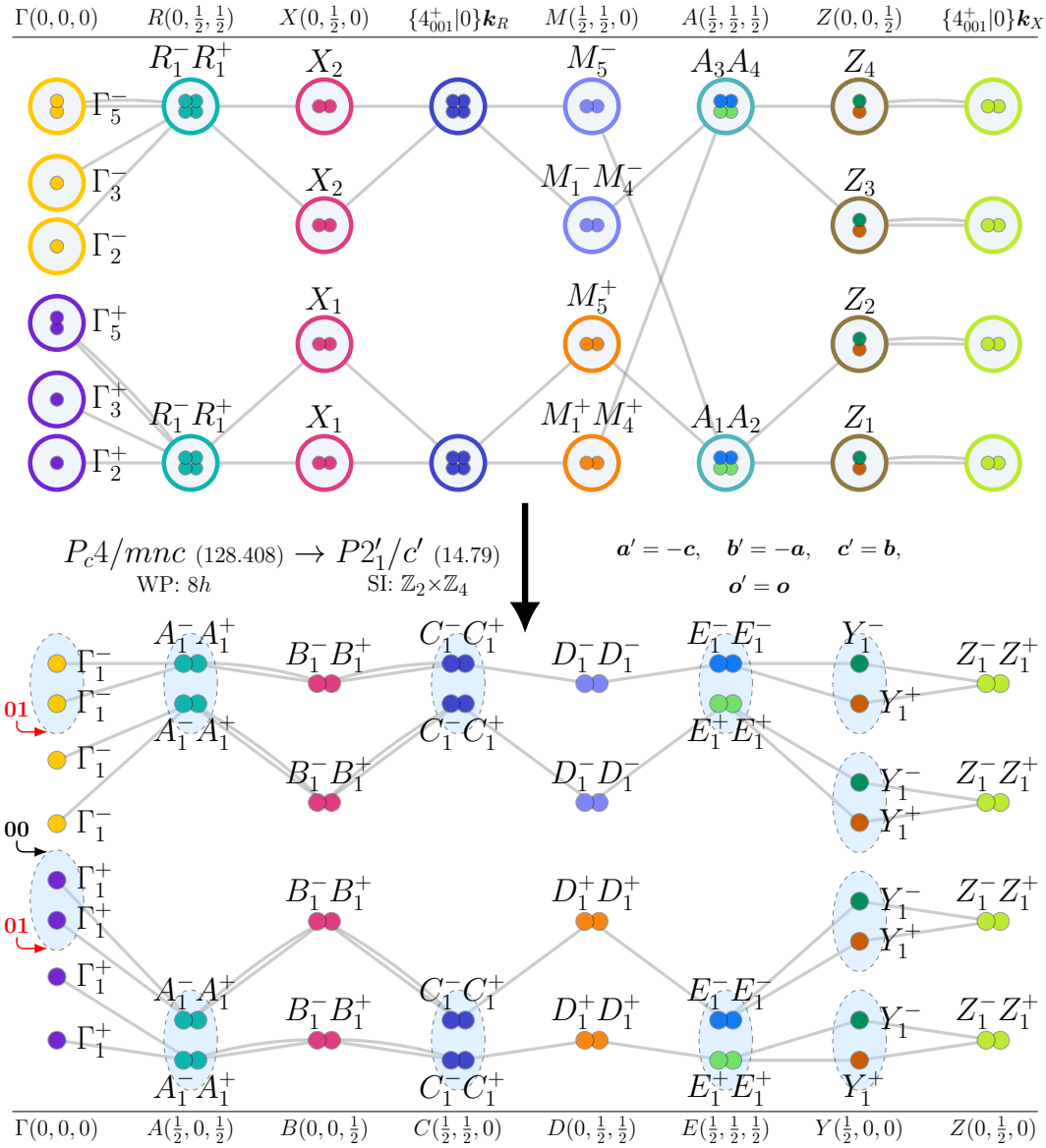
Supplementary Table 82: Possible SI values in each gap. Supergroup: P_c4/mnc (128.408), WP: $8h$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

11.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 47: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff position $8h$ of supergroup P_c4/mnc (128.408).

Supplementary Table 83: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mnc (128.408), WP: $8h$, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

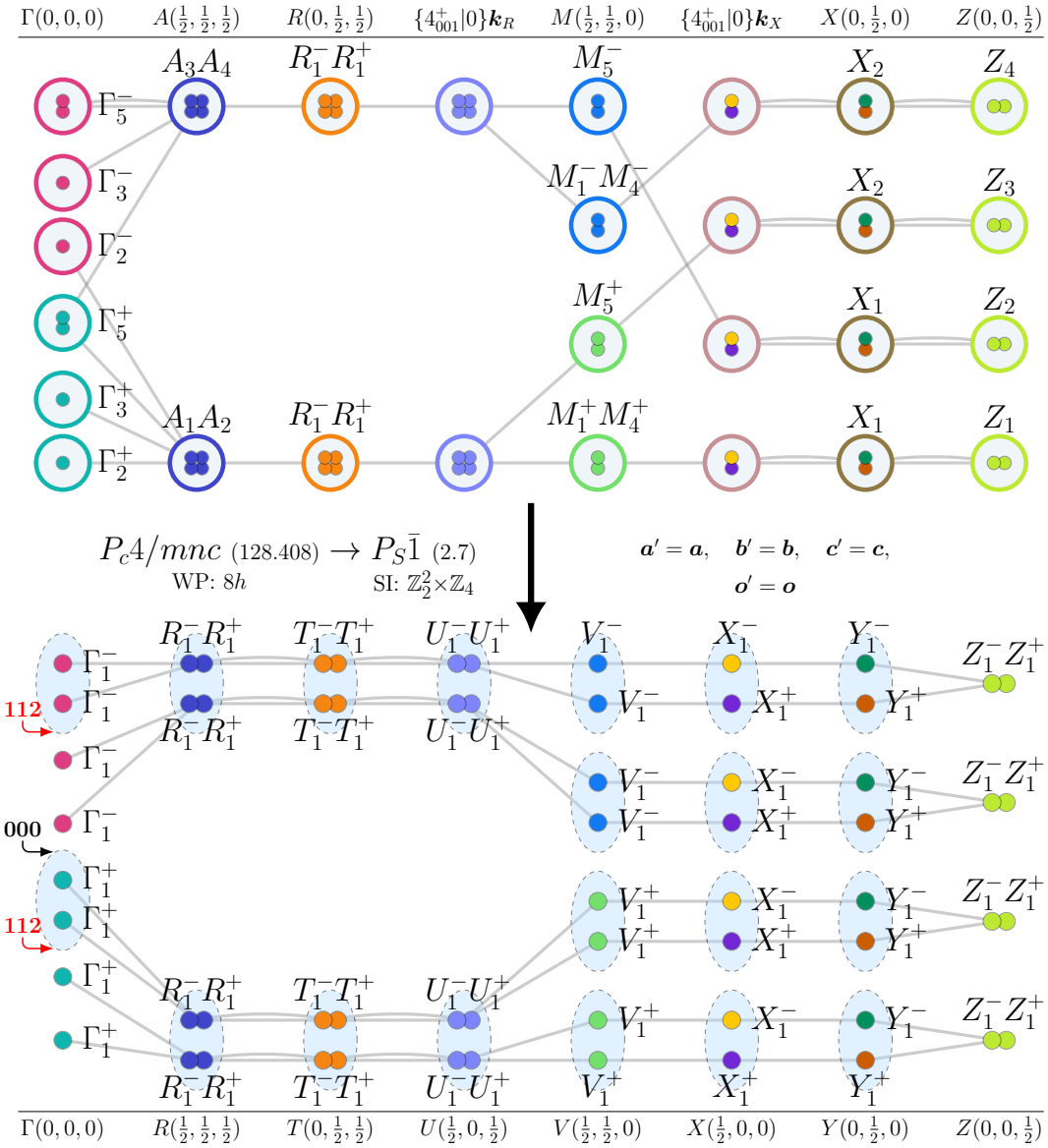
Supplementary Table 84: Possible SI values in each gap. Supergroup: P_c4/mnc (128.408), WP: $8h$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

11.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 48: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8h$ of supergroup P_c4/mnc (128.408).

Supplementary Table 85: Number of times a given SI can appear in the band structure. Supergroup: P_c4/mnc (128.408), WP: $8h$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 86: Possible SI values in each gap. Supergroup: P_c4/mnc (128.408), WP: $8h$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

12 MSG P_c4/ncc (130.432)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2'_1/c'$ (14.79), $P2'_1/m'$ (11.54), $P_S\bar{1}$ (2.7), $Ab'm'2$ (39.199), $C2/c$ (15.85), $Cm'ca'$ (64.476), C_c2/c (15.90), $P2$ (3.1), $Cm'm'2$ (35.168), $Pm'm'2$ (25.60), P_b2 (3.5), C_ccc2 (37.184), P_ccc2 (27.82), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pm'm'n$ (59.409), P_b2/c (13.71), C_cca (68.518), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_c2_1/c (14.82), P_ccn (56.373), $P4m'm'$ (99.167), P_c4cc (103.200), $P4/nm'm'$ (129.417).

Trivial-SI Subgroups: Cm' (8.34), Pc' (7.26), Pm' (6.20), $C2'$ (5.15), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Cc (9.37), $Cm'c2'_1$ (36.174), C_cc (9.40), Pc (7.24), $Abm'2'$ (39.198), $Pm'n2'_1$ (31.125), P_bc (7.29), Pc (7.24), $Pm'c2'_1$ (26.68), P_cc (7.28), $C2$ (5.13), C_c2 (5.16), A_aba2 (41.216), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_ana2_1 (33.149).

12.1 WP: $4c$

BCS Materials: USb_2 (206 K)[37], UP_2 (203 K)[38], UGeS (90 K)[39], CeSbTe (2.3 K)[40].

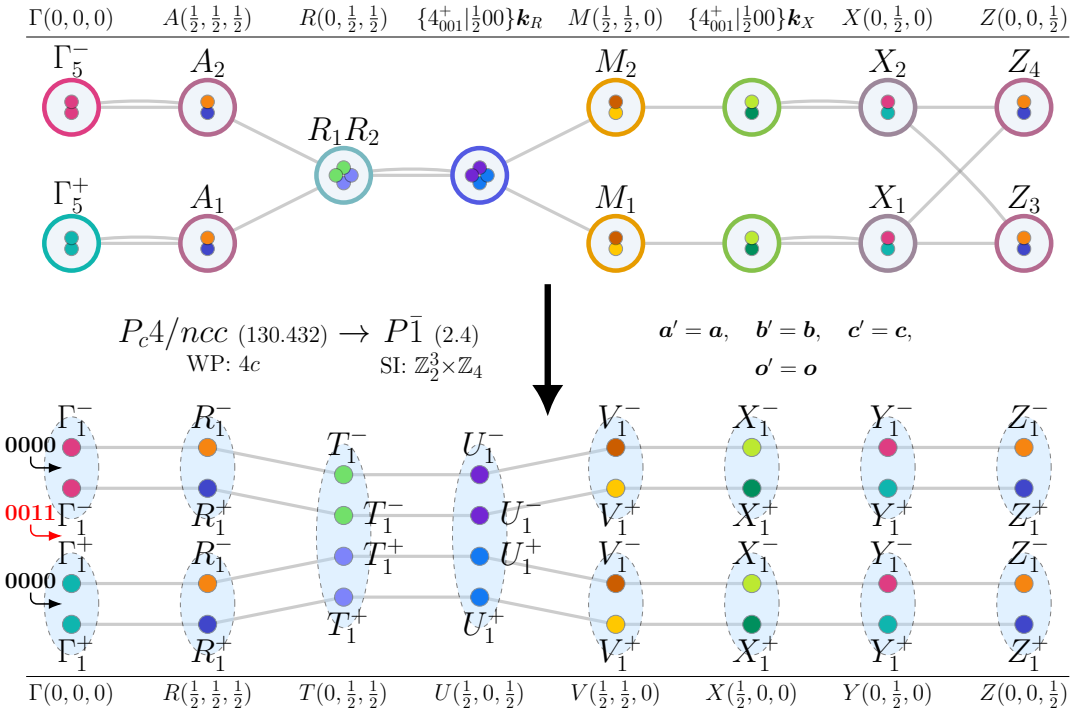
12.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,

[37]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.384`
[38]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.215`
[39]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.426`
[40]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.271`

- B $\perp$ [110] and strain $\parallel$ [100],
- B $\perp$ [110] and strain $\perp$ [001],
- B $\perp$ [110] and strain $\perp$ [100],
- B $\perp$ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 49: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position 4c of supergroup P_c4/ncc (130.432).

Supplementary Table 87: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: 4c, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2, 3
0012	0, 1, 2
0013	0, 1, 2, 3
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2, 3
1011	0, 1, 2
1012	0, 1, 2, 3

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SI	Possible #Gaps
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1, 2, 3
1112	0, 1, 2
1113	0, 1, 2, 3
gapless	0
nontrivial	1, 2, 3

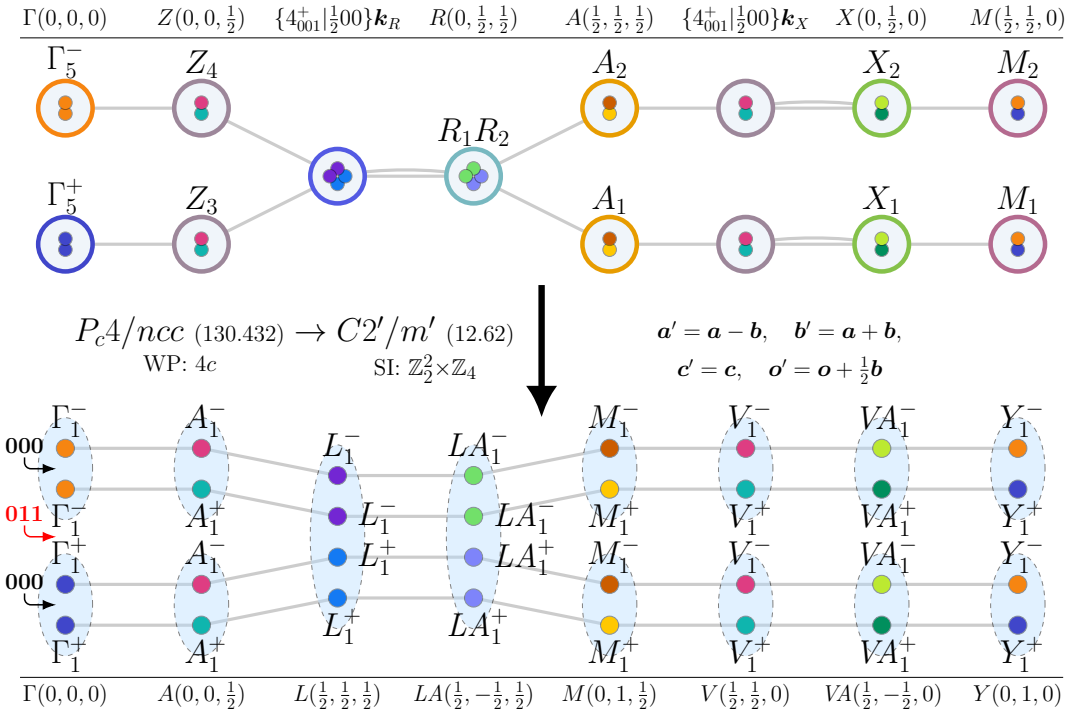
Supplementary Table 88: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0011, 0013, 0110, 0112, 1010 1012, 1111, 1113
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

12.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 50: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 89: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
011	0, 1, 2, 3
012	0, 1, 2
013	0, 1, 2, 3
100	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1, 2, 3
112	0, 1, 2
113	0, 1, 2, 3
gapless	0
nontrivial	1, 2, 3

Supplementary Table 90: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113
2	011, 013, 111, 113
3	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113

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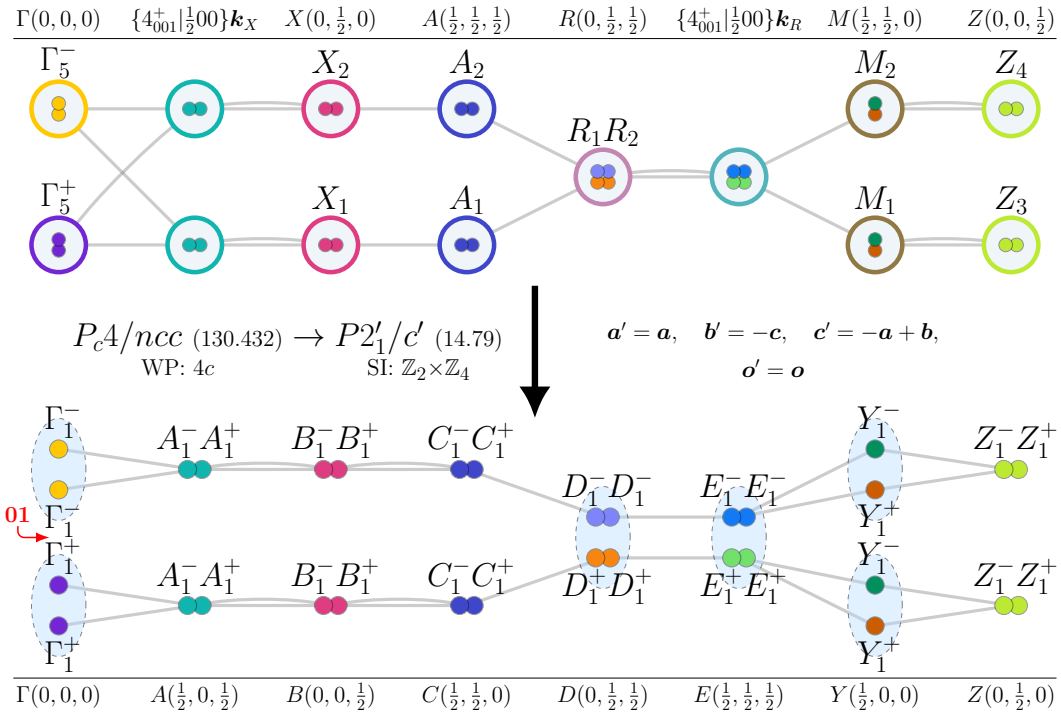
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Gap#	Possible SI Values
4	000

12.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 51: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 91: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

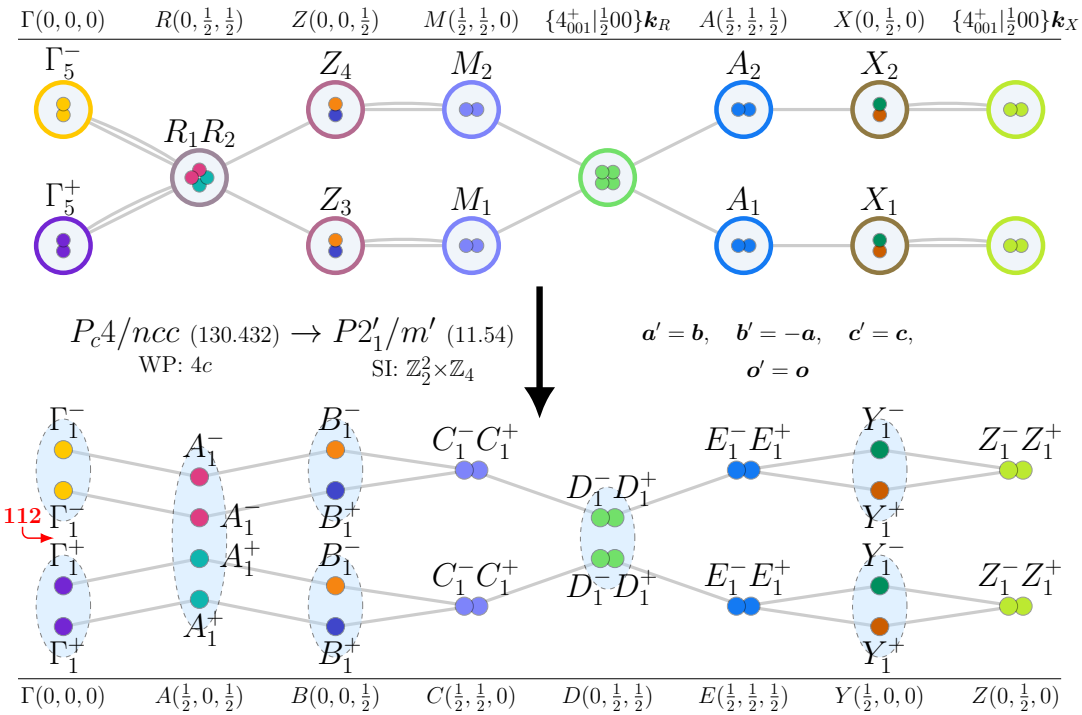
Supplementary Table 92: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

12.1.4 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 52: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 93: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
110	0, 1
111	0, 1
112	0, 1
113	0, 1
gapless	2
nontrivial	1

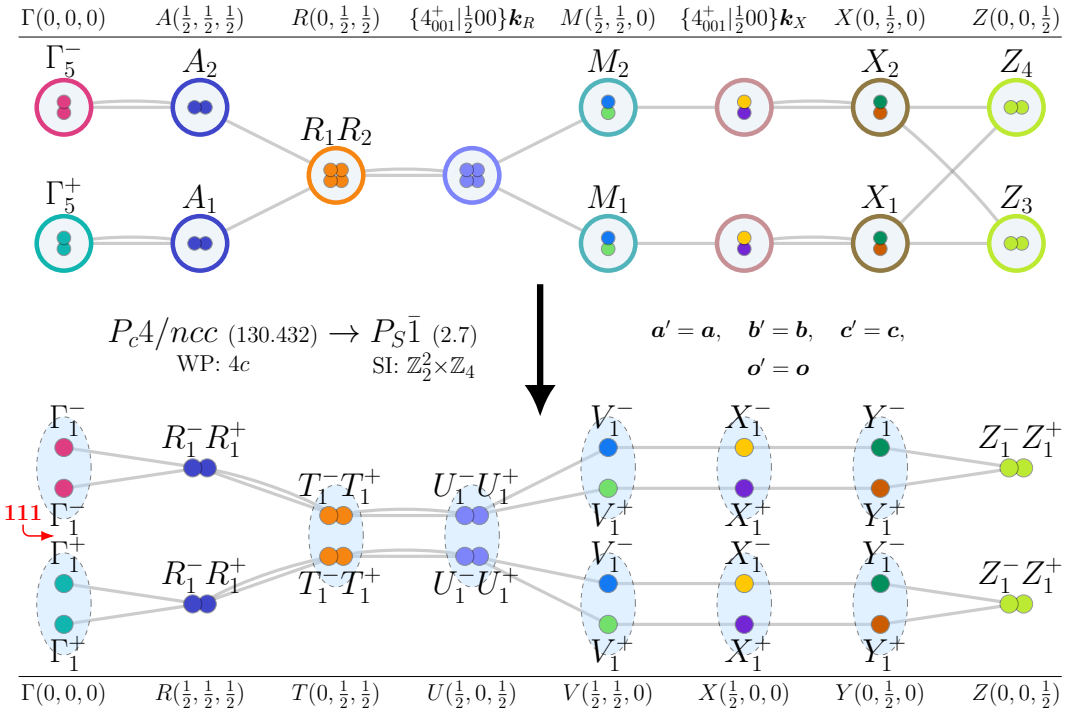
Supplementary Table 94: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: 4c, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	110, 111, 112, 113
3	G
4	000

12.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 53: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position 4c of supergroup P_c4/ncc (130.432).

Supplementary Table 95: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: 4c, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
111	0, 1
113	0, 1
gapless	2
nontrivial	1

Supplementary Table 96: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: 4c, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G

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Gap#	Possible SI Values
2	111, 113
3	G
4	000

12.2 WP: $4c + 4c$

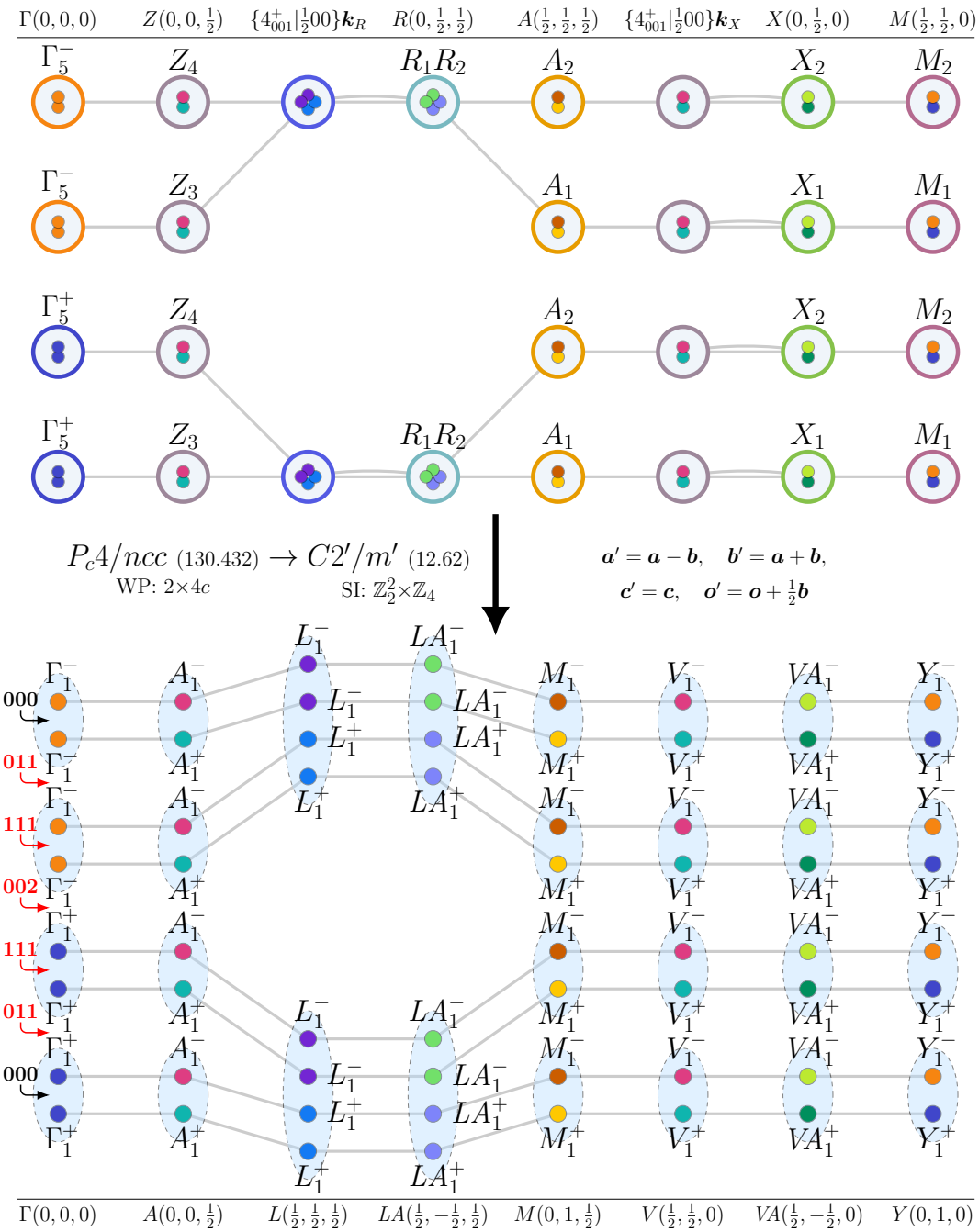
BCS Materials: UPd_2Ge_2 (80 K)[41].

12.2.1 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].

[41] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.535`



Supplementary Figure 54: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 97: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: 4c+4c, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2, 3, 4, 5
001	0, 1, 2, 3, 4
002	0, 1, 2, 3
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
011	0, 1, 2, 3, 4, 5, 6
012	0, 1, 2, 3, 4
013	0, 1, 2, 3, 4, 5, 6
100	0, 1, 2, 3, 4
101	0, 1, 2, 3, 4

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SI	Possible #Gaps
102	0, 1, 2
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	0, 1, 2, 3, 4, 5, 6
112	0, 1, 2, 3, 4
113	0, 1, 2, 3, 4, 5, 6
gapless	0
nontrivial	2, 3, 4, 5, 6, 7

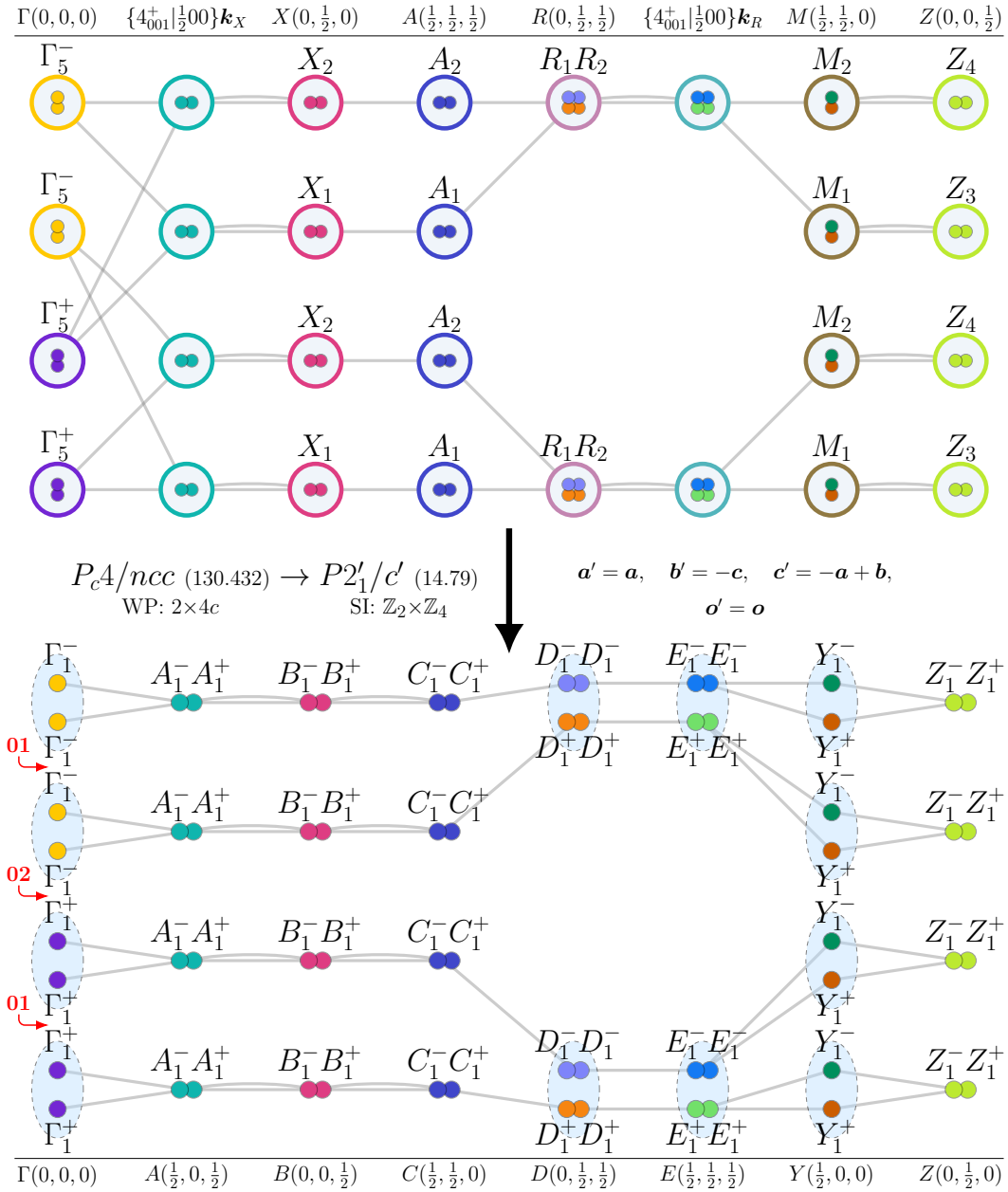
Supplementary Table 98: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: 4c+4c, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113
2	011, 013, 111, 113
3	000, 001, 002, 003, 010 011, 012, 013, 100, 101 102, 103, 110, 111, 112 113
4	000, 002
5	000, 001, 002, 003, 010 011, 012, 013, 100, 101 102, 103, 110, 111, 112 113
6	011, 013, 111, 113
7	000, 001, 003, 010, 011 012, 013, 100, 101, 103 110, 111, 112, 113
8	000

12.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 55: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 99: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: 4c+4c, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
gapless	4
nontrivial	2, 3

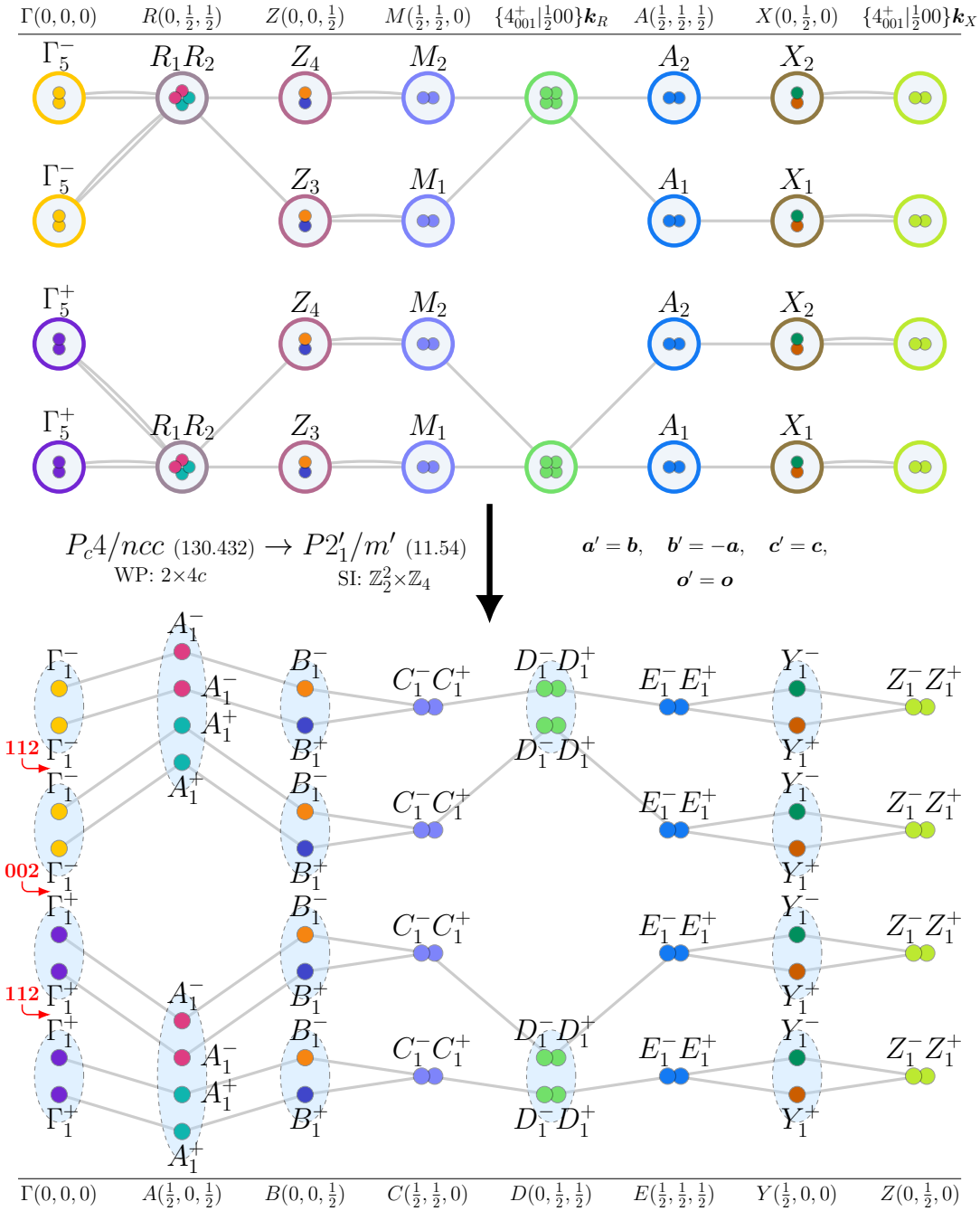
Supplementary Table 100: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: $4c$+$4c$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00, 02
5	G
6	01, 03
7	G
8	00

12.2.3 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 56: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 101: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: 4c+4c, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
002	0, 1
110	0, 1, 2
111	0, 1, 2
112	0, 1, 2
113	0, 1, 2
gapless	4
nontrivial	2, 3

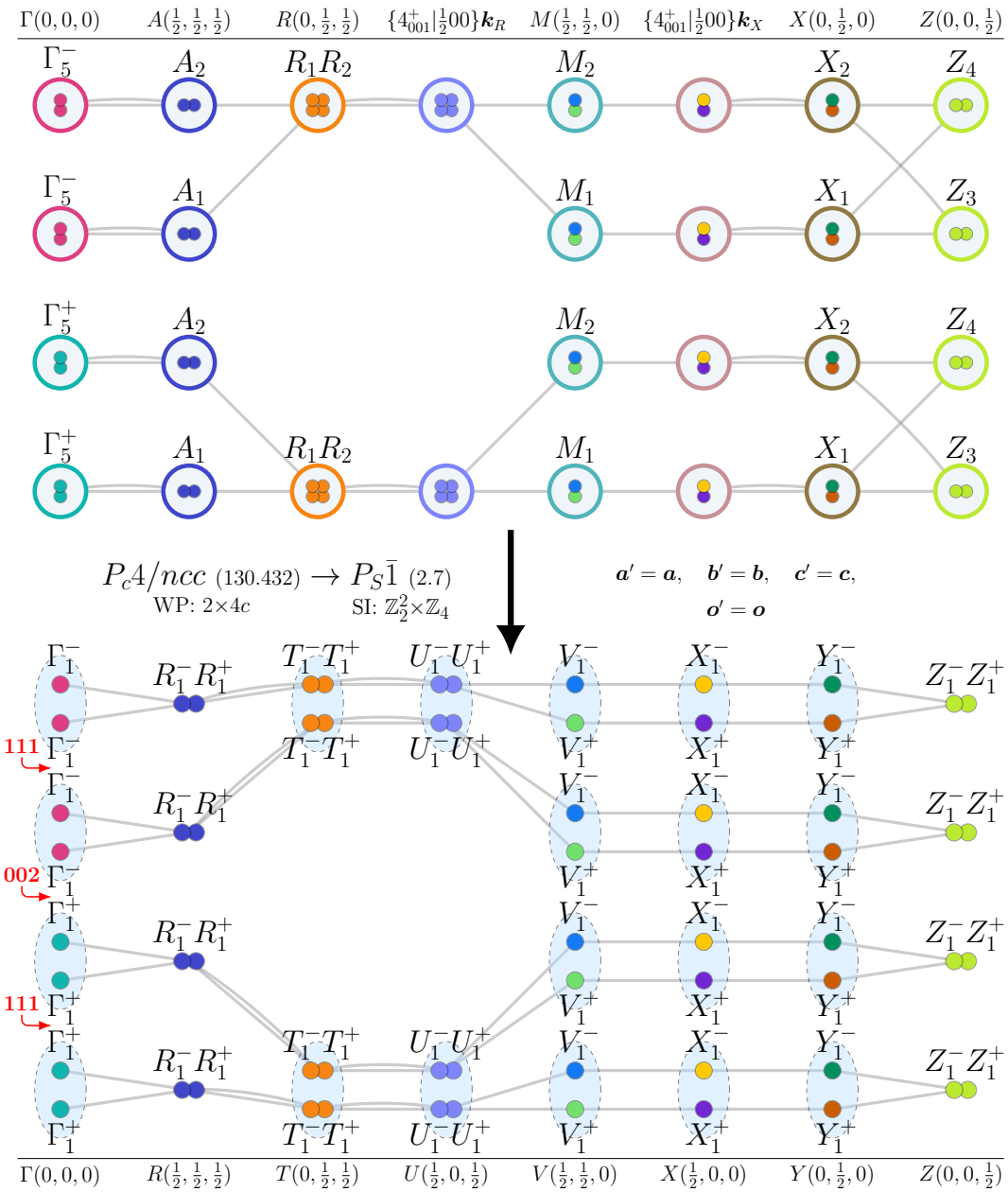
Supplementary Table 102: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: 4*c*+4*c*, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	110, 111, 112, 113
3	G
4	000, 002
5	G
6	110, 111, 112, 113
7	G
8	000

12.2.4 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 57: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup P_c4/ncc (130.432).

Supplementary Table 103: Number of times a given SI can appear in the band structure. Supergroup: P_c4/ncc (130.432), WP: 4c+4c, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1
111	0, 1, 2
113	0, 1, 2
gapless	4
nontrivial	2, 3

Supplementary Table 104: Possible SI values in each gap. Supergroup: P_c4/ncc (130.432), WP: 4*c*+4*c*, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000, 002
5	G
6	111, 113
7	G
8	000

13 MSG P_c4_2/mcm (132.456)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $P2'_1/m'$ (11.54), $P2'/m'$ (10.46), $P_S\bar{1}$ (2.7), $C2/m$ (12.58), $Cmc'm'$ (63.463), C_c2/m (12.63), $P2$ (3.1), $Cc'c'2$ (37.183), $Pm'm'2$ (25.60), P_b2 (3.5), P_cc2 (27.82), $P2/m$ (10.42), $Cc'c'm$ (66.495), $Pm'm'm$ (47.252), P_b2/m (10.48), $P2$ (3.1), $Pm'm'2$ (25.60), P_a2 (3.4), $P2/c$ (13.65), $Pm'm'a$ (51.294), P_c2/c (13.72), P_ccm (49.273), $P4_2m'c'$ (105.215), $P4_2/mm'c'$ (131.441).

Trivial-SI Subgroups: Cc' (9.39), Pm' (6.20), Pm' (6.20), $C2'$ (5.15), $P2'_1$ (4.9), $P2'$ (3.3), P_S1 (1.3), Cm (8.32), $Cmc'2'_1$ (36.175), C_cm (8.35), Pm (6.18), $Ama'2'$ (40.206), $Pm'm2'$ (25.59), P_bm (6.22), Pc (7.24), $Pm'c2'_1$ (26.68), P_cc (7.28), $C2$ (5.13), $Am'a'2$ (40.207), C_c2 (5.16), A_amm2 (38.192), C_cmm2 (35.169), C_cmmm (65.488), P_ama2 (28.92), P_c4_2cm (101.184).

13.1 WP: $4f$

BCS Materials: Mn_3Pt (475 K)[42].

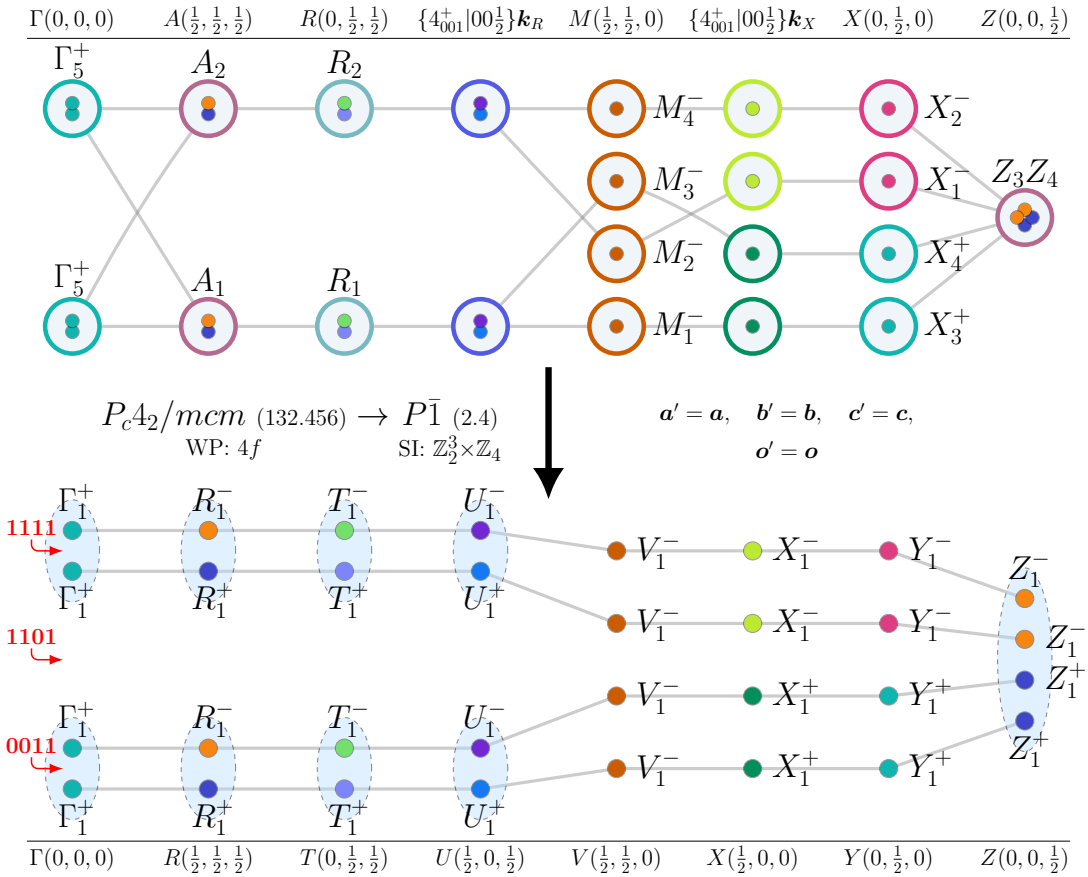
13.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],

[42]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.143`

- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 58: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4f$ of supergroup P_c4_2/mcm (132.456).

Supplementary Table 105: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1
0002	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1
0012	0, 1, 2
0013	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1
1101	0, 1
1103	0, 1
1110	0, 1, 2

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SI	Possible #Gaps
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	3

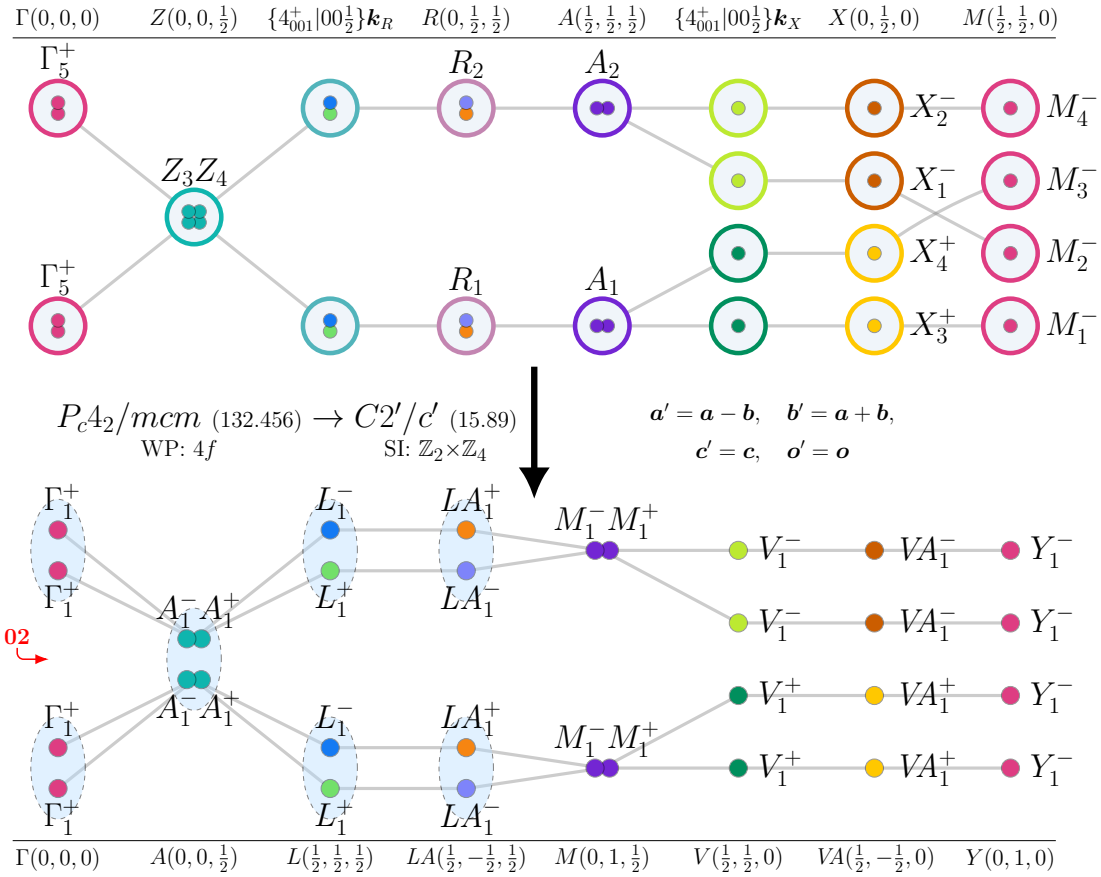
Supplementary Table 106: Possible SI values in each gap. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0110, 0111 0112, 0113, 1010, 1011, 1012 1013, 1110, 1112, 1113
2	0001, 0002, 0003, 1100, 1101 1103
3	0010, 0012, 0013, 0110, 0111 0112, 0113, 1010, 1011, 1012 1013, 1110, 1111, 1112
4	0000

13.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 59: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $4f$ of supergroup P_c4_2/mcm (132.456).

Supplementary Table 107: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	0, 1
10	0, 1
gapless	2
nontrivial	1

Supplementary Table 108: Possible SI values in each gap. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $C2'/c'$ (15.89).

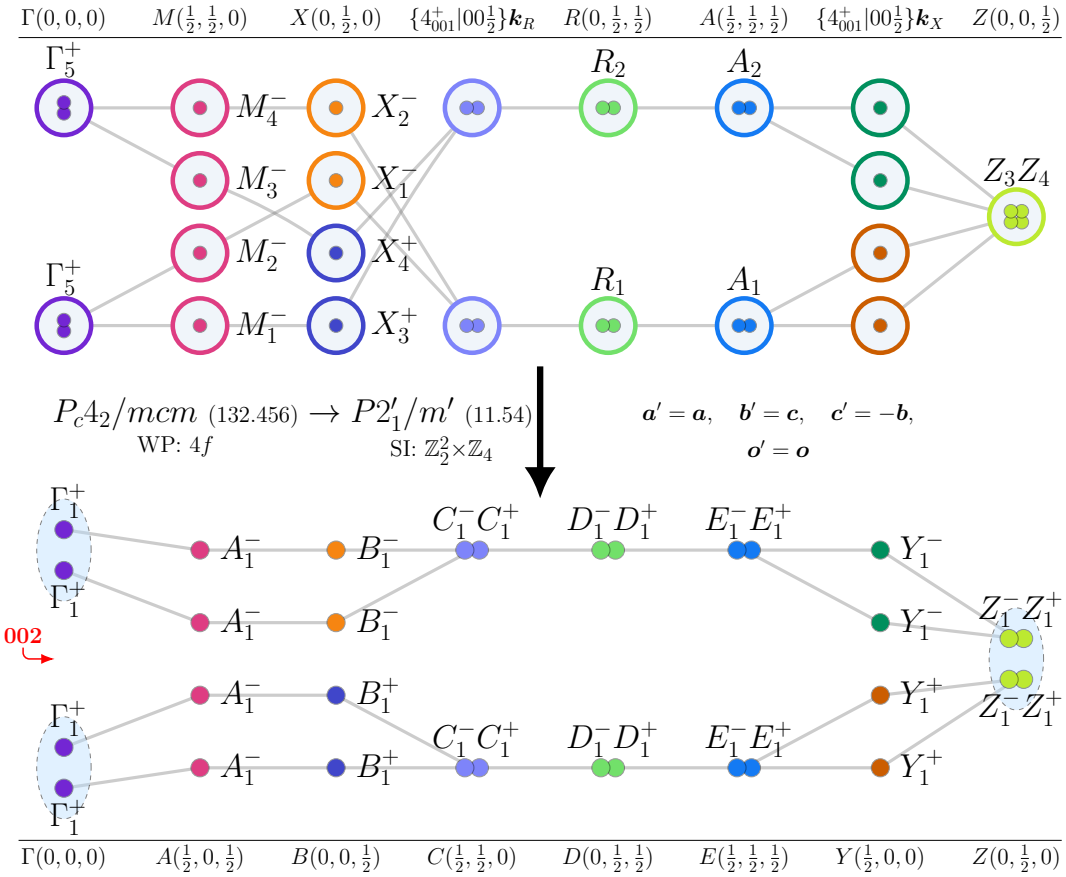
Gap#	Possible SI Values
1	G
2	02, 10
3	G
4	00

13.1.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],

- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 60: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $4f$ of supergroup P_c4_2/mcm (132.456).

Supplementary Table 109: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0
002	0, 1
110	0, 1
gapless	2
nontrivial	1

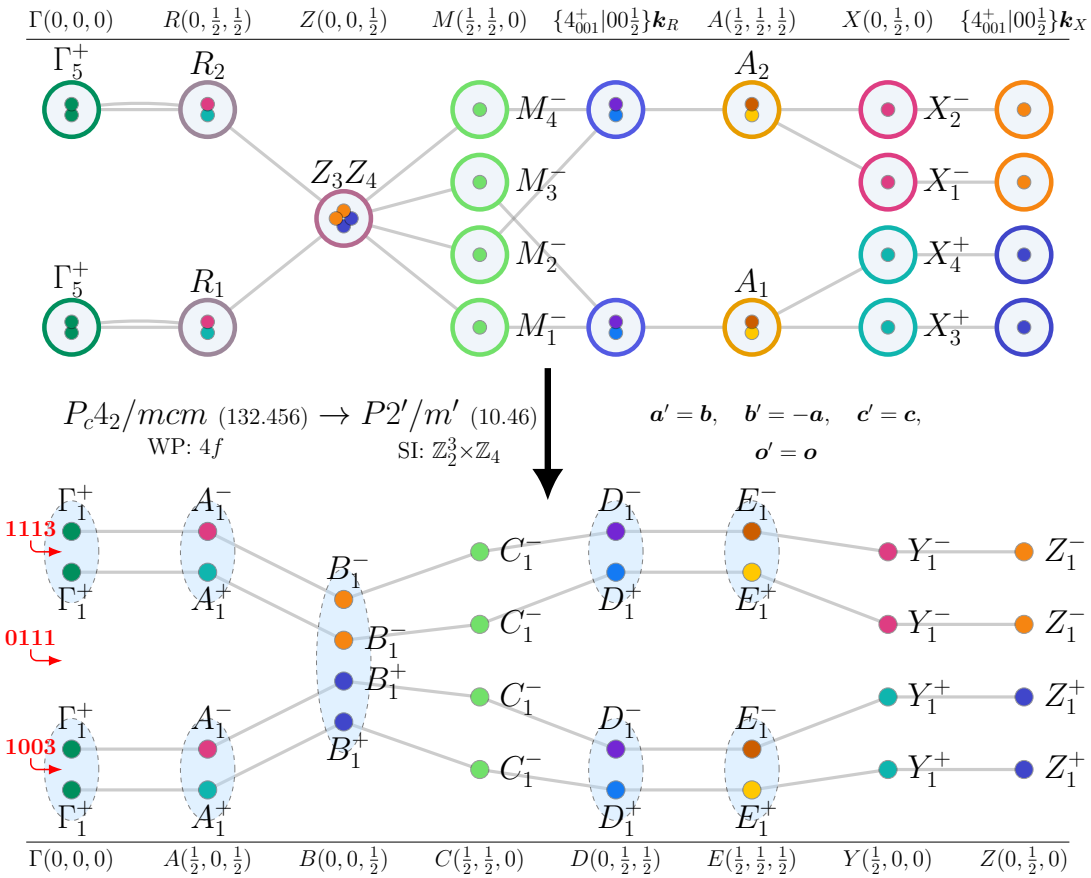
Supplementary Table 110: Possible SI values in each gap. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	002, 110
3	G
4	000

13.1.4 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 61: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4f$ of supergroup P_c4_2/mcm (132.456).

Supplementary Table 111: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0
0001	0, 1
0002	0, 1
0003	0, 1
0110	0, 1
0111	0, 1
0113	0, 1
1000	0, 1, 2
1001	0, 1
1002	0, 1, 2
1003	0, 1
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2

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SI	Possible #Gaps
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	3

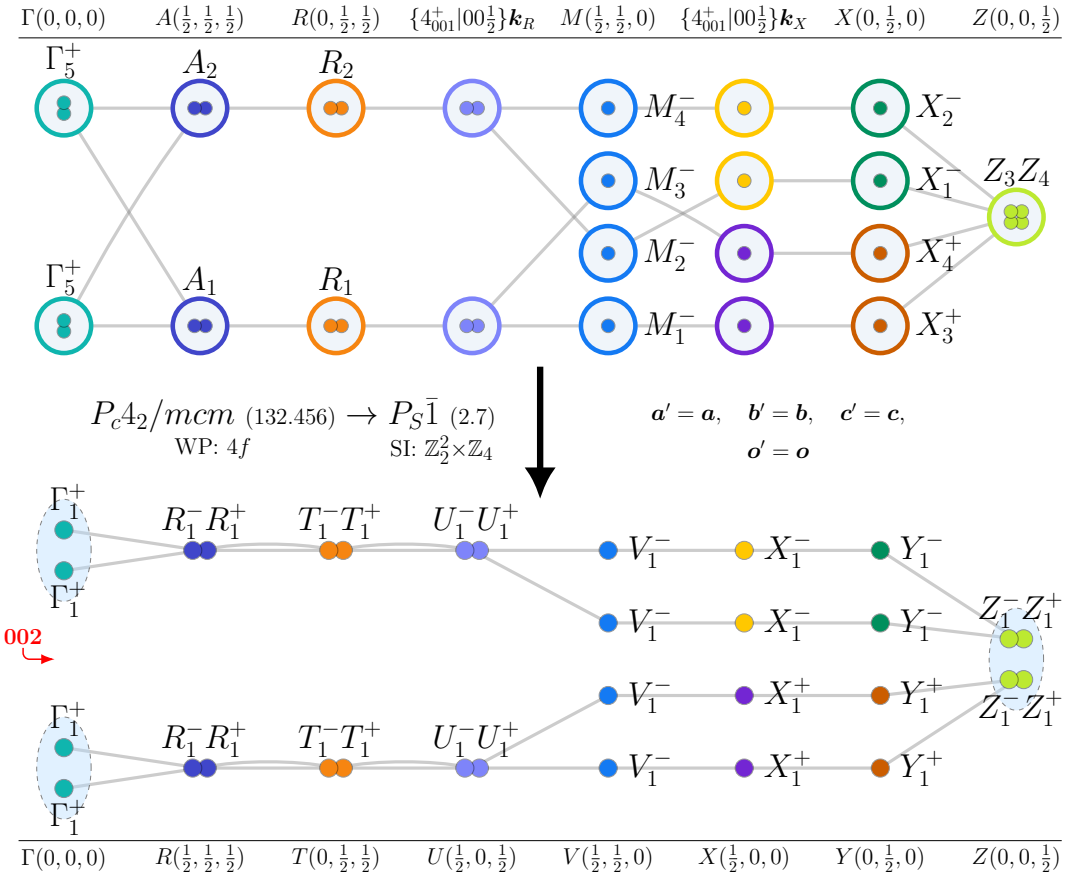
Supplementary Table 112: Possible SI values in each gap. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	1000, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111, 1112
2	0001, 0002, 0003, 0110, 0111 0113
3	1000, 1001, 1002, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1112, 1113
4	0000

13.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 62: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4f$ of supergroup P_c4_2/mcm (132.456).

Supplementary Table 113: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	0, 1
110	0, 1
gapless	2
nontrivial	1

Supplementary Table 114: Possible SI values in each gap. Supergroup: P_c4_2/mcm (132.456), WP: $4f$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	002, 110
3	G
4	000

14 MSG P_C4_2/nnm (134.481)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2'/m'$ (10.46), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), C_a2 (5.17), $C2/m$ (12.58), $Cmm'm'$ (65.486), C_a2/m (12.64), $P2$ (3.1), $Cm'm'2$ (35.168), $Pc'c'2$ (27.81), P_a2 (3.4), P_Cnn2 (34.163), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pc'c'n$ (56.369),

P_c2/c (13.72), $P2$ (3.1), $Pm'a'2$ (28.91), $P2/c$ (13.65), $Pm'na'$ (53.328), P_C2/c (13.74), P_Cnnn (48.263), $P4_2c'm'$ (101.183), $P4_2/nc'm'$ (138.525).

Trivial-SI Subgroups: Cm' (8.34), Pm' (6.20), Pc' (7.26), $C2'$ (5.15), $P2'$ (3.3), $P2'_1$ (4.9), P_S1 (1.3), Cm (8.32), $Cm'm2'$ (35.167), C_am (8.36), Pc (7.24), $Abm'2'$ (39.198), $Pna'2'_1$ (33.147), P_cc (7.28), Pc (7.24), $Pnc'2'$ (30.114), P_Cc (7.30), $C2$ (5.13), $Am'm'2$ (38.191), A_bbm2 (39.201), C_amm2 (35.170), C_amma (67.509), P_C2 (3.6), P_Ann2 (34.162), P_C4_2nm (102.193).

14.1 WP: $4d$

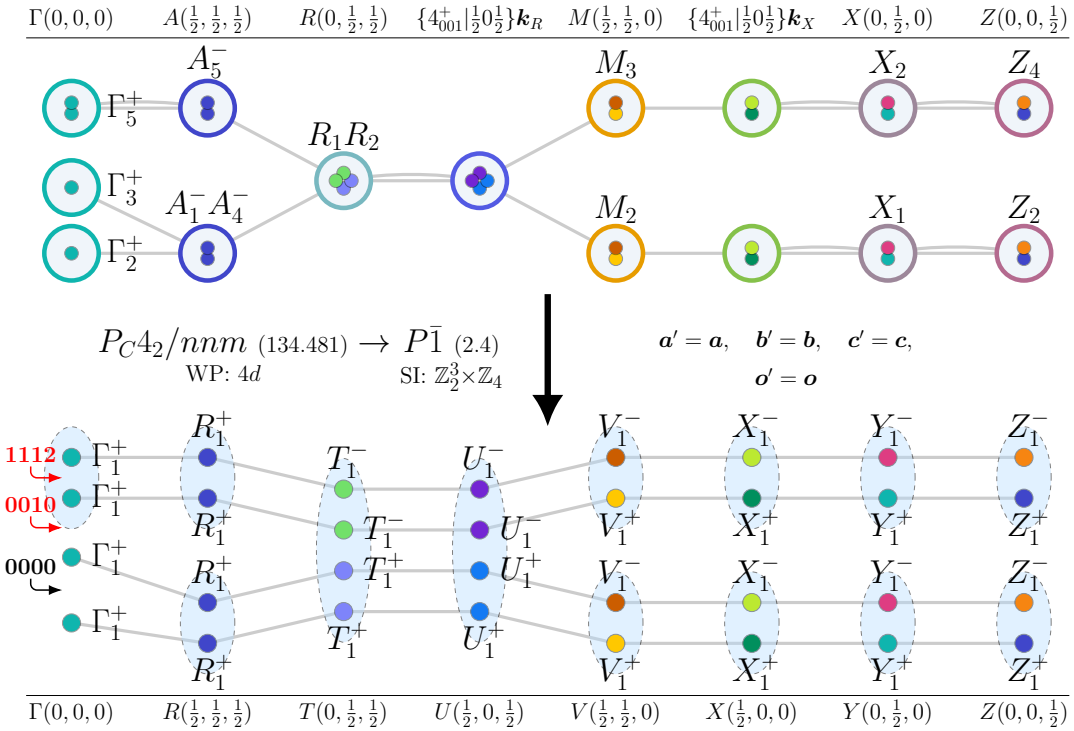
BCS Materials: $LaSrFeO_4$ (380 K)[43], Nd_2CuO_4 (276 K)[44], Pr_2CuO_4 (270 K)[45].

14.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.

[43] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.42`
[44] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.6`
[45] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.48`



Supplementary Figure 63: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 115: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	0, 1, 2, 3
1113	0, 1

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

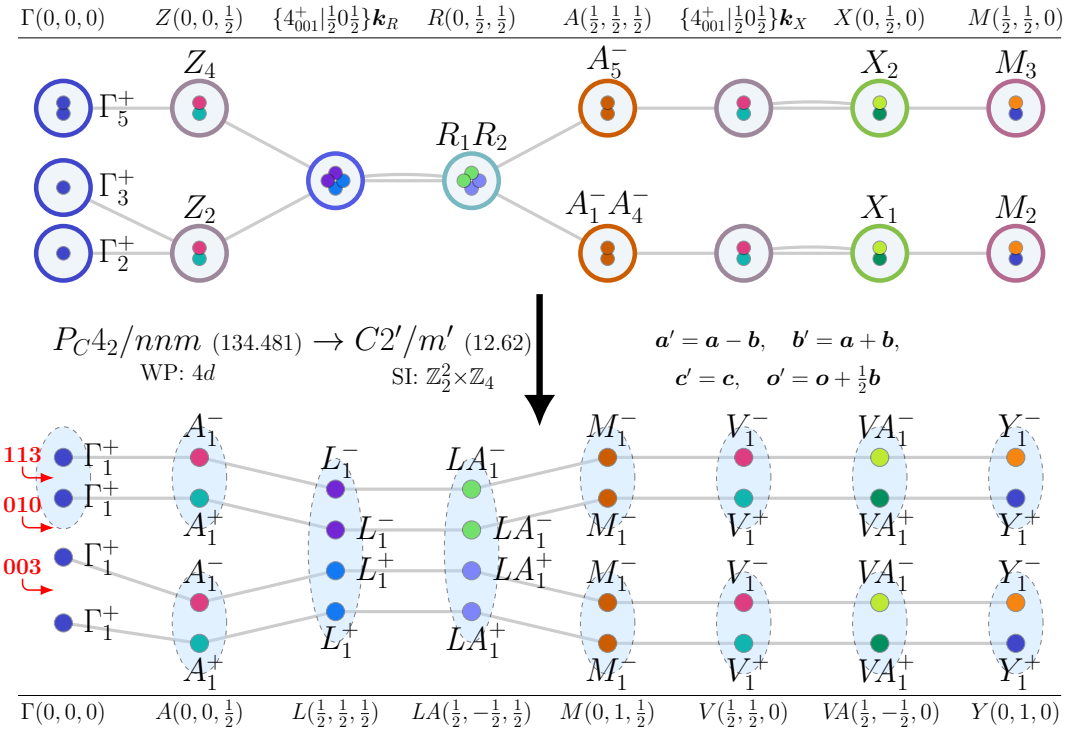
Supplementary Table 116: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	0010, 0012, 0111, 0113, 1011 1013, 1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

14.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 64: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 117: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
010	0, 1, 2, 3
011	0, 1
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
110	0, 1, 2, 3
111	0, 1
113	0, 1
gapless	0
nontrivial	1, 2, 3

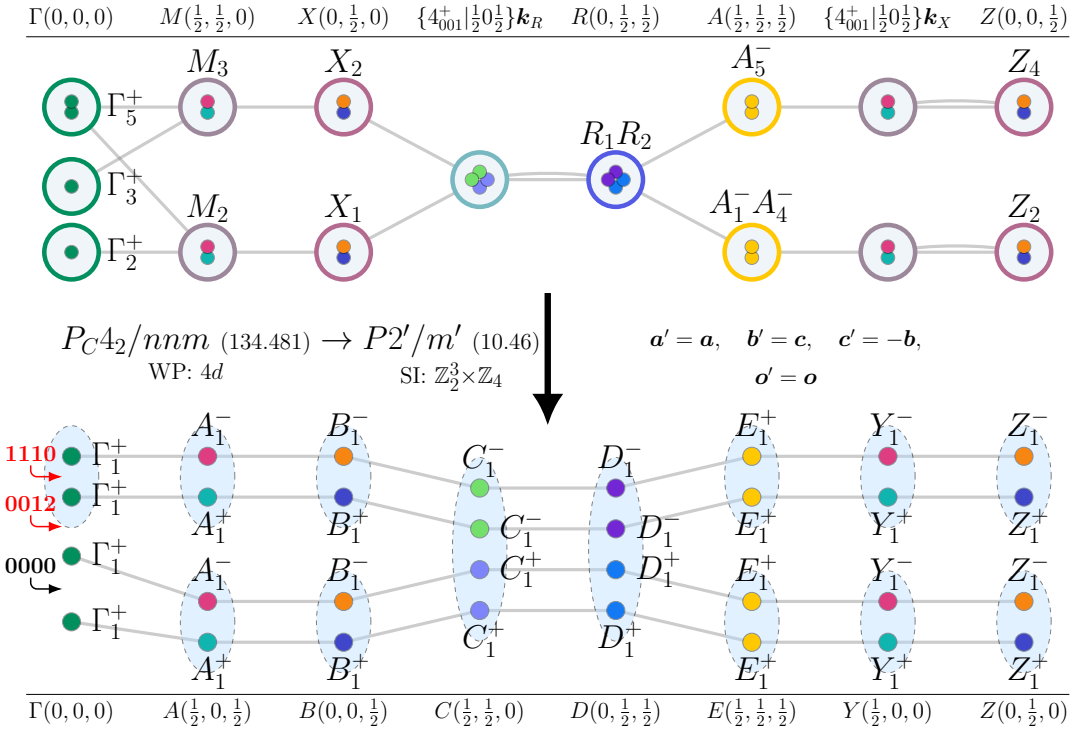
Supplementary Table 118: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 003, 010, 011, 100 103, 110, 111
2	010, 110
3	000, 001, 010, 013, 100 101, 110, 113
4	000

14.1.3 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 65: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 119: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2

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SI	Possible #Gaps
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2, 3
1111	0, 1
1113	0, 1
gapless	0
nontrivial	1, 2, 3

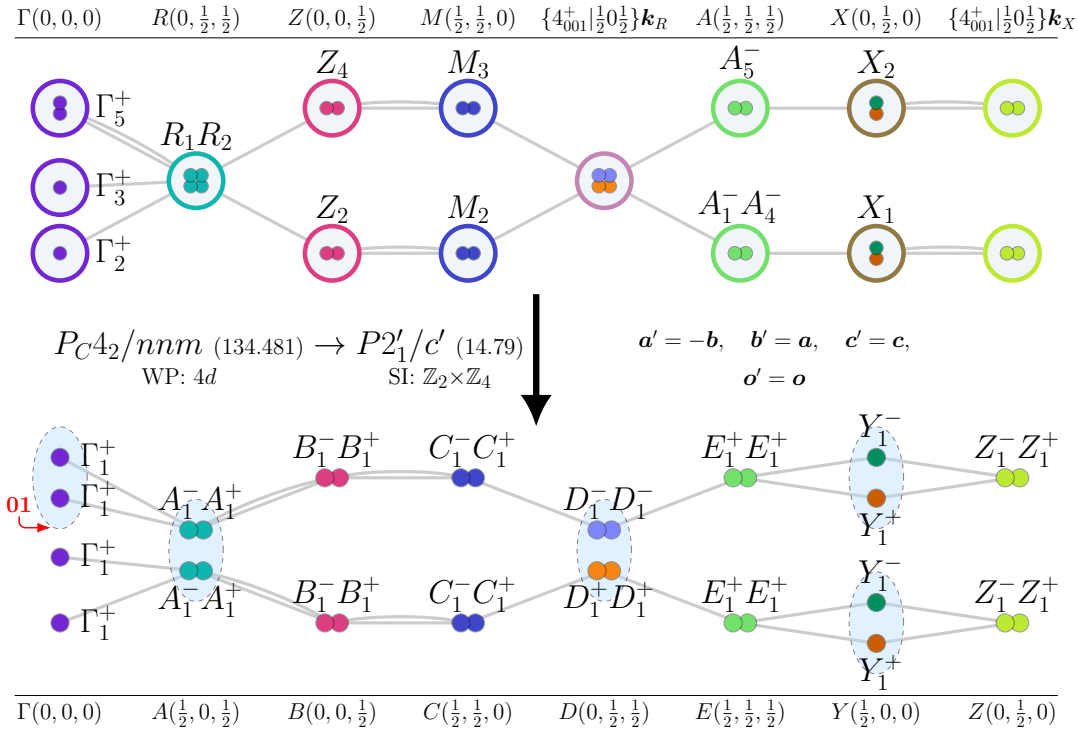
Supplementary Table 120: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
2	0010, 0012, 0111, 0113, 1011 1013, 1110
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
4	0000

14.1.4 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 66: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 121: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

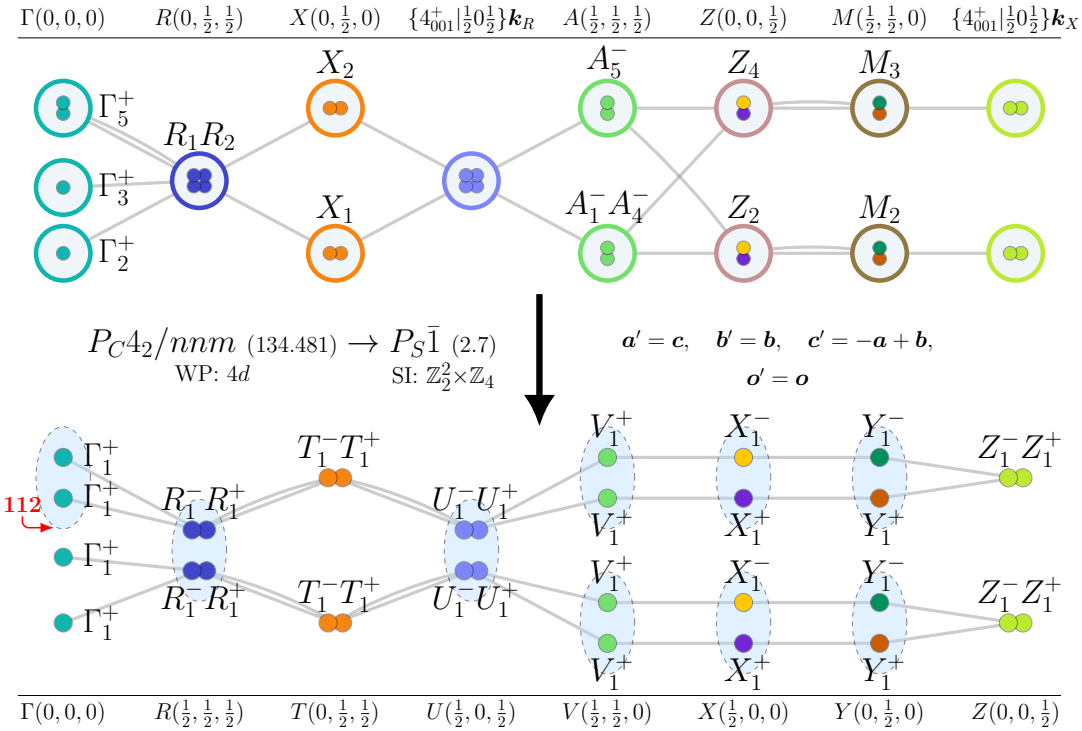
Supplementary Table 122: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

14.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 67: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 123: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 124: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

14.2 WP: $4d+8i$

BCS Materials: Pr_2CuO_4 (284 K)[46].

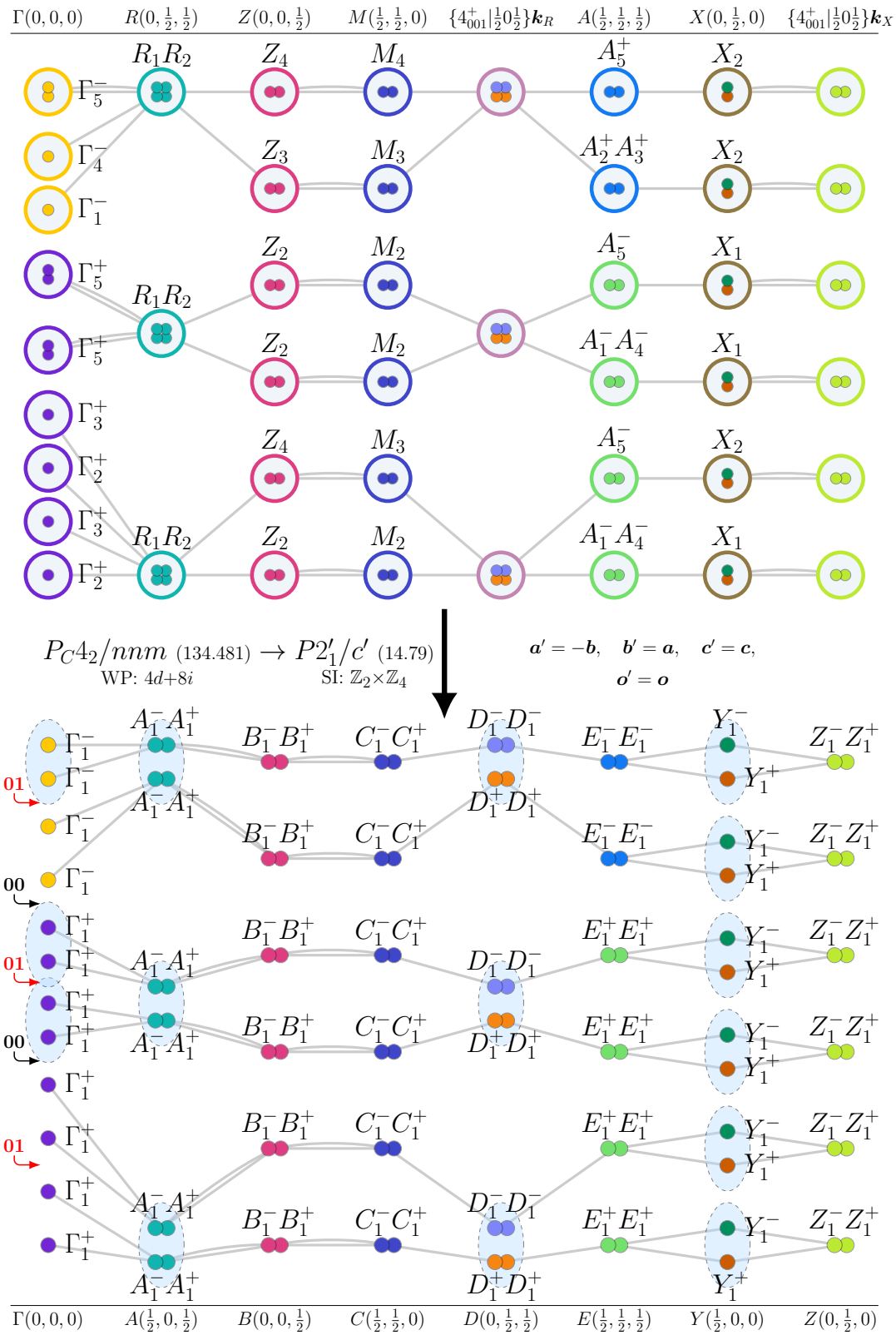
14.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],

[46] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.79`

- B $\perp$ [100].



Supplementary Figure 68: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4d + 8i$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 125: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

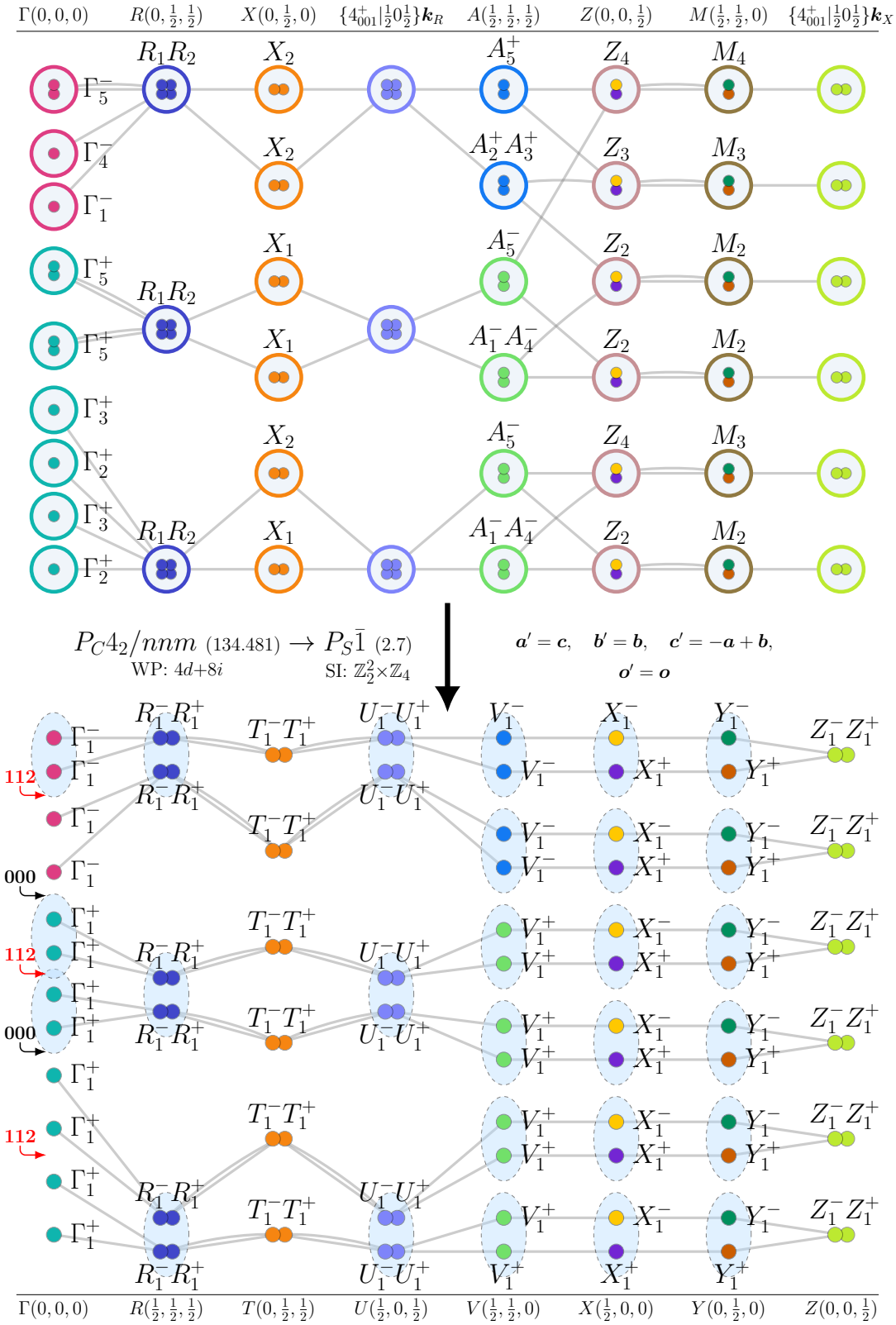
Supplementary Table 126: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4d$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

14.2.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 69: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4d+8i$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 127: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4d$+$8i$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3

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SI	Possible #Gaps
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 128: Possible SI values in each gap. Supergroup: $P_C 4_2/nnm$ (134.481), WP: $4d$+$8i$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

14.3 WP: $4c$

BCS Materials: Nd_2CuO_4 (245 K)[47], UP (22.5 K)[48].

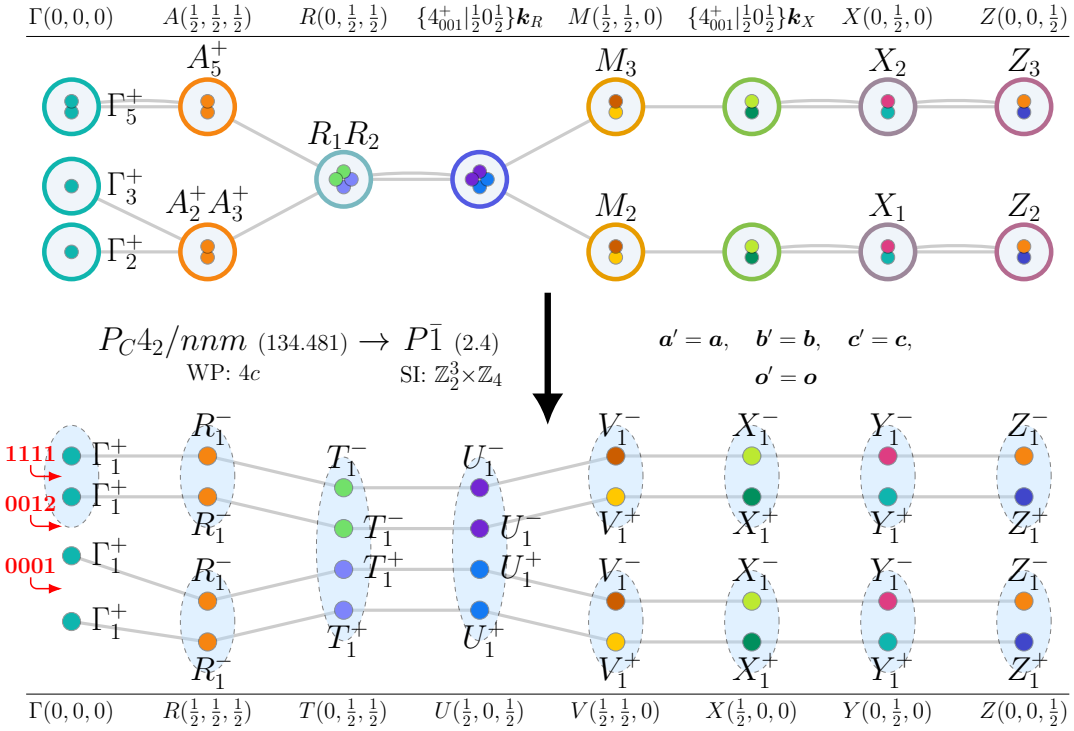
14.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],

[47] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.78`
[48] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.13`

- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 70: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 129: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2
1001	0, 1, 2

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SI	Possible #Gaps
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2, 3
1111	0, 1
1113	0, 1
gapless	0
nontrivial	1, 2, 3

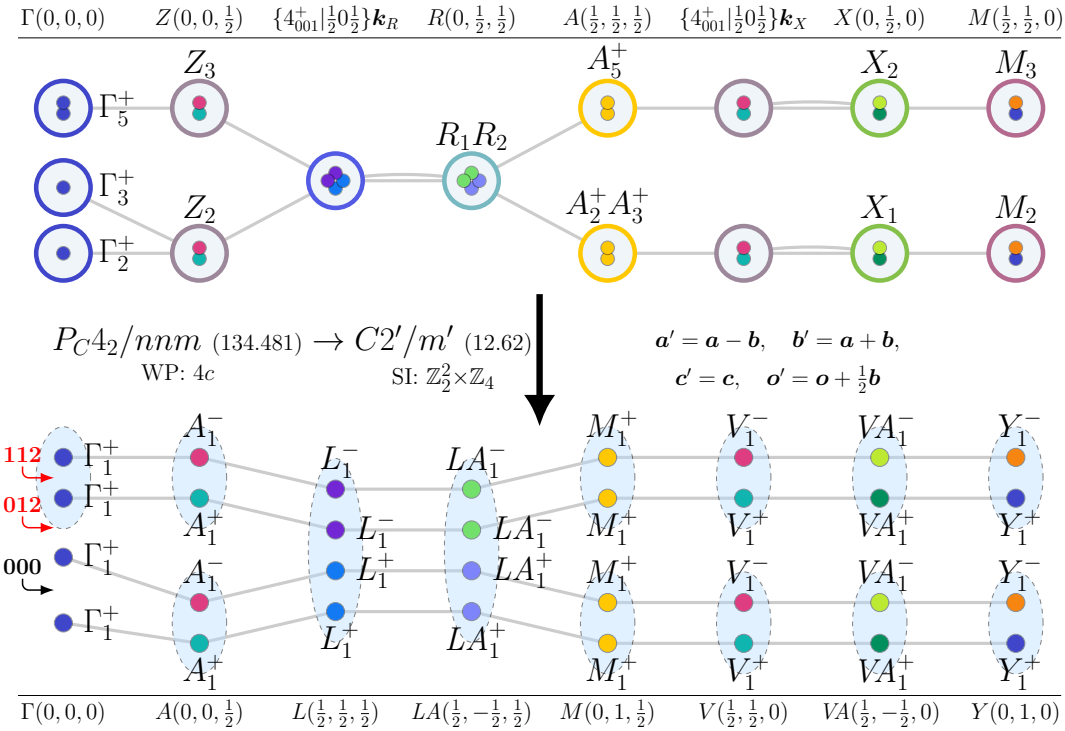
Supplementary Table 130: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
2	0010, 0012, 0111, 0113, 1011 1013, 1110
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
4	0000

14.3.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 71: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 131: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	0, 1, 2, 3
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
111	0, 1
112	0, 1, 2, 3
113	0, 1
gapless	0
nontrivial	1, 2, 3

1498

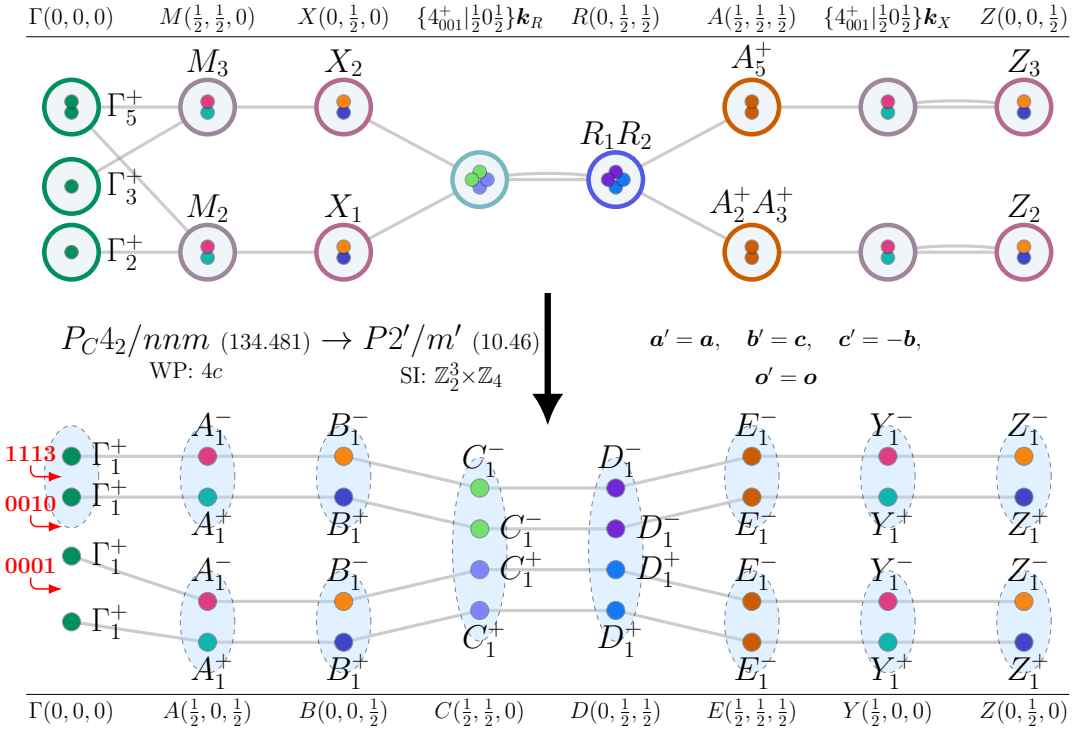
Supplementary Table 132: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	012, 112
3	000, 003, 012, 013, 100 103, 112, 113
4	000

14.3.3 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 72: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 133: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2

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SI	Possible #Gaps
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	0, 1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

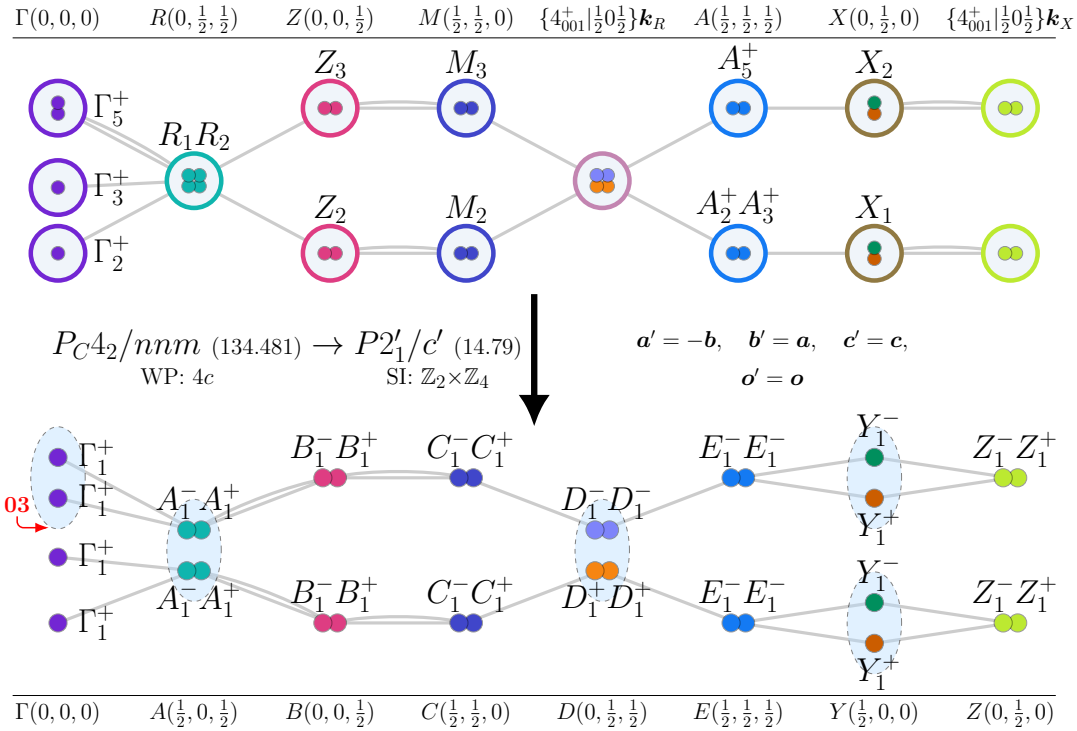
Supplementary Table 134: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
2	0010, 0012, 0111, 0113, 1011 1013, 1112
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
4	0000

14.3.4 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 73: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 135: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

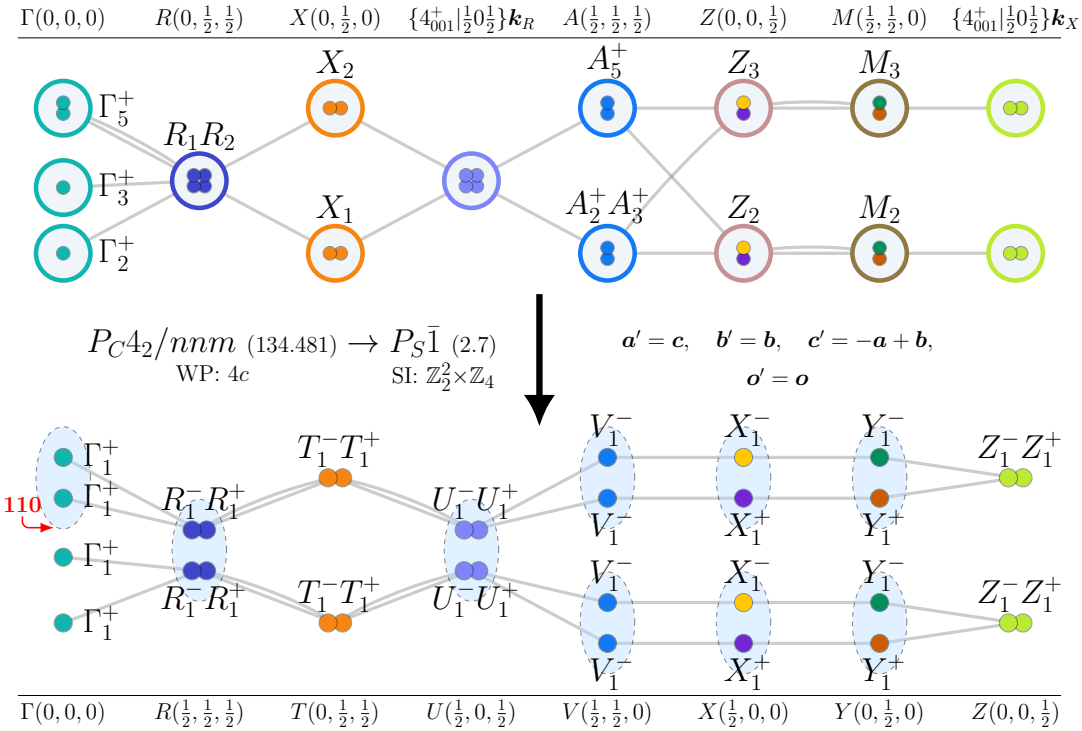
Supplementary Table 136: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

14.3.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 74: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/nnm (134.481).

Supplementary Table 137: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

Supplementary Table 138: Possible SI values in each gap. Supergroup: P_C4_2/nnm (134.481), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

15 MSG P_c4_2/mbc (135.492)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2_1'/m'$ (11.54), $P2_1'/c'$ (14.79), $P_S\bar{1}$ (2.7), $C2/c$ (15.85), $Cm'cm'$ (63.464), C_c2/c (15.90), $P2$ (3.1), $Cm'm'2$ (35.168), P_b2 (3.5), C_ccc2 (37.184), P_cba2 (32.139), $P2/m$ (10.42), $Cm'm'm$ (65.485), $Pn'n'm$ (58.397), P_b2/m (10.48), C_ccm (66.498), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_a2_1/c (14.80), $P4_2n'm'$ (102.191), $P4_2/mn'm'$ (136.501).

Trivial-SI Subgroups: Cm' (8.34), Pm' (6.20), Pc' (7.26), $C2'$ (5.15), $P2_1'$ (4.9), $P2_1'$ (4.9), P_S1 (1.3), Cc (9.37), $Cm'c2_1'$ (36.174), C_cc (9.40), Pm (6.18), $Amm'2'$ (38.190), $Pmn'2_1'$ (31.126), P_bm (6.22), Pc (7.24), $Pn'a2_1'$ (33.146), P_ac (7.27), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_ama2 (40.208), $Pn'n'2$ (34.159), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_amc2_1 (26.71), P_cbam (55.361), P_c4_2bc (106.224).

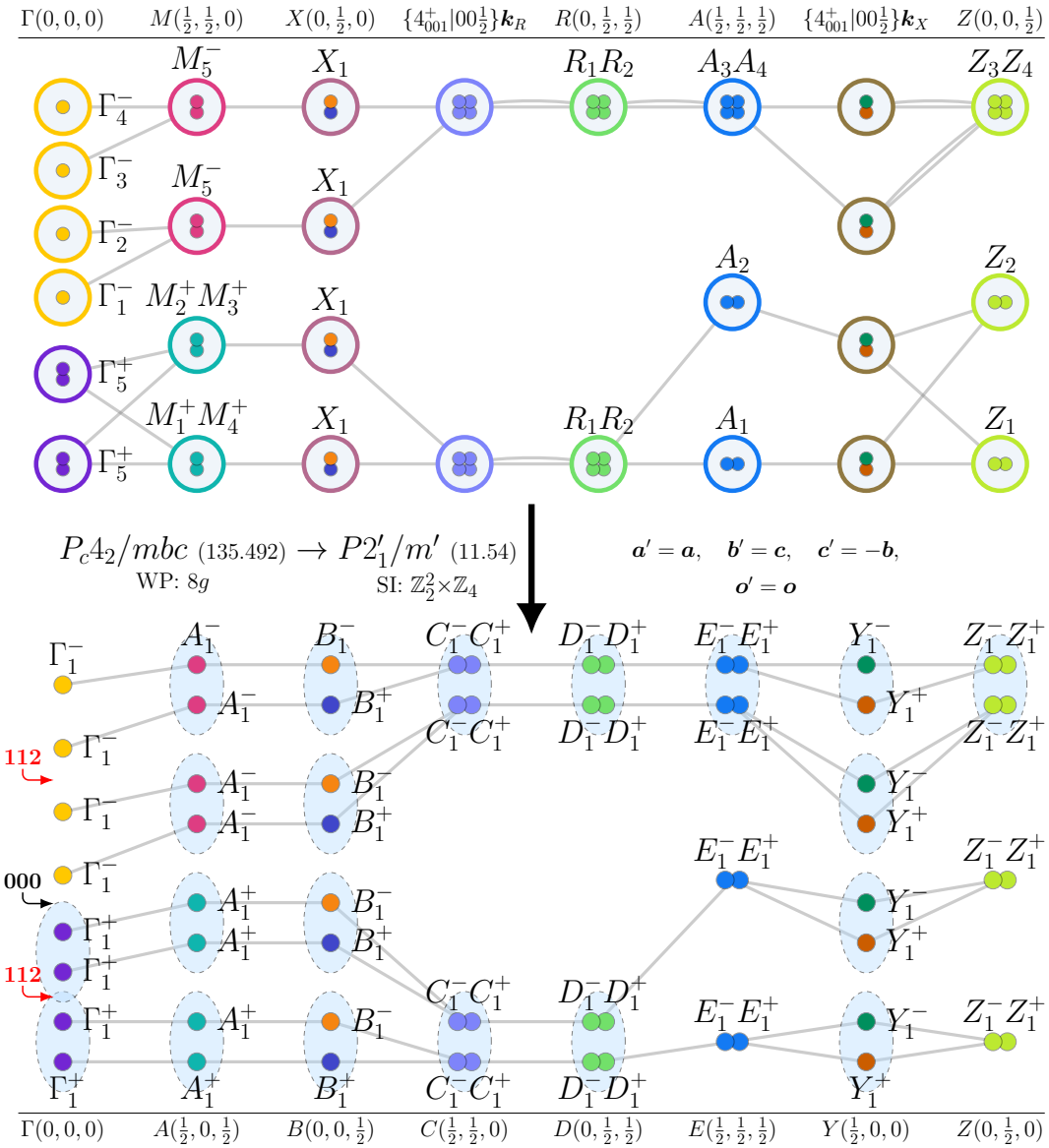
15.1 WP: $8g$

BCS Materials: U_2Rh_2Sn (28 K)[49], U_2Ni_2Sn (25 K)[50].

15.1.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 75: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $8g$ of supergroup P_c4_2/mbc (135.492).

[49]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.103`
[50]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.479`

Supplementary Table 139: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mbc (135.492), WP: $8g$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

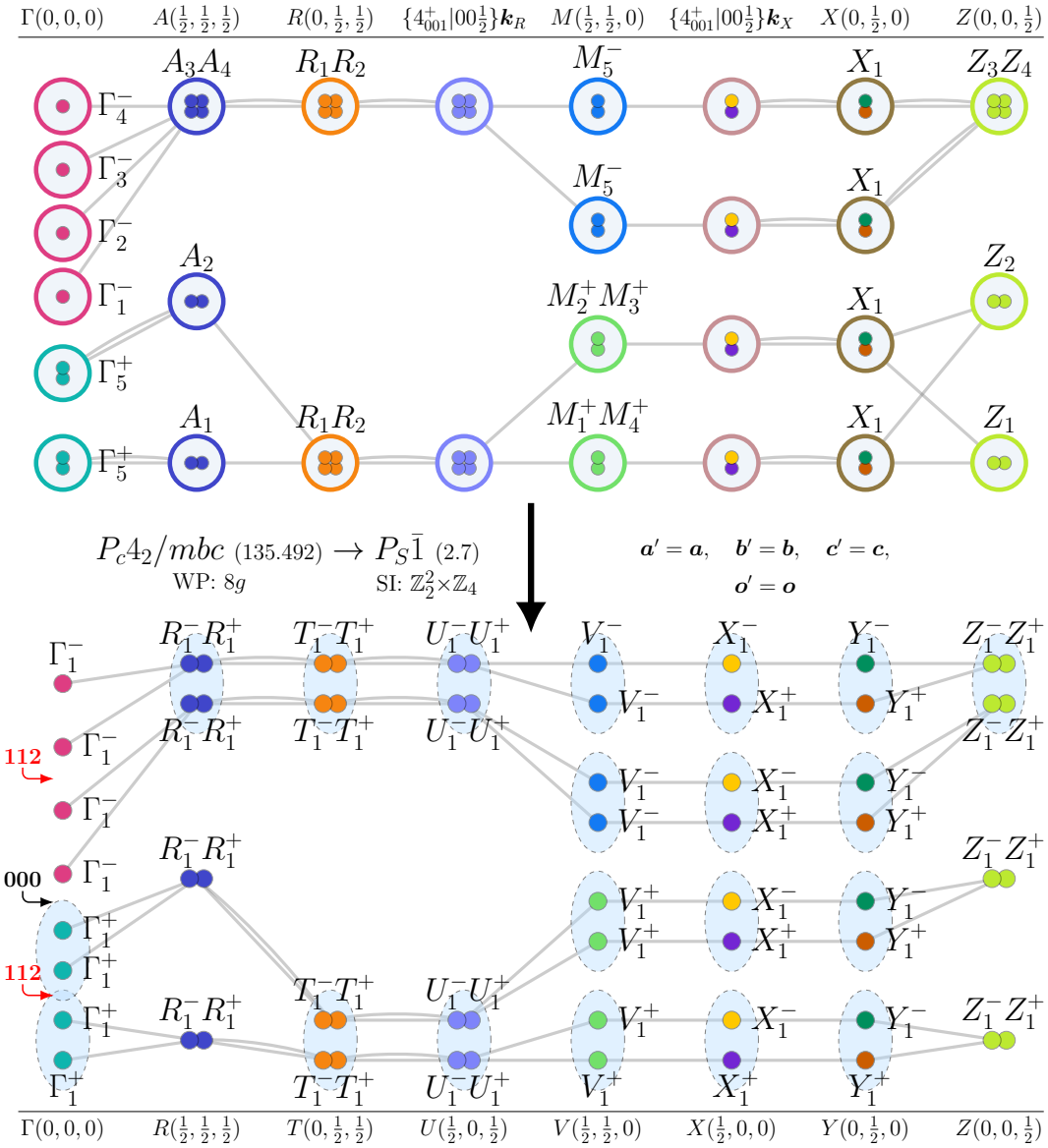
Supplementary Table 140: Possible SI values in each gap. Supergroup: P_c4_2/mbc (135.492), WP: $8g$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

15.1.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 76: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8g$ of supergroup P_c4_2/mbc (135.492).

Supplementary Table 141: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/mbc (135.492), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

1544

Supplementary Table 142: Possible SI values in each gap. Supergroup: P_c4_2/mbc (135.492), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

16 MSG P_I4_2/mnm (136.506)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $P2_1'/c'$ (14.79), $P2'/m'$ (10.46), $P_S\bar{1}$ (2.7), $Ab'a'2$ (41.215), $C2/m$ (12.58), $Cmc'a'$ (64.475), C_c2/m (12.63), $P2$ (3.1), $Cc'c'2$ (37.183), $Pm'm'2$ (25.60), $P2/m$ (10.42), $Cc'c'm$ (66.495), $Pm'm'm$ (47.252), P_C2/m (10.49), $P2_1/c$ (14.75), $Pnn'm'$ (58.398), P_A2_1/c (14.83), P_Innm (58.404), $P4_2m'c'$ (105.215), $P4_2/mm'c'$ (131.441).

Trivial-SI Subgroups: Cc' (9.39), Pc' (7.26), Pm' (6.20), $C2'$ (5.15), $P2_1'$ (4.9), $P2'$ (3.3), P_S1 (1.3), Cm (8.32), $Cmc'2_1'$ (36.175), C_cm (8.35), Pm (6.18), $Ama'2'$ (40.206), $Pm'm2'$ (25.59), P_Cm (6.23), Pc (7.24), $Pm'n2_1'$ (31.125), P_Ac (7.31), $C2$ (5.13), C_c2 (5.16), A_Bmm2 (38.194), P_C2 (3.6), C_Amm2 (35.171), P_Inn2 (34.164), C_Ammm (65.490), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_C2_1 (4.12), P_Imn2_1 (31.134), P_I4_2nm (102.194).

16.1 WP: $4d$

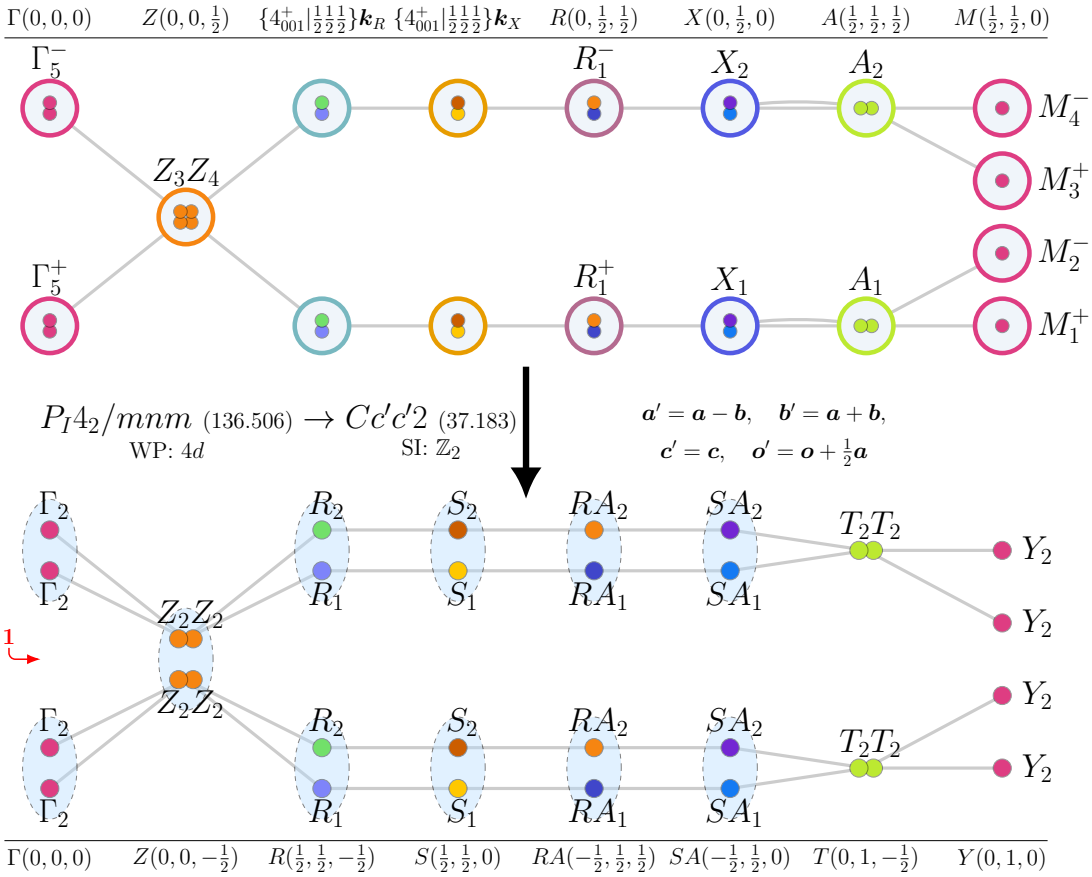
BCS Materials: $Sr_2Cr_3As_2O_2$ (590 K)[51].

16.1.1 Topological bands in subgroup $Cc'c'2$ (37.183)

Perturbation:

- E ∥ [001] and B ∥ [001] and strain ∥ [110].

[51] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.461`



Supplementary Figure 77: Topological magnon bands in subgroup $Cc'c'2$ (37.183) for magnetic moments on Wyckoff position $4d$ of supergroup P_I4_2/mnm (136.506).

Supplementary Table 143: Number of times a given SI can appear in the band structure. Supergroup: P_I4_2/mnm (136.506), WP: $4d$, Subgroup: $Cc'c'2$ (37.183).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 144: Possible SI values in each gap. Supergroup: P_I4_2/mnm (136.506), WP: $4d$, Subgroup: $Cc'c'2$ (37.183).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

16.2 WP: $4d + 4d$

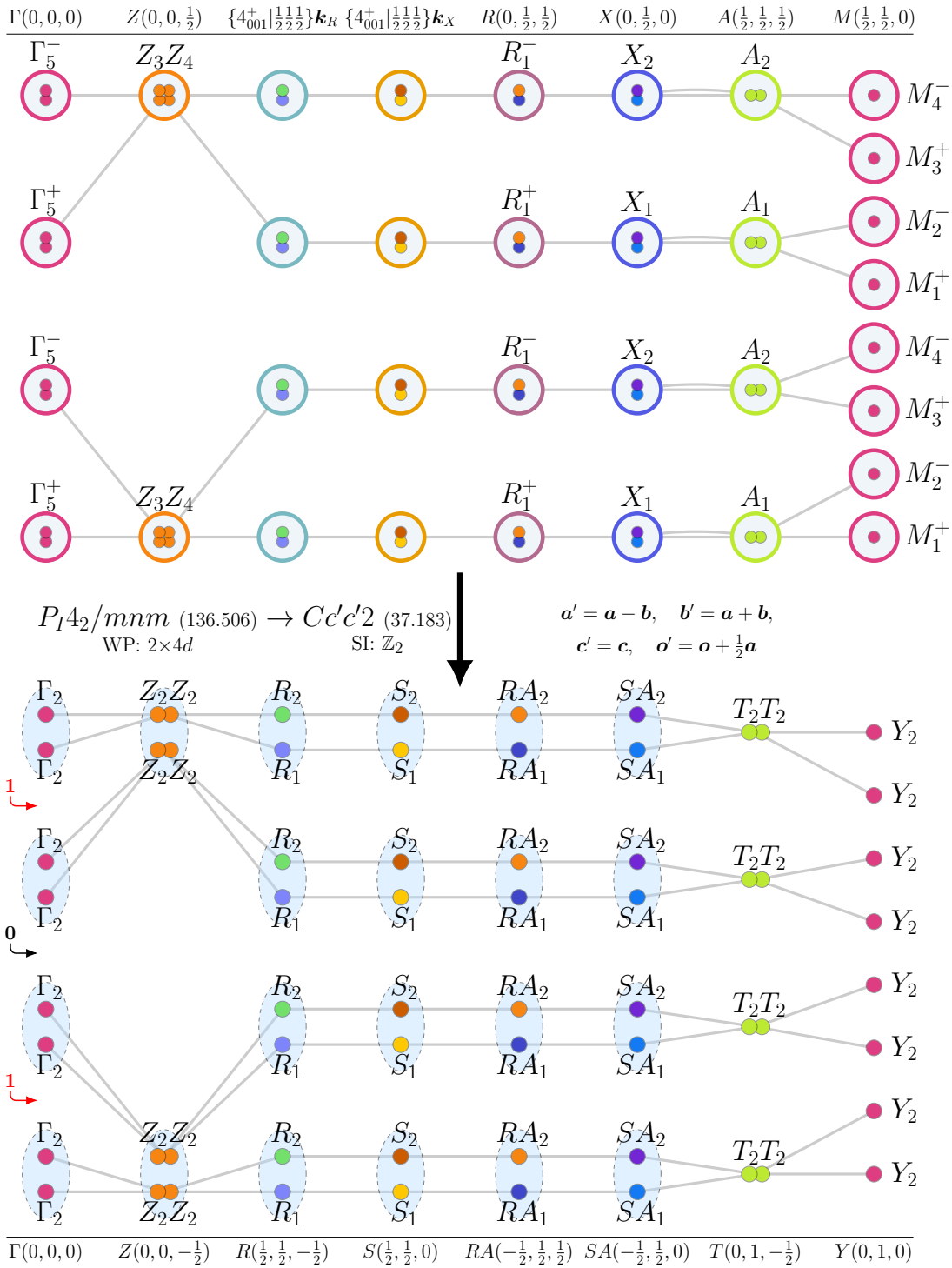
BCS Materials: $Ni_{1.64}Co_{0.36}Mn_{1.28}Ga_{0.72}$ (450 K)[52].

[52] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.198`

16.2.1 Topological bands in subgroup $Cc'c'2$ (37.183)

Perturbation:

- E ∥ [001] and B ∥ [001] and strain ∥ [110].



Supplementary Figure 78: Topological magnon bands in subgroup $Cc'c'2$ (37.183) for magnetic moments on Wyckoff positions $4d + 4d$ of supergroup P_I4_2/mnm (136.506).

Supplementary Table 145: Number of times a given SI can appear in the band structure. Supergroup: P_I4_2/mnm (136.506), WP: $4d$+$4d$, Subgroup: $Cc'c'2$ (37.183).

SI	Possible #Gaps
0	1
1	2
gapless	4
nontrivial	2

Supplementary Table 146: Possible SI values in each gap. Supergroup: P_I4_2/mnm (136.506), WP: $4d$+$4d$, Subgroup: $Cc'c'2$ (37.183).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0
5	G
6	1
7	G
8	0

17 MSG P_C2/c (13.74)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2$ (3.1), $P2/c$ (13.65).

Trivial-SI Subgroups: Pc' (7.26), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), P_Cc (7.30), P_C2 (3.6).

17.1 WP: $4a$

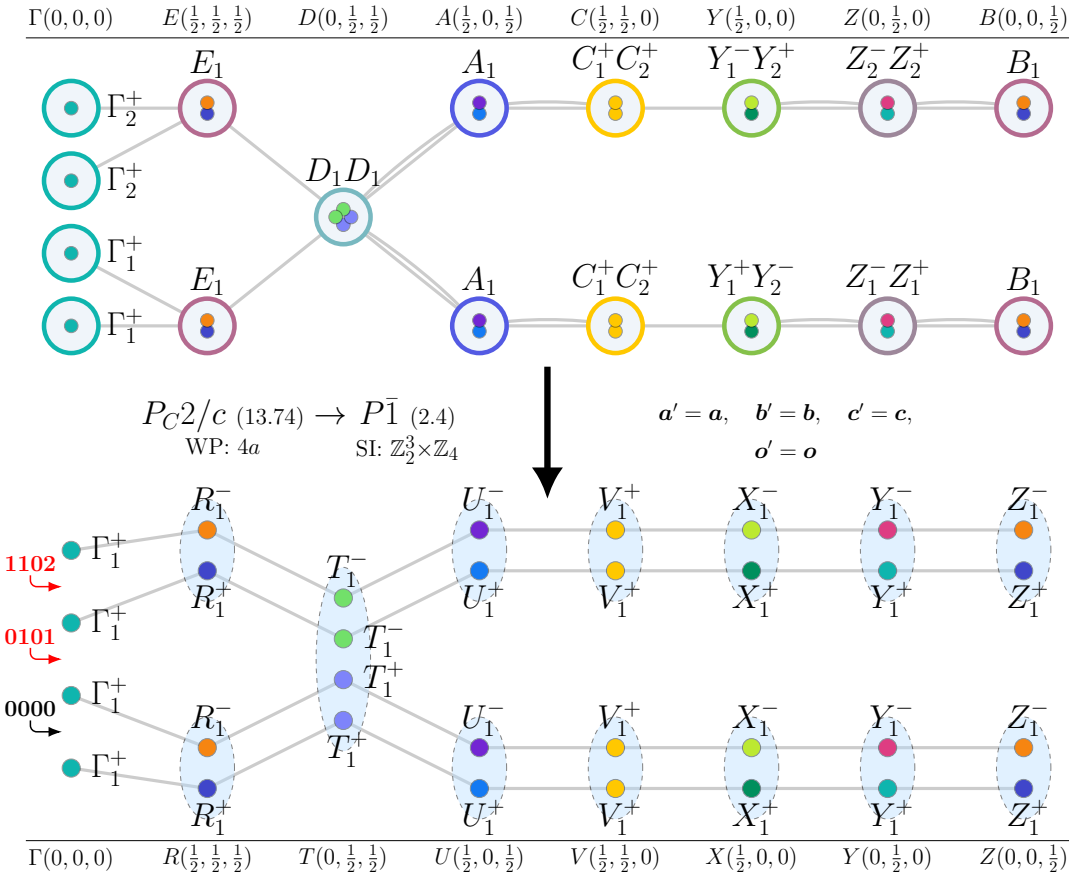
BCS Materials: $NaFeSO_4F$ (36 K)[53], $NaCoSO_4F$ (29 K)[54], Sr_3ZnIrO_6 (17 K)[55], Ca_4IrO_6 (12 K)[56].

17.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[53]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.121`
[54]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.126`
[55]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.649`
[56]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.114`



Supplementary Figure 79: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_C2/c (13.74).

Supplementary Table 147: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1
0012	0, 1, 2
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2, 3
1103	0, 1, 2

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...continued

SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

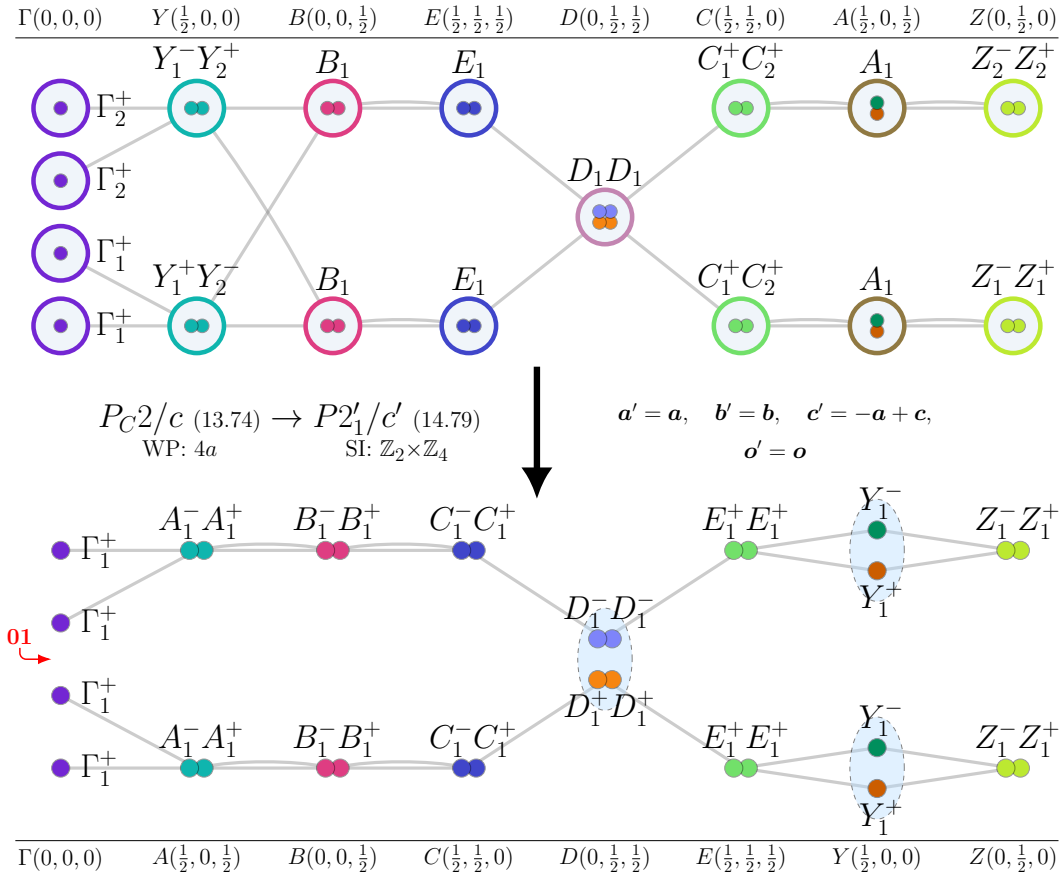
Supplementary Table 148: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1112, 1113
2	0101, 0103, 1102
3	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1111, 1112
4	0000

17.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 80: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_C2/c (13.74).

Supplementary Table 149: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

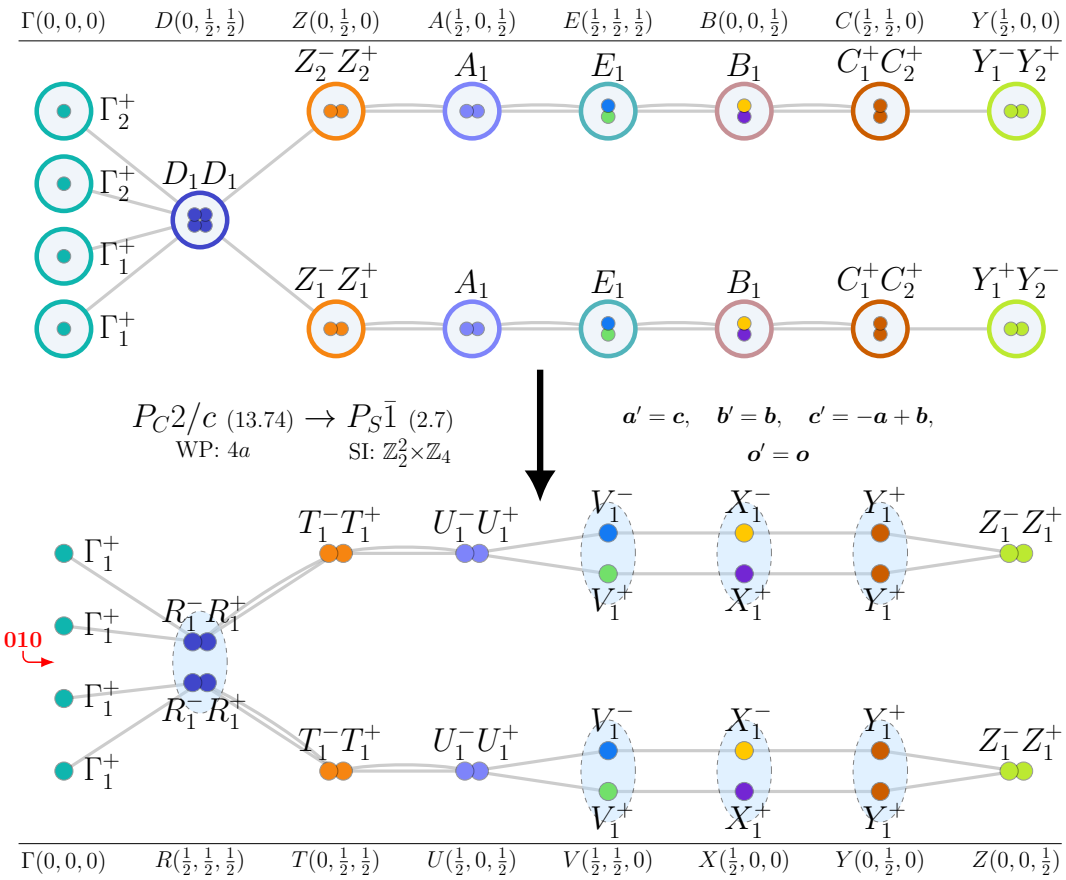
Supplementary Table 150: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

17.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 81: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_C2/c (13.74).

Supplementary Table 151: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
010	1
gapless	2
nontrivial	1

Supplementary Table 152: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	010
3	G
4	000

17.2 WP: $8f$

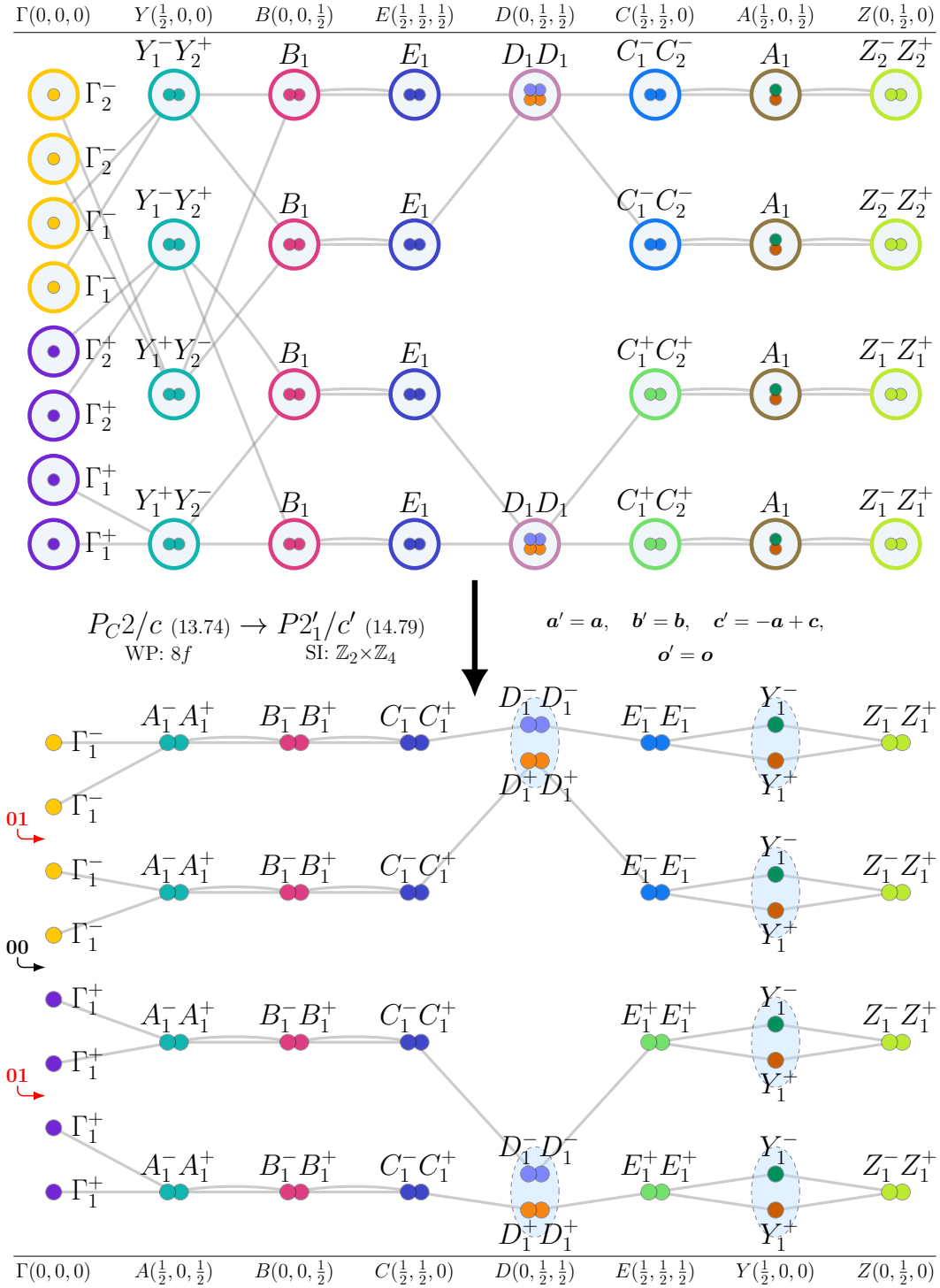
BCS Materials: $U_2Rh_3Si_5$ (26 K)[57], $Cu_3Ni_2SbO_6$ (22.3 K)[58].

[57] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.564`
[58] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.259`

17.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 82: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8f$ of supergroup P_C2/c (13.74).

Supplementary Table 153: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $8f$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2

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...continued

SI	Possible #Gaps
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

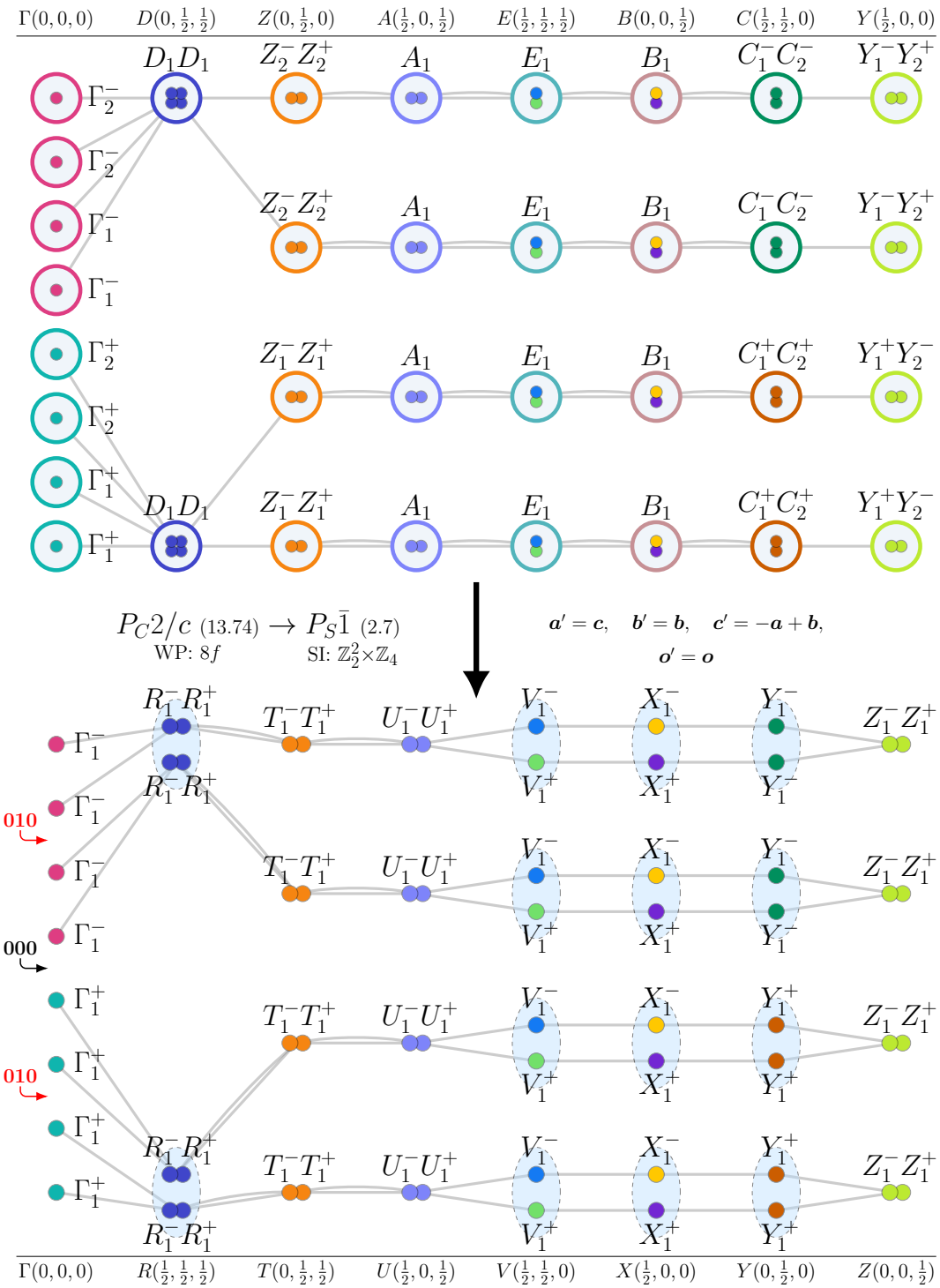
Supplementary Table 154: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $8f$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

17.2.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 83: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8f$ of supergroup P_C2/c (13.74).

Supplementary Table 155: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $8f$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1
010	0, 1, 2
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
111	0, 1
113	0, 1

continued on next page...

...continued

SI	Possible #Gaps
gapless	4
nontrivial	2, 3

Supplementary Table 156: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $8f$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	010, 012, 101, 103
3	G
4	000, 002, 111, 113
5	G
6	010, 012, 101, 103
7	G
8	000

17.3 WP: $4b$

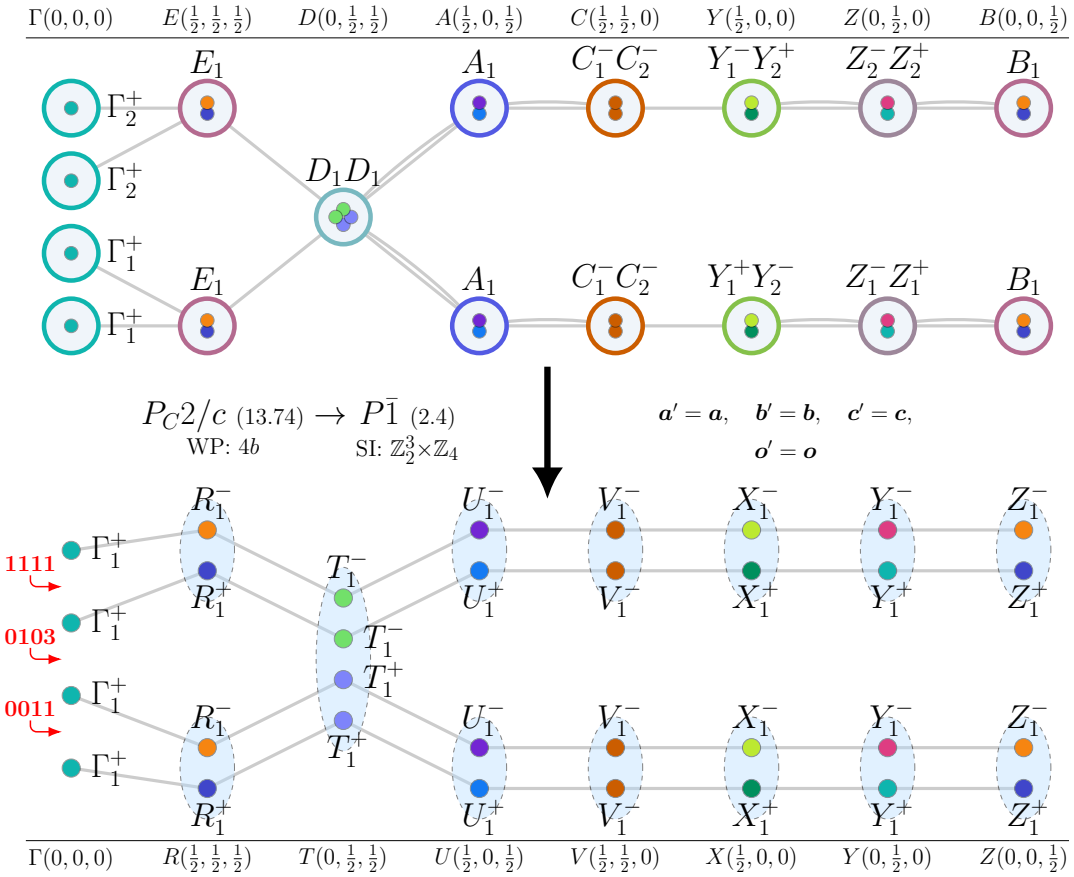
BCS Materials: $TlMnF_4$ (4.2 K)[59].

17.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[59]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.346`



Supplementary Figure 84: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup P_C2/c (13.74).

Supplementary Table 157: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1
0012	0, 1, 2
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2, 3
1101	0, 1, 2
1103	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

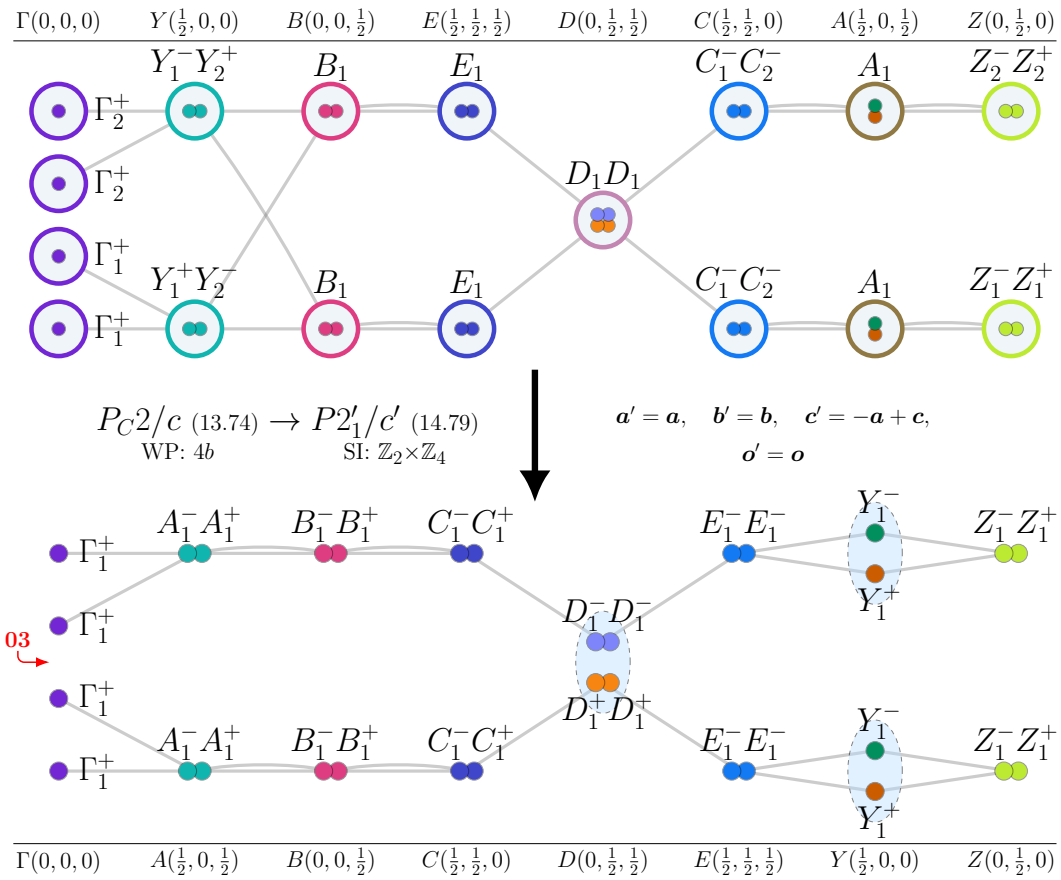
Supplementary Table 158: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1103 1110, 1112, 1113
2	0101, 0103, 1100
3	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1103 1110, 1111, 1112
4	0000

17.3.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 85: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup P_C2/c (13.74).

Supplementary Table 159: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

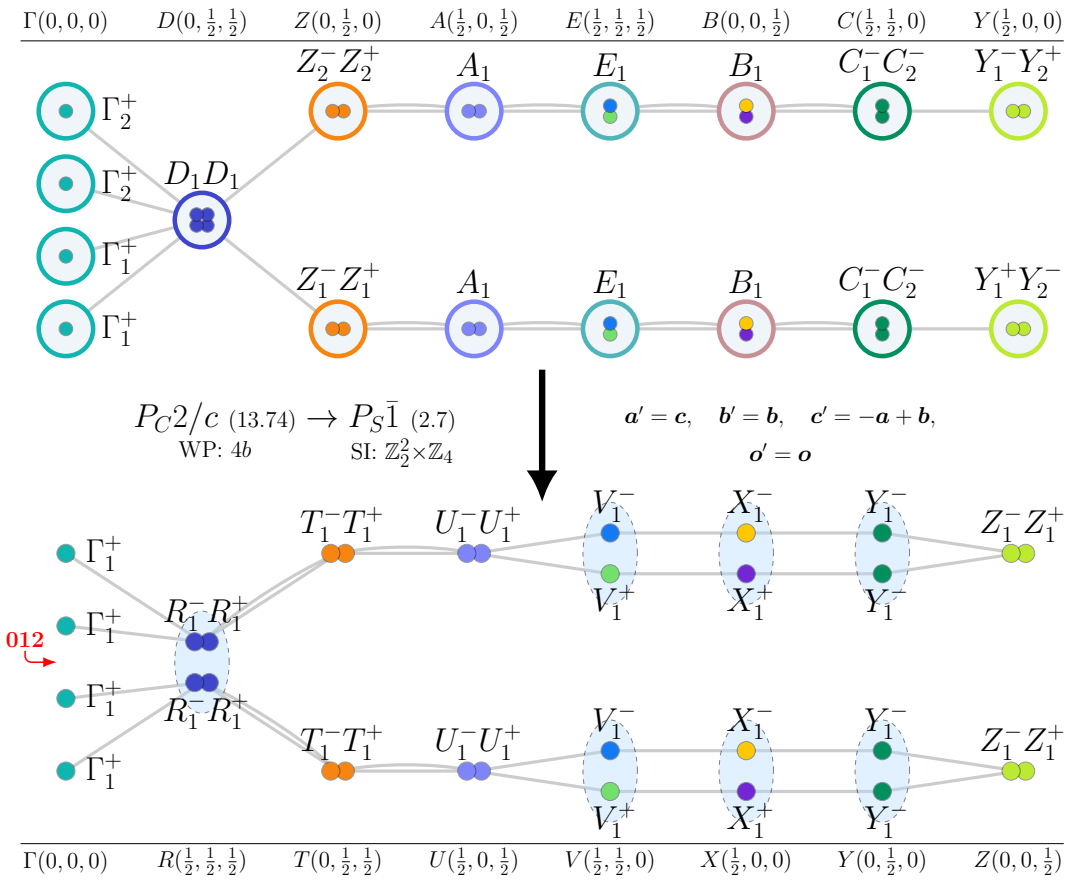
Supplementary Table 160: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

17.3.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 86: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4b$ of supergroup P_C2/c (13.74).

Supplementary Table 161: Number of times a given SI can appear in the band structure. Supergroup: P_C2/c (13.74), WP: $4b$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
012	1
gapless	2
nontrivial	1

Supplementary Table 162: Possible SI values in each gap. Supergroup: P_C2/c (13.74), WP: $4b$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	012
3	G
4	000

18 MSG $P4_2/nc'm'$ (138.525)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P2_1'/c'$ (14.79), $P2_1/c'$ (14.79), $P2$ (3.1), $Cm'm'2$ (35.168), $Pc'c'2$ (27.81), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pc'c'n$ (56.369), $P4_2c'm'$ (101.183).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Pc' (7.26), Pc' (7.26), $C2'$ (5.15), $P2_1'$ (4.9), Pc (7.24), $Abm'2'$ (39.198), $Pna'2_1'$ (33.147).

18.1 WP: $4d + 8i$

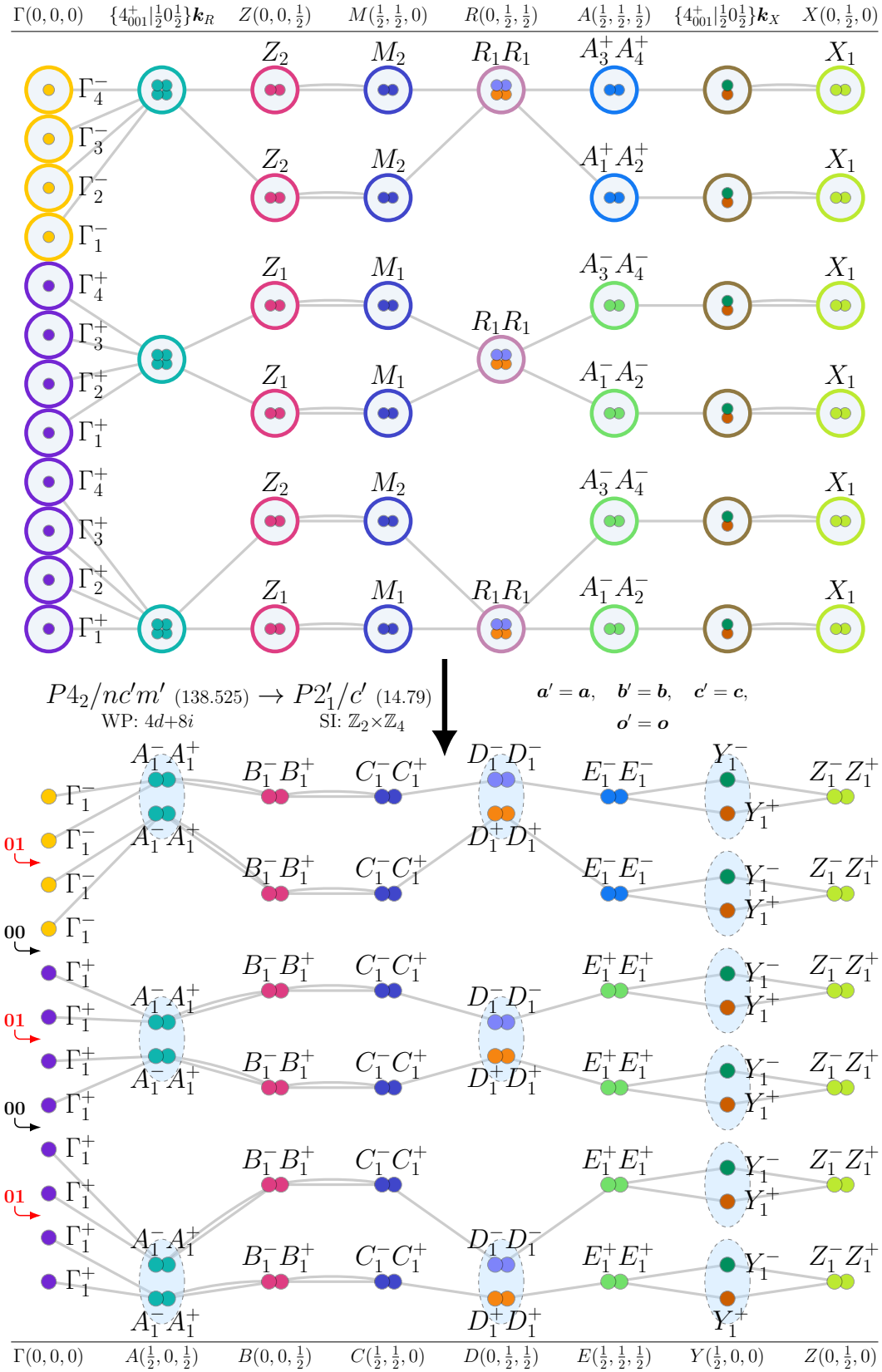
BCS Materials: Nd_2NiO_4 (130 K)[60], $Nd_2NiO_{4.11}$ (53 K)[61].

18.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B ∥ [100].

[60] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.349
[61] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.247



Supplementary Figure 87: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4d+8i$ of supergroup $P4_2/nc'm'$ (138.525).

Supplementary Table 163: Number of times a given SI can appear in the band structure. Supergroup: $P4_2/nc'm'$ (138.525), WP: $4d$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

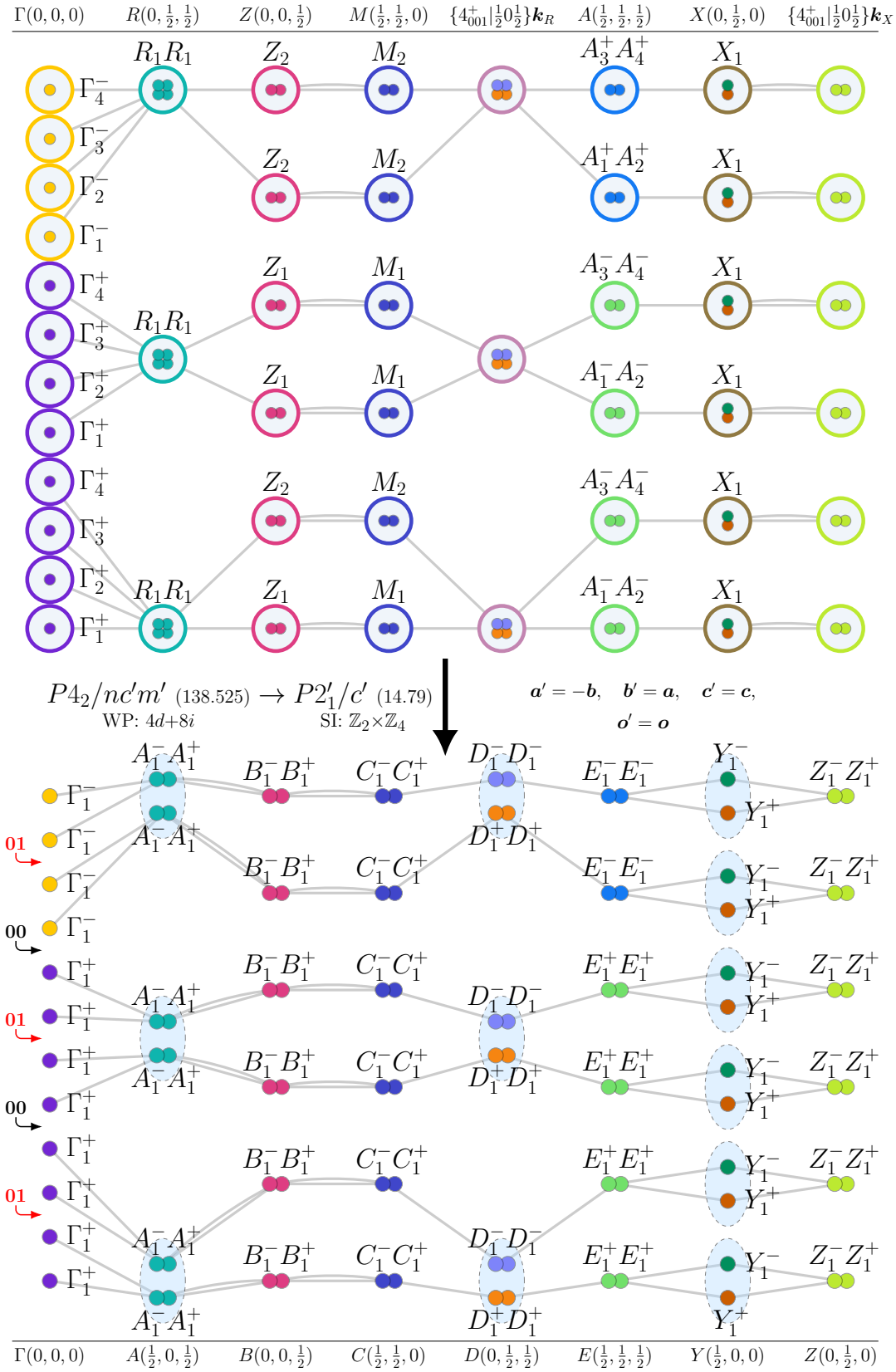
Supplementary Table 164: Possible SI values in each gap. Supergroup: $P4_2/nc'm'$ (138.525), WP: $4d$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

18.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 88: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4d+8i$ of supergroup $P4_2/nc'm'$ (138.525).

Supplementary Table 165: Number of times a given SI can appear in the band structure. Supergroup: $P4_2/nc'm'$ (138.525), WP: $4d$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 166: Possible SI values in each gap. Supergroup: $P4_2/nc'm'$ (138.525), WP: $4d$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

19 MSG P_c4_2/ncm (138.528)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $P2_1'/c'$ (14.79), $P2_1'/m'$ (11.54), $P_S\bar{1}$ (2.7), $Ab'a'2$ (41.215), $C2/m$ (12.58), $Cmc'a'$ (64.475), C_c2/m (12.63), $P2$ (3.1), $Cc'c'2$ (37.183), $Pm'm'2$ (25.60), P_b2 (3.5), P_ccc2 (27.82), $P2/c$ (13.65), $Cc'c'a$ (68.515), $Pm'm'n$ (59.409), P_b2/c (13.71), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_c2_1/c (14.82), P_ccn (56.373), $P4_2m'c'$ (105.215), $P4_2/nm'c'$ (137.513)

Trivial-SI Subgroups: Cc' (9.39), Pc' (7.26), Pm' (6.20), $C2'$ (5.15), $P2_1'$ (4.9), $P2_1'$ (4.9), P_S1 (1.3), Cm (8.32), $Cmc'2_1'$ (36.175), C_cm (8.35), Pc (7.24), $Aba'2'$ (41.214), $Pm'n2_1'$ (31.125), P_bc (7.29), Pc (7.24), $Pm'c2_1'$ (26.68), P_cc (7.28), $C2$ (5.13), C_c2 (5.16), A_abm2 (39.200), C_cmm2 (35.169), C_cmma (67.508), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_ana2_1 (33.149), P_c4_2cm (101.184).

19.1 WP: $4b$

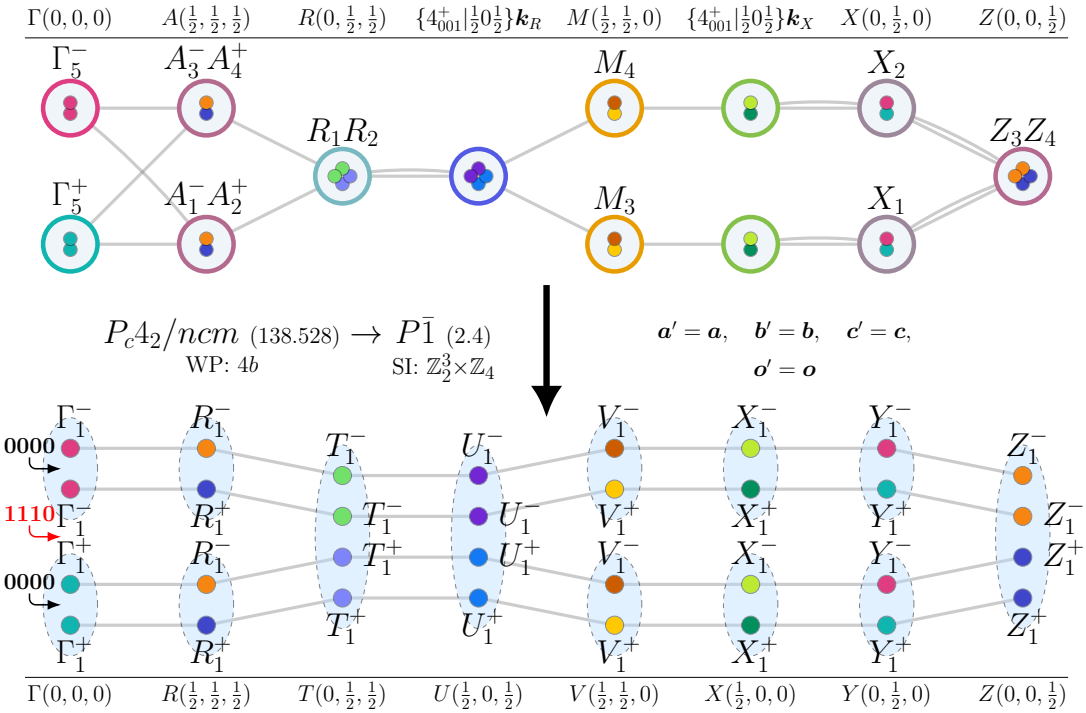
BCS Materials: LiMnAs (393 K)[62], LiMnAs (393 K)[63], LiMnAs (393 K)[64], CsMnP (295 K)[65], CsMnP (295 K)[66], CsMnBi (295 K)[67], RbMnAs (295 K)[68], RbMnP (295 K)[69], RbMnP (295 K)[70], KMnP (295 K)[71], KMnP (295 K)[72], KMnAs (293 K)[73], KMnAs (293 K)[74], RbMnAs (293 K)[75], RbMnBi[76], LaCrAsO[77].

19.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.

[62]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.552
[63]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.551
[64]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.550
[65]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.548
[66]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.547
[67]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.546
[68]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.543
[69]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.542
[70]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.541
[71]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.540
[72]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.539
[73]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.554
[74]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.553
[75]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.544
[76]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.545
[77]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.146



Supplementary Figure 89: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup P_c4_2/ncm (138.528).

Supplementary Table 167: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

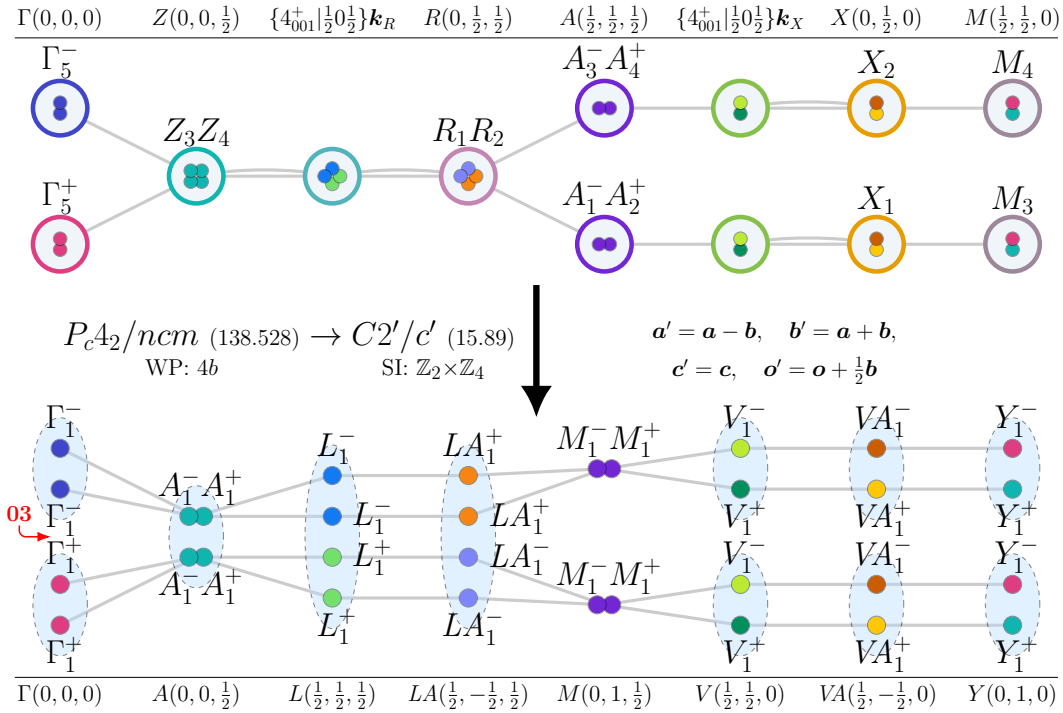
Supplementary Table 168: Possible SI values in each gap. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1010, 1011 1012, 1013, 1110, 1111, 1112 1113
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

19.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 90: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $4b$ of supergroup P_c4_2/ncm (138.528).

Supplementary Table 169: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
11	0, 1
13	0, 1
gapless	2
nontrivial	1

Supplementary Table 170: Possible SI values in each gap. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $C2'/c'$ (15.89).

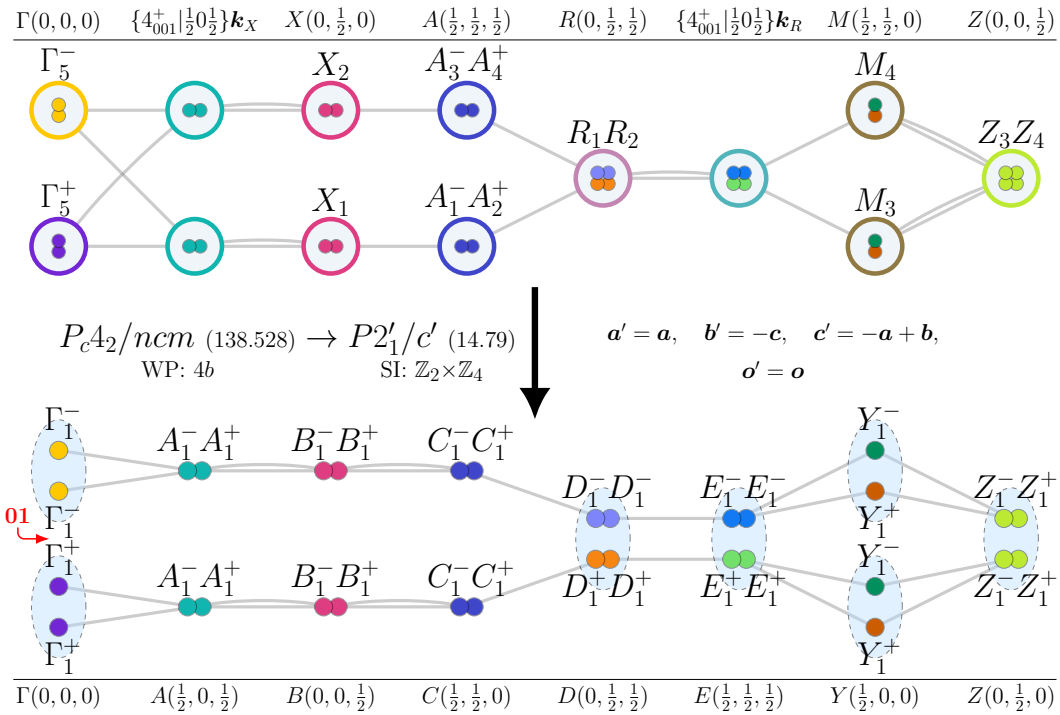
Gap#	Possible SI Values
1	G
2	01, 03, 11, 13
3	G
4	00

19.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],

- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 91: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup P_c4_2/ncm (138.528).

Supplementary Table 171: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

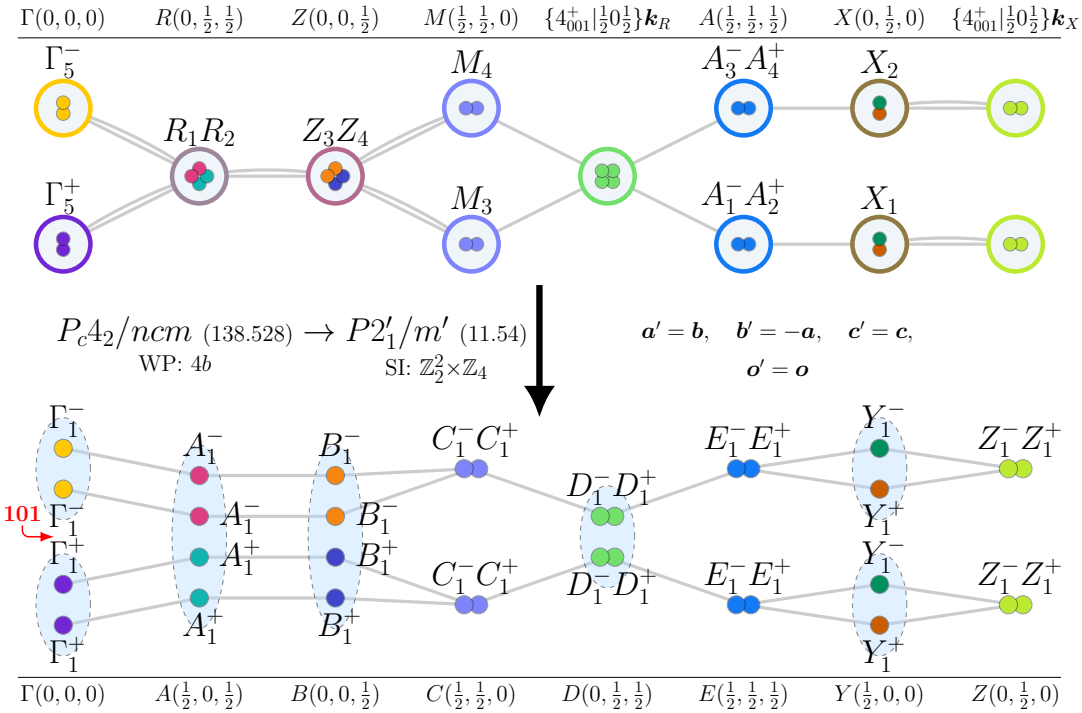
Supplementary Table 172: Possible SI values in each gap. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

19.1.4 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 92: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4b$ of supergroup P_c4_2/ncm (138.528).

Supplementary Table 173: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
100	0, 1
101	0, 1
102	0, 1
103	0, 1
110	0, 1
111	0, 1
112	0, 1
113	0, 1
gapless	2
nontrivial	1

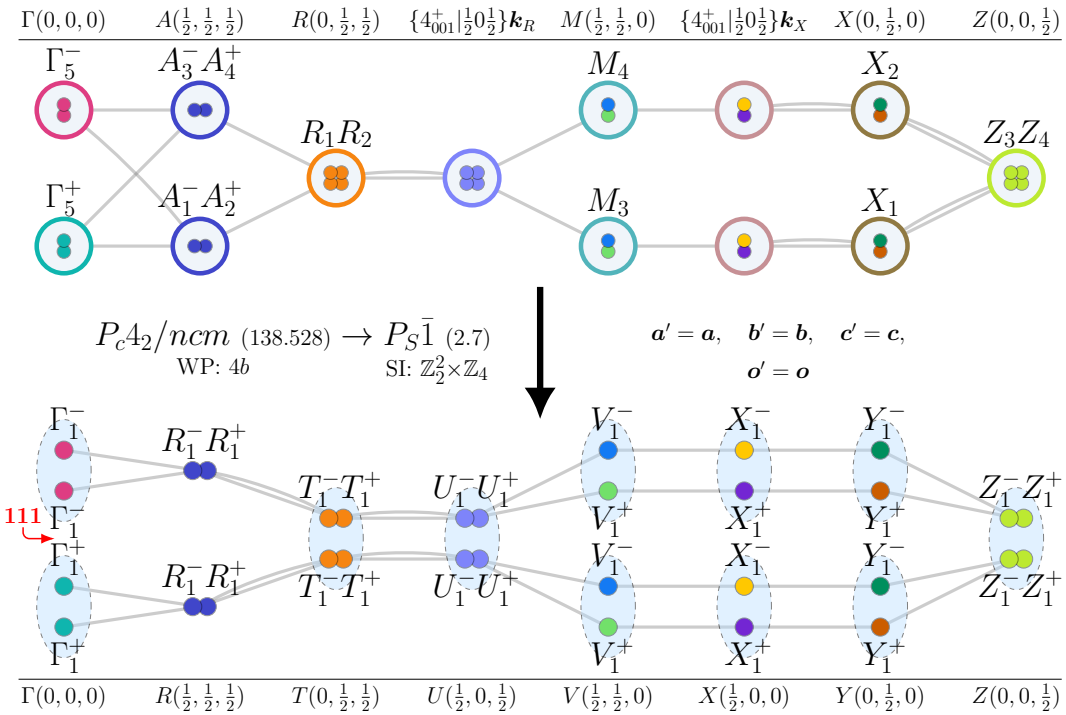
Supplementary Table 174: Possible SI values in each gap. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	100, 101, 102, 103, 110 111, 112, 113
3	G
4	000

19.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 93: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4b$ of supergroup P_c4_2/ncm (138.528).

Supplementary Table 175: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
111	0, 1
113	0, 1
gapless	2
nontrivial	1

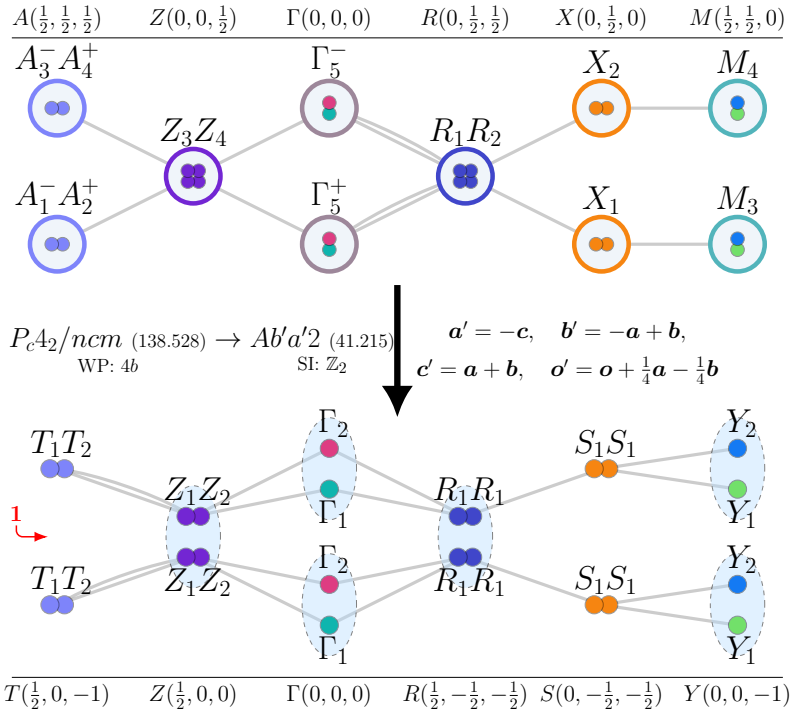
Supplementary Table 176: Possible SI values in each gap. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000

19.1.6 Topological bands in subgroup $Ab'a'2$ (41.215)

Perturbation:

- E $\parallel$ [110] and B $\parallel$ [110].



Supplementary Figure 94: Topological magnon bands in subgroup $Ab'a'2$ (41.215) for magnetic moments on Wyckoff position $4b$ of supergroup P_c4_2/ncm (138.528).

Supplementary Table 177: Number of times a given SI can appear in the band structure. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $Ab'a'2$ (41.215).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 178: Possible SI values in each gap. Supergroup: P_c4_2/ncm (138.528), WP: $4b$, Subgroup: $Ab'a'2$ (41.215).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

20 MSG P_C4_2/ncm (138.529)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $P2'/m'$ (10.46), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), C_a2 (5.17), $C2/m$ (12.58), $Cmm'm'$ (65.486) C_a2/m (12.64), $P2$ (3.1), $Cm'm'2$ (35.168), P_a2 (3.4), P_Ccc2 (27.84), $P2/c$ (13.65), $Cm'm'a$ (67.505), $Pn'n'n$ (48.260), P_c2/c (13.72), $P2_1/c$ (14.75), $Pm'n'a$ (53.326), P_C2_1/c (14.84), P_Cccn (56.375), $P4_2n'm'$ (102.191), $P4_2/nn'm'$ (134.477).

Trivial-SI Subgroups: Cm' (8.34), Pm' (6.20), Pc' (7.26), $C2'$ (5.15), $P2'$ (3.3), $P2'$ (3.3), P_S1 (1.3), Cm (8.32), $Cm'm2'$ (35.167), C_am (8.36), Pc (7.24), $Abm'2'$ (39.198), $Pn'n2'$ (34.158), P_cc (7.28), Pc (7.24), $Pn'c2'$ (30.113), P_Cc (7.30), $C2$ (5.13), $Am'm'2$ (38.191), A_bbm2 (39.201), $Pn'n'2$ (34.159), C_amm2 (35.170), C_amma (67.509), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_C2_1 (4.12), P_Ana2_1 (33.152),

$P_C 4_2 cm$ (101.185).

20.1 WP: $4d$

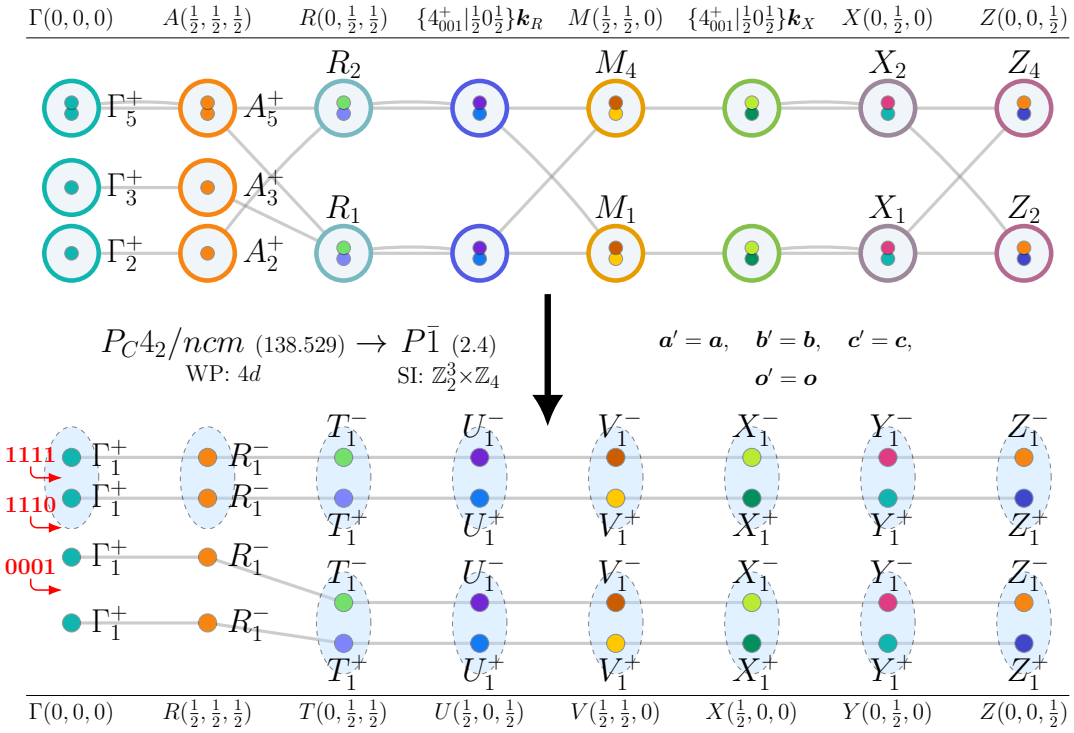
BCS Materials: $LaSrFeO_4$ (380 K)[78], $LaCaFeO_4$ (373 K)[79], $LaBaFeO_4$[80].

20.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.

[78] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.41
[79] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.39
[80] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.40



Supplementary Figure 95: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 179: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	1, 2, 3
1111	0, 1
1113	0, 1

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

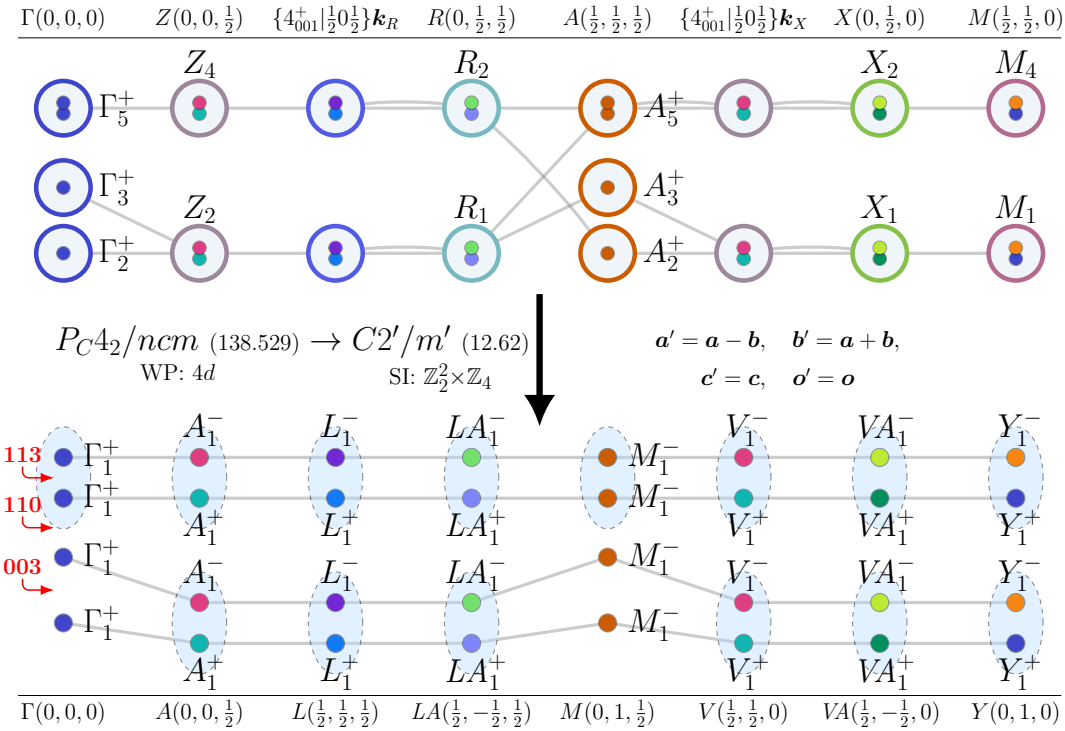
Supplementary Table 180: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
2	1110
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
4	0000

20.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 96: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 181: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
010	0, 1, 2
011	0, 1
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
110	1, 2, 3
111	0, 1
113	0, 1
gapless	0
nontrivial	1, 2, 3

1754

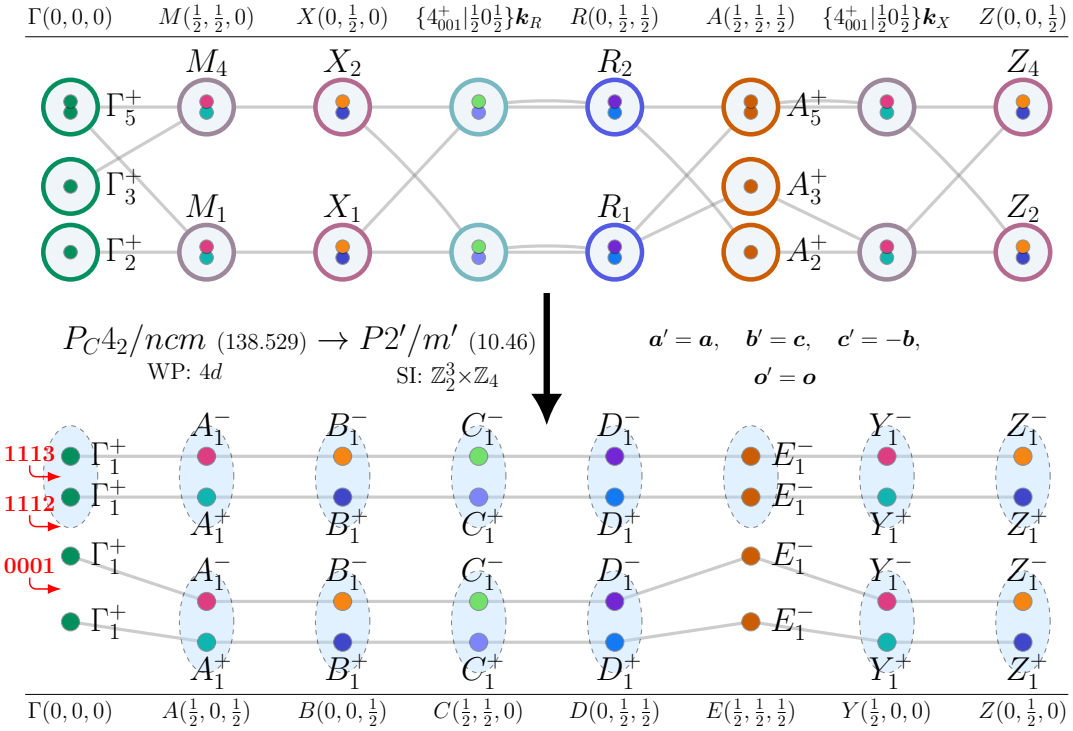
Supplementary Table 182: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 003, 010, 011, 100 103, 110, 111
2	110
3	000, 001, 010, 013, 100 101, 110, 113
4	000

20.1.3 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ⊥ [001].



Supplementary Figure 97: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 183: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2

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SI	Possible #Gaps
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

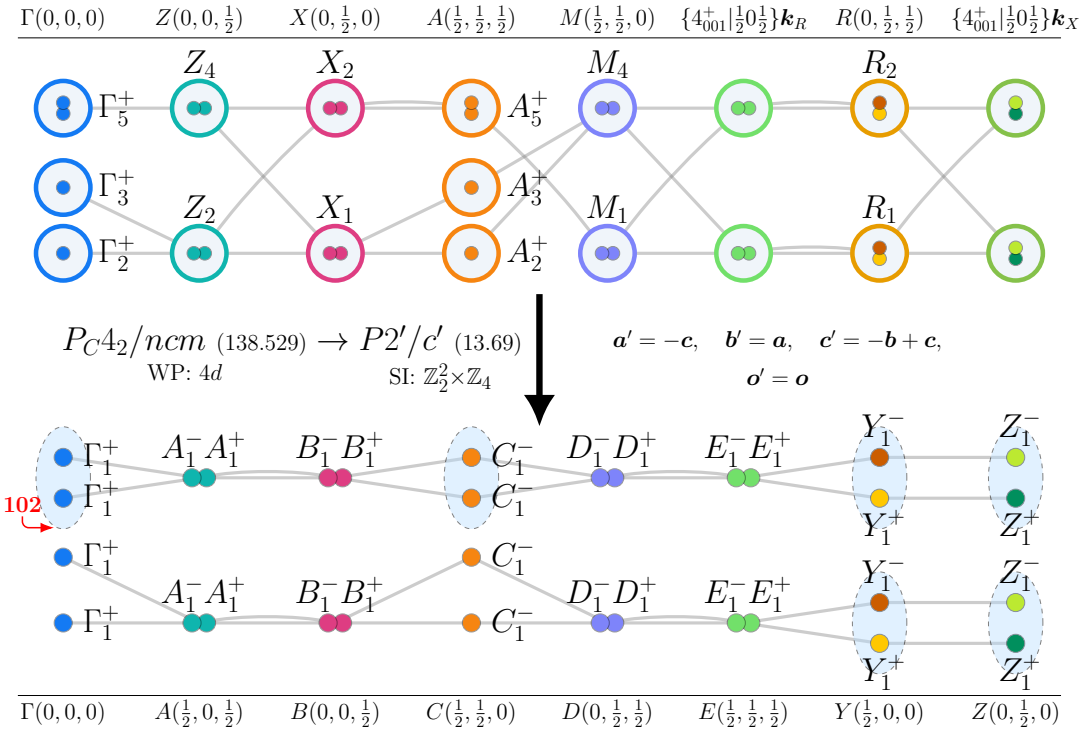
Supplementary Table 184: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
2	1112
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
4	0000

20.1.4 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 98: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 185: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
102	1
gapless	2
nontrivial	1

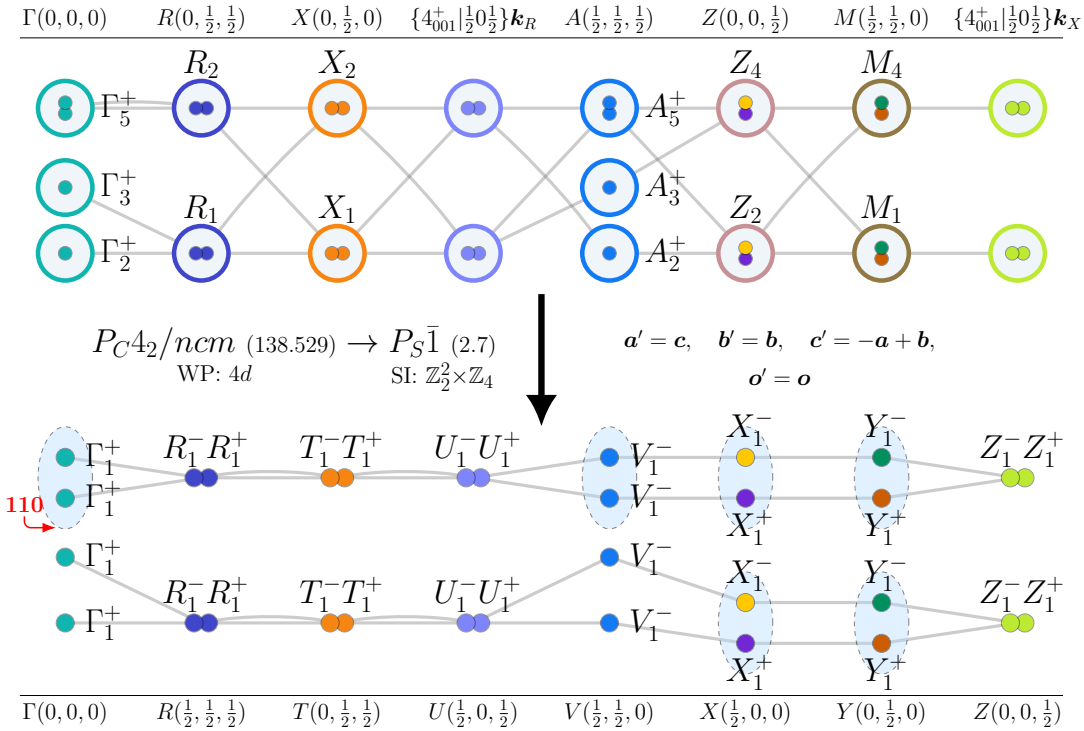
Supplementary Table 186: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	102
3	G
4	000

20.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 99: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4d$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 187: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

Supplementary Table 188: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

20.2 WP: $4c$

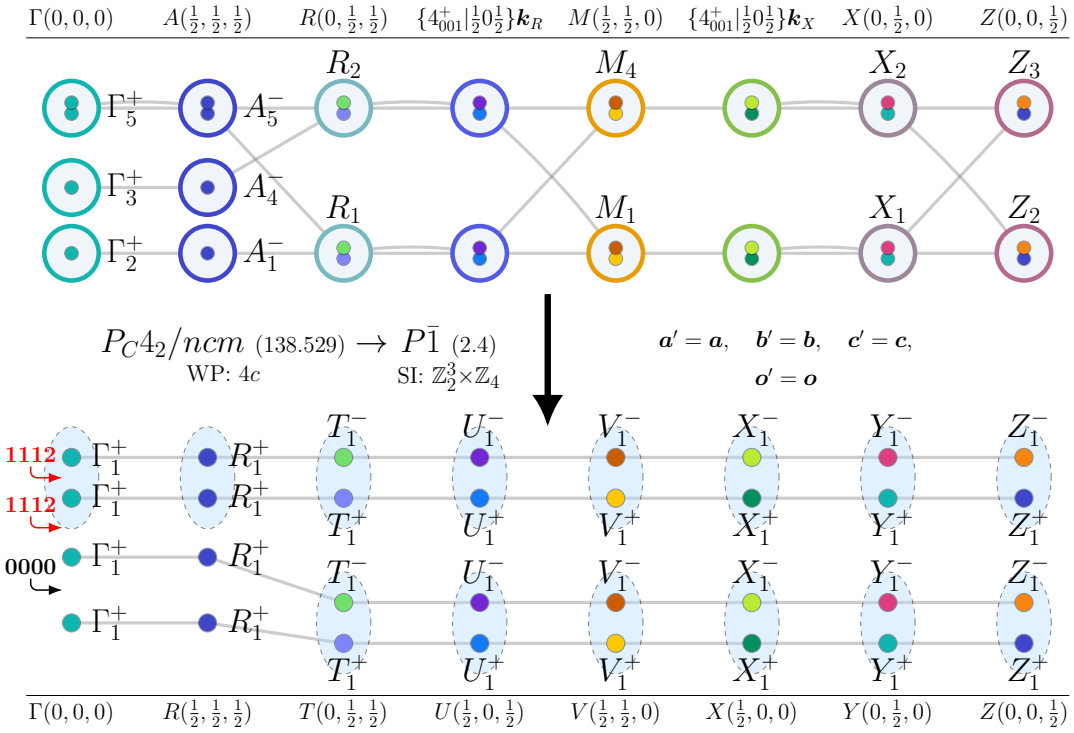
BCS Materials: Sm_2CuO_4 (280 K)[81], Eu_2CuO_4 (265 K)[82].

20.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

[81] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.7`
[82] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.77`

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 100: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 189: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 190: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

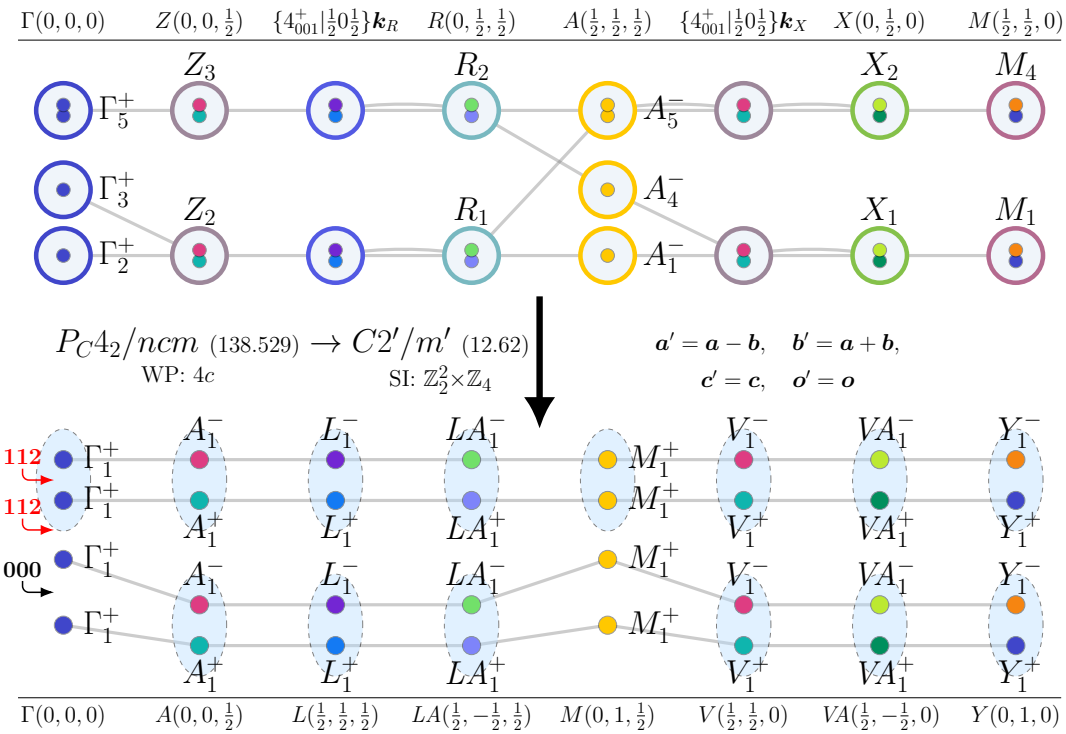
Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

20.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],

- B $\perp$ [110].



Supplementary Figure 101: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 191: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
111	0, 1
112	1, 2, 3
113	0, 1
gapless	0
nontrivial	1, 2, 3

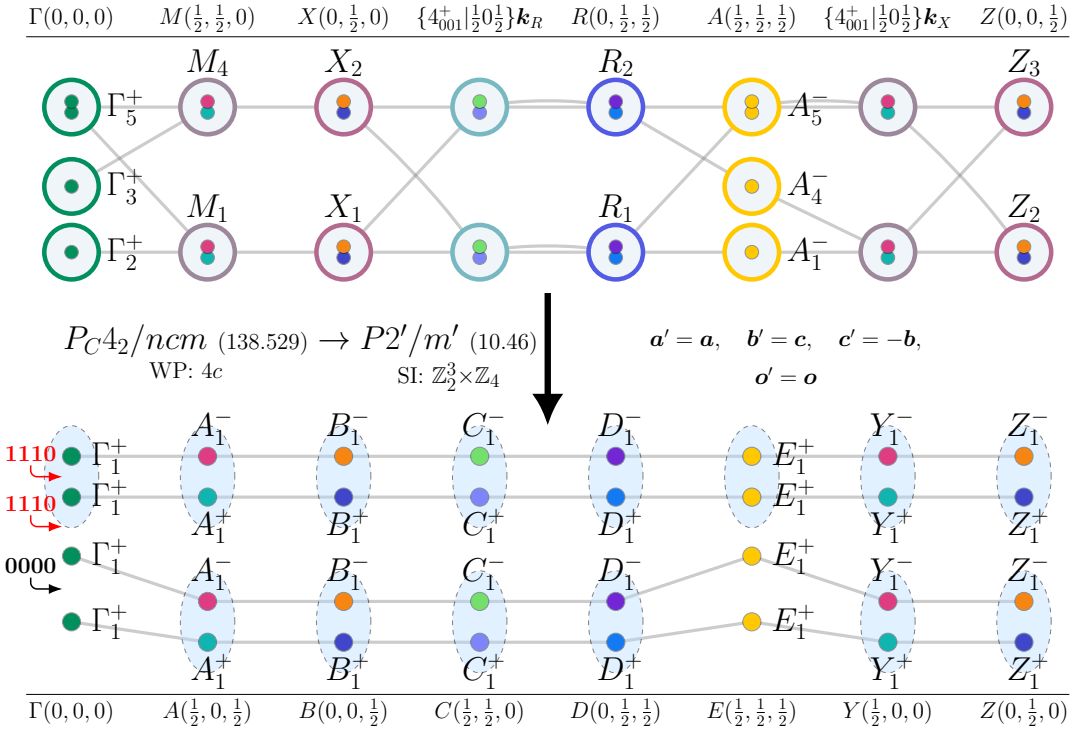
Supplementary Table 192: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	112
3	000, 003, 012, 013, 100 103, 112, 113
4	000

20.2.3 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 102: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 193: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2

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SI	Possible #Gaps
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	1, 2, 3
1111	0, 1
1113	0, 1
gapless	0
nontrivial	1, 2, 3

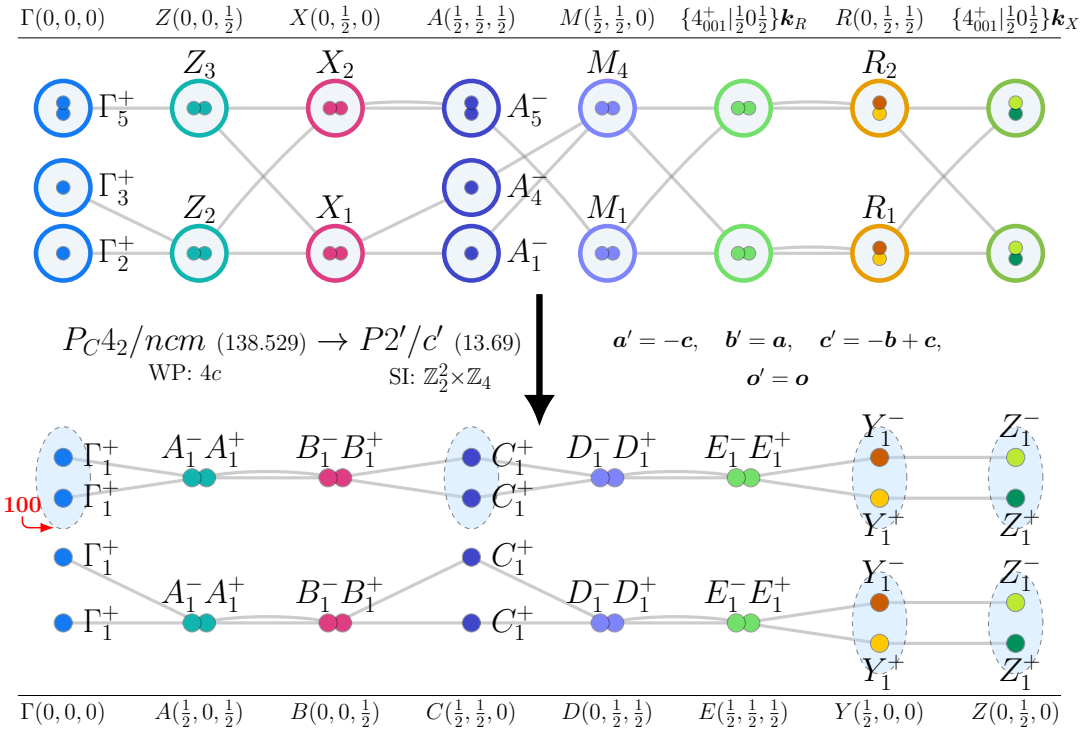
Supplementary Table 194: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
2	1110
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
4	0000

20.2.4 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 103: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 195: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

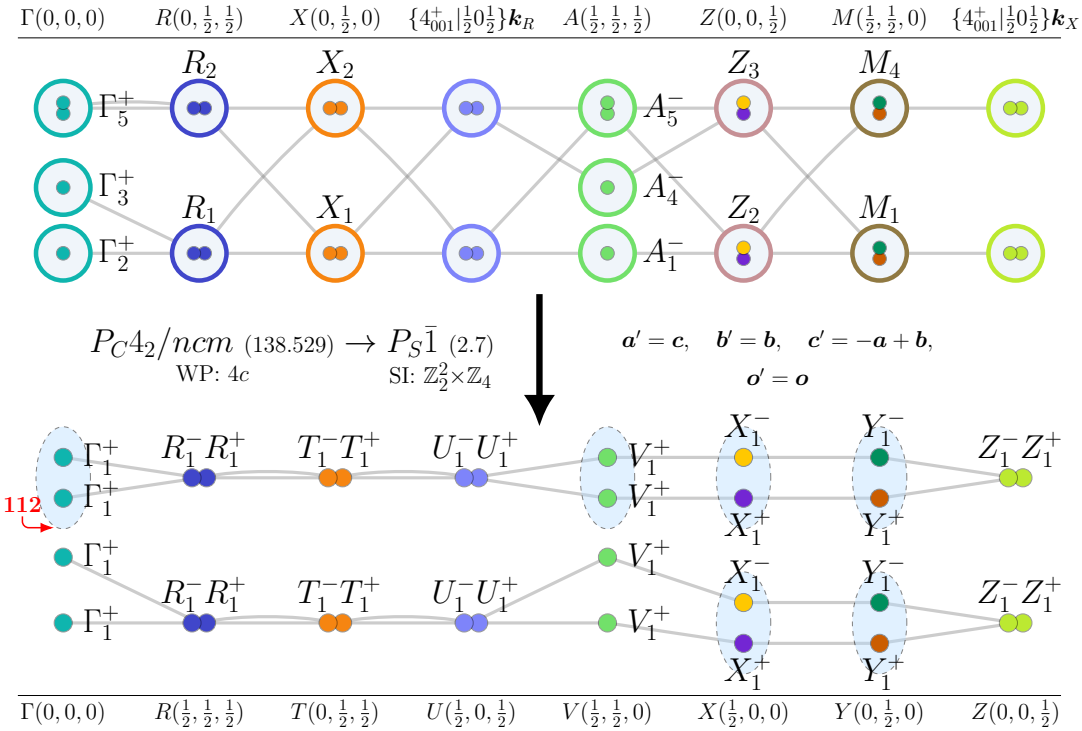
Supplementary Table 196: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

20.2.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 104: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/ncm (138.529).

Supplementary Table 197: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 198: Possible SI values in each gap. Supergroup: P_C4_2/ncm (138.529), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

21 MSG $I4'/mmm'$ (139.535)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $Fm'm'2$ (42.222), $C2/m$ (12.58), $Fm'm'm$ (69.524), $C2/m$ (12.58).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), $C2'$ (5.15), Cm (8.32), $Fm'm2'$ (42.221), Cm (8.32), $C2$ (5.13), $Imm2$ (44.229), $I4'mm'$ (107.230), $C2$ (5.13), $Imm2$ (44.229), $Immm$ (71.533).

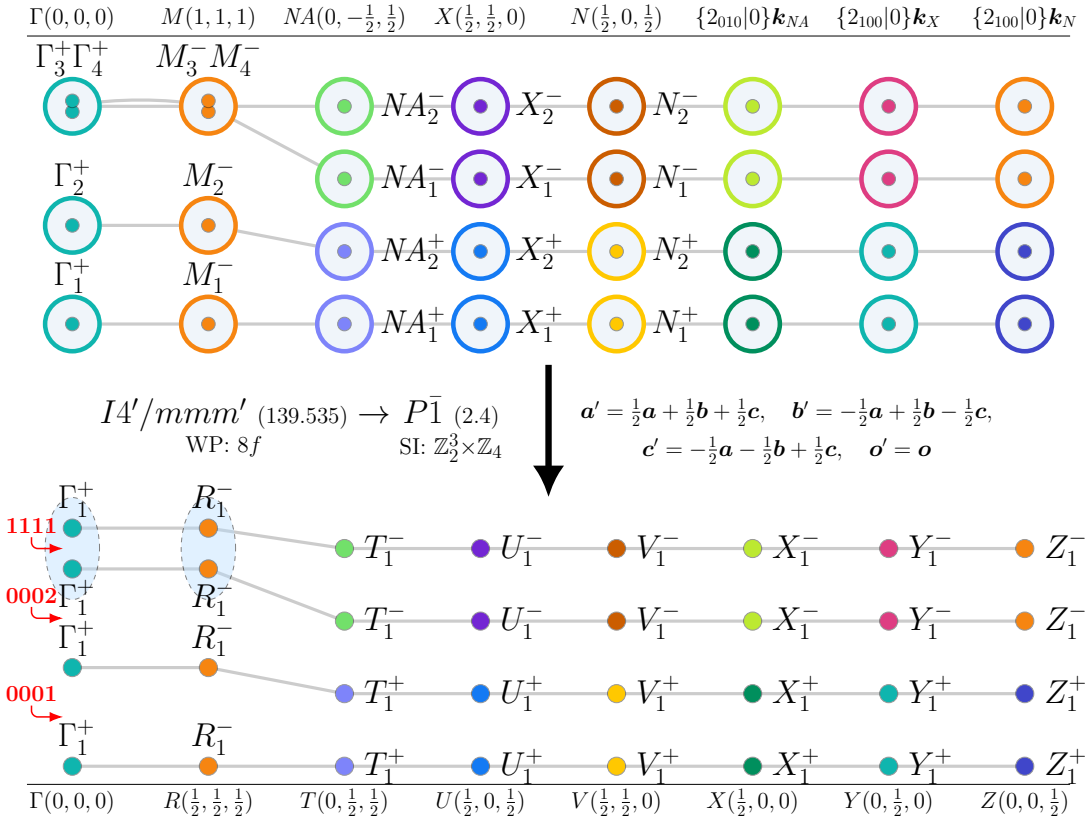
21.1 WP: $8f$

BCS Materials: $CaFe_4Al_8$ (180 K)[83].

21.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [001] and strain ⊥ [100],
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].



Supplementary Figure 105: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8f$ of supergroup $I4'/mmm'$ (139.535).

[83]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.236`

Supplementary Table 199: Number of times a given SI can appear in the band structure. Supergroup: $I4'/mmm'$ (139.535), WP: $8f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1
0002	0, 1
0003	0, 1
1110	0, 1
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

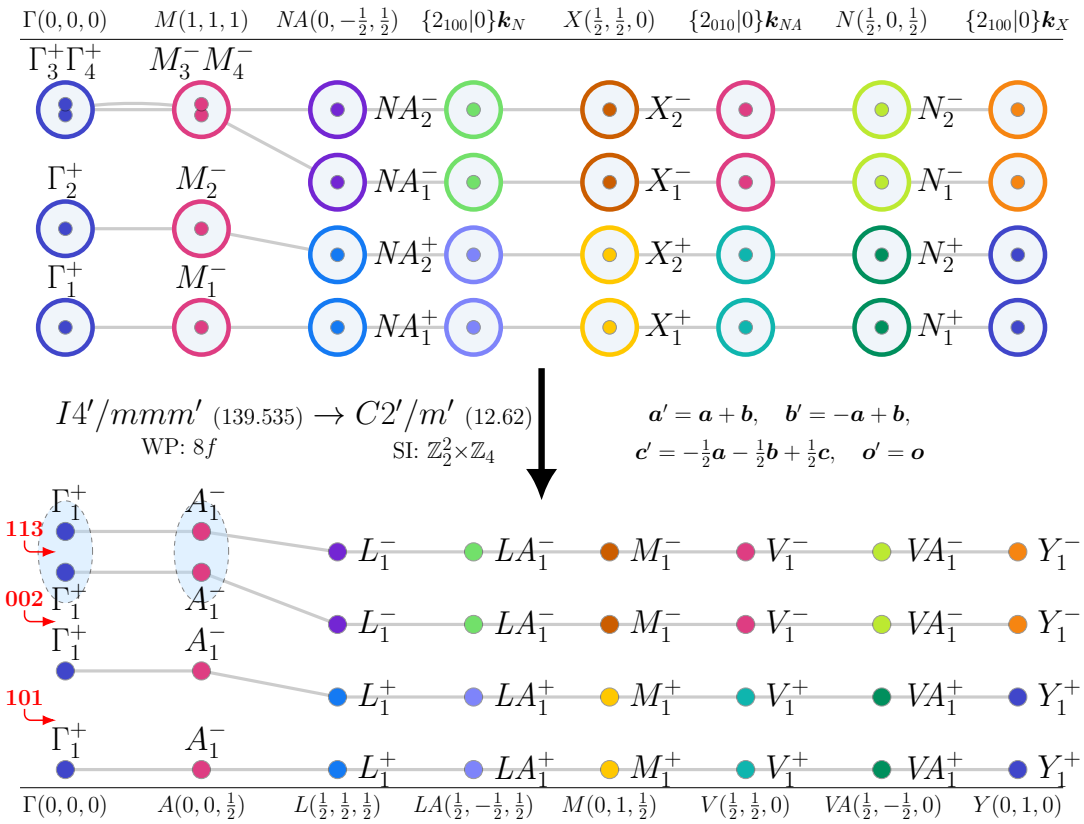
Supplementary Table 200: Possible SI values in each gap. Supergroup: $I4'/mmm'$ (139.535), WP: $8f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 1113
2	0002, 1110
3	0003, 1111
4	0000

21.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [110].



Supplementary Figure 106: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8f$ of supergroup $I4'/mmm'$ (139.535).

Supplementary Table 201: Number of times a given SI can appear in the band structure. Supergroup: $I4'/mmm'$ (139.535), WP: $8f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
002	0, 1
012	0, 1
101	0, 1
103	0, 1
111	0, 1
113	0, 1
gapless	0
nontrivial	3

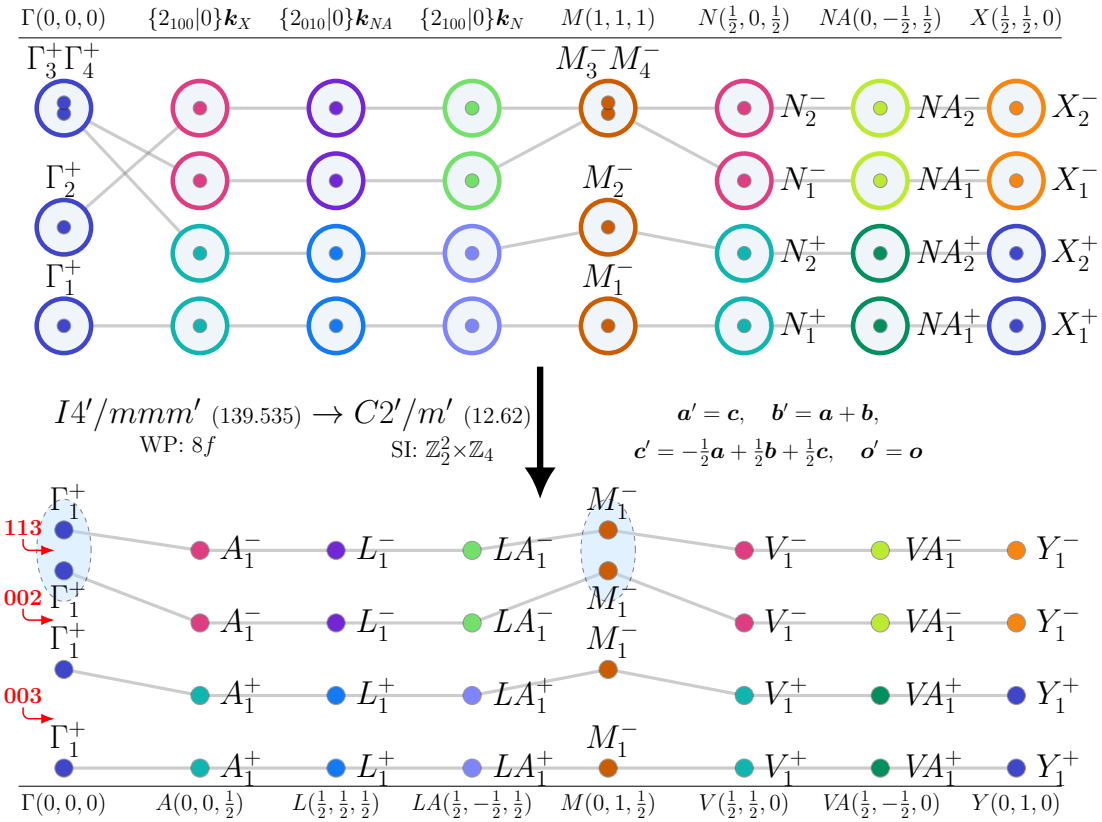
Supplementary Table 202: Possible SI values in each gap. Supergroup: $I4'/mmm'$ (139.535), WP: $8f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	101, 111
2	002, 012
3	103, 113
4	000

21.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 107: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8f$ of supergroup $I4'/mmm'$ (139.535).

Supplementary Table 203: Number of times a given SI can appear in the band structure. Supergroup: $I4'/mmm'$ (139.535), WP: $8f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1
002	0, 1
003	0, 1
110	0, 1
111	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 204: Possible SI values in each gap. Supergroup: $I4'/mmm'$ (139.535), WP: $8f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	003, 111
2	002, 110
3	001, 113
4	000

22 MSG I_c4/mcm (140.550)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P_S\bar{1}$ (2.7), $Fm'm'2$ (42.222), C_a2 (5.17), $C2/m$ (12.58), $Fm'm'm$ (69.524), C_a2/m (12.64), $Fm'm'2$ (42.222), C_a2 (5.17), I_cba2 (45.239), $C2/m$ (12.58), $Fm'm'm$ (69.524), $Im'm'm$ (71.536), C_a2/m (12.64), $C2/c$ (15.85), $Im'm'a$ (74.558), C_c2/c (15.90), $I4m'm'$ (107.231), $I4/mm'm'$ (139.537).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Cm (8.32), $Fm'm2'$ (42.221), C_am (8.36), Cm (8.32), $Fm'm2'$ (42.221), $Im'm2'$ (44.231), C_am (8.36), Cc (9.37), $Im'a2'$ (46.243), C_cc (9.40), $C2$ (5.13), F_Smm2 (42.223), $C2$ (5.13), $Im'm'2$ (44.232), F_Smm2 (42.223), F_Smmm (69.526), $C2$ (5.13), $Im'm'2$ (44.232), C_c2 (5.16), I_ama2 (46.247), I_cbam (72.546), I_c4cm (108.238).

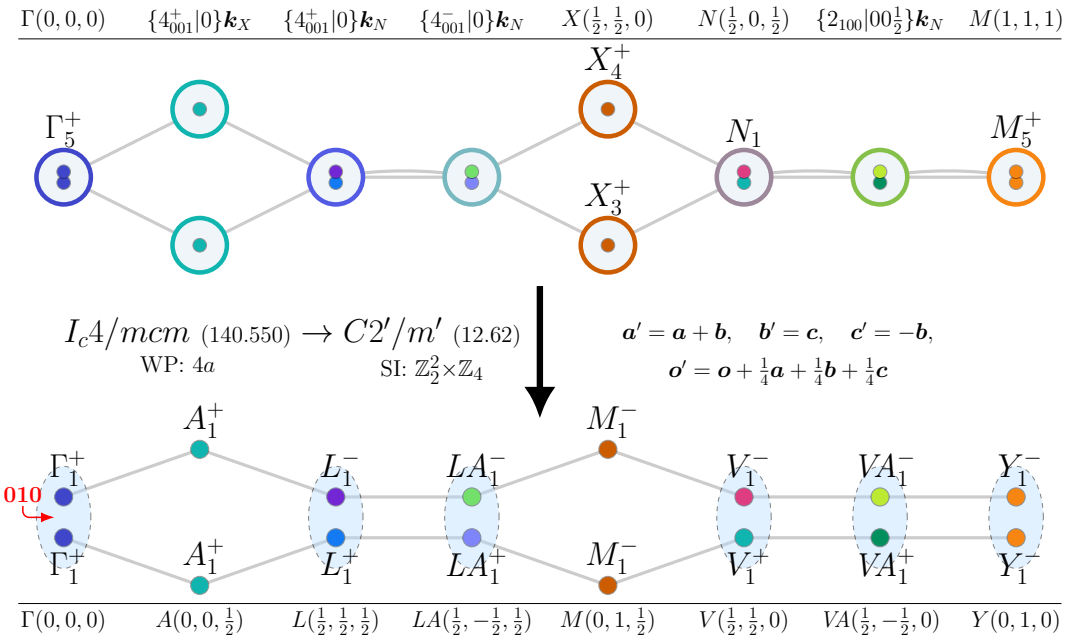
22.1 WP: $4a$

BCS Materials: $SrFeO_2F$ (710 K)[84], $Pb_{0.7}Bi_{0.3}Fe_{0.762}W_{0.231}O_3$ (504 K)[85], $Pb_{0.8}Bi_{0.2}Fe_{0.728}W_{0.264}O_3$ (435 K)[86], $KNiF_3$ (275 K)[87], $UNiGa_5$ (86 K)[88], $CeIr(In_{0.97}Cd_{0.03})_5$ (3 K)[89].

22.1.1 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 108: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4a$ of supergroup I_c4/mcm (140.550).

[84] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.84
[85] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.591
[86] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.590
[87] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.250
[88] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.254
[89] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.598

Supplementary Table 205: Number of times a given SI can appear in the band structure. Supergroup: I_c4/mcm (140.550), WP: $4a$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
010	0, 1
110	0, 1
gapless	0
nontrivial	1

Supplementary Table 206: Possible SI values in each gap. Supergroup: I_c4/mcm (140.550), WP: $4a$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	010, 110
2	000

22.2 WP: $8g$

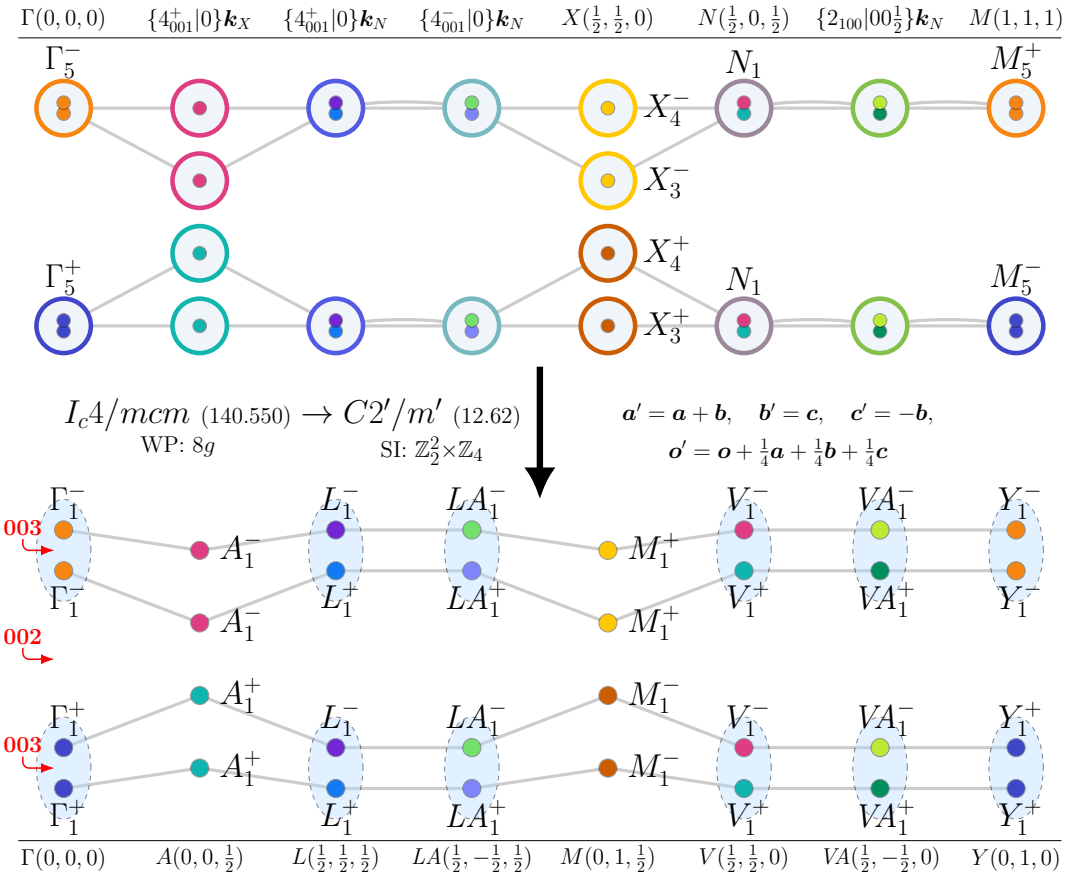
BCS Materials: Tb_2CoGa_8 (28.5 K)[90], Dy_2CoGa_8 (15.2 K)[91], Nd_2RhIn_8 (10.63 K)[92].

22.2.1 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].

[90] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.87`
[91] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.80`
[92] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.82`



Supplementary Figure 109: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8g$ of supergroup I_c4/mcm (140.550).

Supplementary Table 207: Number of times a given SI can appear in the band structure. Supergroup: I_c4/mcm (140.550), WP: $8g$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
010	0, 1, 2
012	0, 1, 2
100	0, 1
101	0, 1, 2
102	0, 1
103	0, 1, 2
110	0, 1, 2
112	0, 1, 2
gapless	0
nontrivial	2, 3

Supplementary Table 208: Possible SI values in each gap. Supergroup: I_c4/mcm (140.550), WP: $8g$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 100, 102

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Gap#	Possible SI Values
3	001, 003, 010, 012, 101 103, 110, 112
4	000

23 MSG $I4_1/amd$ (141.551)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2/c$ (15.85), $C2/c$ (15.85), $Fddd$ (70.527), $C2/m$ (12.58), $I4_1/a$ (88.81).

Trivial-SI Subgroups: Cc (9.37), Cc (9.37), Cm (8.32), $C2$ (5.13), $Fdd2$ (43.224), $C2$ (5.13), $Fdd2$ (43.224), $Imm2$ (44.229), $C2$ (5.13), $Ima2$ (46.241), $Imma$ (74.554), $I4_1$ (80.29), $I4_1md$ (109.239).

23.1 WP: $8d$

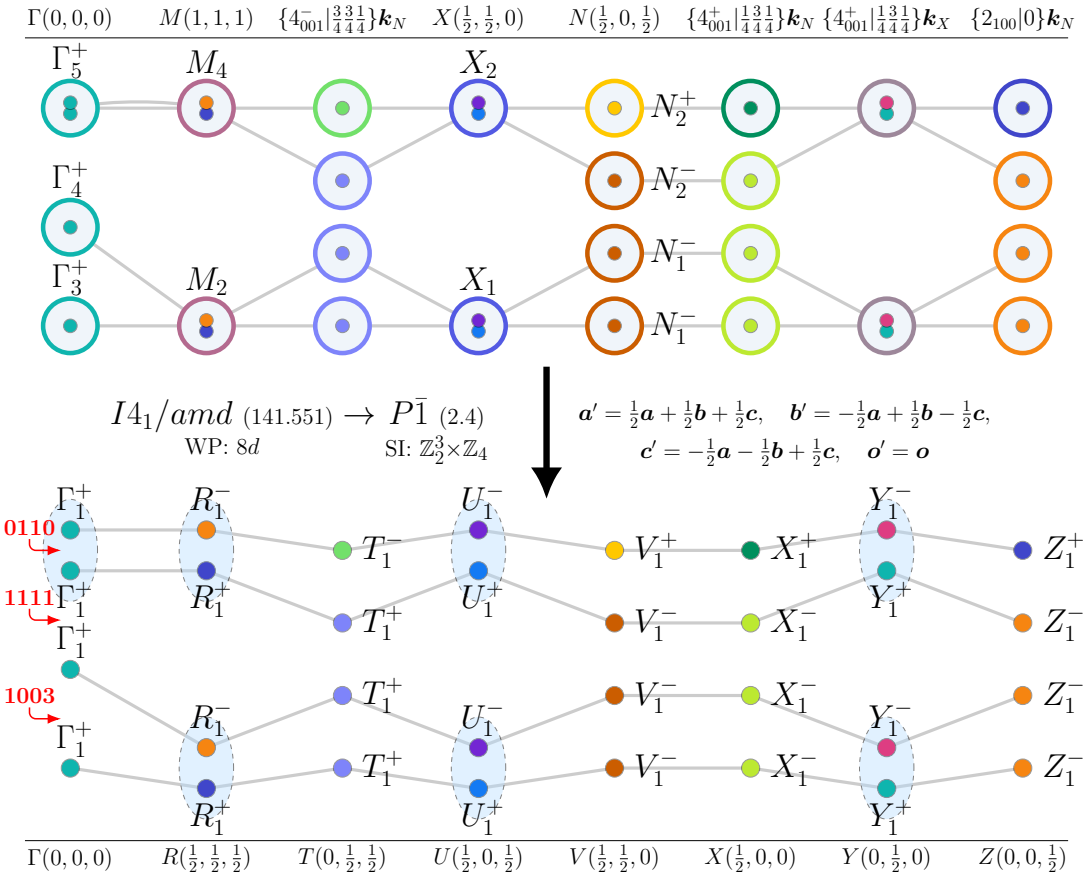
BCS Materials: $CdYb_2S_4$ (1.92 K)[93], $CdYb_2Se_4$ (1.75 K)[94].

23.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [001] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [110],
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B in generic direction.

[93] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.324`
[94] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.325`



Supplementary Figure 110: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1/amd$ (141.551).

Supplementary Table 209: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/amd$ (141.551), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1
0013	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1
1003	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1
1103	0, 1, 2
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

Supplementary Table 210: Possible SI values in each gap. Supergroup: $I4_1/amd$ (141.551), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

24 MSG $I4'_1/am'd$ (141.554)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2/c$ (15.85), $C2/c$ (15.85), $Im'm'a$ (74.558), $Fddd$ (70.527).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), $C2'$ (5.15), Cc (9.37), Cc (9.37), $Im'a2'$ (46.243), $C2$ (5.13), $Fdd2$ (43.224), $C2$ (5.13), $Im'm'2$ (44.232), $Fdd2$ (43.224), $I4'_1m'd$ (109.241).

24.1 WP: $8c + 8d$

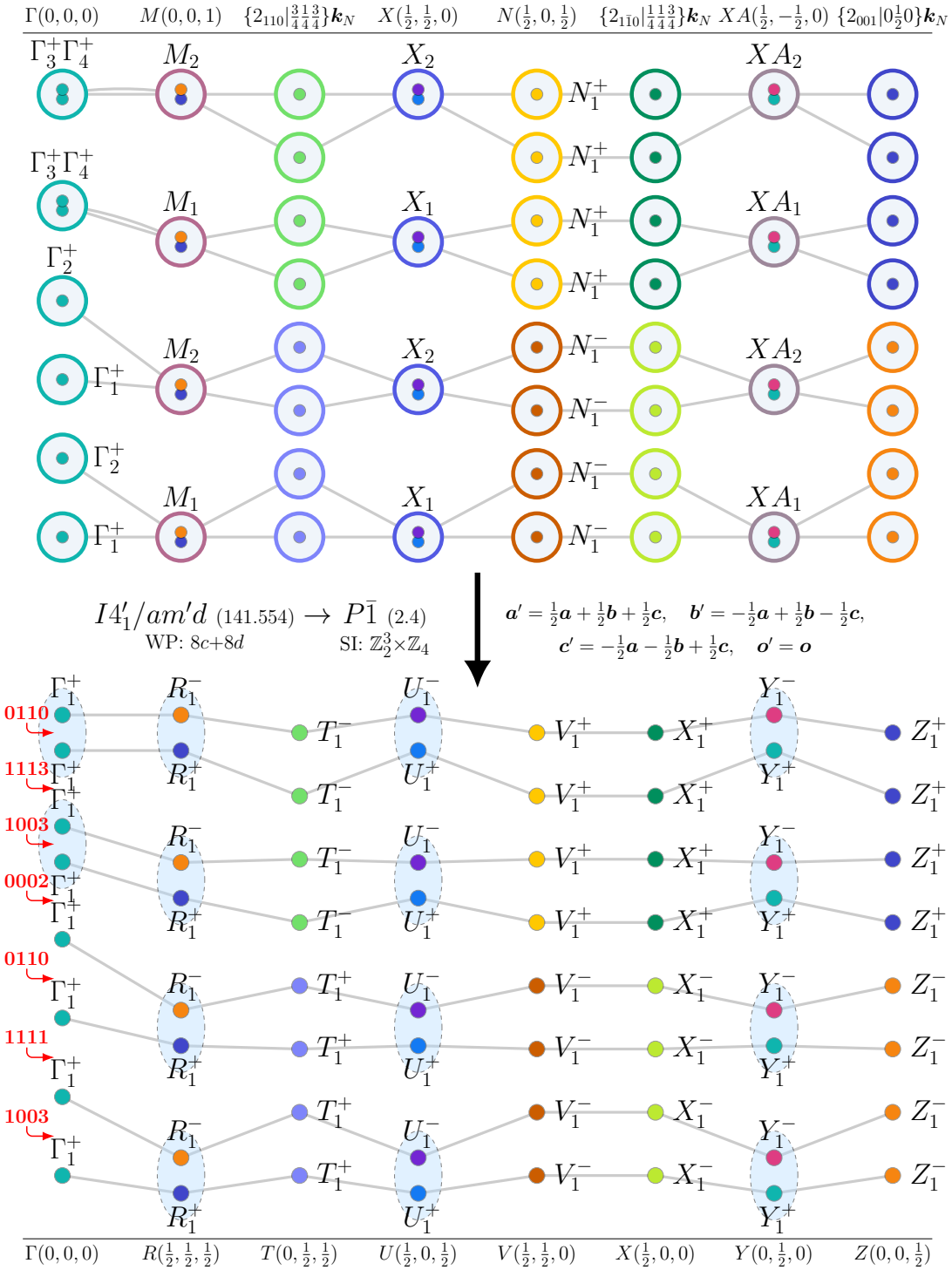
BCS Materials: $Er_2Ru_2O_7$ (90 K)[95].

24.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [001] and strain ⊥ [110],
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110].

[95] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.154`



Supplementary Figure 111: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1'/am'd$ (141.554).

Supplementary Table 211: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/am'd$ (141.554), WP: 8c+8d, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4

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SI	Possible #Gaps
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1111	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	6, 7

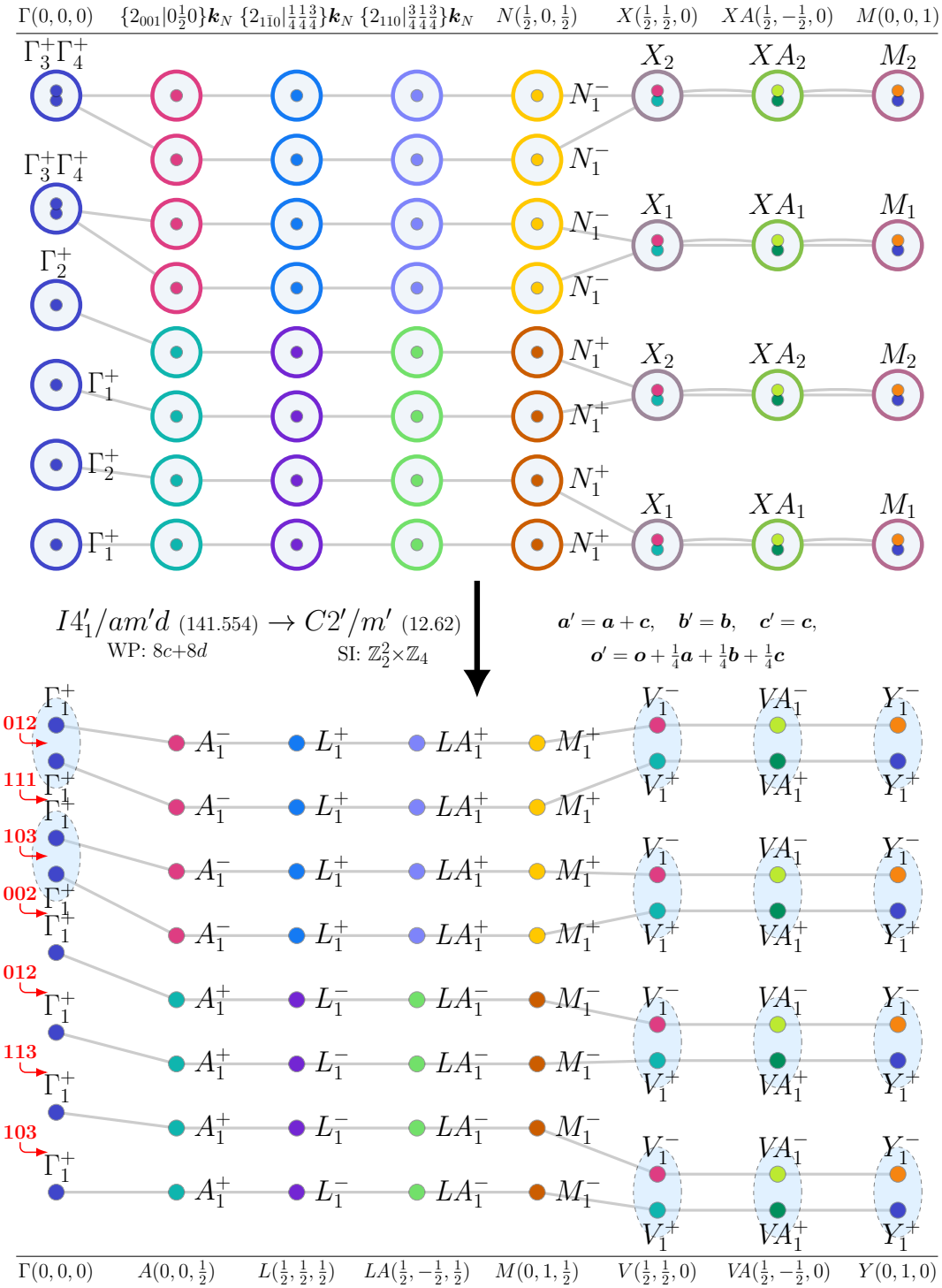
Supplementary Table 212: Possible SI values in each gap. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$+$8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0002
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	1111, 1113
7	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
8	0000

24.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 112: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1'/am'd$ (141.554).

Supplementary Table 213: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/am'd$ (141.554), WP: 8c+8d, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3, 4
002	0, 1
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
012	0, 1, 2, 3, 4

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...continued

SI	Possible #Gaps
101	0, 1, 2, 3, 4
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	0, 1, 2
112	0, 1, 2, 3, 4
113	0, 1, 2
gapless	0
nontrivial	6, 7

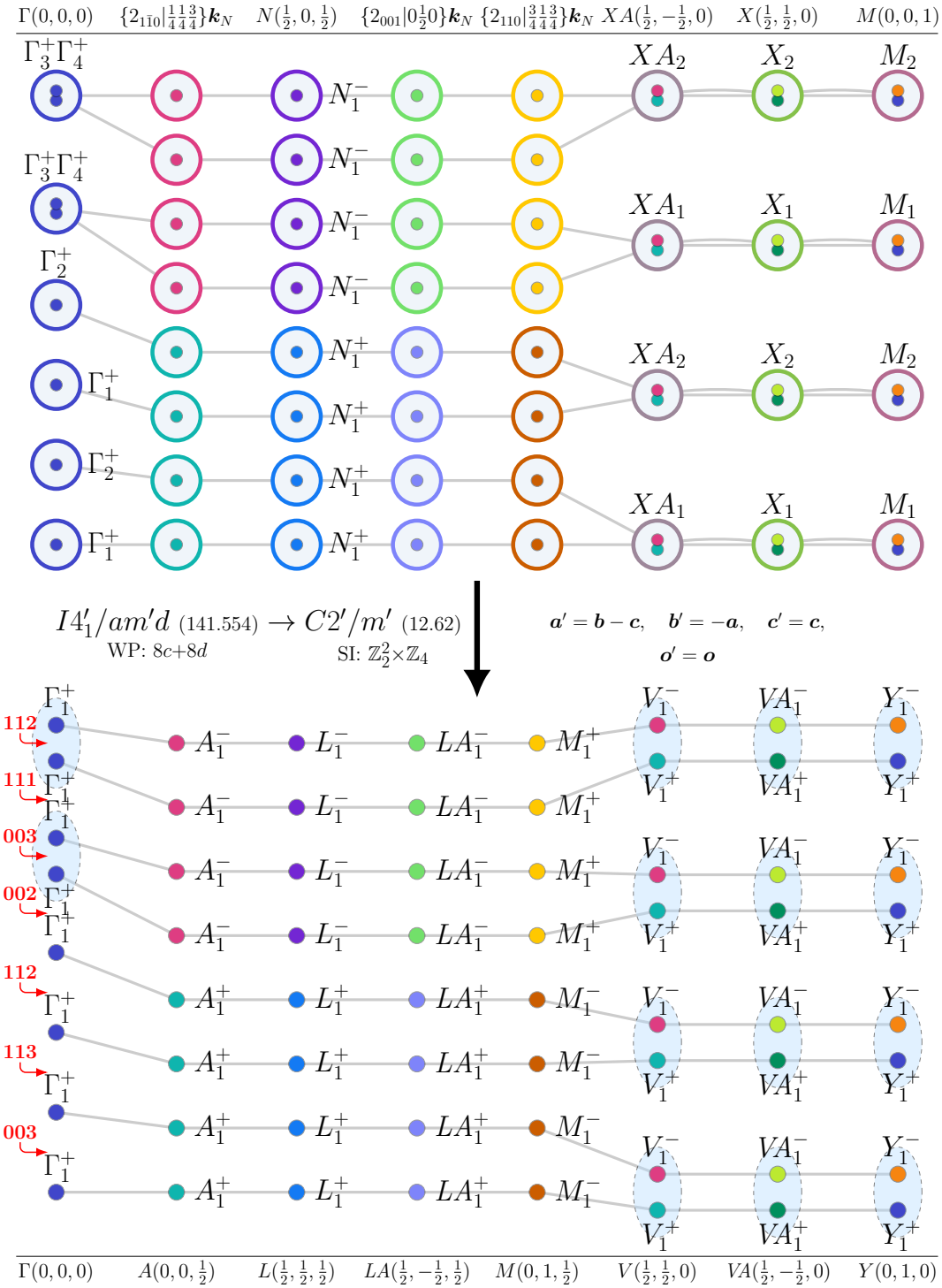
Supplementary Table 214: Possible SI values in each gap. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002
5	001, 003, 010, 012, 101 103, 110, 112
6	111, 113
7	001, 003, 010, 012, 101 103, 110, 112
8	000

24.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 113: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1'/am'd$ (141.554).

Supplementary Table 215: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3, 4
002	0, 1
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
012	0, 1, 2, 3, 4

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SI	Possible #Gaps
101	0, 1, 2, 3, 4
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	0, 1, 2
112	0, 1, 2, 3, 4
113	0, 1, 2
gapless	0
nontrivial	6, 7

Supplementary Table 216: Possible SI values in each gap. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002
5	001, 003, 010, 012, 101 103, 110, 112
6	111, 113
7	001, 003, 010, 012, 101 103, 110, 112
8	000

24.2 WP: $8c$

BCS Materials: $Er_2Ti_2O_7$ (1.173 K)[96].

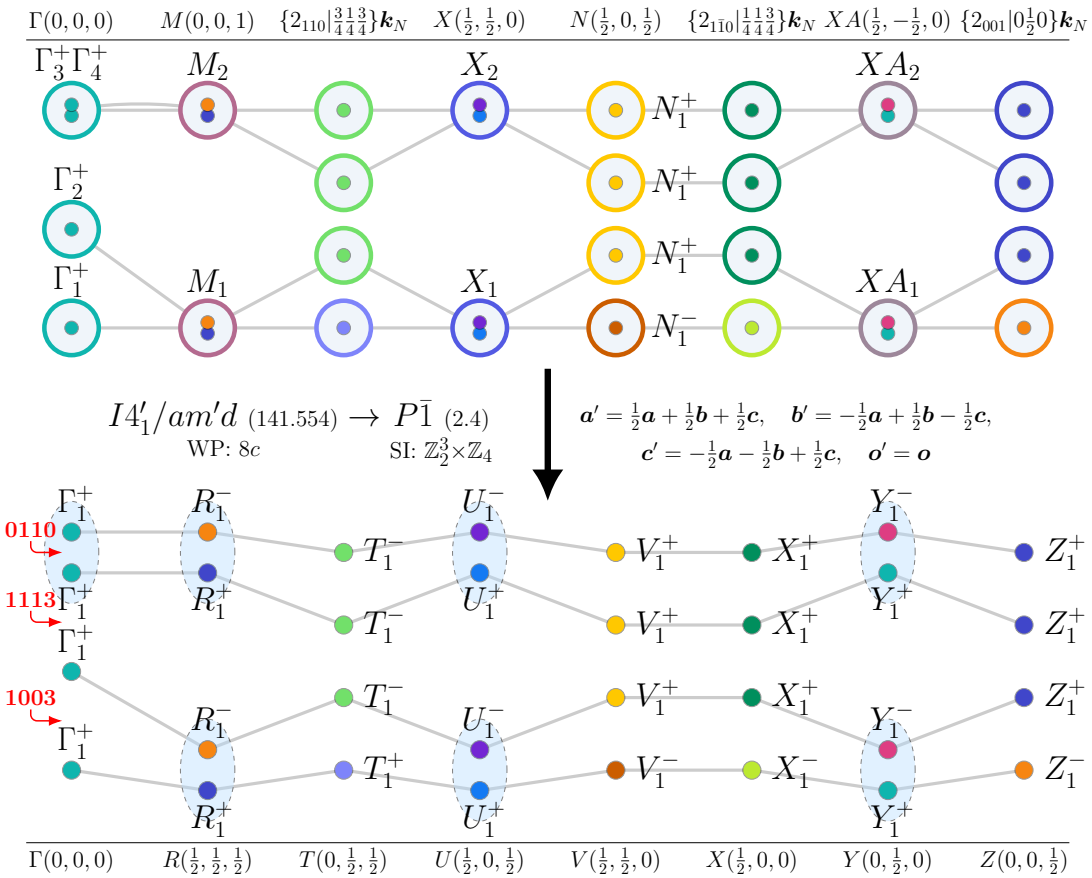
24.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B $\parallel$ [001] and strain $\perp$ [110],
- B $\parallel$ [100] and strain $\parallel$ [110],
- B $\parallel$ [100] and strain $\perp$ [001],
- B $\parallel$ [100] and strain $\perp$ [100],
- B $\parallel$ [100] and strain $\perp$ [110],
- B $\parallel$ [110] and strain $\parallel$ [100],
- B $\parallel$ [110] and strain $\perp$ [001],
- B $\parallel$ [110] and strain $\perp$ [100],
- B in generic direction,
- B $\perp$ [100] and strain $\parallel$ [110],

[96]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.29`

- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110].



Supplementary Figure 114: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1'/am'd$ (141.554).

Supplementary Table 217: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0010	0, 1
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1
0113	0, 1, 2
1000	0, 1
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1100	0, 1
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

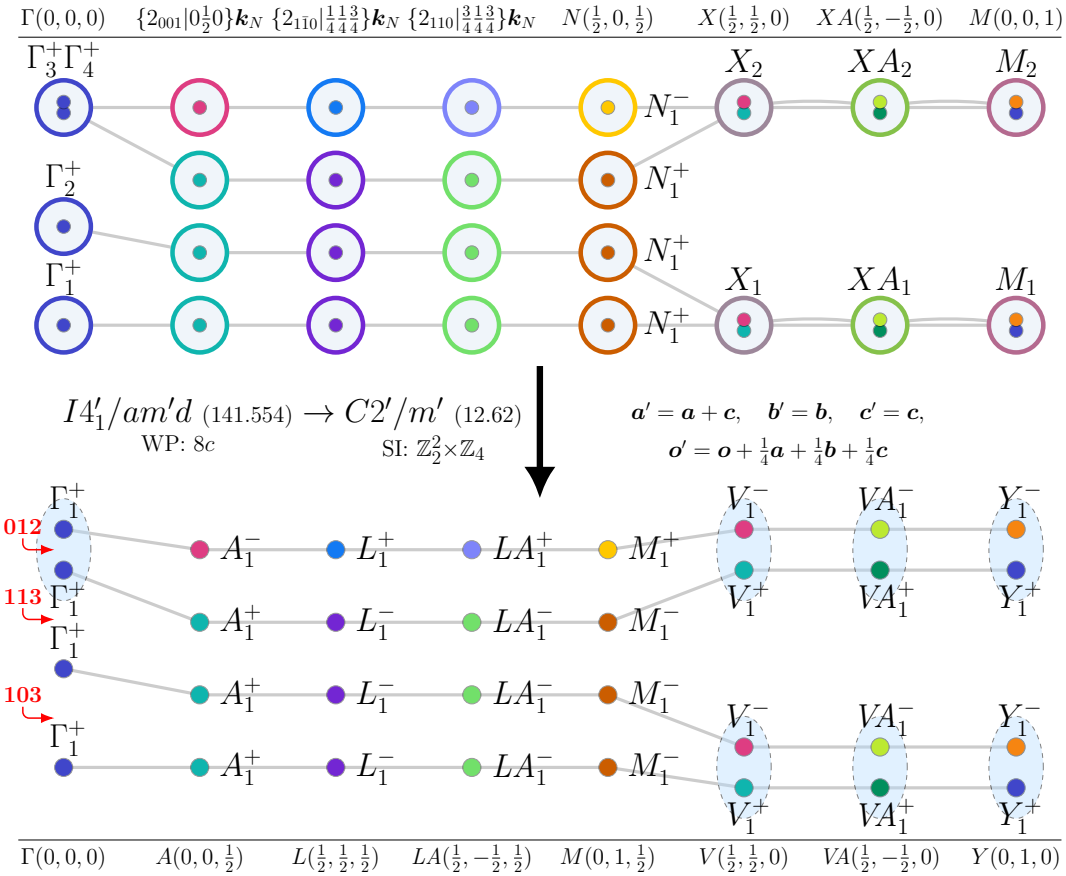
Supplementary Table 218: Possible SI values in each gap. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

24.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 115: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1'/am'd$ (141.554).

Supplementary Table 219: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

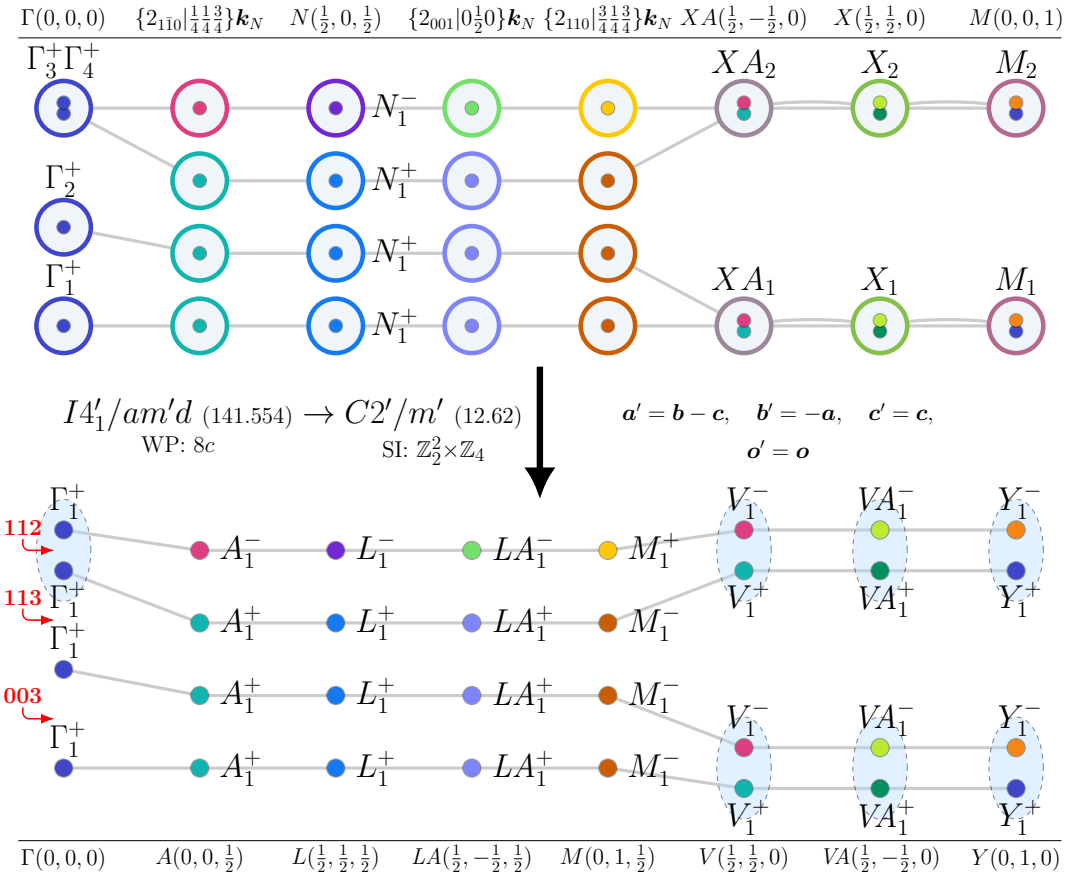
Supplementary Table 220: Possible SI values in each gap. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

24.2.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 116: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1'/am'd$ (141.554).

Supplementary Table 221: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 222: Possible SI values in each gap. Supergroup: $I4_1'/am'd$ (141.554), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112

continued on next page...

...continued

Gap#	Possible SI Values
4	000

25 MSG $I4_1'/amd'$ (141.555)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $C2/c$ (15.85), $Fd'd'd$ (70.530), $C2/m$ (12.58).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), $C2'$ (5.15), Cc (9.37), $Fd'd2'$ (43.226), Cm (8.32), $C2$ (5.13), $Fd'd'2$ (43.227), $Imm2$ (44.229), $I4_1'md'$ (109.242), $C2$ (5.13), $Ima2$ (46.241), $Imma$ (74.554).

25.1 WP: $8d$

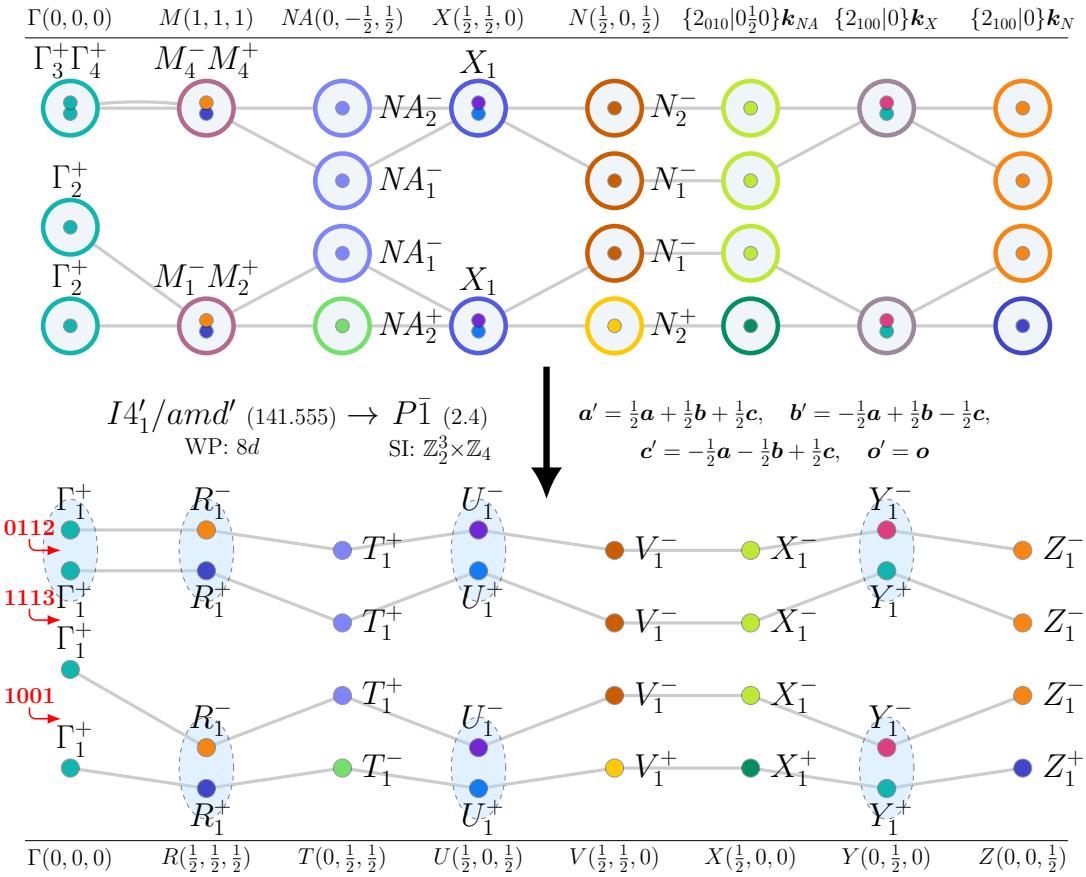
BCS Materials: $Gd_2Sn_2O_7$ (1 K)[97], $Er_2Pt_2O_7$ (0.38 K)[98], $Er_2Sn_2O_7$ (0.1 K)[99].

25.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [001] and strain ⊥ [100],
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].

[97] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.47
[98] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.238
[99] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.237



Supplementary Figure 117: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1'/amd'$ (141.555).

Supplementary Table 223: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/amd'$ (141.555), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1
0013	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1
1003	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1
1103	0, 1, 2
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

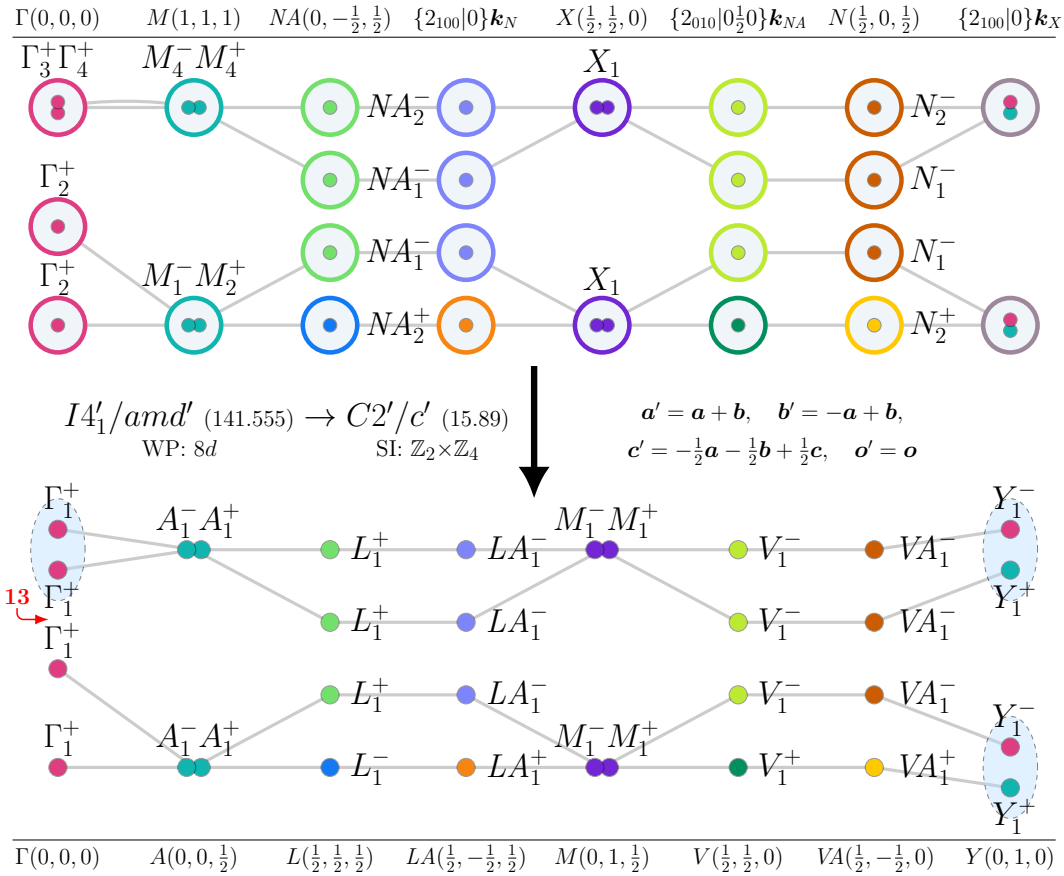
Supplementary Table 224: Possible SI values in each gap. Supergroup: $I4_1'/amd'$ (141.555), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

25.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B ∥ [110].



Supplementary Figure 118: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1'/amd'$ (141.555).

Supplementary Table 225: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/amd'$ (141.555), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

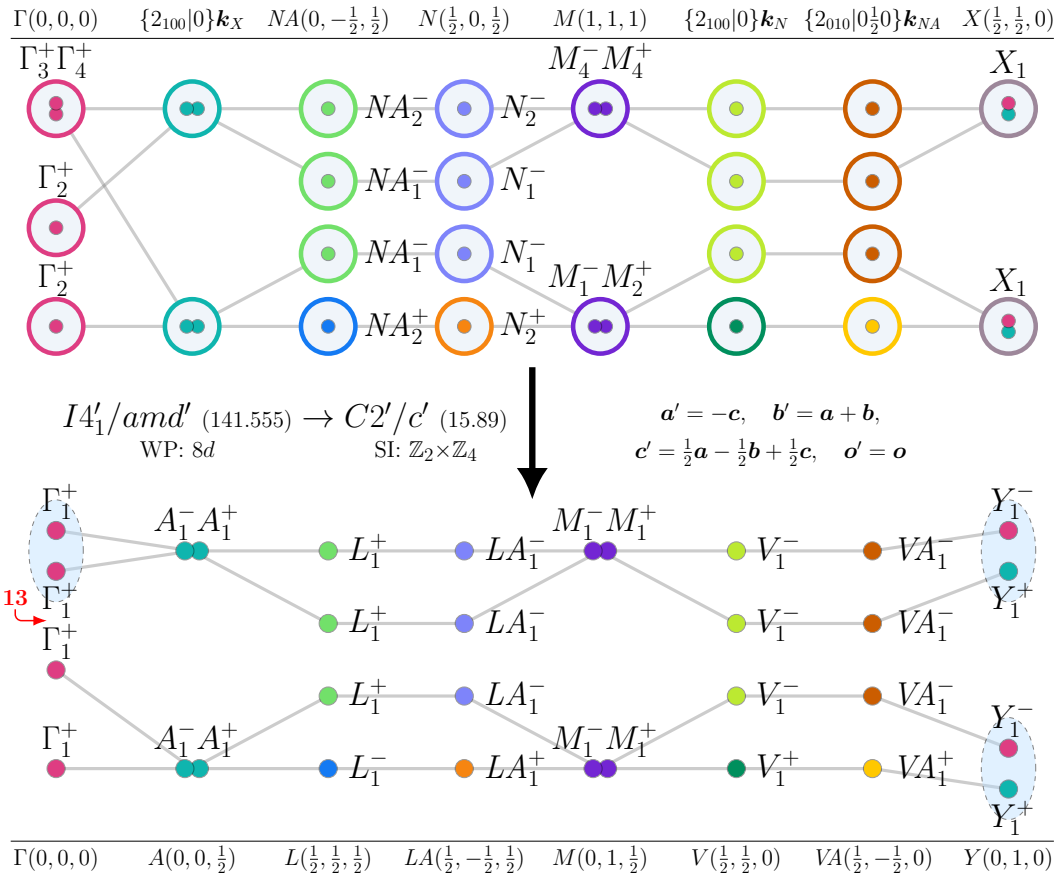
Supplementary Table 226: Possible SI values in each gap. Supergroup: $I4_1'/amd'$ (141.555), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

25.1.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 119: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1'/amd'$ (141.555).

Supplementary Table 227: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/amd'$ (141.555), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

Supplementary Table 228: Possible SI values in each gap. Supergroup: $I4_1'/amd'$ (141.555), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

26 MSG $I4_1/am'd'$ (141.557)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2/c$ (15.85), $Fd'd'd$ (70.530), $Im'm'a$ (74.558).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), Cc (9.37), $Fd'd2'$ (43.226), $Im'a2'$ (46.243), $C2$ (5.13), $Fd'd'2$ (43.227), $Im'm'2$ (44.232), $I4_1m'd'$ (109.243).

26.1 WP: $4a + 8d + 8d$

BCS Materials: $NiFe_2O_4$ (858 K)[100].

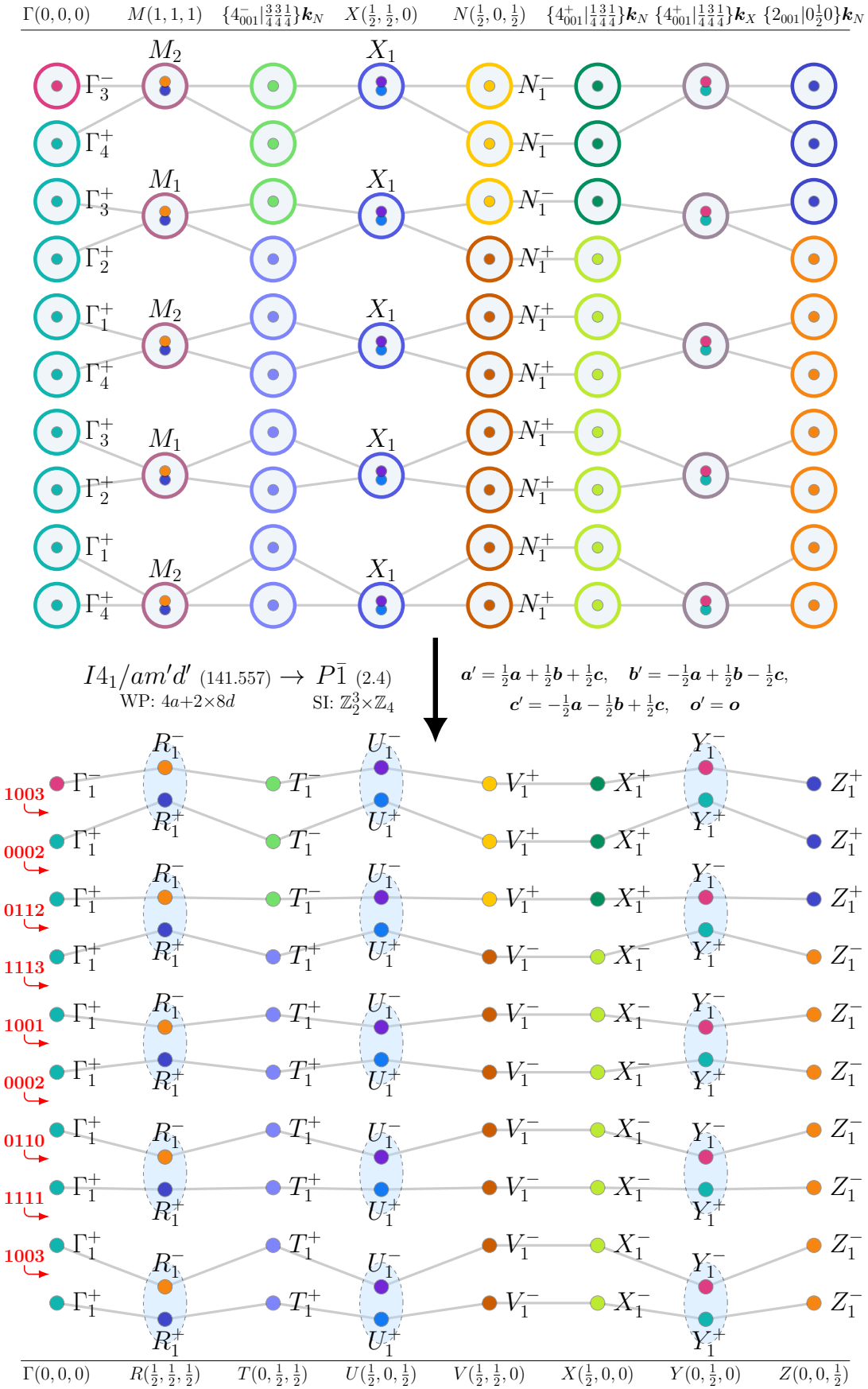
26.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [100] and strain ∥ [110],

[100]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.713`

- B $\perp$ [100] and strain $\perp$ [001],
- B $\perp$ [100] and strain $\perp$ [110],
- B $\perp$ [110] and strain $\parallel$ [100],
- B $\perp$ [110] and strain $\perp$ [001],
- B $\perp$ [110] and strain $\perp$ [100].



Supplementary Figure 120: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a + 8d + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 229: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0002	0, 1, 2
0010	0, 1, 2, 3, 4, 5
0011	0, 1, 2, 3, 4, 5
0012	0, 1, 2, 3, 4, 5
0013	0, 1, 2, 3, 4, 5
0110	0, 1, 2, 3, 4, 5
0111	0, 1, 2, 3, 4, 5
0112	0, 1, 2, 3, 4, 5
0113	0, 1, 2, 3, 4, 5
1000	0, 1, 2, 3, 4, 5
1001	0, 1, 2, 3, 4, 5
1002	0, 1, 2, 3, 4, 5
1003	0, 1, 2, 3, 4, 5
1100	0, 1, 2, 3, 4, 5
1101	0, 1, 2, 3, 4, 5
1102	0, 1, 2, 3, 4, 5
1103	0, 1, 2, 3, 4, 5
1111	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	7, 8, 9

2054

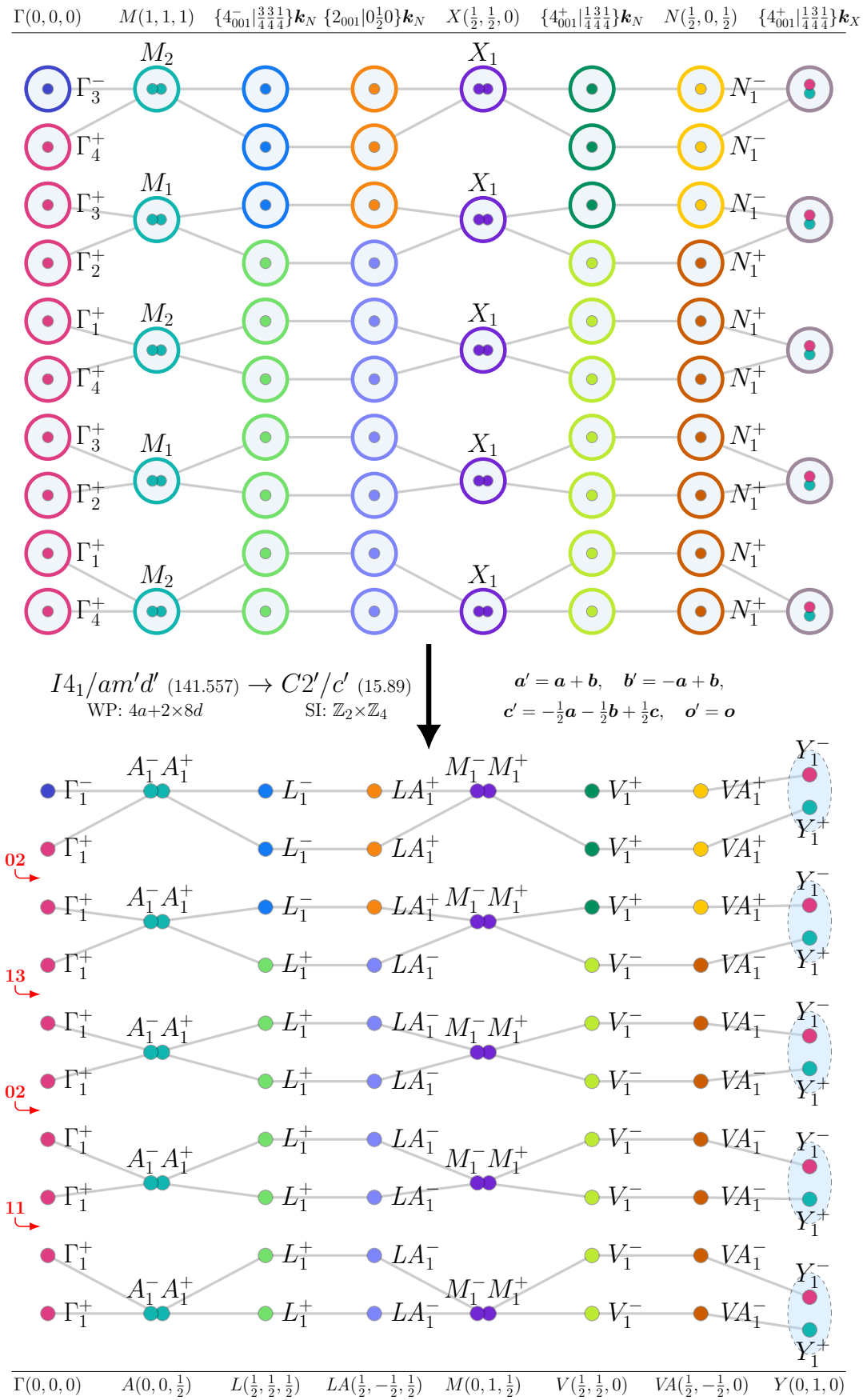
Supplementary Table 230: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0002, 1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0002, 1111, 1113
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	0000, 0002, 1111, 1113
7	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
8	0000, 0002, 1111, 1113
9	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
10	0000

26.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B ∥ [110].



Supplementary Figure 121: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4a + 8d + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 231: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1, 2
02	0, 1, 2
11	0, 1, 2, 3
13	0, 1, 2, 3
gapless	5
nontrivial	2, 3, 4

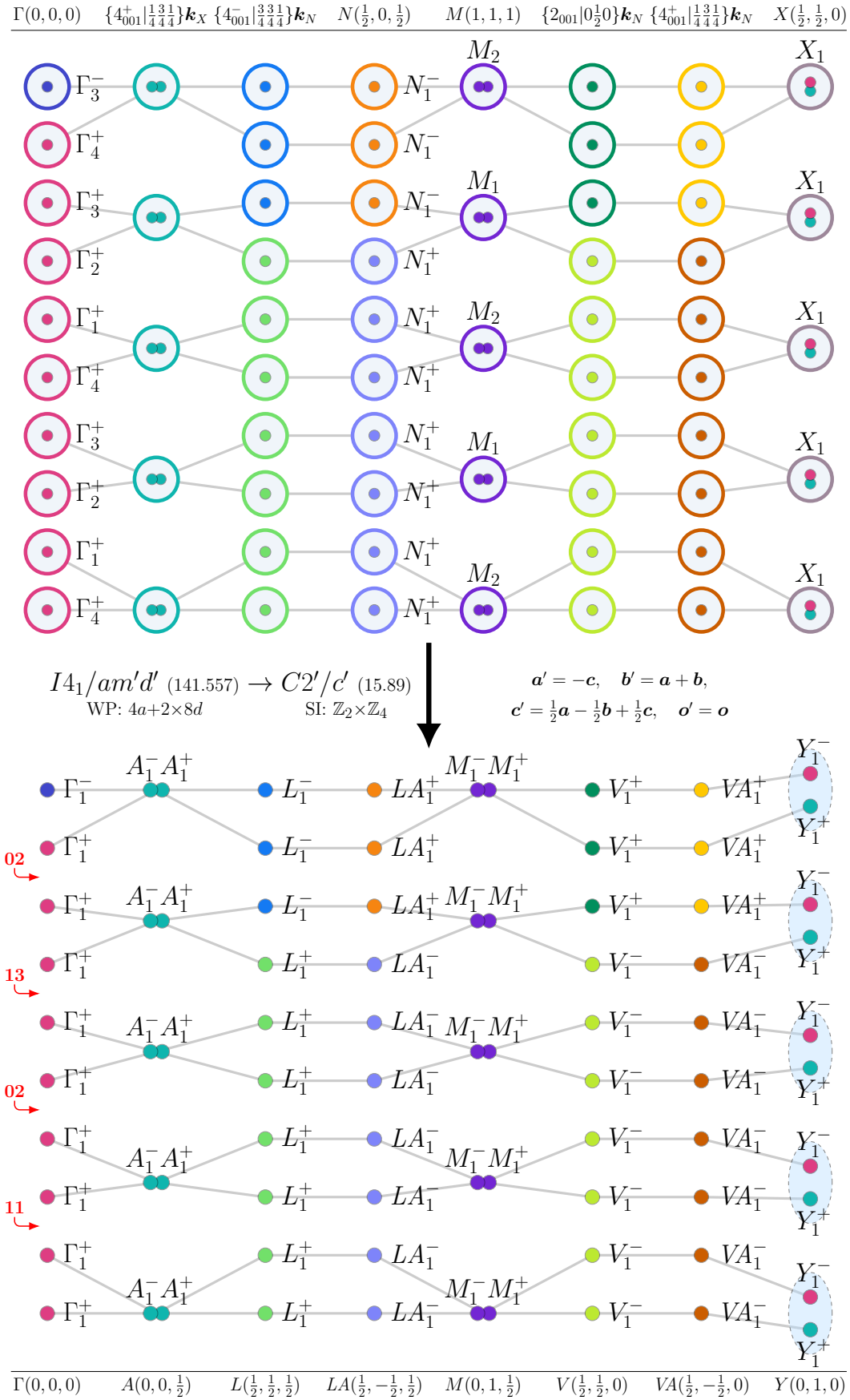
Supplementary Table 232: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G
4	00, 02, 11, 13
5	G
6	00, 02, 11, 13
7	G
8	00, 02, 11, 13
9	G
10	00

26.1.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 122: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4a+8d+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 233: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1, 2
02	0, 1, 2
11	0, 1, 2, 3
13	0, 1, 2, 3
gapless	5
nontrivial	2, 3, 4

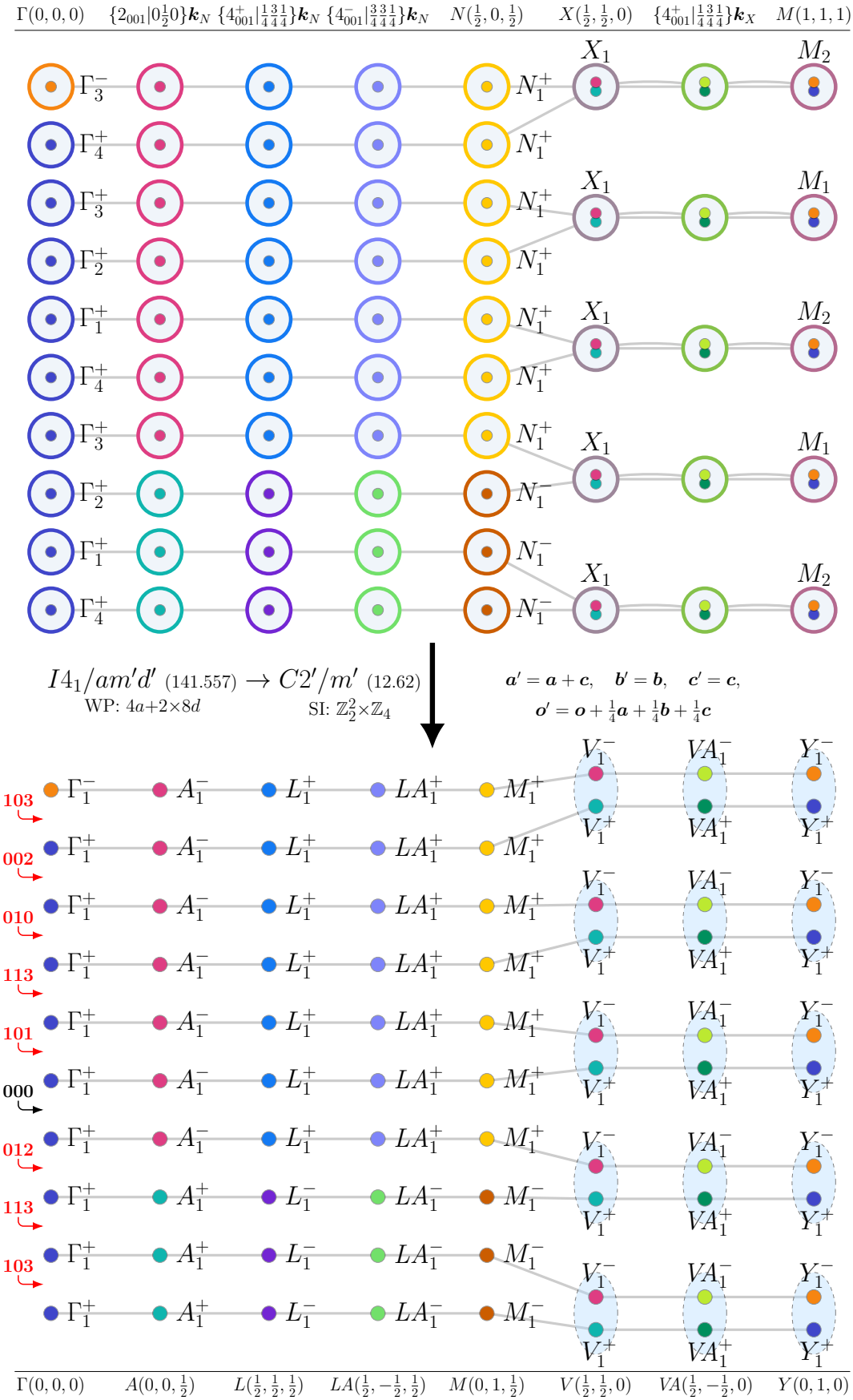
Supplementary Table 234: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G
4	00, 02, 11, 13
5	G
6	00, 02, 11, 13
7	G
8	00, 02, 11, 13
9	G
10	00

26.1.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 123: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4a+8d+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 235: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3, 4, 5
002	0, 1, 2
003	0, 1, 2, 3, 4, 5
010	0, 1, 2, 3, 4, 5
012	0, 1, 2, 3, 4, 5
101	0, 1, 2, 3, 4, 5
103	0, 1, 2, 3, 4, 5
110	0, 1, 2, 3, 4, 5
111	0, 1, 2, 3
112	0, 1, 2, 3, 4, 5
113	0, 1, 2, 3
gapless	0
nontrivial	7, 8, 9

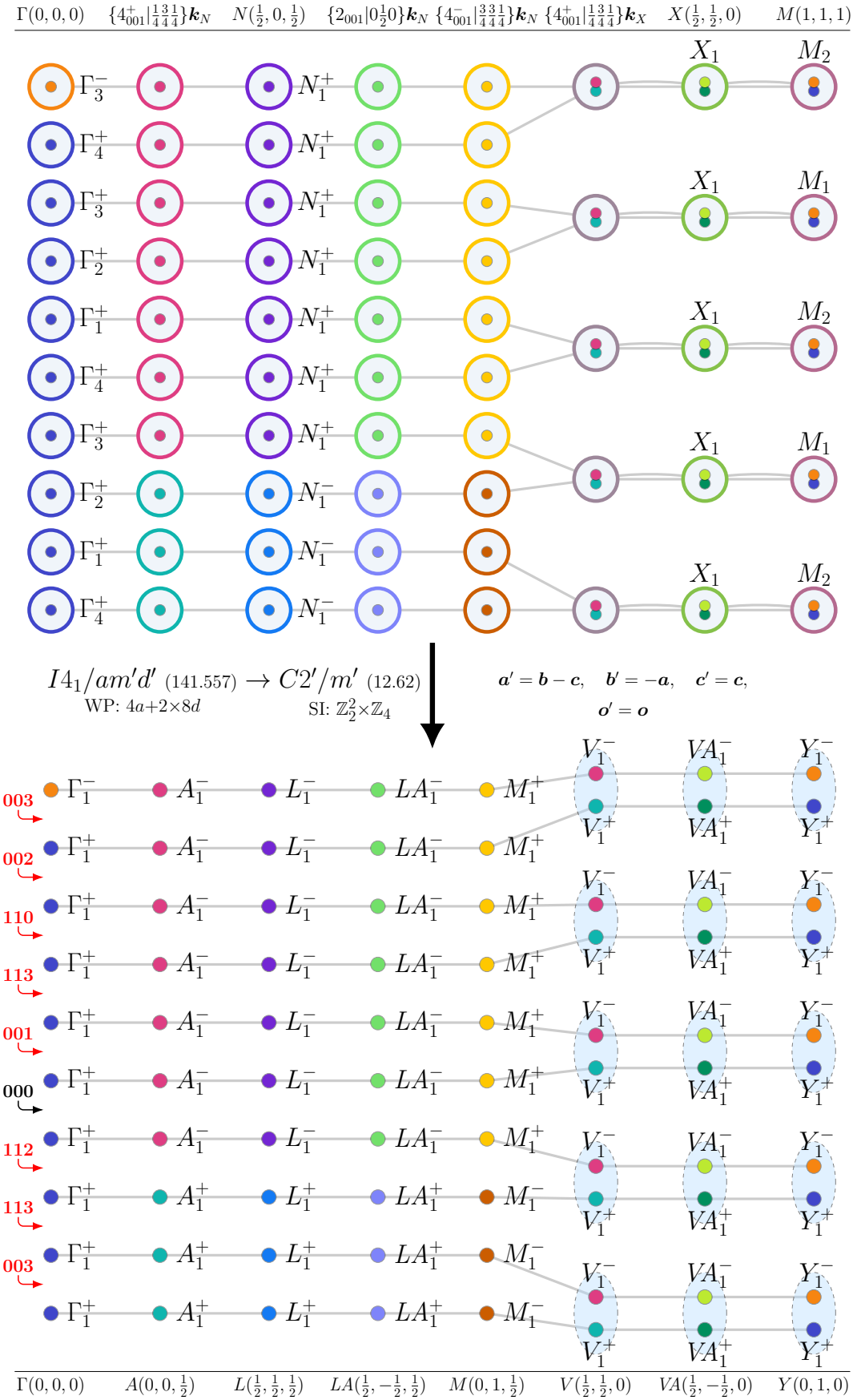
Supplementary Table 236: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000, 002, 111, 113
7	001, 003, 010, 012, 101 103, 110, 112
8	000, 002, 111, 113
9	001, 003, 010, 012, 101 103, 110, 112
10	000

26.1.5 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 124: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4a + 8d + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 237: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3, 4, 5
002	0, 1, 2
003	0, 1, 2, 3, 4, 5
010	0, 1, 2, 3, 4, 5
012	0, 1, 2, 3, 4, 5
101	0, 1, 2, 3, 4, 5
103	0, 1, 2, 3, 4, 5
110	0, 1, 2, 3, 4, 5
111	0, 1, 2, 3
112	0, 1, 2, 3, 4, 5
113	0, 1, 2, 3
gapless	0
nontrivial	7, 8, 9

Supplementary Table 238: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000, 002, 111, 113
7	001, 003, 010, 012, 101 103, 110, 112
8	000, 002, 111, 113
9	001, 003, 010, 012, 101 103, 110, 112
10	000

26.2 WP: $4a + 8d$

BCS Materials: $Li_{0.5}FeCr_{1.5}O_4$ (417 K)[101], $FeCr_2S_4$ (172 K)[102], Ce_5TeO_8 (40 K)[103].

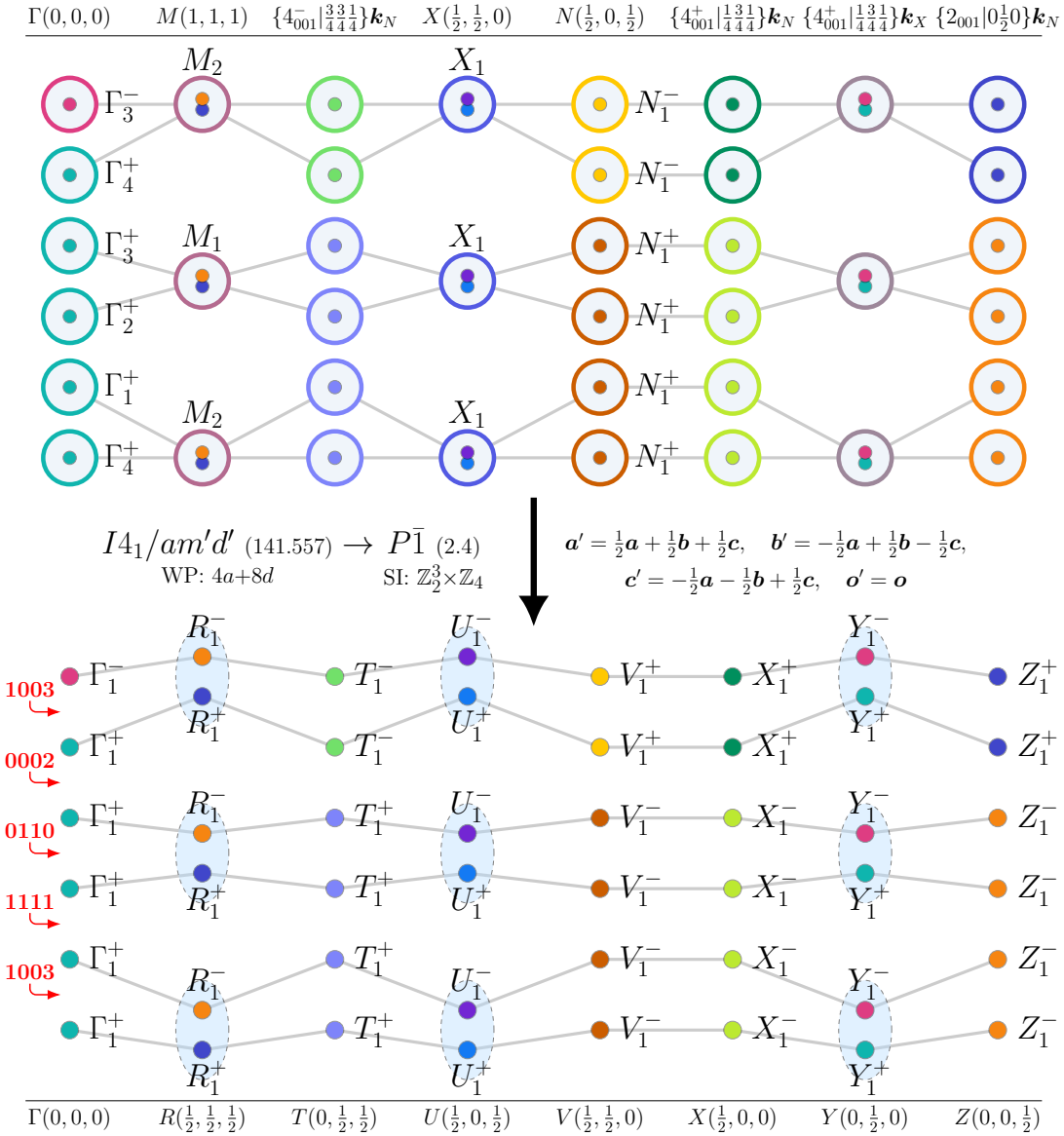
26.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],

[101] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.570`
[102] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.613`
[103] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.725`

- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].



Supplementary Figure 125: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 239: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3
1102	0, 1, 2, 3
1103	0, 1, 2, 3
1111	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	4, 5

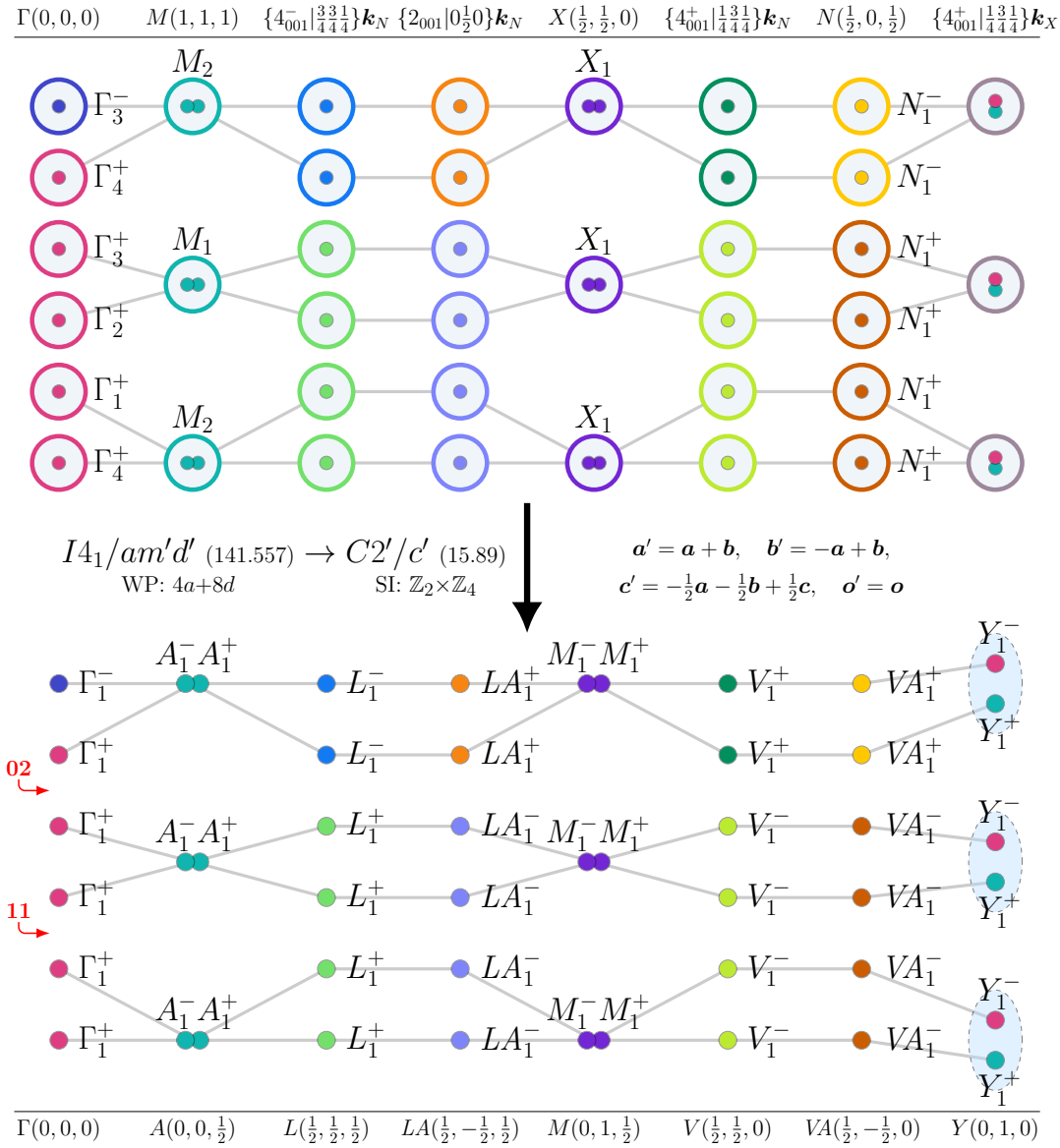
Supplementary Table 240: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0002, 1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0002, 1111, 1113
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	0000

26.2.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B ∥ [110].



Supplementary Figure 126: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4a+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 241: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	3
nontrivial	1, 2

Supplementary Table 242: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G

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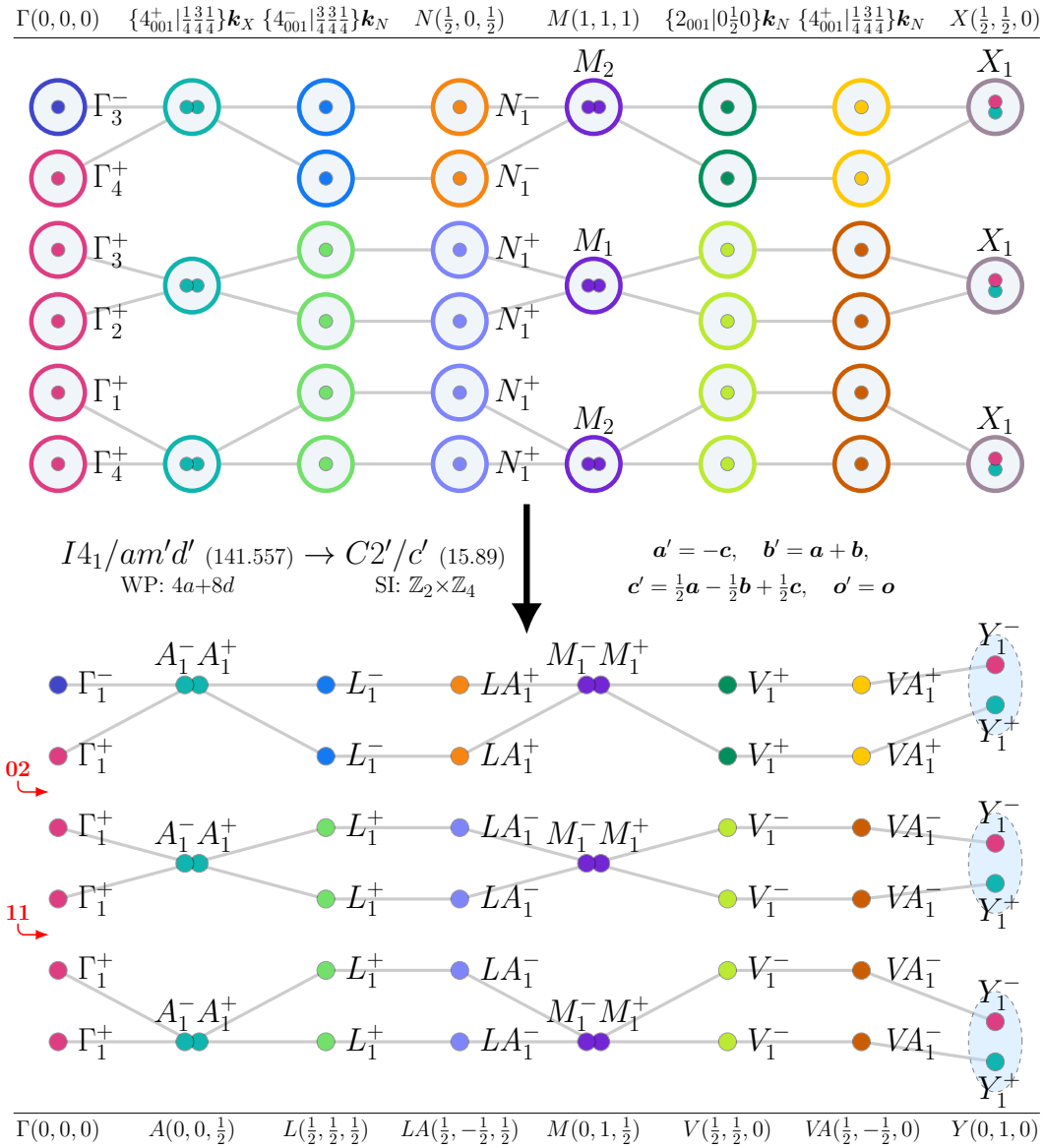
...continued

Gap#	Possible SI Values
4	00, 02, 11, 13
5	G
6	00

26.2.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 127: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4a+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 243: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	3
nontrivial	1, 2

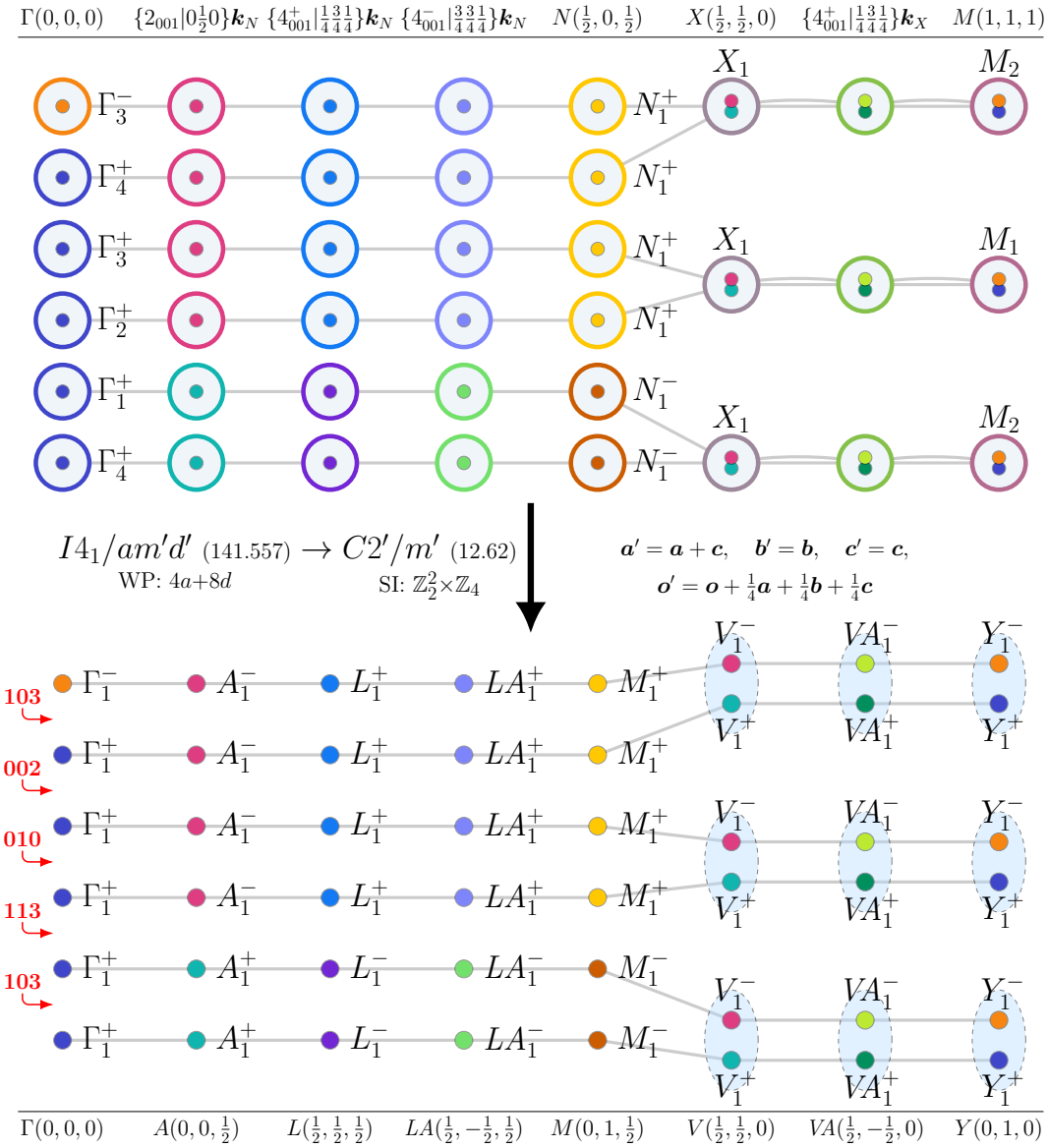
Supplementary Table 244: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G
4	00, 02, 11, 13
5	G
6	00

26.2.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 128: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4a + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 245: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3
002	0, 1
003	0, 1, 2, 3
010	0, 1, 2, 3
012	0, 1, 2, 3
101	0, 1, 2, 3
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	0
nontrivial	4, 5

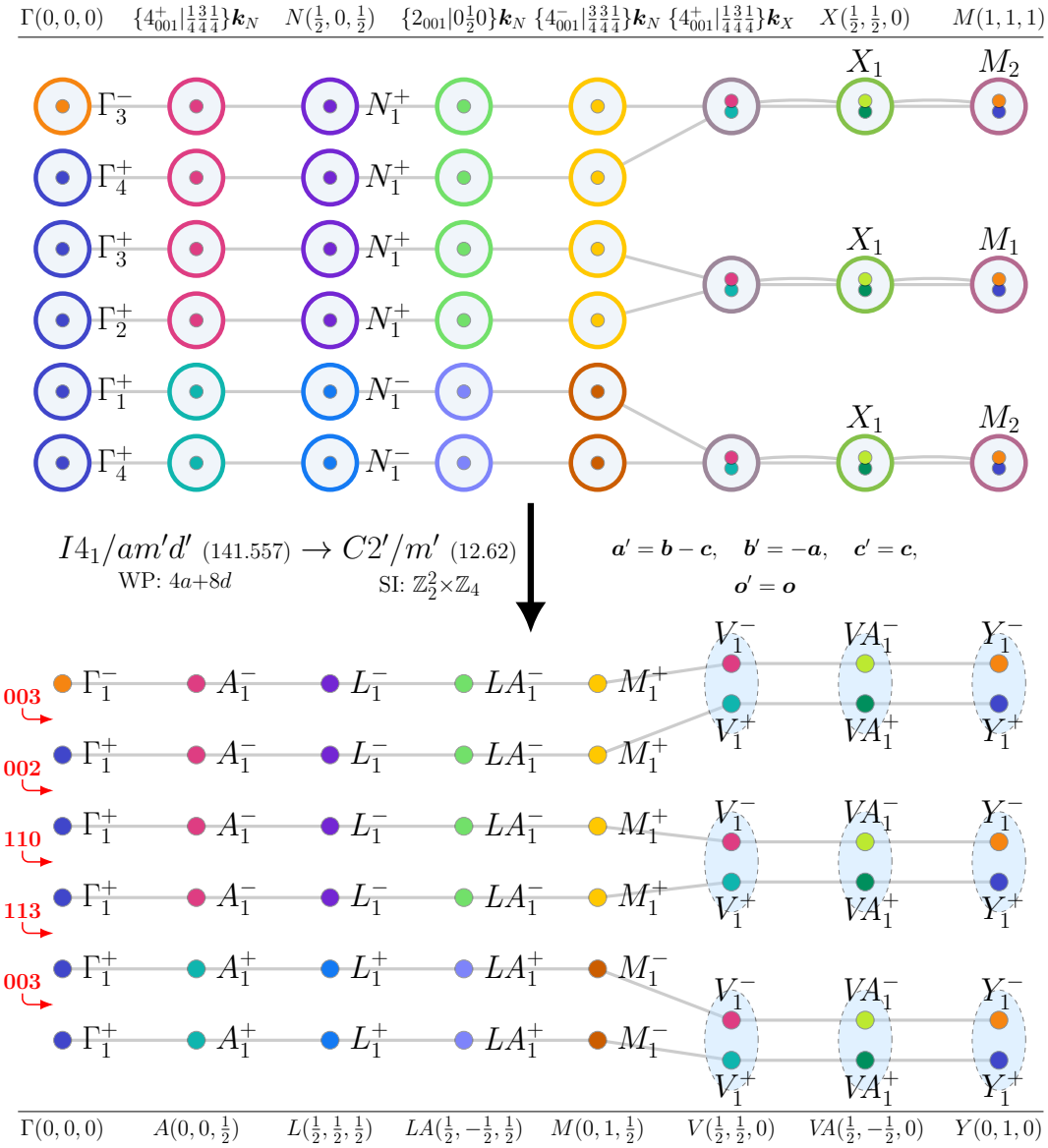
Supplementary Table 246: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000

26.2.5 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 129: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4a + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 247: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: 4a+8d, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3
002	0, 1
003	0, 1, 2, 3
010	0, 1, 2, 3
012	0, 1, 2, 3
101	0, 1, 2, 3
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	0
nontrivial	4, 5

Supplementary Table 248: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4a$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000

26.3 WP: $4b + 8c$

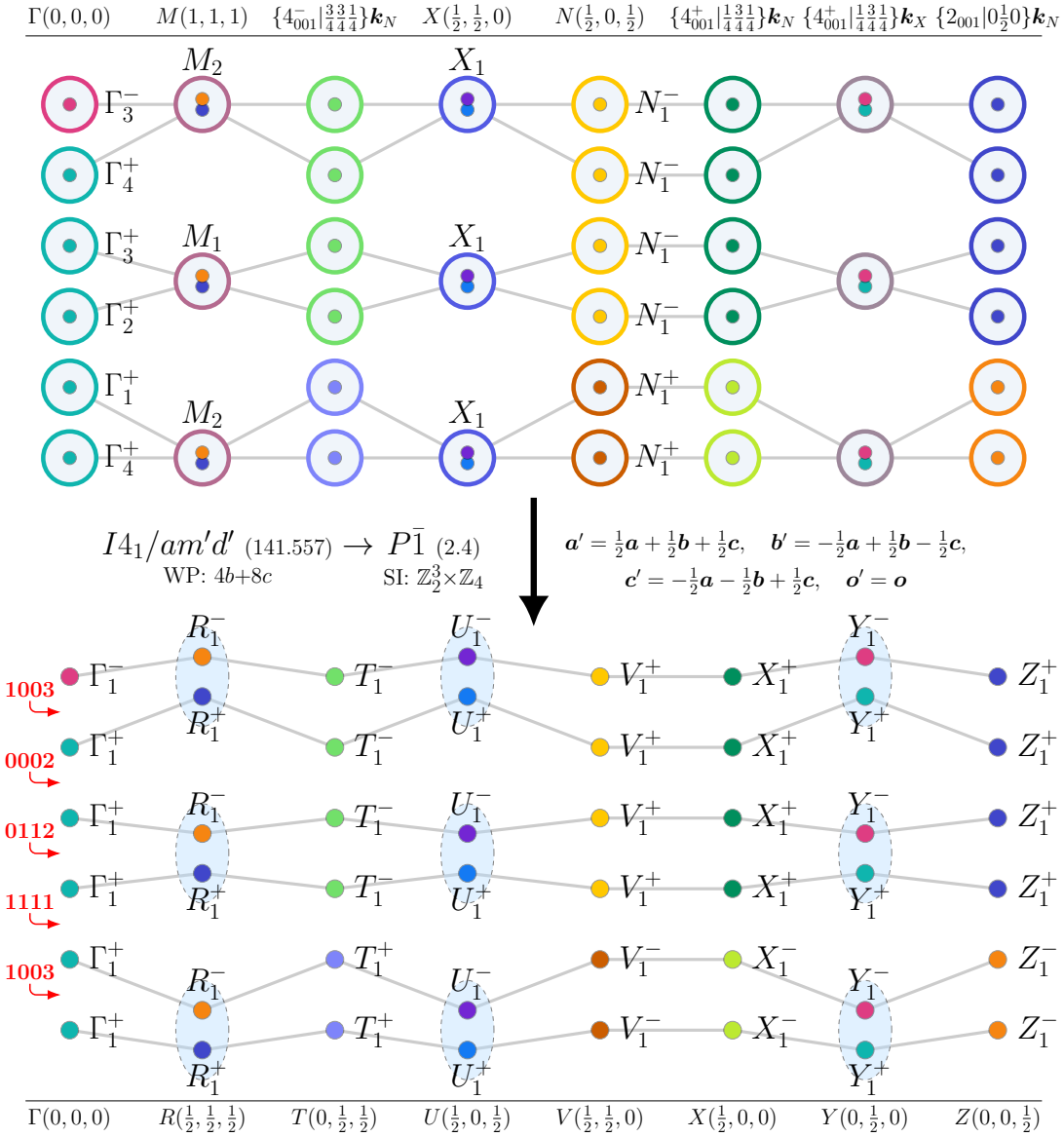
BCS Materials: $FeCr_2S_4$ (180 K)[104], $FeCr_2S_4$ (180 K)[105], $NdCo_2$ (100 K)[106].

26.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].

[104]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.615`
[105]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.614`
[106]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.227`



Supplementary Figure 130: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b+8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 249: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1100	0, 1, 2, 3

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SI	Possible #Gaps
1101	0, 1, 2, 3
1102	0, 1, 2, 3
1103	0, 1, 2, 3
1111	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	4, 5

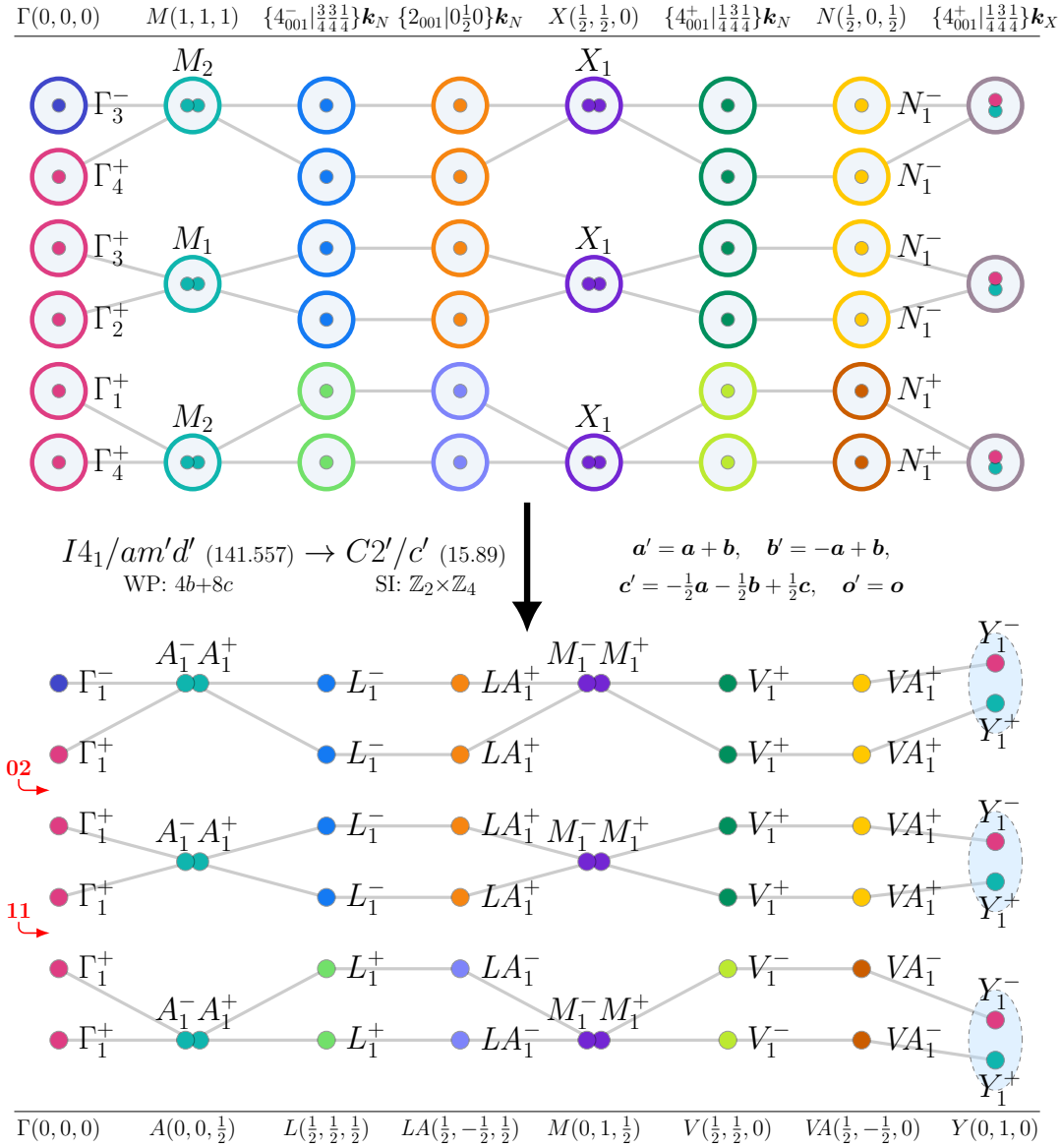
Supplementary Table 250: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0002, 1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0002, 1111, 1113
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	0000

26.3.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B ∥ [110].



Supplementary Figure 131: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4b+8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 251: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	3
nontrivial	1, 2

Supplementary Table 252: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G

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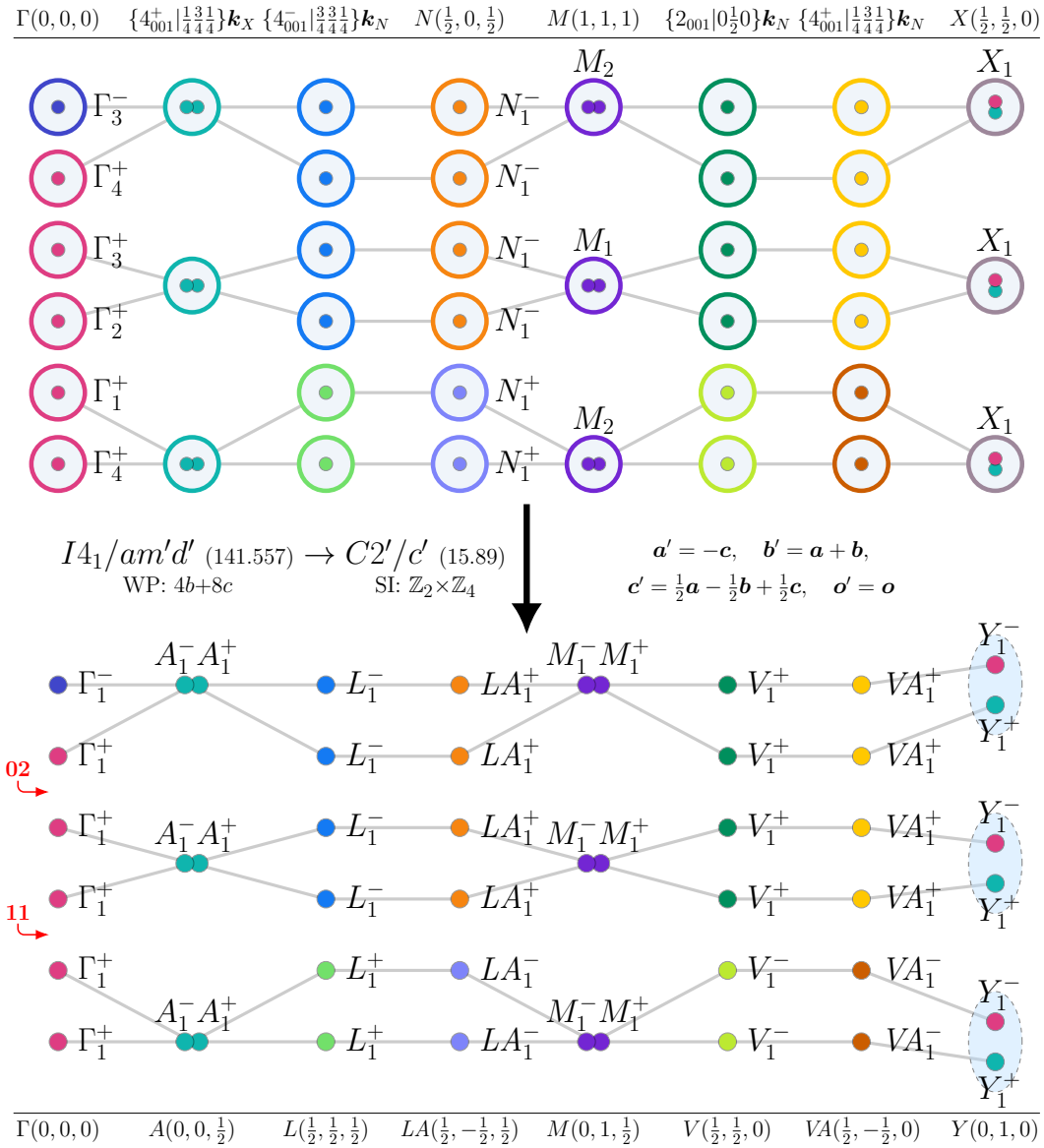
...continued

Gap#	Possible SI Values
4	00, 02, 11, 13
5	G
6	00

26.3.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 132: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4b+8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 253: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	3
nontrivial	1, 2

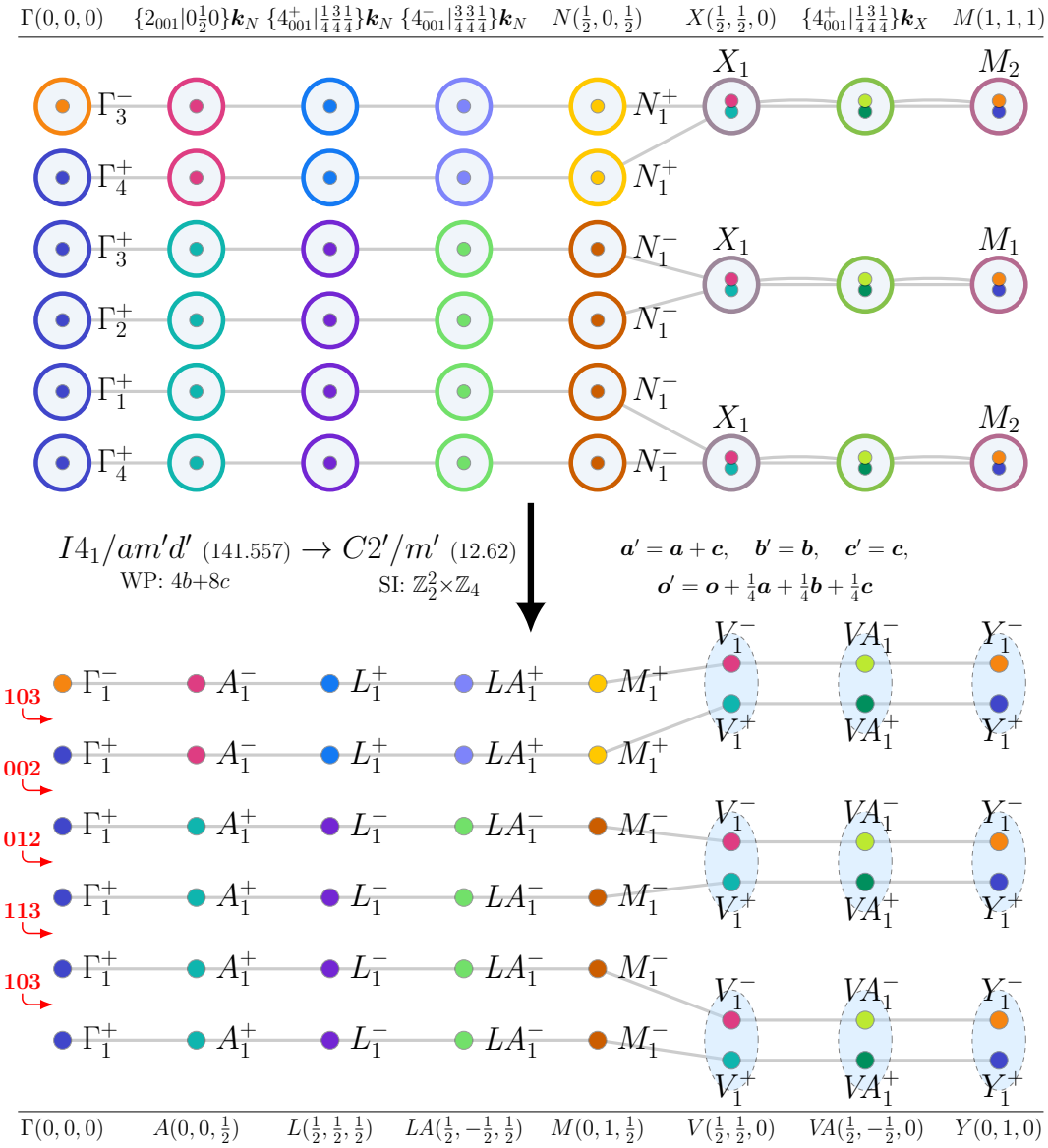
Supplementary Table 254: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G
4	00, 02, 11, 13
5	G
6	00

26.3.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 133: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b+8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 255: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: 4*b*+8*c*, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3
002	0, 1
003	0, 1, 2, 3
010	0, 1, 2, 3
012	0, 1, 2, 3
101	0, 1, 2, 3
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	0
nontrivial	4, 5

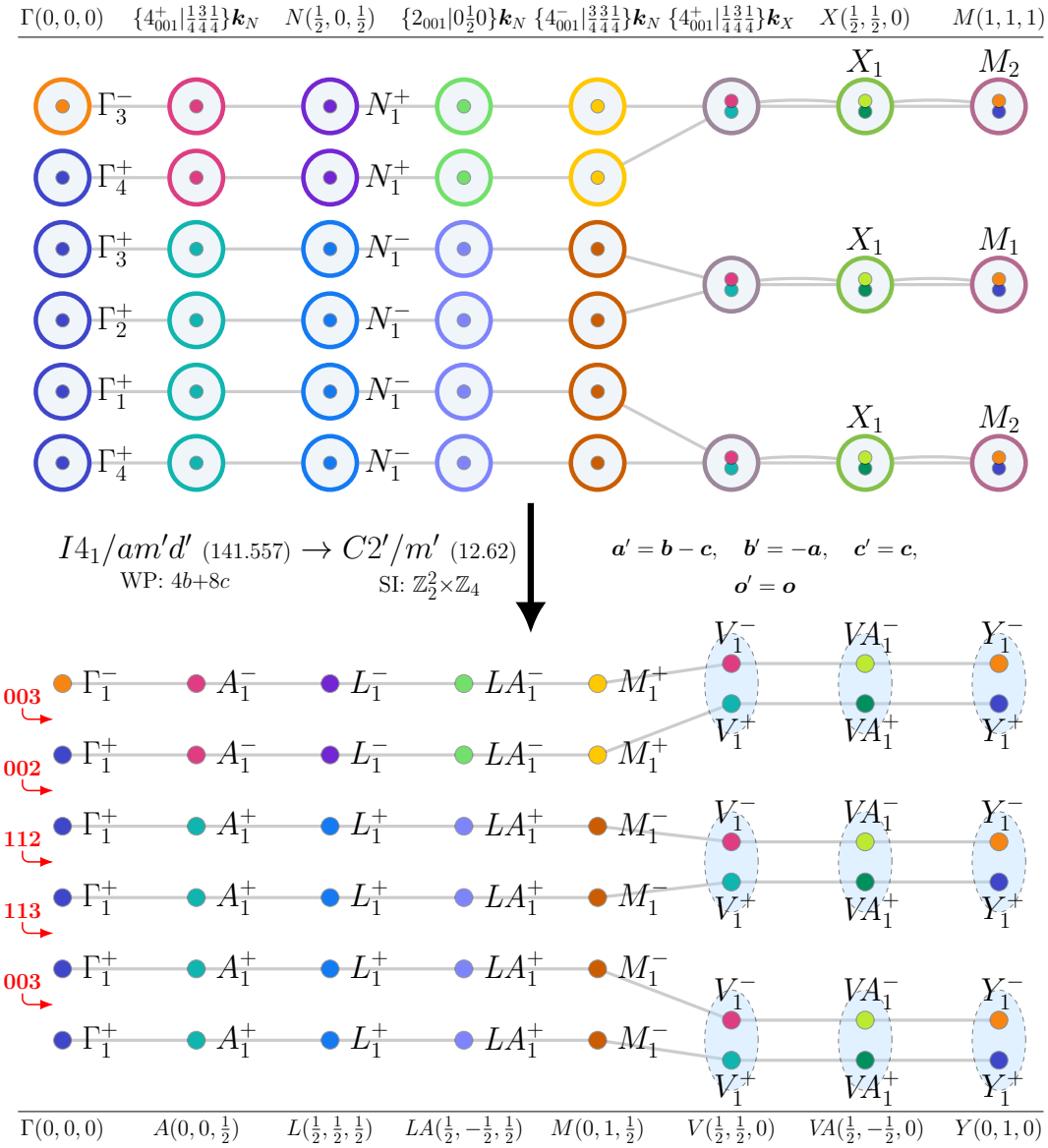
Supplementary Table 256: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000

26.3.5 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 134: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b+8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 257: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3
002	0, 1
003	0, 1, 2, 3
010	0, 1, 2, 3
012	0, 1, 2, 3
101	0, 1, 2, 3
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	0
nontrivial	4, 5

Supplementary Table 258: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $4b$+$8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000

26.4 WP: $8c$

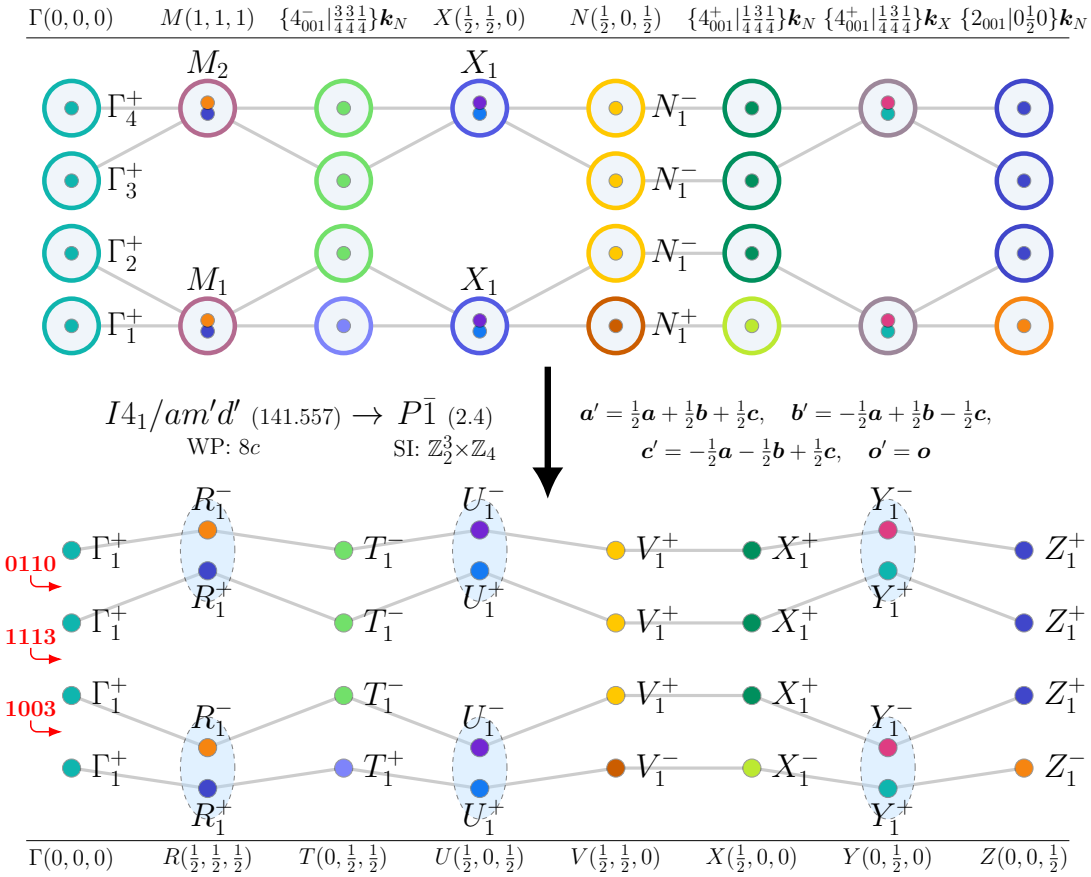
BCS Materials: $Ho_2Ru_2O_7$ (95 K)[107].

26.4.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].

[107] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.49`



Supplementary Figure 135: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 259: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0010	0, 1
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1
0113	0, 1, 2
1000	0, 1
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1100	0, 1
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

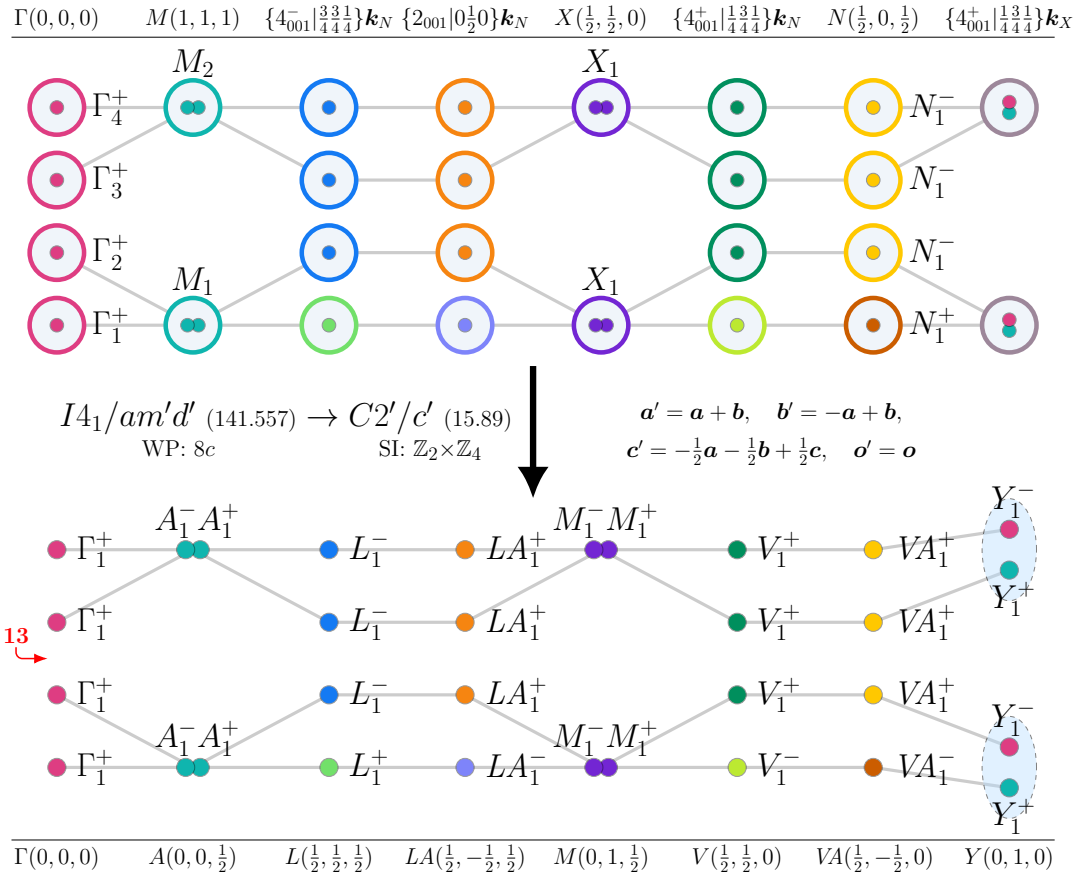
Supplementary Table 260: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

26.4.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B ∥ [110].



Supplementary Figure 136: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 261: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

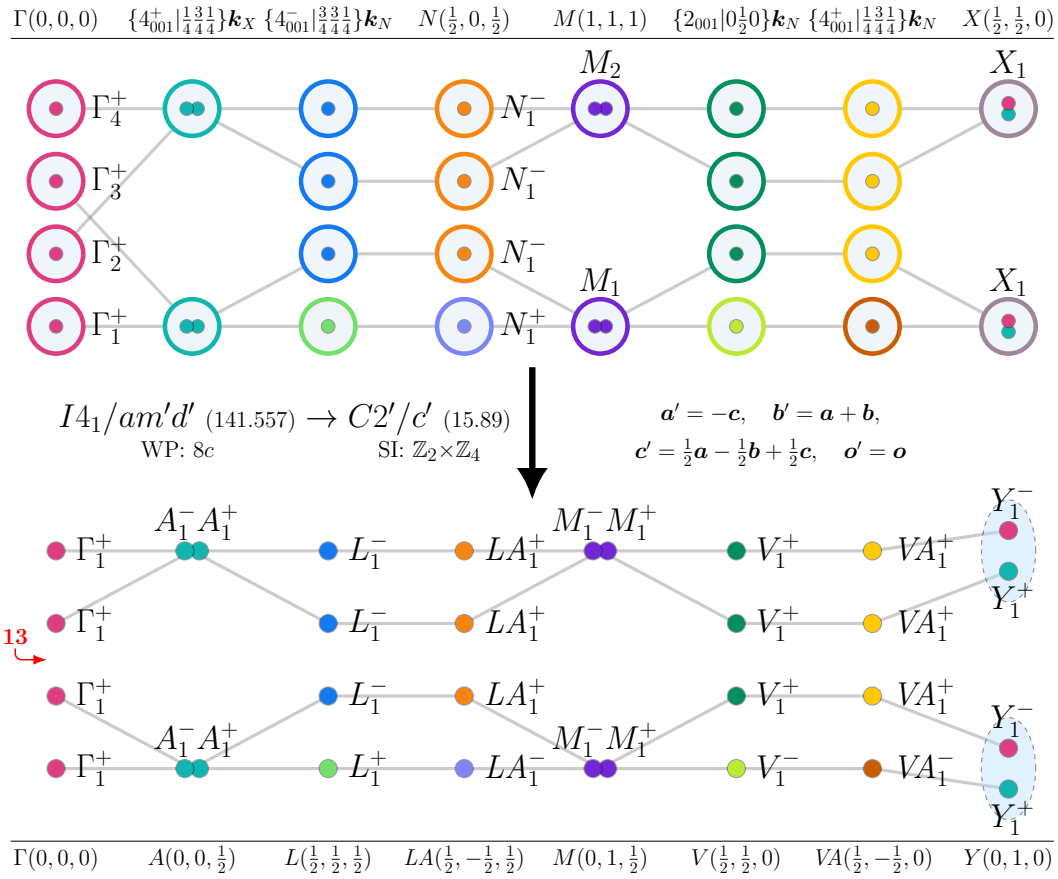
Supplementary Table 262: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

26.4.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 137: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 263: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

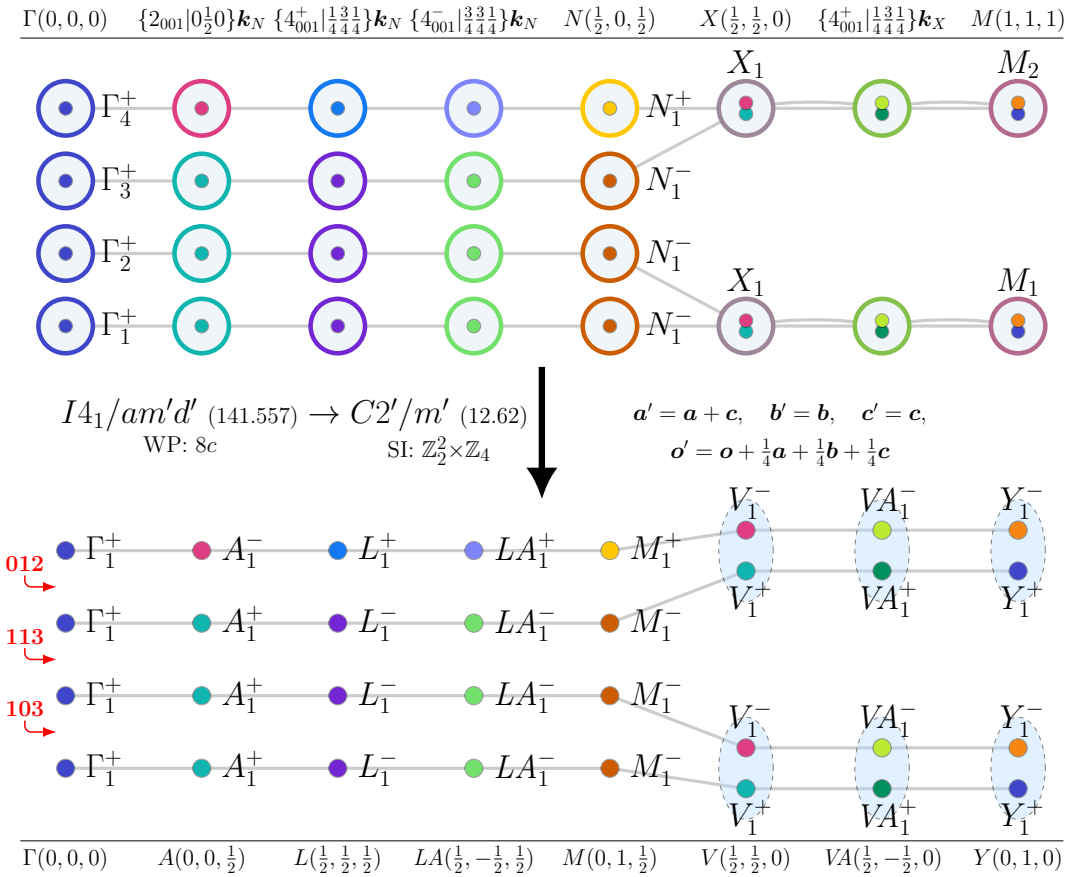
Supplementary Table 264: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

26.4.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 138: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 265: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

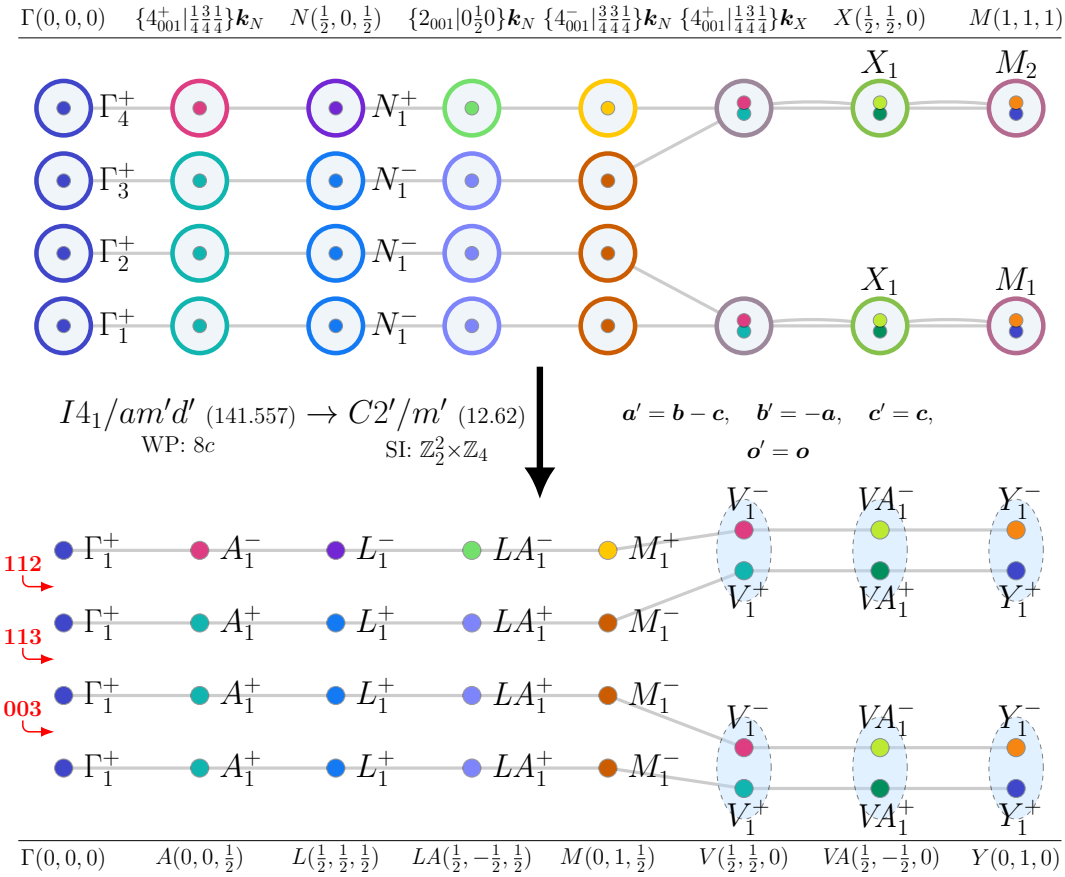
Supplementary Table 266: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

26.4.5 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 139: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8c$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 267: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 268: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112

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Gap#	Possible SI Values
4	000

26.5 WP: $8c + 8d$

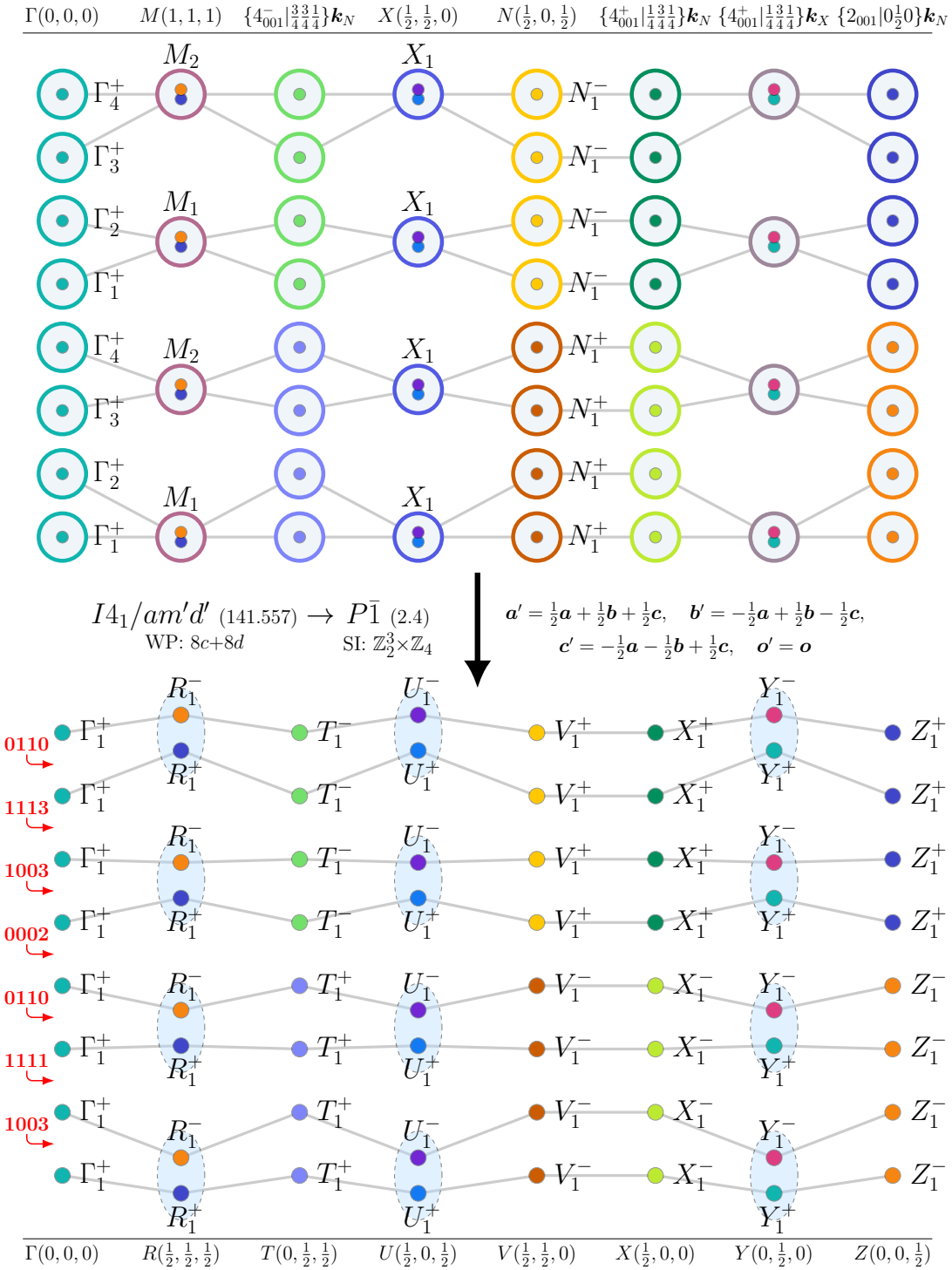
BCS Materials: $Tm_2Mn_2O_7$ (25 K)[108], Ho_2CrSbO_7 (13 K)[109], $Ho_2Ru_2O_7$ (1.4 K)[110].

26.5.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100].

[108] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.151`
[109] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.63`
[110] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.51`



Supplementary Figure 140: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 269: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c+8d, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4

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SI	Possible #Gaps
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1111	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	6, 7

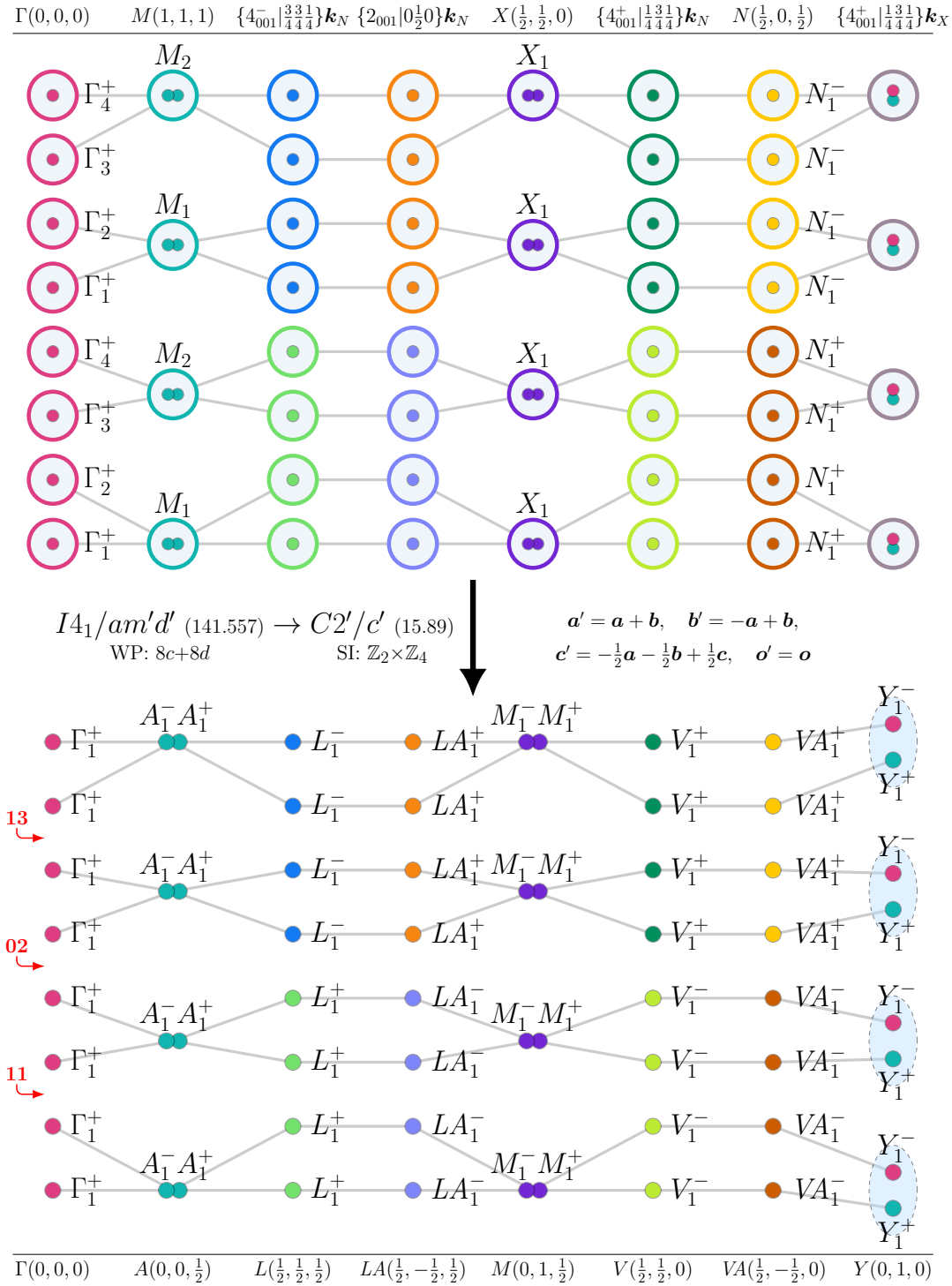
Supplementary Table 270: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$+$8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0002
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	1111, 1113
7	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
8	0000

26.5.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B $\parallel$ [110].



Supplementary Figure 141: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $8c + 8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 271: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c+8d, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	4
nontrivial	2, 3

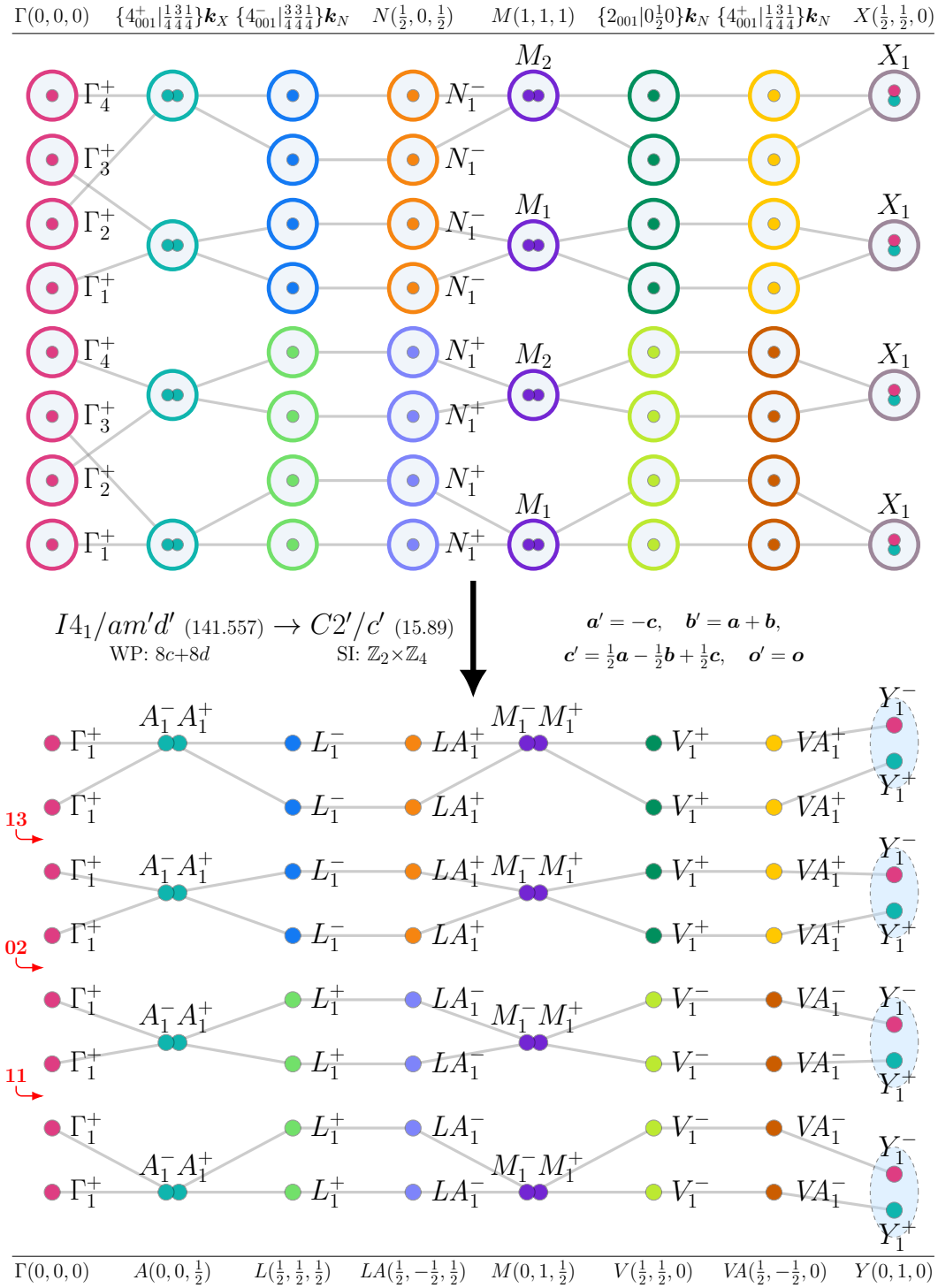
Supplementary Table 272: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c+8d, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00, 02
5	G
6	11, 13
7	G
8	00

26.5.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 142: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 273: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c+8d, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	4
nontrivial	2, 3

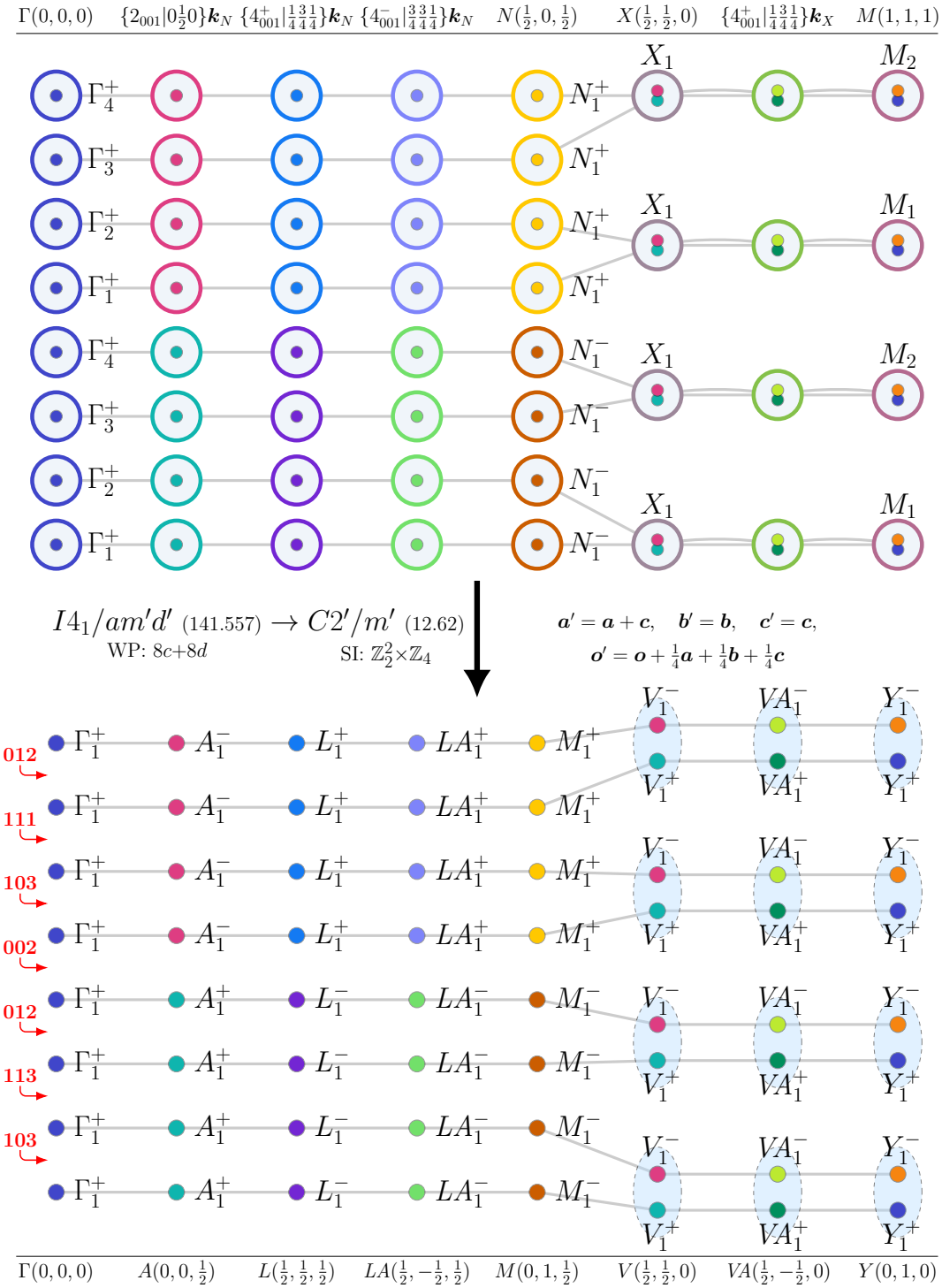
Supplementary Table 274: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: 8c+8d, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00, 02
5	G
6	11, 13
7	G
8	00

26.5.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 143: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 275: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3, 4
002	0, 1
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
012	0, 1, 2, 3, 4

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SI	Possible #Gaps
101	0, 1, 2, 3, 4
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	0, 1, 2
112	0, 1, 2, 3, 4
113	0, 1, 2
gapless	0
nontrivial	6, 7

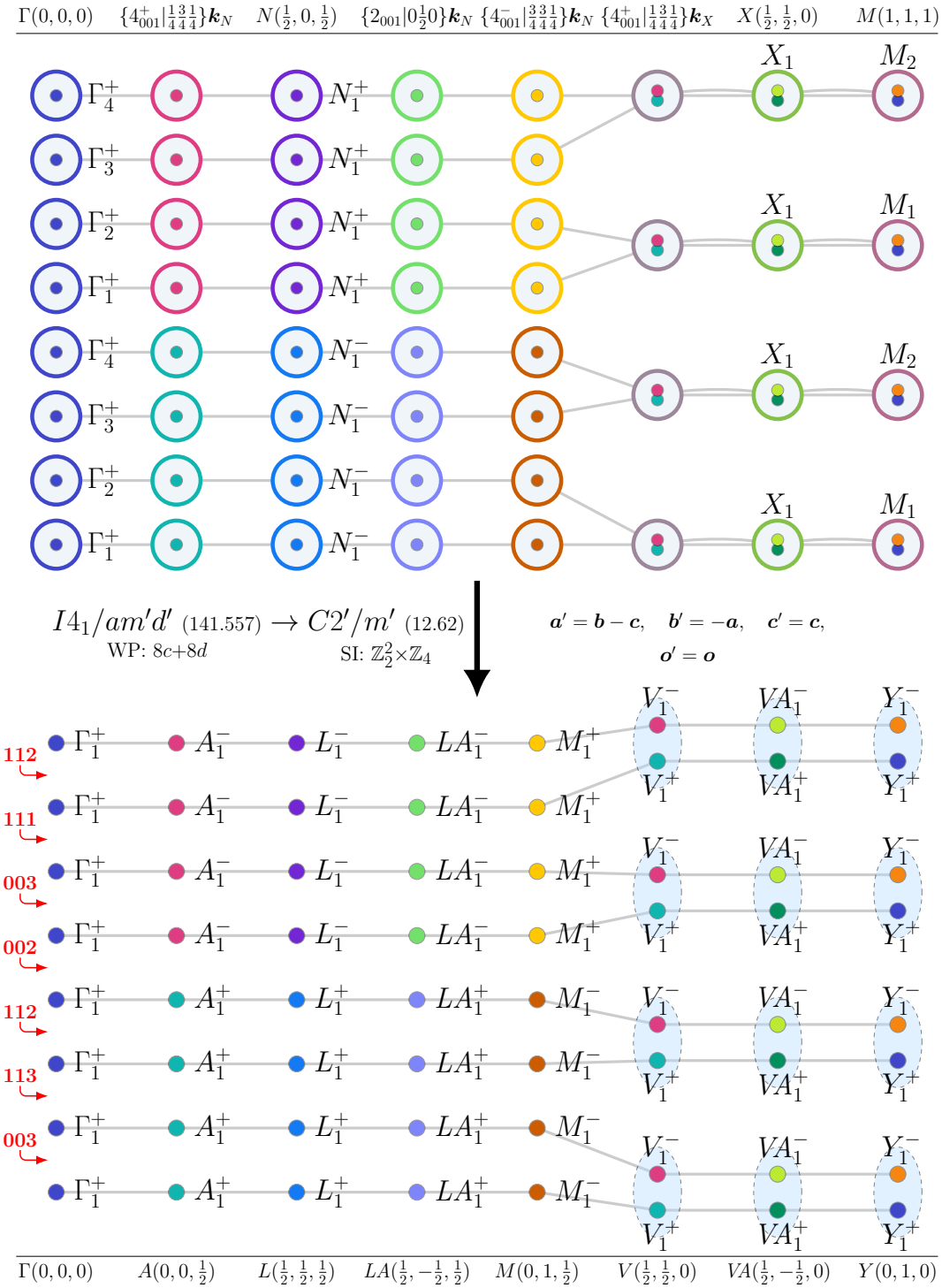
Supplementary Table 276: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002
5	001, 003, 010, 012, 101 103, 110, 112
6	111, 113
7	001, 003, 010, 012, 101 103, 110, 112
8	000

26.5.5 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 144: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $8c+8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 277: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2, 3, 4
002	0, 1
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
012	0, 1, 2, 3, 4

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SI	Possible #Gaps
101	0, 1, 2, 3, 4
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	0, 1, 2
112	0, 1, 2, 3, 4
113	0, 1, 2
gapless	0
nontrivial	6, 7

Supplementary Table 278: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8c$+$8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002
5	001, 003, 010, 012, 101 103, 110, 112
6	111, 113
7	001, 003, 010, 012, 101 103, 110, 112
8	000

26.6 WP: $8d$

BCS Materials: $Tb_2Sn_2O_7$ (0.87 K)[111], $Yb_2Ti_2O_7$ (0.26 K)[112], $Yb_2Sn_2O_7$ (0.15 K)[113].

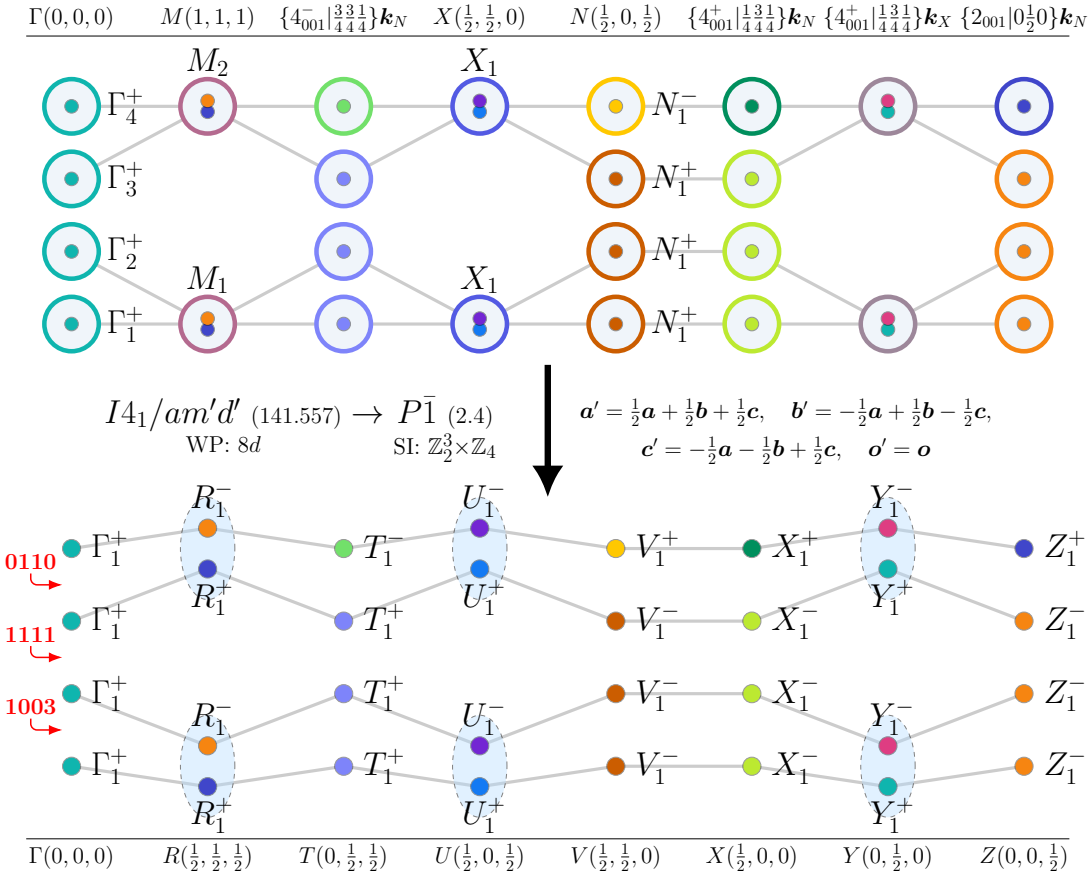
26.6.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [100] and strain ⊥ [100],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],

[111]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.48`
[112]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.158`
[113]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.157`

- B in generic direction,
- B $\perp$ [100] and strain $\parallel$ [110],
- B $\perp$ [100] and strain $\perp$ [001],
- B $\perp$ [100] and strain $\perp$ [110],
- B $\perp$ [110] and strain $\parallel$ [100],
- B $\perp$ [110] and strain $\perp$ [001],
- B $\perp$ [110] and strain $\perp$ [100].



Supplementary Figure 145: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 279: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1
0013	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1
1003	0, 1, 2
1100	0, 1, 2

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SI	Possible #Gaps
1101	0, 1, 2
1102	0, 1
1103	0, 1, 2
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

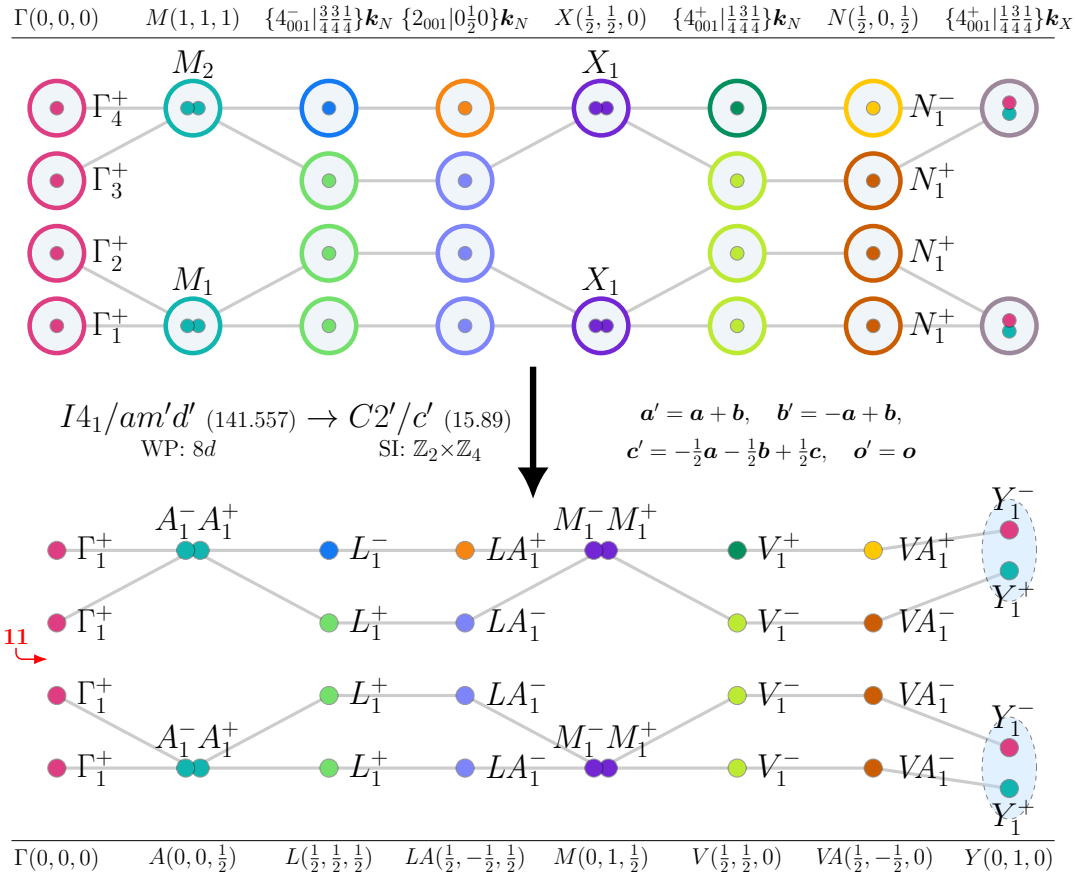
Supplementary Table 280: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

26.6.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B ∥ [110].



Supplementary Figure 146: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 281: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

Supplementary Table 282: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

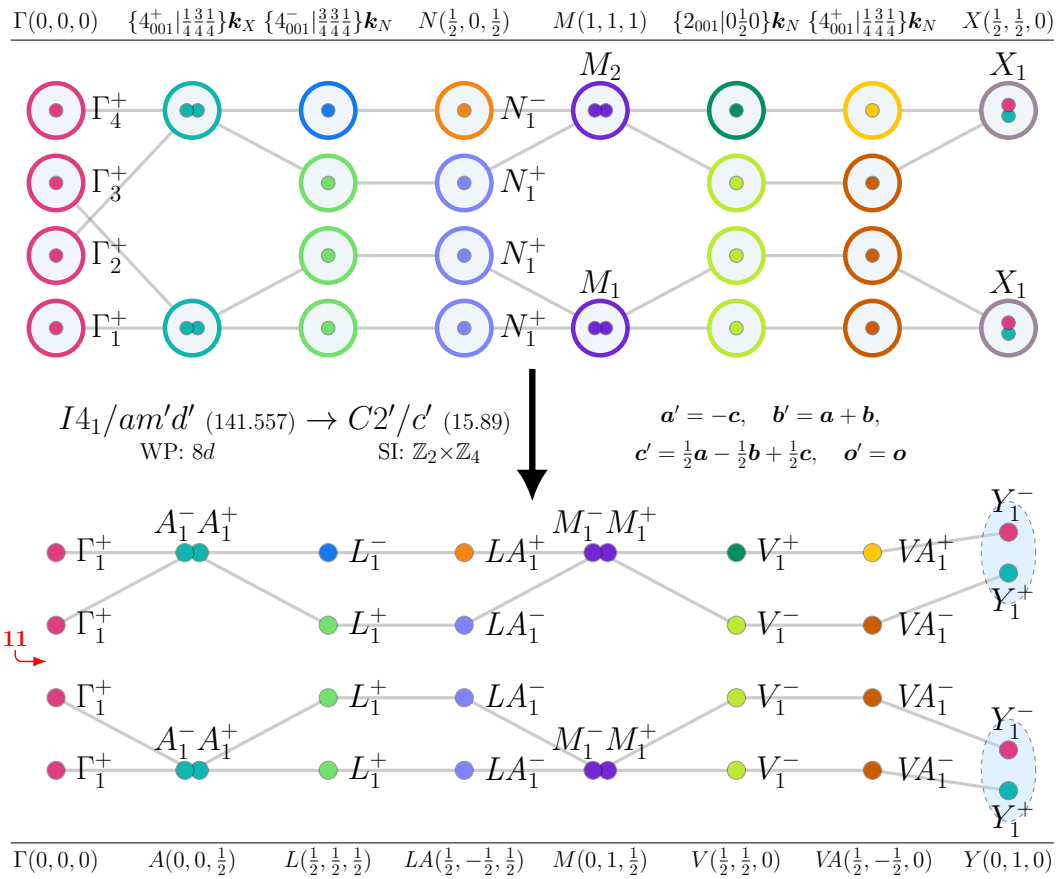
Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

26.6.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain ⊥ [110],

- B $\perp$ [110].



Supplementary Figure 147: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 283: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

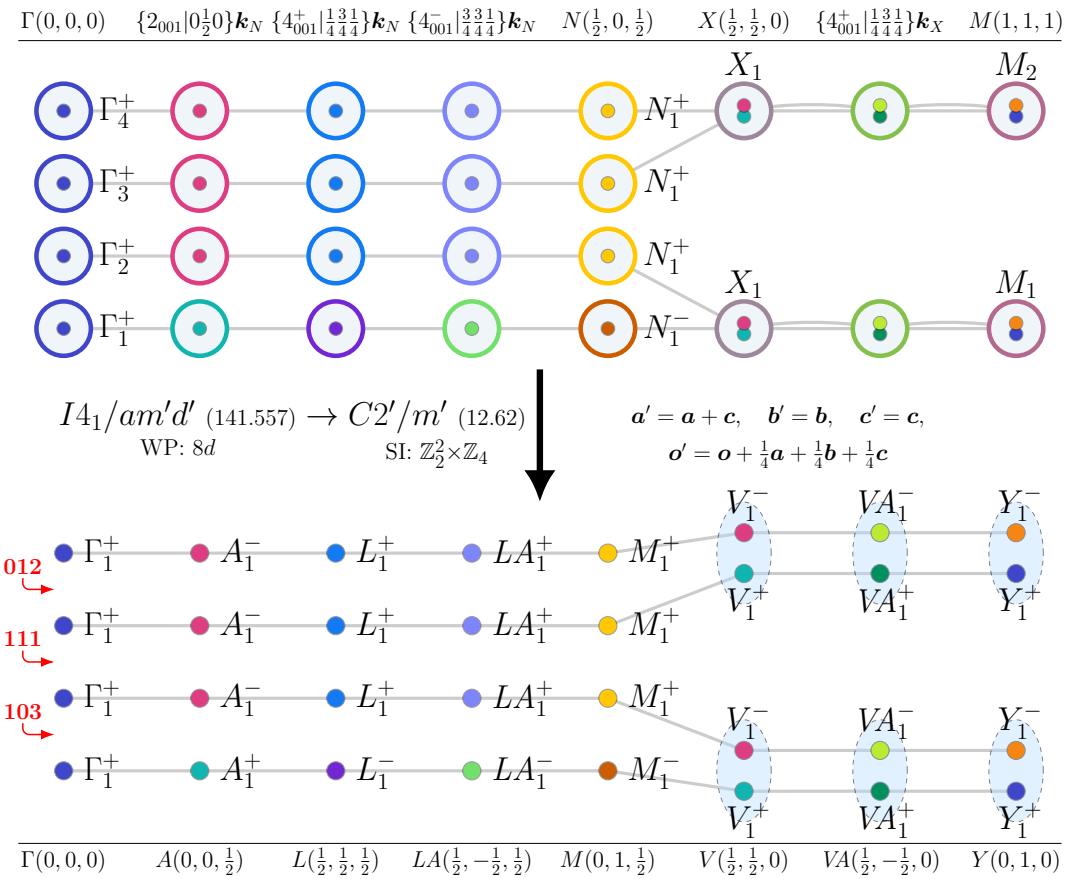
Supplementary Table 284: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

26.6.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B ∥ [100].



Supplementary Figure 148: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 285: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 286: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113

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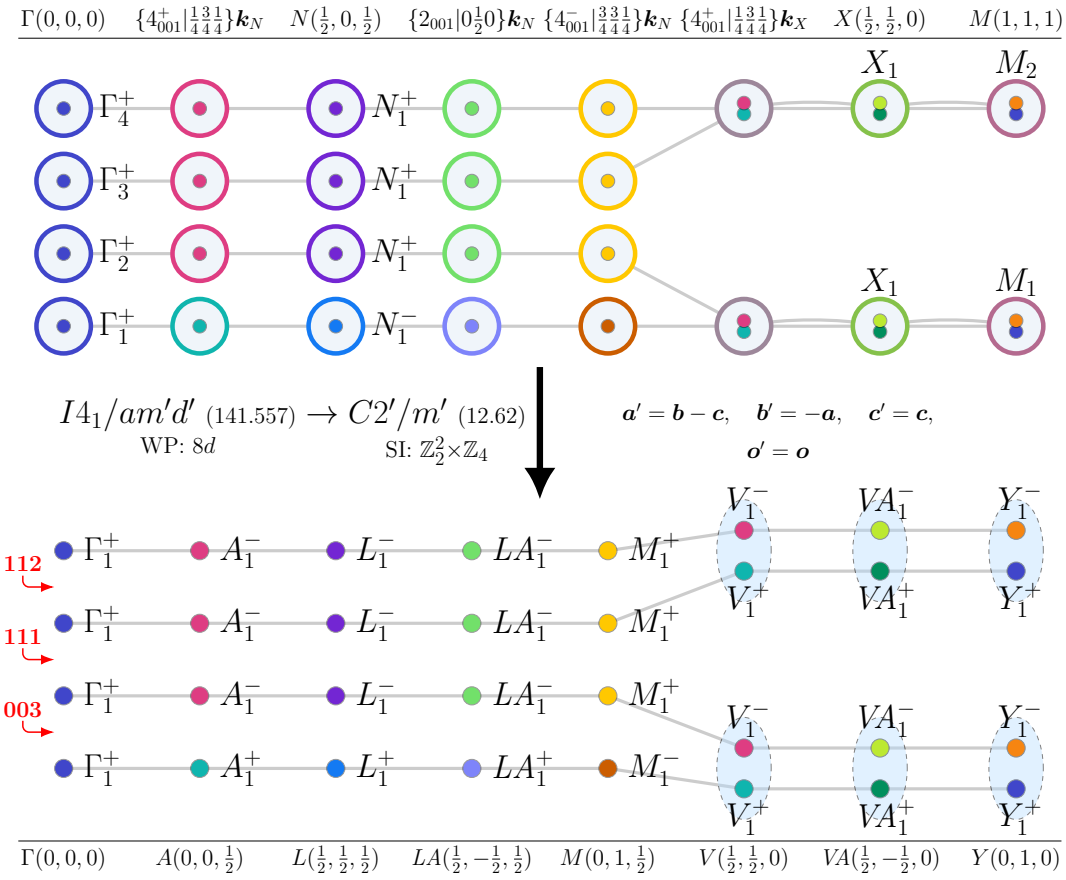
...continued

Gap#	Possible SI Values
3	001, 003, 010, 012, 101 103, 110, 112
4	000

26.6.5 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 149: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8d$ of supergroup $I4_1/am'd'$ (141.557).

Supplementary Table 287: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1

continued on next page...

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SI	Possible #Gaps
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 288: Possible SI values in each gap. Supergroup: $I4_1/am'd'$ (141.557), WP: $8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

27 MSG $I4_1'/a'cd'$ (142.568)

Nontrivial-SI Subgroups: $Iba2$ (45.235), $Ib'ca$ (73.550), $I4_1'cd'$ (110.248), $I\bar{4}2'd'$ (122.337).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), Cc' (9.39), $P\bar{1}'$ (2.6), $C2'$ (5.15), $Ib'a2'$ (45.237), Cc (9.37), $Ib'a2'$ (45.237), $C2'/c$ (15.87), $C2$ (5.13), $Fd'd'2$ (43.227), $C2/c'$ (15.88), $C2$ (5.13), $Fd'd'2$ (43.227), $C2/c'$ (15.88), $I2_1'2_1'2_1$ (24.55), $Fd'd'd'$ (70.531).

27.1 WP: $8a$

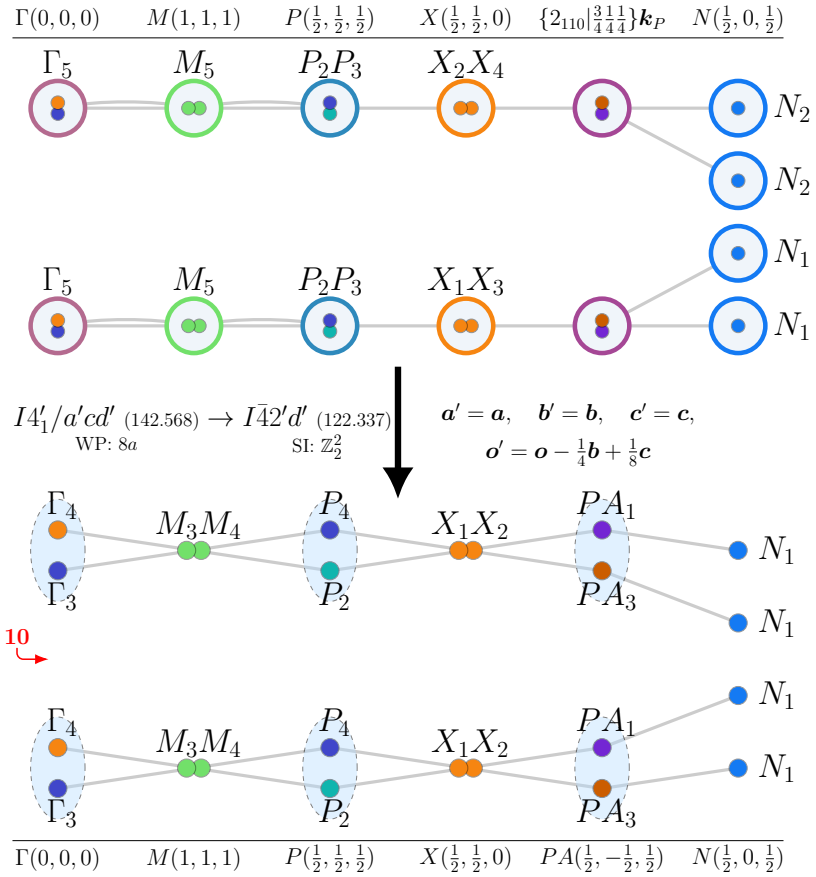
BCS Materials: Ca_2MnO_4 (110 K)[114].

27.1.1 Topological bands in subgroup $I\bar{4}2'd'$ (122.337)

Perturbation:

- B ∥ [001].

[114]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.211`



Supplementary Figure 150: Topological magnon bands in subgroup $I\bar{4}2'd'$ (122.337) for magnetic moments on Wyckoff position $8a$ of supergroup $I4_1'/a'cd'$ (142.568).

Supplementary Table 289: Number of times a given SI can appear in the band structure. Supergroup: $I4_1'/a'cd'$ (142.568), WP: $8a$, Subgroup: $I\bar{4}2'd'$ (122.337).

SI	Possible #Gaps
00	0
10	1
gapless	2
nontrivial	1

Supplementary Table 290: Possible SI values in each gap. Supergroup: $I4_1'/a'cd'$ (142.568), WP: $8a$, Subgroup: $I\bar{4}2'd'$ (122.337).

Gap#	Possible SI Values
1	G
2	10
3	G
4	00

28 MSG I_c4_1/acd (142.570)

Nontrivial-SI Subgroups: C_ac (9.41), C_ac (9.41), $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $C2'/m'$ (12.62), $P_S\bar{1}$ (2.7), C_a2 (5.17), F_Sdd2 (43.228), $C2/c$ (15.85), $Fd'd'd$ (70.530), C_a2/c (15.91), C_a2 (5.17), F_Sdd2 (43.228), I_cba2 (45.239), $C2/c$ (15.85), $Fd'd'd$ (70.530), $Im'm'a$ (74.558), C_a2/c (15.91), F_Sddd (70.532), I_aba2 (45.240), $C2/c$ (15.85), $Iba'm'$ (72.544), C_c2/c (15.90), I_cbca (73.553),

I_c4_1cd (110.250), $I4_1/am'd'$ (141.557).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Cc (9.37), $Fd'd2'$ (43.226), Cc (9.37), $Fd'd2'$ (43.226), $Im'a2'$ (46.243), Cc (9.37), $Im'a2'$ (46.243), C_cc (9.40), $C2$ (5.13), $Fd'd'2$ (43.227), $C2$ (5.13), $Fd'd'2$ (43.227), $Im'm'2$ (44.232), $C2$ (5.13), $Im'a'2$ (46.245), C_c2 (5.16), $I4_1m'd'$ (109.243).

28.1 WP: $16e$

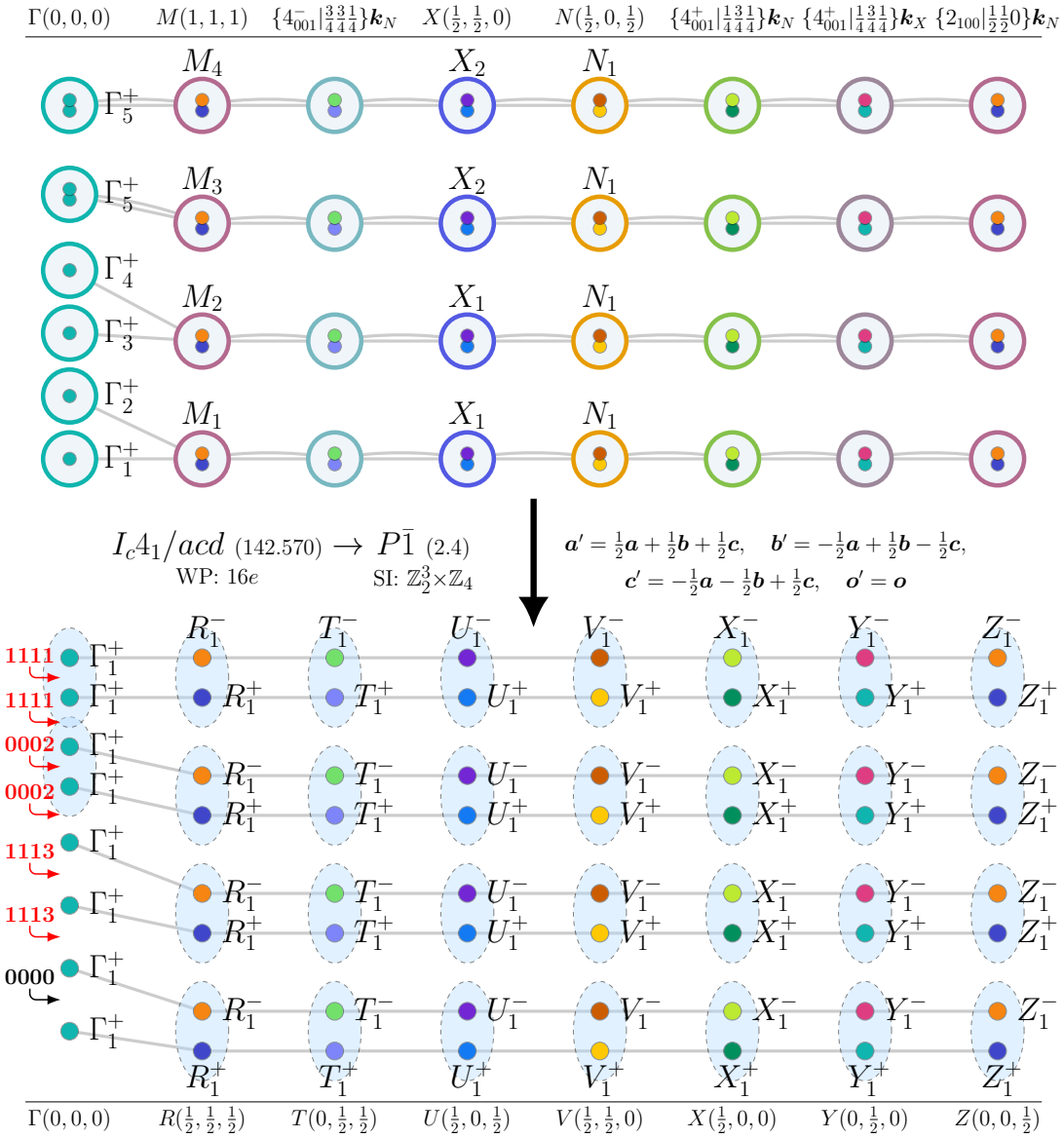
BCS Materials: CoO (290 K)[115].

28.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.

[115] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.19`



Supplementary Figure 151: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $16e$ of supergroup I_c4_1/acd (142.570).

Supplementary Table 291: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/acd (142.570), WP: $16e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3, 4
0002	1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4

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SI	Possible #Gaps
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	1, 2, 3
1112	0, 1, 2, 3, 4
1113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

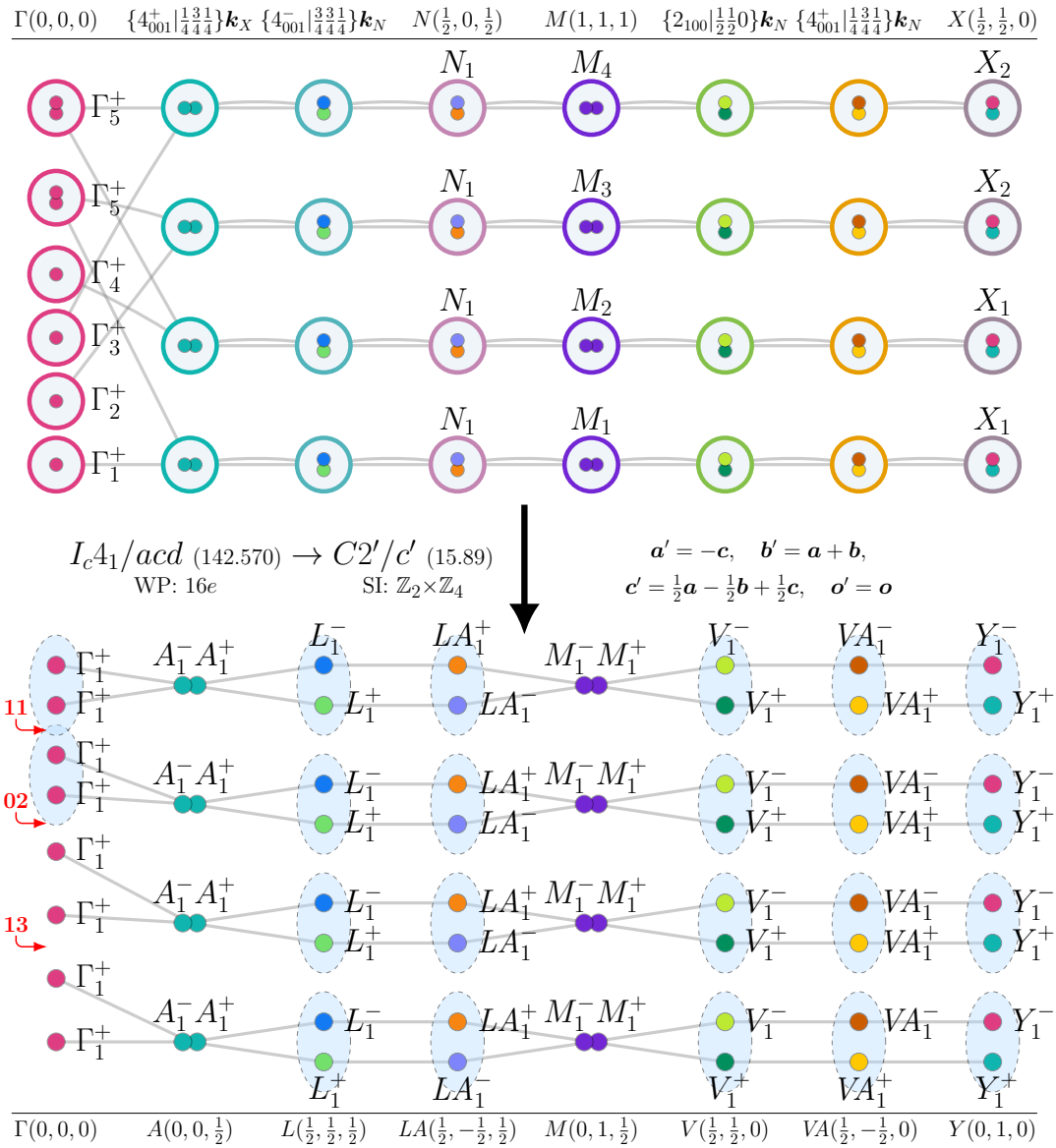
Supplementary Table 292: Possible SI values in each gap. Supergroup: I_c4_1/acd (142.570), WP: $16e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
2	1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
4	0002
5	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
6	1111
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
8	0000

28.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [001] and strain ⊥ [110],
- B ⊥ [110].



Supplementary Figure 152: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 16e of supergroup I_c4_1/acd (142.570).

Supplementary Table 293: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/acd (142.570), WP: 16e, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	1
11	1
13	1
gapless	4
nontrivial	3

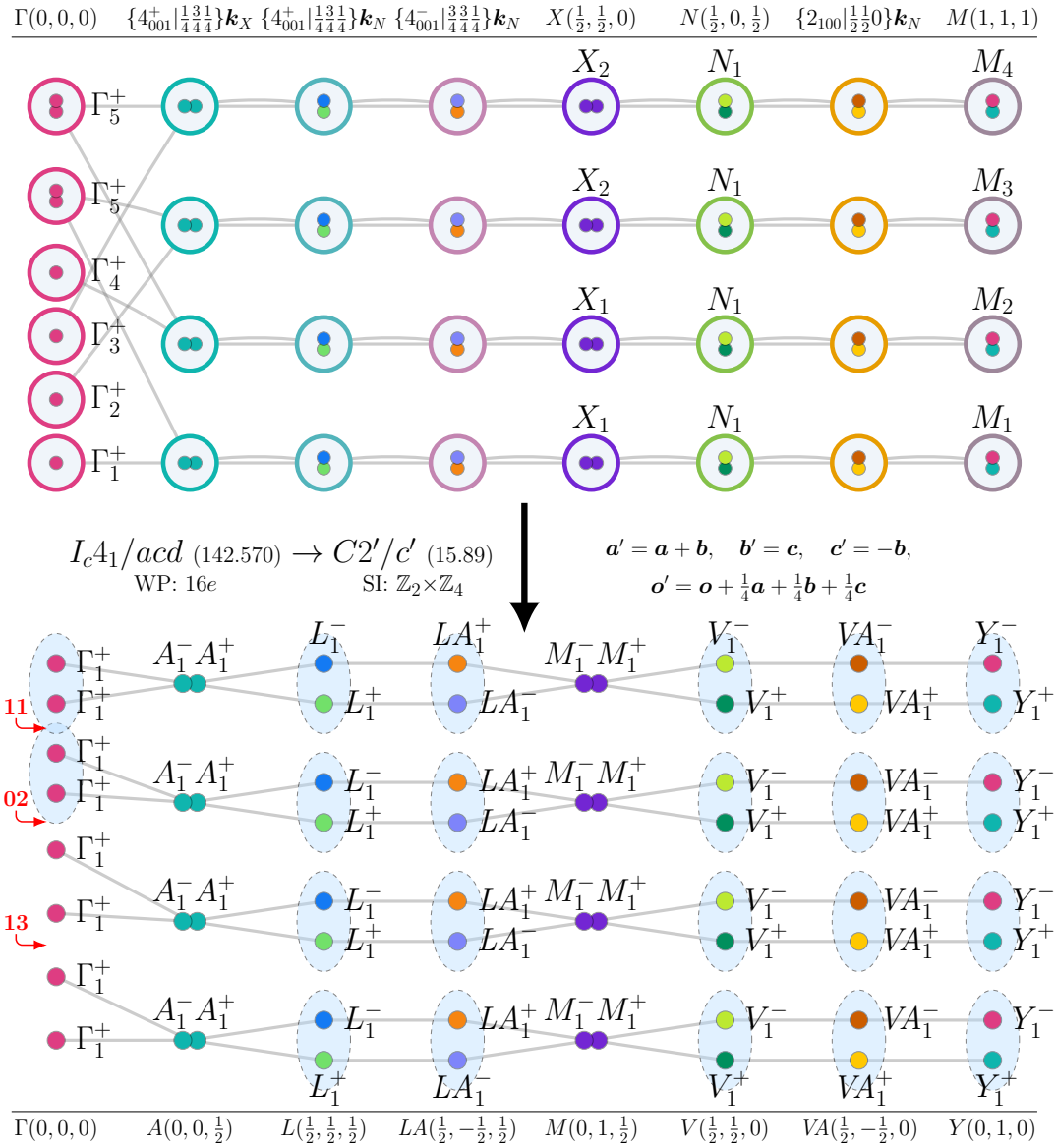
Supplementary Table 294: Possible SI values in each gap. Supergroup: I_c4_1/acd (142.570), WP: $16e$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	13
3	G
4	02
5	G
6	11
7	G
8	00

28.1.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 153: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 16e of supergroup I_c4_1/acd (142.570).

Supplementary Table 295: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/acd (142.570), WP: 16e, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	1
11	1
13	1
gapless	4
nontrivial	3

Supplementary Table 296: Possible SI values in each gap. Supergroup: I_c4_1/acd (142.570), WP: 16e, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	13
3	G

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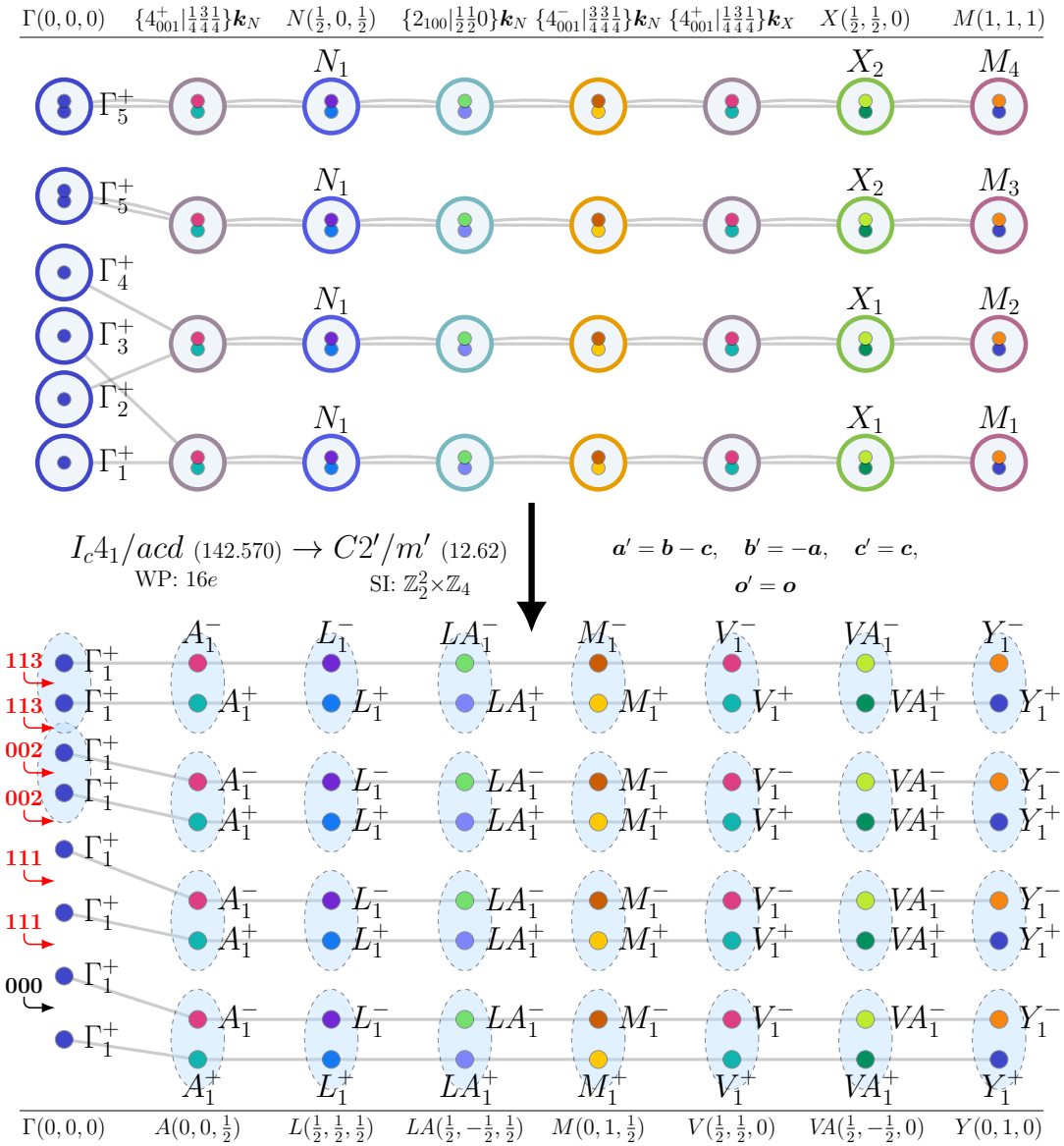
...continued

Gap#	Possible SI Values
4	02
5	G
6	11
7	G
8	00

28.1.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 154: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position 16e of supergroup I_c4_1/acd (142.570).

Supplementary Table 297: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/acd (142.570), WP: $16e$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3, 4
002	1, 2, 3
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
011	0, 1, 2
012	0, 1, 2, 3, 4
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3, 4
102	0, 1, 2
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	1, 2, 3
112	0, 1, 2, 3, 4
113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

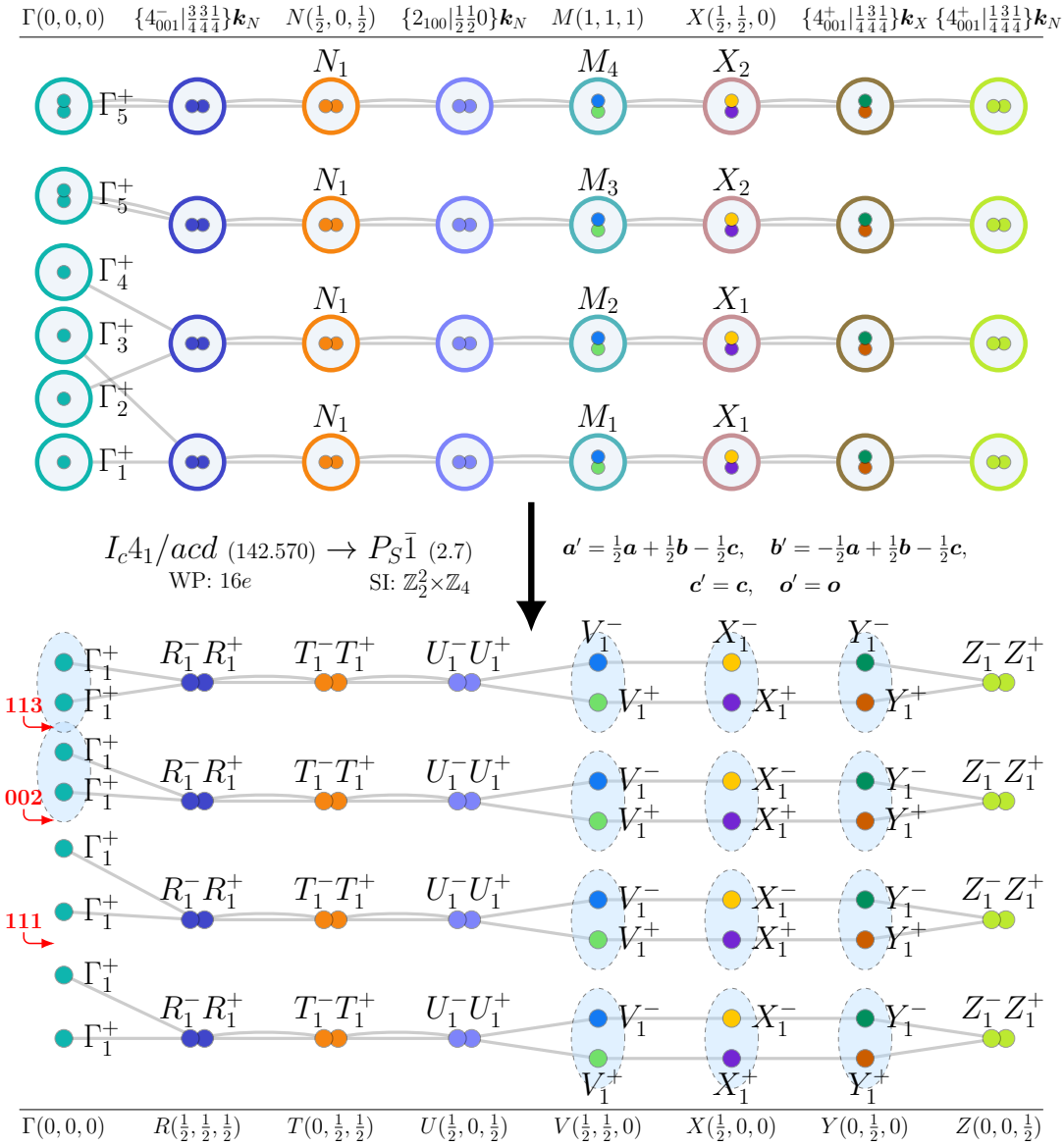
Supplementary Table 298: Possible SI values in each gap. Supergroup: I_c4_1/acd (142.570), WP: $16e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	111
3	001, 002, 003, 010, 011 012, 101, 102, 103, 110 111, 112
4	002
5	001, 002, 003, 010, 012 013, 101, 102, 103, 110 112, 113
6	113
7	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
8	000

28.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 155: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position 16e of supergroup I_c4_1/acd (142.570).

Supplementary Table 299: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/acd (142.570), WP: 16e, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

Supplementary Table 300: Possible SI values in each gap. Supergroup: I_c4_1/acd (142.570), WP: 16e, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111
3	G

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...continued

Gap#	Possible SI Values
4	002
5	G
6	113
7	G
8	000

29 MSG P_a2_1/c (14.80)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pc' (7.26), $P2_1'$ (4.9), P_S1 (1.3), Pc (7.24), P_ac (7.27), $P2_1$ (4.7), P_a2_1 (4.10).

29.1 WP: $8e$

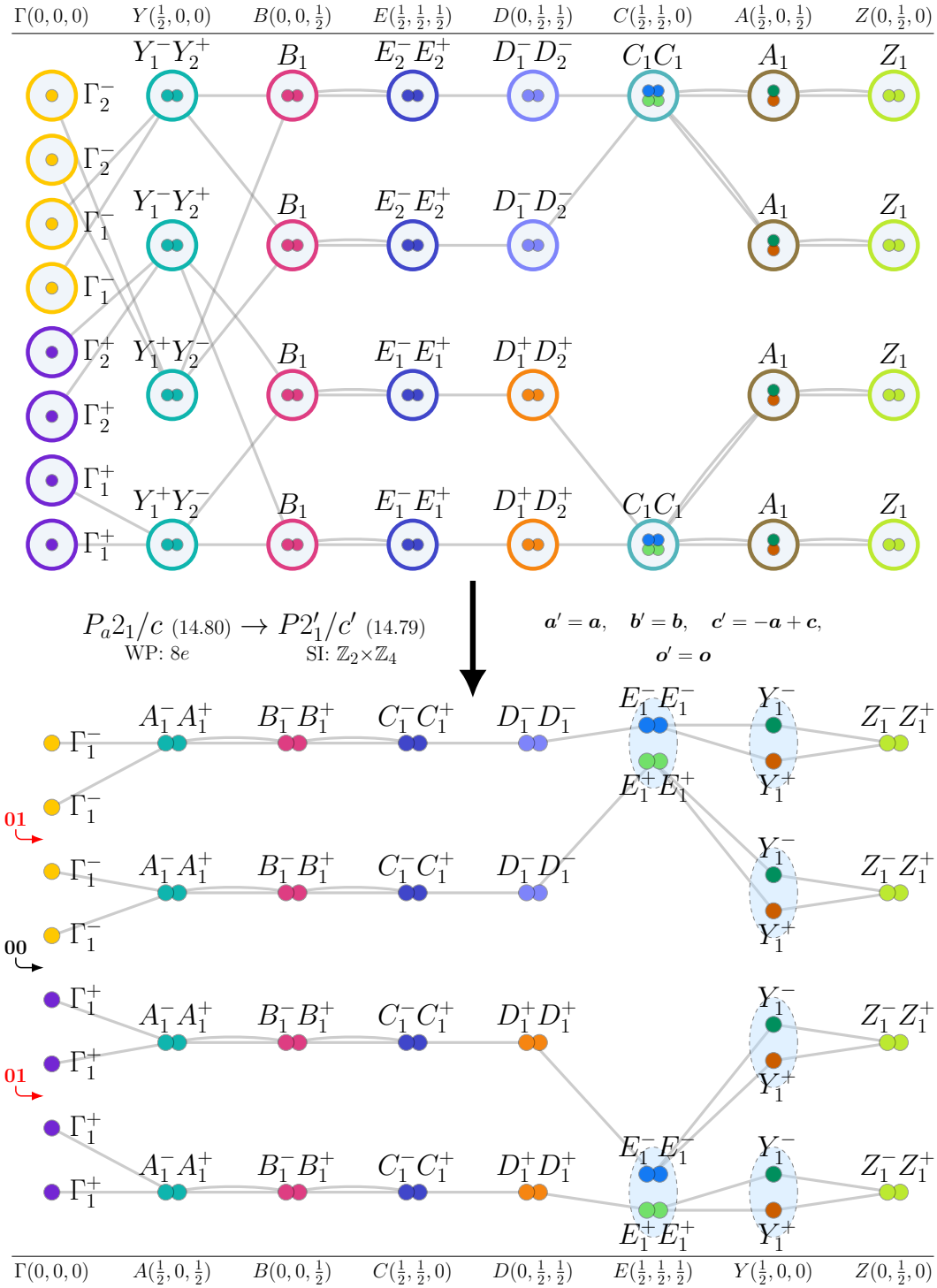
BCS Materials: CuO (213 K)[116], Ba_2CoO_4 (26 K)[117], Ba_2CoO_4 (25 K)[118], Ba_2CoO_4 (23 K)[119], $LiFeGe_2O_6$ (20 K)[120], $Na_{0.5}Li_{0.5}FeGe_2O_6$ (18 K)[121], $BiNiO(PO_4)$ (17.5 K)[122], $Li_{0.31}Na_{0.69}FeGe_2O_6$ (15 K)[123], $BiCoO(PO_4)$ (15 K)[124], Li_2MnSiO_4 (12 K)[125], $BiMnTeO_6$ (10 K)[126], MnV_2O_6 (4.7 K)[127], ErNiGe (2.9 K)[128], $BaNd_2O_4$ (1.7 K)[129], $BaNd_2O_4$ (1.7 K)[130], $GdPO_4$ (0.8 K)[131], $CoNb_2O_6$[132], Y_2BaCuO_5[133].

29.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].

[116]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.62
[117]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.302
[118]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.476
[119]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.477
[120]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.39
[121]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.276
[122]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.127
[123]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.331
[124]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.128
[125]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.78
[126]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.301
[127]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.196
[128]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.379
[129]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.96
[130]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.95
[131]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.118
[132]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.656
[133]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.445



Supplementary Figure 156: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8e$ of supergroup P_a2_1/c (14.80).

Supplementary Table 301: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $8e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

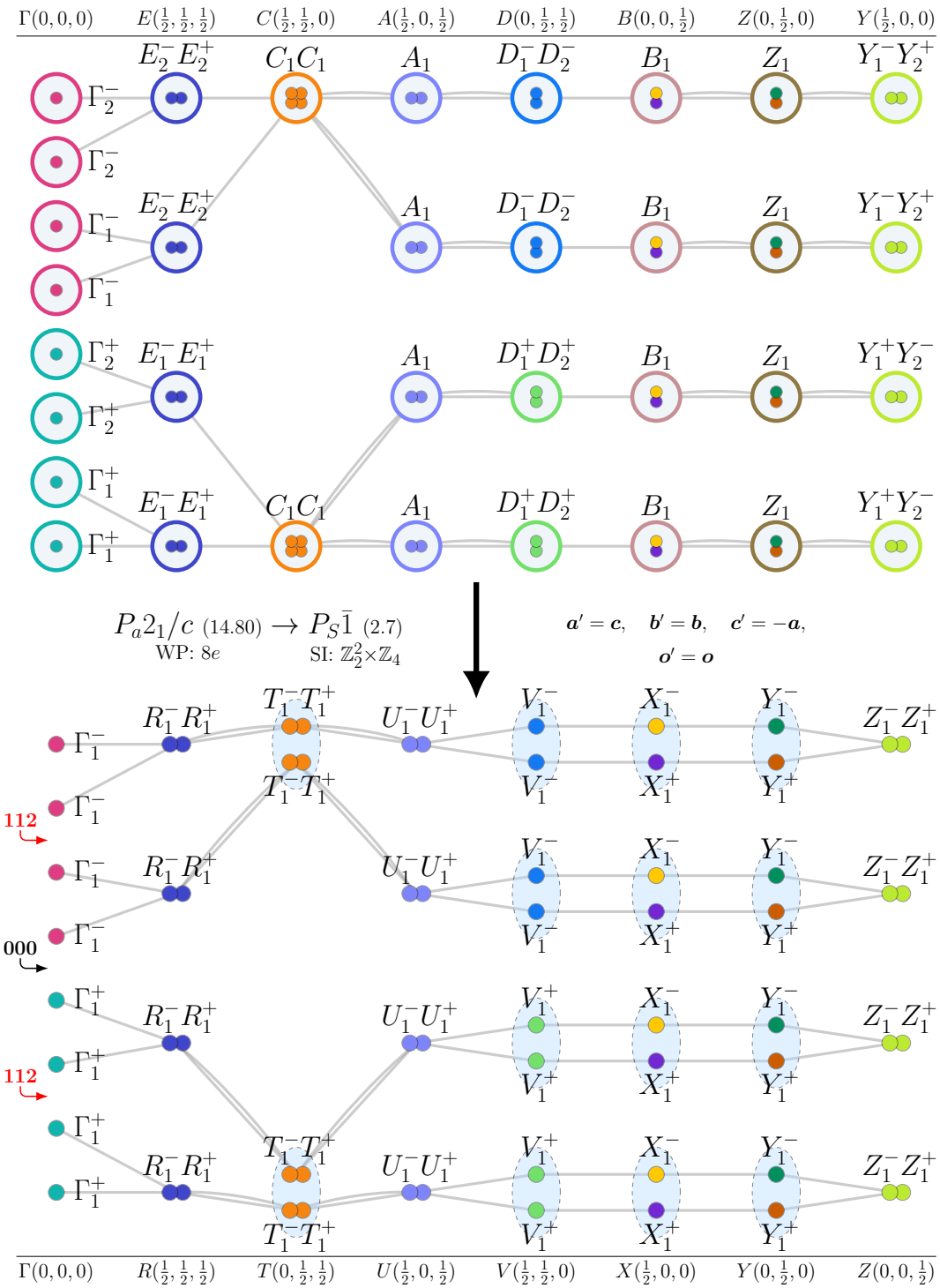
Supplementary Table 302: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $8e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

29.1.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 157: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position 8e of supergroup P_a2_1/c (14.80).

Supplementary Table 303: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: 8e, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1

continued on next page...

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SI	Possible #Gaps
gapless	4
nontrivial	2, 3

Supplementary Table 304: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

29.2 WP: $4a$

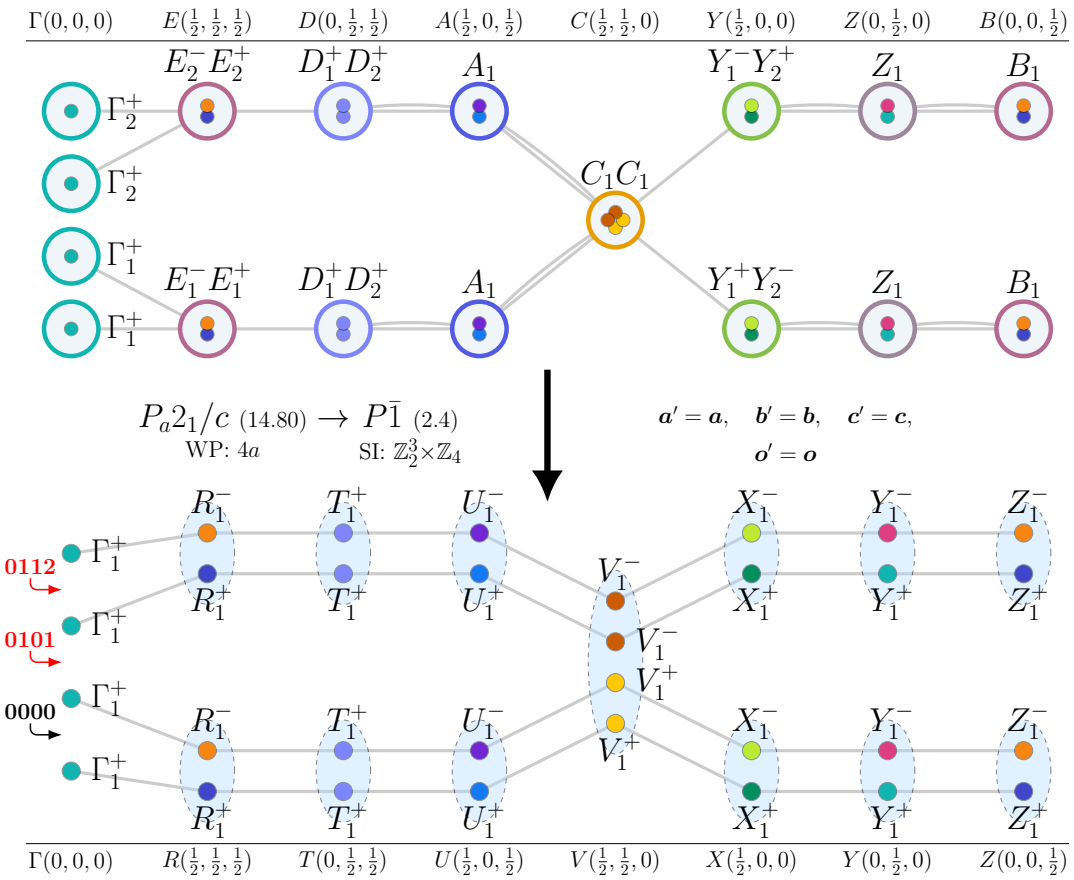
BCS Materials: $LiCoF_4$ (150 K)[134], $NaMnF_4$ (13 K)[135], $CuSb_2O_6$ (8.7 K)[136], $MnPb_4Sb_6S_{14}$ (6 K)[137], $FePb_4Sb_6S_{14}$ (5 K)[138], $Li_2Fe(SO_4)_2$ (4 K)[139].

29.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B $\parallel$ [010] and strain in generic direction,
- B $\perp$ [010] and strain in generic direction,
- B in generic direction.

[134]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.526
[135]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.345
[136]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.133
[137]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.63
[138]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.660
[139]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.147



Supplementary Figure 158: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_a2_1/c (14.80).

Supplementary Table 305: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1
1002	0, 1, 2
1003	0, 1
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

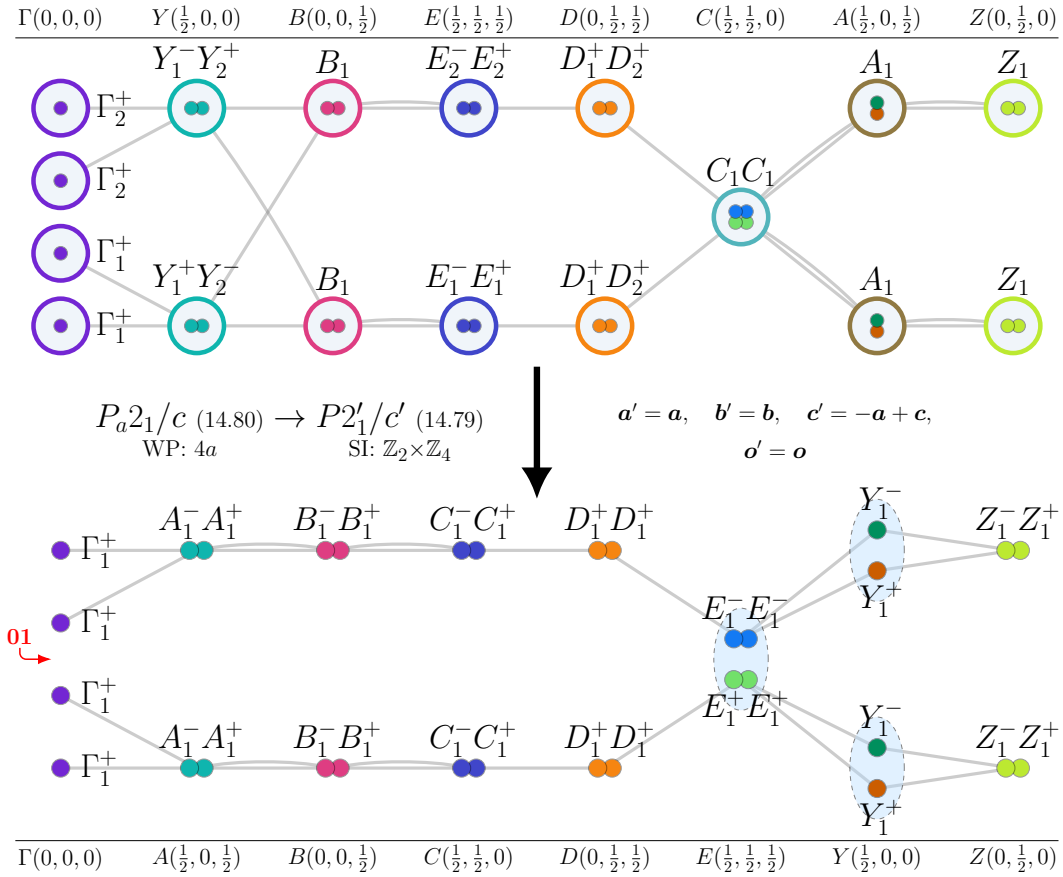
Supplementary Table 306: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0111, 0112, 0113, 1000 1002, 1003, 1010, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	0101, 0103, 0112
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0111, 0112, 0113, 1000 1001, 1002, 1010, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

29.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 159: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_a2_1/c (14.80).

Supplementary Table 307: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

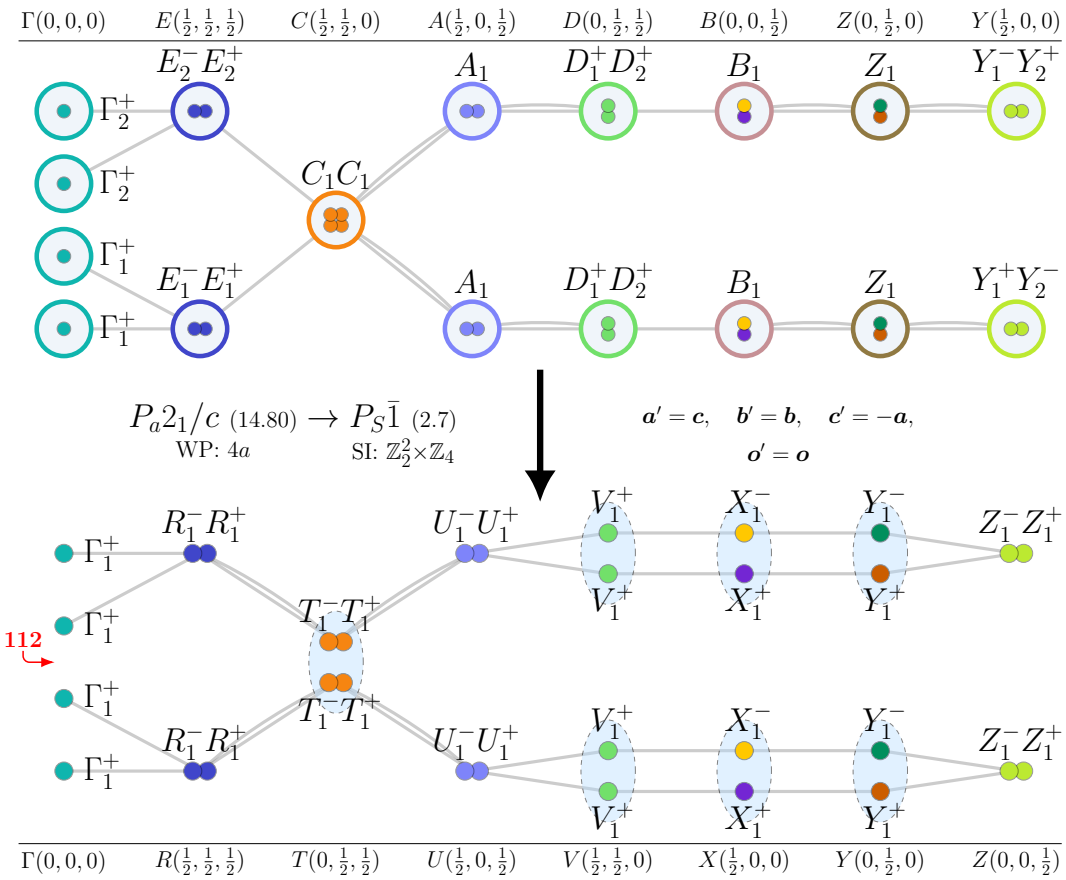
Supplementary Table 308: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

29.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 160: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_a2_1/c (14.80).

Supplementary Table 309: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 310: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

29.3 WP: $4a+8e$

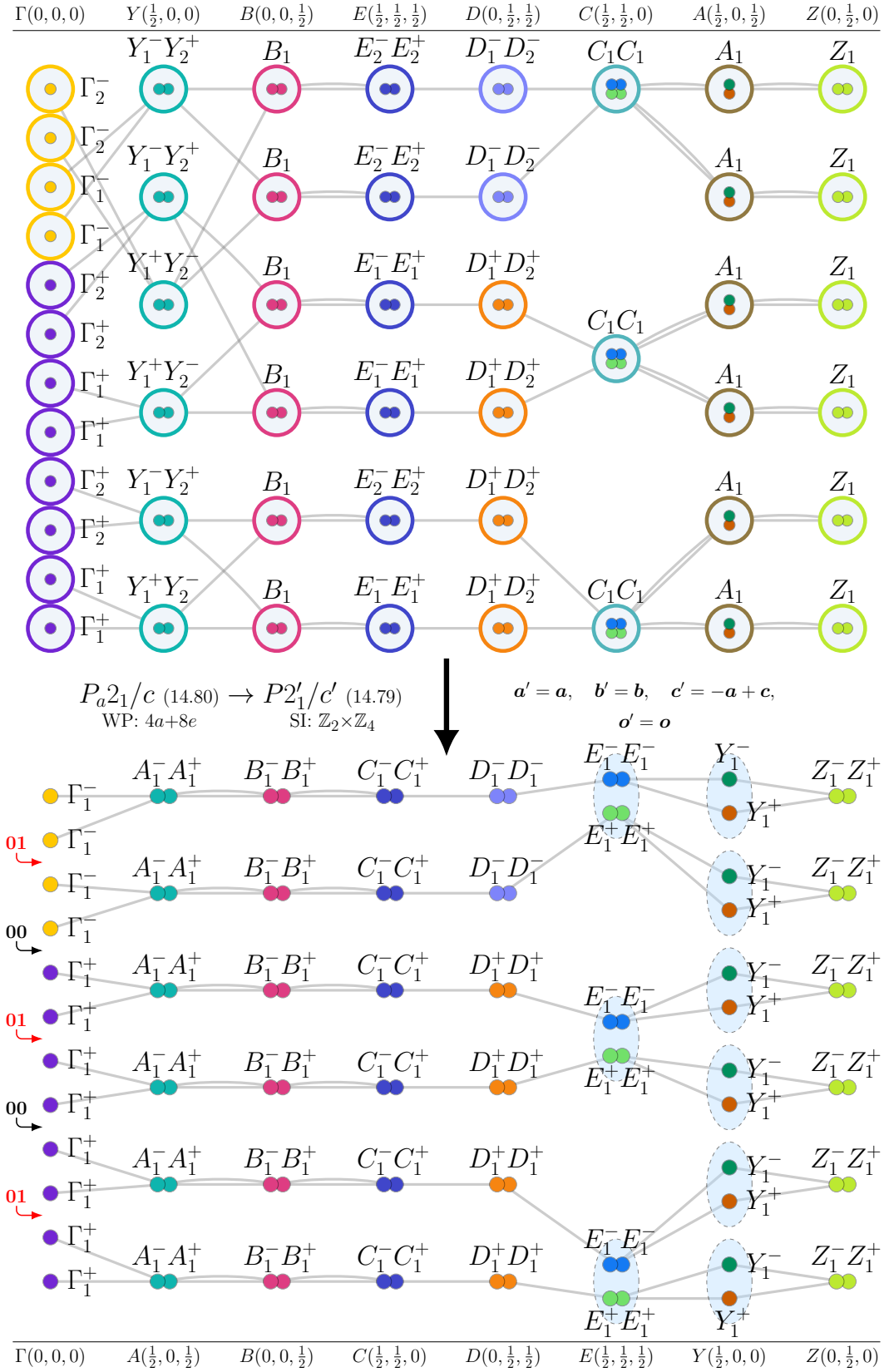
BCS Materials: Cr_2ReO_6 (67 K)[140], $Co_3(PO_4)_2$ (30 K)[141], $CuFe_2(P_2O_7)_2$ (15.5 K)[142].

[140]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.201`
[141]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.342`
[142]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.297`

29.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 161: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a+8e$ of supergroup P_a2_1/c (14.80).

Supplementary Table 311: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4a$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

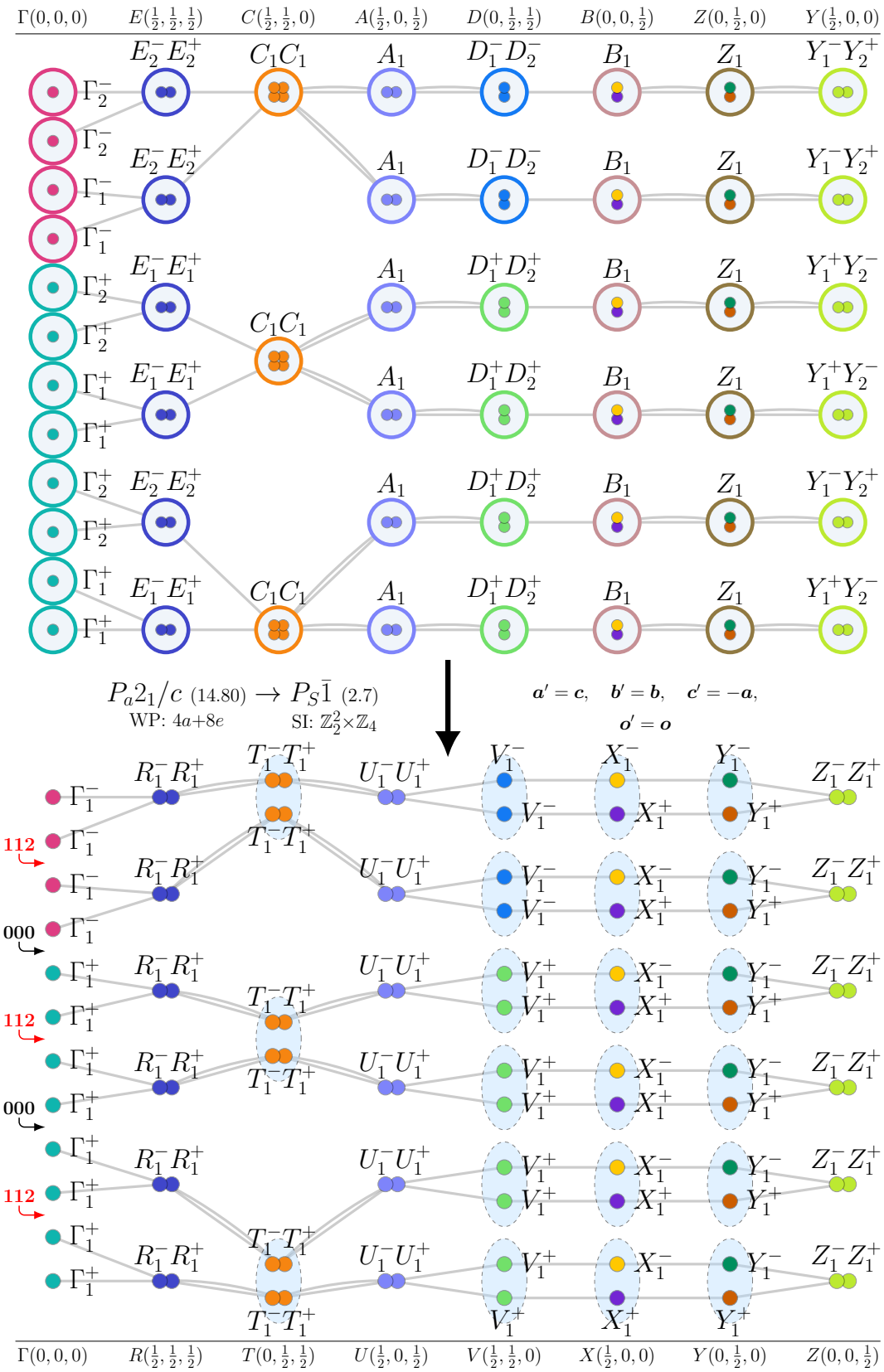
Supplementary Table 312: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4a$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

29.3.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 162: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4a+8e$ of supergroup P_a2_1/c (14.80).

Supplementary Table 313: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4a$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 314: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4a$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

29.4 WP: $4c + 8e$

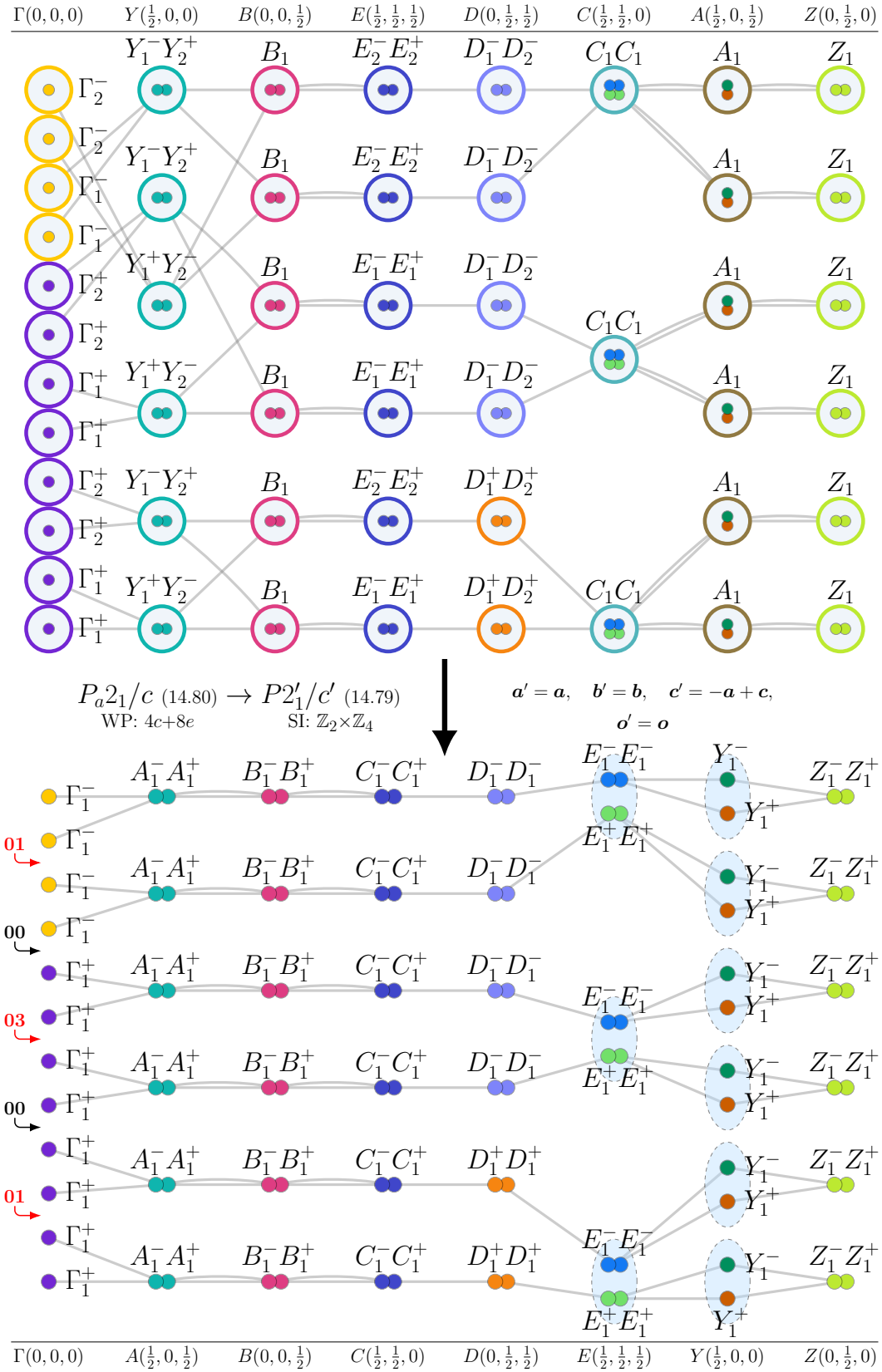
BCS Materials: Mn_3TeO_6 (35.8 K)[143].

29.4.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].

[143] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.485`



Supplementary Figure 163: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+8e$ of supergroup P_a2_1/c (14.80).

Supplementary Table 315: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4c$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

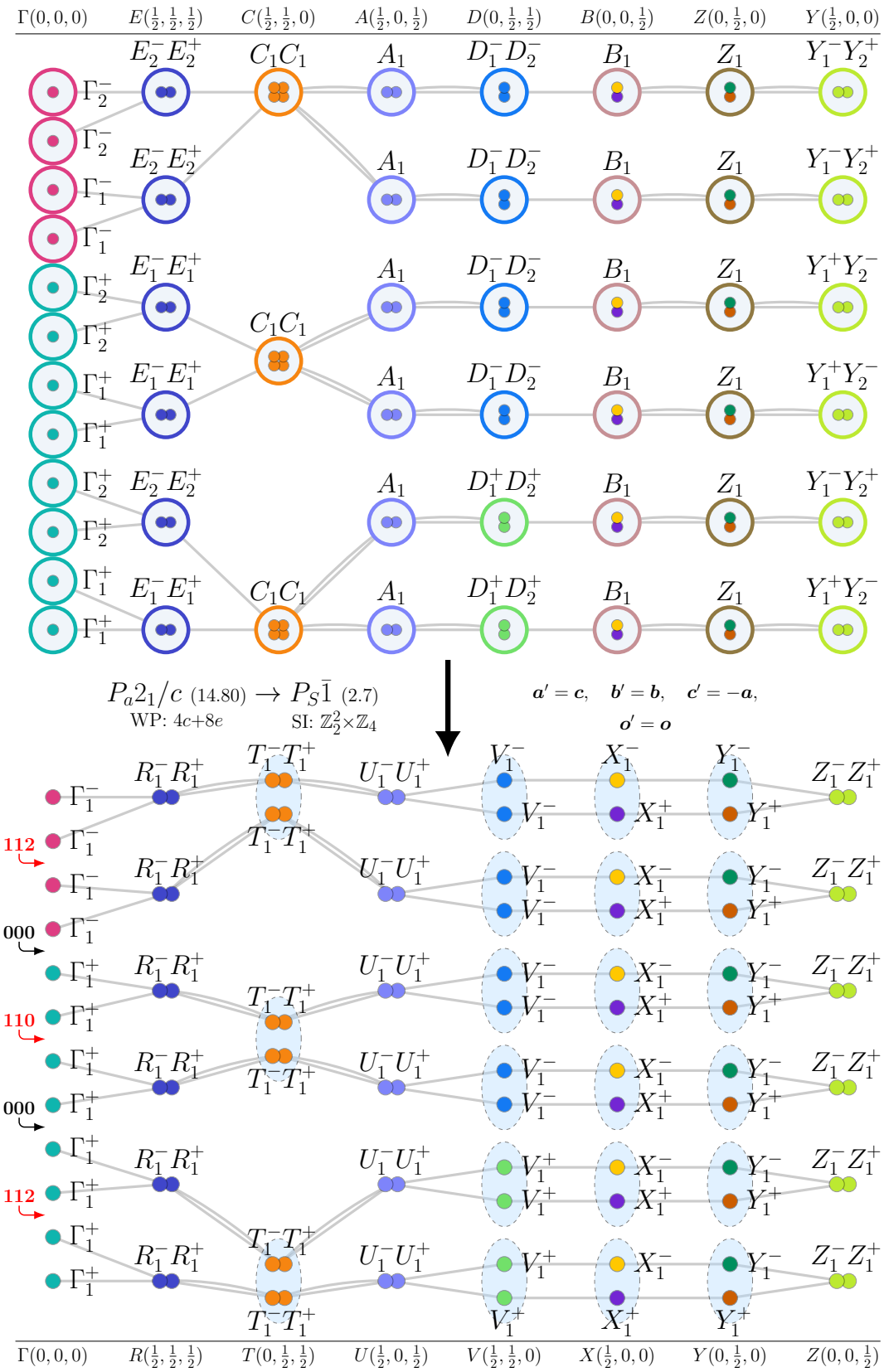
Supplementary Table 316: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4c$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

29.4.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 164: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4c + 8e$ of supergroup P_a2_1/c (14.80).

Supplementary Table 317: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4c$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 318: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4c$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

29.5 WP: $4c$

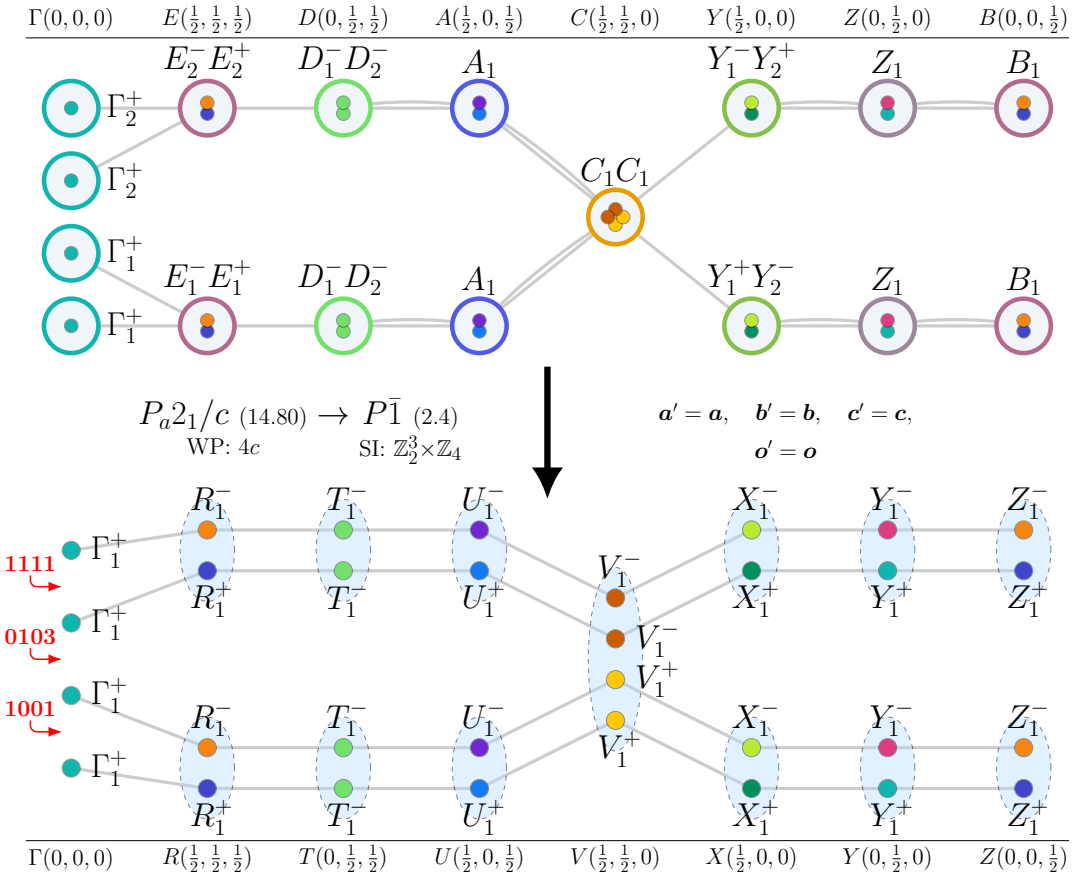
BCS Materials: Sc_2NiMnO_6 (35 K)[144], La_2CoPtO_6 (28 K)[145].

29.5.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[144]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.199`
[145]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.462`



Supplementary Figure 165: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup P_a2_1/c (14.80).

Supplementary Table 319: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1
1002	0, 1, 2
1003	0, 1
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2

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SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

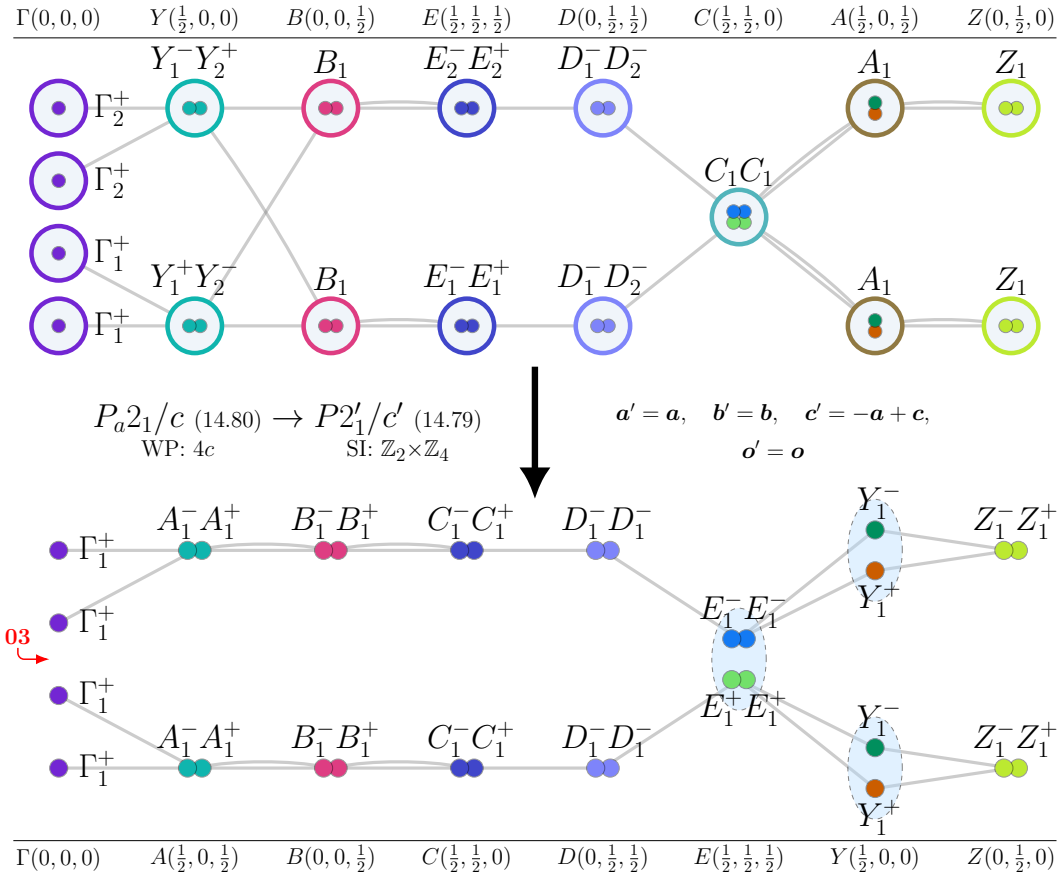
Supplementary Table 320: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0113, 1000 1001, 1002, 1010, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	0101, 0103, 0110
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0113, 1000 1002, 1003, 1010, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

29.5.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 166: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup P_a2_1/c (14.80).

Supplementary Table 321: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

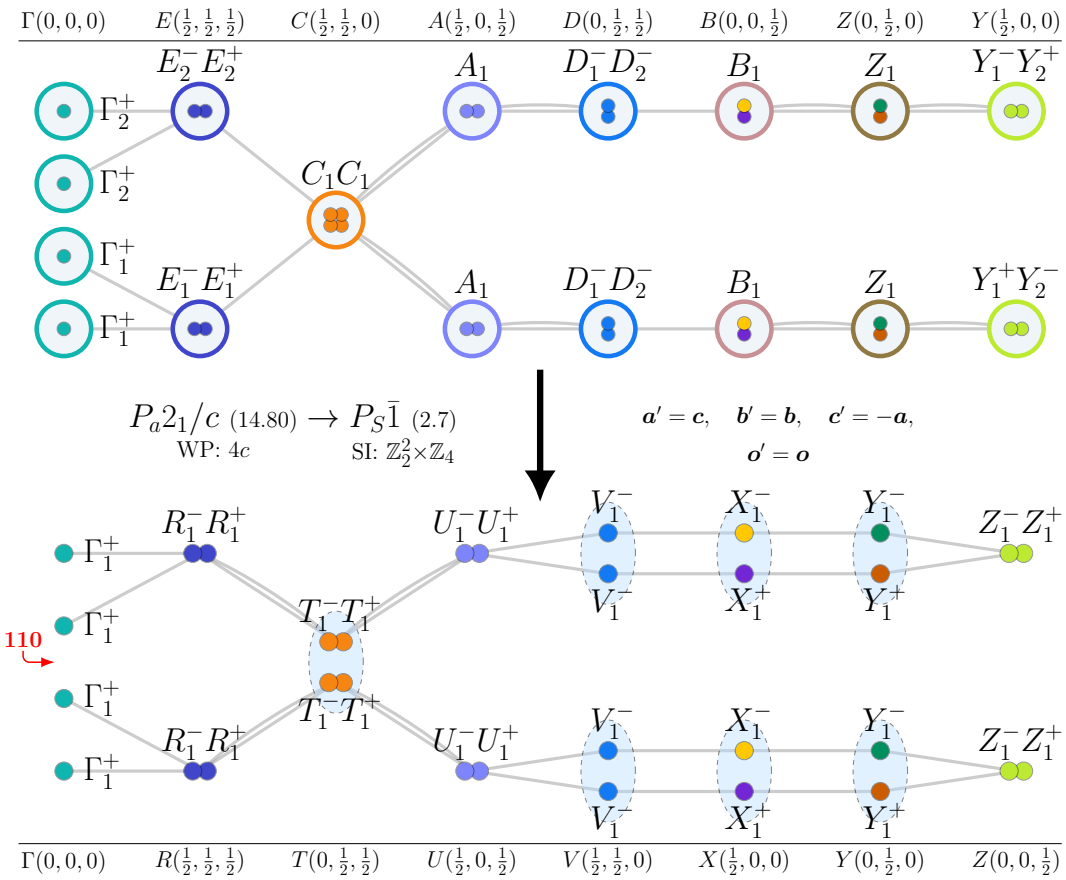
Supplementary Table 322: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

29.5.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 167: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4c$ of supergroup P_a2_1/c (14.80).

Supplementary Table 323: Number of times a given SI can appear in the band structure. Supergroup: P_a2_1/c (14.80), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

Supplementary Table 324: Possible SI values in each gap. Supergroup: P_a2_1/c (14.80), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

30 MSG $R_I\bar{3}$ (148.20)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P_S\bar{1}$ (2.7), $R\bar{3}$ (148.17).

Trivial-SI Subgroups: P_S1 (1.3), $R3$ (146.10), R_I3 (146.12).

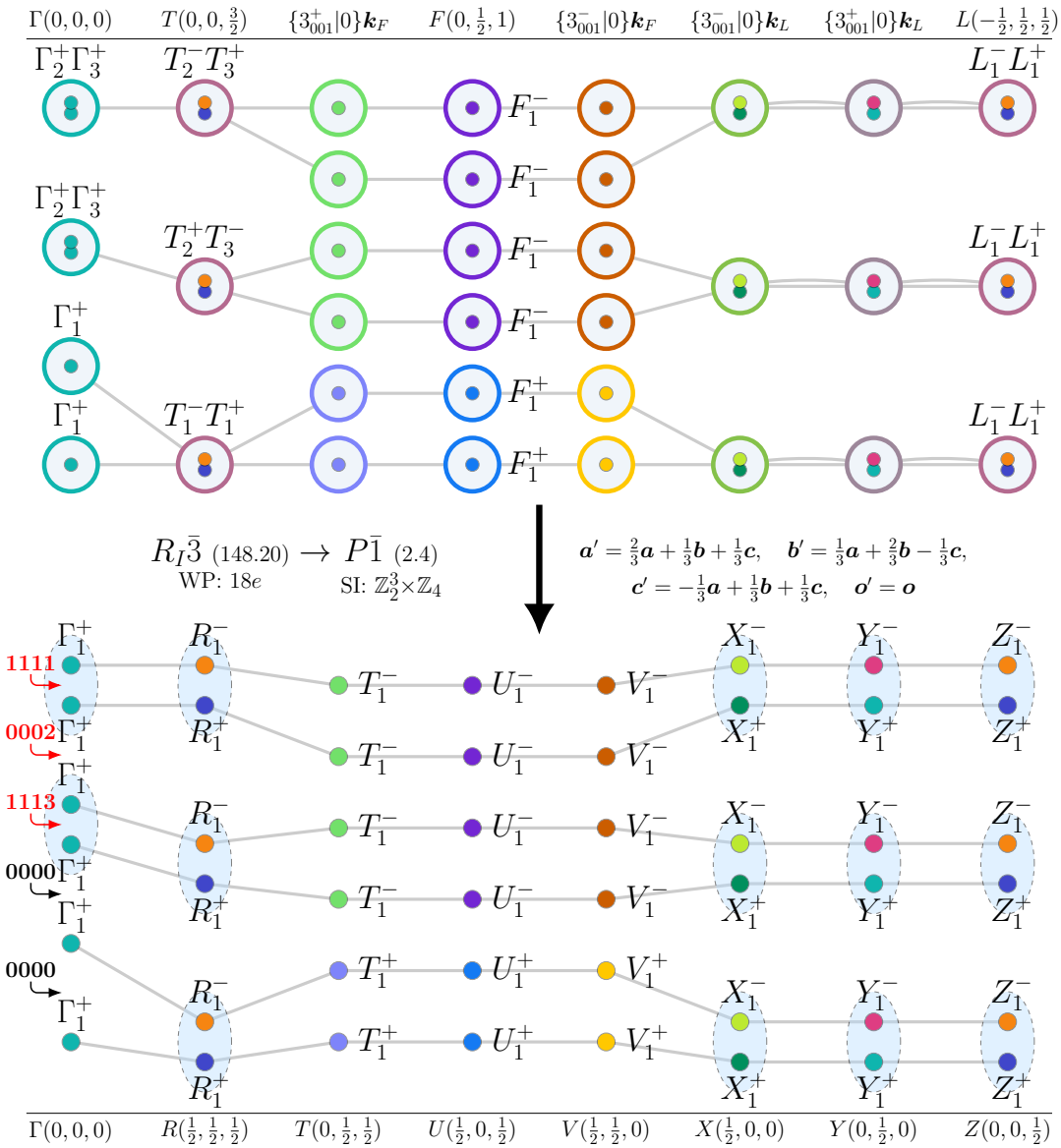
30.1 WP: $18e$

BCS Materials: $LaMn_3V_4O_{12}$ (44 K)[146].

30.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 168: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}$ (148.20).

Supplementary Table 325: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}$ (148.20), WP: $18e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3

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[146]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.119`

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SI	Possible #Gaps
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2
0102	0, 1, 2, 3
0103	0, 1, 2
0110	0, 1
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2
1002	0, 1, 2, 3
1003	0, 1, 2
1010	0, 1
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

2476

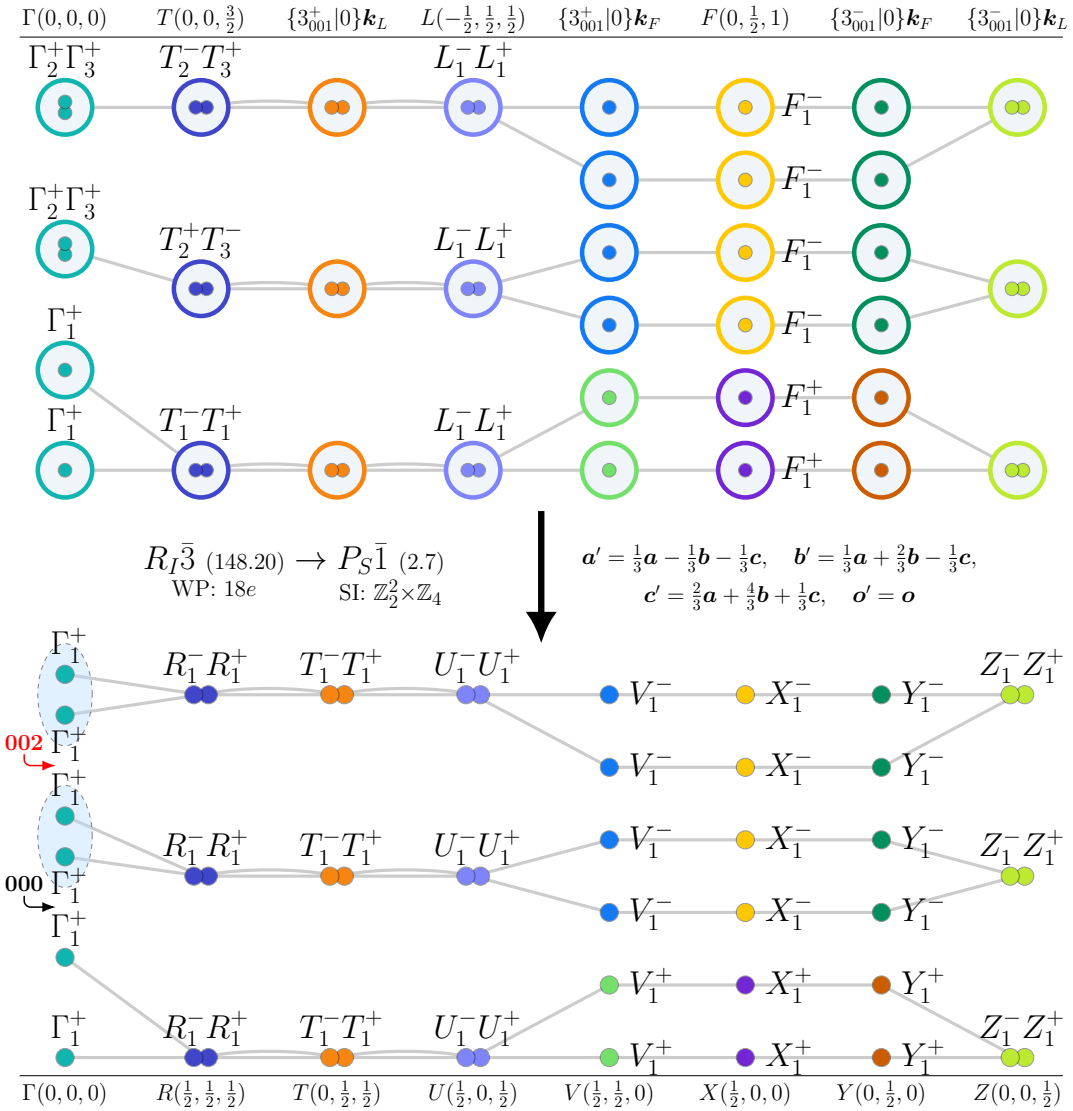
Supplementary Table 326: Possible SI values in each gap. Supergroup: $R_I\bar{3}$ (148.20), WP: $18e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0111 0112, 0113, 1000, 1001, 1002 1011, 1012, 1013, 1101, 1102 1103, 1110, 1112, 1113
2	0000, 0002, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1103, 1110 1111, 1112, 1113
4	0000, 0002, 1111
5	0000, 0001, 0003, 0010, 0012 0013, 0100, 0102, 0103, 0111 0112, 0113, 1000, 1002, 1003 1011, 1012, 1013, 1101, 1102 1103, 1110, 1111, 1112
6	0000

30.1.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 169: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}$ (148.20).

Supplementary Table 327: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}$ (148.20), WP: $18e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1, 2
111	0, 1
113	0, 1
gapless	3
nontrivial	1, 2

Supplementary Table 328: Possible SI values in each gap. Supergroup: $R_I\bar{3}$ (148.20), WP: $18e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 002, 111
3	G
4	000, 002, 113
5	G
6	000

31 MSG P_c2_1/c (14.82)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/m'$ (11.54), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pm' (6.20), $P2_1'$ (4.9), P_S1 (1.3), Pc (7.24), P_cc (7.28), $P2_1$ (4.7), P_a2_1 (4.10).

31.1 WP: $4c + 4d$

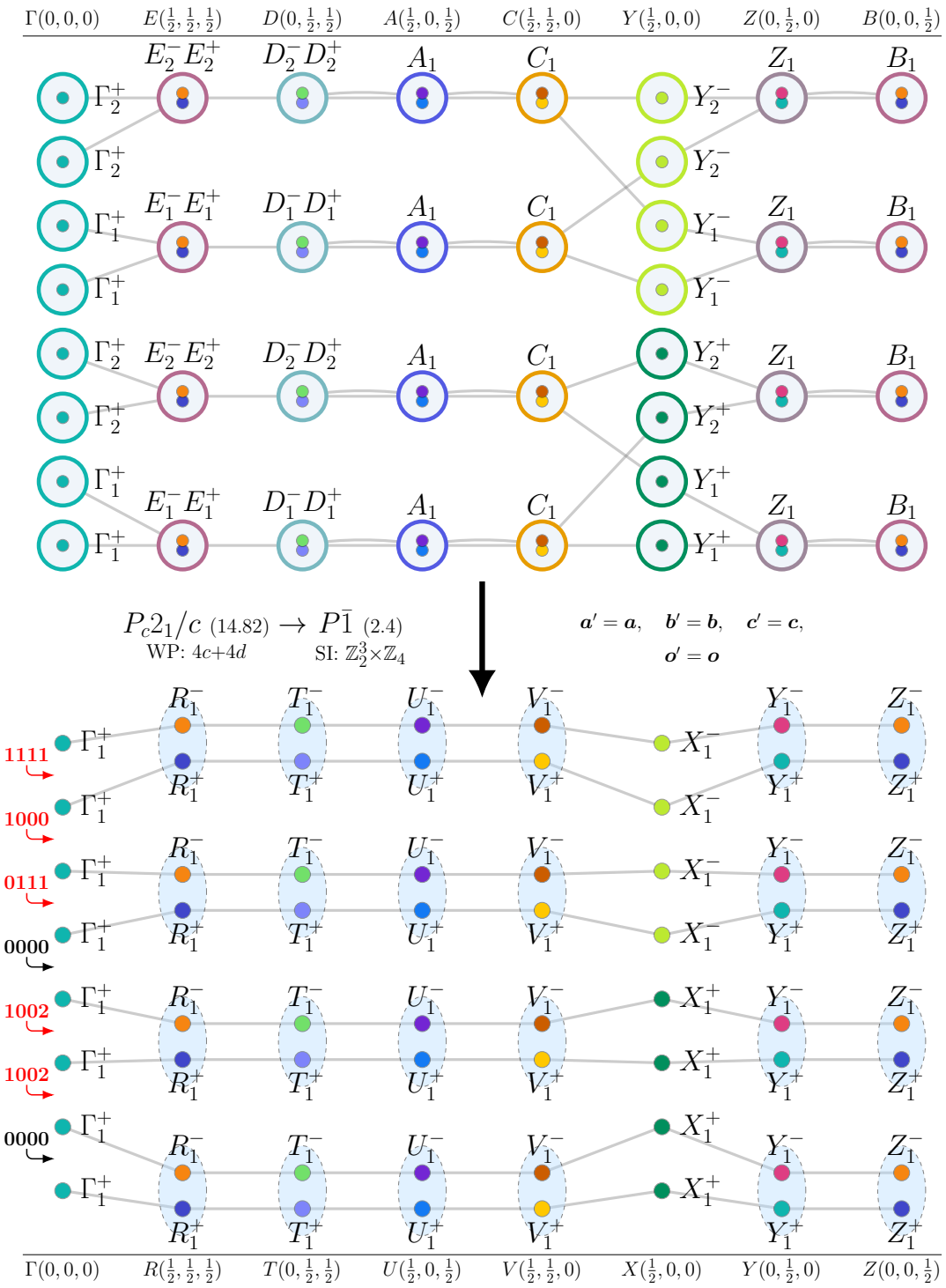
BCS Materials: $FeSO_4$ (14 K)[147], $NaFePO_4$ (13 K)[148].

31.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[147]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.573`
[148]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.117`



Supplementary Figure 170: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4c + 4d$ of supergroup P_c2_1/c (14.82).

Supplementary Table 329: Number of times a given SI can appear in the band structure. Supergroup: P_c2_1/c (14.82), WP: 4c+4d, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2, 3, 4, 5
0001	0, 1, 2, 3, 4
0002	0, 1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4

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...continued

SI	Possible #Gaps
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4, 5
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4, 5
1000	0, 1, 2, 3, 4, 5, 6
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4, 5, 6
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	0, 1, 2, 3, 4
1112	0, 1, 2, 3, 4
1113	0, 1, 2, 3, 4
gapless	0
nontrivial	2, 3, 4, 5, 6, 7

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Supplementary Table 330: Possible SI values in each gap. Supergroup: P_c2_1/c (14.82), WP: $4c$+$4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
2	1000, 1002, 1113
3	0000, 0001, 0002, 0003, 0010 0011, 0012, 0013, 0100, 0101 0102, 0103, 0110, 0111, 0112 0113, 1000, 1001, 1002, 1003 1010, 1011, 1012, 1013, 1100 1101, 1102, 1103, 1110, 1111 1112, 1113
4	0000, 0002, 0111, 0113
5	0000, 0001, 0002, 0003, 0010 0011, 0012, 0013, 0100, 0101 0102, 0103, 0110, 0111, 0112 0113, 1000, 1001, 1002, 1003 1010, 1011, 1012, 1013, 1100 1101, 1102, 1103, 1110, 1111 1112, 1113
6	1000, 1002, 1111

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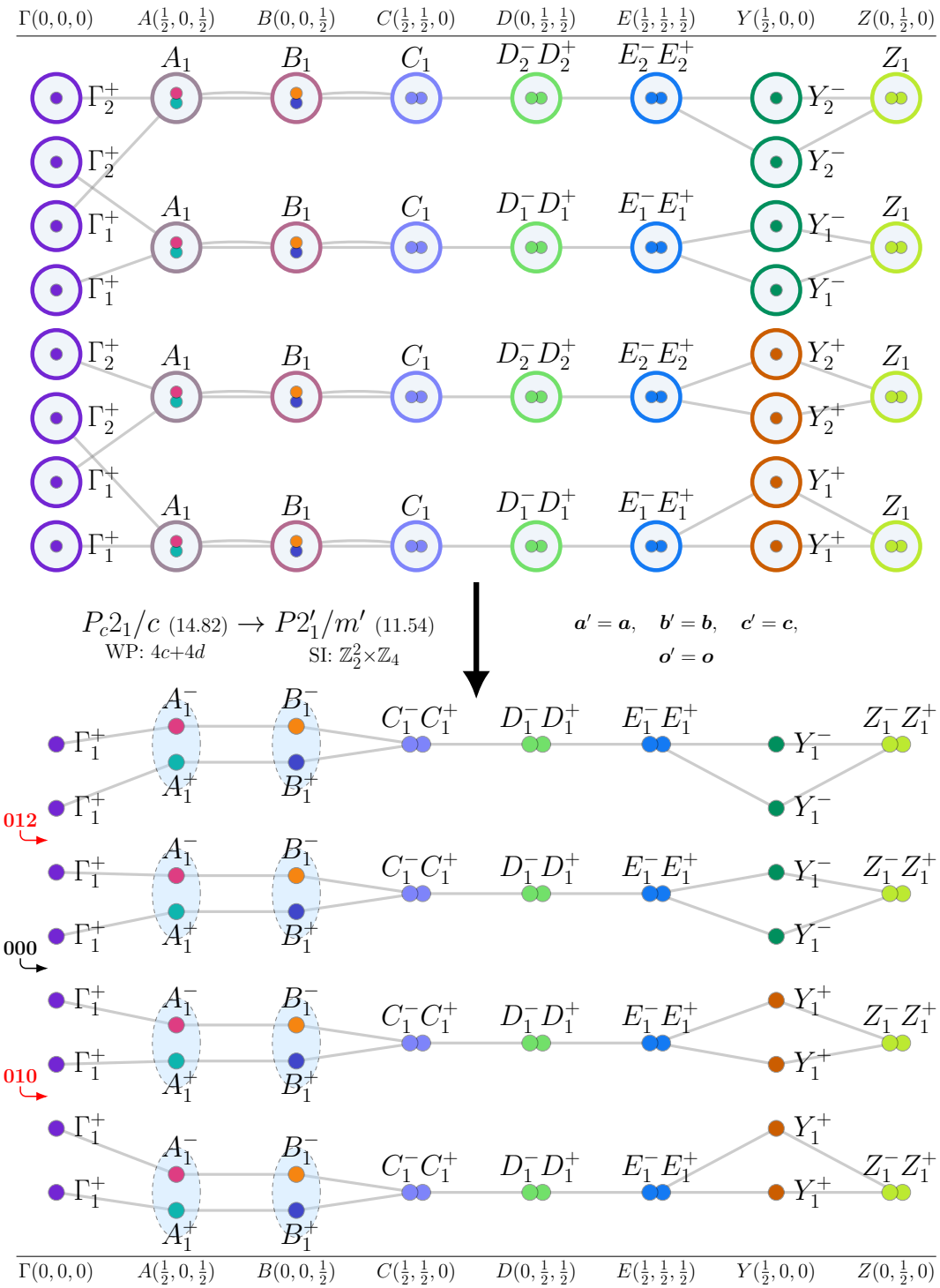
...continued

Gap#	Possible SI Values
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
8	0000

31.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 171: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff positions $4c + 4d$ of supergroup P_c2_1/c (14.82).

Supplementary Table 331: Number of times a given SI can appear in the band structure. Supergroup: P_c2_1/c (14.82), WP: 4c+4d, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
002	0, 1
010	0, 1, 2
012	0, 1, 2
101	0, 1
103	0, 1

continued on next page...

...continued

SI	Possible #Gaps
111	0, 1
113	0, 1
gapless	4
nontrivial	2, 3

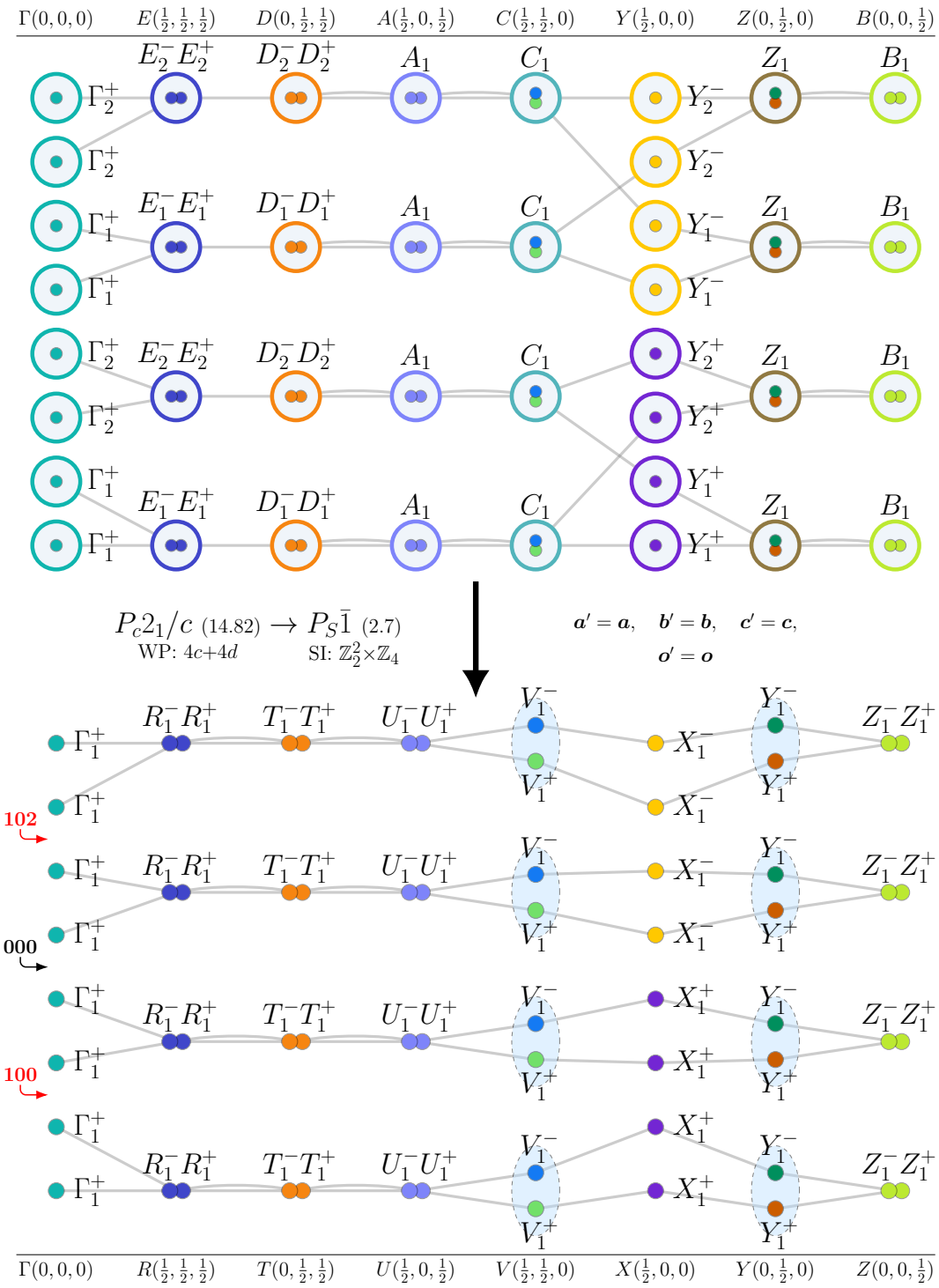
Supplementary Table 332: Possible SI values in each gap. Supergroup: P_c2_1/c (14.82), WP: $4c$+$4d$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	010, 012, 111
3	G
4	000, 002, 101, 103
5	G
6	010, 012, 113
7	G
8	000

31.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 172: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4c + 4d$ of supergroup P_c2_1/c (14.82).

Supplementary Table 333: Number of times a given SI can appear in the band structure. Supergroup: P_c2_1/c (14.82), WP: 4c+4d, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1
011	0, 1
013	0, 1
100	0, 1, 2
102	0, 1, 2
111	0, 1

continued on next page...

...continued

SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 334: Possible SI values in each gap. Supergroup: P_c2_1/c (14.82), WP: $4c$+$4d$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	100, 102, 111
3	G
4	000, 002, 011, 013
5	G
6	100, 102, 113
7	G
8	000

32 MSG P_C2_1/c (14.84)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pc' (7.26), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), P_Cc (7.30), $P2_1$ (4.7), P_C2_1 (4.12).

32.1 WP: $4c$

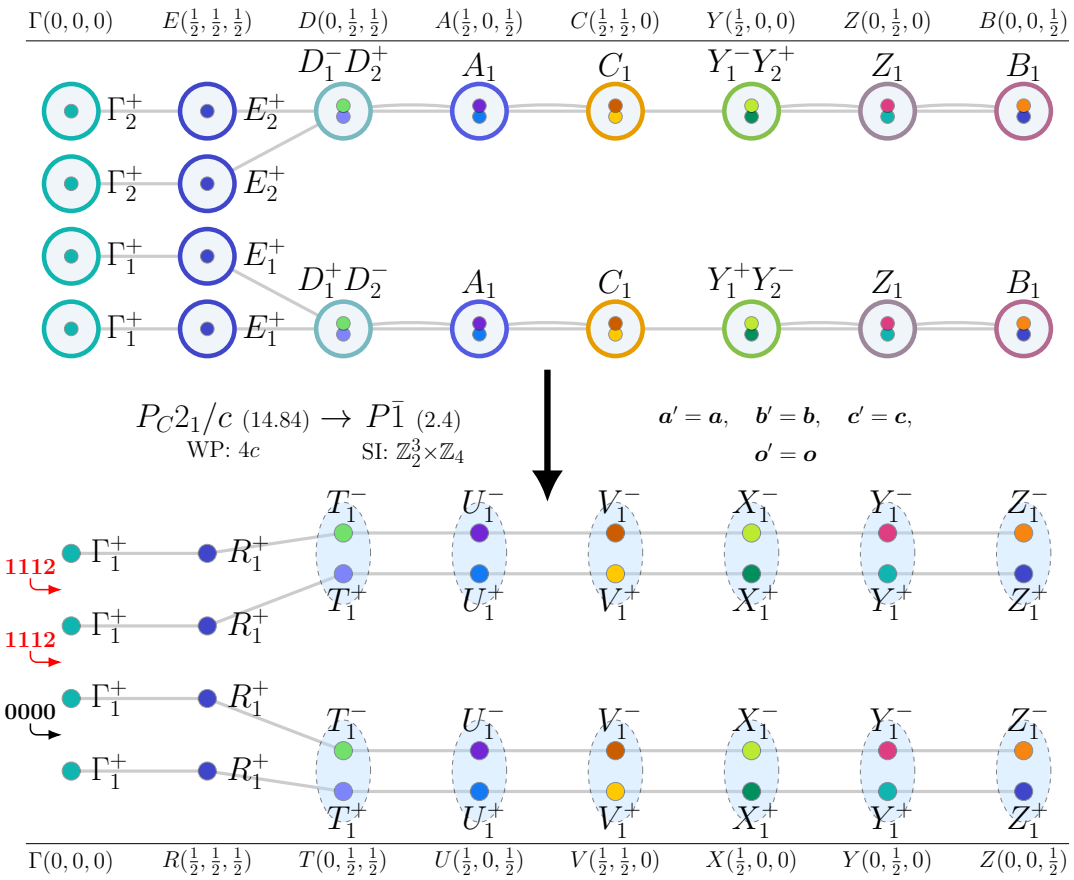
BCS Materials: $CuSe_2O_5$ (17 K)[149], $NH_4FeCl_2(HCOO)$ (6 K)[150], $La_2NiO_3F_{1.93}$[151].

32.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.

[149]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.2`
[150]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.144`
[151]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.390`



Supplementary Figure 173: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup P_C2_1/c (14.84).

Supplementary Table 335: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/c (14.84), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

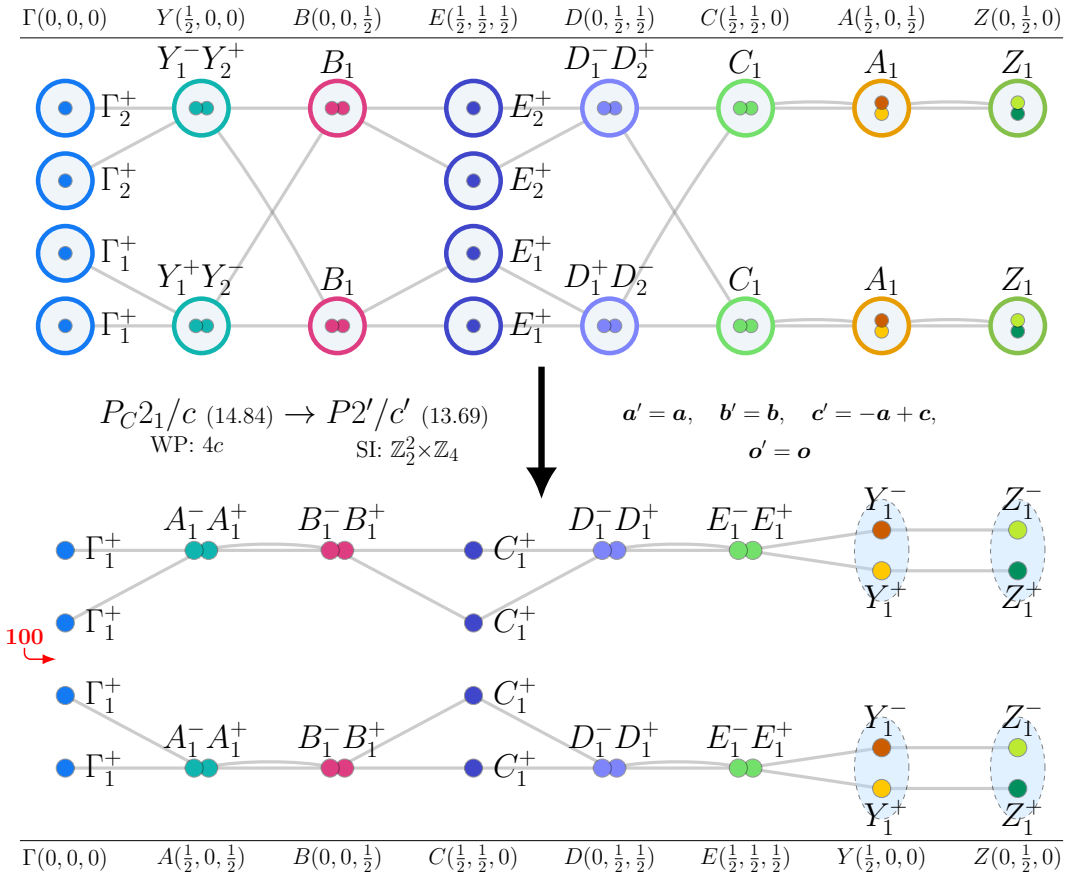
Supplementary Table 336: Possible SI values in each gap. Supergroup: P_C2_1/c (14.84), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

32.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 174: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4c$ of supergroup P_C2_1/c (14.84).

Supplementary Table 337: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/c (14.84), WP: $4c$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

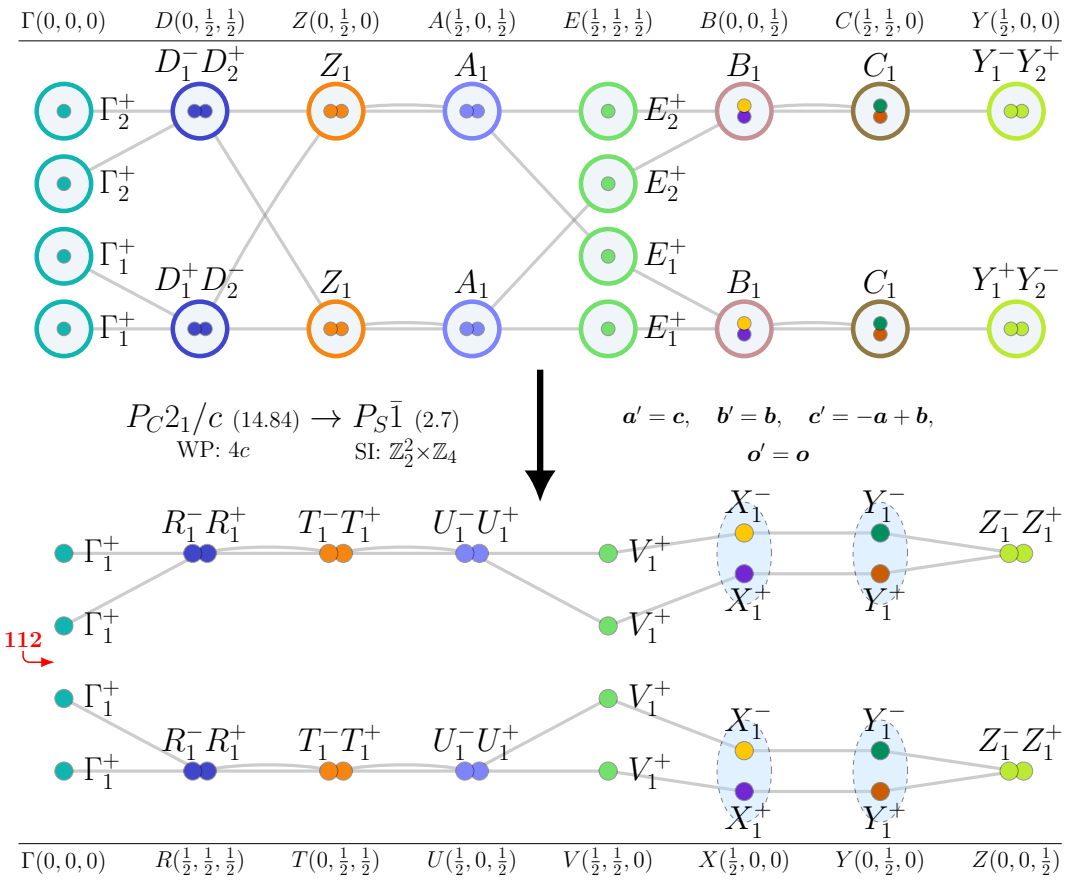
Supplementary Table 338: Possible SI values in each gap. Supergroup: P_C2_1/c (14.84), WP: $4c$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

32.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 175: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4c$ of supergroup P_C2_1/c (14.84).

Supplementary Table 339: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/c (14.84), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 340: Possible SI values in each gap. Supergroup: P_C2_1/c (14.84), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

32.2 WP: $4d$

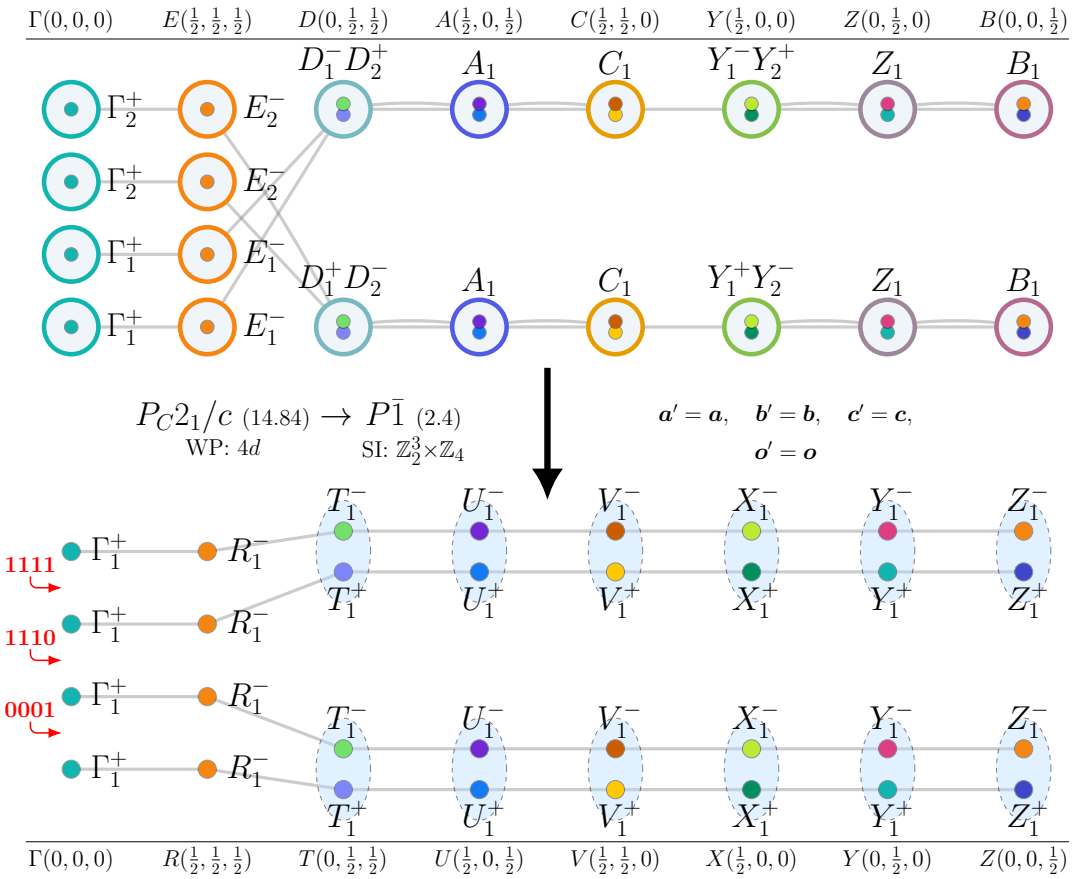
BCS Materials: $Na_2BaMn(VO_4)_2$ (1.3 K)[152].

[152]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.306`

32.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [010] and strain in generic direction,
- B ⊥ [010] and strain in generic direction,
- B in generic direction.



Supplementary Figure 176: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position 4*d* of supergroup P_C2_1/c (14.84).

Supplementary Table 341: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/c (14.84), WP: 4*d*, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2

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SI	Possible #Gaps
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	1, 2, 3
1111	0, 1
1113	0, 1
gapless	0
nontrivial	1, 2, 3

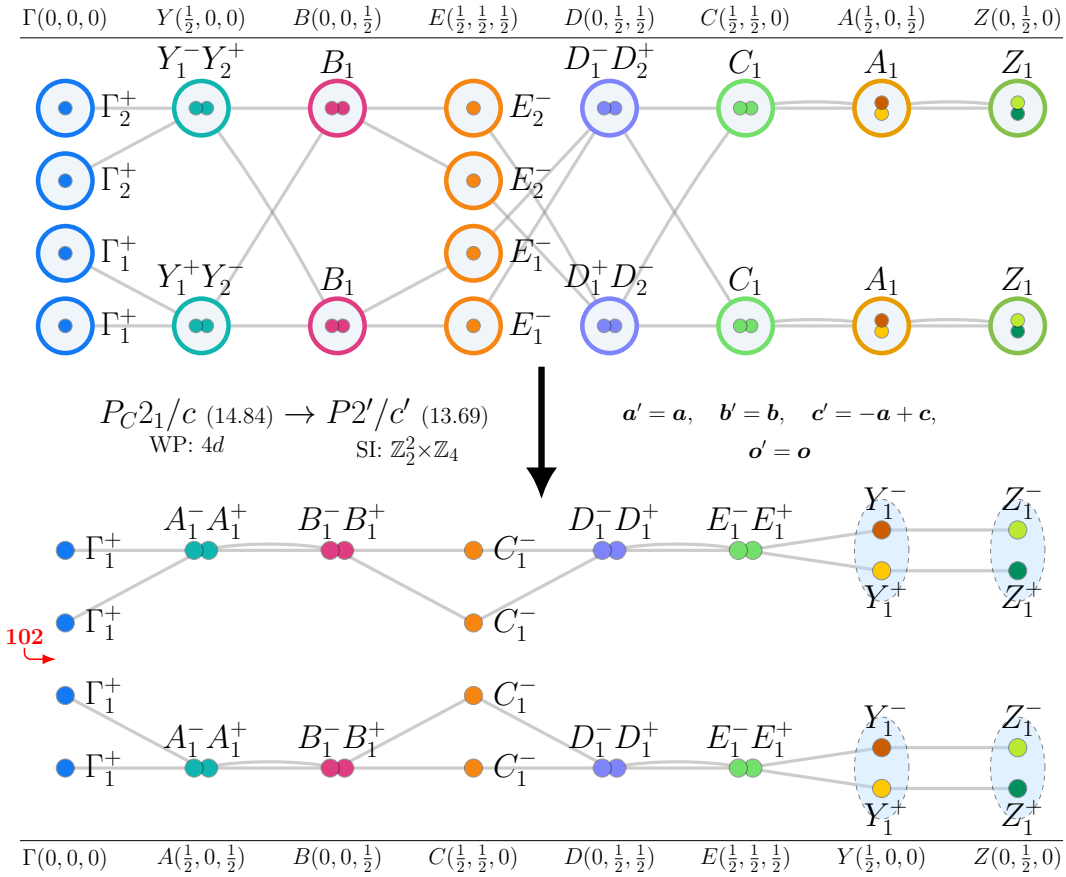
Supplementary Table 342: Possible SI values in each gap. Supergroup: P_C2_1/c (14.84), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
2	1110
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
4	0000

32.2.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbation:

- B $\perp$ [010].



Supplementary Figure 177: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4d$ of supergroup P_C2_1/c (14.84).

Supplementary Table 343: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/c (14.84), WP: $4d$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
102	1
gapless	2
nontrivial	1

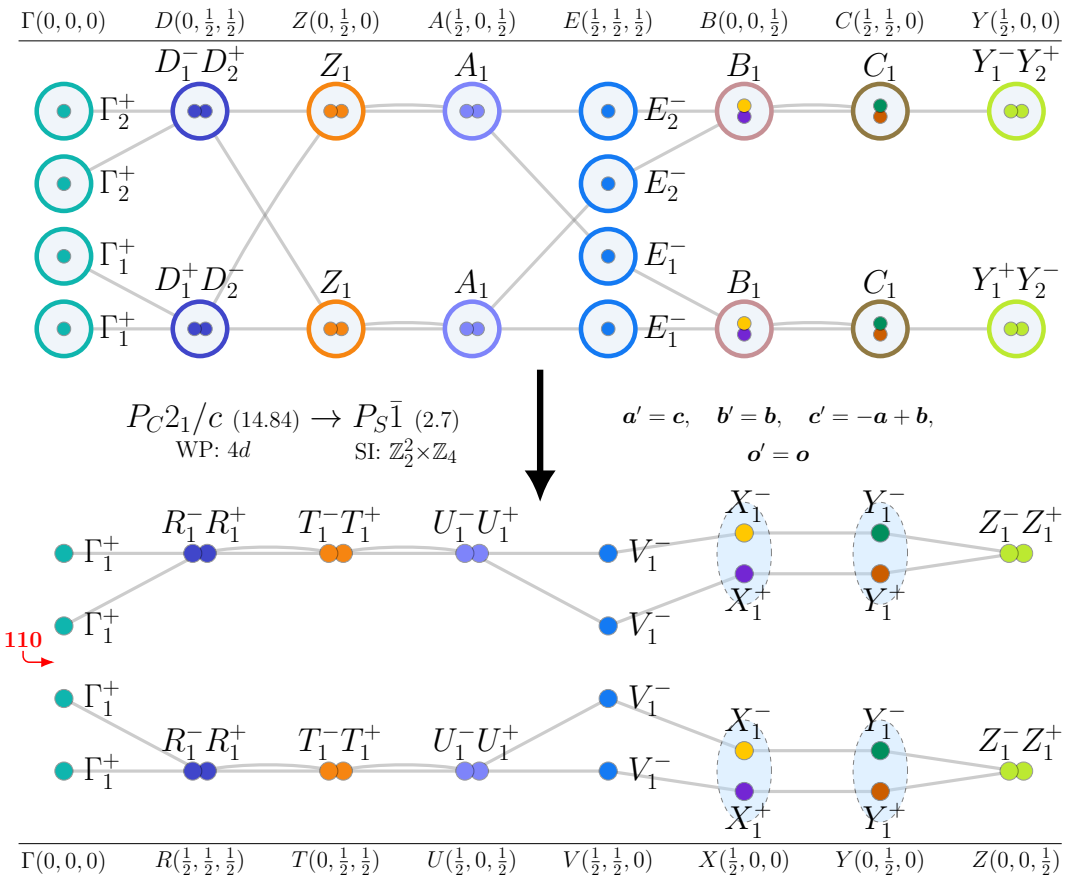
Supplementary Table 344: Possible SI values in each gap. Supergroup: P_C2_1/c (14.84), WP: $4d$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	102
3	G
4	000

32.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 178: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4d$ of supergroup P_C2_1/c (14.84).

Supplementary Table 345: Number of times a given SI can appear in the band structure. Supergroup: P_C2_1/c (14.84), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

Supplementary Table 346: Possible SI values in each gap. Supergroup: P_C2_1/c (14.84), WP: $4d$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

33 MSG $C2/c$ (15.85)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4).

Trivial-SI Subgroups: Cc (9.37), $C2$ (5.13).

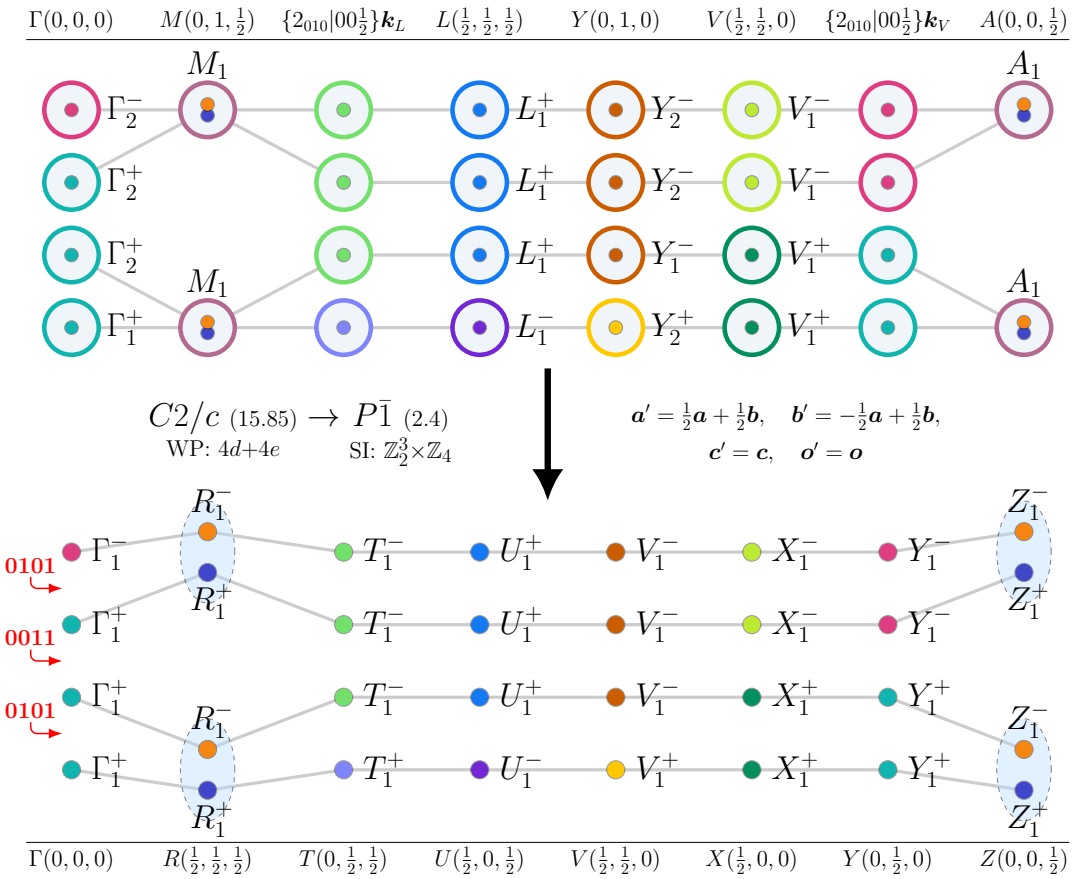
33.1 WP: $4d + 4e$

BCS Materials: $BiCrO_3$ (114 K)[153].

33.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.



Supplementary Figure 179: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4d + 4e$ of supergroup $C2/c$ (15.85).

Supplementary Table 347: Number of times a given SI can appear in the band structure. Supergroup: $C2/c$ (15.85), WP: $4d$+$4e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0011	0, 1
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2

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[153]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.138`

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SI	Possible #Gaps
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1
1102	0, 1
1111	0, 1
1113	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 348: Possible SI values in each gap. Supergroup: $C2/c$ (15.85), WP: $4d$+$4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
2	0000, 0002, 0011, 0013, 1100 1102, 1111, 1113
3	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
4	0000

33.2 WP: $4c + 4d$

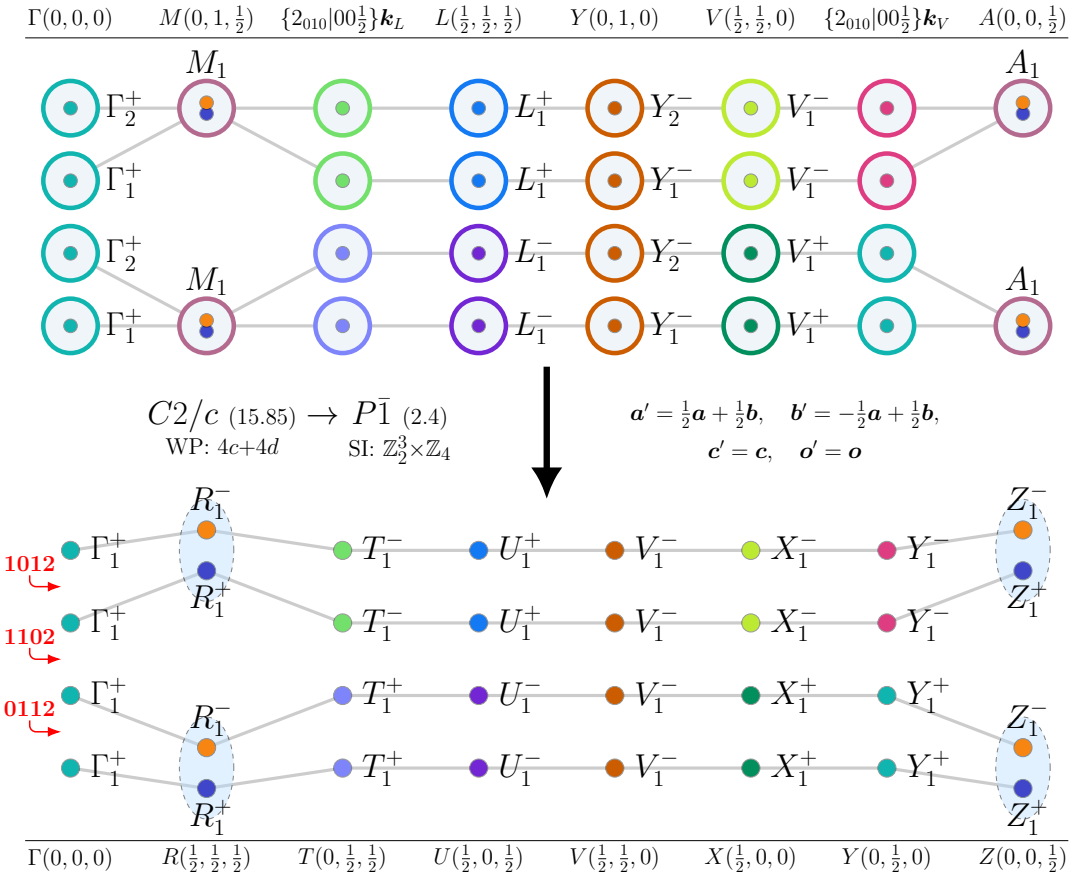
BCS Materials: Sr_2CoOsO_6 (108 K)[154].

33.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[154]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.210`



Supplementary Figure 180: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4c+4d$ of supergroup $C2/c$ (15.85).

Supplementary Table 349: Number of times a given SI can appear in the band structure. Supergroup: $C2/c$ (15.85), WP: $4c$+$4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1
1102	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 350: Possible SI values in each gap. Supergroup: $C2/c$ (15.85), WP: $4c$+$4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0110, 0111, 0112, 0113, 1010 1011, 1012, 1013
2	0000, 0002, 1100, 1102
3	0110, 0111, 0112, 0113, 1010 1011, 1012, 1013

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Gap#	Possible SI Values
4	0000

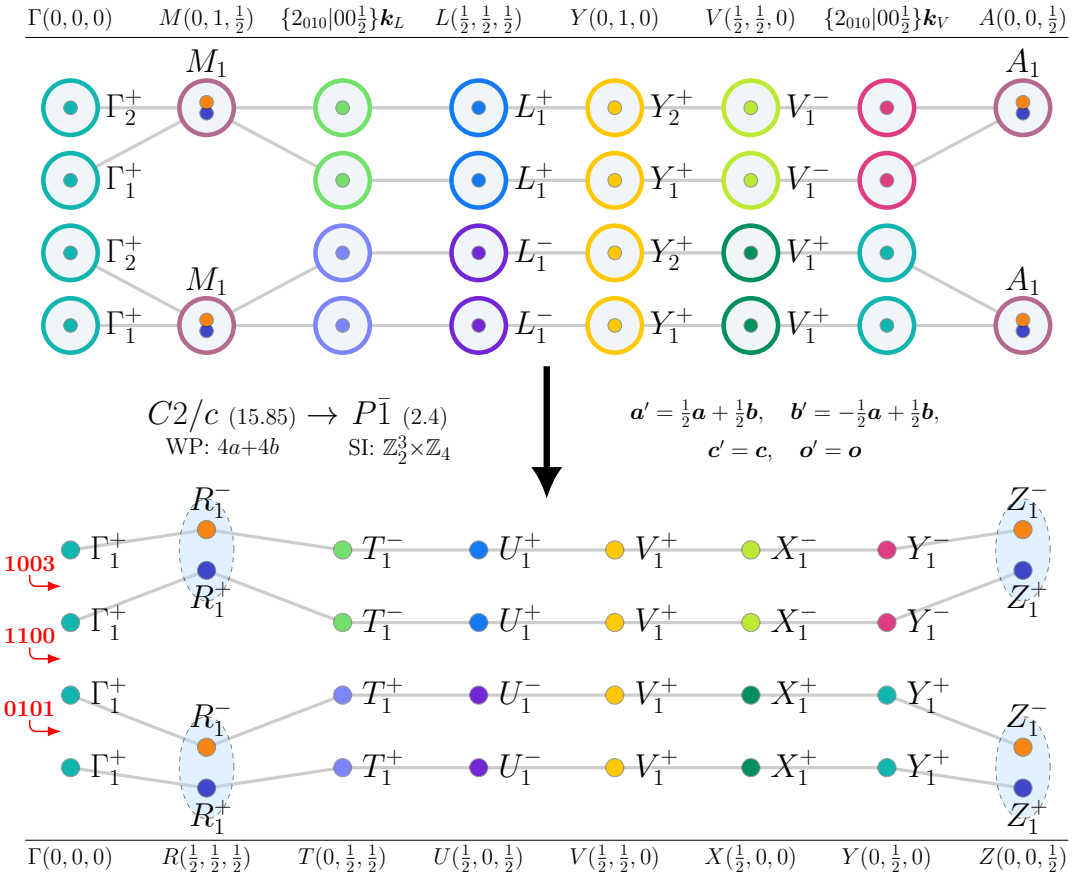
33.3 WP: $4a + 4b$

BCS Materials: Cr_2F_5 (40 K)[155].

33.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.



Supplementary Figure 181: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a + 4b$ of supergroup $C2/c$ (15.85).

Supplementary Table 351: Number of times a given SI can appear in the band structure. Supergroup: $C2/c$ (15.85), WP: $4a$+$4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2

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[155]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.576`

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SI	Possible #Gaps
0103	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1100	0, 1
1102	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 352: Possible SI values in each gap. Supergroup: $C2/c$ (15.85), WP: $4a$+$4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 1000 1001, 1002, 1003
2	0000, 0002, 1100, 1102
3	0100, 0101, 0102, 0103, 1000 1001, 1002, 1003
4	0000

33.4 WP: $4c$

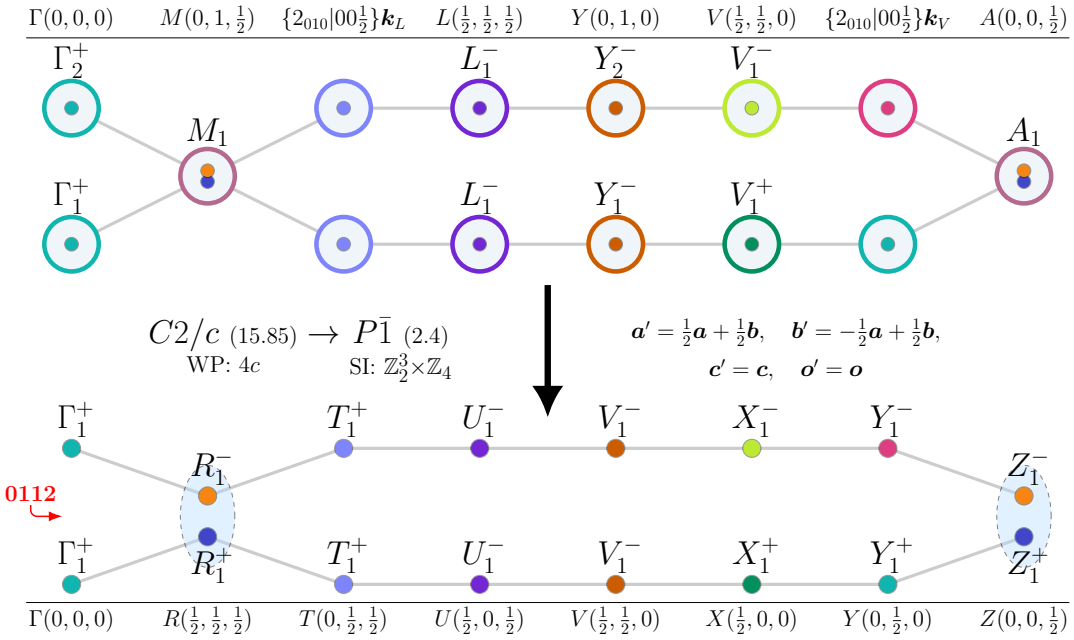
BCS Materials: $MnCO_3$ (32 K)[156], $NiCO_3$ (30 K)[157], $CoCO_3$ (18 K)[158].

33.4.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[156]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.115
[157]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.113
[158]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.114



Supplementary Figure 182: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $C2/c$ (15.85).

Supplementary Table 353: Number of times a given SI can appear in the band structure. Supergroup: $C2/c$ (15.85), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0111	0, 1
0112	0, 1
0113	0, 1
1010	0, 1
1011	0, 1
1013	0, 1
gapless	0
nontrivial	1

Supplementary Table 354: Possible SI values in each gap. Supergroup: $C2/c$ (15.85), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0111, 0112, 0113, 1010, 1011 1013
2	0000

34 MSG $C2'/c'$ (15.89)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4).

Trivial-SI Subgroups: Cc' (9.39), $C2'$ (5.15).

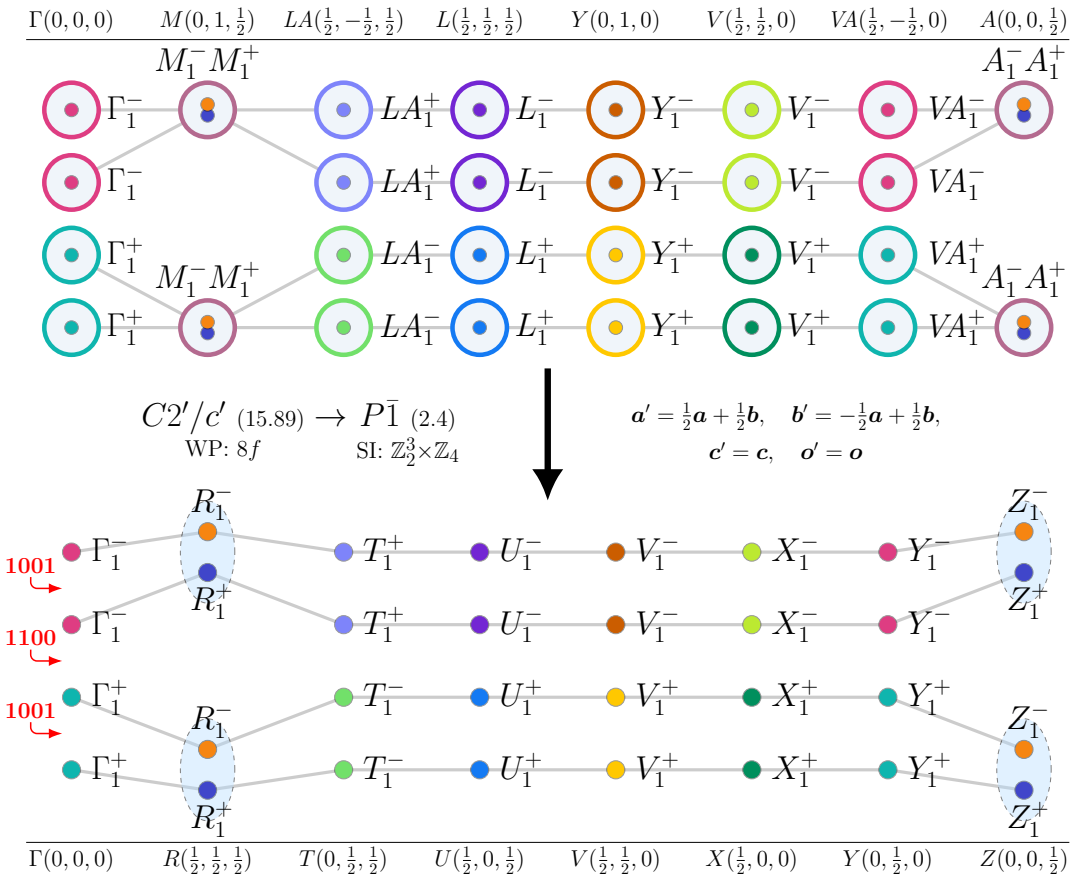
34.1 WP: $8f$

BCS Materials: α-Fe_2O_3 (955 K)[159], $Ho(Co_{0.667}Ga_{0.333})_2$ (31 K)[160].

34.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.



Supplementary Figure 183: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8f$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 355: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $8f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0011	0, 1
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2

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[159]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.65`
[160]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.493`

...continued

SI	Possible #Gaps
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1
1102	0, 1
1111	0, 1
1113	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 356: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $8f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
2	0000, 0002, 0011, 0013, 1100 1102, 1111, 1113
3	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
4	0000

34.2 WP: $4d$

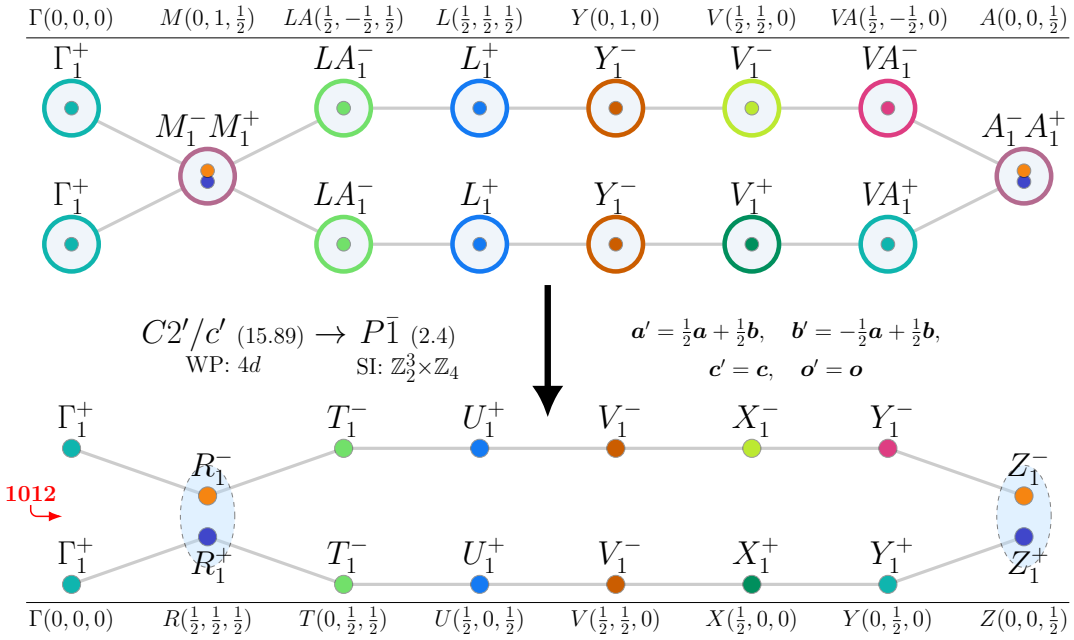
BCS Materials: FeF_3 (394 K)[161], Ca_3LiRuO_6 (117 K)[162], $FeSO_4F$ (100 K)[163], Sr_3LiRuO_6 (90 K)[164], Sr_3NaRuO_6 (70 K)[165], HoP (5.5 K)[166], FeF_3[167].

34.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[161] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.581
[162] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.239
[163] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.128
[164] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.361
[165] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.404
[166] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.10
[167] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.335



Supplementary Figure 184: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4d$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 357: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0110	0, 1
0111	0, 1
0113	0, 1
1011	0, 1
1012	0, 1
1013	0, 1
gapless	0
nontrivial	1

Supplementary Table 358: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0110, 0111, 0113, 1011, 1012 1013
2	0000

34.3 WP: $4c$

BCS Materials: $FeBO_3$ (348 K)[168], $FeOHSO_4$ (125 K)[169], Ca_3LiOsO_6 (117.1 K)[170].

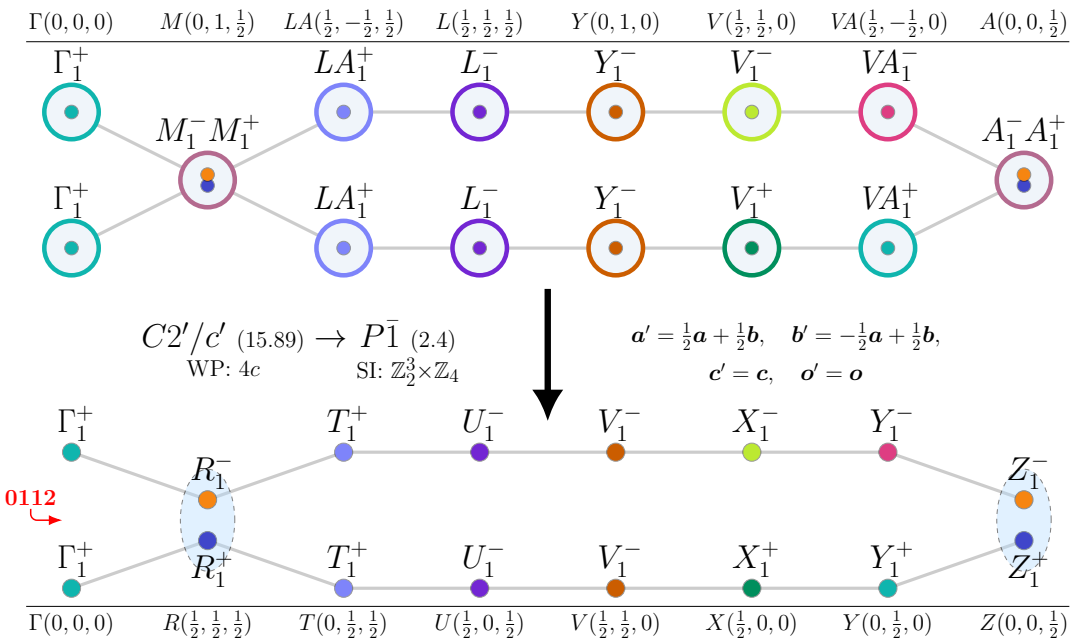
34.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,

[168]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.112`
[169]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.760`
[170]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.3`

- B in generic direction.



Supplementary Figure 185: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 359: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0111	0, 1
0112	0, 1
0113	0, 1
1010	0, 1
1011	0, 1
1013	0, 1
gapless	0
nontrivial	1

Supplementary Table 360: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0111, 0112, 0113, 1010, 1011 1013
2	0000

34.4 WP: $4a$

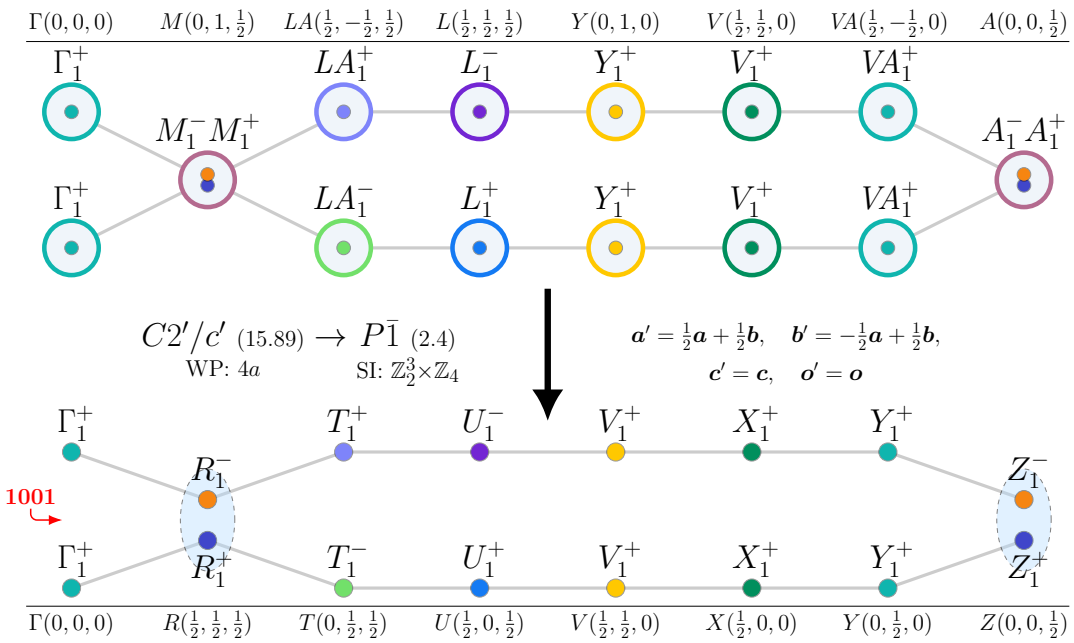
BCS Materials: GdMg (110 K)[171].

34.4.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

[171] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.70`

- strain in generic direction,
- B in generic direction.



Supplementary Figure 186: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 361: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0101	0, 1
0102	0, 1
0103	0, 1
1001	0, 1
1002	0, 1
1003	0, 1
gapless	0
nontrivial	1

Supplementary Table 362: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0101, 0102, 0103, 1001, 1002 1003
2	0000

34.5 WP: $4b+4d+4e$

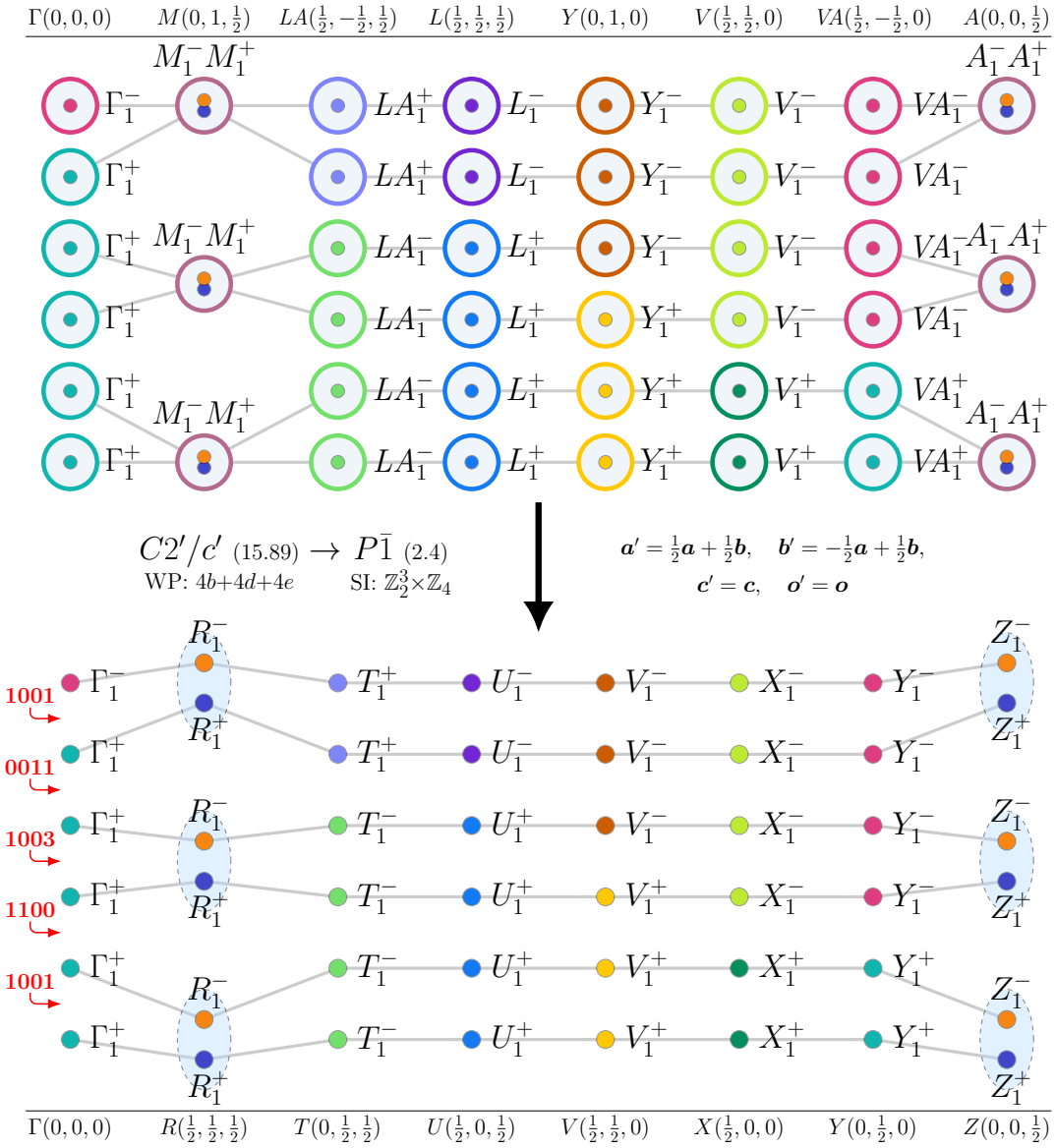
BCS Materials: $NdCo_2$ (42 K)[172].

[172] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.226`

34.5.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.



Supplementary Figure 187: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b + 4d + 4e$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 363: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: 4*b*+4*d*+4*e*, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0002	0, 1, 2
0011	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2, 3
0102	0, 1, 2, 3
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3

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SI	Possible #Gaps
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1102	0, 1, 2
1111	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	3, 4, 5

Supplementary Table 364: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4b$+$4d$+$4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
2	0000, 0002, 0011, 0013, 1100 1102, 1111, 1113
3	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
4	0000, 0002, 0011, 0013, 1100 1102, 1111, 1113
5	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
6	0000

34.6 WP: $4a + 4e$

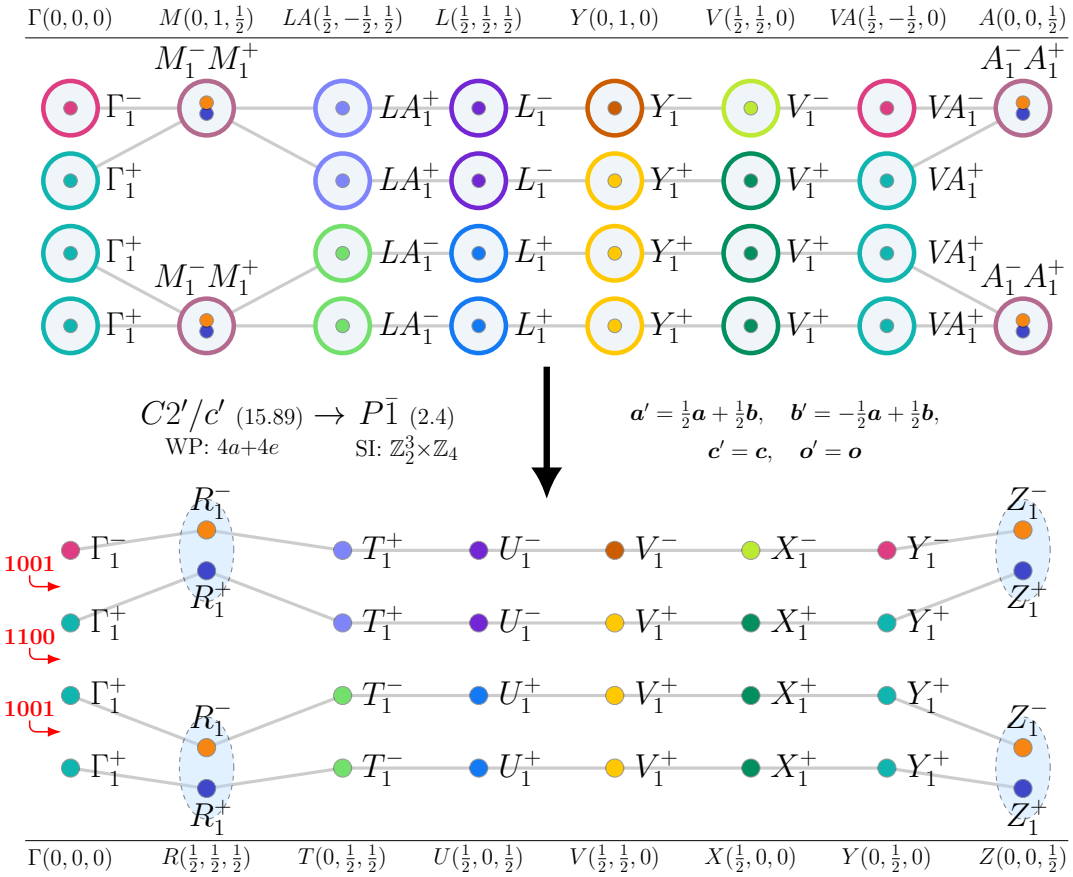
BCS Materials: $(NH_2(CH_3)_2)(FeMn(HCOO)_6)$ (35 K)[173], $(NH_2(CH_3)_2)(FeCo(HCOO)_6)$ (32 K)[174].

34.6.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[173]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.251`
[174]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.250`



Supplementary Figure 188: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a+4e$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 365: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4a$+$4e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0011	0, 1
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1
1102	0, 1
1111	0, 1
1113	0, 1
gapless	0

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SI	Possible #Gaps
nontrivial	2, 3

Supplementary Table 366: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4a$+$4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
2	0000, 0002, 0011, 0013, 1100 1102, 1111, 1113
3	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
4	0000

34.7 WP: $4a + 4c$

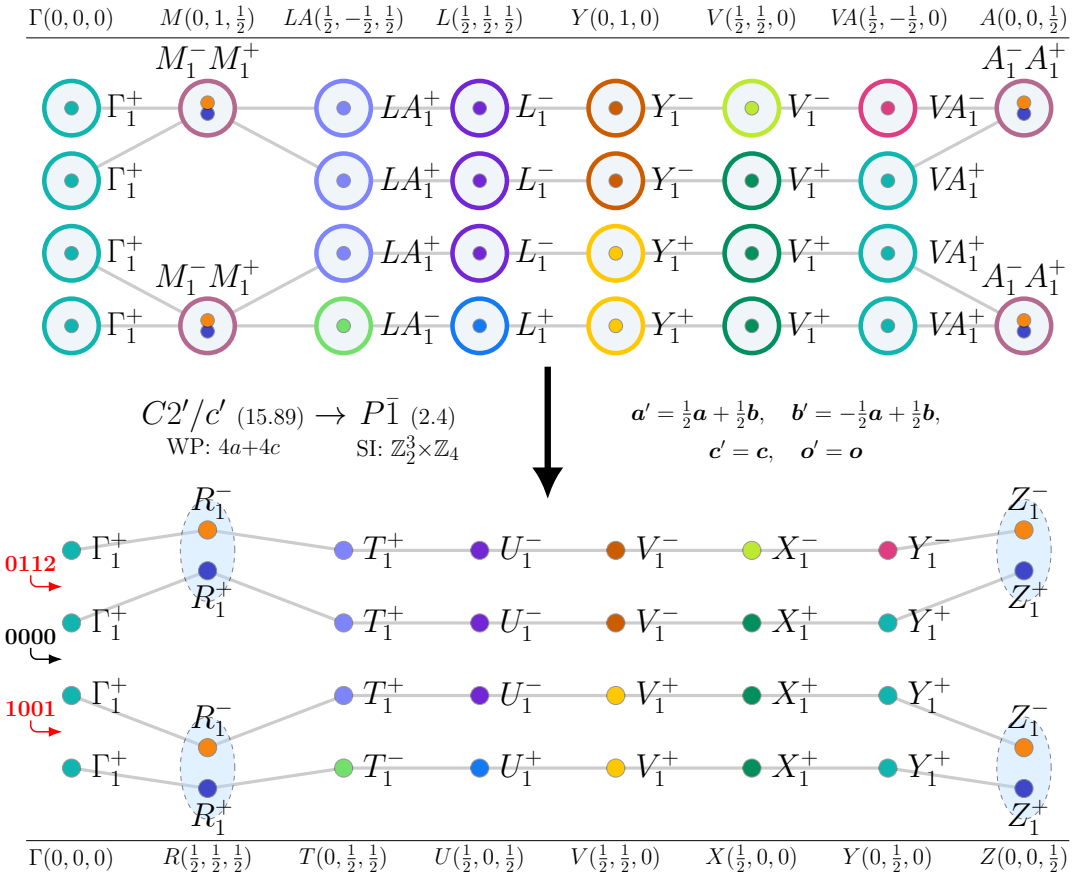
BCS Materials: $Fe_2F_5(H_2O)_2$ (26 K)[175].

34.7.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[175] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.584`



Supplementary Figure 189: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a+4c$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 367: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4a$+$4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0011	0, 1
0013	0, 1
0100	0, 1
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1
1013	0, 1, 2
1100	0, 1
1102	0, 1
1111	0, 1
1113	0, 1
gapless	0

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SI	Possible #Gaps
nontrivial	2, 3

Supplementary Table 368: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4a$+$4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
2	0000, 0002, 0011, 0013, 1100 1102, 1111, 1113
3	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
4	0000

34.8 WP: $4e$

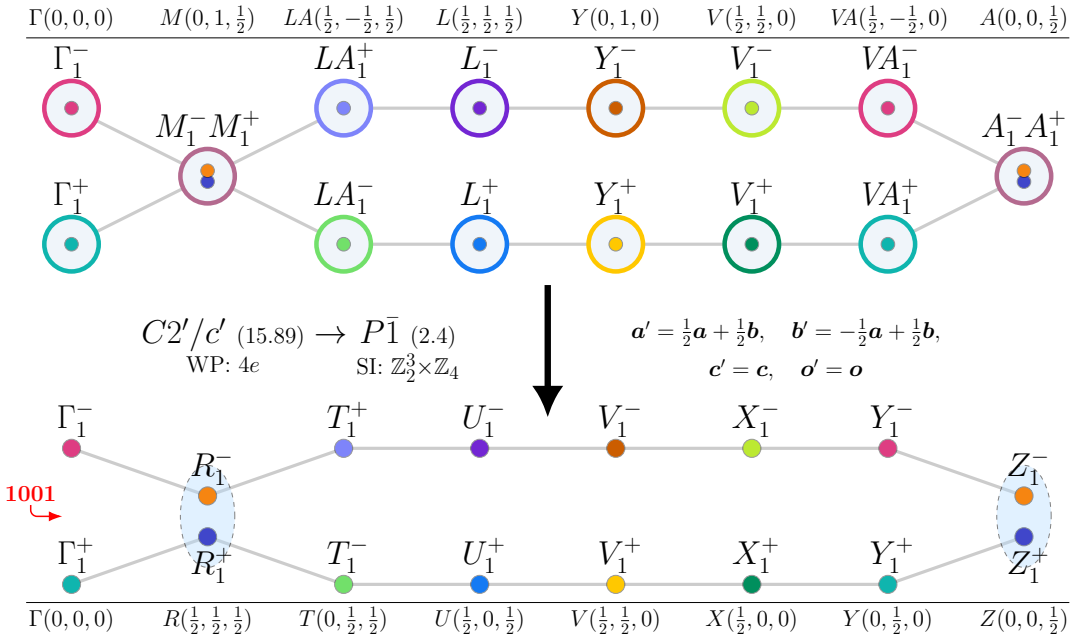
BCS Materials: NdPt (23 K)[176], $NaCrGe_2O_6$ (6 K)[177].

34.8.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[176]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.690`
[177]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.297`



Supplementary Figure 190: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4e$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 369: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0100	0, 1
0101	0, 1
0102	0, 1
0103	0, 1
0110	0, 1
0111	0, 1
0112	0, 1
0113	0, 1
1000	0, 1
1001	0, 1
1002	0, 1
1003	0, 1
1010	0, 1
1011	0, 1
1012	0, 1
1013	0, 1
gapless	0
nontrivial	1

Supplementary Table 370: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
2	0000

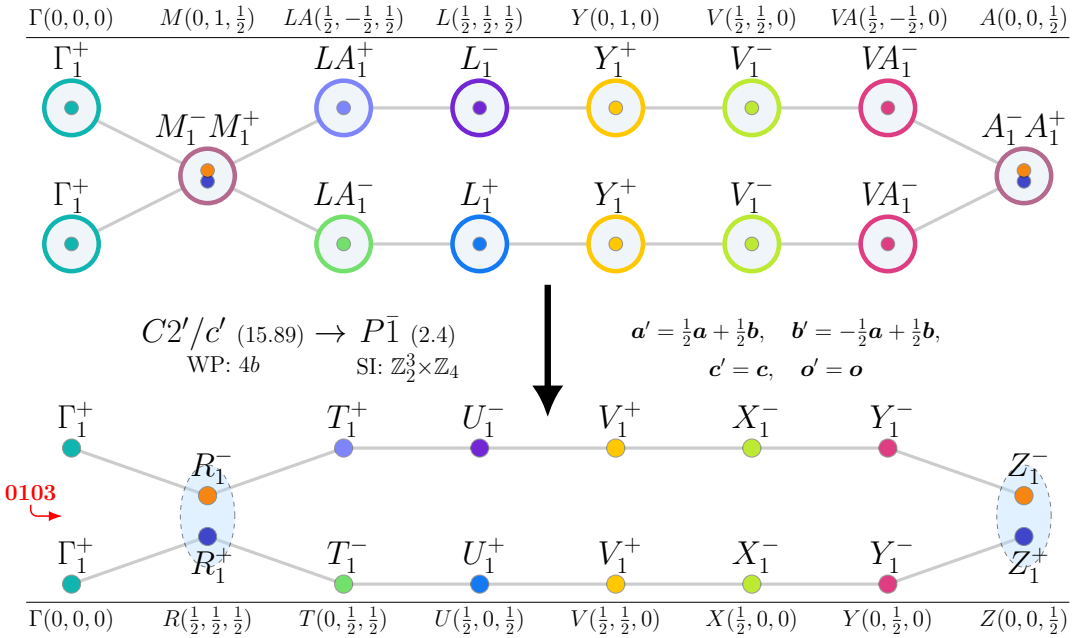
34.9 WP: $4b$

BCS Materials: $Na_2BaFe(VO_4)_2$ (7 K)[178].

34.9.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.



Supplementary Figure 191: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $C2'/c'$ (15.89).

Supplementary Table 371: Number of times a given SI can appear in the band structure. Supergroup: $C2'/c'$ (15.89), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0100	0, 1
0101	0, 1
0103	0, 1
1000	0, 1
1001	0, 1
1003	0, 1
gapless	0
nontrivial	1

Supplementary Table 372: Possible SI values in each gap. Supergroup: $C2'/c'$ (15.89), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0100, 0101, 0103, 1000, 1001 1003
2	0000

[178] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.298`

35 MSG $R_I\bar{3}m$ (166.102)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), $C2/m$ (12.58), C_c2/m (12.63), $C2/m$ (12.58), C_c2/m (12.63), $R\bar{3}c'$ (167.107).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Cm (8.32), C_cm (8.35), Cm (8.32), C_cm (8.35), $C2$ (5.13), C_c2 (5.16), $C2$ (5.13), C_c2 (5.16), $R3c'$ (161.71), R_I3m (160.68).

35.1 WP: $18e$

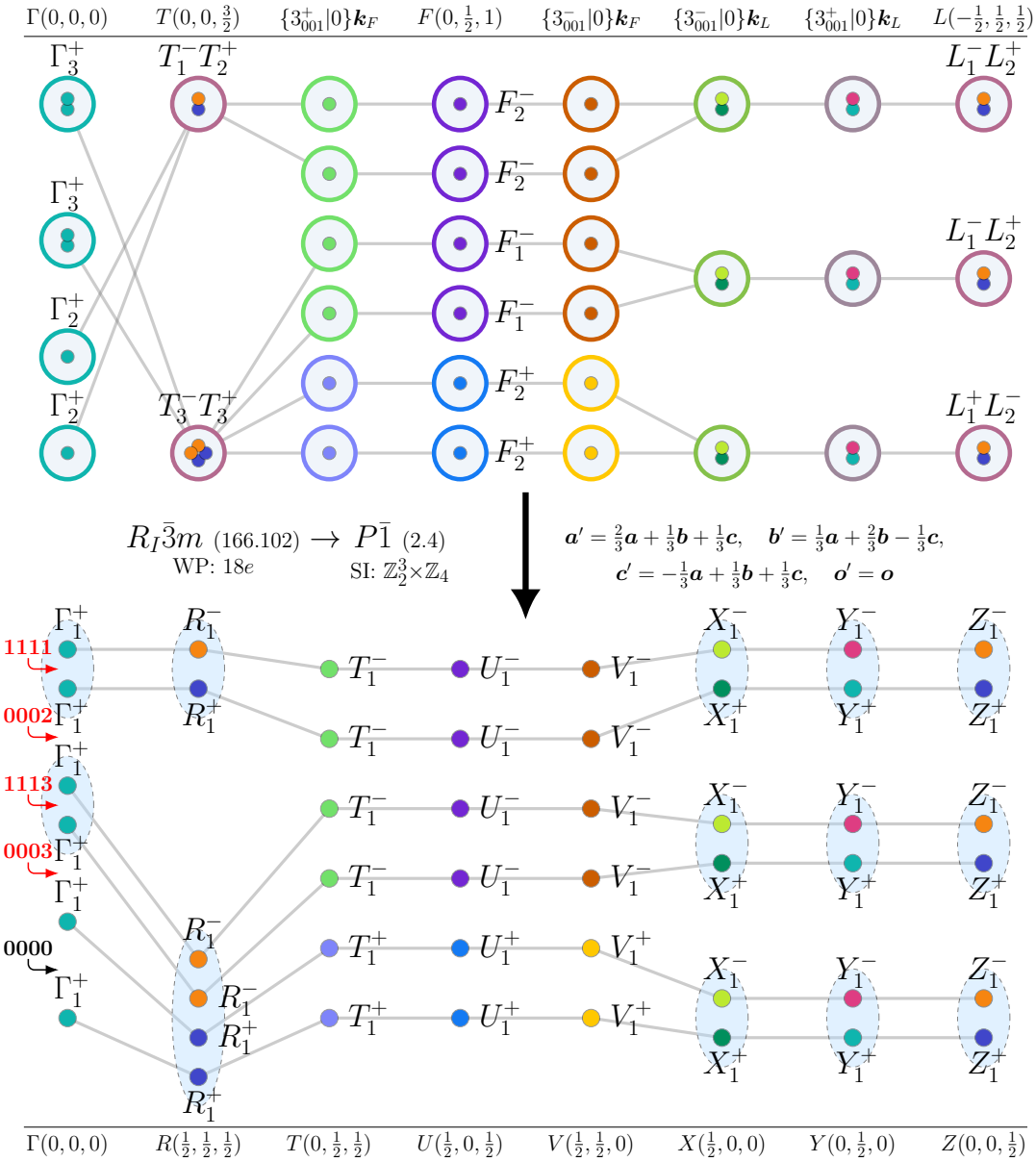
BCS Materials: $Gd_2Ti_2O_7$ (0.97 K)[179].

35.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and (strain ∥ [110] or strain ⊥ [110]),
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and (strain ∥ [100] or strain ⊥ [100]),
- B ∥ [110] and strain in generic direction,
- B in generic direction,
- B ⊥ [100] and (strain ∥ [110] or strain ⊥ [110]),
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and (strain ∥ [100] or strain ⊥ [100]),
- B ⊥ [110] and strain in generic direction.

[179] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.56`



Supplementary Figure 192: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}m$ (166.102).

Supplementary Table 373: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2
0102	0, 1, 2, 3
0103	0, 1, 2
0110	0, 1
0111	0, 1, 2, 3
0112	0, 1, 2

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SI	Possible #Gaps
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2
1002	0, 1, 2, 3
1003	0, 1, 2
1010	0, 1
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3, 4
1111	0, 1, 2, 3
1112	0, 1, 2, 3, 4
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

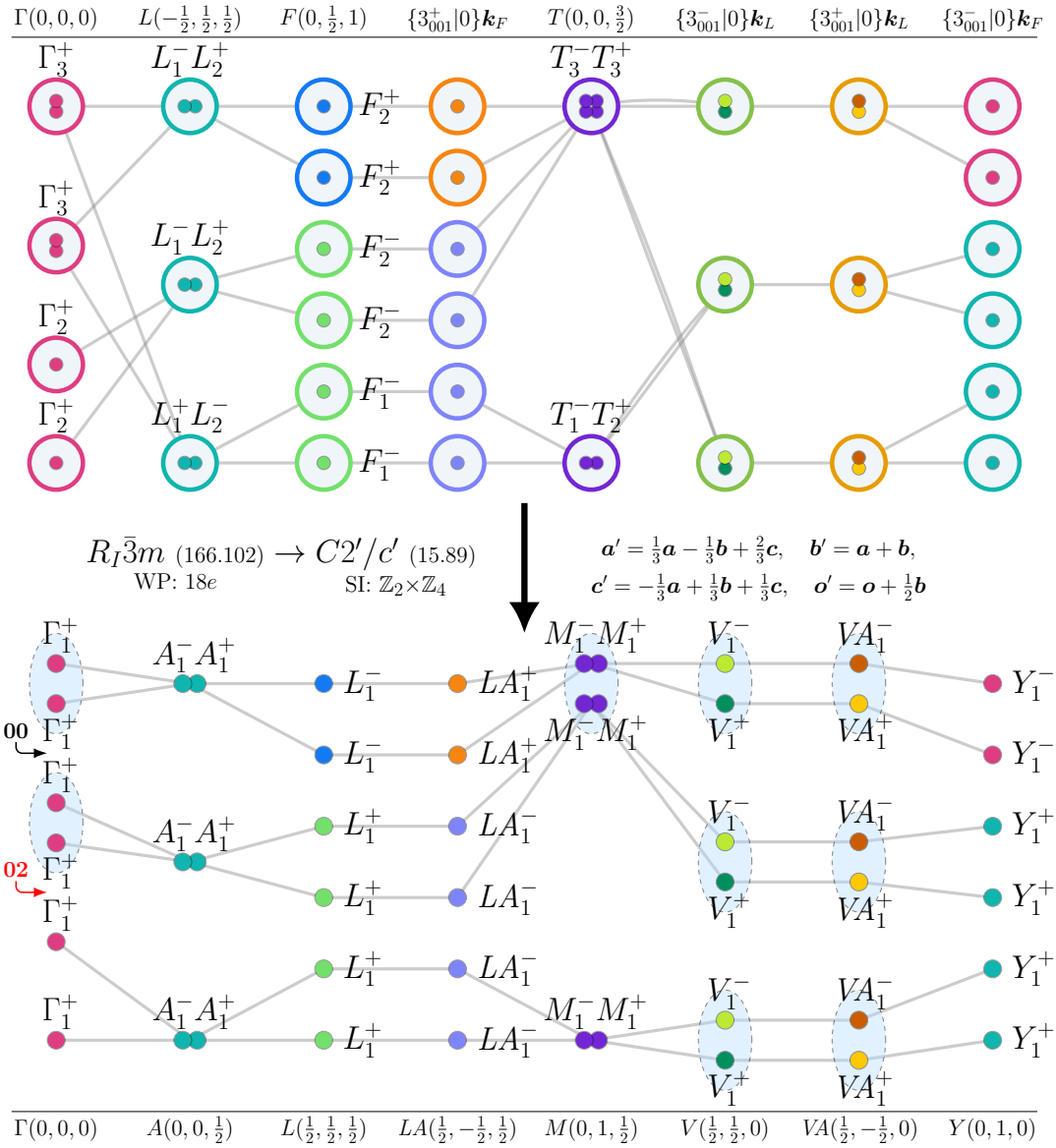
Supplementary Table 374: Possible SI values in each gap. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0111 0112, 0113, 1000, 1001, 1002 1011, 1012, 1013, 1101, 1102 1103, 1110, 1112, 1113
2	0000, 0001, 0002, 0003, 1110 1112, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1103, 1110 1111, 1112, 1113
4	0000, 0001, 0002, 0003, 1110 1111, 1112
5	0000, 0001, 0003, 0010, 0012 0013, 0100, 0102, 0103, 0111 0112, 0113, 1000, 1002, 1003 1011, 1012, 1013, 1101, 1102 1103, 1110, 1111, 1112
6	0000

35.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [001] and (strain ∥ [110] or strain ⊥ [110]),
- B ⊥ [110].



Supplementary Figure 193: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}m$ (166.102).

Supplementary Table 375: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1, 2
11	0, 1
13	0, 1
gapless	3
nontrivial	1, 2

Supplementary Table 376: Possible SI values in each gap. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 13
3	G

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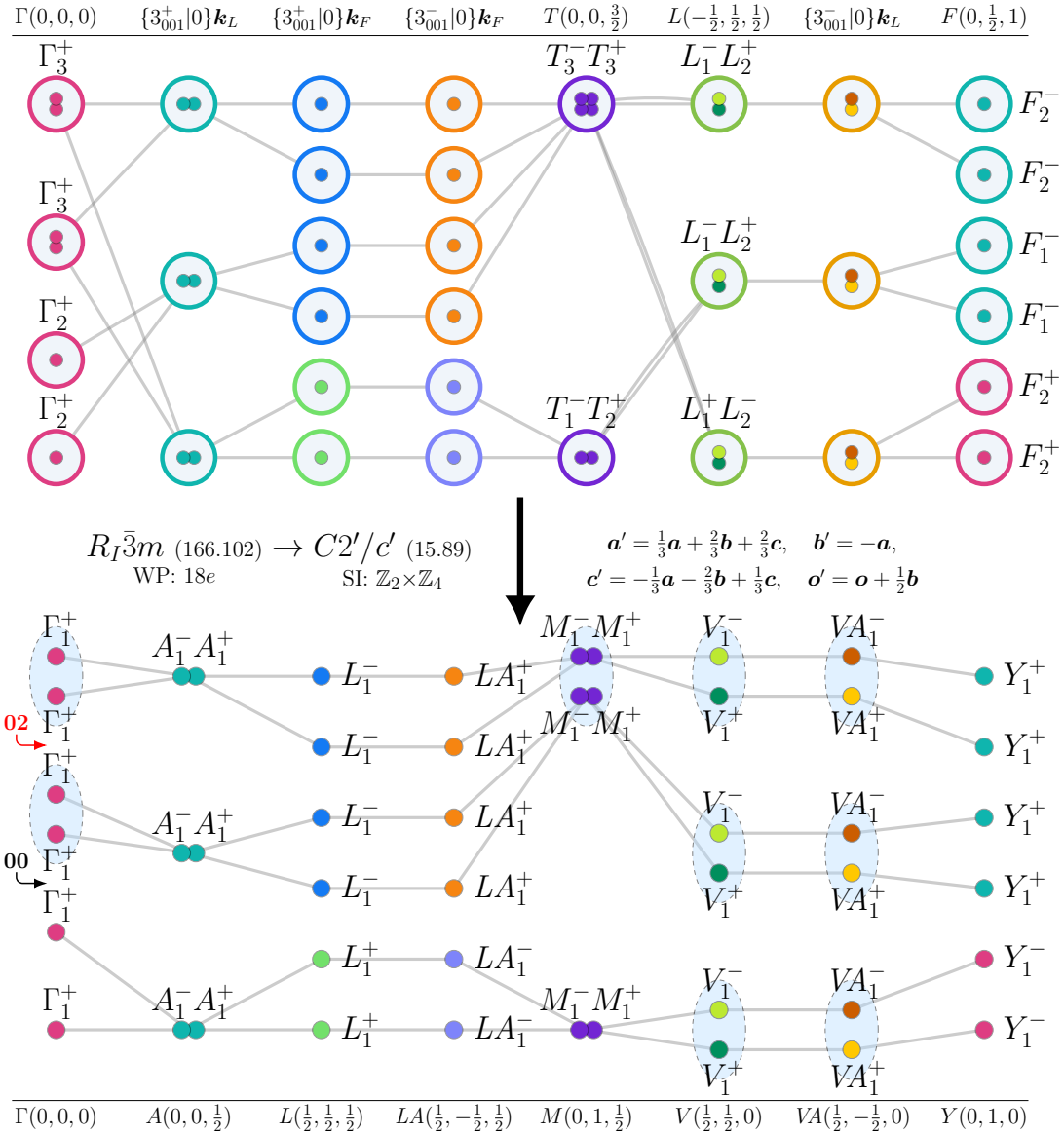
...continued

Gap#	Possible SI Values
4	00, 02, 11
5	G
6	00

35.1.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [001] and (strain ∥ [100] or strain ⊥ [100]),
- B ⊥ [100].



Supplementary Figure 194: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 18e of supergroup $R_I\bar{3}m$ (166.102).

Supplementary Table 377: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1, 2
11	0, 1
13	0, 1
gapless	3
nontrivial	1, 2

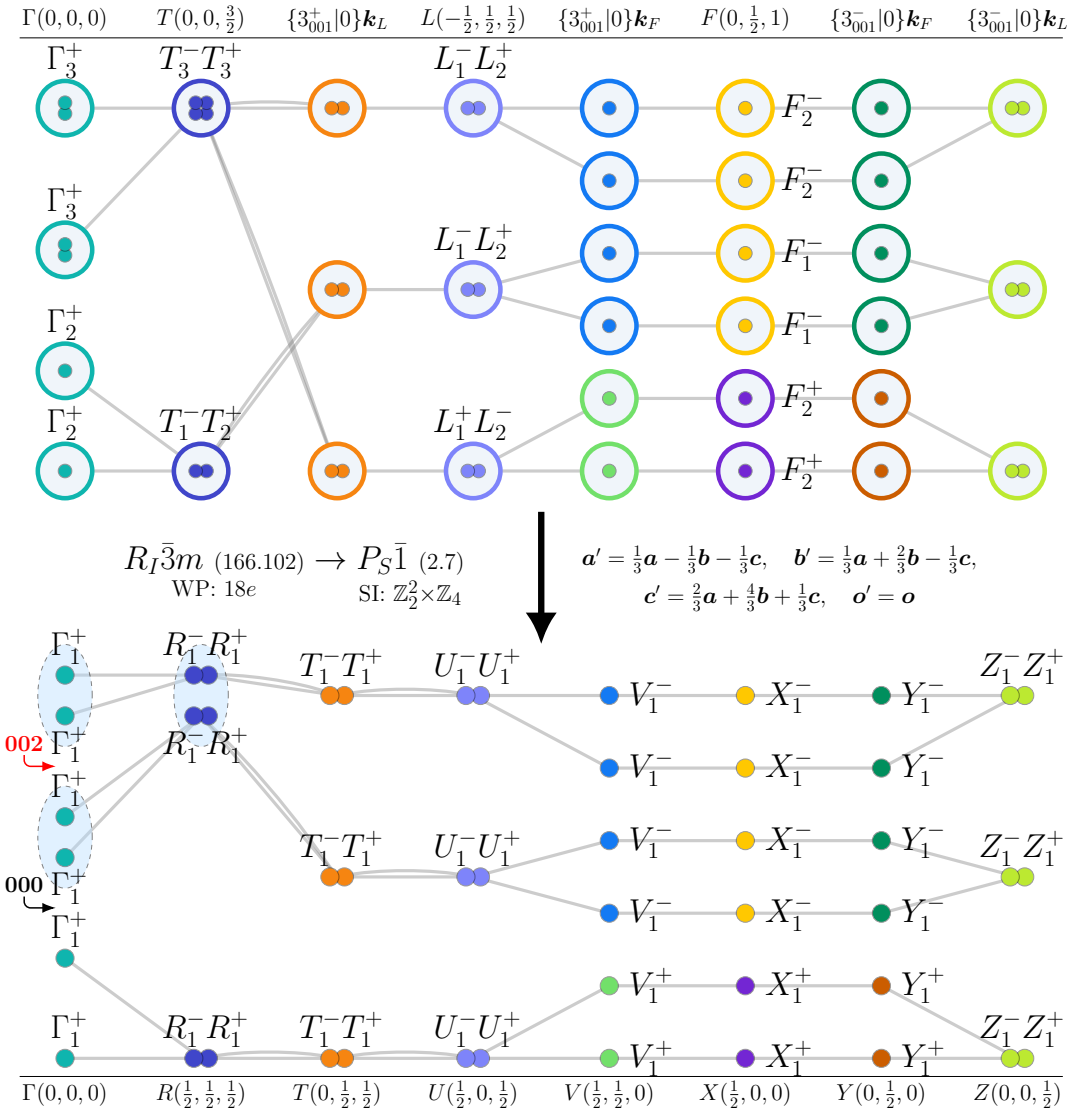
Supplementary Table 378: Possible SI values in each gap. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 13
3	G
4	00, 02, 11
5	G
6	00

35.1.4 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 195: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}m$ (166.102).

Supplementary Table 379: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1, 2
111	0, 1
113	0, 1
gapless	3
nontrivial	1, 2

Supplementary Table 380: Possible SI values in each gap. Supergroup: $R_I\bar{3}m$ (166.102), WP: $18e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 002, 111
3	G
4	000, 002, 113

continued on next page...

...continued

Gap#	Possible SI Values
5	G
6	000

36 MSG $R\bar{3}m$ (166.97)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2/m$ (12.58), $C2/m$ (12.58), $R\bar{3}$ (148.17).

Trivial-SI Subgroups: Cm (8.32), Cm (8.32), $C2$ (5.13), $C2$ (5.13), $R3$ (146.10), $R3m$ (160.65).

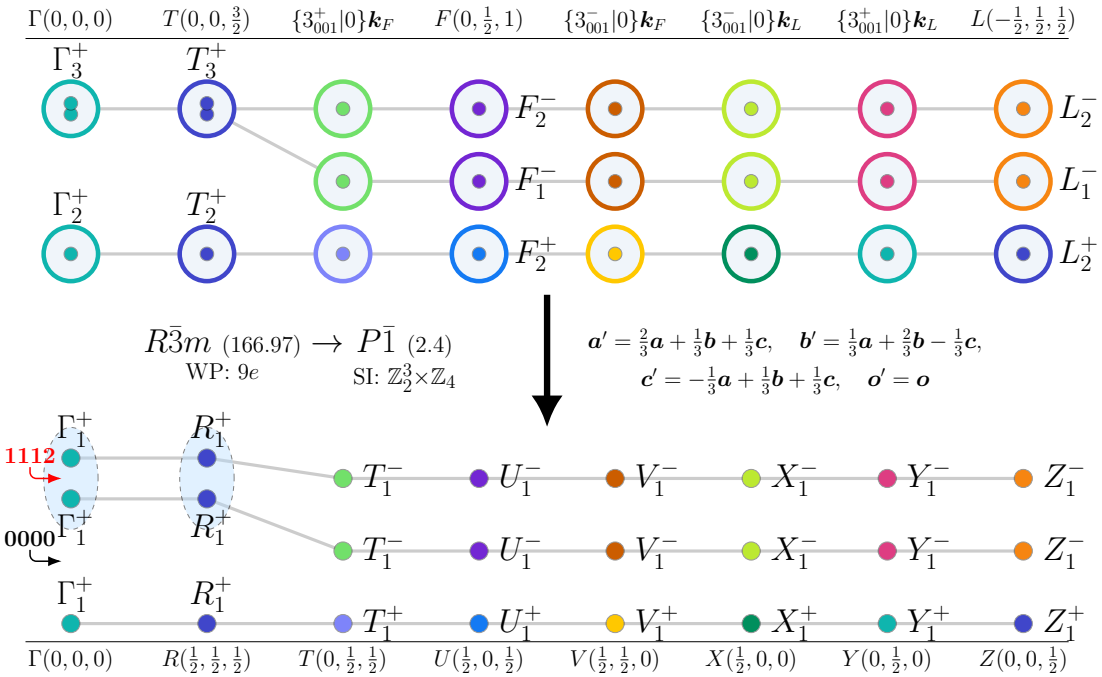
36.1 WP: $9e$

BCS Materials: $Mn_3Cu_{0.5}Ge_{0.5}N$ (380 K)[180], Mn_3GaN (298 K)[181], Mn_3ZnN (183 K)[182].

36.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [001] and (strain ∥ [100] or strain ⊥ [100]),
- B ∥ [001] and (strain ∥ [110] or strain ⊥ [110]),
- B ∥ [100] and (strain ∥ [110] or strain ⊥ [110]),
- B ∥ [110] and (strain ∥ [100] or strain ⊥ [100]),
- B in generic direction.



Supplementary Figure 196: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $9e$ of supergroup $R\bar{3}m$ (166.97).

[180]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.74
[181]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.177
[182]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.273

Supplementary Table 381: Number of times a given SI can appear in the band structure. Supergroup: $R\bar{3}m$ (166.97), WP: $9e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0001	0, 1
0003	0, 1
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2

Supplementary Table 382: Possible SI values in each gap. Supergroup: $R\bar{3}m$ (166.97), WP: $9e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 1112, 1113
2	0000, 0001, 1111, 1112
3	0000

37 MSG $R_I\bar{3}c$ (167.108)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P_S\bar{1}$ (2.7), $C2/c$ (15.85), C_c2/c (15.90), $C2/c$ (15.85), C_c2/c (15.90), $R\bar{3}m'$ (166.101).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Cc (9.37), C_cc (9.40), Cc (9.37), C_cc (9.40), $C2$ (5.13), C_c2 (5.16), $C2$ (5.13), C_c2 (5.16), $R3m'$ (160.67), R_I3c (161.72).

37.1 WP: $18e$

BCS Materials: Mn_3GaC (164 K)[183], $KFe_3(OH)_6(SO_4)_2$ (64 K)[184], $NaFe_3(SO_4)_2(OH)_6$ (50 K)[185], $AgFe_3(SO_4)_2(OD)_6$[186].

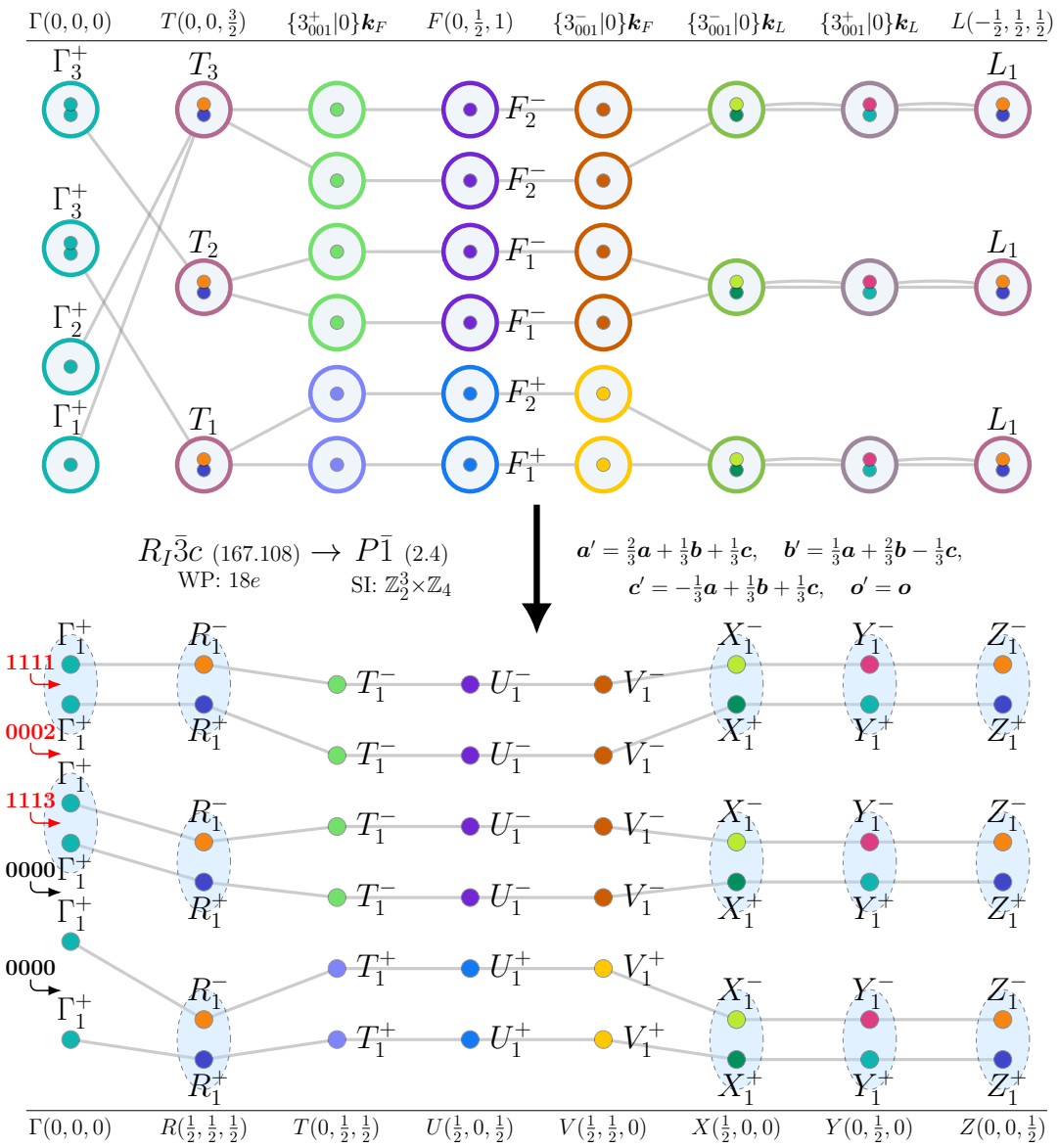
37.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and (strain ∥ [110] or strain ⊥ [110]),
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and (strain ∥ [100] or strain ⊥ [100]),
- B ∥ [110] and strain in generic direction,
- B in generic direction,
- B ⊥ [100] and (strain ∥ [110] or strain ⊥ [110]),
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and (strain ∥ [100] or strain ⊥ [100]),

[183]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.153`
[184]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.25`
[185]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.441`
[186]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.129`

- B $\perp$ [110] and strain in generic direction.



Supplementary Figure 197: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position 18e of supergroup $R_I\bar{3}c$ (167.108).

Supplementary Table 383: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}c$ (167.108), WP: 18e, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2
0102	0, 1, 2, 3
0103	0, 1, 2
0110	0, 1
0111	0, 1, 2, 3

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SI	Possible #Gaps
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2
1002	0, 1, 2, 3
1003	0, 1, 2
1010	0, 1
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

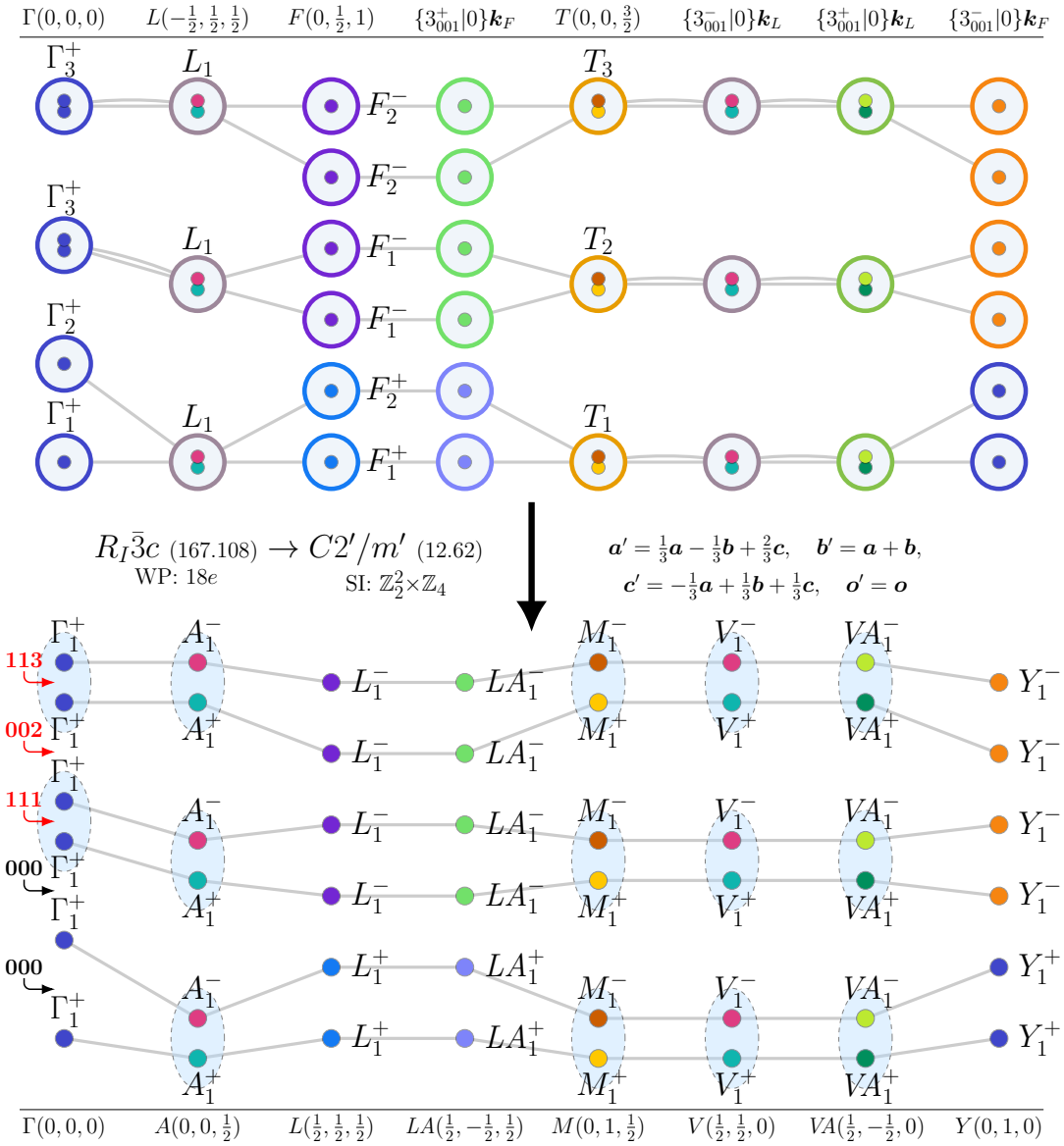
Supplementary Table 384: Possible SI values in each gap. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0111 0112, 0113, 1000, 1001, 1002 1011, 1012, 1013, 1101, 1102 1103, 1110, 1112, 1113
2	0000, 0002, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1103, 1110 1111, 1112, 1113
4	0000, 0002, 1111
5	0000, 0001, 0003, 0010, 0012 0013, 0100, 0102, 0103, 0111 0112, 0113, 1000, 1002, 1003 1011, 1012, 1013, 1101, 1102 1103, 1110, 1111, 1112
6	0000

37.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [001] and (strain ∥ [110] or strain ⊥ [110]),
- B ⊥ [110].



Supplementary Figure 198: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}c$ (167.108).

Supplementary Table 385: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
010	0, 1, 2, 3
011	0, 1, 2
012	0, 1, 2, 3
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3
102	0, 1
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3

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SI	Possible #Gaps
113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

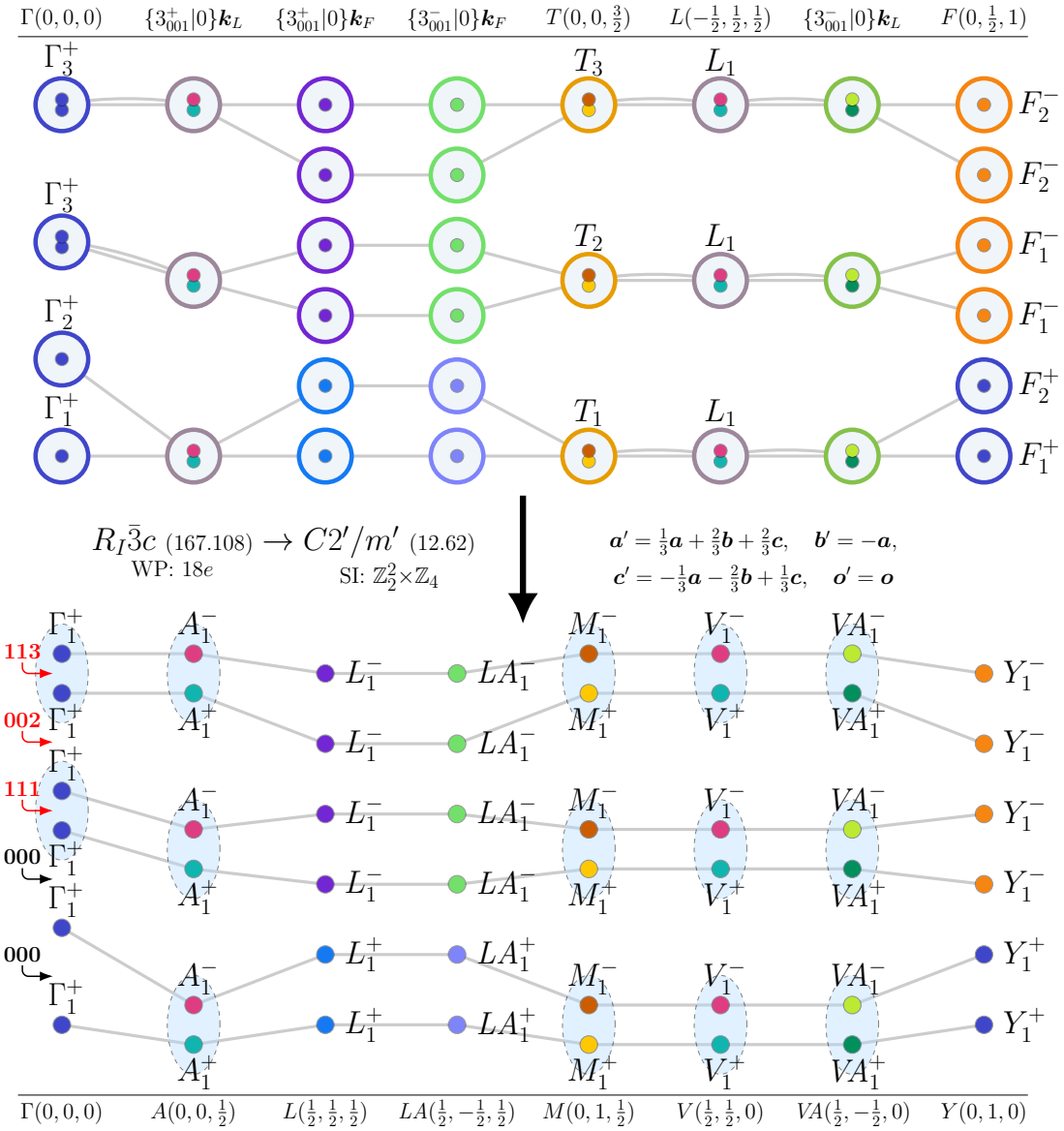
Supplementary Table 386: Possible SI values in each gap. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	000, 002, 111
3	001, 002, 003, 010, 011 012, 013, 101, 102, 103 110, 111, 112, 113
4	000, 002, 113
5	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
6	000

37.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and (strain $\parallel$ [100] or strain $\perp$ [100]),
- B $\perp$ [100].



Supplementary Figure 199: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}c$ (167.108).

Supplementary Table 387: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
010	0, 1, 2, 3
011	0, 1, 2
012	0, 1, 2, 3
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3
102	0, 1
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3

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SI	Possible #Gaps
113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

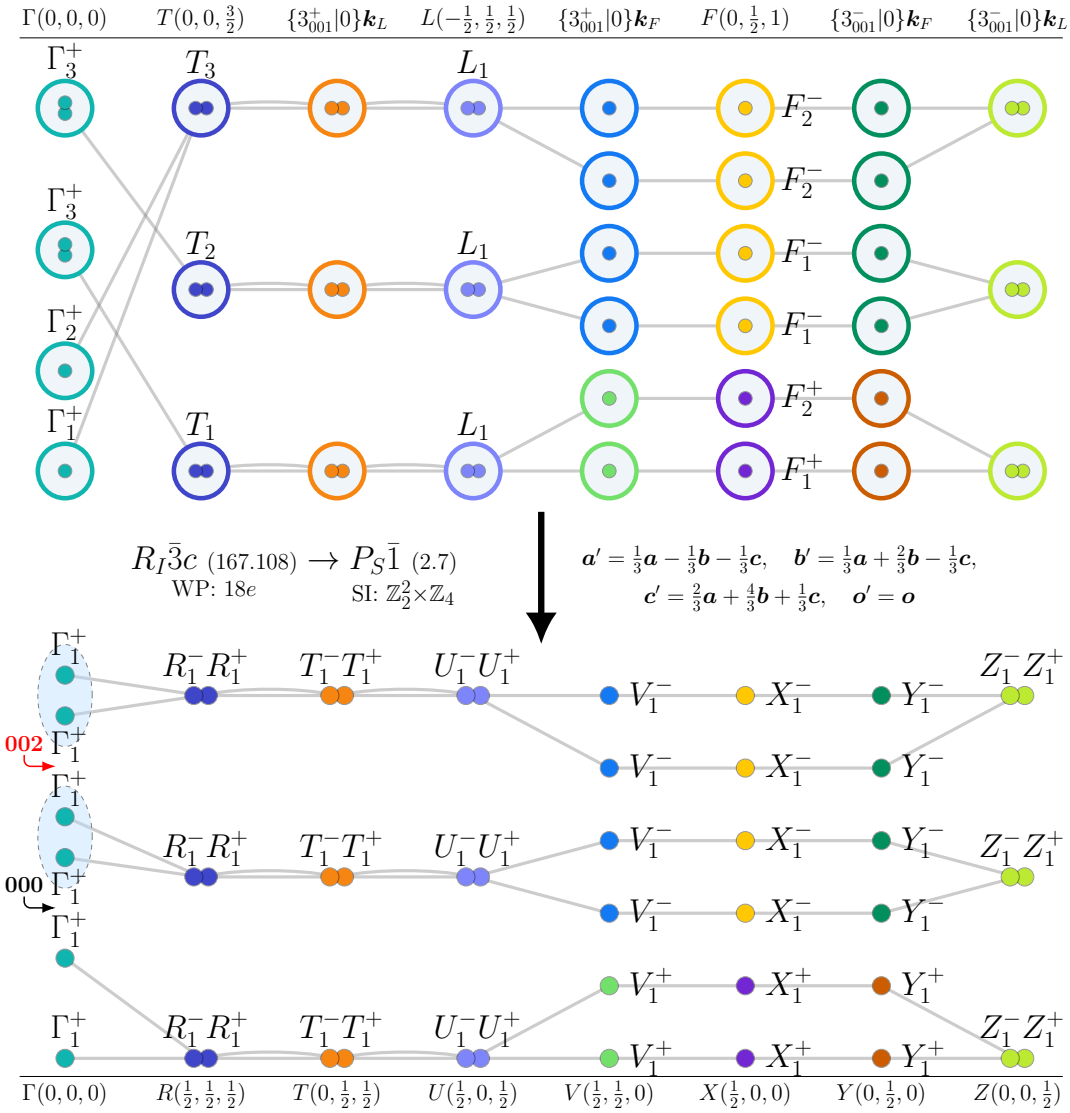
Supplementary Table 388: Possible SI values in each gap. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	000, 002, 111
3	001, 002, 003, 010, 011 012, 013, 101, 102, 103 110, 111, 112, 113
4	000, 002, 113
5	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
6	000

37.1.4 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 200: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $18e$ of supergroup $R_I\bar{3}c$ (167.108).

Supplementary Table 389: Number of times a given SI can appear in the band structure. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1, 2
111	0, 1
113	0, 1
gapless	3
nontrivial	1, 2

2779

Supplementary Table 390: Possible SI values in each gap. Supergroup: $R_I\bar{3}c$ (167.108), WP: $18e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 002, 111
3	G
4	000, 002, 113

continued on next page...

...continued

Gap#	Possible SI Values
5	G
6	000

38 MSG $P6_3/m$ (176.143)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1/m$ (11.50), $P6_3$ (173.129).

Trivial-SI Subgroups: Pm (6.18), $P2_1$ (4.7).

38.1 WP: $6g$

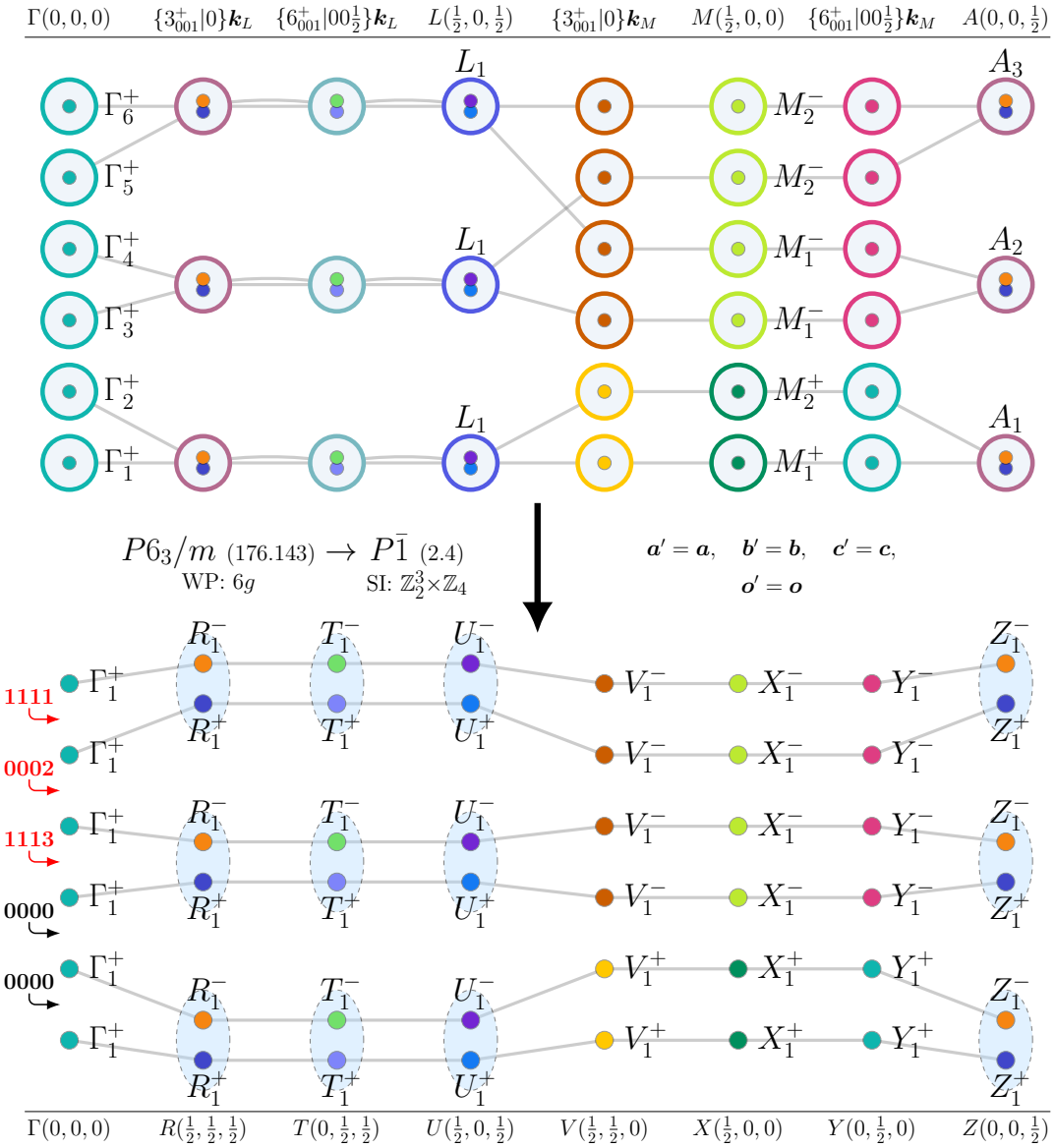
BCS Materials: FeF_3 (97 K)[187].

38.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[187] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.0.33`



Supplementary Figure 201: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $6g$ of supergroup $P6_3/m$ (176.143).

Supplementary Table 391: Number of times a given SI can appear in the band structure. Supergroup: $P6_3/m$ (176.143), WP: $6g$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
1000	0, 1
1001	0, 1, 2, 3
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2
1012	0, 1, 2, 3
1013	0, 1, 2
1100	0, 1
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

Supplementary Table 392: Possible SI values in each gap. Supergroup: $P6_3/m$ (176.143), WP: $6g$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0101, 0102, 0103, 0110 0111, 0112, 1001, 1002, 1003 1010, 1011, 1012, 1101, 1102 1103, 1110, 1112, 1113
2	0000, 0002, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0103 0110, 0111, 0112, 0113, 1000 1001, 1003, 1010, 1011, 1012 1013, 1100, 1101, 1103, 1110 1111, 1112, 1113
4	0000, 0002, 1111
5	0000, 0001, 0003, 0010, 0012 0013, 0101, 0102, 0103, 0110 0112, 0113, 1001, 1002, 1003 1010, 1012, 1013, 1101, 1102 1103, 1110, 1111, 1112
6	0000

39 MSG $P_B2_12_12$ (18.22)

Nontrivial-SI Subgroups: $P2$ (3.1), $P22'2'_1$ (17.10).

Trivial-SI Subgroups: $P2'_1$ (4.9), $P2'_1$ (4.9), $P2'$ (3.3), P_S1 (1.3), P_C2 (3.6), $P2_1$ (4.7), $P2_12'_12'$ (18.19), P_a2_1 (4.10), $P2_1$ (4.7), $P2'_12'_12_1$ (19.27), P_C2_1 (4.12).

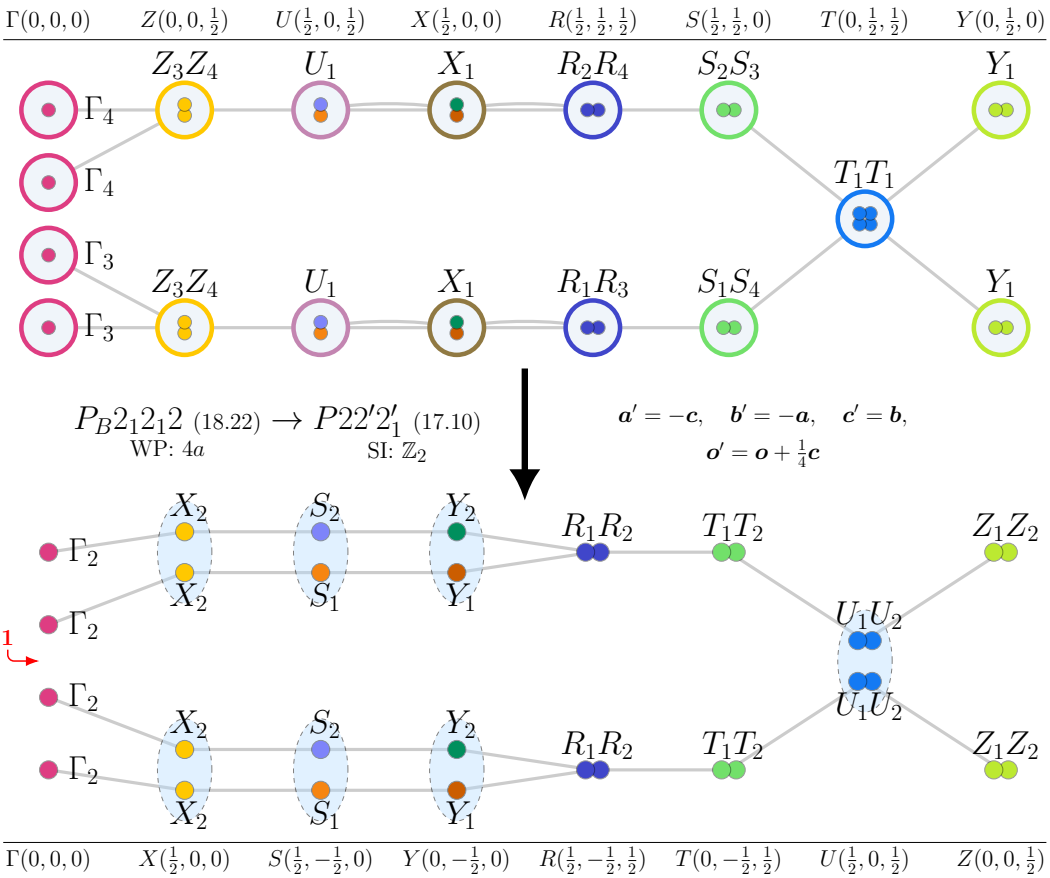
39.1 WP: $4a$

BCS Materials: $CoNb_3S_6$ (25 K)[188].

39.1.1 Topological bands in subgroup $P22'2'_1$ (17.10)

Perturbation:

- B ∥ [001].



Supplementary Figure 202: Topological magnon bands in subgroup $P22'2'_1$ (17.10) for magnetic moments on Wyckoff position $4a$ of supergroup $P_B2_12_12$ (18.22).

Supplementary Table 393: Number of times a given SI can appear in the band structure. Supergroup: $P_B2_12_12$ (18.22), WP: $4a$, Subgroup: $P22'2'_1$ (17.10).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 394: Possible SI values in each gap. Supergroup: $P_B2_12_12$ (18.22), WP: $4a$, Subgroup: $P22'2'_1$ (17.10).

Gap#	Possible SI Values
1	G
2	1

continued on next page...

[188]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.349`

...continued

Gap#	Possible SI Values
3	G
4	0

40 MSG P_c6/mcc (192.252)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P2'_1/m'$ (11.54), $P_S\bar{1}$ (2.7), $C2/c$ (15.85), $Cm'cm'$ (63.464), C_c2/c (15.90), $C2/c$ (15.85), $Cm'cm'$ (63.464), C_c2/c (15.90), $P2$ (3.1), $Cm'm'2$ (35.168), $Cm'm'2$ (35.168), P_b2 (3.5), C_ccc2 (37.184), C_ccc2 (37.184), $P2/m$ (10.42), $Cm'm'm$ (65.485), $Cm'm'm$ (65.485), P_b2/m (10.48), C_ccm (66.498), C_ccm (66.498), $P6m'm'$ (183.189), P_c6cc (184.196), $P6/mm'm'$ (191.240).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Pm' (6.20), $C2'$ (5.15), $C2'$ (5.15), $P2'_1$ (4.9), P_S1 (1.3), Cc (9.37), $Cm'c2'_1$ (36.174), C_cc (9.40), Cc (9.37), $Cm'c2'_1$ (36.174), C_cc (9.40), Pm (6.18), $Amm'2'$ (38.190), $Amm'2'$ (38.190), P_bm (6.22), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_ama2 (40.208), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_ama2 (40.208).

40.1 WP: $6f$

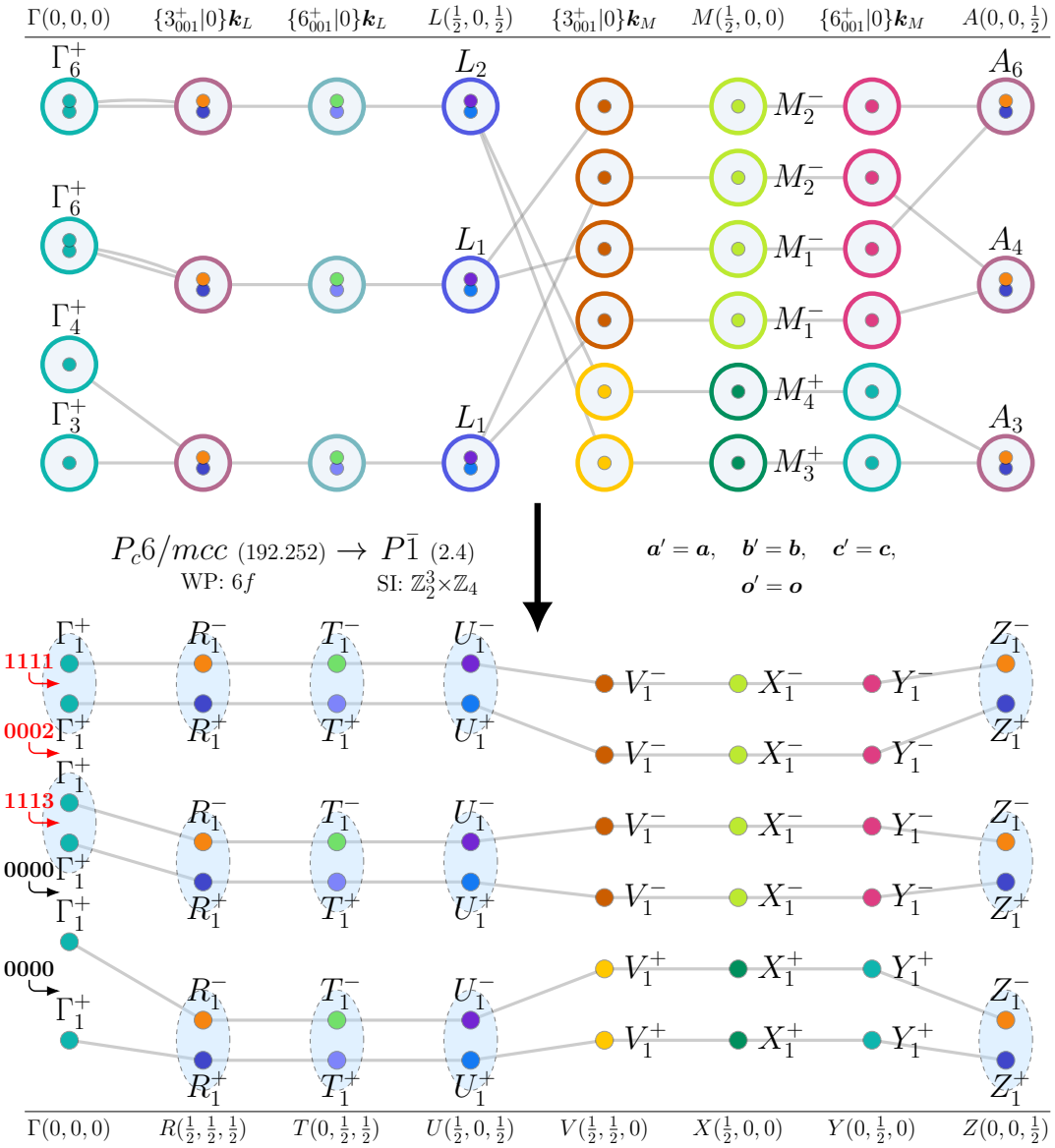
BCS Materials: FeGe (410 K)[189].

40.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [110],
- B ⊥ [001] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ⊥ [001],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.

[189] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.629`



Supplementary Figure 203: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $6f$ of supergroup P_c6/mcc (192.252).

Supplementary Table 395: Number of times a given SI can appear in the band structure. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
1000	0, 1
1001	0, 1, 2, 3
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2
1012	0, 1, 2, 3
1013	0, 1, 2
1100	0, 1
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

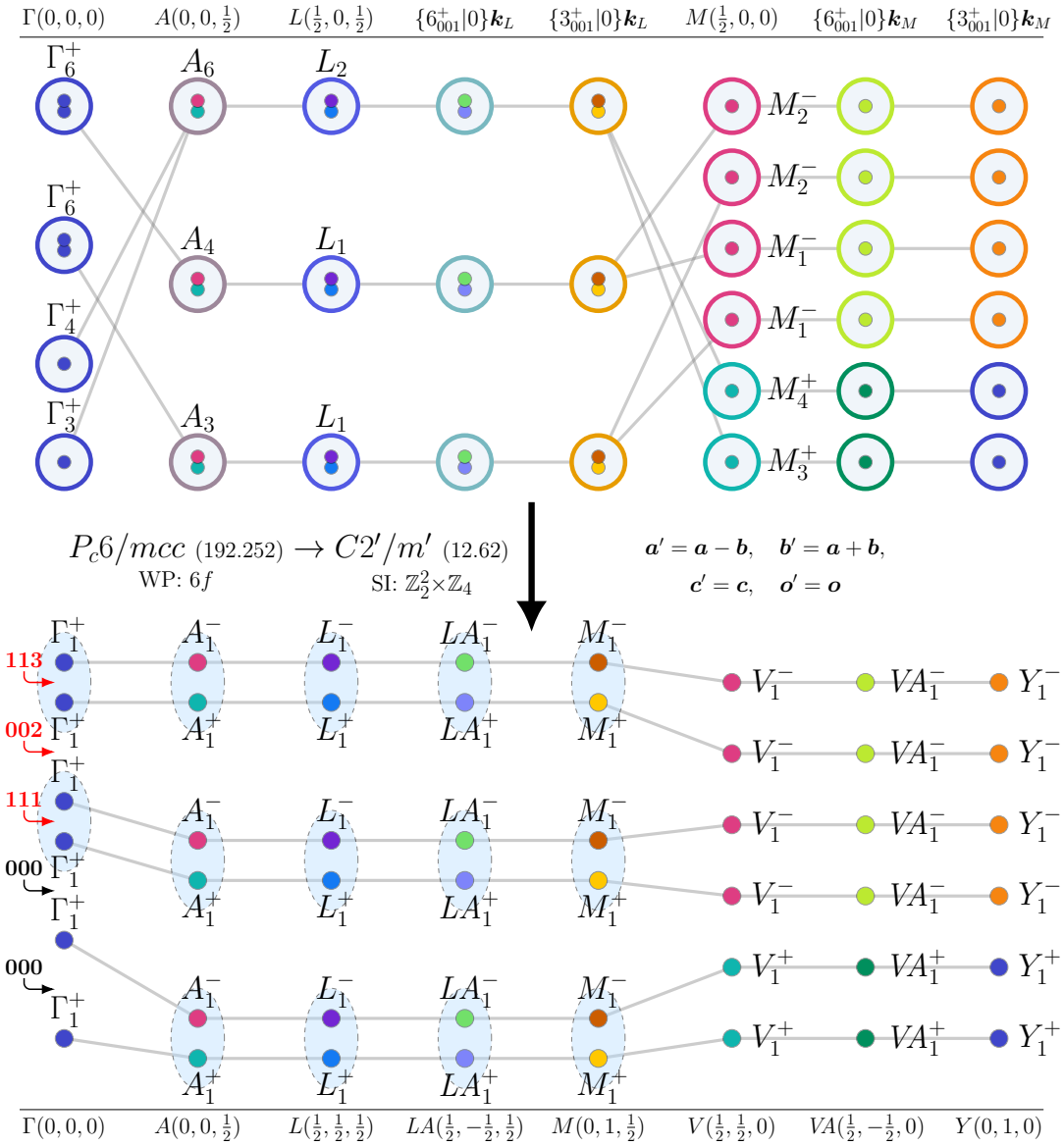
Supplementary Table 396: Possible SI values in each gap. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0101, 0102, 0103, 0110 0111, 0112, 1001, 1002, 1003 1010, 1011, 1012, 1101, 1102 1103, 1110, 1112, 1113
2	0000, 0002, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0103 0110, 0111, 0112, 0113, 1000 1001, 1003, 1010, 1011, 1012 1013, 1100, 1101, 1103, 1110 1111, 1112, 1113
4	0000, 0002, 1111
5	0000, 0001, 0003, 0010, 0012 0013, 0101, 0102, 0103, 0110 0112, 0113, 1001, 1002, 1003 1010, 1012, 1013, 1101, 1102 1103, 1110, 1111, 1112
6	0000

40.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 204: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $6f$ of supergroup P_c6/mcc (192.252).

Supplementary Table 397: Number of times a given SI can appear in the band structure. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
010	0, 1, 2, 3
011	0, 1, 2
012	0, 1, 2, 3
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3
102	0, 1
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3

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SI	Possible #Gaps
113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

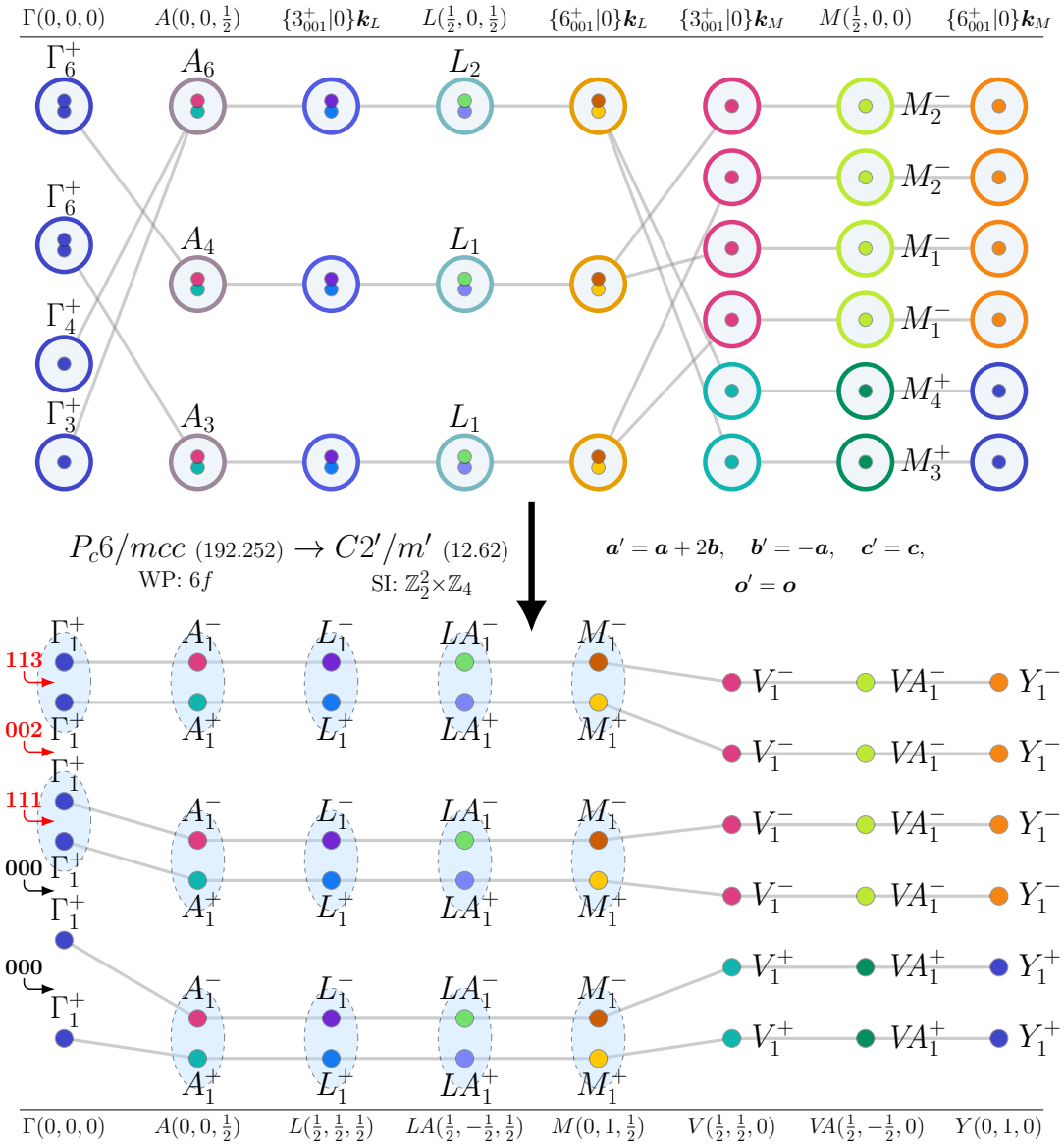
Supplementary Table 398: Possible SI values in each gap. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	000, 002, 111
3	001, 002, 003, 010, 011 012, 013, 101, 102, 103 110, 111, 112, 113
4	000, 002, 113
5	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
6	000

40.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 205: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $6f$ of supergroup P_c6/mcc (192.252).

Supplementary Table 399: Number of times a given SI can appear in the band structure. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
010	0, 1, 2, 3
011	0, 1, 2
012	0, 1, 2, 3
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3
102	0, 1
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3

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SI	Possible #Gaps
113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

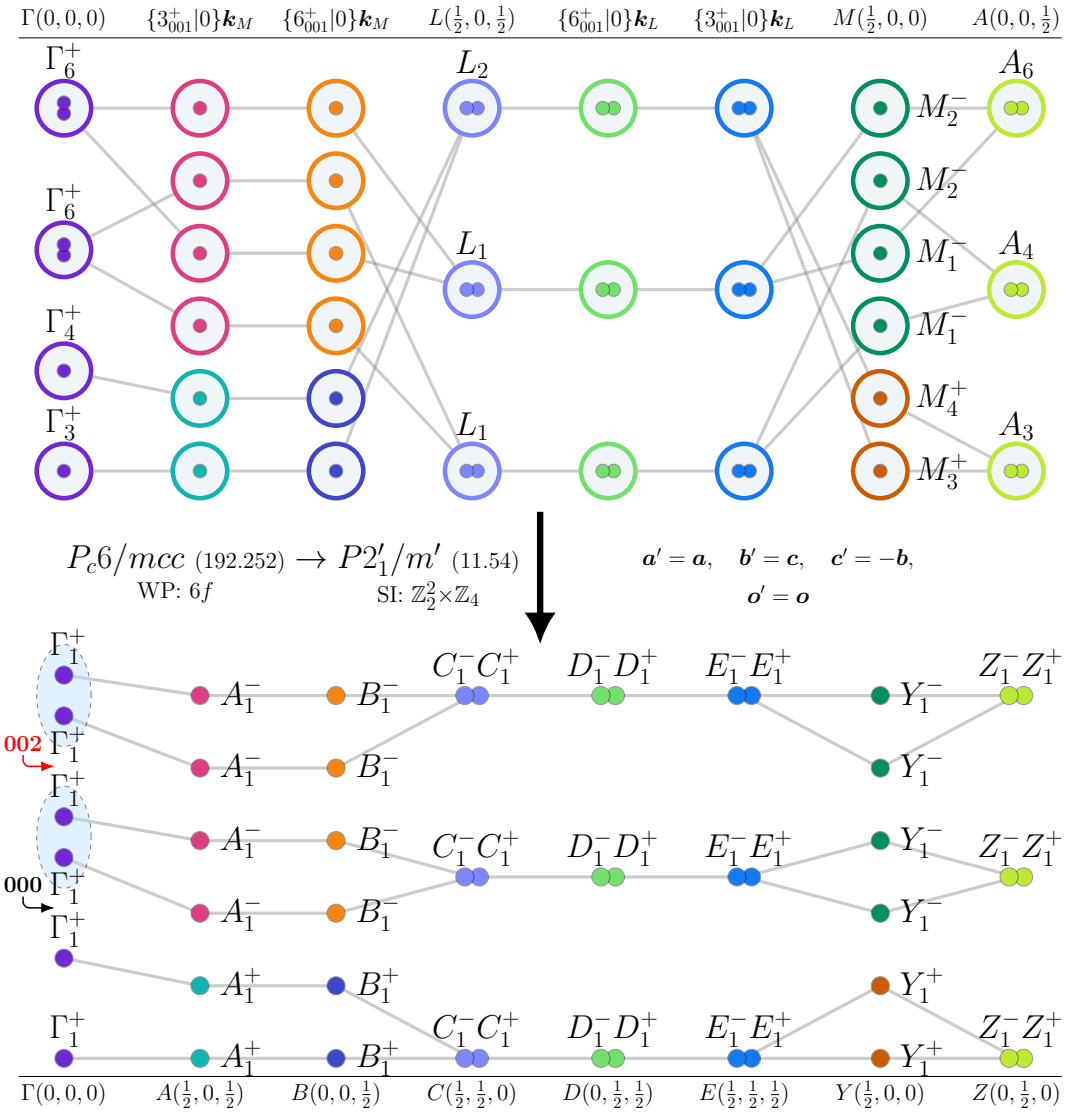
Supplementary Table 400: Possible SI values in each gap. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	000, 002, 111
3	001, 002, 003, 010, 011 012, 013, 101, 102, 103 110, 111, 112, 113
4	000, 002, 113
5	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
6	000

40.1.4 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 206: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $6f$ of supergroup P_c6/mcc (192.252).

Supplementary Table 401: Number of times a given SI can appear in the band structure. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
002	0, 1, 2
111	0, 1
113	0, 1
gapless	3
nontrivial	1, 2

Supplementary Table 402: Possible SI values in each gap. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	000, 002, 111
3	G
4	000, 002, 113

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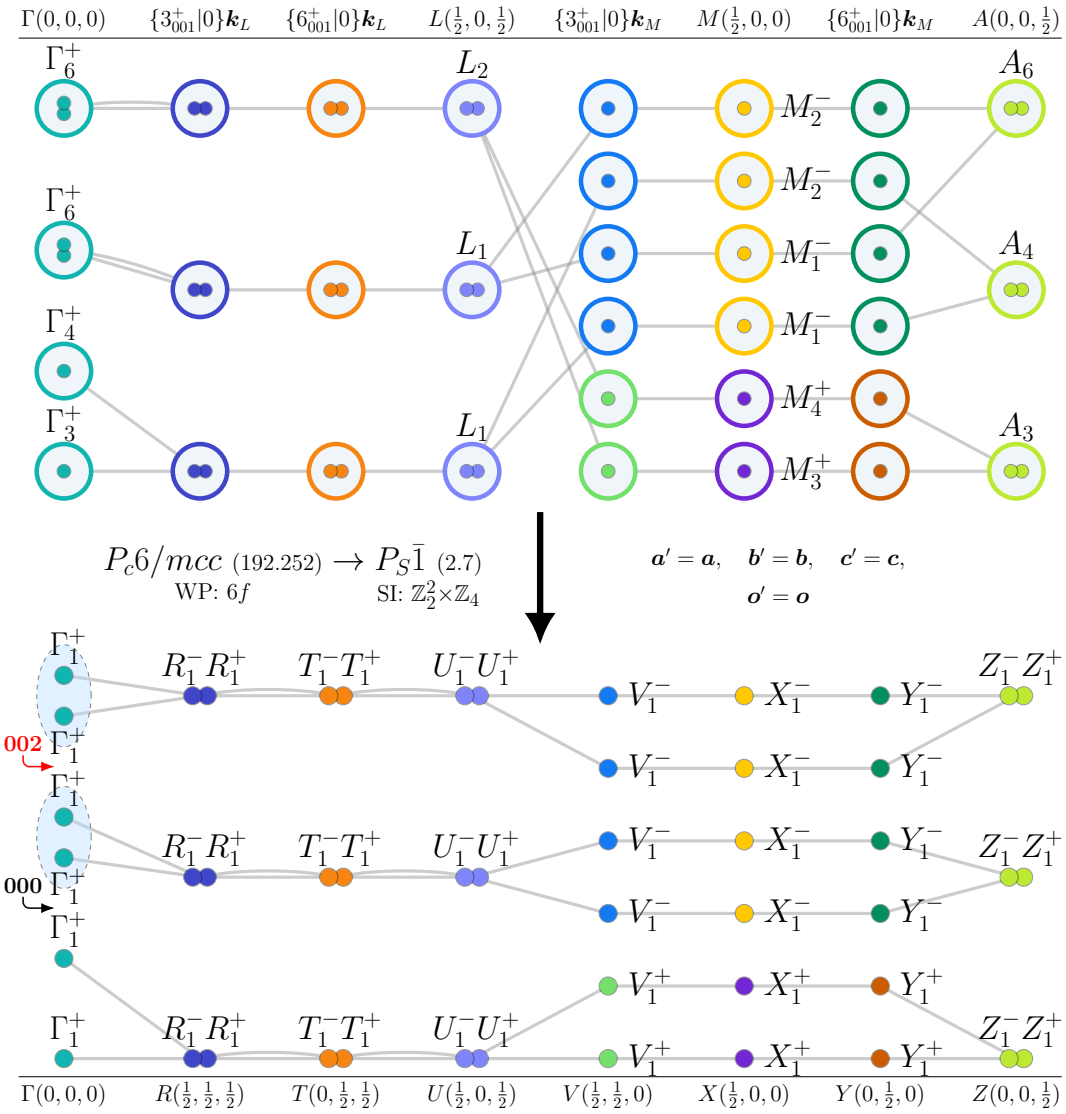
...continued

Gap#	Possible SI Values
5	G
6	000

40.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 207: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $6f$ of supergroup P_c6/mcc (192.252).

Supplementary Table 403: Number of times a given SI can appear in the band structure. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1, 2
111	0, 1
113	0, 1

continued on next page...

...continued

SI	Possible #Gaps
gapless	3
nontrivial	1, 2

Supplementary Table 404: Possible SI values in each gap. Supergroup: P_c6/mcc (192.252), WP: $6f$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 002, 111
3	G
4	000, 002, 113
5	G
6	000

41 MSG $Fd\bar{3}$ (203.26)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $R\bar{3}$ (148.17), $C2/c$ (15.85), $C2/c$ (15.85), $Fddd$ (70.527).

Trivial-SI Subgroups: Cc (9.37), Cc (9.37), $R3$ (146.10), $C2$ (5.13), $Fdd2$ (43.224).

41.1 WP: $16c$

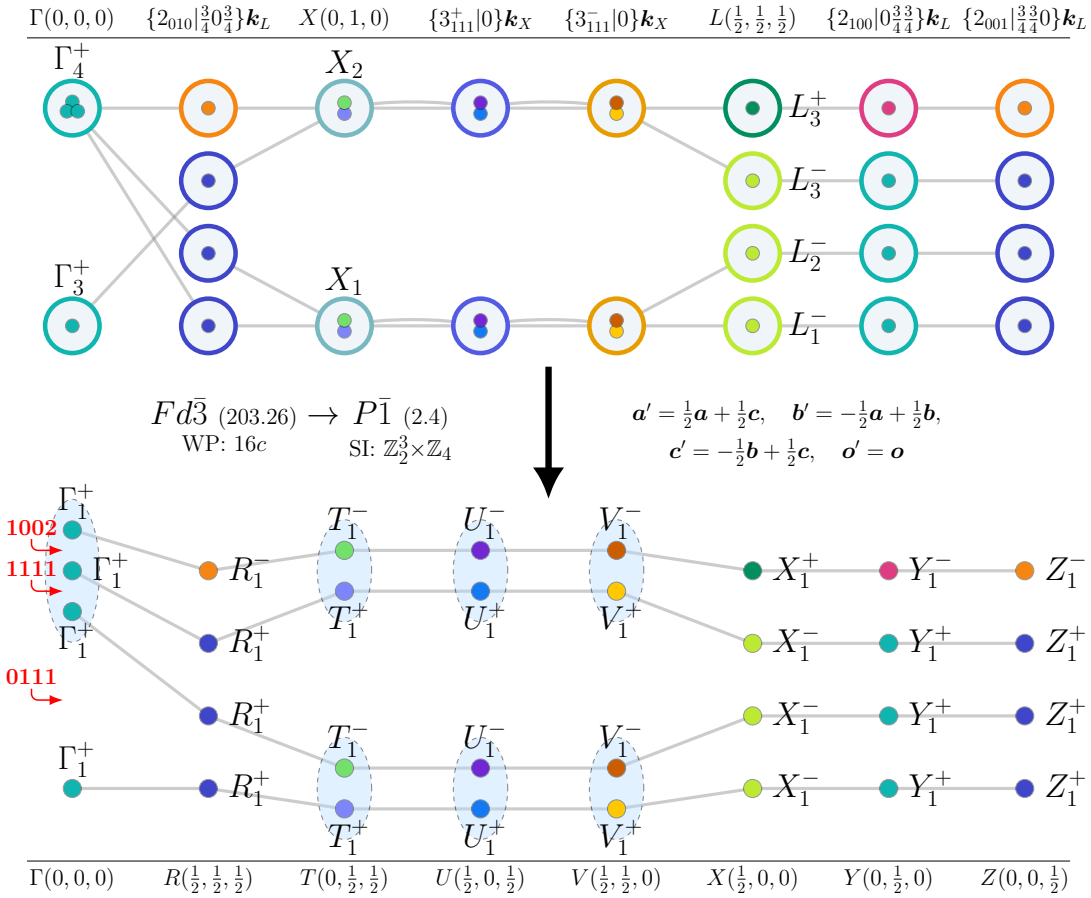
BCS Materials: $Na_3Co(CO_3)_2Cl$ (1.5 K)[190].

41.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ∥ [111],
- B in generic direction,
- B ∥ [111] and strain ∥ [100],
- B ∥ [111] and strain ∥ [110],
- B ∥ [111] and strain ⊥ [100].

[190]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.70`



Supplementary Figure 208: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $16c$ of supergroup $Fd\bar{3}$ (203.26).

Supplementary Table 405: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}$ (203.26), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1
0012	0, 1, 2
0100	0, 1
0102	0, 1, 2
0111	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1002	0, 1
1011	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1103	0, 1, 2
1110	0, 1
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	3

Supplementary Table 406: Possible SI values in each gap. Supergroup: $Fd\bar{3}$ (203.26), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
2	1111, 1113
3	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
4	0000

42 MSG $Pa\bar{3}$ (205.33)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $R\bar{3}$ (148.17), $P2_1/c$ (14.75), $P2_1/c$ (14.75), $Pbca$ (61.433).

Trivial-SI Subgroups: Pc (7.24), Pc (7.24), $R3$ (146.10), $P2_1$ (4.7), $Pca2_1$ (29.99).

42.1 WP: $4a$

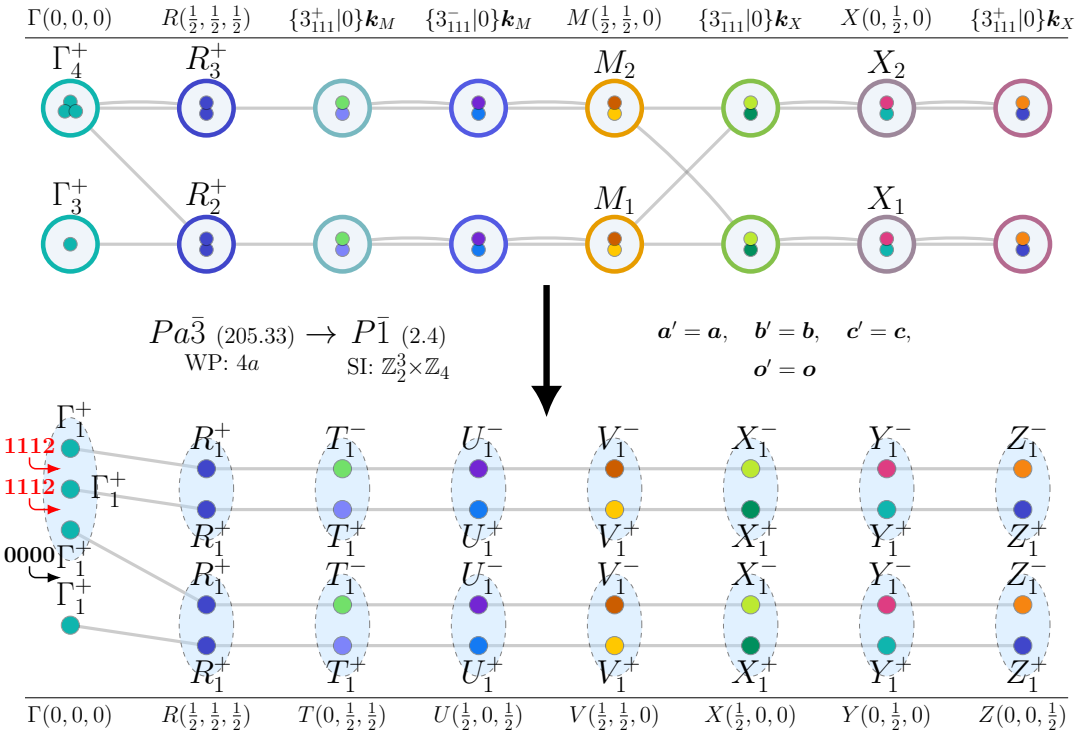
BCS Materials: $MnTe_2$ (86.5 K)[191], NiS_2 (39 K)[192].

42.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ∥ [111],
- B in generic direction,
- B ∥ [111] and strain ∥ [100],
- B ∥ [111] and strain ∥ [110],
- B ∥ [111] and strain ⊥ [100].

[191] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.20`
[192] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.150`



Supplementary Figure 209: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $Pa\bar{3}$ (205.33).

Supplementary Table 407: Number of times a given SI can appear in the band structure. Supergroup: $Pa\bar{3}$ (205.33), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1
gapless	0

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SI	Possible #Gaps
nontrivial	1, 2, 3

Supplementary Table 408: Possible SI values in each gap. Supergroup: $Pa\bar{3}$ (205.33), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

43 MSG $P_Ia\bar{3}$ (205.36)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $R\bar{3}$ (148.17), $R_I\bar{3}$ (148.20), P_C2_1/c (14.84), $P2_1/c$ (14.75), $Pcc'a'$ (54.343), P_C2_1/c (14.84), P_Ibca (61.440).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), $P2'$ (3.3), P_S1 (1.3), P_Cc (7.30), Pc (7.24), P_Cc (7.30), $R3$ (146.10), R_I3 (146.12), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_C2_1 (4.12), P_Ica2_1 (29.110).

43.1 WP: $8b$

BCS Materials: HoRh (3.2 K)[193].

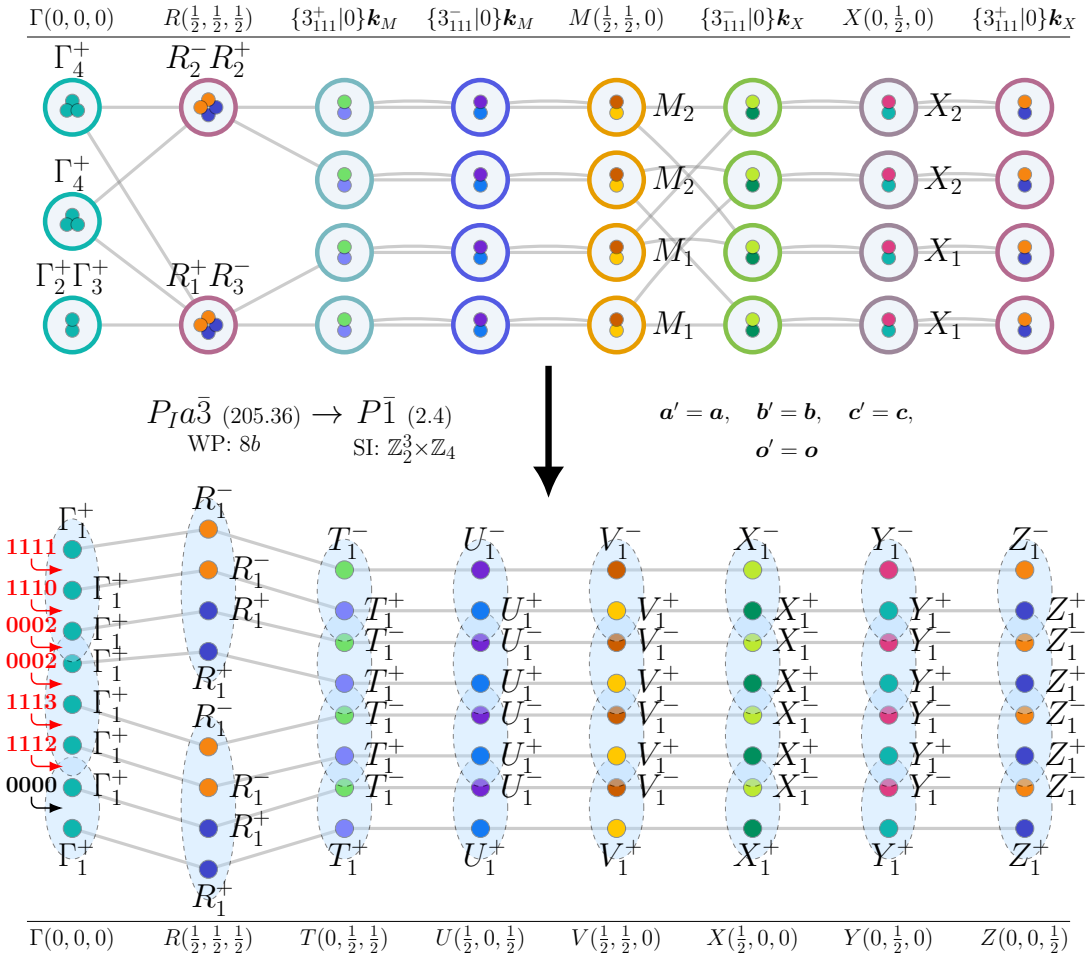
43.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ∥ [111],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ∥ [111] and strain ∥ [100],
- B ∥ [111] and strain ∥ [110],
- B ∥ [111] and strain ⊥ [100],

[193] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.18`

- B ∥ [111] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],
- B ⊥ [100] and strain ∥ [111],
- B ⊥ [100] and strain in generic direction,
- B in generic direction.



Supplementary Figure 210: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position 8*b* of supergroup $P_Ia\bar{3}$ (205.36).

Supplementary Table 409: Number of times a given SI can appear in the band structure. Supergroup: $P_Ia\bar{3}$ (205.36), WP: 8*b*, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3, 4
0002	1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4

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SI	Possible #Gaps
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4, 5, 6
1111	0, 1, 2, 3
1112	0, 1, 2, 3, 4, 5, 6
1113	0, 1, 2, 3
gapless	0
nontrivial	5, 6, 7

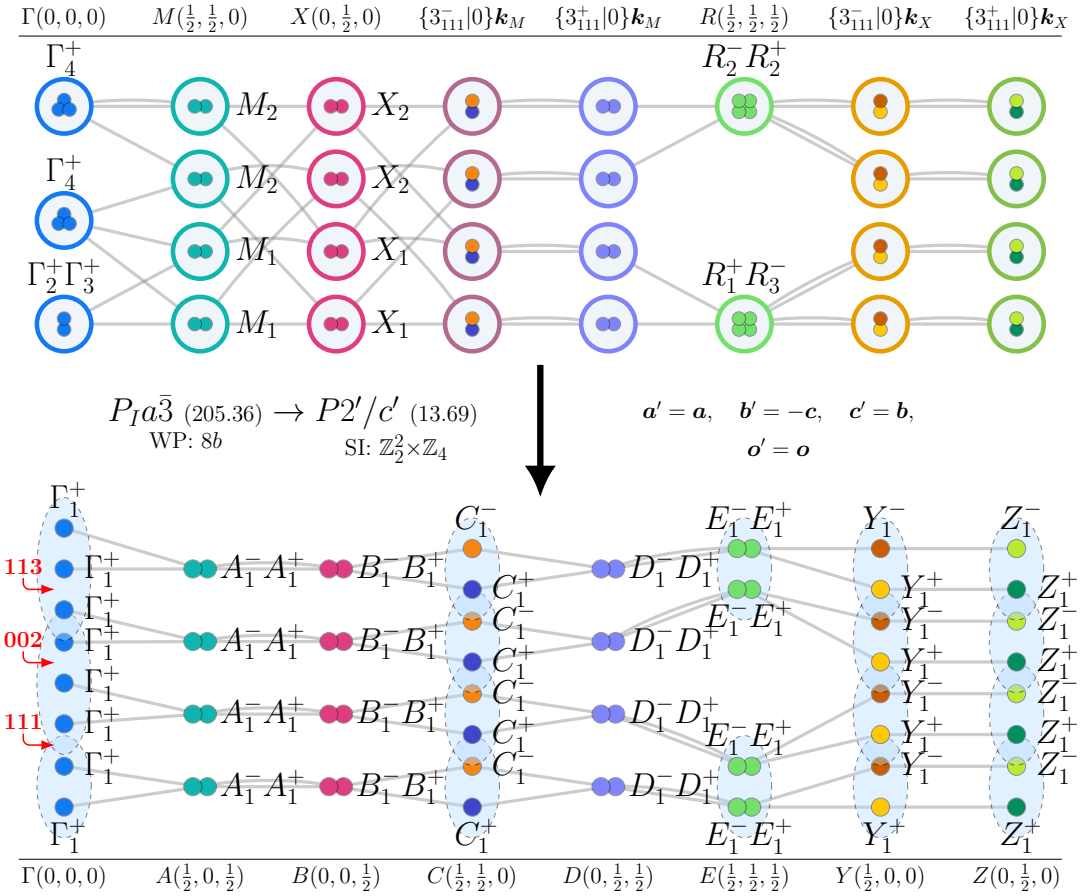
Supplementary Table 410: Possible SI values in each gap. Supergroup: $P_I a\bar{3}$ (205.36), WP: $8b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
2	1110, 1112, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
4	0002
5	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
6	1110, 1111, 1112
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
8	0000

43.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [110].



Supplementary Figure 211: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8b$ of supergroup $P_Ia\bar{3}$ (205.36).

Supplementary Table 411: Number of times a given SI can appear in the band structure. Supergroup: $P_Ia\bar{3}$ (205.36), WP: $8b$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

Supplementary Table 412: Possible SI values in each gap. Supergroup: $P_Ia\bar{3}$ (205.36), WP: $8b$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002

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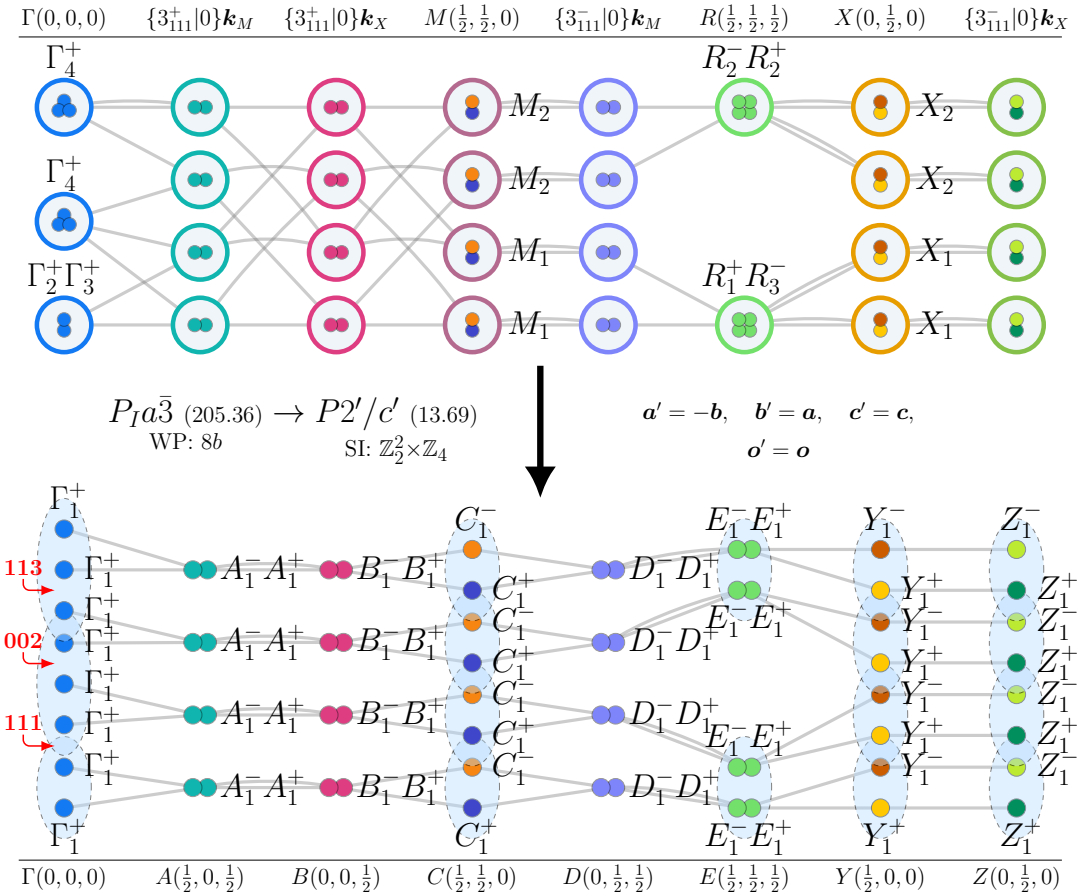
...continued

Gap#	Possible SI Values
5	G
6	113
7	G
8	000

43.1.3 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbation:

- B $\perp$ [100].



Supplementary Figure 212: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8b$ of supergroup $P_Ia\bar{3}$ (205.36).

Supplementary Table 413: Number of times a given SI can appear in the band structure. Supergroup: $P_Ia\bar{3}$ (205.36), WP: $8b$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

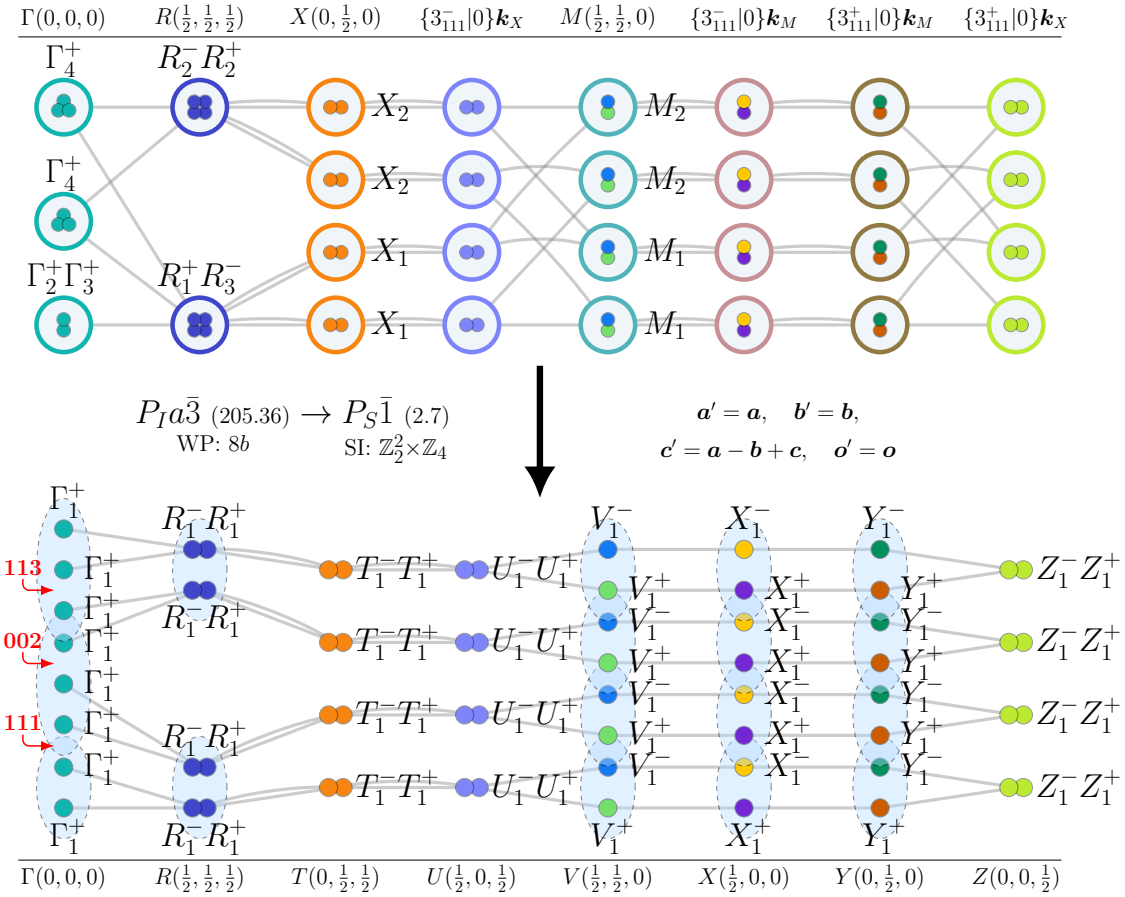
Supplementary Table 414: Possible SI values in each gap. Supergroup: $P_Ia\bar{3}$ (205.36), WP: $8b$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

43.1.4 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 213: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8b$ of supergroup $P_Ia\bar{3}$ (205.36).

Supplementary Table 415: Number of times a given SI can appear in the band structure. Supergroup: $P_Ia\bar{3}$ (205.36), WP: $8b$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	1
111	1

continued on next page...

...continued

SI	Possible #Gaps
113	1
gapless	4
nontrivial	3

Supplementary Table 416: Possible SI values in each gap. Supergroup: $P_Ia\bar{3}$ (205.36), WP: $8b$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

44 MSG $P_In\bar{3}n$ (222.103)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P2'_1/m'$ (11.54), $P2'_1/m'$ (11.54), $P_S\bar{1}$ (2.7), $R\bar{3}m'$ (166.101), $R_I\bar{3}c$ (167.108), $C2/c$ (15.85), $Cm'cm'$ (63.464), C_c2/c (15.90), C_Acca (68.520), $P2$ (3.1), $P2/c$ (13.65), P_A2/c (13.73), $P4m'm'$ (99.167), P_I4nc (104.210), $P4/nm'm'$ (129.417), P_I4/nnc (126.386).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), Pm' (6.20), Pm' (6.20), $C2'$ (5.15), $P2'_1$ (4.9), P_S1 (1.3), C_cc (9.40), C_cc (9.40), Cc (9.37), C_cc (9.40), P_Ac (7.31), Pc (7.24), P_Ac (7.31), $R3m'$ (160.67), R_I3c (161.72), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_Bba2 (41.218), P_C2 (3.6).

44.1 WP: $8c$

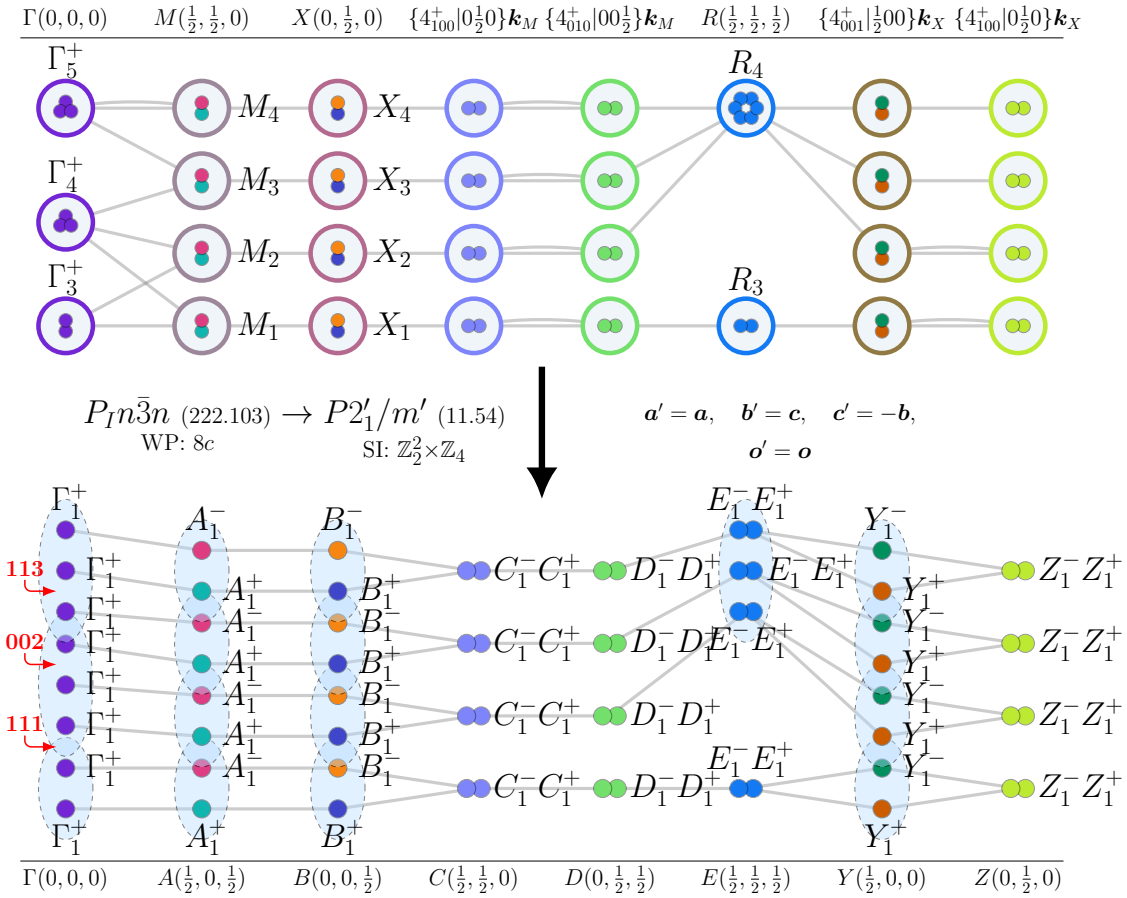
BCS Materials: NdZn (70 K)[194].

44.1.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [110] and strain ∥ [100].

[194]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.8`



Supplementary Figure 214: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position 8c of supergroup $P_In\bar{3}n$ (222.103).

Supplementary Table 417: Number of times a given SI can appear in the band structure. Supergroup: $P_In\bar{3}n$ (222.103), WP: 8c, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

2966

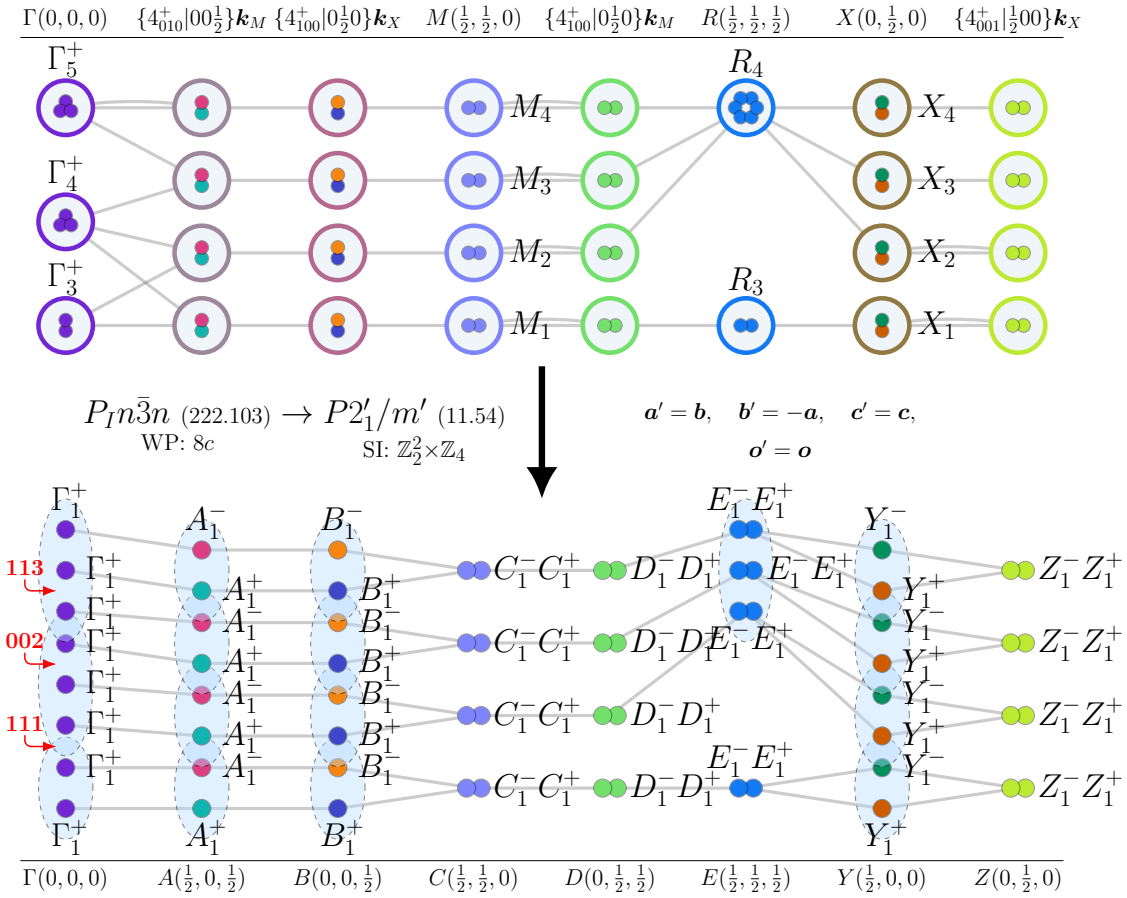
Supplementary Table 418: Possible SI values in each gap. Supergroup: $P_In\bar{3}n$ (222.103), WP: 8c, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

44.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbation:

- B $\perp$ [100].



Supplementary Figure 215: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position 8c of supergroup $P_In\bar{3}n$ (222.103).

Supplementary Table 419: Number of times a given SI can appear in the band structure. Supergroup: $P_In\bar{3}n$ (222.103), WP: 8c, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

Supplementary Table 420: Possible SI values in each gap. Supergroup: $P_In\bar{3}n$ (222.103), WP: 8c, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	111
3	G

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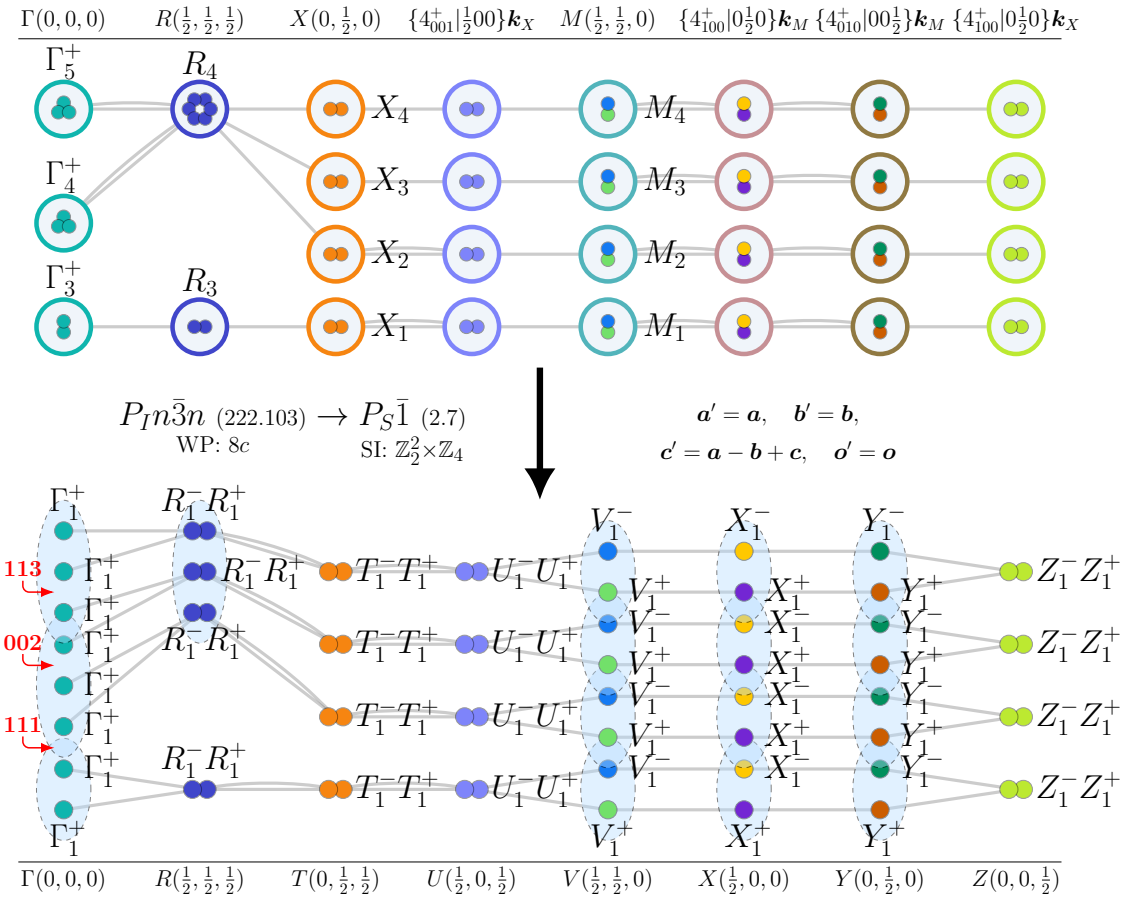
...continued

Gap#	Possible SI Values
4	002
5	G
6	113
7	G
8	000

44.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 216: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8c$ of supergroup $P_In\bar{3}n$ (222.103).

Supplementary Table 421: Number of times a given SI can appear in the band structure. Supergroup: $P_In\bar{3}n$ (222.103), WP: $8c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

Supplementary Table 422: Possible SI values in each gap. Supergroup: $P_I n\bar{3}n$ (222.103), WP: $8c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

45 MSG $Pn\bar{3}m'$ (224.113)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $R\bar{3}m'$ (166.101), $Cm'm'a$ (67.505), $P2$ (3.1), $Cm'm'2$ (35.168), $P4'_2nm'$ (102.190), $P2/c$ (13.65), $Cm'm'a$ (67.505), $P4'_2/nnm'$ (134.475).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), Pc (7.24), $Abm'2'$ (39.198), Pc (7.24), $R3m'$ (160.67).

45.1 WP: $4b$

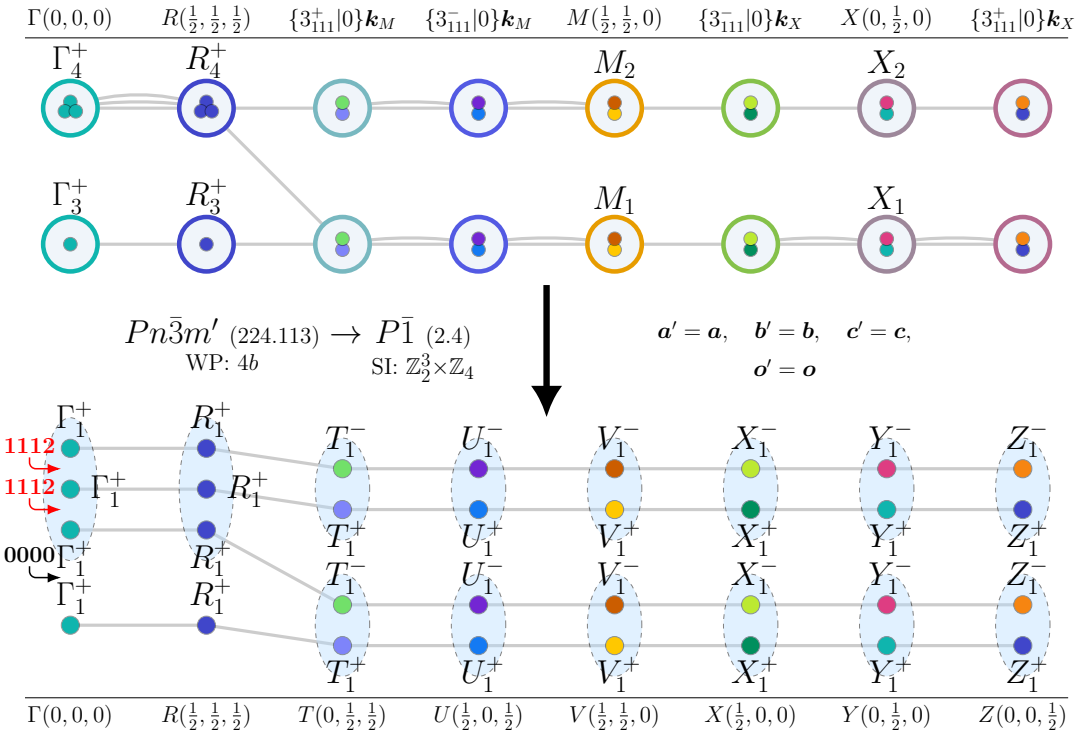
BCS Materials: USb (213 K)[195], NpBi (192.5 K)[196], $Fe_{0.7}Mn_{0.3}$[197].

45.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B ∥ [111] and strain ⊥ [100],
- B ∥ [111] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ∥ [111],
- B ⊥ [110] and strain ⊥ [100].

[195]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.12`
[196]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.7`
[197]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.5`



Supplementary Figure 217: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 423: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

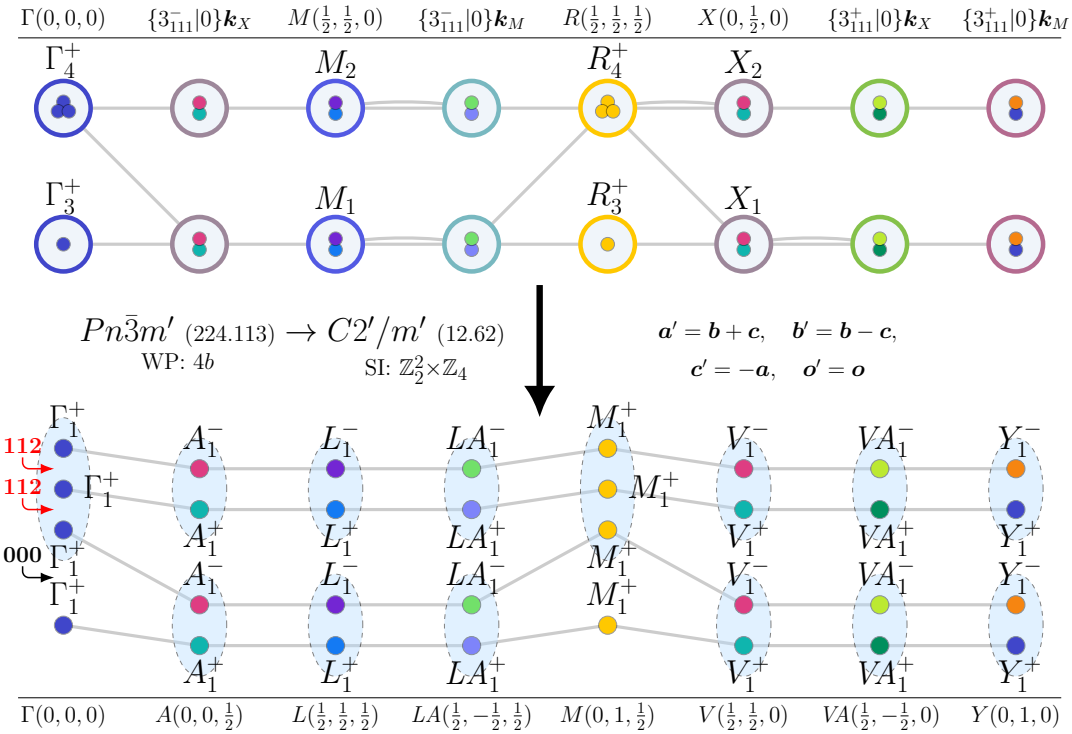
Supplementary Table 424: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

45.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].



Supplementary Figure 218: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 425: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
111	0, 1
112	1, 2, 3
113	0, 1
gapless	0
nontrivial	1, 2, 3

3007

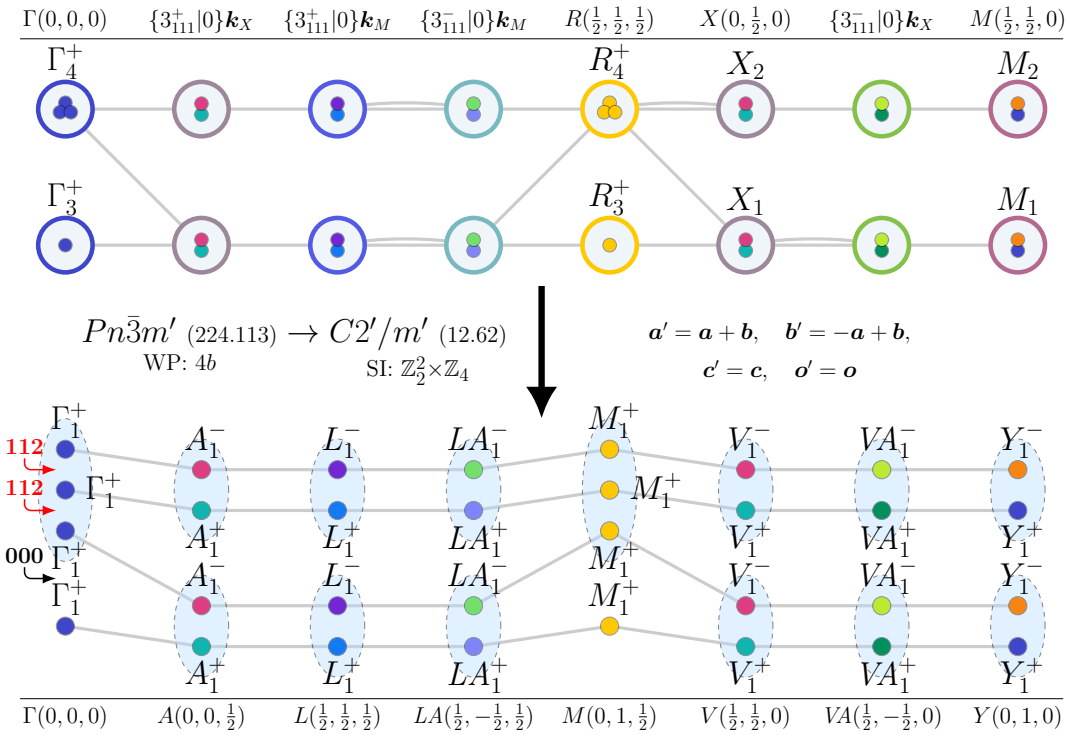
Supplementary Table 426: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	112
3	000, 003, 012, 013, 100 103, 112, 113
4	000

45.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [110],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 219: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 427: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
111	0, 1
112	1, 2, 3
113	0, 1
gapless	0
nontrivial	1, 2, 3

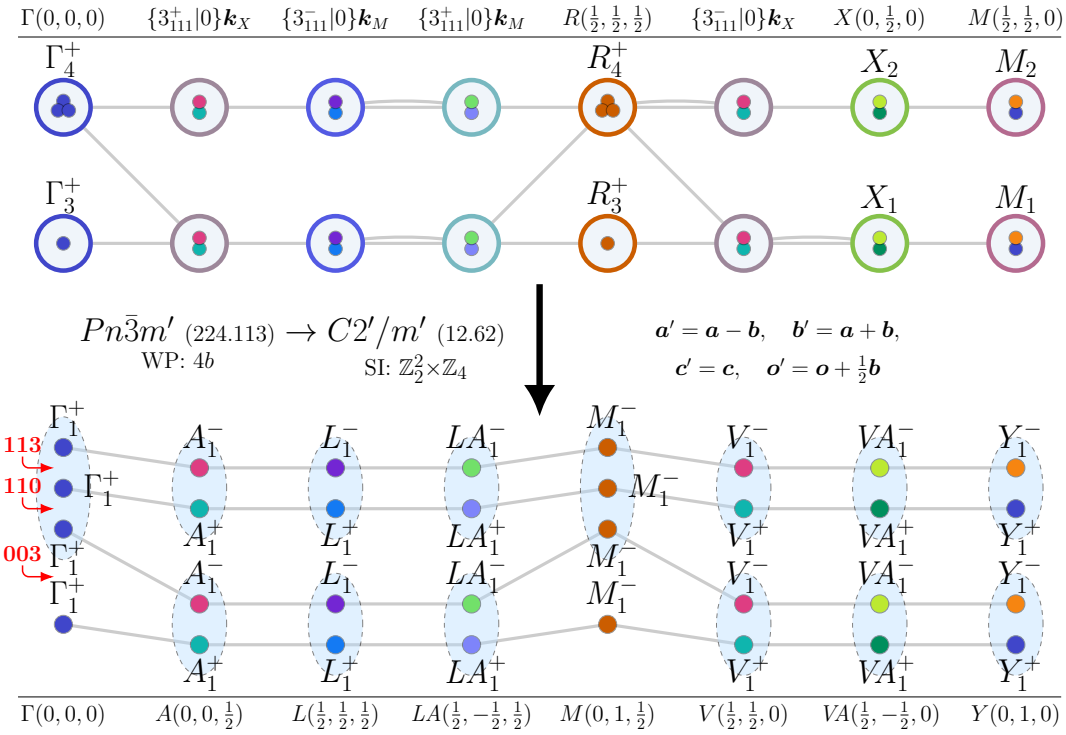
Supplementary Table 428: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	112
3	000, 003, 012, 013, 100 103, 112, 113
4	000

45.1.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 220: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 429: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
010	0, 1, 2
011	0, 1
013	0, 1
100	0, 1, 2
101	0, 1

continued on next page...

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SI	Possible #Gaps
103	0, 1
110	1, 2, 3
111	0, 1
113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 430: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 003, 010, 011, 100 103, 110, 111
2	110
3	000, 001, 010, 013, 100 101, 110, 113
4	000

45.2 WP: $4c$

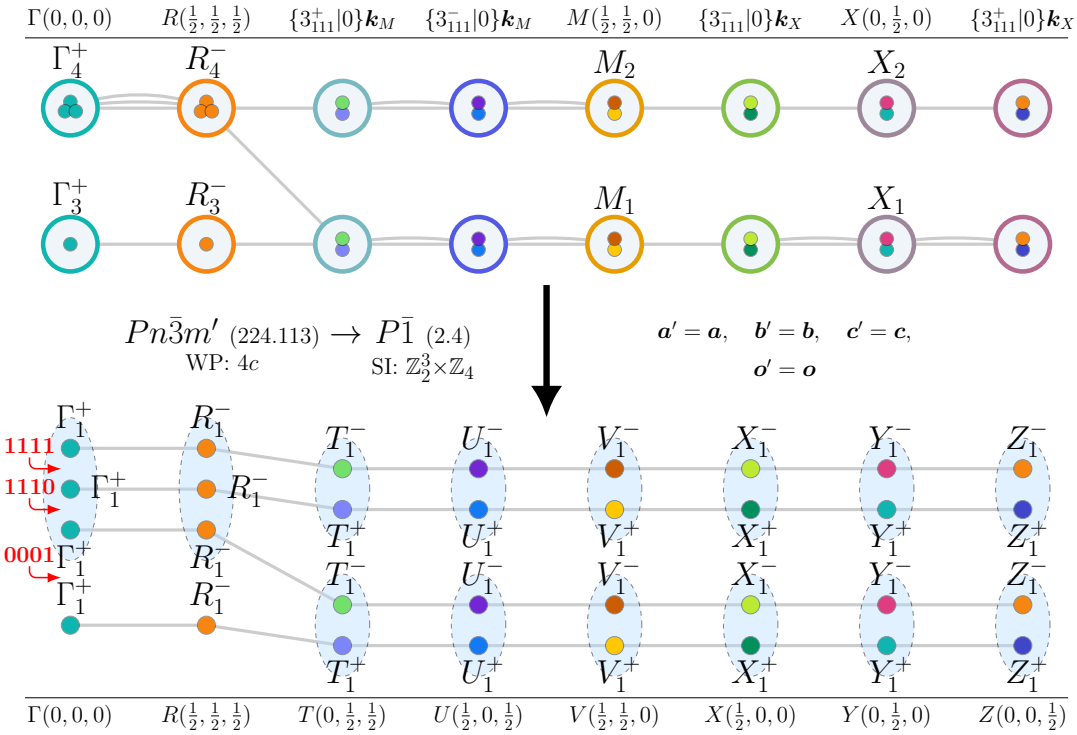
BCS Materials: UO_2 (30.8 K)[198].

45.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B ∥ [111] and strain ⊥ [100],
- B ∥ [111] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ∥ [111],
- B ⊥ [110] and strain ⊥ [100].

[198] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.2`



Supplementary Figure 221: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 431: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	1, 2, 3
1111	0, 1
1113	0, 1

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

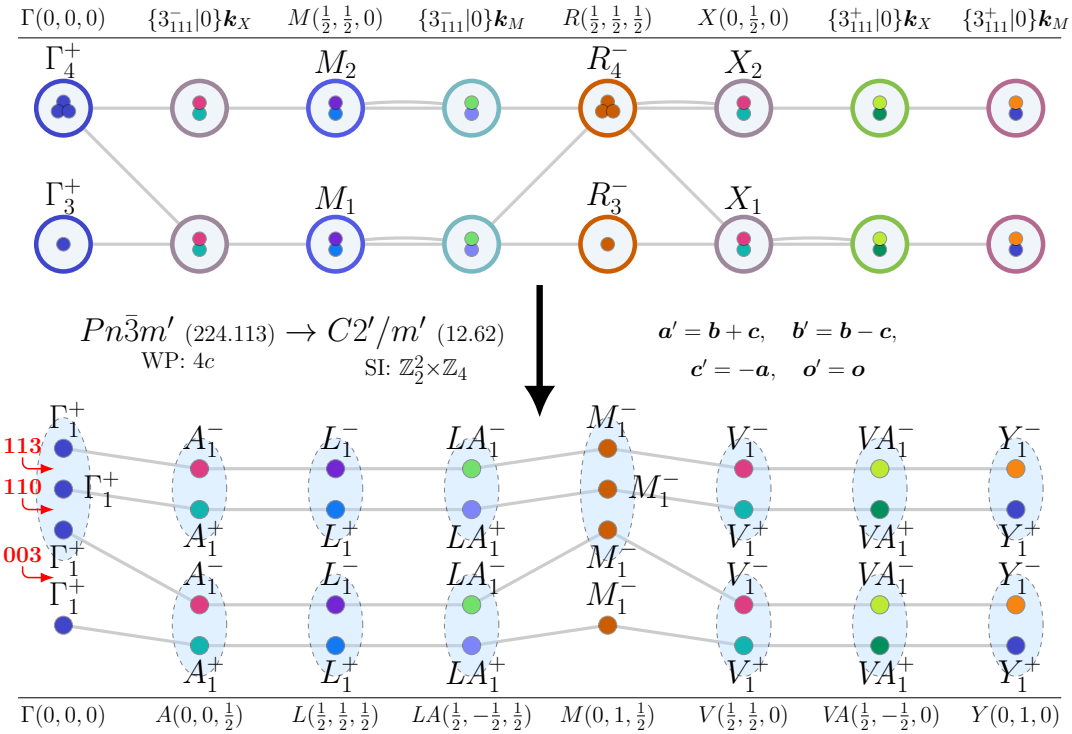
Supplementary Table 432: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
2	1110
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
4	0000

45.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].



Supplementary Figure 222: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 433: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
010	0, 1, 2
011	0, 1
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
110	1, 2, 3
111	0, 1
113	0, 1
gapless	0
nontrivial	1, 2, 3

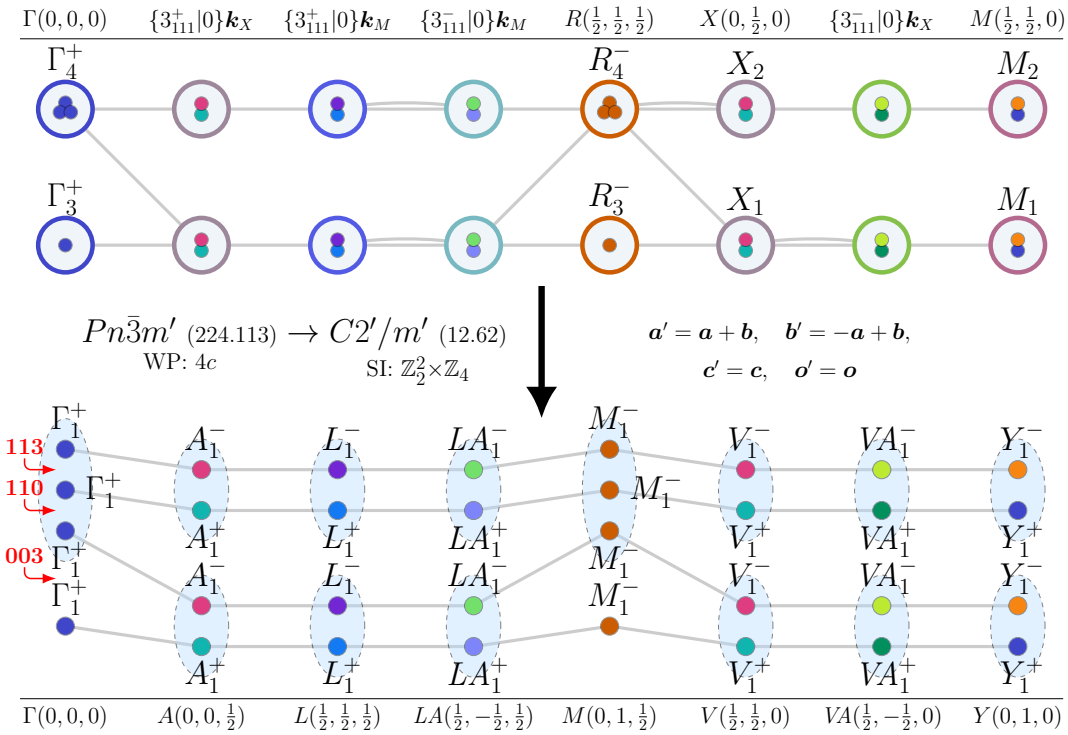
Supplementary Table 434: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 003, 010, 011, 100 103, 110, 111
2	110
3	000, 001, 010, 013, 100 101, 110, 113
4	000

45.2.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [110],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 223: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 435: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
010	0, 1, 2
011	0, 1
013	0, 1
100	0, 1, 2
101	0, 1
103	0, 1
110	1, 2, 3
111	0, 1
113	0, 1
gapless	0
nontrivial	1, 2, 3

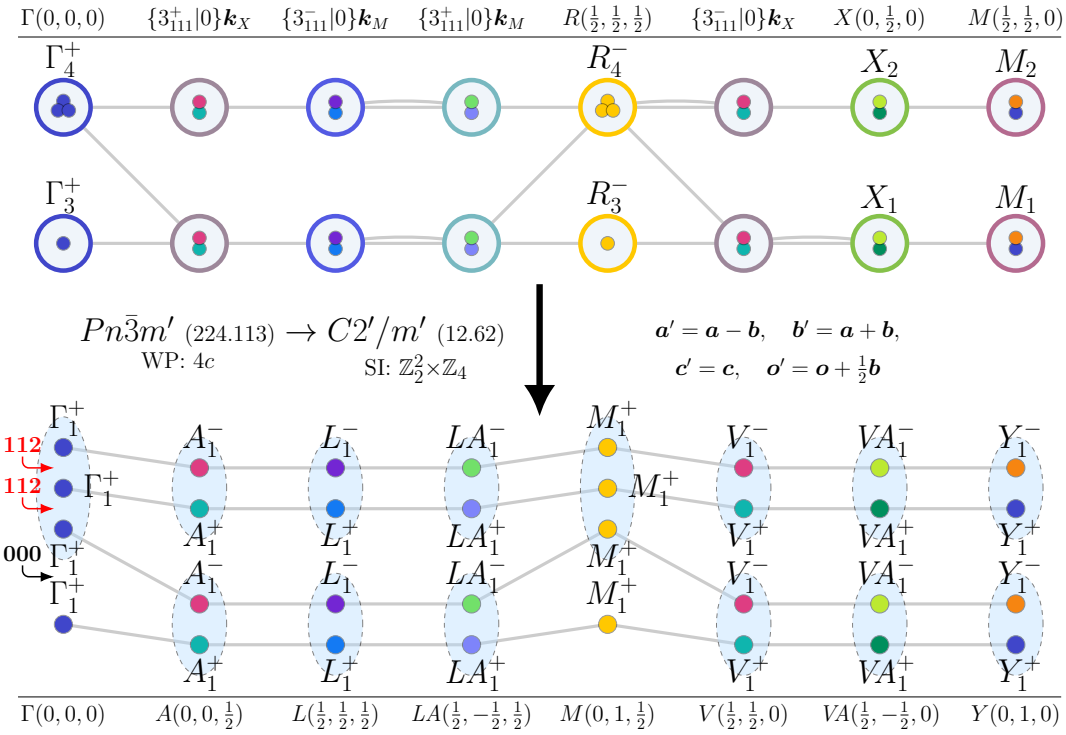
Supplementary Table 436: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 003, 010, 011, 100 103, 110, 111
2	110
3	000, 001, 010, 013, 100 101, 110, 113
4	000

45.2.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 224: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn\bar{3}m'$ (224.113).

Supplementary Table 437: Number of times a given SI can appear in the band structure. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1
003	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1, 2
101	0, 1

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SI	Possible #Gaps
103	0, 1
111	0, 1
112	1, 2, 3
113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 438: Possible SI values in each gap. Supergroup: $Pn\bar{3}m'$ (224.113), WP: $4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 011, 012, 100 101, 111, 112
2	112
3	000, 003, 012, 013, 100 103, 112, 113
4	000

46 MSG $Fd\bar{3}m'$ (227.131)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $R\bar{3}m'$ (166.101), $Im'm'a$ (74.558), $C2/c$ (15.85), $Im'm'a$ (74.558), $I4_1'/am'd$ (141.554).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), Cc (9.37), $Im'a2'$ (46.243), Cc (9.37), $R3m'$ (160.67), $C2$ (5.13), $Im'm'2$ (44.232), $I4_1'm'd$ (109.241).

46.1 WP: $16c$

BCS Materials: $Cd_2Os_2O_7$ (227 K)[199].

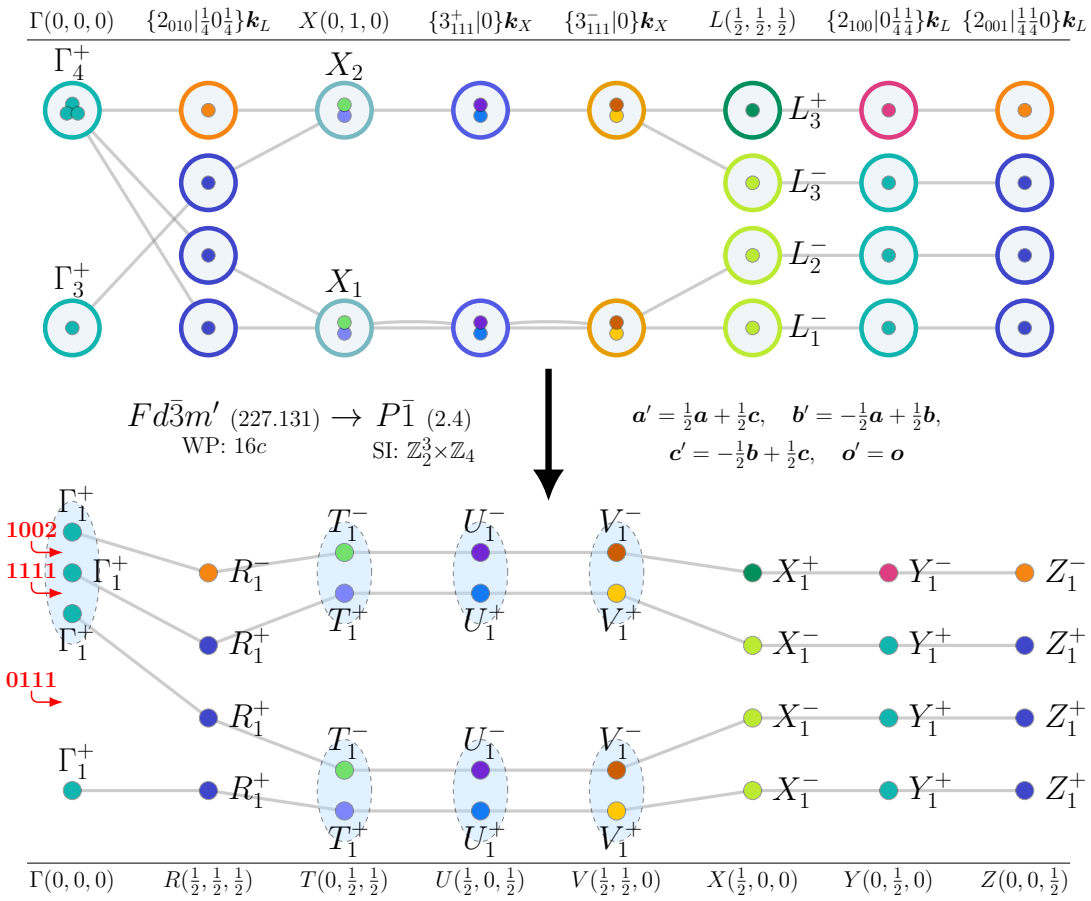
46.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B ∥ [111] and strain ⊥ [100],
- B ∥ [111] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],

[199] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.2`

- B $\perp$ [110] and strain $\parallel$ [111],
- B $\perp$ [110] and strain $\perp$ [100].



Supplementary Figure 225: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $16c$ of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 439: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1
0012	0, 1, 2
0100	0, 1
0102	0, 1, 2
0111	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1002	0, 1
1011	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1103	0, 1, 2
1110	0, 1
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	3

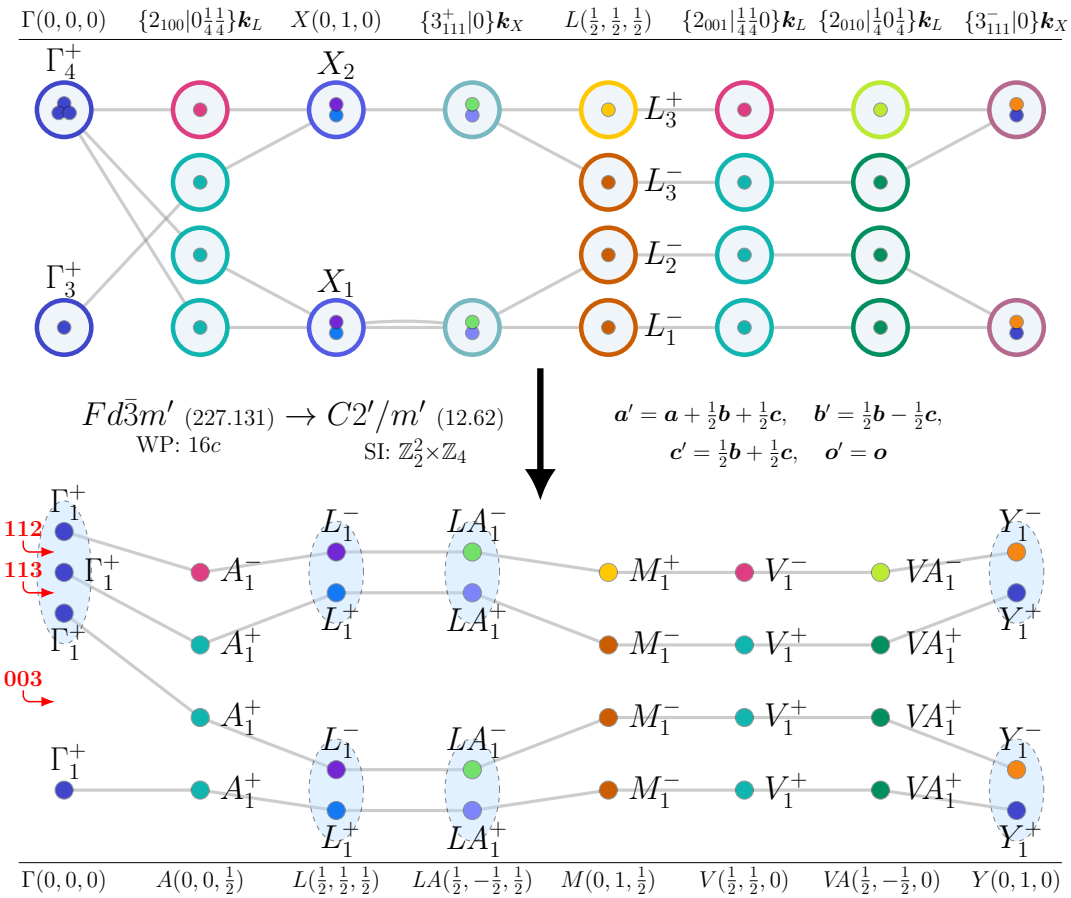
Supplementary Table 440: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: 16c, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
2	1111, 1113
3	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
4	0000

46.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].



Supplementary Figure 226: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position 16c of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 441: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: 16c, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

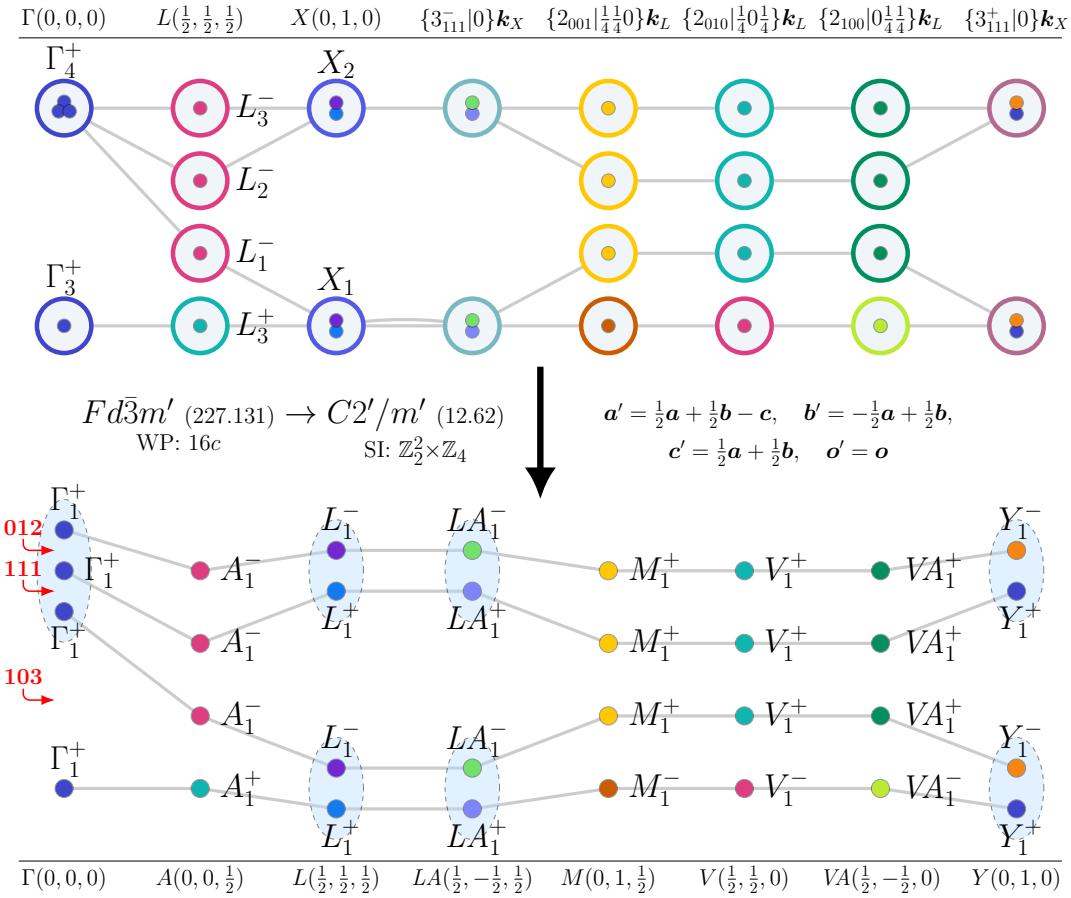
Supplementary Table 442: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: 16c, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

46.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [110],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 227: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $16c$ of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 443: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 444: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113

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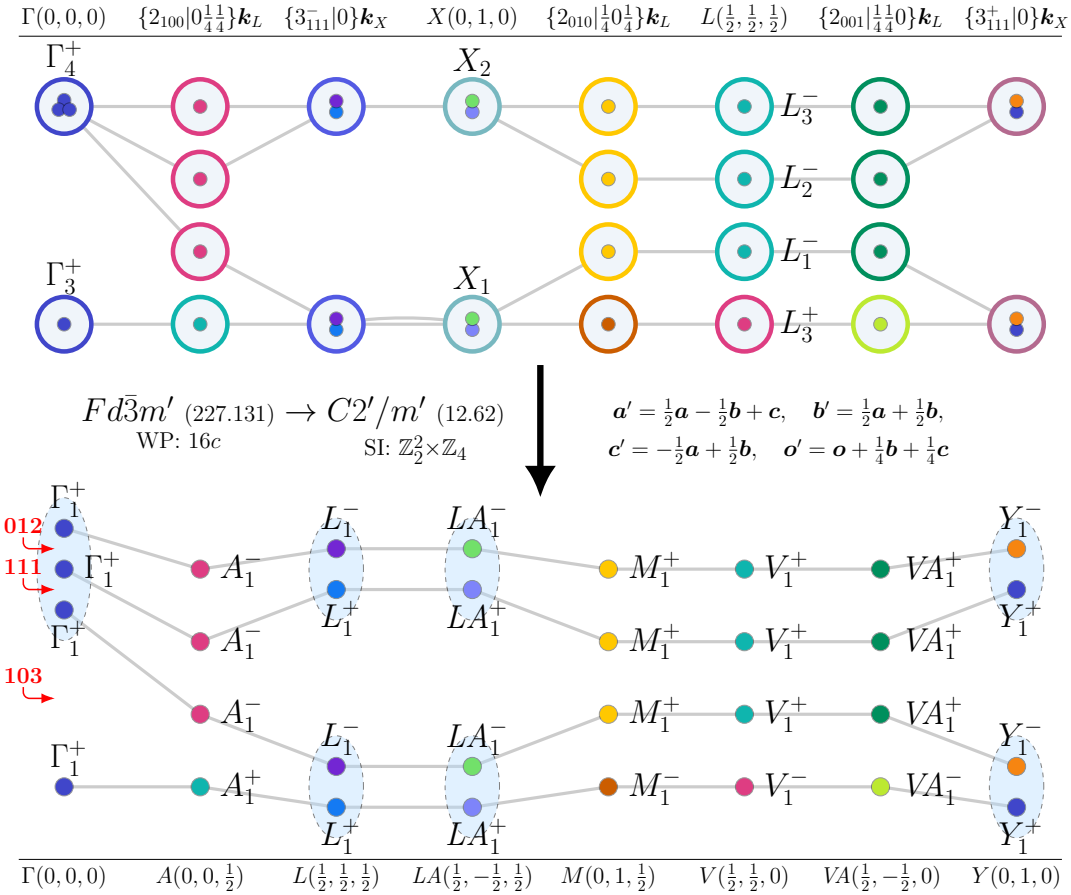
...continued

Gap#	Possible SI Values
3	001, 003, 010, 012, 101 103, 110, 112
4	000

46.1.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 228: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position 16c of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 445: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: 16c, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 446: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: 16c, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

46.2 WP: 16d

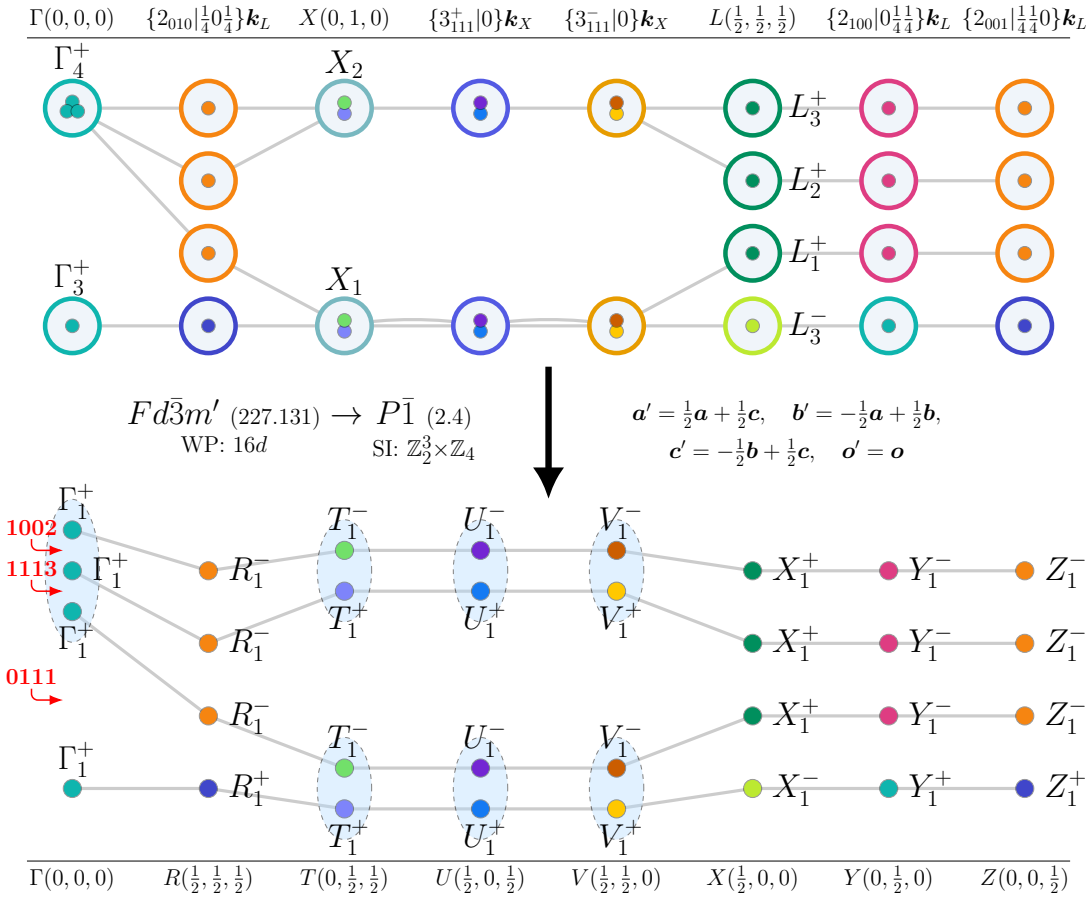
BCS Materials: $Nd_2Sn_2O_7$ (0.91 K)[200], $Nd_2Hf_2O_7$ (0.55 K)[201], $Nd_2Zr_2O_7$ (0.4 K)[202], Nd_2ScNbO_7 (0.371 K)[203], $Sm_2Ti_2O_7$ (0.35 K)[204].

46.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B $\parallel$ [100] and strain $\parallel$ [110],
- B $\parallel$ [100] and strain $\perp$ [110],
- B $\parallel$ [110] and strain $\parallel$ [100],
- B $\parallel$ [110] and strain $\perp$ [100],
- B $\parallel$ [110] and strain $\perp$ [110],
- B $\parallel$ [111] and strain $\perp$ [100],
- B $\parallel$ [111] and strain $\perp$ [110],
- B in generic direction,
- B $\perp$ [110] and strain $\parallel$ [100],
- B $\perp$ [110] and strain $\parallel$ [111],
- B $\perp$ [110] and strain $\perp$ [100].

[200]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.326`
[201]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.339`
[202]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.340`
[203]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.822`
[204]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.427`



Supplementary Figure 229: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $16d$ of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 447: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0012	0, 1
0100	0, 1, 2
0102	0, 1
0111	0, 1, 2
0113	0, 1, 2
1000	0, 1
1002	0, 1, 2
1011	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1
1113	0, 1
gapless	0
nontrivial	3

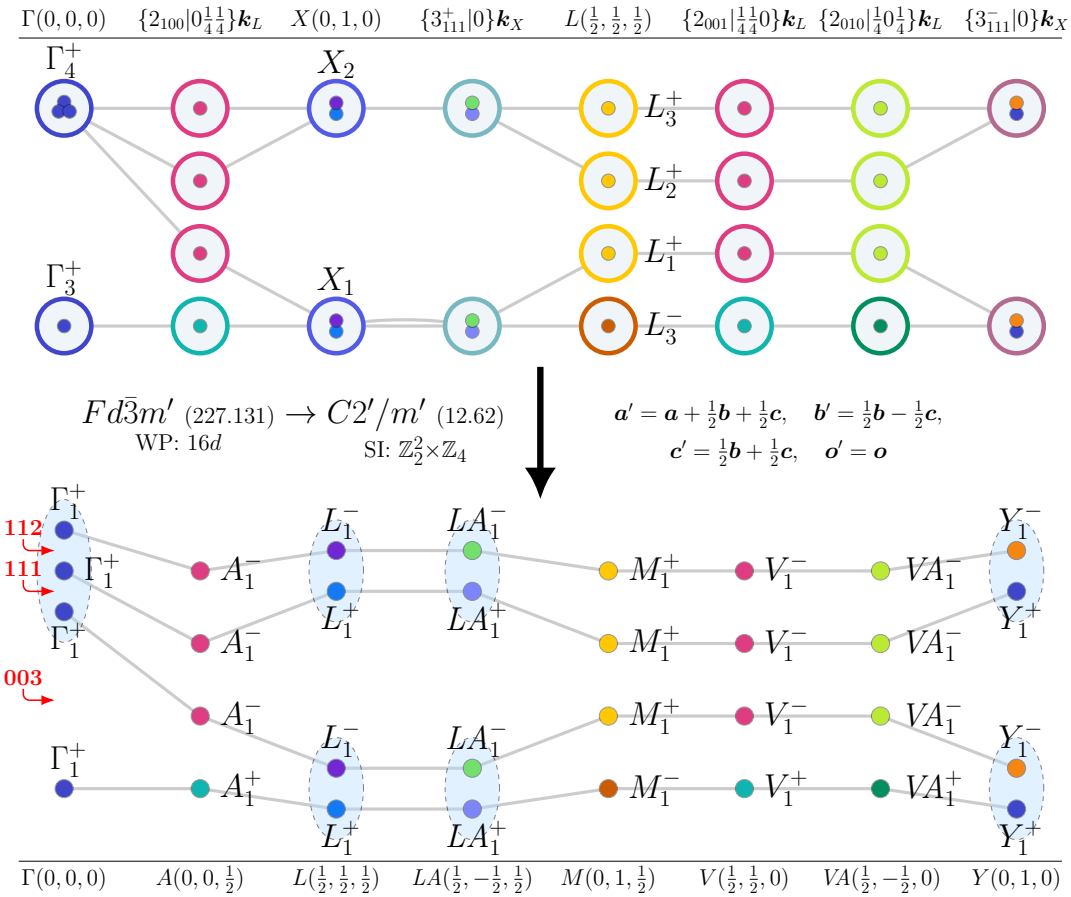
Supplementary Table 448: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
2	1111, 1113
3	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
4	0000

46.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].



Supplementary Figure 230: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $16d$ of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 449: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	3

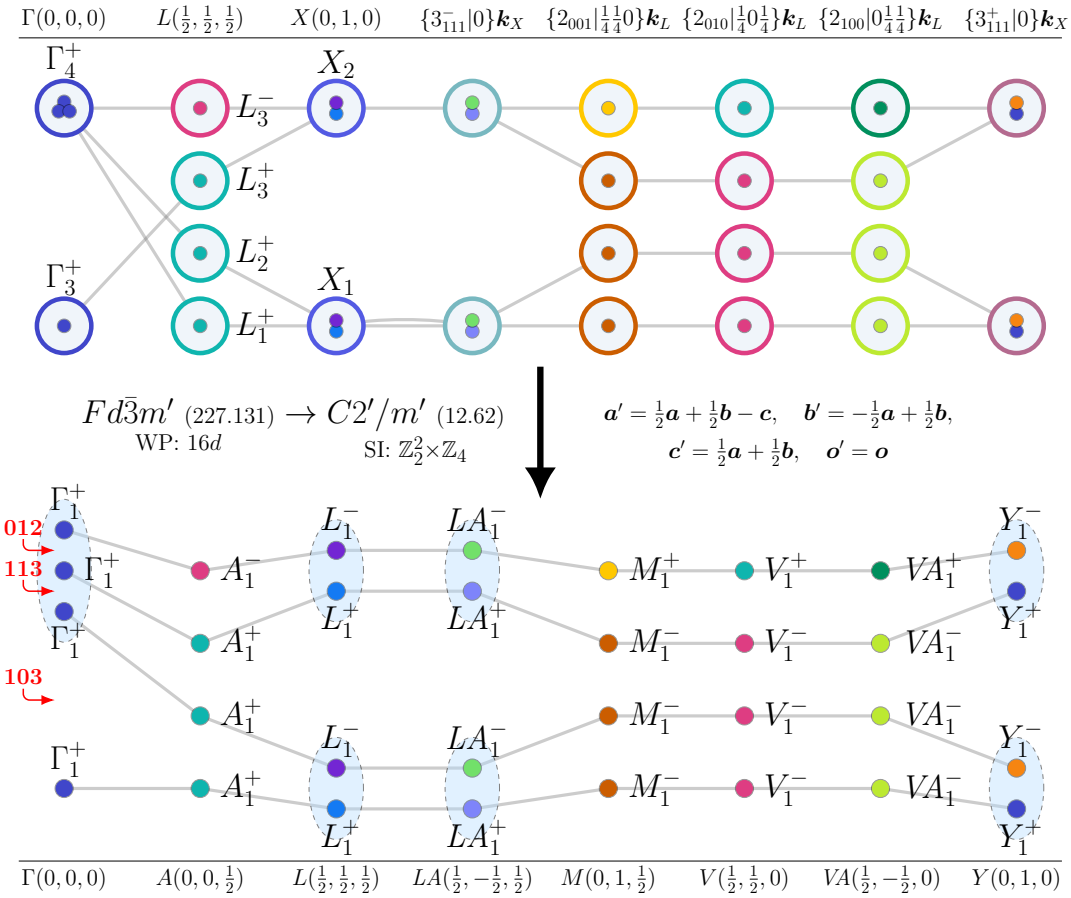
Supplementary Table 450: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

46.2.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [110],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 231: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $16d$ of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 451: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 452: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113

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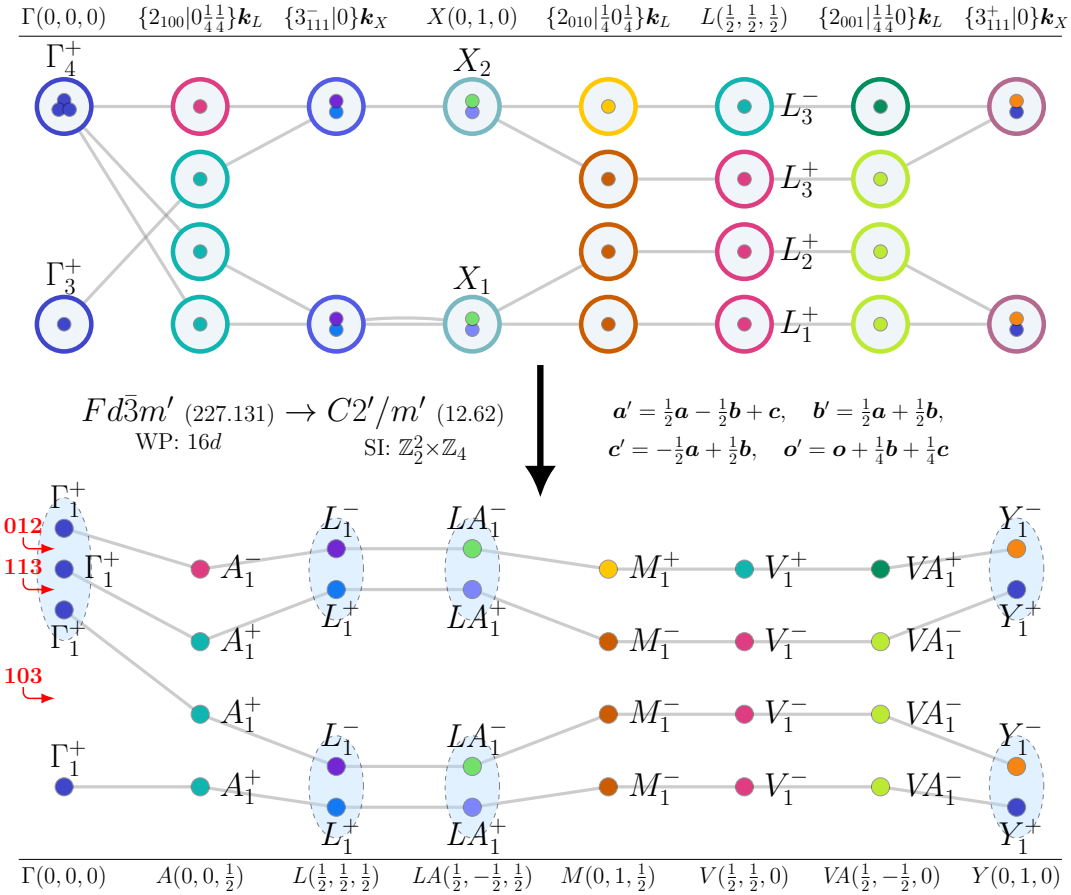
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Gap#	Possible SI Values
3	001, 003, 010, 012, 101 103, 110, 112
4	000

46.2.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 232: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position 16d of supergroup $Fd\bar{3}m'$ (227.131).

Supplementary Table 453: Number of times a given SI can appear in the band structure. Supergroup: $Fd\bar{3}m'$ (227.131), WP: 16d, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1, 2
003	0, 1, 2
010	0, 1, 2
012	0, 1
101	0, 1, 2
103	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 454: Possible SI values in each gap. Supergroup: $Fd\bar{3}m'$ (227.131), WP: $16d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

47 MSG $F_Sd\bar{3}c$ (228.139)

Nontrivial-SI Subgroups: C_ac (9.41), C_ac (9.41), $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), $R\bar{3}m'$ (166.101), $R_I\bar{3}c$ (167.108), I_aba2 (45.240), $C2/c$ (15.85), $Iba'm'$ (72.544), C_c2/c (15.90), I_cbca (73.553), C_a2 (5.17), $C2/c$ (15.85), C_a2/c (15.91), I_c4_1cd (110.250), $I4_1/am'd'$ (141.557), I_c4_1/acd (142.570).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), Cc' (9.39), Cc' (9.39), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), C_cc (9.40), C_cc (9.40), Cc (9.37), C_cc (9.40), Cc (9.37), $R3m'$ (160.67), R_I3c (161.72), $C2$ (5.13), $Im'a'2$ (46.245), C_c2 (5.16), $C2$ (5.13), $I4_1m'd'$ (109.243).

47.1 WP: $32b$

BCS Materials: NpTe (40 K)[205], NpSe (38 K)[206], NpS (23 K)[207].

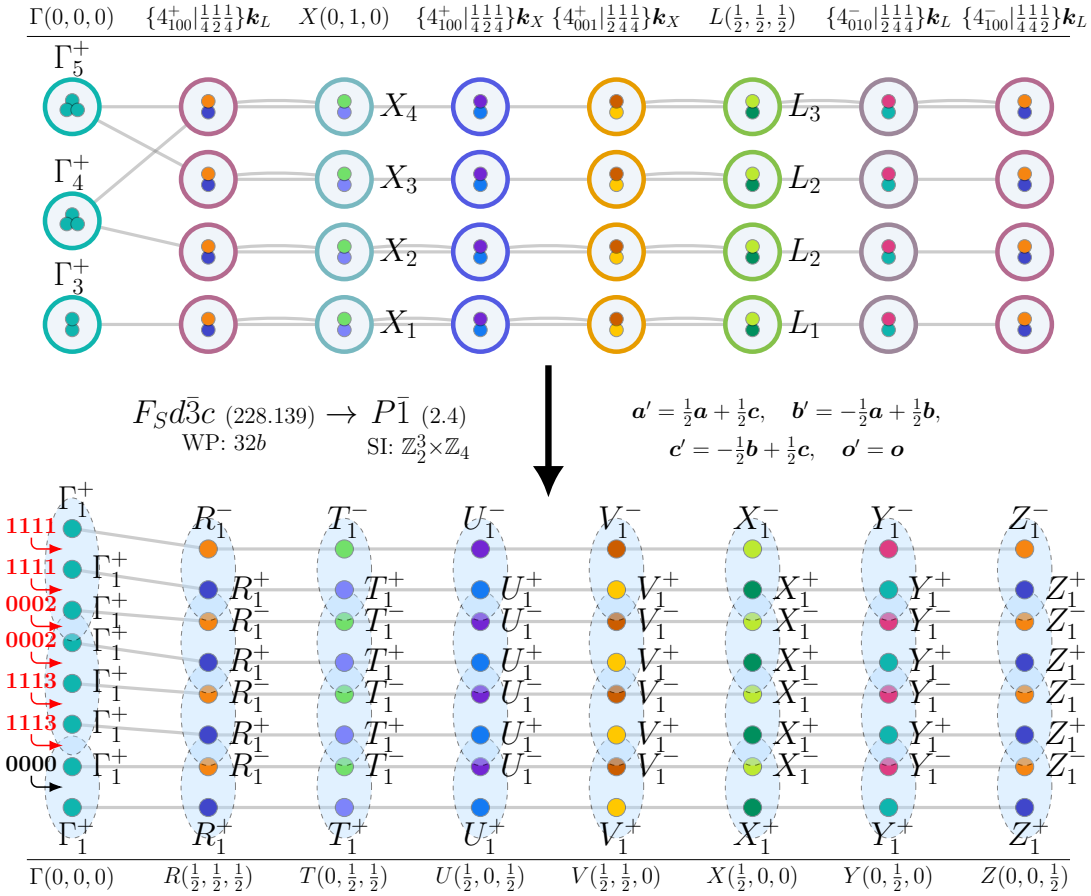
47.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain ⊥ [110],
- B ∥ [100] and strain in generic direction,
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain in generic direction,
- B ∥ [111] and strain ⊥ [100],
- B ∥ [111] and strain ⊥ [110],
- B ∥ [111] and strain in generic direction,
- B ⊥ [100] and strain ∥ [110],

[205]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.11`
[206]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.10`
[207]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.9`

- B ⊥ [100] and strain ∥ [111],
- B ⊥ [100] and strain ⊥ [110],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ∥ [111],
- B ⊥ [110] and strain ⊥ [100],
- B ⊥ [110] and strain in generic direction,
- B in generic direction.



Supplementary Figure 233: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position 32b of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 455: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3, 4
0002	1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4

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SI	Possible #Gaps
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	1, 2, 3
1112	0, 1, 2, 3, 4
1113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

Supplementary Table 456: Possible SI values in each gap. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
2	1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
4	0002
5	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
6	1111
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112

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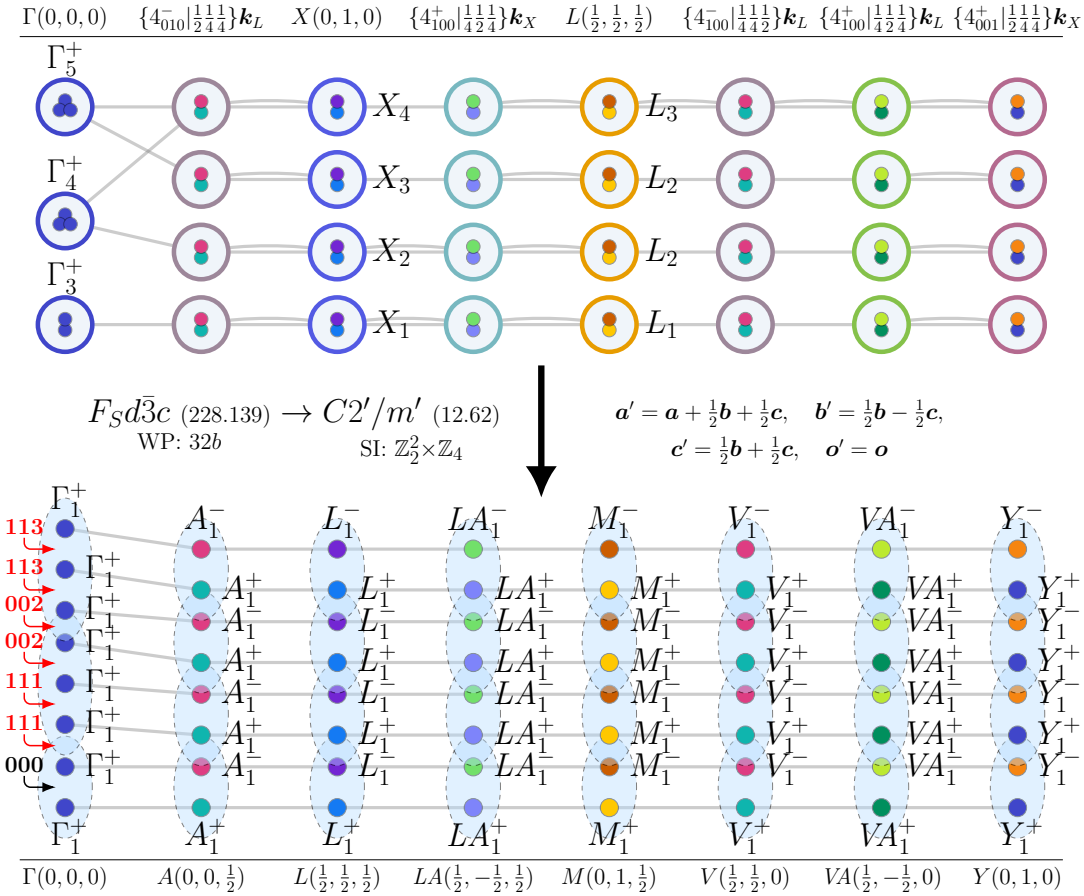
...continued

Gap#	Possible SI Values
8	0000

47.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].



Supplementary Figure 234: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position 32b of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 457: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3, 4
002	1, 2, 3
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
011	0, 1, 2
012	0, 1, 2, 3, 4
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3, 4

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...continued

SI	Possible #Gaps
102	0, 1, 2
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	1, 2, 3
112	0, 1, 2, 3, 4
113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

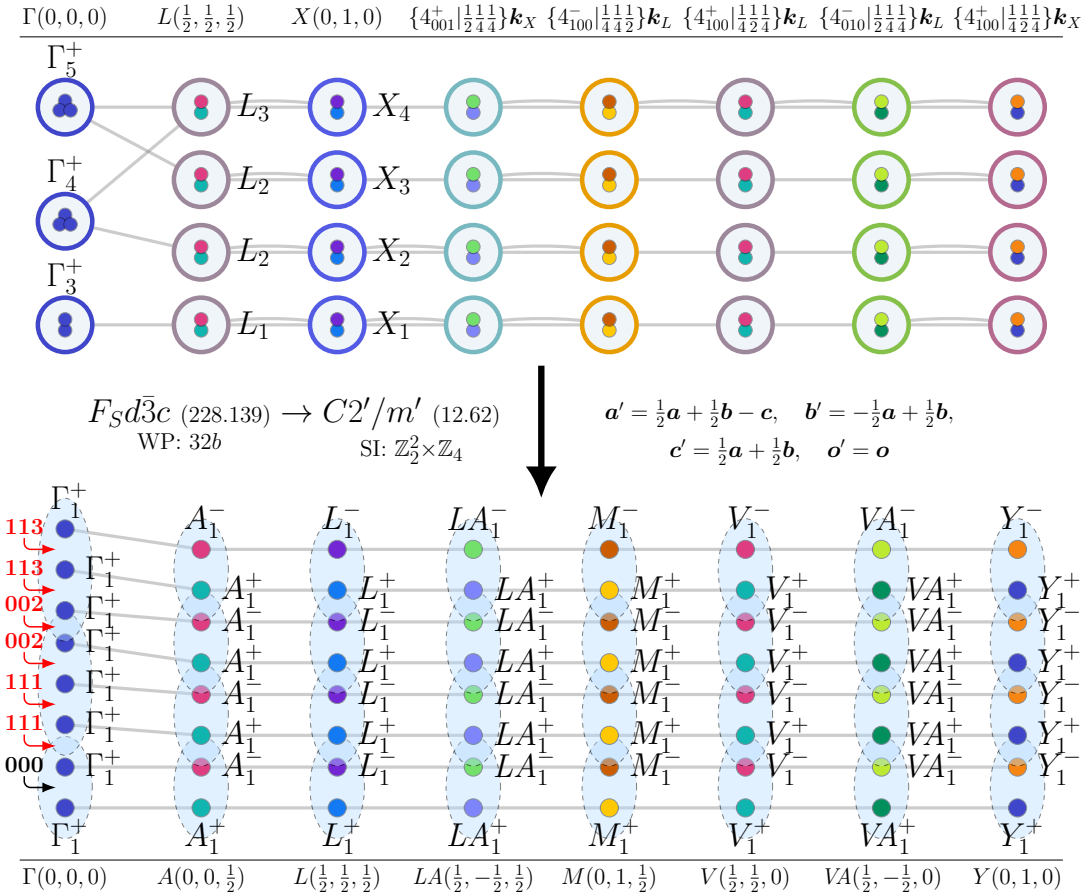
Supplementary Table 458: Possible SI values in each gap. Supergroup: $F_S d\bar{3}c$ (228.139), WP: 32*b*, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	111
3	001, 002, 003, 010, 011 012, 101, 102, 103, 110 111, 112
4	002
5	001, 002, 003, 010, 012 013, 101, 102, 103, 110 112, 113
6	113
7	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
8	000

47.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [110] and strain ∥ [111],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 235: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $32b$ of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 459: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: $32b$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3, 4
002	1, 2, 3
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
011	0, 1, 2
012	0, 1, 2, 3, 4
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3, 4
102	0, 1, 2
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	1, 2, 3
112	0, 1, 2, 3, 4
113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

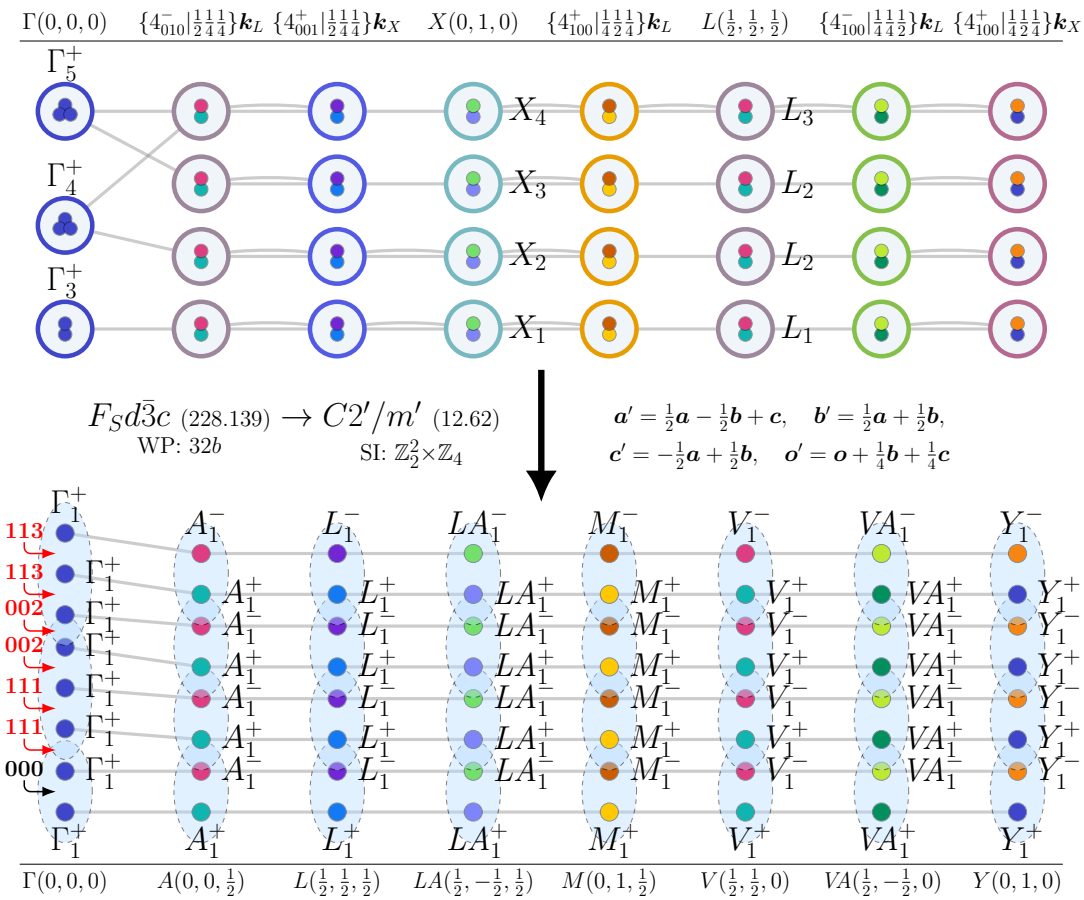
Supplementary Table 460: Possible SI values in each gap. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: $32b$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	111
3	001, 002, 003, 010, 011 012, 101, 102, 103, 110 111, 112
4	002
5	001, 002, 003, 010, 012 013, 101, 102, 103, 110 112, 113
6	113
7	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
8	000

47.1.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbation:

- B $\perp$ [110].



Supplementary Figure 236: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $32b$ of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 461: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3, 4
002	1, 2, 3
003	0, 1, 2, 3, 4
010	0, 1, 2, 3, 4
011	0, 1, 2
012	0, 1, 2, 3, 4
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3, 4
102	0, 1, 2
103	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	1, 2, 3
112	0, 1, 2, 3, 4
113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

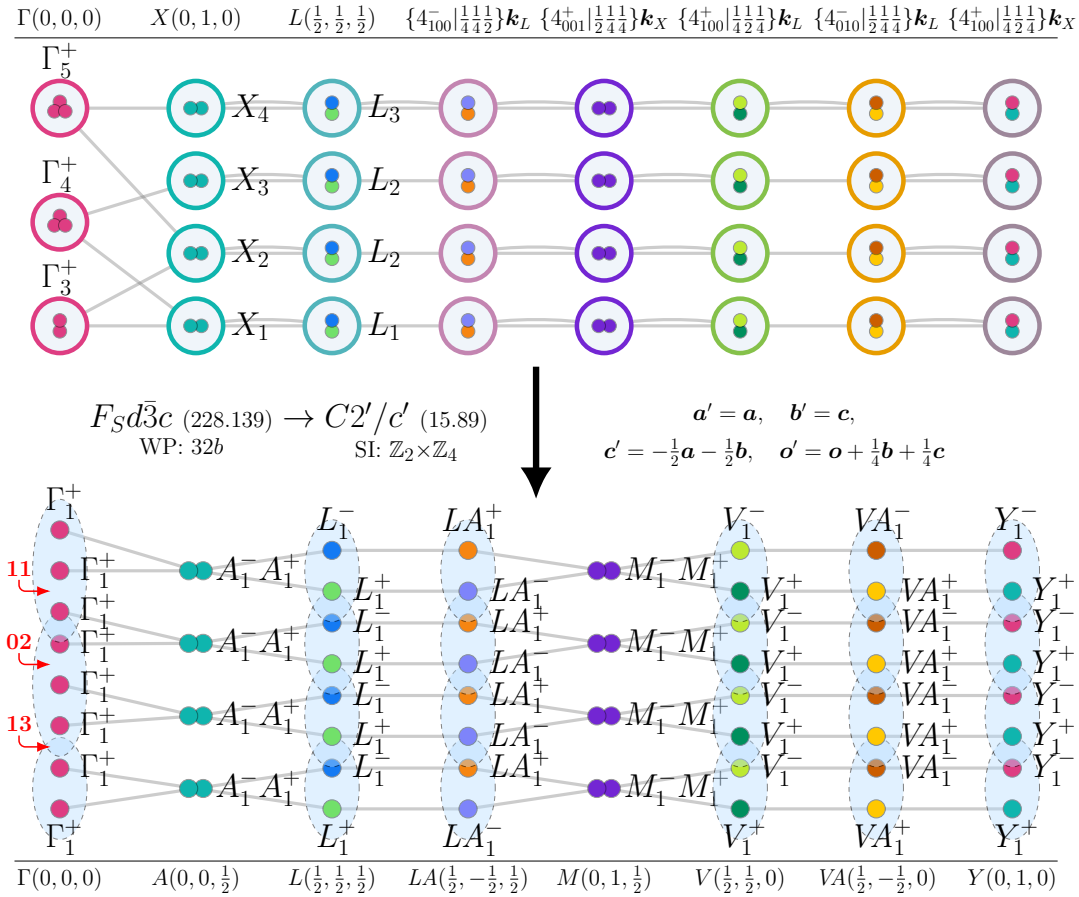
Supplementary Table 462: Possible SI values in each gap. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	111
3	001, 002, 003, 010, 011 012, 101, 102, 103, 110 111, 112
4	002
5	001, 002, 003, 010, 012 013, 101, 102, 103, 110 112, 113
6	113
7	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
8	000

47.1.5 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [100] and strain ∥ [110],
- B ∥ [110] and strain ∥ [100].



Supplementary Figure 237: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $32b$ of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 463: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: $32b$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	1
11	1
13	1
gapless	4
nontrivial	3

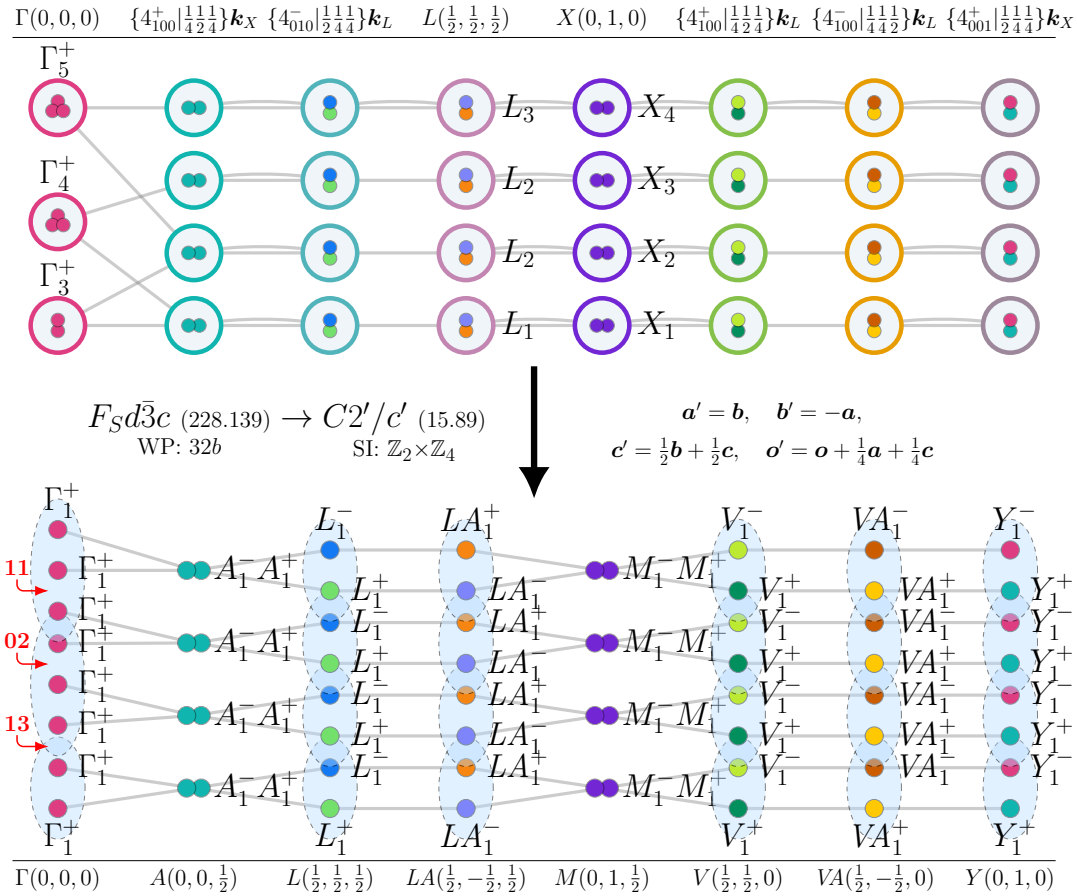
Supplementary Table 464: Possible SI values in each gap. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: $32b$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	13
3	G
4	02
5	G
6	11
7	G
8	00

47.1.6 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- B $\perp$ [100].



Supplementary Figure 238: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 32b of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 465: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	1
11	1
13	1
gapless	4
nontrivial	3

Supplementary Table 466: Possible SI values in each gap. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	13
3	G

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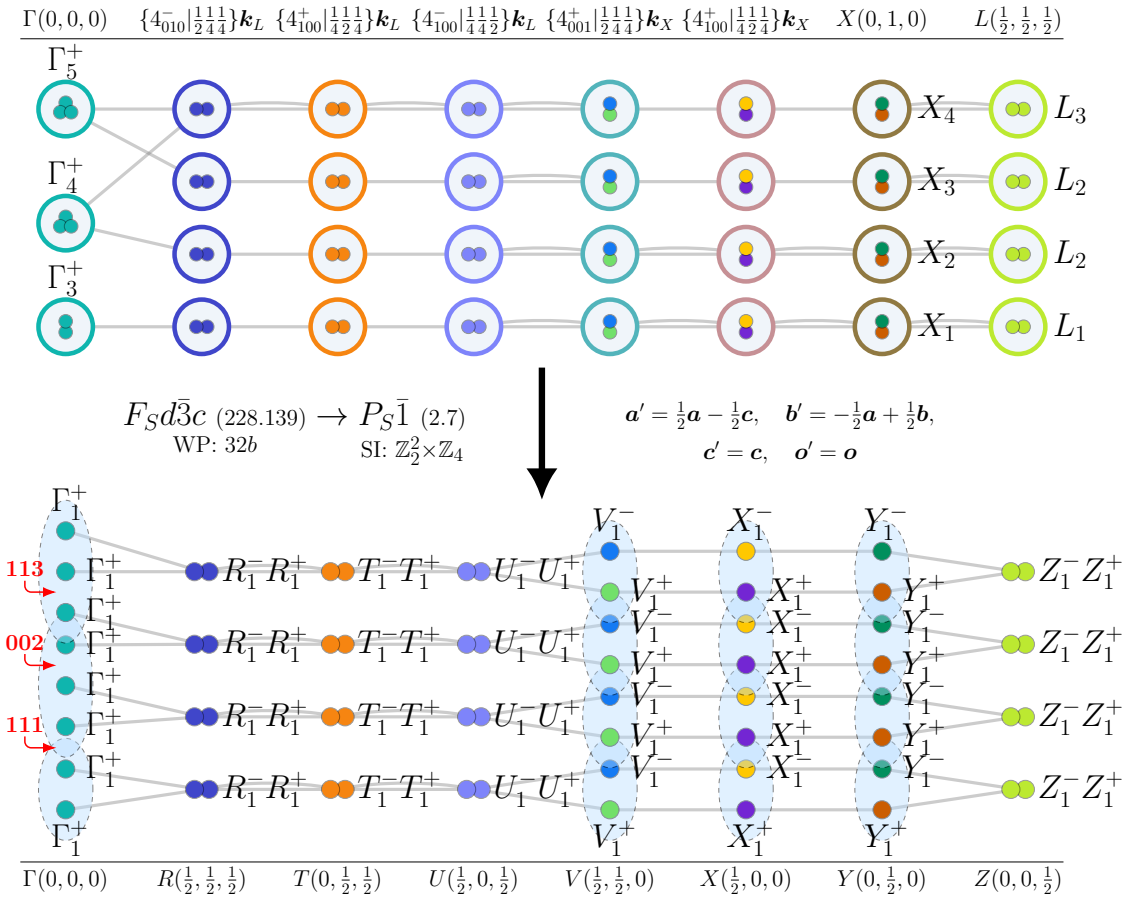
...continued

Gap#	Possible SI Values
4	02
5	G
6	11
7	G
8	00

47.1.7 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 239: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position 32b of supergroup $F_Sd\bar{3}c$ (228.139).

Supplementary Table 467: Number of times a given SI can appear in the band structure. Supergroup: $F_Sd\bar{3}c$ (228.139), WP: 32b, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

Supplementary Table 468: Possible SI values in each gap. Supergroup: $F_S d\bar{3}c$ (228.139), WP: $32b$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

48 MSG $Im\bar{3}m'$ (229.143)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $R\bar{3}m'$ (166.101), $Fm'm'm$ (69.524), $Fm'm'2$ (42.222), $C2/m$ (12.58), $Fm'm'm$ (69.524).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), Cm (8.32), $Fm'm2'$ (42.221), Cm (8.32), $R3m'$ (160.67), $C2$ (5.13), $I4'mm'$ (107.230), $I4'/mmm'$ (139.535).

48.1 WP: $8c$

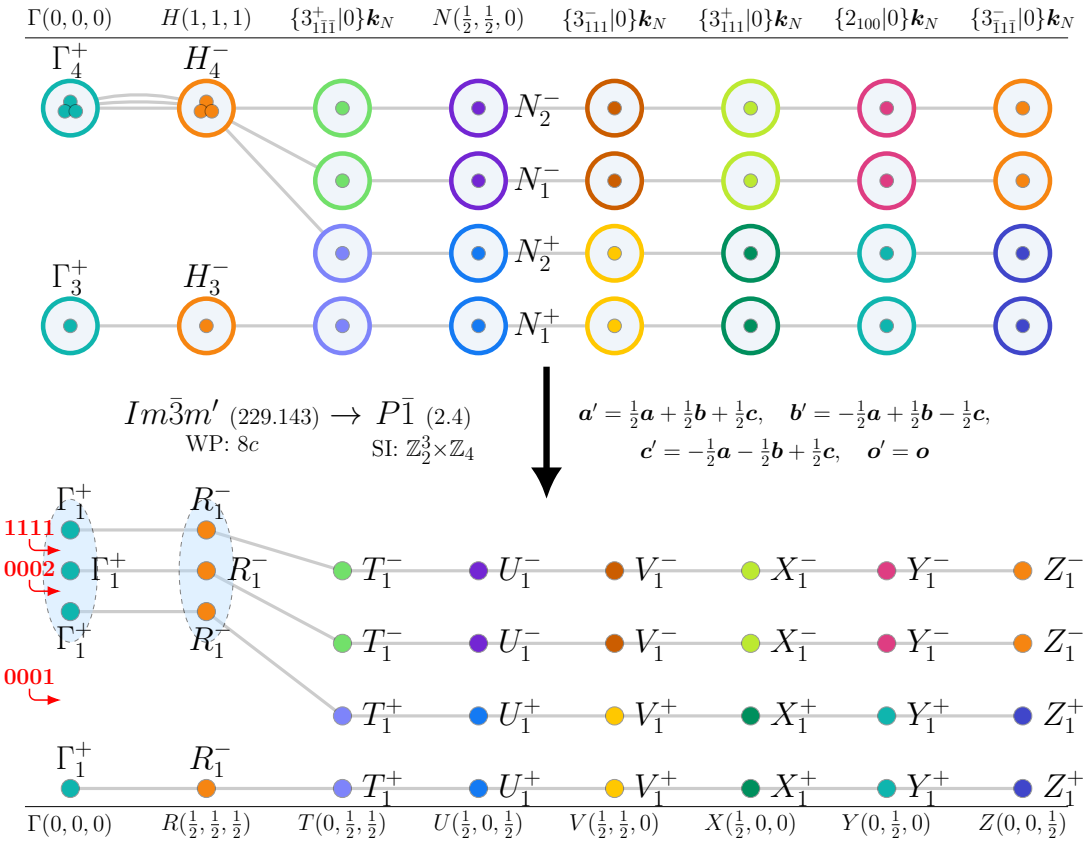
BCS Materials: DyCu (64 K)[208].

48.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ∥ [110],
- B ∥ [100] and strain ⊥ [110],
- B ∥ [110] and strain ∥ [100],
- B ∥ [110] and strain ⊥ [100],
- B ∥ [110] and strain ⊥ [110],
- B ∥ [111] and strain ⊥ [100],
- B ∥ [111] and strain ⊥ [110],
- B in generic direction,
- B ⊥ [110] and strain ∥ [100],
- B ⊥ [110] and strain ∥ [111],
- B ⊥ [110] and strain ⊥ [100].

[208] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=3.6`



Supplementary Figure 240: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8c$ of supergroup $Im\bar{3}m'$ (229.143).

Supplementary Table 469: Number of times a given SI can appear in the band structure. Supergroup: $Im\bar{3}m'$ (229.143), WP: $8c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1
0002	0, 1
0003	0, 1
1110	0, 1
1111	0, 1
1113	0, 1
gapless	0
nontrivial	3

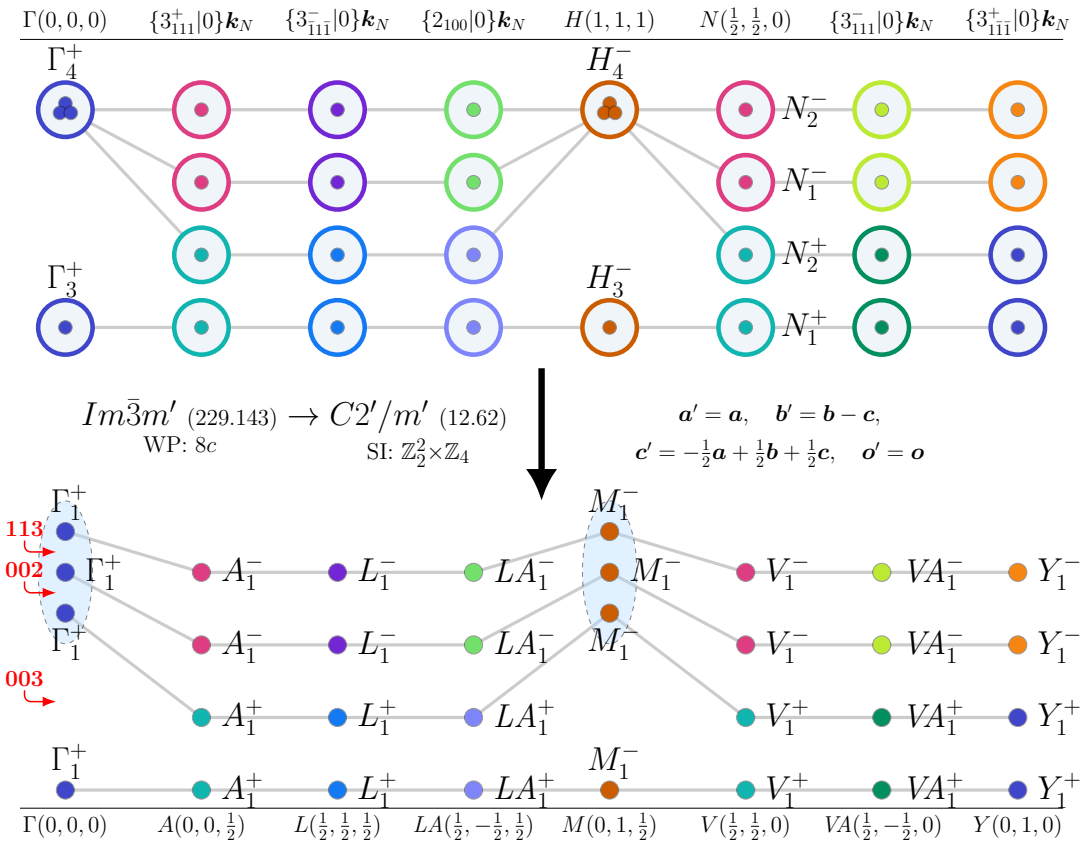
Supplementary Table 470: Possible SI values in each gap. Supergroup: $Im\bar{3}m'$ (229.143), WP: $8c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 1113
2	0002, 1110
3	0003, 1111
4	0000

48.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].



Supplementary Figure 241: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8c$ of supergroup $Im\bar{3}m'$ (229.143).

Supplementary Table 471: Number of times a given SI can appear in the band structure. Supergroup: $Im\bar{3}m'$ (229.143), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1
002	0, 1
003	0, 1
110	0, 1
111	0, 1
113	0, 1
gapless	0
nontrivial	3

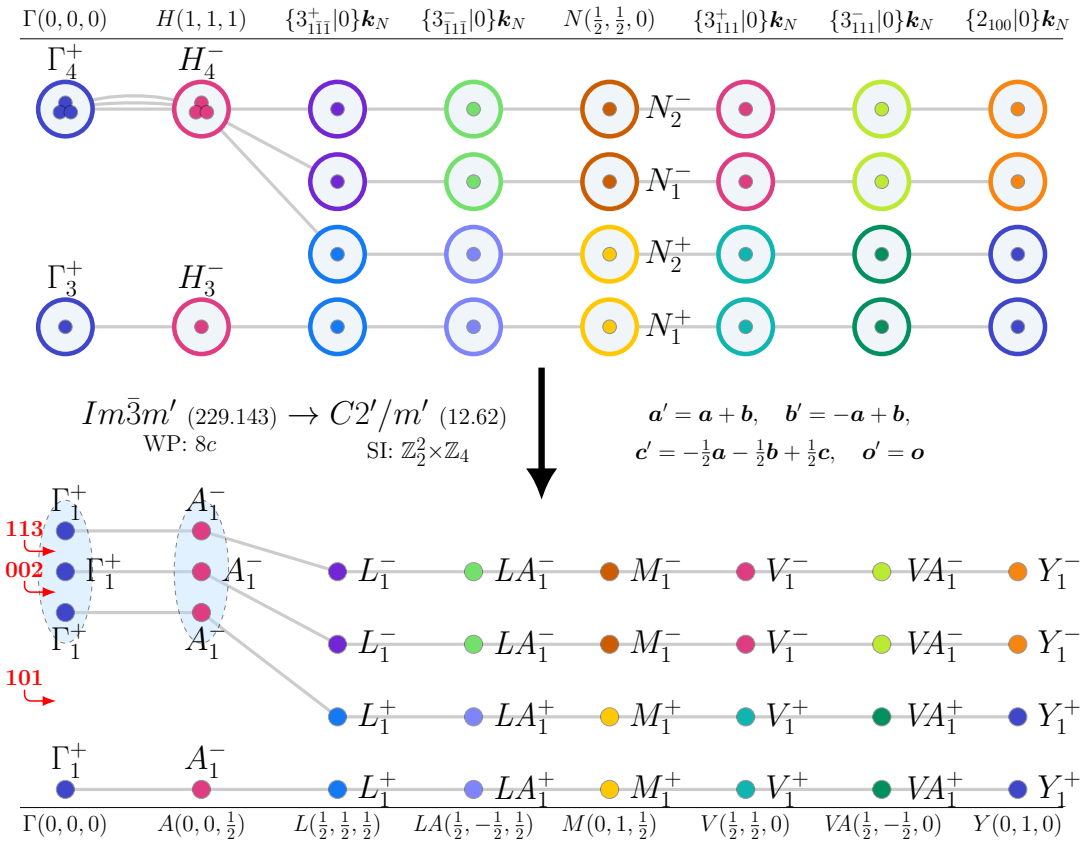
Supplementary Table 472: Possible SI values in each gap. Supergroup: $Im\bar{3}m'$ (229.143), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	003, 111
2	002, 110
3	001, 113
4	000

48.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [110],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 242: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position 8c of supergroup $Im\bar{3}m'$ (229.143).

Supplementary Table 473: Number of times a given SI can appear in the band structure. Supergroup: $Im\bar{3}m'$ (229.143), WP: 8c, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
002	0, 1
012	0, 1
101	0, 1
103	0, 1
111	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 474: Possible SI values in each gap. Supergroup: $Im\bar{3}m'$ (229.143), WP: 8c, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	101, 111

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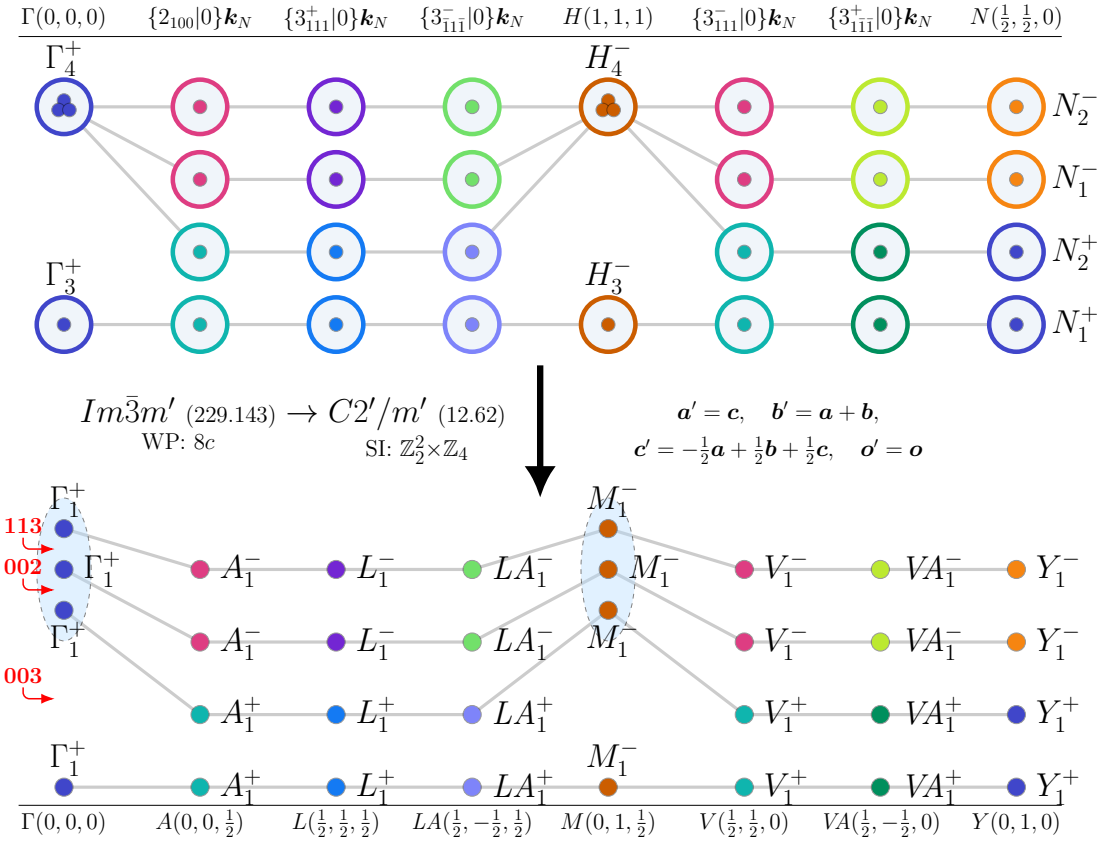
...continued

Gap#	Possible SI Values
2	002, 012
3	103, 113
4	000

48.1.4 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 243: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8c$ of supergroup $Im\bar{3}m'$ (229.143).

Supplementary Table 475: Number of times a given SI can appear in the band structure. Supergroup: $Im\bar{3}m'$ (229.143), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1
002	0, 1
003	0, 1
110	0, 1
111	0, 1
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 476: Possible SI values in each gap. Supergroup: $Im\bar{3}m'$ (229.143), WP: $8c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	003, 111
2	002, 110
3	001, 113
4	000

49 MSG $Ia\bar{3}d'$ (230.148)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $R\bar{3}c'$ (167.107), $Fd'd'd$ (70.530), $I4_1'cd'$ (110.248), $C2/c$ (15.85), $Fd'd'd$ (70.530), $I4_1'/acd'$ (142.565).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), Cc' (9.39), $C2'$ (5.15), Cc (9.37), $Fd'd2'$ (43.226), Cc (9.37), $R3c'$ (161.71), $C2$ (5.13), $Fd'd'2$ (43.227).

49.1 WP: $24c$

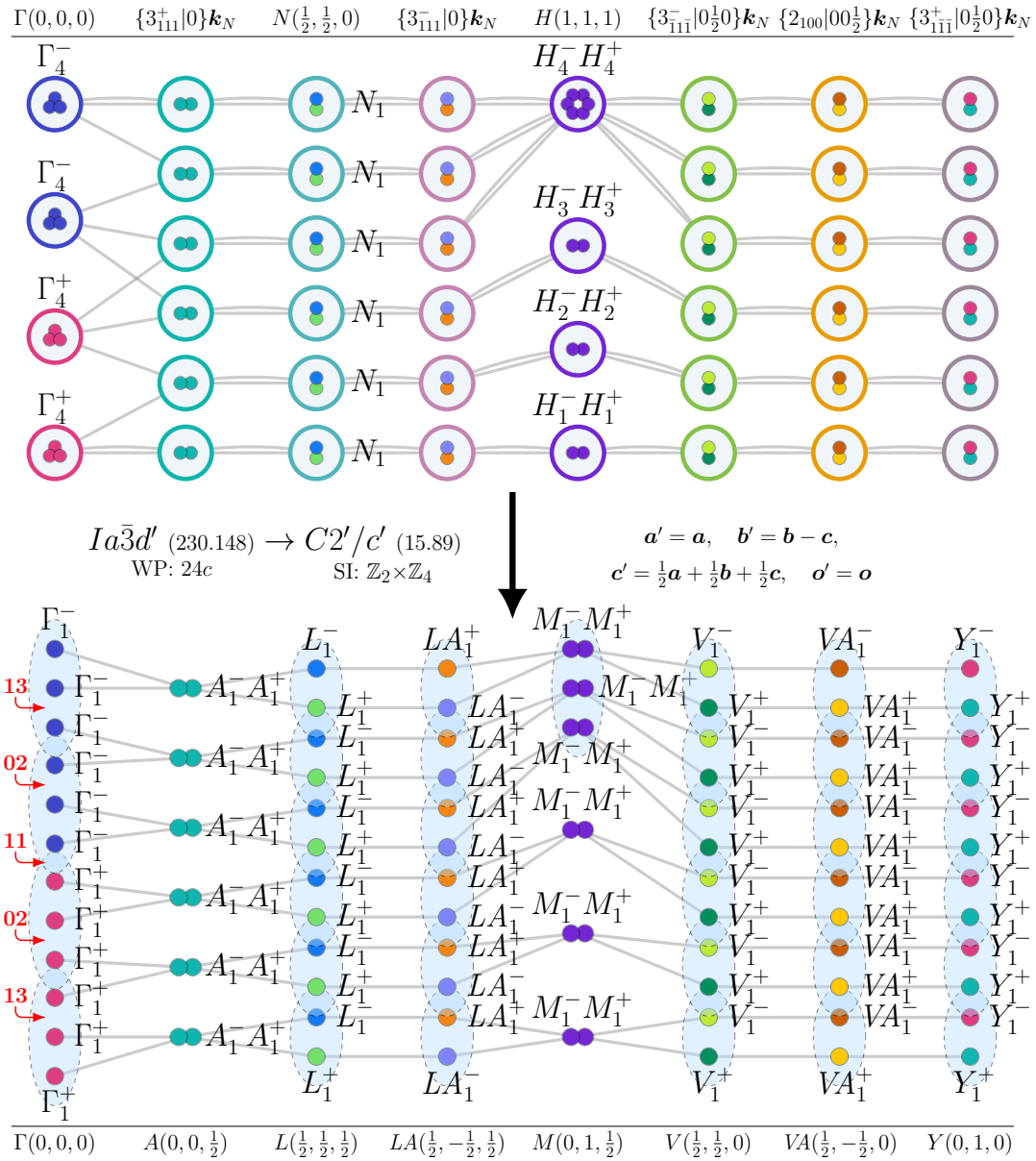
BCS Materials: $Dy_3Al_5O_{12}$ (2.49 K)[209], $Tb_3Al_5O_{12}$ (1.35 K)[210], $Ho_3Al_5O_{12}$ (0.85 K)[211], $Er_3Ga_5O_{12}$ (0.8 K)[212], $Dy_3Ga_5O_{12}$ (0.37 K)[213], $Tb_3Ga_5O_{12}$ (0.25 K)[214], $Tb_3Ga_5O_{12}$ (0.24 K)[215], $Ho_3Ga_5O_{12}$ (0.15 K)[216].

49.1.1 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [100] and strain ∥ [111],
- B ∥ [111] and strain ∥ [100].

[209]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.127`
[210]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.744`
[211]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.743`
[212]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.741`
[213]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.740`
[214]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.742`
[215]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.746`
[216]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.745`



Supplementary Figure 244: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 24c of supergroup $Ia\bar{3}d'$ (230.148).

Supplementary Table 477: Number of times a given SI can appear in the band structure. Supergroup: $Ia\bar{3}d'$ (230.148), WP: 24c, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 2
11	0, 1, 2, 4
13	0, 1, 2, 4
gapless	6
nontrivial	4, 5

3263

Supplementary Table 478: Possible SI values in each gap. Supergroup: $Ia\bar{3}d'$ (230.148), WP: 24c, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13

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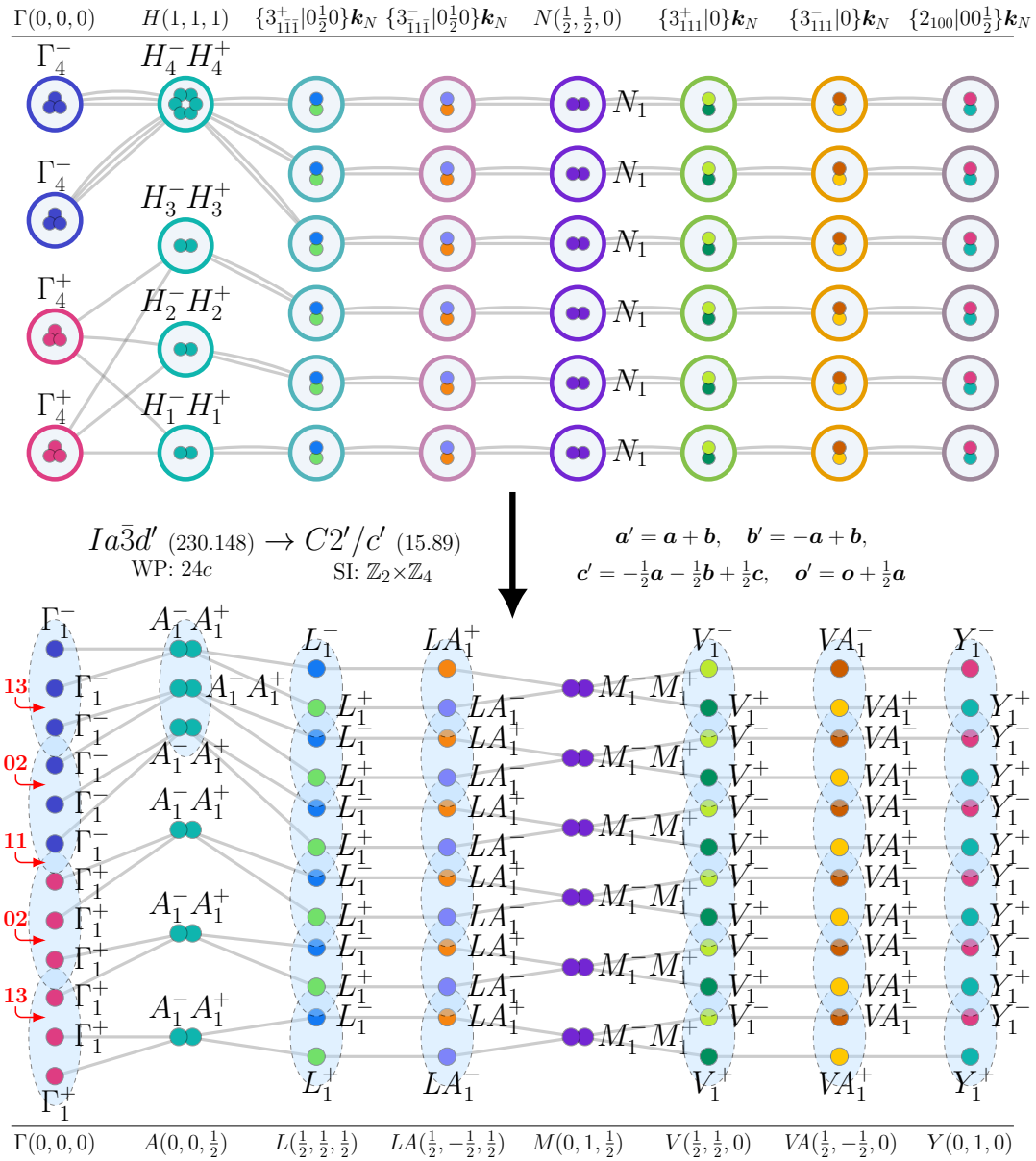
...continued

Gap#	Possible SI Values
3	G
4	02, 11, 13
5	G
6	00, 11, 13
7	G
8	02, 11, 13
9	G
10	11, 13
11	G
12	00

49.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [110],
- B ∥ [111] and strain ∥ [110].



Supplementary Figure 245: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 24c of supergroup $Ia\bar{3}d'$ (230.148).

Supplementary Table 479: Number of times a given SI can appear in the band structure. Supergroup: $Ia\bar{3}d'$ (230.148), WP: 24c, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 2
11	0, 1, 2, 4
13	0, 1, 2, 4
gapless	6
nontrivial	4, 5

Supplementary Table 480: Possible SI values in each gap. Supergroup: $Ia\bar{3}d'$ (230.148), WP: 24c, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13

continued on next page...

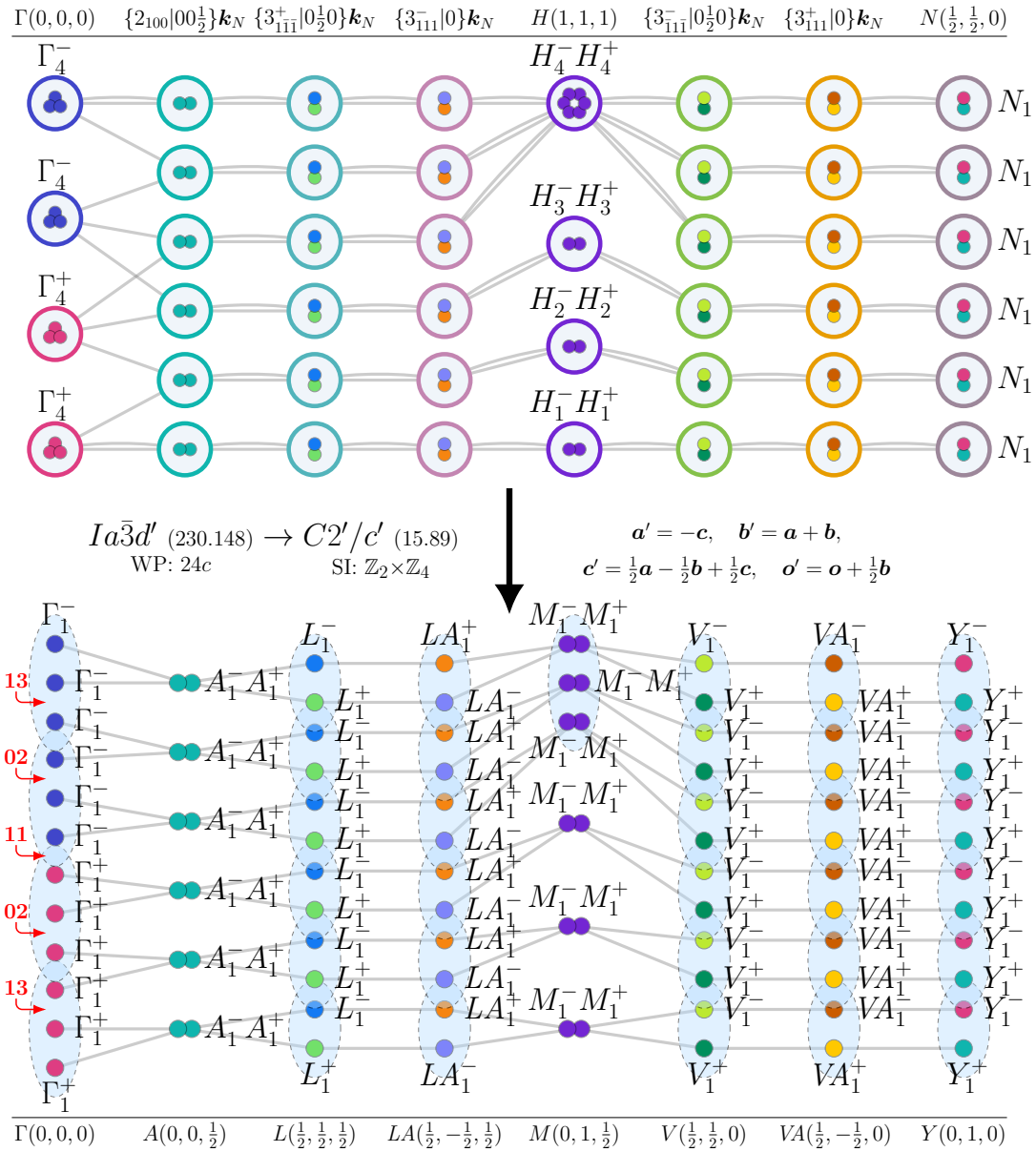
...continued

Gap#	Possible SI Values
3	G
4	02, 11, 13
5	G
6	00, 11, 13
7	G
8	02, 11, 13
9	G
10	11, 13
11	G
12	00

49.1.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [110],
- B $\perp$ [110].



Supplementary Figure 246: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $24c$ of supergroup $Ia\bar{3}d'$ (230.148).

Supplementary Table 481: Number of times a given SI can appear in the band structure. Supergroup: $Ia\bar{3}d'$ (230.148), WP: $24c$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 2
11	0, 1, 2, 4
13	0, 1, 2, 4
gapless	6
nontrivial	4, 5

Supplementary Table 482: Possible SI values in each gap. Supergroup: $Ia\bar{3}d'$ (230.148), WP: $24c$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13

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Gap#	Possible SI Values
3	G
4	02, 11, 13
5	G
6	00, 11, 13
7	G
8	02, 11, 13
9	G
10	11, 13
11	G
12	00

50 MSG $Pn'n'n$ (48.260)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'/c'$ (13.69), $P2$ (3.1), $P2/c$ (13.65).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2'$ (3.3), Pc (7.24), $Pn'n2'$ (34.158), $Pn'n2'$ (34.158), $Pn'n'2$ (34.159).

50.1 WP: $2b + 2c + 2d + 4f$

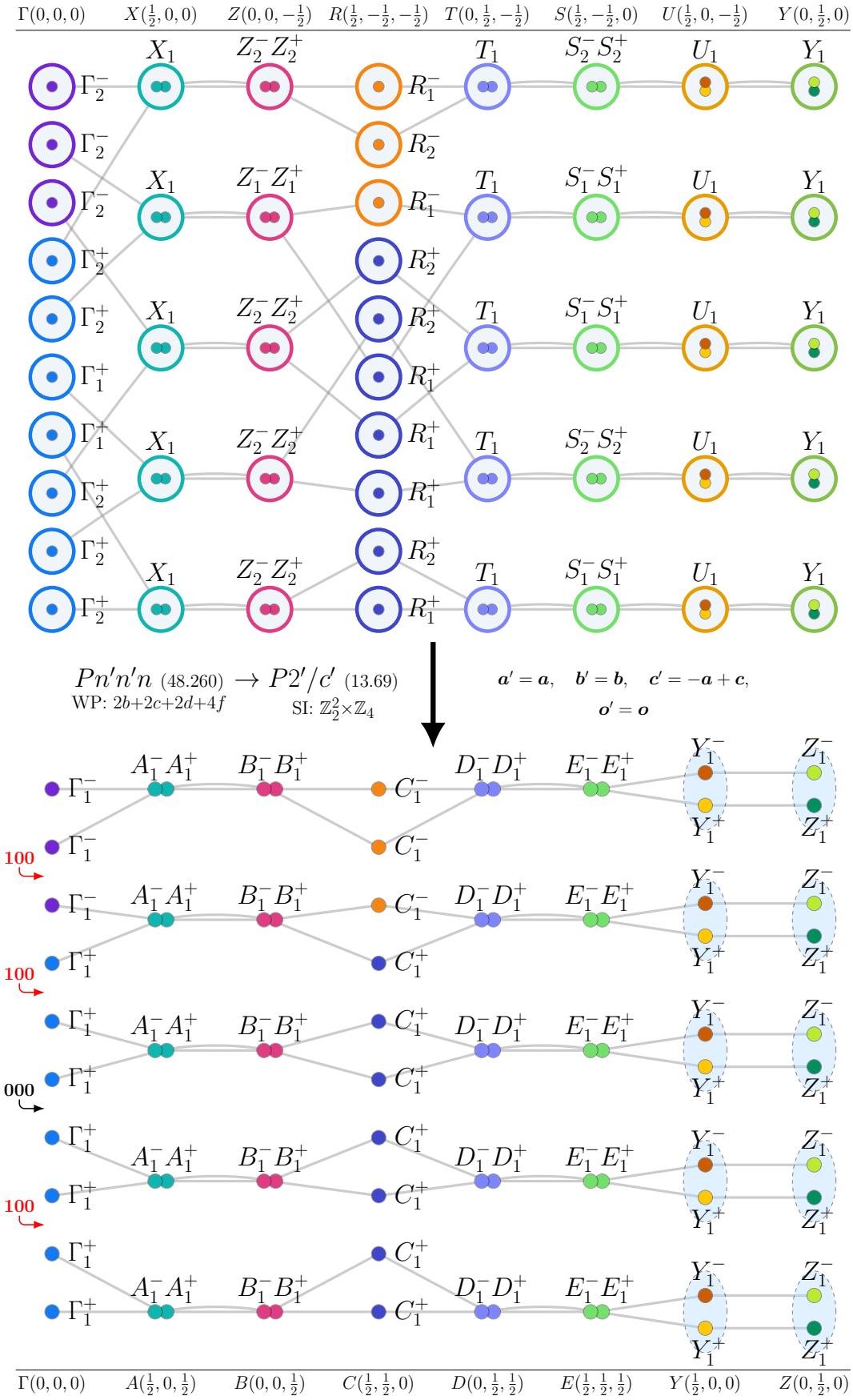
BCS Materials: $CaCu_3Fe_2Sb_2O_{12}$ (160 K)[217].

50.1.1 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).

[217] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.672`



Supplementary Figure 247: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $2b+2c+2d+4f$ of supergroup $Pn'n'n$ (48.260).

Supplementary Table 483: Number of times a given SI can appear in the band structure. Supergroup: $Pn'n'n$ (48.260), WP: 2b+2c+2d+4f, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2, 3
002	0, 1, 2
011	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2, 3, 4
102	0, 1, 2, 3
111	0, 1, 2, 3
113	0, 1, 2, 3
gapless	5
nontrivial	1, 2, 3, 4

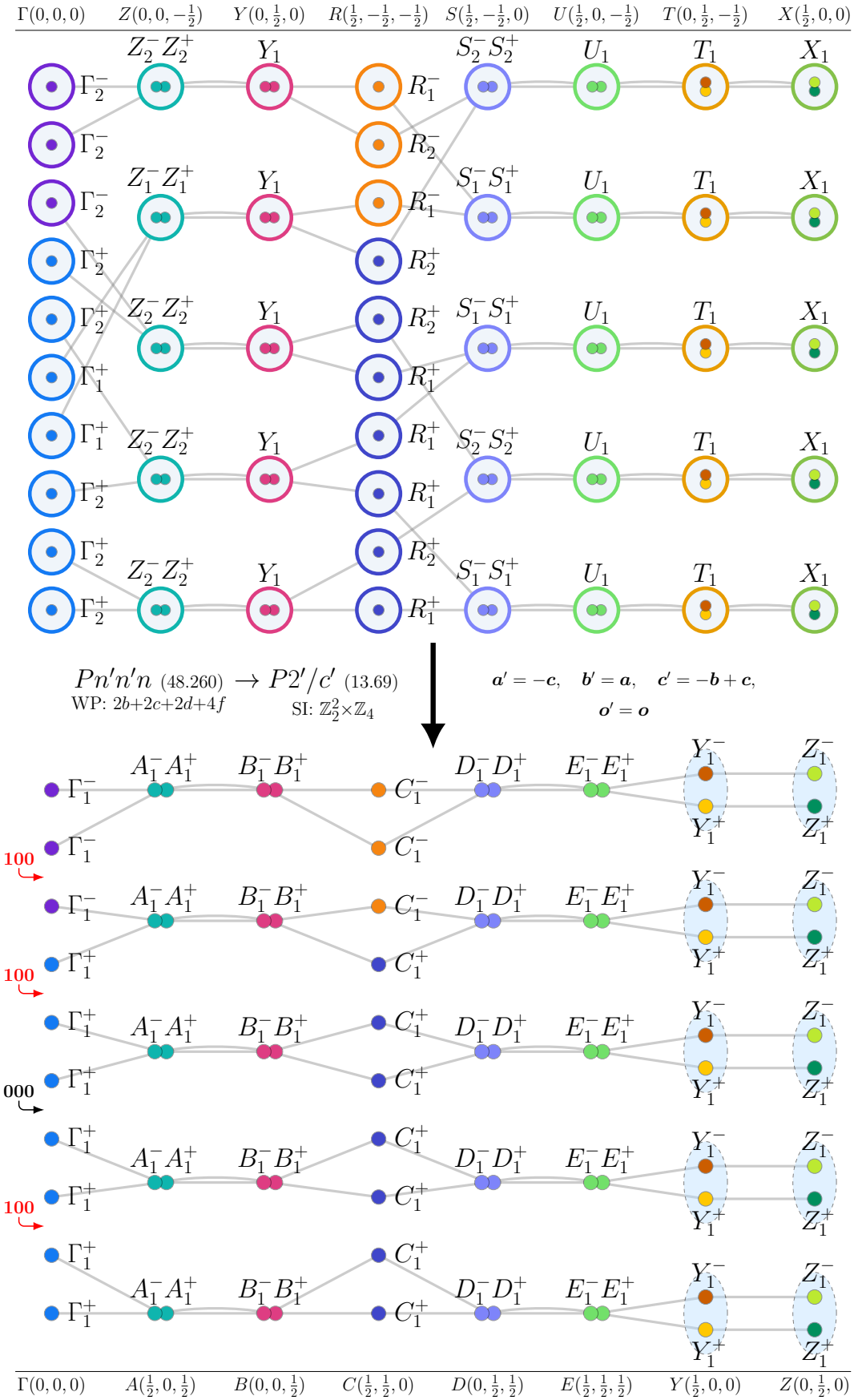
Supplementary Table 484: Possible SI values in each gap. Supergroup: $Pn'n'n$ (48.260), WP: 2b+2c+2d+4f, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	000, 011, 013, 100, 102 111, 113
3	G
4	000, 002, 011, 013, 100 102, 111, 113
5	G
6	000, 002, 011, 013, 100 102, 111, 113
7	G
8	000, 011, 013, 100, 102 111, 113
9	G
10	000

50.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 248: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $2b + 2c + 2d + 4f$ of supergroup $Pn'n'n$ (48.260).

Supplementary Table 485: Number of times a given SI can appear in the band structure. Supergroup: $Pn'n'n$ (48.260), WP: 2b+2c+2d+4f, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2, 3
002	0, 1, 2
011	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2, 3, 4
102	0, 1, 2, 3
111	0, 1, 2, 3
113	0, 1, 2, 3
gapless	5
nontrivial	1, 2, 3, 4

Supplementary Table 486: Possible SI values in each gap. Supergroup: $Pn'n'n$ (48.260), WP: 2b+2c+2d+4f, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	000, 011, 013, 100, 102 111, 113
3	G
4	000, 002, 011, 013, 100 102, 111, 113
5	G
6	000, 002, 011, 013, 100 102, 111, 113
7	G
8	000, 011, 013, 100, 102 111, 113
9	G
10	000

51 MSG $Pb'an'$ (50.282)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'/c'$ (13.69), $P2$ (3.1), $P2/c$ (13.65).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2'$ (3.3), Pc (7.24), $Pb'a2'$ (32.137), $Pn'c2'$ (30.113), $Pn'c'2$ (30.115).

51.1 WP: $4f + 4f$

BCS Materials: $CeMnCoGe_4O_{12}$ (6 K)[218].

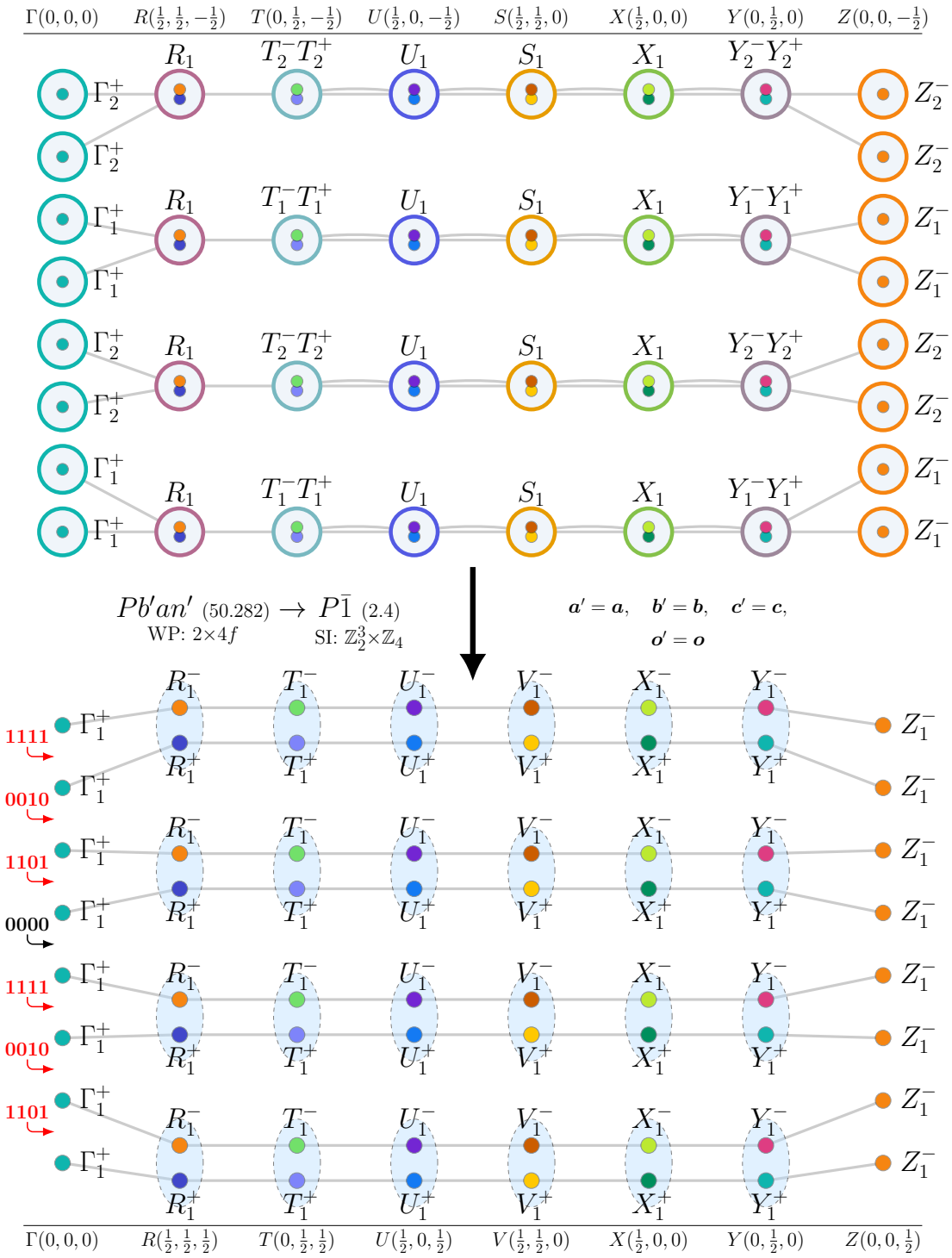
51.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,

[218]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.190`

- (B ∥ [100] or B ⊥ [001]) and strain ⊥ [100],
- (B ∥ [100] or B ⊥ [001]) and strain ⊥ [010],
- (B ∥ [001] or B ⊥ [100]) and strain ⊥ [010],
- (B ∥ [001] or B ⊥ [100]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 249: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4f + 4f$ of supergroup $Pb'an'$ (50.282).

Supplementary Table 487: Number of times a given SI can appear in the band structure. Supergroup: $Pb'an'$ (50.282), WP: $4f$+$4f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	1, 2, 3, 4, 5
0001	0, 1, 2, 3, 4
0003	0, 1, 2, 3, 4
0010	2, 3, 4, 5, 6
0011	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2
1102	0, 1, 2, 3, 4
1103	0, 1, 2
1110	0, 1, 2, 3, 4
1111	0, 1, 2
1112	0, 1, 2, 3, 4
1113	0, 1, 2
gapless	0
nontrivial	2, 3, 4, 5, 6

Supplementary Table 488: Possible SI values in each gap. Supergroup: $Pb'an'$ (50.282), WP: $4f$+$4f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1112, 1113
2	0010
3	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1111, 1112
4	0000

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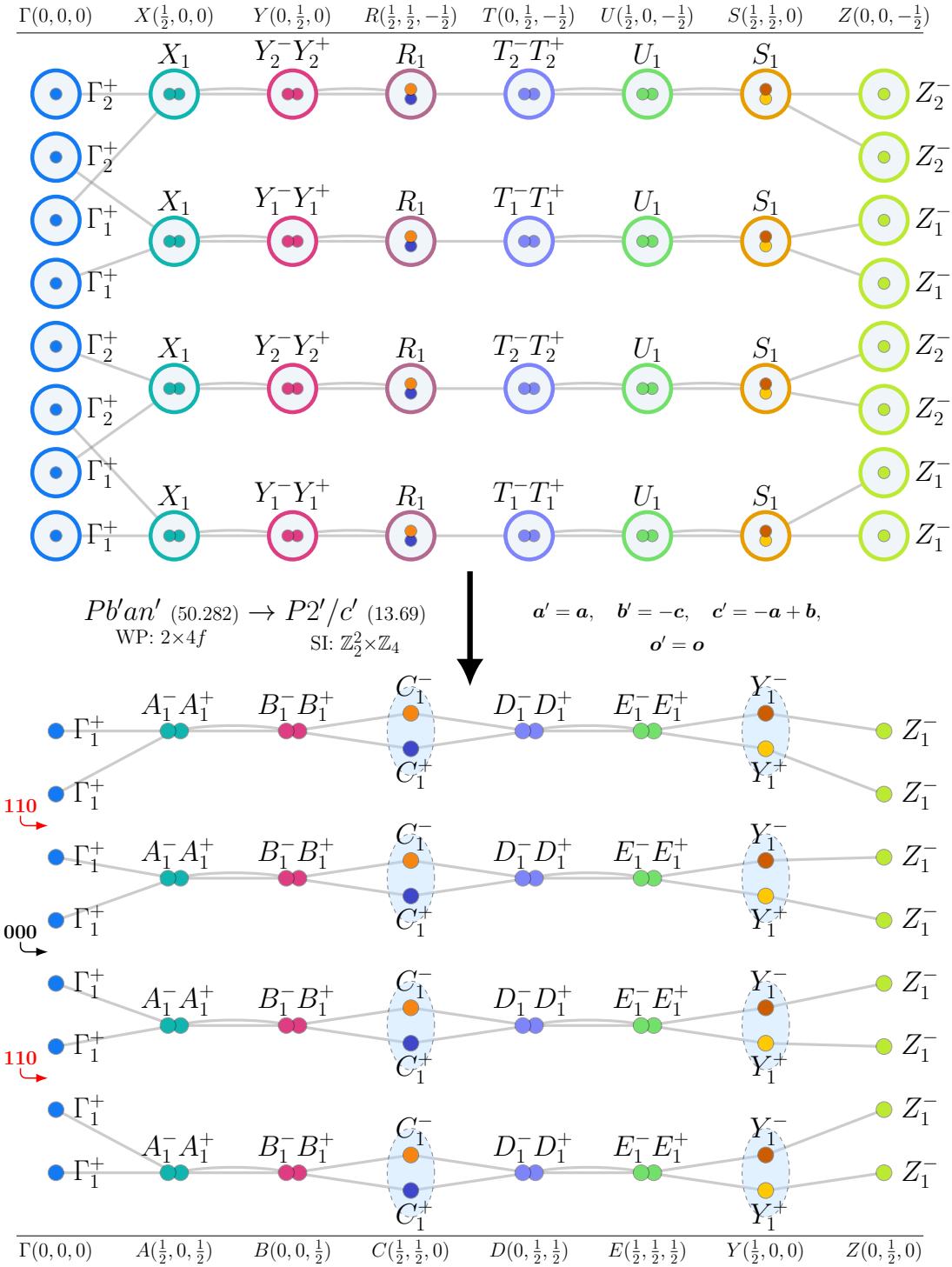
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Gap#	Possible SI Values
5	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1112, 1113
6	0010
7	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1111, 1112
8	0000

51.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).



Supplementary Figure 250: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4f+4f$ of supergroup $Pb'an'$ (50.282).

Supplementary Table 489: Number of times a given SI can appear in the band structure. Supergroup: $Pb'an'$ (50.282), WP: $4f$+$4f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	1
110	2
gapless	4
nontrivial	2

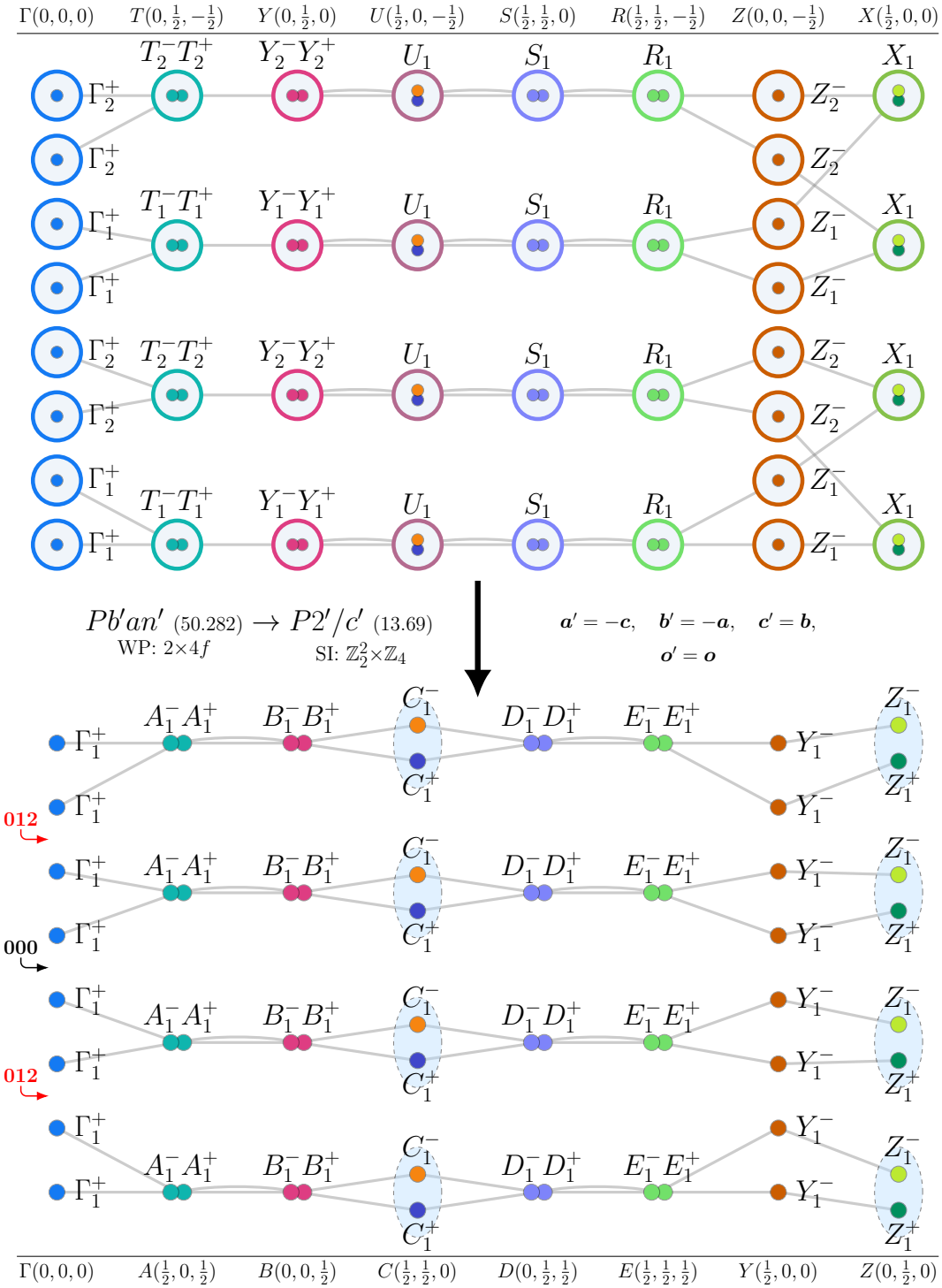
Supplementary Table 490: Possible SI values in each gap. Supergroup: $Pb'an'$ (50.282), WP: $4f$+$4f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000
5	G
6	110
7	G
8	000

51.1.3 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [100]).



Supplementary Figure 251: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4f + 4f$ of supergroup $Pb'an'$ (50.282).

Supplementary Table 491: Number of times a given SI can appear in the band structure. Supergroup: $Pb'an'$ (50.282), WP: $4f$+$4f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	1
012	2
gapless	4
nontrivial	2

Supplementary Table 492: Possible SI values in each gap. Supergroup: $Pb'an'$ (50.282), WP: $4f$+$4f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	012
3	G
4	000
5	G
6	012
7	G
8	000

51.2 WP: $4f$

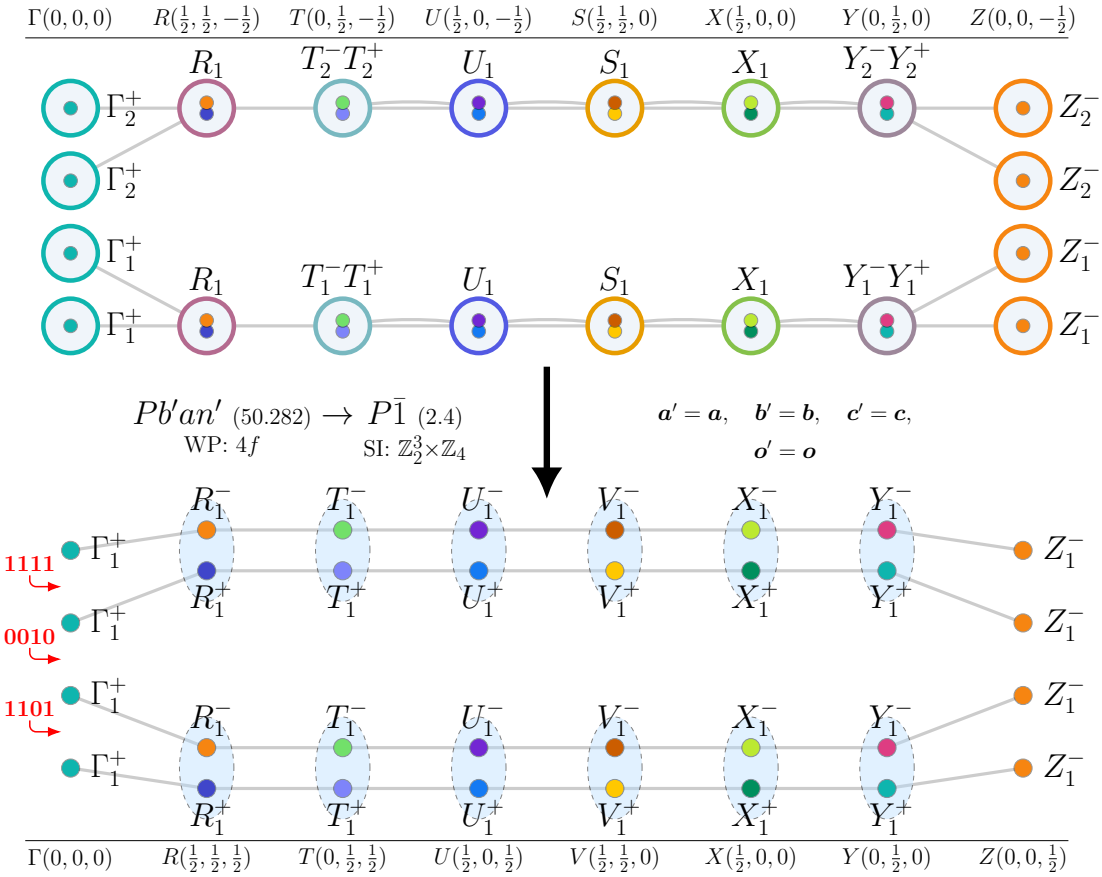
BCS Materials: $ZrCo_2Ge_4O_{12}$ (3.5 K)[219].

51.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[219] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.314`



Supplementary Figure 252: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4f$ of supergroup $Pb'an'$ (50.282).

Supplementary Table 493: Number of times a given SI can appear in the band structure. Supergroup: $Pb'an'$ (50.282), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	1, 2, 3
0011	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

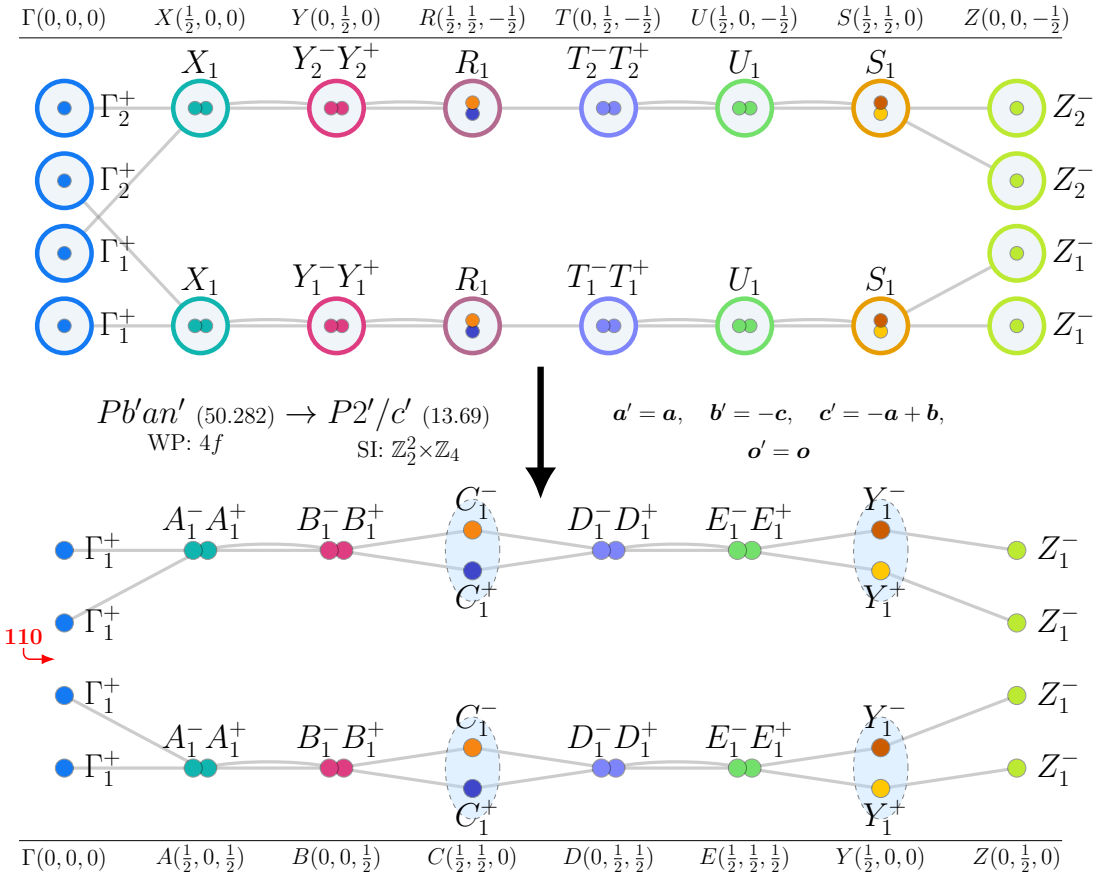
Supplementary Table 494: Possible SI values in each gap. Supergroup: $Pb'an'$ (50.282), WP: $4f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1112, 1113
2	0010
3	0000, 0001, 0003, 0010, 0011 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1111, 1112
4	0000

51.2.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).



Supplementary Figure 253: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4f$ of supergroup $Pb'an'$ (50.282).

Supplementary Table 495: Number of times a given SI can appear in the band structure. Supergroup: $Pb'an'$ (50.282), WP: $4f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

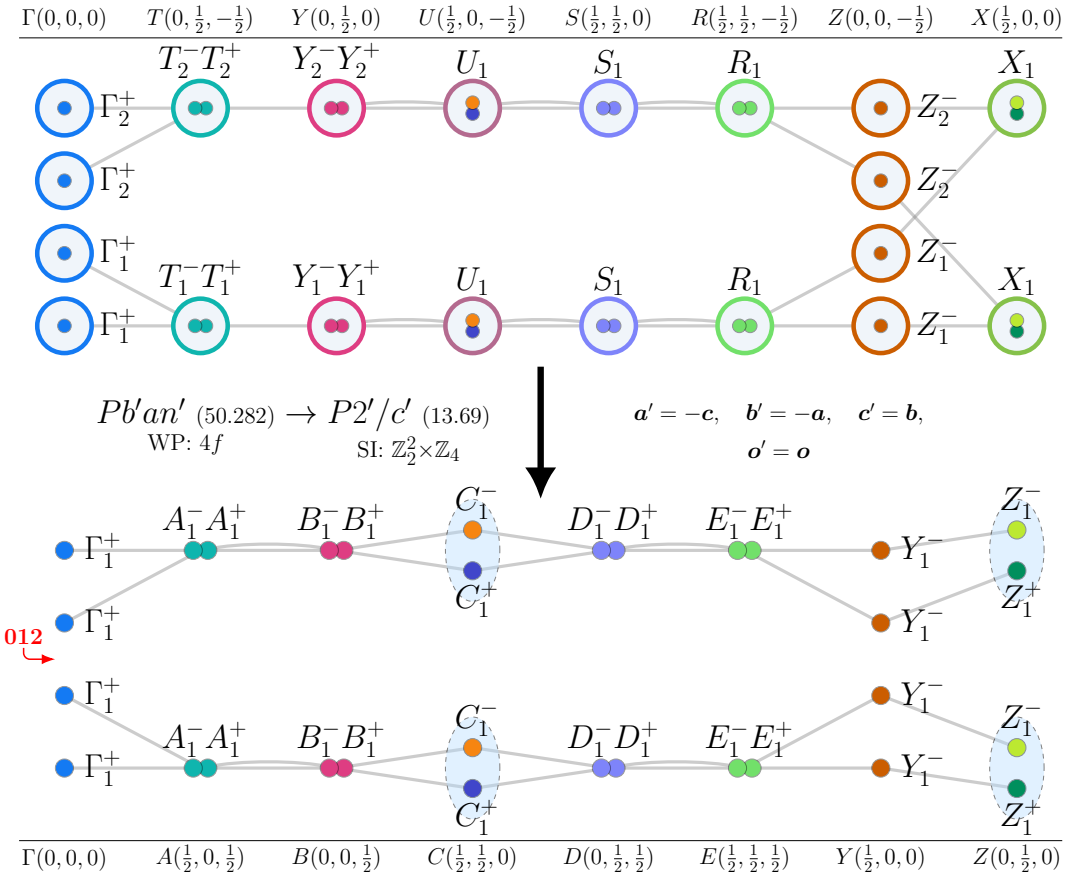
Supplementary Table 496: Possible SI values in each gap. Supergroup: $Pb'an'$ (50.282), WP: $4f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

51.2.3 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain ⊥ [100],
- (B ∥ [001] or B ⊥ [100]).



Supplementary Figure 254: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4f$ of supergroup $Pb'an'$ (50.282).

Supplementary Table 497: Number of times a given SI can appear in the band structure. Supergroup: $Pb'an'$ (50.282), WP: $4f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
012	1
gapless	2
nontrivial	1

Supplementary Table 498: Possible SI values in each gap. Supergroup: $Pb'an'$ (50.282), WP: $4f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	012
3	G
4	000

52 MSG $Pn'n'a$ (52.310)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'/c'$ (13.69), $P2$ (3.1), $P2/c$ (13.65).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), $P2'_1$ (4.9), $P2'$ (3.3), Pc (7.24), $Pn'a2'_1$ (33.146), $Pn'c2'$ (30.113), $Pn'n'2$ (34.159).

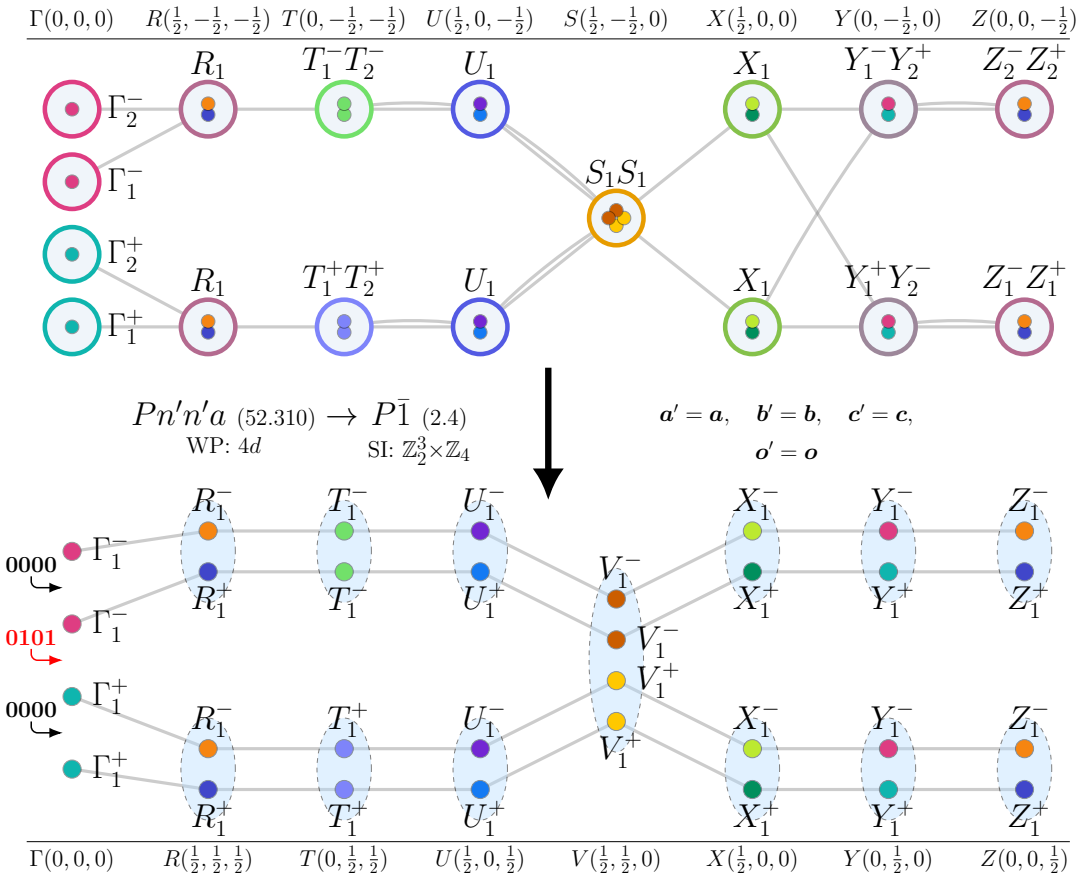
52.1 WP: $4d$

BCS Materials: $[C(ND_2)_3]Mn(DCOO)_3$ (8.73 K)[220].

52.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [100],
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [001],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [010],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 255: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4d$ of supergroup $Pn'n'a$ (52.310).

Supplementary Table 499: Number of times a given SI can appear in the band structure. Supergroup: $Pn'n'a$ (52.310), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2

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[220]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.256`

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SI	Possible #Gaps
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2, 3
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2
1012	0, 1, 2, 3
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	1, 2, 3

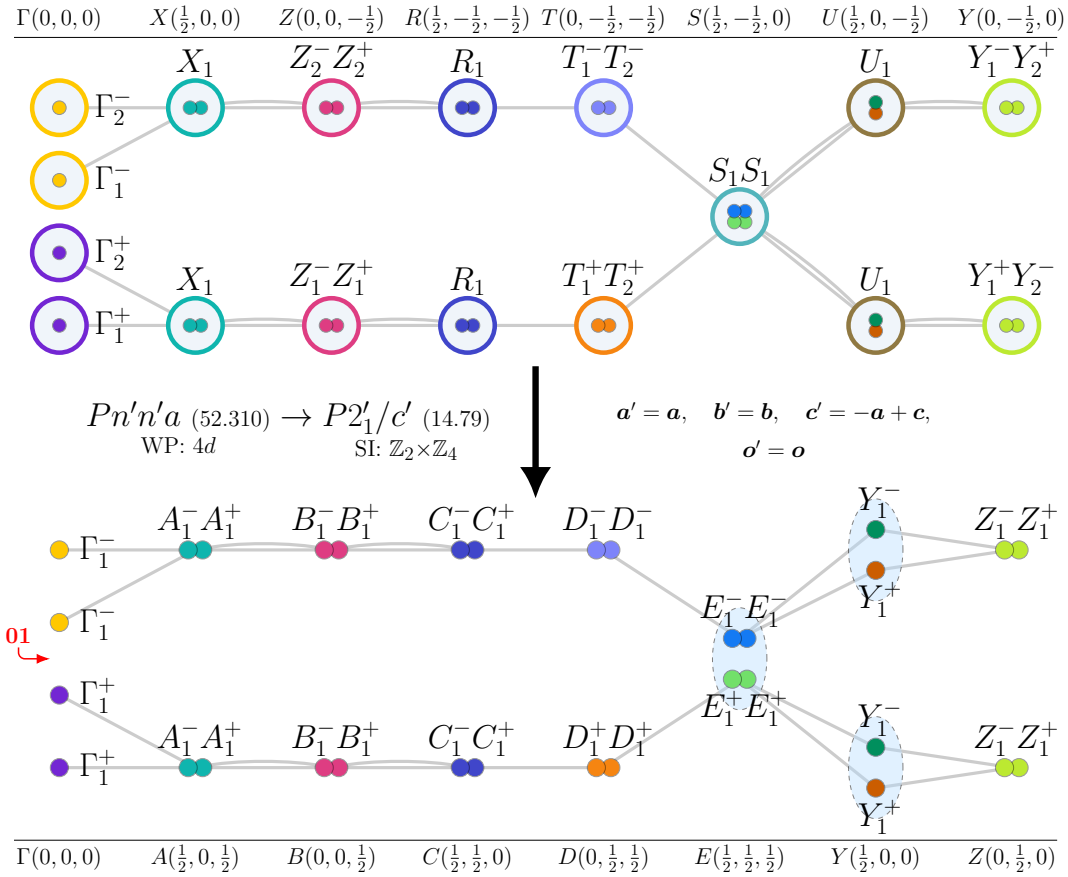
Supplementary Table 500: Possible SI values in each gap. Supergroup: $Pn'n'a$ (52.310), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0101, 0103, 0110, 0112, 1001 1003, 1010, 1012
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

52.1.2 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- strain ⊥ [010],
- (B ∥ [100] or B ⊥ [010]).



Supplementary Figure 256: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4d$ of supergroup $Pn'n'a$ (52.310).

Supplementary Table 501: Number of times a given SI can appear in the band structure. Supergroup: $Pn'n'a$ (52.310), WP: $4d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

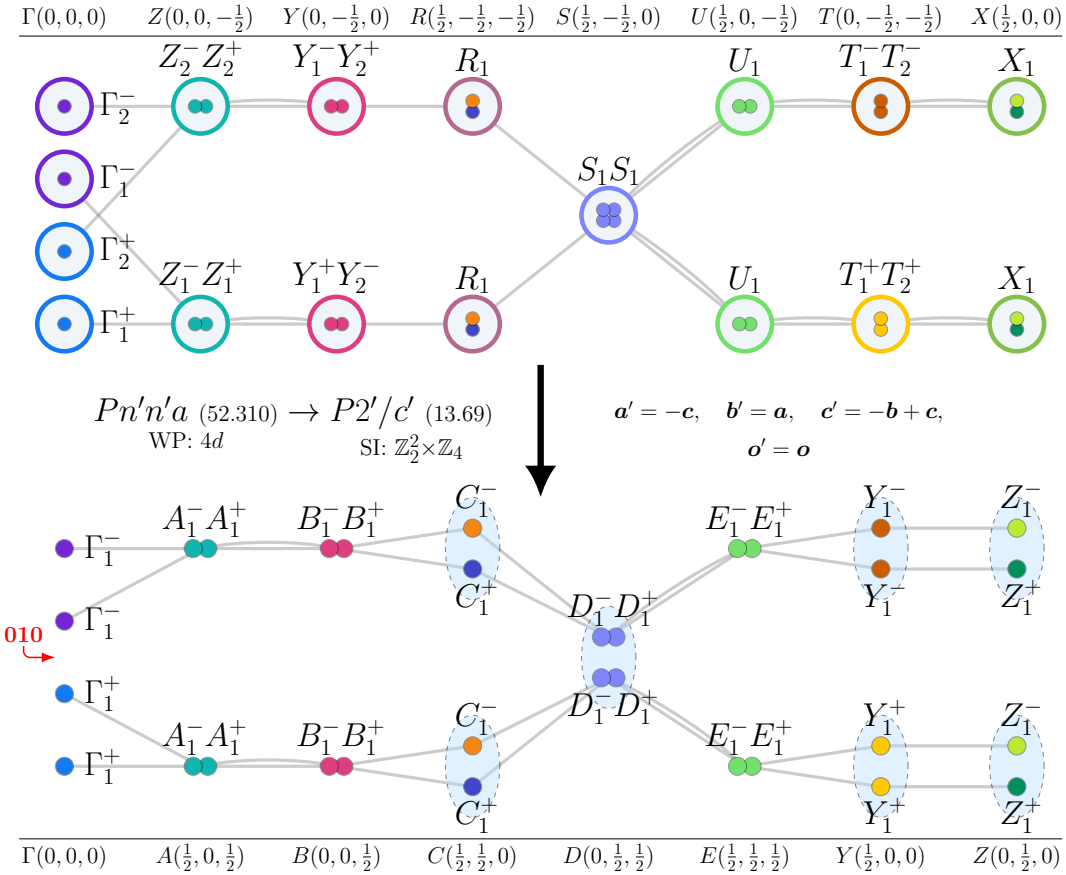
Supplementary Table 502: Possible SI values in each gap. Supergroup: $Pn'n'a$ (52.310), WP: $4d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

52.1.3 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 257: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4d$ of supergroup $Pn'n'a$ (52.310).

Supplementary Table 503: Number of times a given SI can appear in the band structure. Supergroup: $Pn'n'a$ (52.310), WP: $4d$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
010	0, 1
012	0, 1
101	0, 1
103	0, 1
gapless	2
nontrivial	1

Supplementary Table 504: Possible SI values in each gap. Supergroup: $Pn'n'a$ (52.310), WP: $4d$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	010, 012, 101, 103
3	G
4	000

53 MSG P_bnna (52.315)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'/c'$ (13.69), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2$ (3.1), P_a2 (3.4), $P2/c$ (13.65), $Pb'an'$ (50.282), P_a2/c (13.70), $P2_1/c$ (14.75), $Pn'na'$ (52.312), P_b2_1/c (14.81), $P2$ (3.1), P_a2 (3.4), $P2/c$ (13.65), $Pn'n'n$ (48.260), P_a2/c (13.70).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2'$ (3.3), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pb'a2'$ (32.137), $Pn'c2'$ (30.113), P_ac (7.27), Pc (7.24), $Pnc'2'$ (30.114), $Pn'n2'$ (34.158), P_bc (7.29), Pc (7.24), $Pn'n2'$ (34.158), $Pn'n2'$ (34.158), P_ac (7.27), $Pn'c'2$ (30.115), P_ann2 (34.160), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_b2_1 (4.11), P_cna2_1 (33.151), $Pn'n'2$ (34.159), P_anc2 (30.116).

53.1 WP: $4c + 8f$

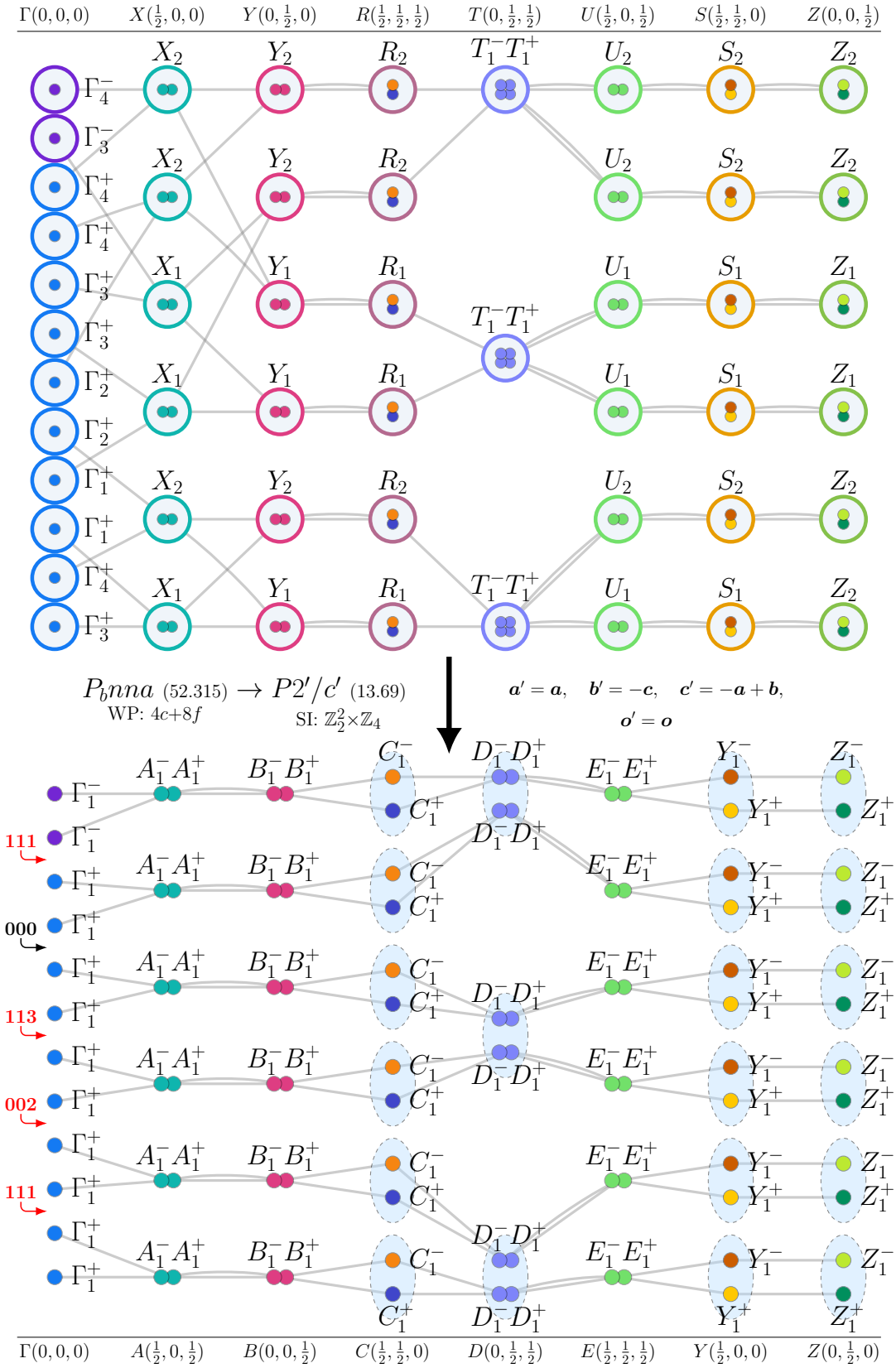
BCS Materials: $GdFeZnGe_4O_{12}$ (13.8 K)[221].

53.1.1 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[221] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.313`



Supplementary Figure 258: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4c+8f$ of supergroup P_bnna (52.315).

Supplementary Table 505: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
111	1, 2, 3
113	1, 2, 3
gapless	6
nontrivial	3, 4, 5

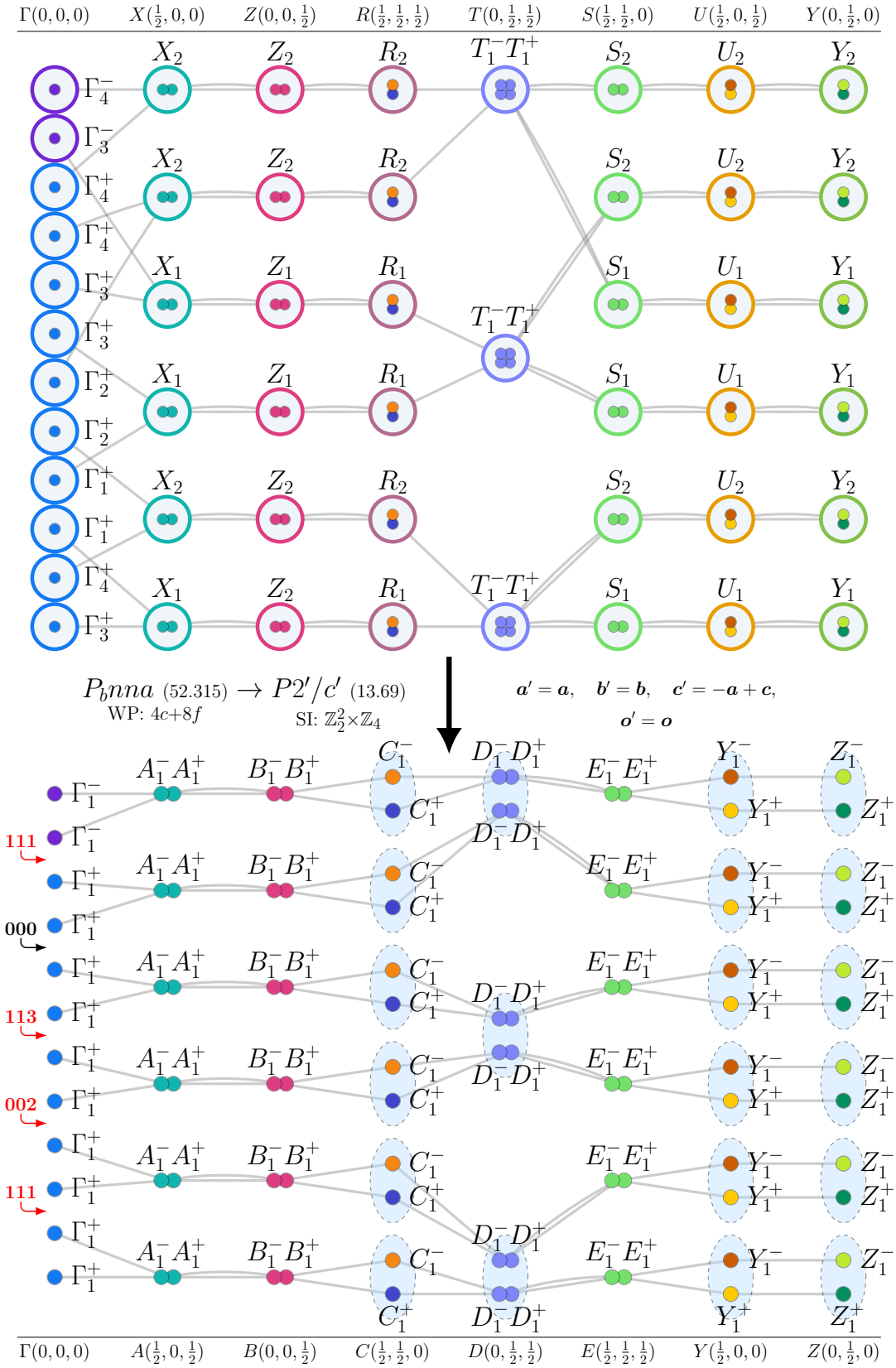
Supplementary Table 506: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	000, 111, 113
3	G
4	000, 002, 111
5	G
6	002, 111, 113
7	G
8	000, 002, 113
9	G
10	000, 111, 113
11	G
12	000

53.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],
- B $\perp$ [010].



Supplementary Figure 259: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4c + 8f$ of supergroup P_bnna (52.315).

Supplementary Table 507: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
111	1, 2, 3
113	1, 2, 3
gapless	6
nontrivial	3, 4, 5

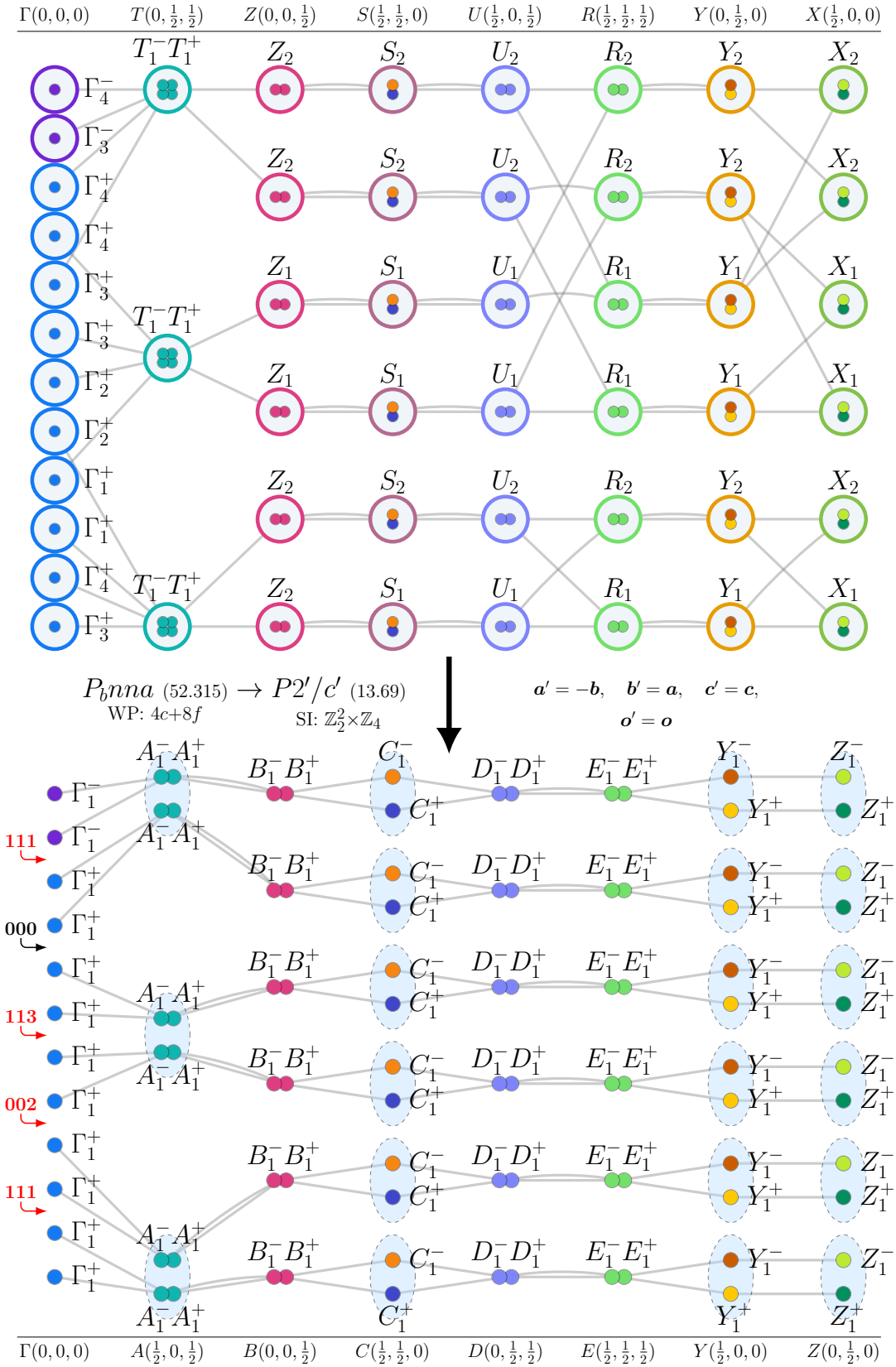
Supplementary Table 508: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	000, 111, 113
3	G
4	000, 002, 111
5	G
6	002, 111, 113
7	G
8	000, 002, 113
9	G
10	000, 111, 113
11	G
12	000

53.1.3 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 260: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4c+8f$ of supergroup P_bnna (52.315).

Supplementary Table 509: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
111	1, 2, 3
113	1, 2, 3
gapless	6
nontrivial	3, 4, 5

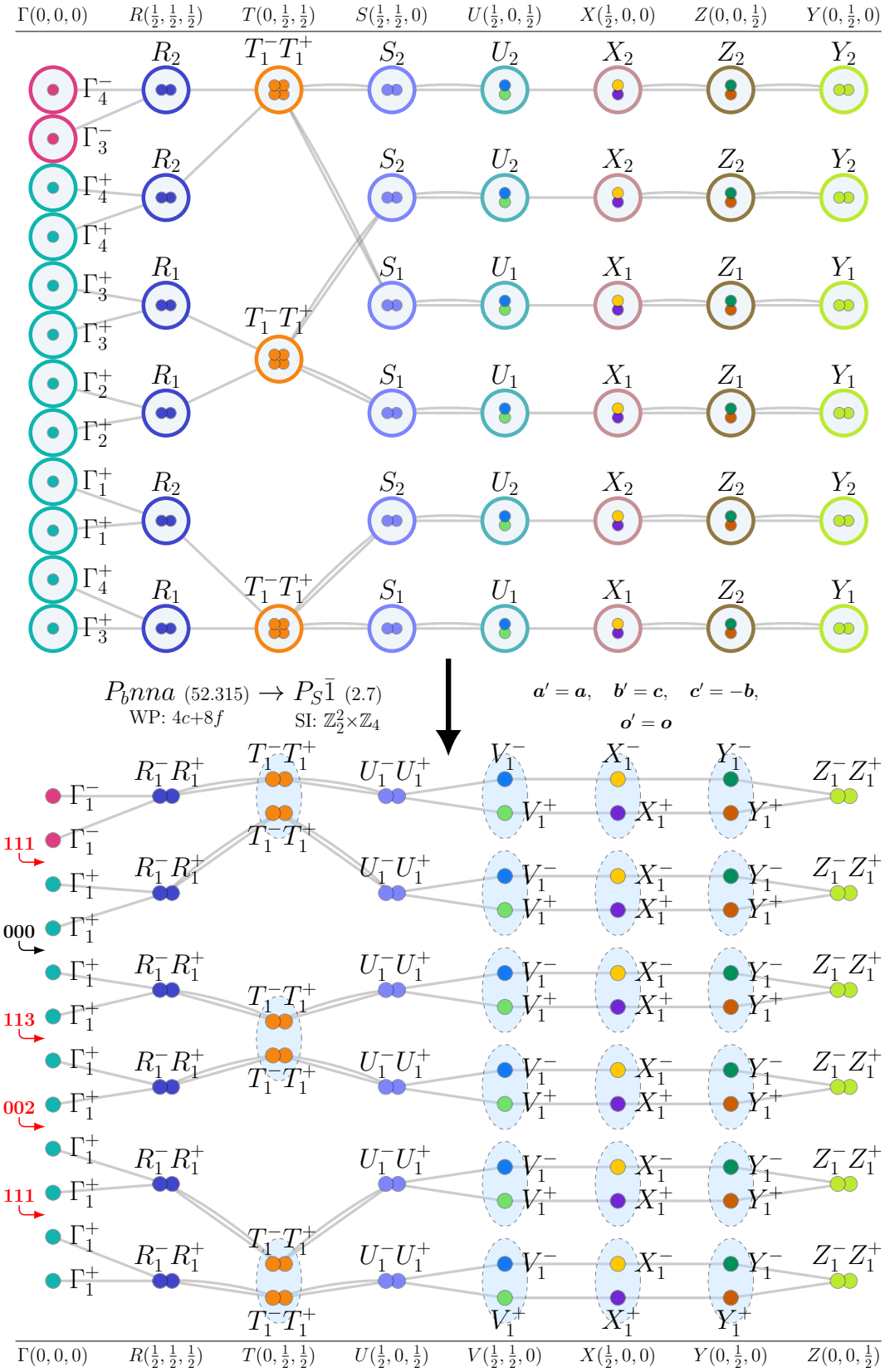
Supplementary Table 510: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	000, 111, 113
3	G
4	000, 002, 111
5	G
6	002, 111, 113
7	G
8	000, 002, 113
9	G
10	000, 111, 113
11	G
12	000

53.1.4 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 261: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4c+8f$ of supergroup P_bnna (52.315).

Supplementary Table 511: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
111	1, 2, 3
113	1, 2, 3
gapless	6
nontrivial	3, 4, 5

Supplementary Table 512: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $4c$+$8f$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 111, 113
3	G
4	000, 002, 111
5	G
6	002, 111, 113
7	G
8	000, 002, 113
9	G
10	000, 111, 113
11	G
12	000

53.2 WP: $8f$

BCS Materials: $CeCo_2Ge_4O_{12}$ (4.5 K)[222].

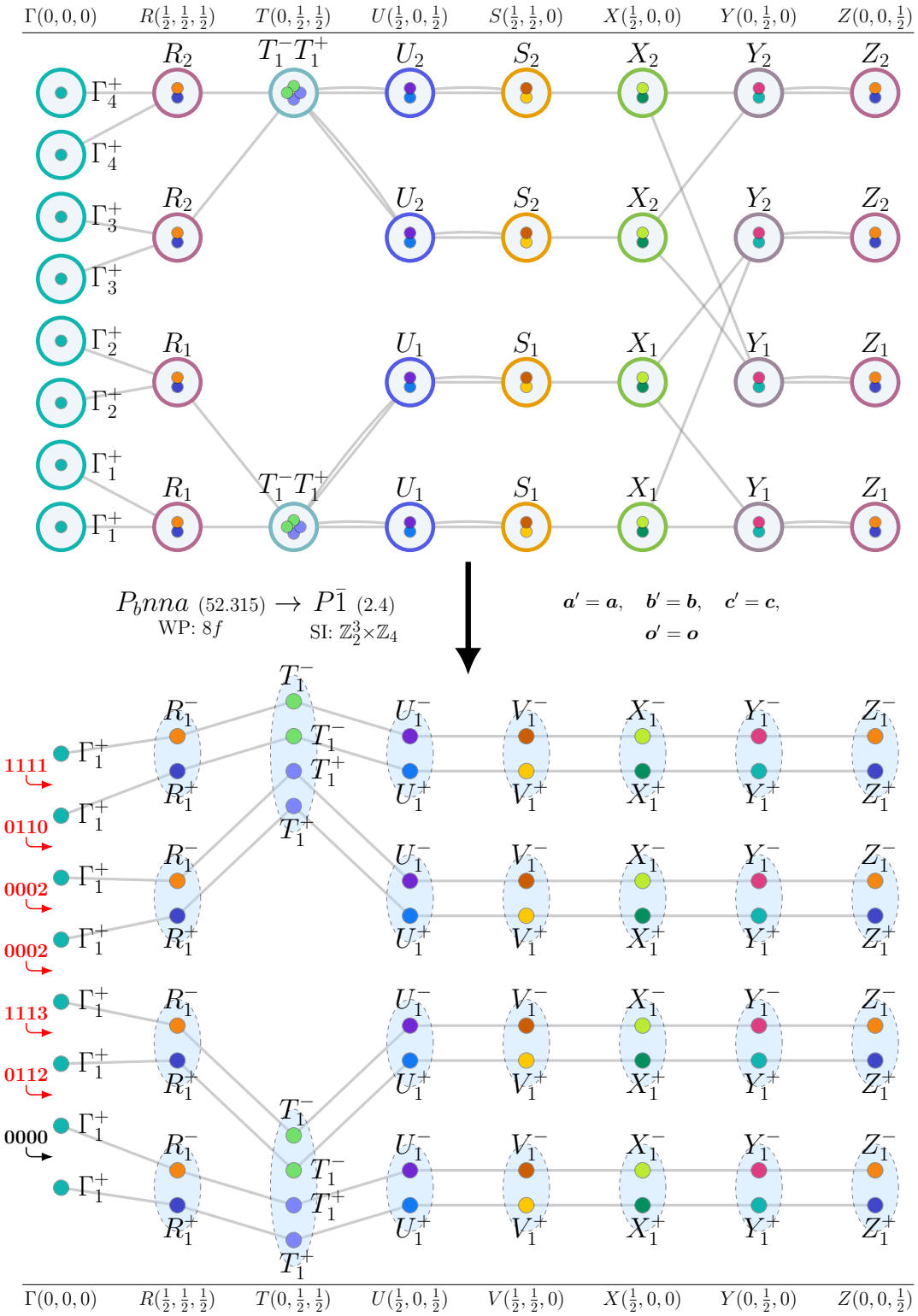
53.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],
- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],

[222]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.226`

- B $\perp$ [001] and strain $\perp$ [010],
- B $\perp$ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 262: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $8f$ of supergroup P_bnna (52.315).

Supplementary Table 513: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3, 4
0002	1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4
0110	0, 1, 2, 3, 4, 5, 6
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4, 5, 6
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	0, 1, 2, 3
1112	0, 1, 2, 3, 4
1113	0, 1, 2, 3
gapless	0
nontrivial	5, 6, 7

Supplementary Table 514: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
2	0110, 0112, 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
4	0002

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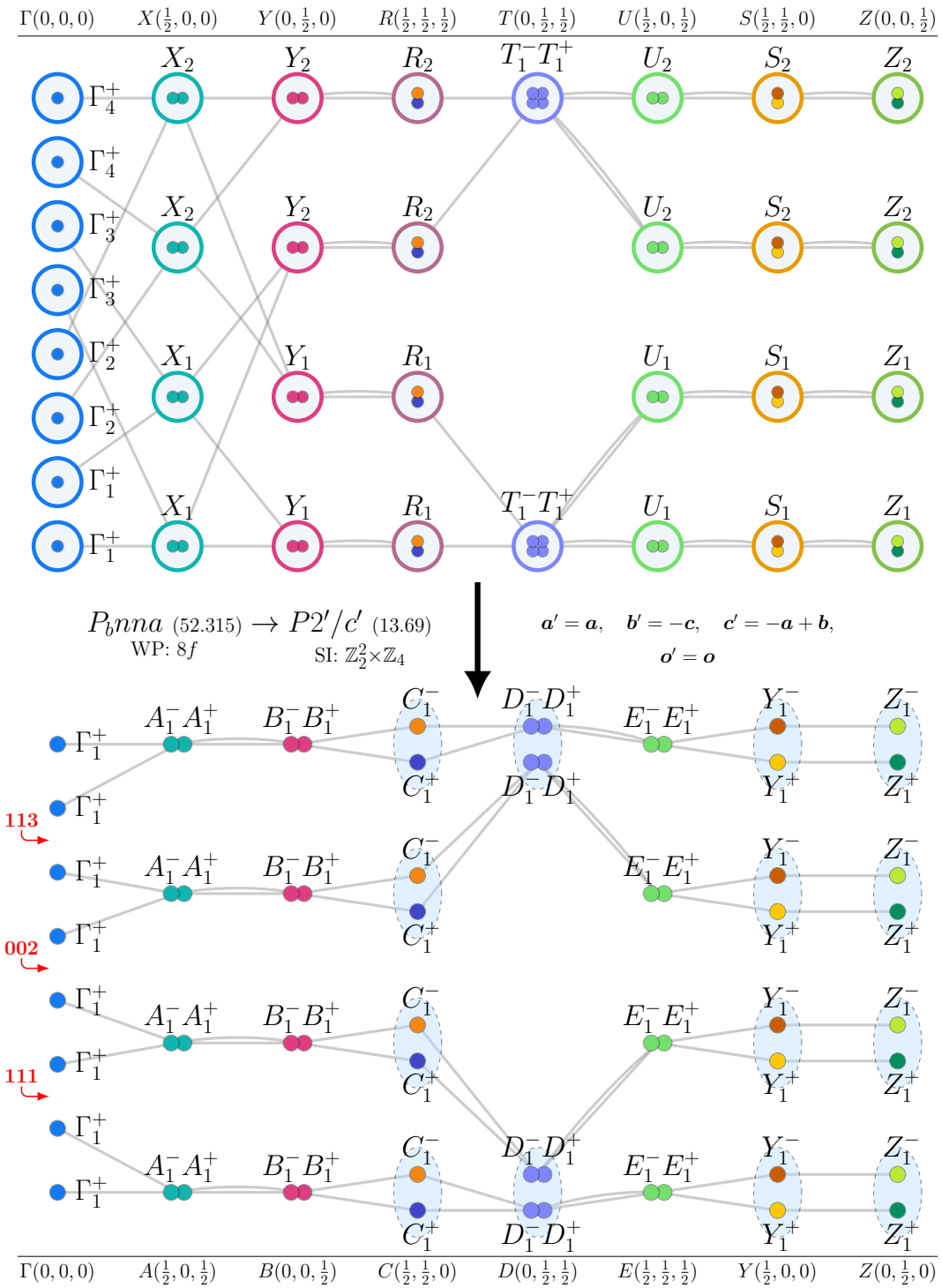
...continued

Gap#	Possible SI Values
5	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
6	0110, 0112, 1111
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
8	0000

53.2.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [001],
- B $\parallel$ [010] and strain $\perp$ [001],
- B $\perp$ [001].



Supplementary Figure 263: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8f$ of supergroup P_bnna (52.315).

Supplementary Table 515: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

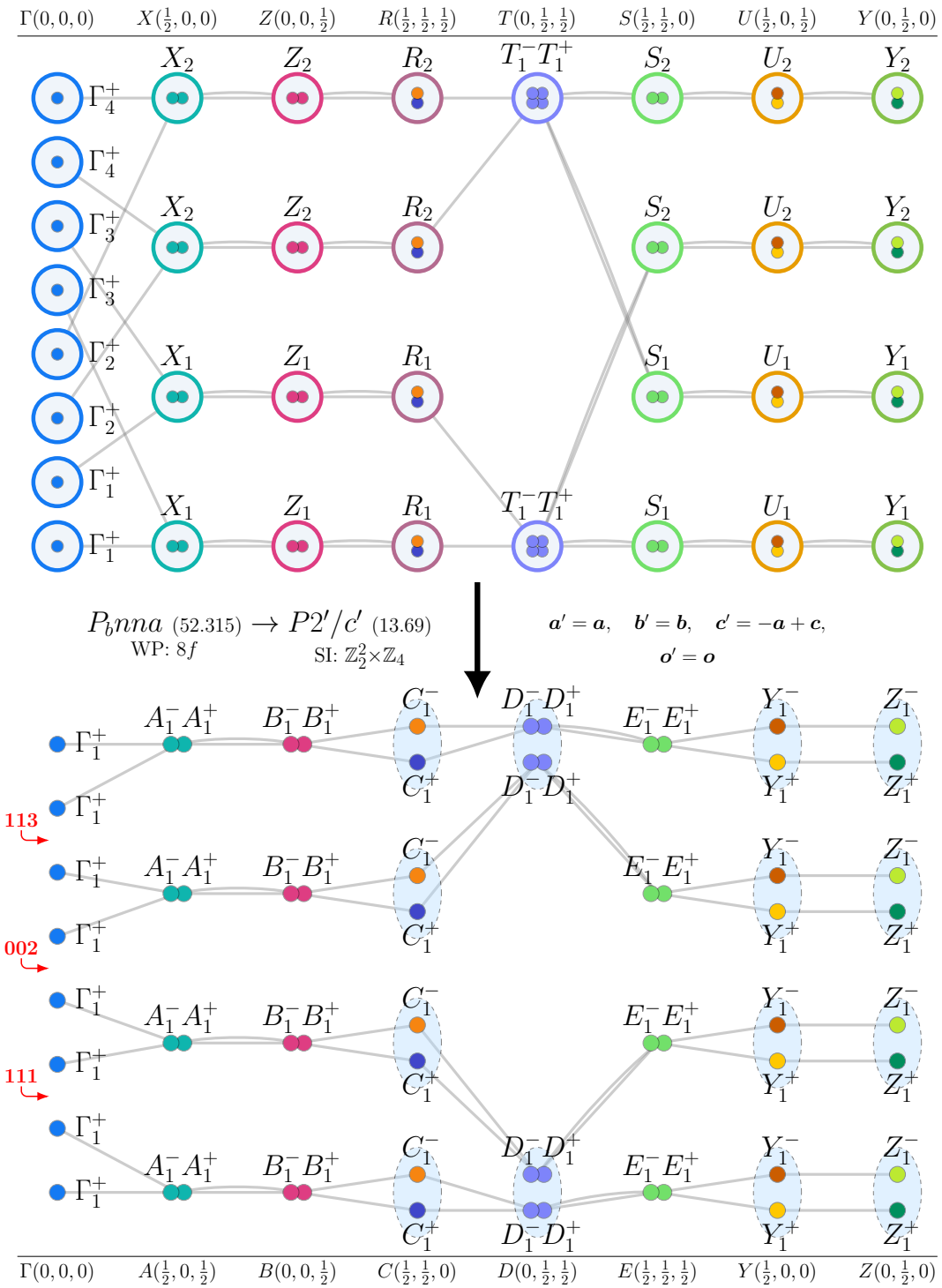
Supplementary Table 516: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

53.2.3 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 264: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8f$ of supergroup P_bnna (52.315).

Supplementary Table 517: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

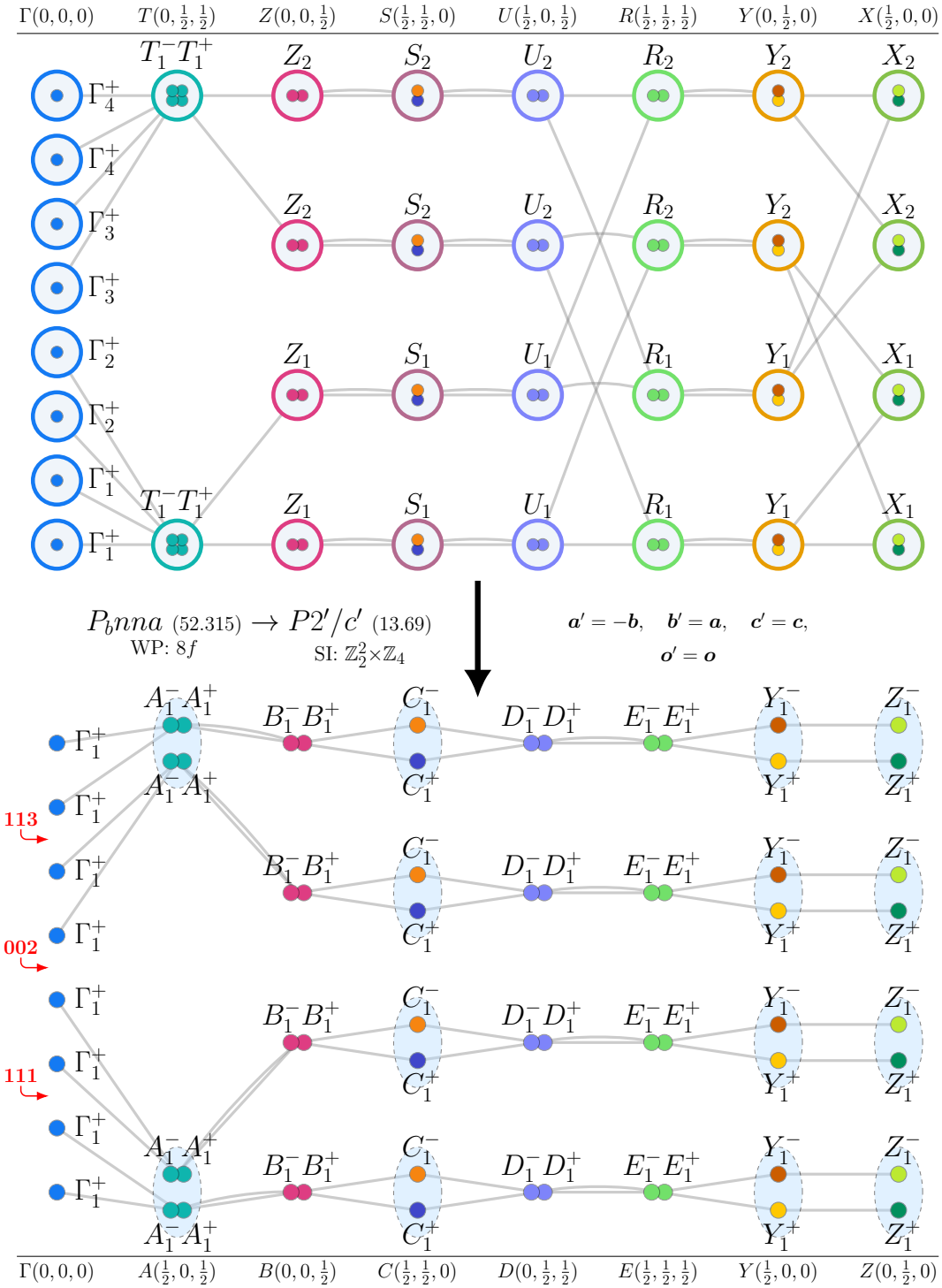
Supplementary Table 518: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

53.2.4 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 265: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8f$ of supergroup P_bnna (52.315).

Supplementary Table 519: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

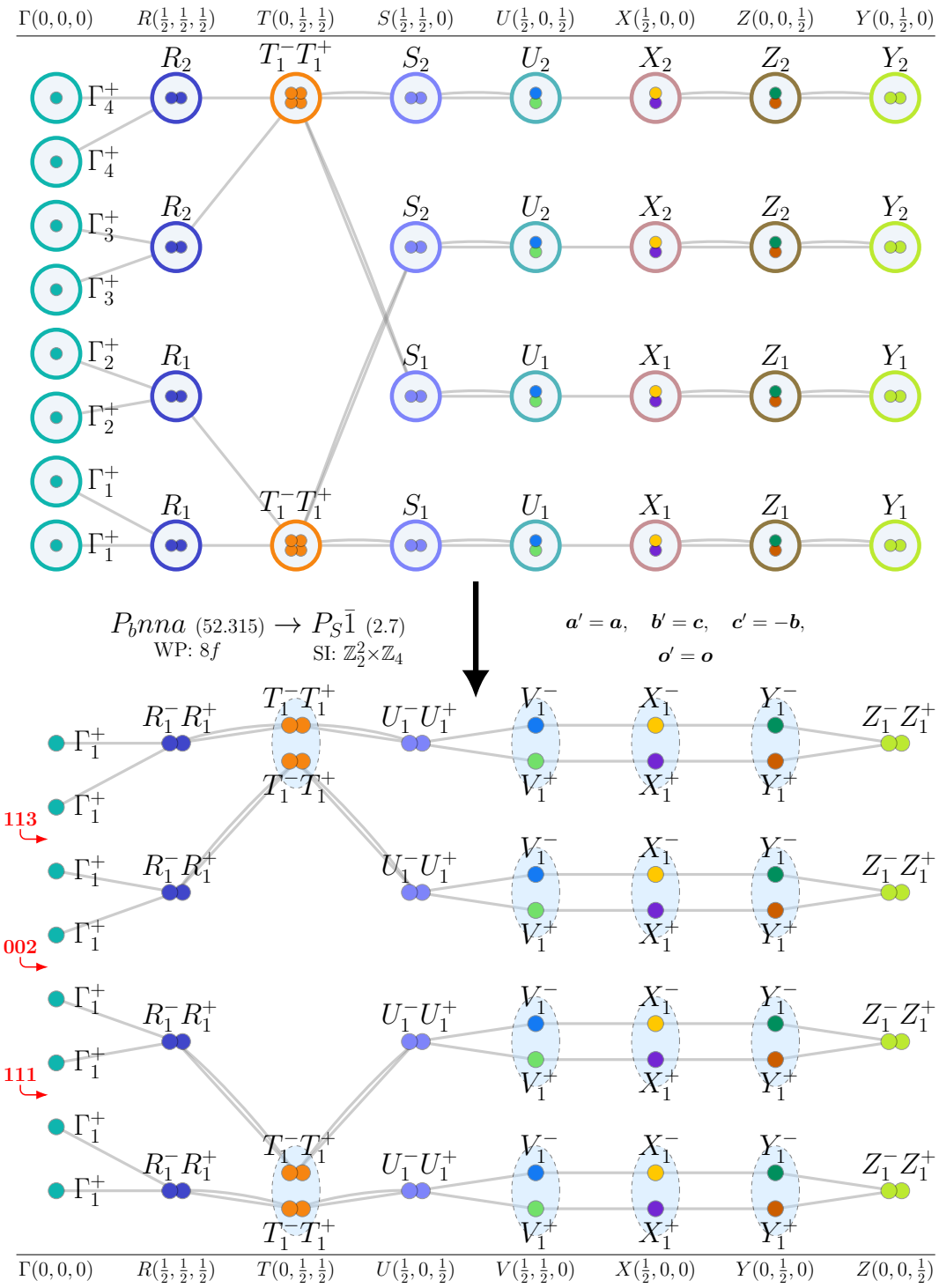
Supplementary Table 520: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

53.2.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 266: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8f$ of supergroup P_bnna (52.315).

Supplementary Table 521: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

Supplementary Table 522: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $8f$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111
3	G
4	002
5	G
6	113
7	G
8	000

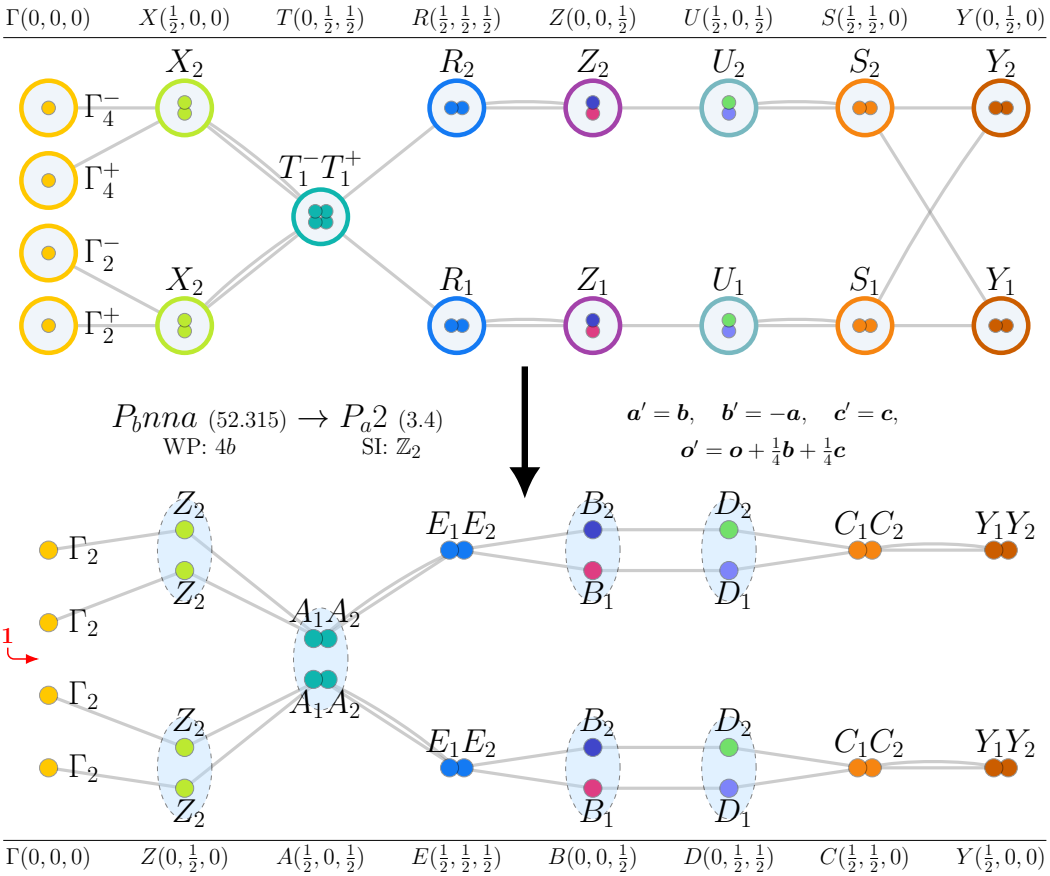
53.3 WP: $4b$

BCS Materials: $CuCl(C_4H_4N_2)_2(BF_4)$ (3.9 K)[223], $CuBr(C_4H_4N_2)_2(BF_4)$ (3.8 K)[224].

53.3.1 Topological bands in subgroup P_a2 (3.4)

Perturbation:

- E ∥ [100] and strain ⊥ [100].



Supplementary Figure 267: Topological magnon bands in subgroup P_a2 (3.4) for magnetic moments on Wyckoff position $4b$ of supergroup P_bnna (52.315).

[223]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.474`
[224]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.473`

Supplementary Table 523: Number of times a given SI can appear in the band structure. Supergroup: P_bnna (52.315), WP: $4b$, Subgroup: P_a2 (3.4).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 524: Possible SI values in each gap. Supergroup: P_bnna (52.315), WP: $4b$, Subgroup: P_a2 (3.4).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

54 MSG P_Bnna (52.318)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/m'$ (11.54), $P2'_1/m'$ (11.54), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Pm'a'2$ (28.91), $P2/c$ (13.65), $Pbc'm'$ (57.383), P_A2/c (13.73), $P2_1/c$ (14.75), $Pnm'a'$ (62.447), P_c2_1/c (14.82), $P2$ (3.1), $Pm'm'2$ (25.60), $P2/c$ (13.65), $Pm'm'n$ (59.409), P_C2/c (13.74).

Trivial-SI Subgroups: Pm' (6.20), Pm' (6.20), Pc' (7.26), $P2'_1$ (4.9), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), $Pc'a2'_1$ (29.101), $Pm'c2'_1$ (26.68), P_Ac (7.31), Pc (7.24), $Pna'2'_1$ (33.147), $Pm'n2'_1$ (31.125), P_cc (7.28), Pc (7.24), $Pm'n2'_1$ (31.125), $Pm'n2'_1$ (31.125), P_Cc (7.30), P_C2 (3.6), P_Ann2 (34.162), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_a2_1 (4.10), P_Cna2_1 (33.154), P_C2 (3.6), P_Anc2 (30.119).

54.1 WP: $4c + 4c$

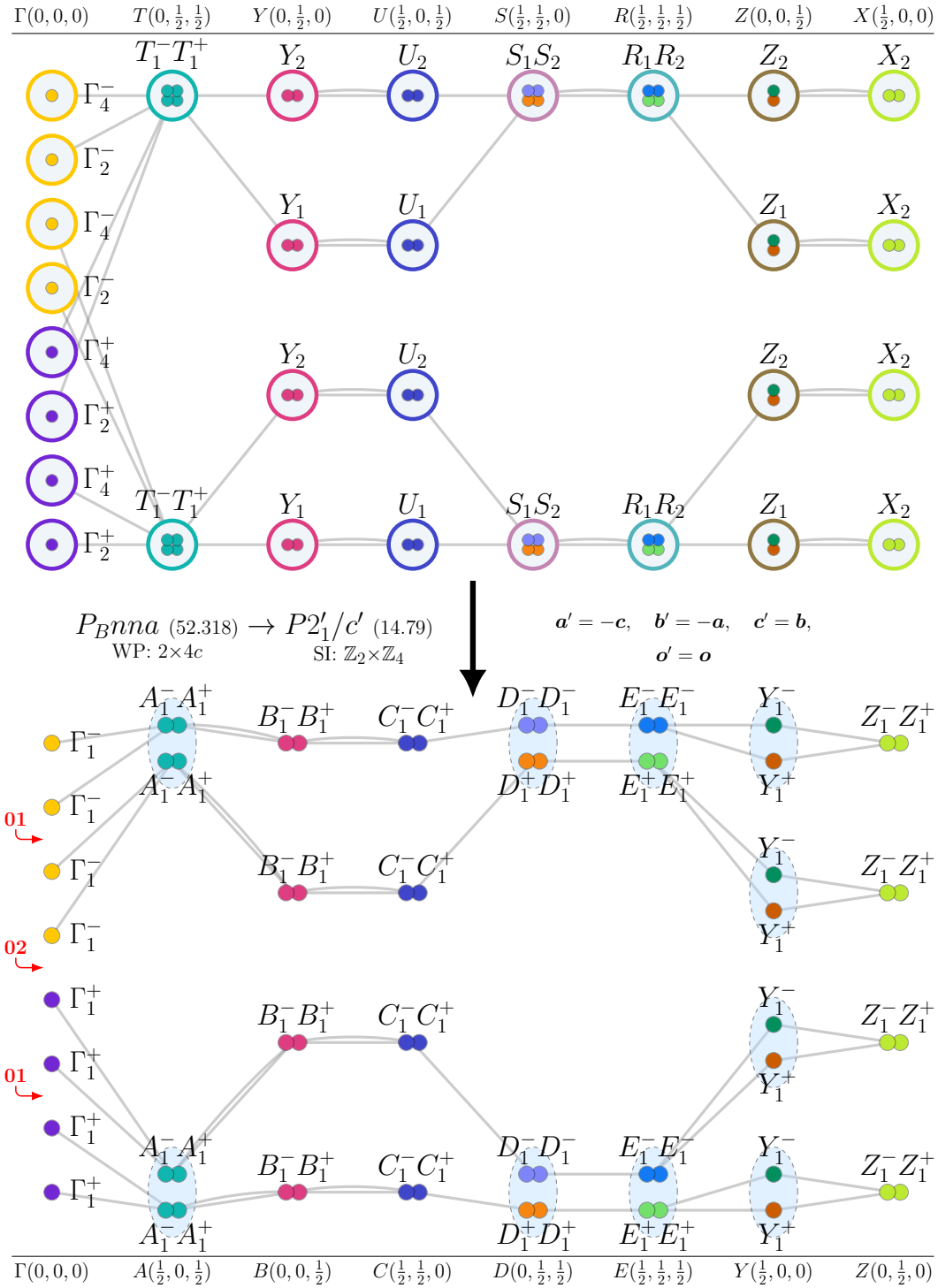
BCS Materials: $PrMnSi_2$ (34 K)[225].

54.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].

[225] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.628`



Supplementary Figure 268: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c + 4c$ of supergroup P_Bnna (52.318).

Supplementary Table 525: Number of times a given SI can appear in the band structure. Supergroup: P_Bnna (52.318), WP: 4c+4c, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

continued on next page...

...continued

SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 526: Possible SI values in each gap. Supergroup: P_Bnna (52.318), WP: $4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

54.2 WP: $8f$

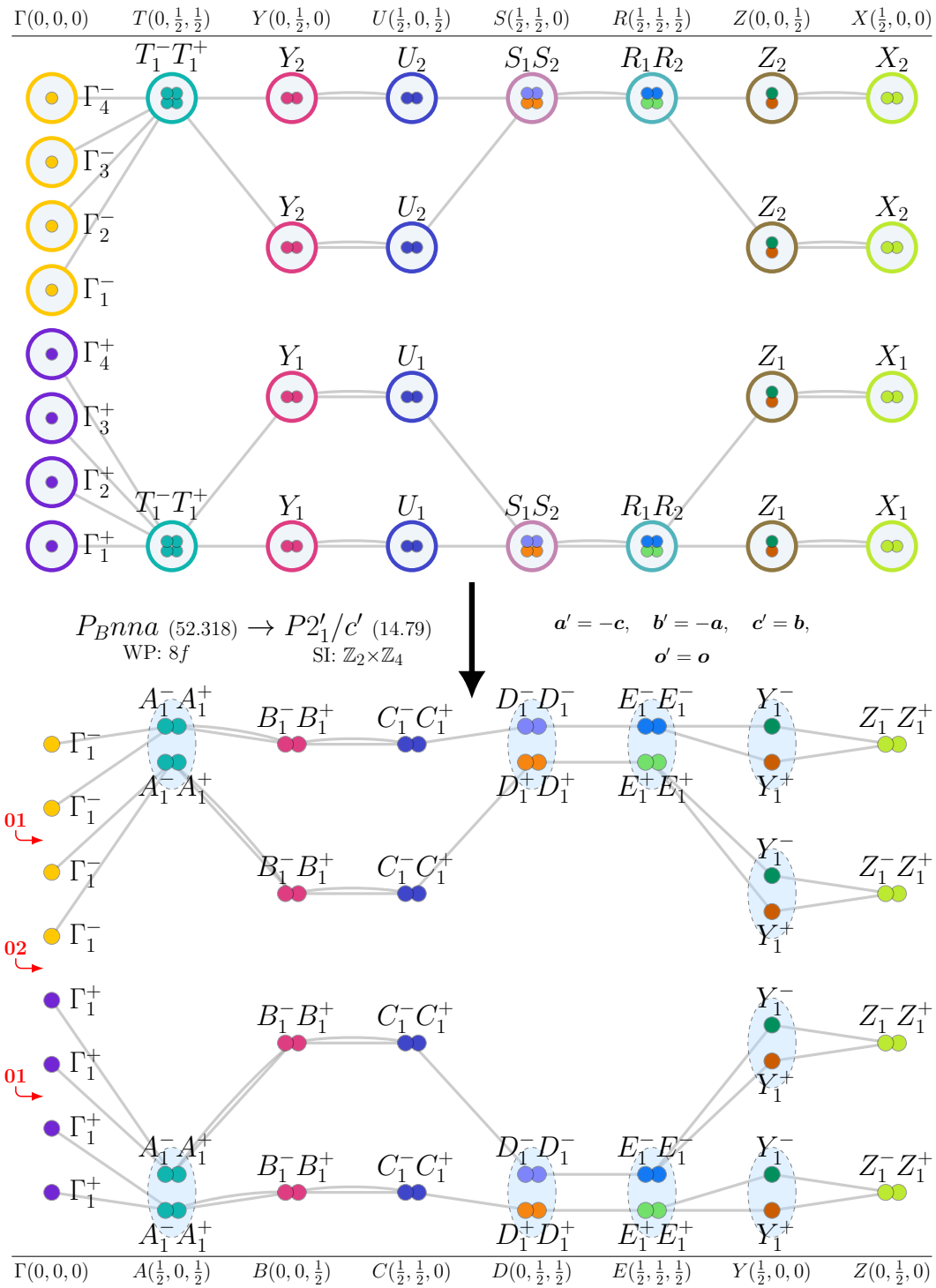
BCS Materials: Ho_3Ge_4 (12 K)[226].

54.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].

[226]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.356`



Supplementary Figure 269: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8f$ of supergroup P_Bnna (52.318).

Supplementary Table 527: Number of times a given SI can appear in the band structure. Supergroup: P_Bnna (52.318), WP: $8f$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 528: Possible SI values in each gap. Supergroup: P_Bnna (52.318), WP: $8f$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

55 MSG P_Amna (53.333)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2_1'/c'$ (14.79), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pc'cn'$ (56.370), P_C2_1/c (14.84), $P2$ (3.1), $P2/c$ (13.65), $Pn'n'n$ (48.260), P_C2/c (13.74), $P2$ (3.1), P_a2 (3.4), $P2/m$ (10.42), $Pmn'a'$ (53.327), P_a2/m (10.47).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2_1'$ (4.9), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pn'a2_1'$ (33.146), $Pc'c2'$ (27.80), P_Cc (7.30), Pc (7.24), $Pn'n2'$ (34.158), $Pn'n2'$ (34.158), P_Cc (7.30), Pm (6.18), $Pma'2'$ (28.90), $Pmn'2_1'$ (31.126), P_am (6.21), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_C2_1 (4.12), P_Amn2_1 (31.131), $Pn'n'2$ (34.159), P_C2 (3.6), P_Ama2 (28.95), $Pn'c'2$ (30.115), P_Cnc2 (30.121).

55.1 WP: $4e$

BCS Materials: $La_2NiO_3F_2$ (55 K)[227].

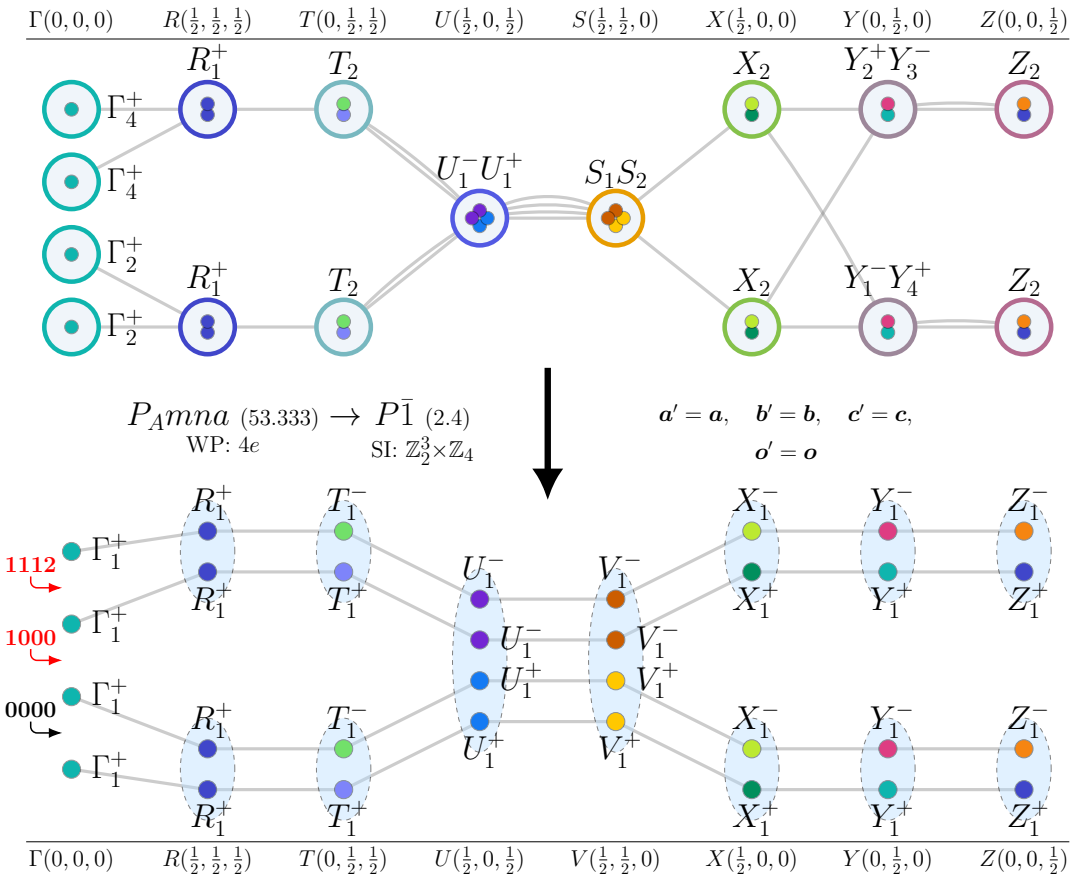
55.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],

[227] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.388`

- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 270: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4e$ of supergroup P_Amna (53.333).

Supplementary Table 529: Number of times a given SI can appear in the band structure. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2, 3
1001	0, 1, 2

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SI	Possible #Gaps
1002	0, 1, 2, 3
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1111	0, 1
1112	0, 1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

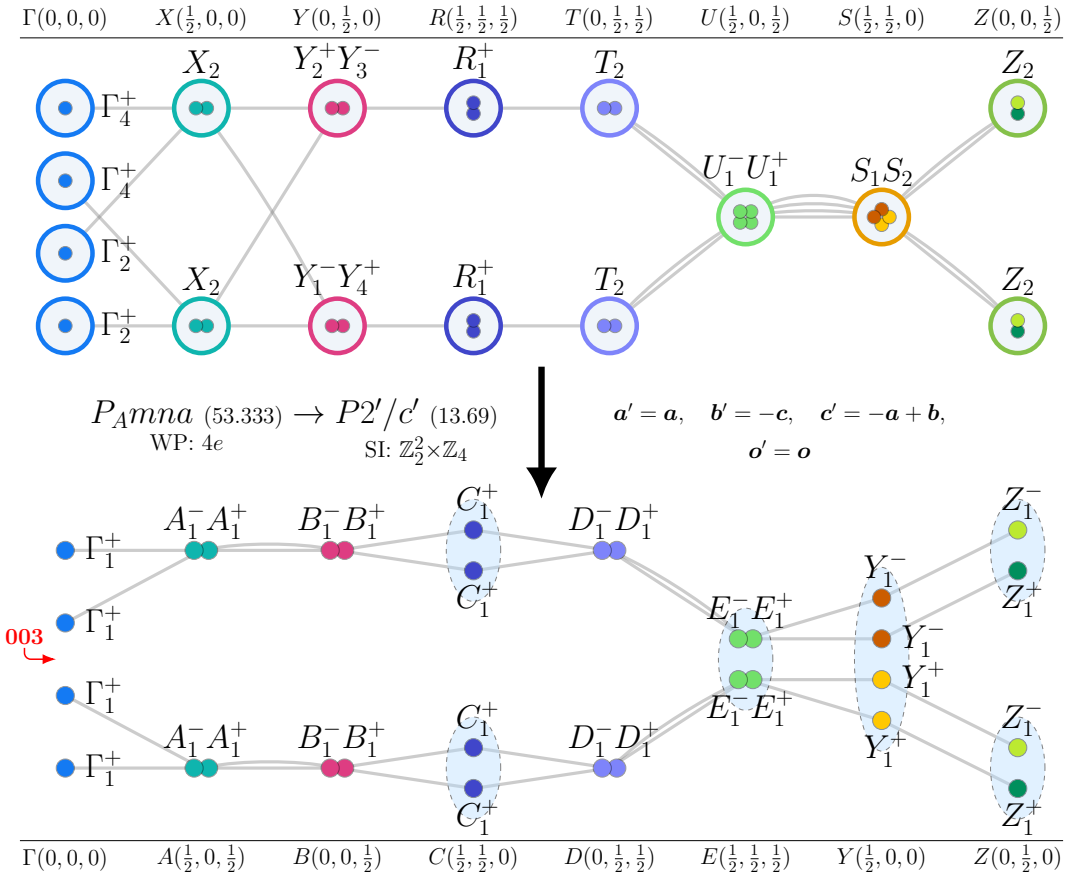
Supplementary Table 530: Possible SI values in each gap. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1000, 1002, 1011, 1013, 1101 1103, 1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

55.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [001],
- B $\parallel$ [010] and strain $\perp$ [001],
- B $\perp$ [001].



Supplementary Figure 271: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4e$ of supergroup P_Amna (53.333).

Supplementary Table 531: Number of times a given SI can appear in the band structure. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
001	0, 1
003	0, 1
100	0, 1
gapless	2
nontrivial	1

Supplementary Table 532: Possible SI values in each gap. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P2'/c'$ (13.69).

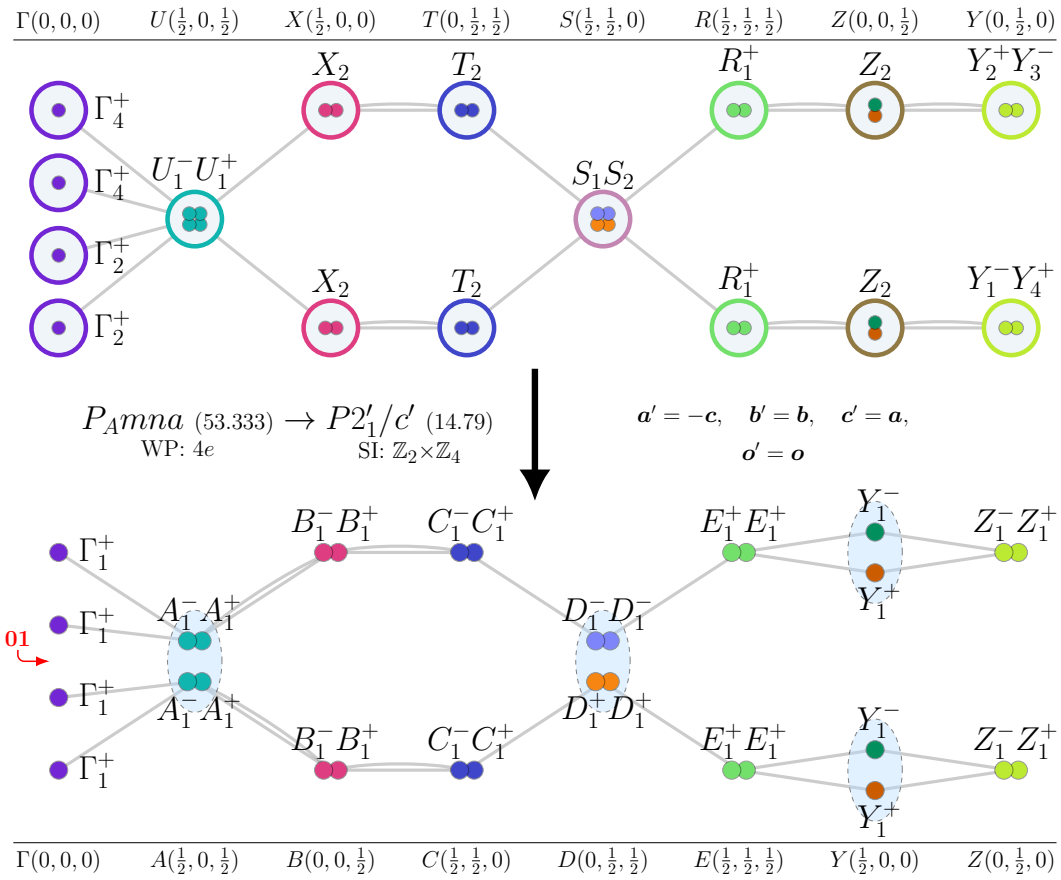
Gap#	Possible SI Values
1	G
2	001, 003, 100
3	G
4	000

55.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],

- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 272: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4e$ of supergroup P_Amna (53.333).

Supplementary Table 533: Number of times a given SI can appear in the band structure. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

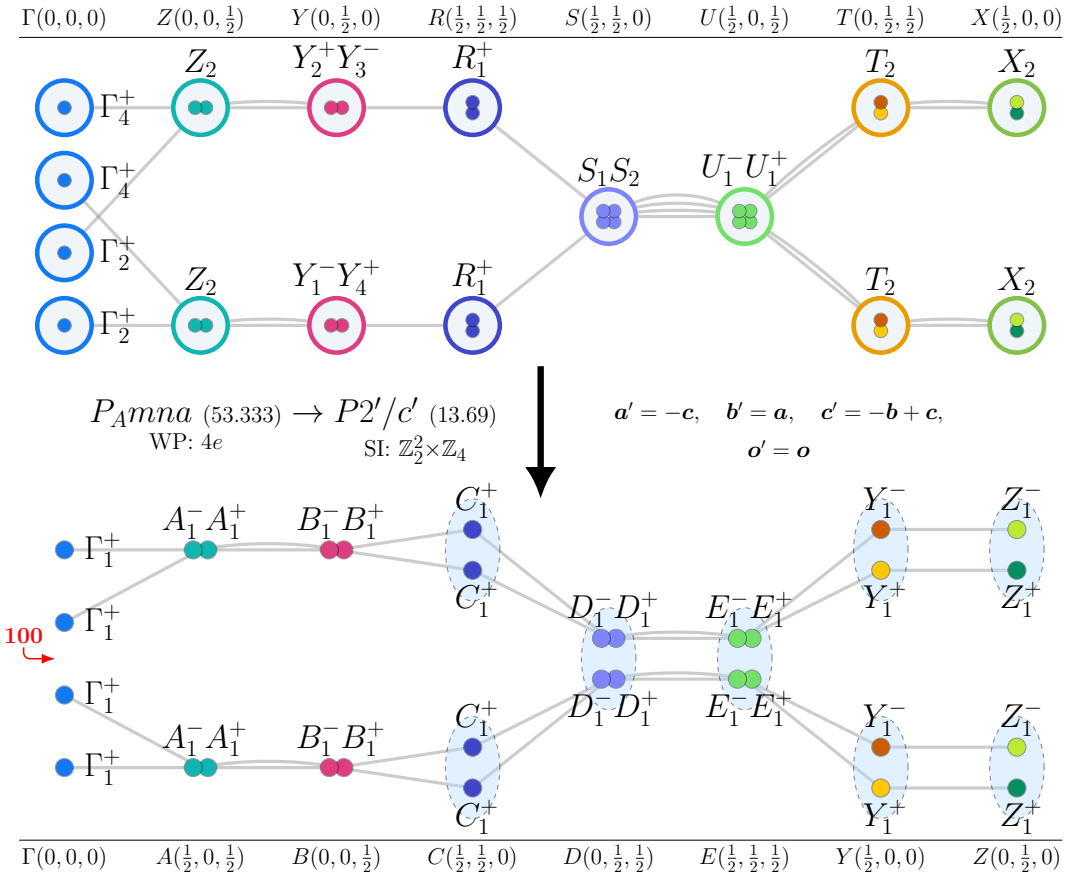
Supplementary Table 534: Possible SI values in each gap. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

55.1.4 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 273: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4e$ of supergroup $PAmna$ (53.333).

Supplementary Table 535: Number of times a given SI can appear in the band structure. Supergroup: $PAmna$ (53.333), WP: $4e$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

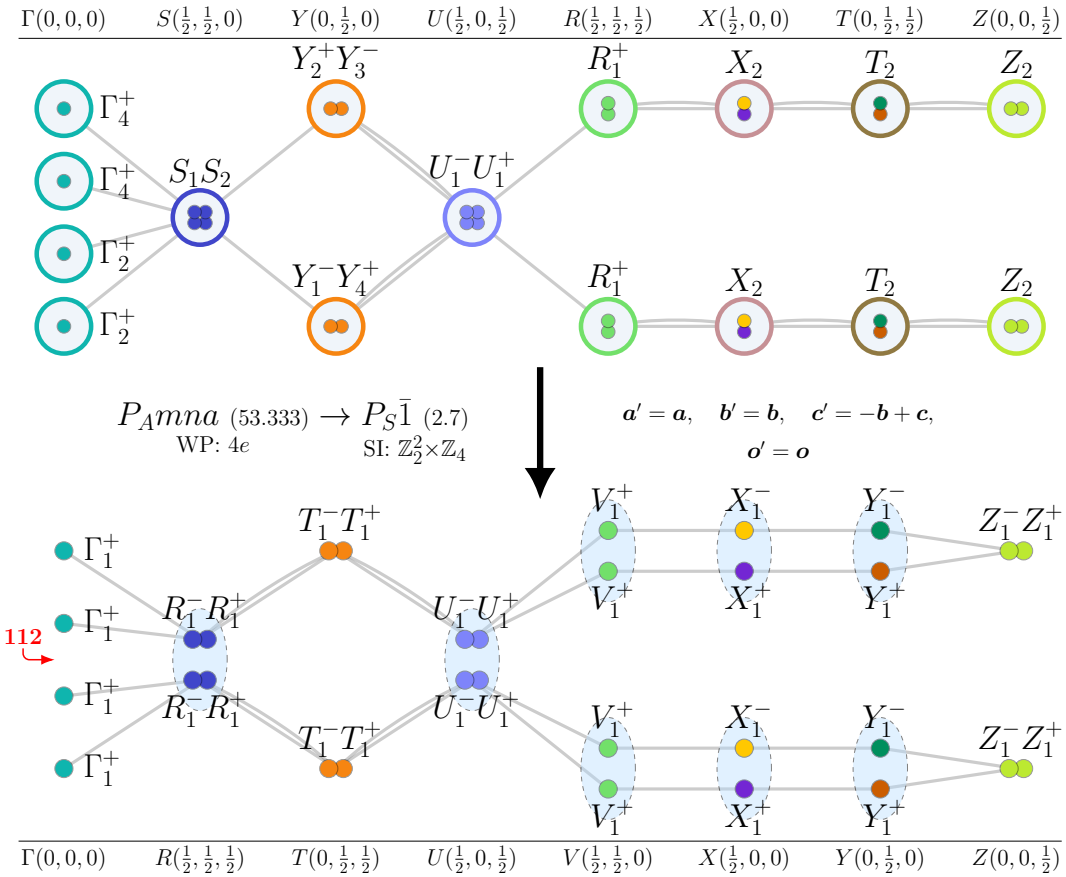
Supplementary Table 536: Possible SI values in each gap. Supergroup: $PAmna$ (53.333), WP: $4e$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

55.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 274: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4e$ of supergroup P_Amna (53.333).

Supplementary Table 537: Number of times a given SI can appear in the band structure. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

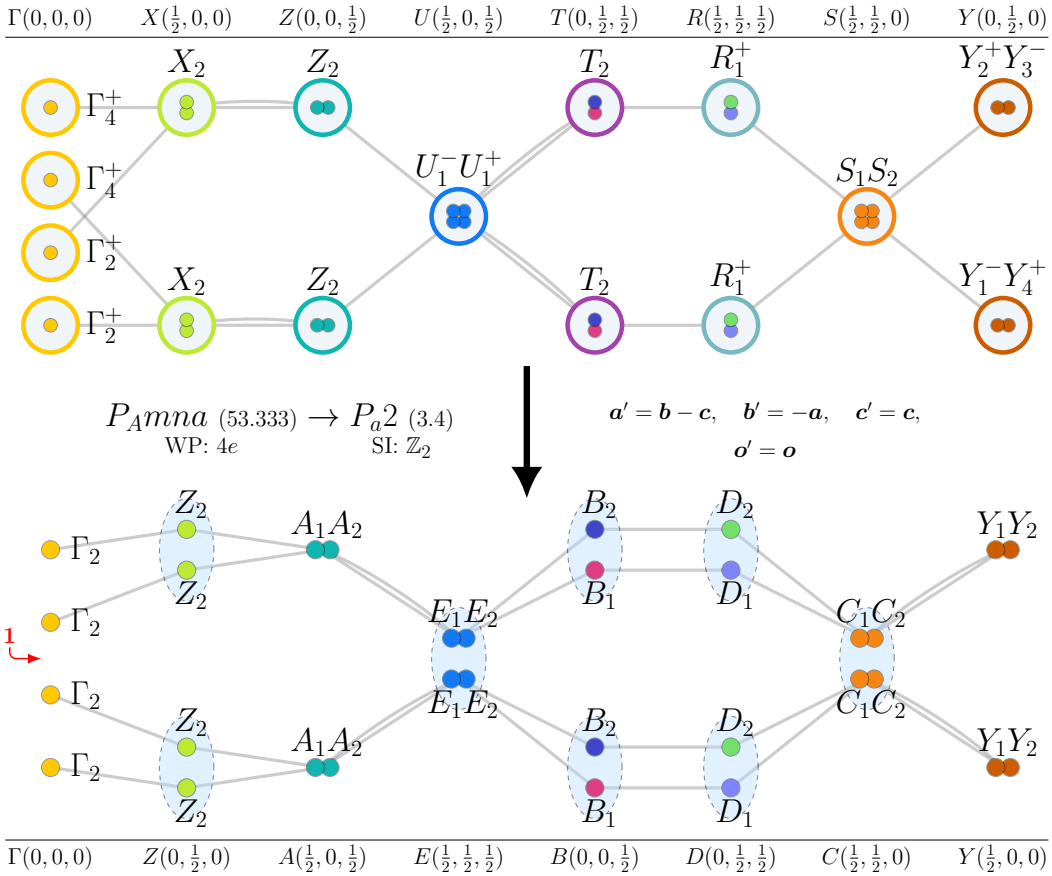
Supplementary Table 538: Possible SI values in each gap. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

55.1.6 Topological bands in subgroup P_a2 (3.4)

Perturbation:

- E ∥ [100] and strain ⊥ [100].



Supplementary Figure 275: Topological magnon bands in subgroup P_a2 (3.4) for magnetic moments on Wyckoff position $4e$ of supergroup P_Amna (53.333).

Supplementary Table 539: Number of times a given SI can appear in the band structure. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: P_a2 (3.4).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 540: Possible SI values in each gap. Supergroup: P_Amna (53.333), WP: $4e$, Subgroup: P_a2 (3.4).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

56 MSG P_Cmna (53.335)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'_1/c'$ (14.79), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pb'c'a$ (61.436), P_a2_1/c (14.80), $P2$ (3.1), $Pc'c'2$ (27.81), $P2/c$ (13.65), $Pc'c'n$ (56.369), P_C2/c (13.74), $P2$ (3.1), $Pb'a'2$ (32.138), P_Bnc2 (30.120), $P2/m$ (10.42), $Pb'a'm$ (55.357), P_C2/m (10.49).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2'_1$ (4.9), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), $Pc'a2'_1$ (29.101), $Pca'2'_1$ (29.102), P_ac (7.27), Pc (7.24), $Pna'2'_1$ (33.147), $Pna'2'_1$ (33.147), P_Cc (7.30), Pm (6.18), $Pmc'2'_1$ (26.69), $Pmc'2'_1$ (26.69), P_Cm (6.23), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_a2_1 (4.10), P_Cmn2_1 (31.133), P_C2 (3.6), P_Bma2 (28.96), P_C2 (3.6).

56.1 WP: $4a$

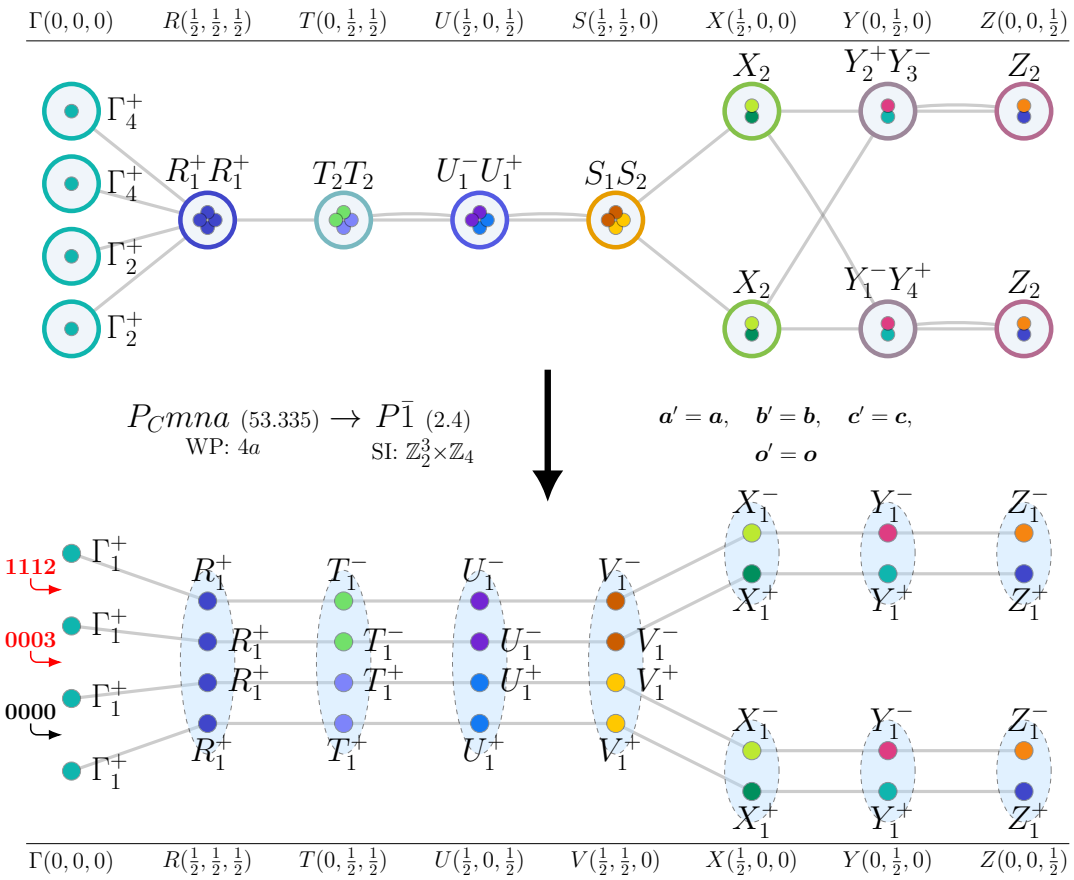
BCS Materials: La_2NiO_4 (330 K)[228], Nd_2NiO_4 (320 K)[229], La_2CoO_4 (275 K)[230].

56.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],
- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.

[228]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.42`
[229]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.371`
[230]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.403`



Supplementary Figure 276: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cmna (53.335).

Supplementary Table 541: Number of times a given SI can appear in the band structure. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2
0102	0, 1, 2, 3
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2
1002	0, 1, 2, 3
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2, 3

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SI	Possible #Gaps
1102	0, 1, 2
1103	0, 1, 2, 3
1111	0, 1
1112	0, 1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

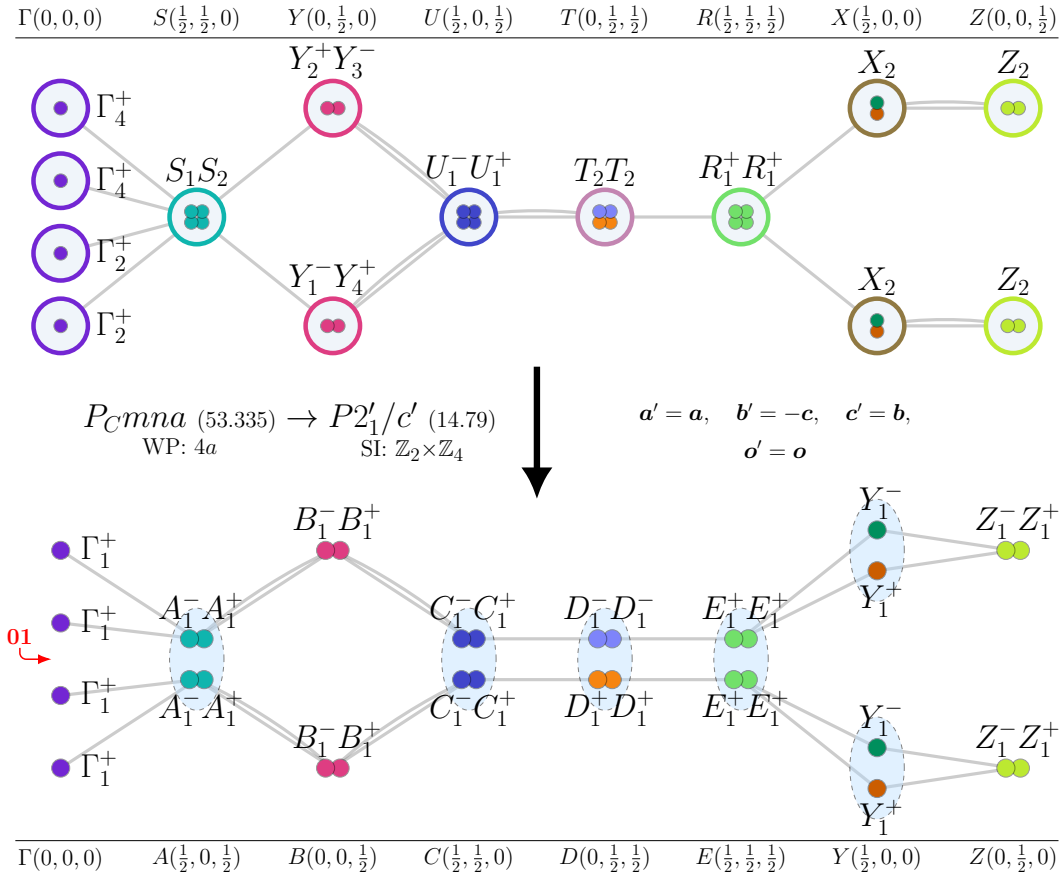
Supplementary Table 542: Possible SI values in each gap. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

56.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [001],
- B $\parallel$ [010] and strain $\perp$ [001],
- B $\perp$ [001].



Supplementary Figure 277: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cmna (53.335).

Supplementary Table 543: Number of times a given SI can appear in the band structure. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 544: Possible SI values in each gap. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P2'_1/c'$ (14.79).

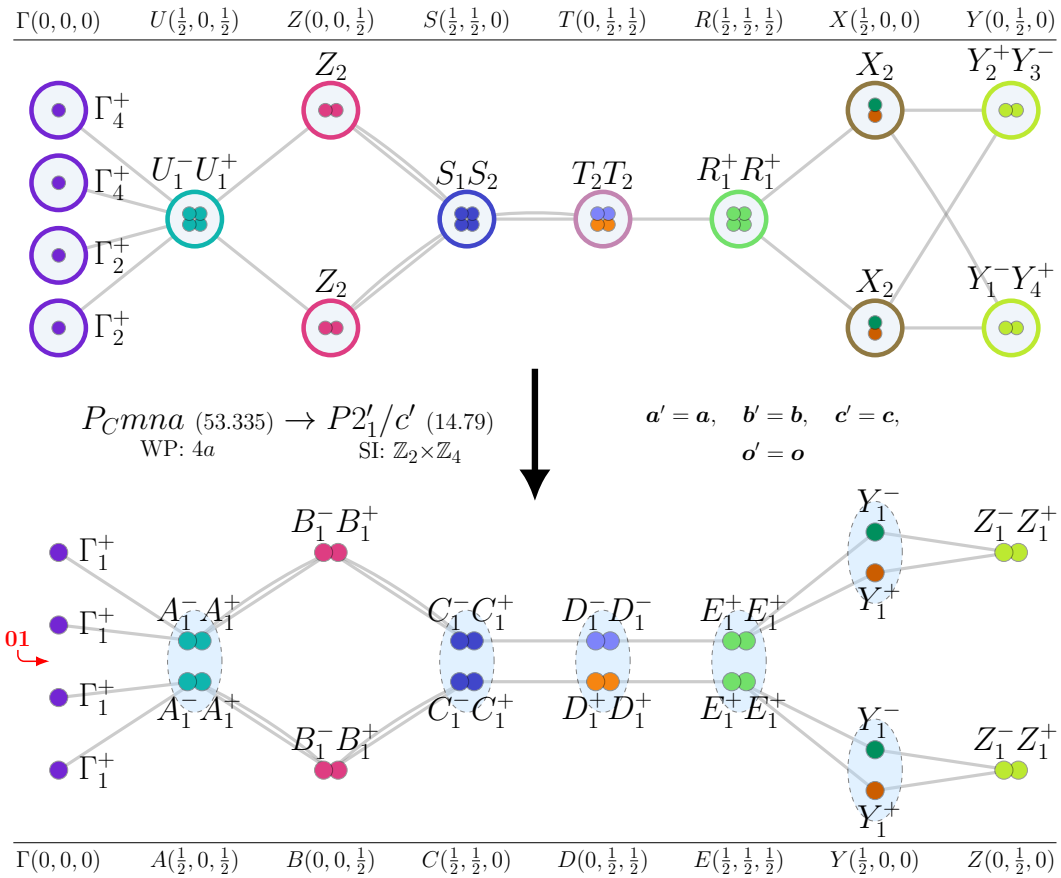
Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

56.1.3 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],

- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 278: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cmna (53.335).

Supplementary Table 545: Number of times a given SI can appear in the band structure. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

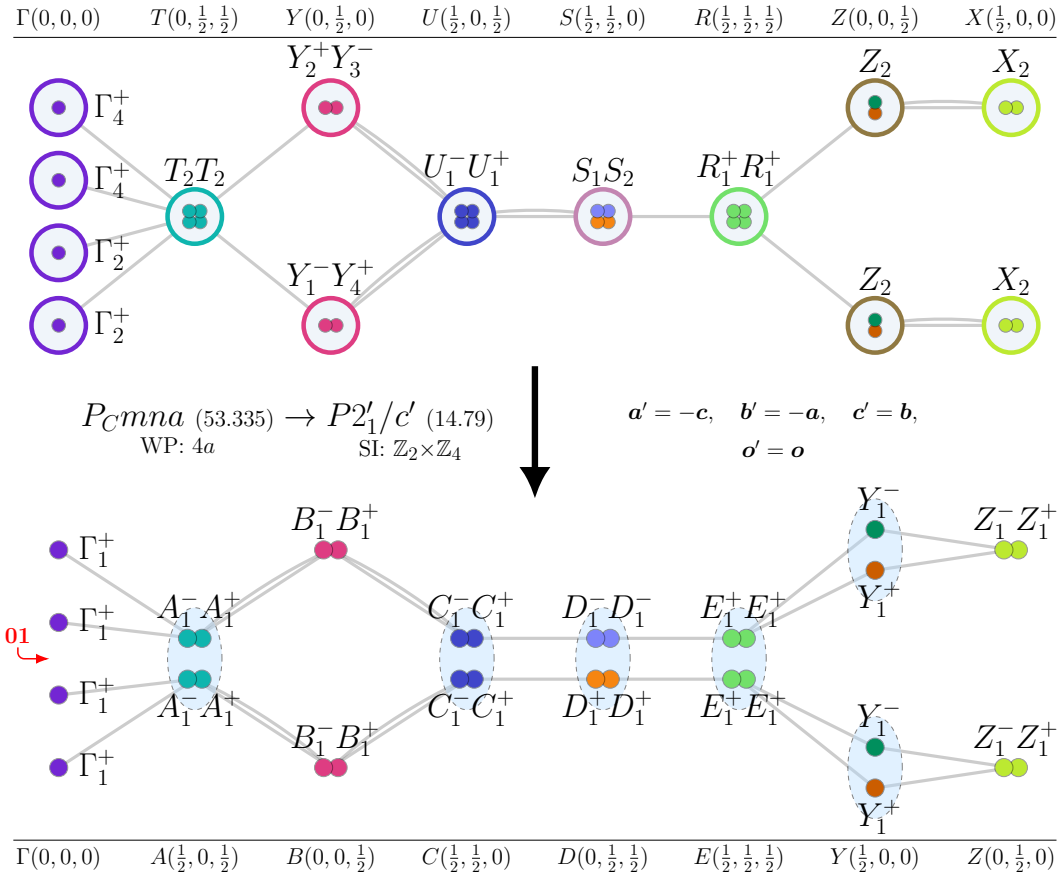
Supplementary Table 546: Possible SI values in each gap. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

56.1.4 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 279: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cmna (53.335).

Supplementary Table 547: Number of times a given SI can appear in the band structure. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

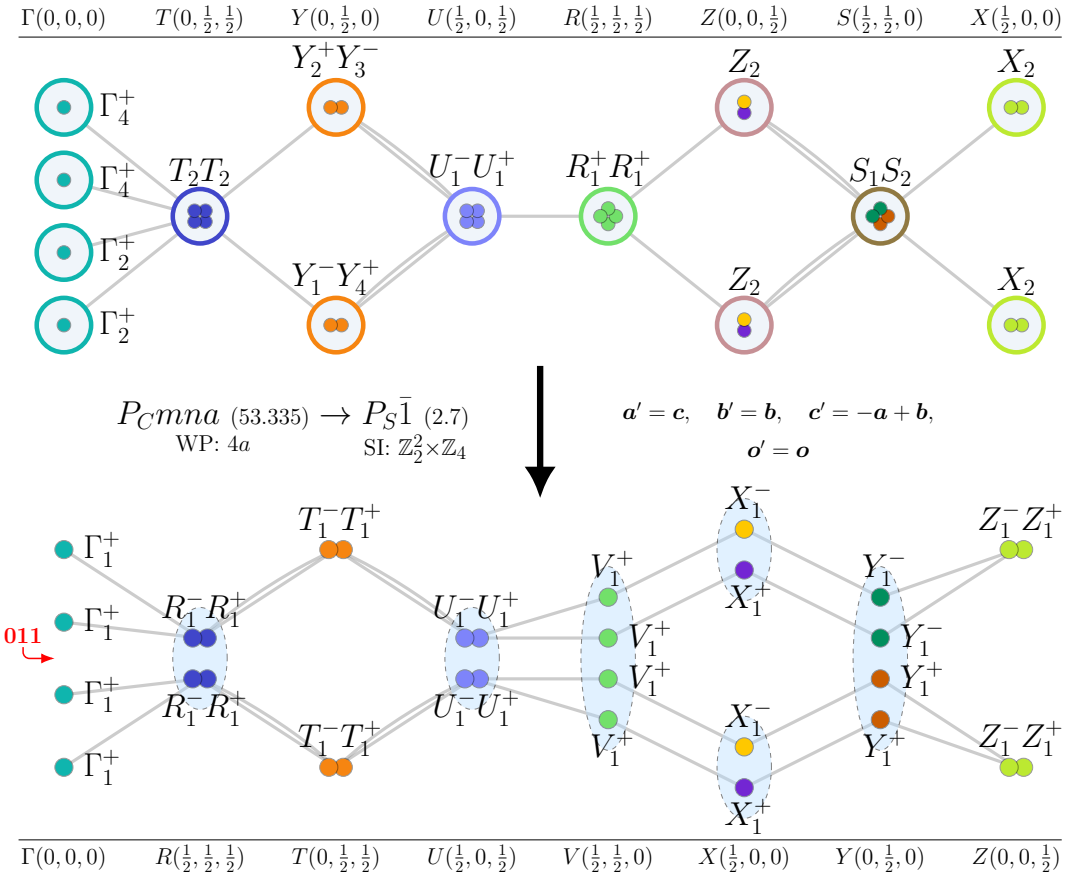
Supplementary Table 548: Possible SI values in each gap. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

56.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 280: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cmna (53.335).

Supplementary Table 549: Number of times a given SI can appear in the band structure. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
011	0, 1
013	0, 1
112	0, 1
gapless	2
nontrivial	1

Supplementary Table 550: Possible SI values in each gap. Supergroup: P_Cmna (53.335), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	011, 013, 112
3	G
4	000

57 MSG P_Imna (53.336)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'_1/m'$ (11.54), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_C2_1/c (14.84), $P2$ (3.1), $P2/c$ (13.65), $Pnn'a'$ (52.311), P_A2/c (13.73), $P2$ (3.1), $Pm'a'2$ (28.91), $P2/m$ (10.42), $Pm'ma'$ (51.296), P_C2/m (10.49).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), Pc' (7.26), $P2'$ (3.3), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), $Pn'a2'_1$ (33.146), $Pm'c2'_1$ (26.68), P_Cc (7.30), Pc (7.24), $Pn'n2'$ (34.158), $Pna'2'_1$ (33.147), P_Ac (7.31), Pm (6.18), $Pm'm2'$ (25.59), $Pmc'2'_1$ (26.69), P_Cm (6.23), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_C2_1 (4.12), P_Imn2_1 (31.134), $Pn'c'2$ (30.115), P_C2 (3.6), P_Ima2 (28.98), P_C2 (3.6), P_Inc2 (30.122).

57.1 WP: $4a + 4a + 8i$

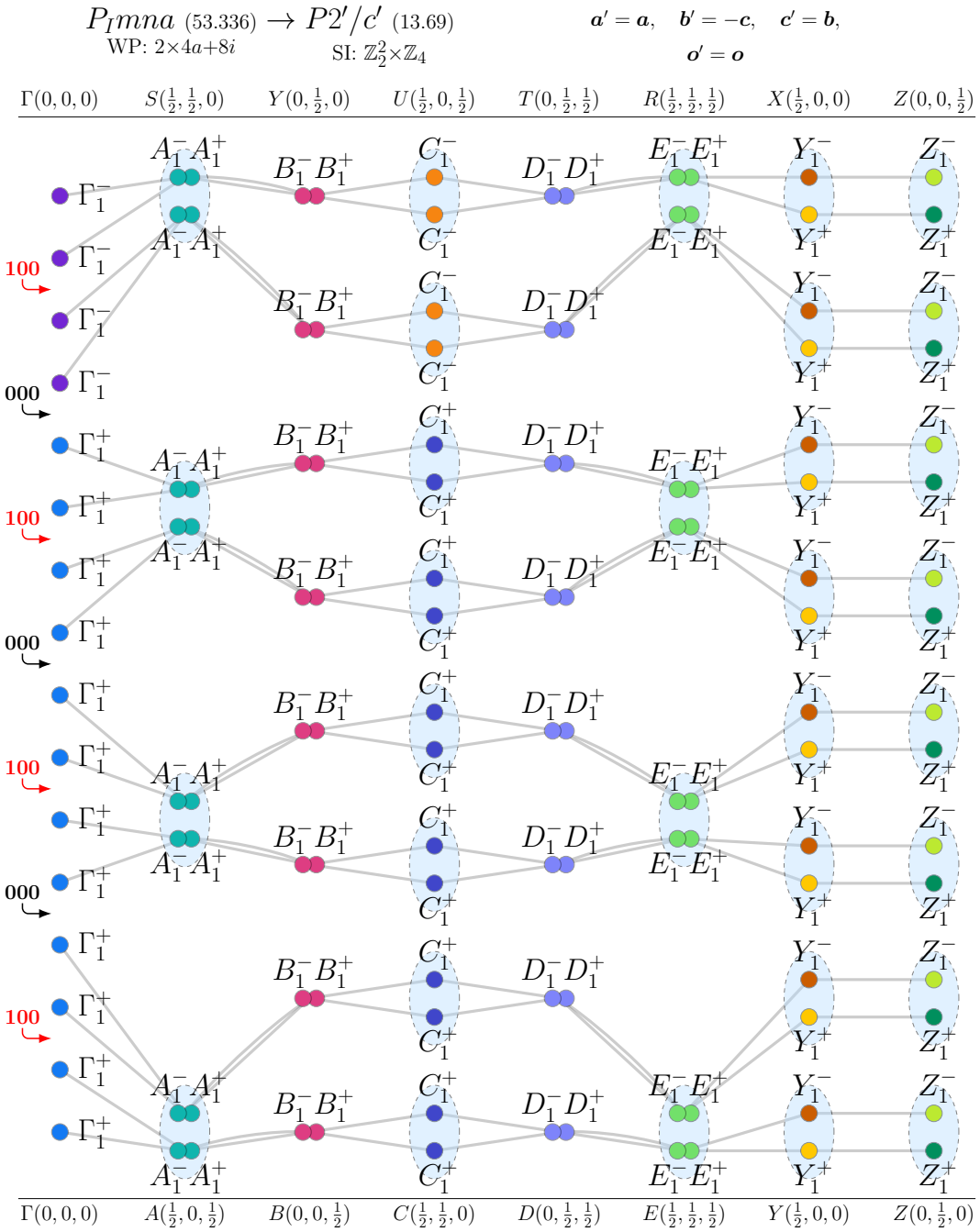
BCS Materials: $Sr_2Fe_{1.9}Cr_{0.1}O_5$ (650 K)[231].

57.1.1 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[231] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.410`



Supplementary Figure 281: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4a + 4a + 8i$ of supergroup P_Imna (53.336).

Supplementary Table 551: Number of times a given SI can appear in the band structure. Supergroup: P_Imna (53.336), WP: 4a+4a+8i, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2, 3
002	0, 1, 2, 3
011	0, 1, 2, 3, 4
013	0, 1, 2, 3, 4
100	0, 1, 2, 3, 4
102	0, 1, 2, 3, 4
111	0, 1, 2, 3
113	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

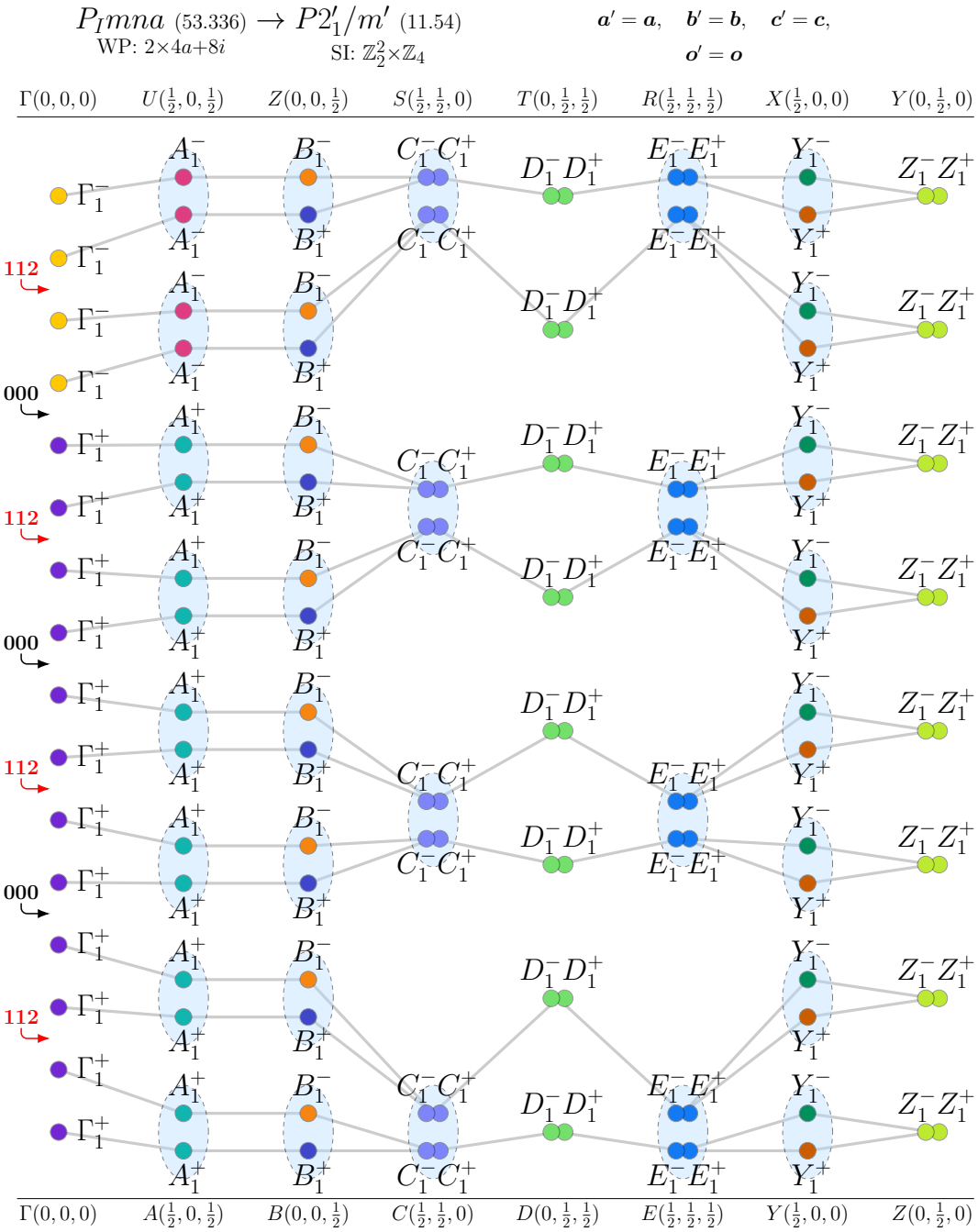
Supplementary Table 552: Possible SI values in each gap. Supergroup: P_Imna (53.336), WP: $4a$+$4a$+$8i$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	011, 013, 100, 102
3	G
4	000, 002, 111, 113
5	G
6	011, 013, 100, 102
7	G
8	000, 002, 111, 113
9	G
10	011, 013, 100, 102
11	G
12	000, 002, 111, 113
13	G
14	011, 013, 100, 102
15	G
16	000

57.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 282: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4a+4a+8i$ of supergroup P_Imna (53.336).

Supplementary Table 553: Number of times a given SI can appear in the band structure. Supergroup: P_Imna (53.336), WP: 4a+4a+8i, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2, 3
001	0, 1, 2, 3, 4
002	0, 1, 2, 3
003	0, 1, 2, 3, 4
110	0, 1, 2, 3, 4
111	0, 1, 2, 3
112	0, 1, 2, 3, 4
113	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

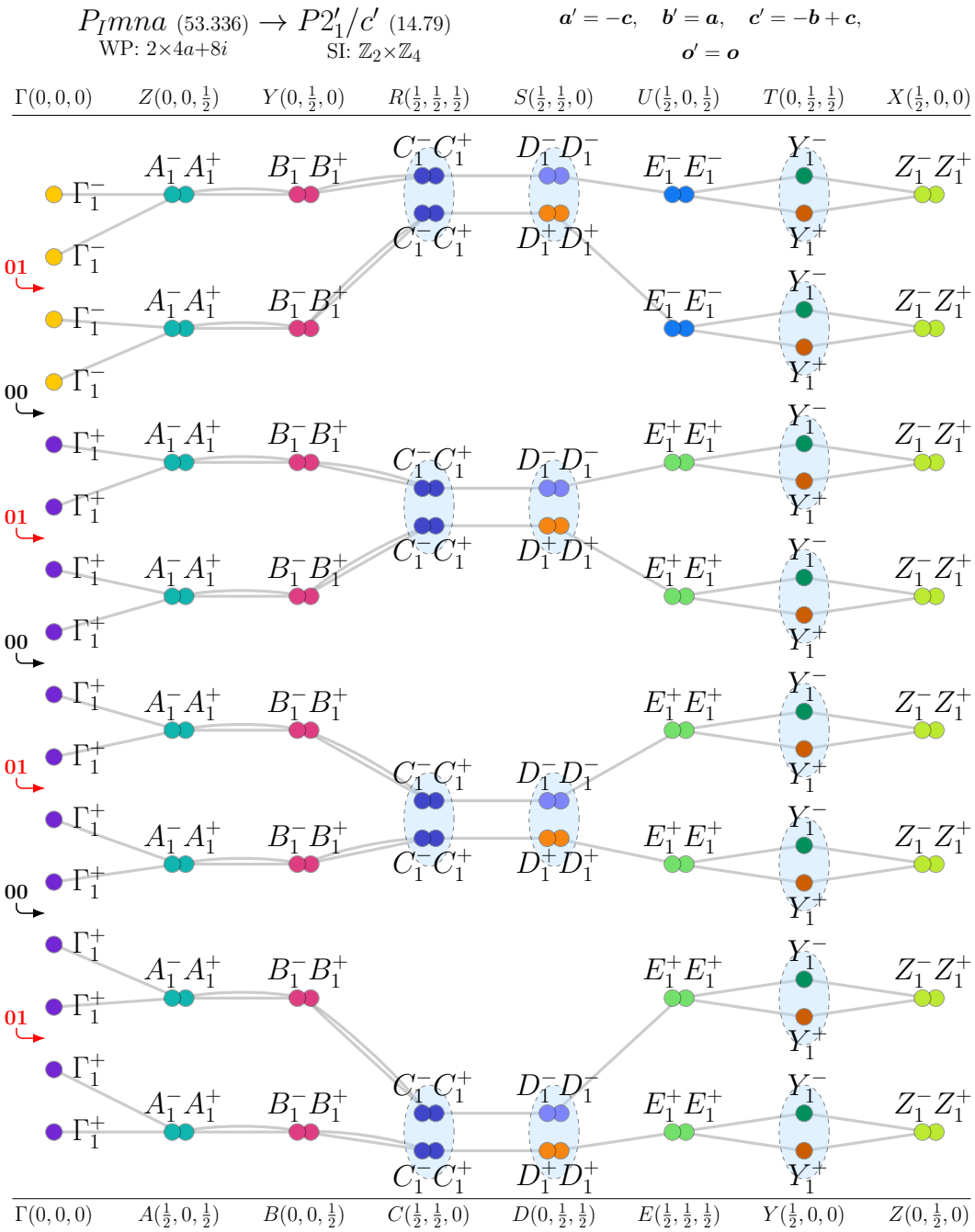
Supplementary Table 554: Possible SI values in each gap. Supergroup: P_Imna (53.336), WP: $4a$+$4a$+$8i$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000, 002, 111, 113
13	G
14	001, 003, 110, 112
15	G
16	000

57.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 283: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a + 4a + 8i$ of supergroup P_Imna (53.336).

Supplementary Table 555: Number of times a given SI can appear in the band structure. Supergroup: P_Imna (53.336), WP: 4a+4a+8i, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2, 3
01	0, 1, 2, 3, 4
02	0, 1, 2, 3
03	0, 1, 2, 3, 4
10	0, 1, 2, 3, 4
11	0, 1, 2, 3
12	0, 1, 2, 3, 4
13	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

Supplementary Table 556: Possible SI values in each gap. Supergroup: P_Imna (53.336), WP: $4a$+$4a$+$8i$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00, 02, 11, 13
13	G
14	01, 03, 10, 12
15	G
16	00

58 MSG P_Bcca (54.350)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/m'$ (11.54), $P2'/c'$ (13.69), $P2'/m'$ (10.46), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Pm'a'2$ (28.91), P_Acc2 (27.85), $P2/c$ (13.65), $Pc'cm'$ (49.270), P_A2/c (13.73), $P2$ (3.1), $Pm'm'2$ (25.60), P_a2 (3.4), $P2/c$ (13.65), $Pm'm'a$ (51.294), P_a2/c (13.70), $P2_1/c$ (14.75), $Pb'cm'$ (57.384), P_A2_1/c (14.83).

Trivial-SI Subgroups: Pm' (6.20), Pc' (7.26), Pm' (6.20), $P2_1'$ (4.9), $P2'$ (3.3), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pm'a2'$ (28.89), $Pc'c2'$ (27.80), P_Ac (7.31), Pc (7.24), $Pm'c2_1'$ (26.68), $Pm'a2'$ (28.89), P_ac (7.27), Pc (7.24), $Pca'2_1'$ (29.102), $Pm'a2'$ (28.89), P_Ac (7.31), P_C2 (3.6), P_Cba2 (32.141), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_C2_1 (4.12), P_Bca2_1 (29.108).

58.1 WP: $4g + 4g$

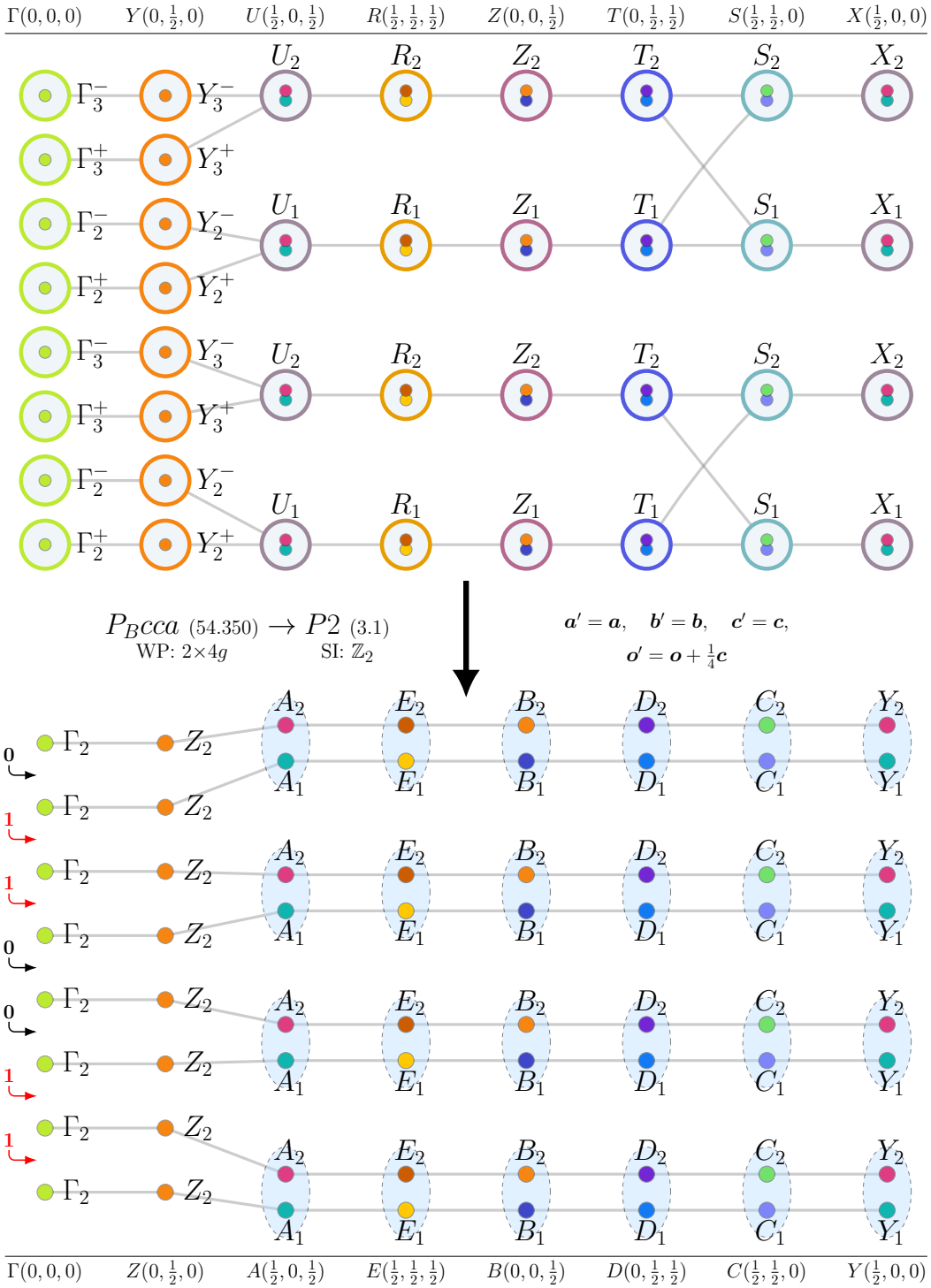
BCS Materials: $CeRh_2Si_2$ (27 K)[232].

58.1.1 Topological bands in subgroup $P2$ (3.1)

Perturbation:

- E ∥ [010] and B ∥ [010] and strain ⊥ [010].

[232]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.30`



Supplementary Figure 284: Topological magnon bands in subgroup $P2$ (3.1) for magnetic moments on Wyckoff positions $4g + 4g$ of supergroup P_Bcca (54.350).

Supplementary Table 557: Number of times a given SI can appear in the band structure. Supergroup: P_Bcca (54.350), WP: $4g$+$4g$, Subgroup: $P2$ (3.1).

SI	Possible #Gaps
0	1, 2, 3, 4, 5
1	2, 3, 4, 5, 6
gapless	0, 1, 2, 3, 4
nontrivial	2, 3, 4, 5, 6

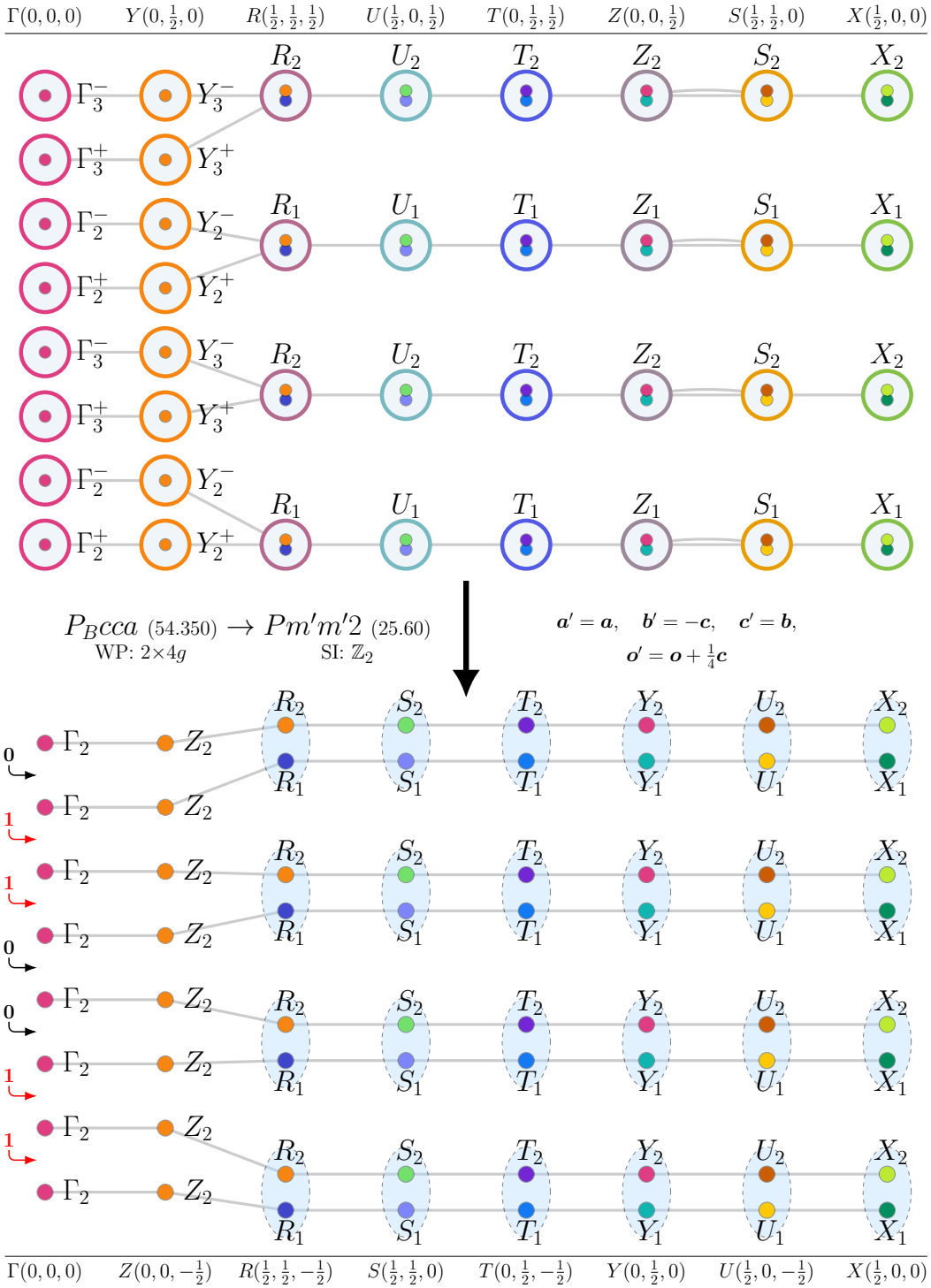
Supplementary Table 558: Possible SI values in each gap. Supergroup: P_Bcca (54.350), WP: $4g$+$4g$, Subgroup: $P2$ (3.1).

Gap#	Possible SI Values
1	0, 1, G
2	1
3	0, 1, G
4	0
5	0, 1, G
6	1
7	0, 1, G
8	0

58.1.2 Topological bands in subgroup $Pm'm'2$ (25.60)

Perturbation:

- E ∥ [010] and B ∥ [010].



Supplementary Figure 285: Topological magnon bands in subgroup $Pm'm'2$ (25.60) for magnetic moments on Wyckoff positions $4g + 4g$ of supergroup P_Bcca (54.350).

Supplementary Table 559: Number of times a given SI can appear in the band structure. Supergroup: P_Bcca (54.350), WP: $4g$+$4g$, Subgroup: $Pm'm'2$ (25.60).

SI	Possible #Gaps
0	1, 2, 3, 4, 5
1	2, 3, 4, 5, 6
gapless	0, 1, 2, 3, 4
nontrivial	2, 3, 4, 5, 6

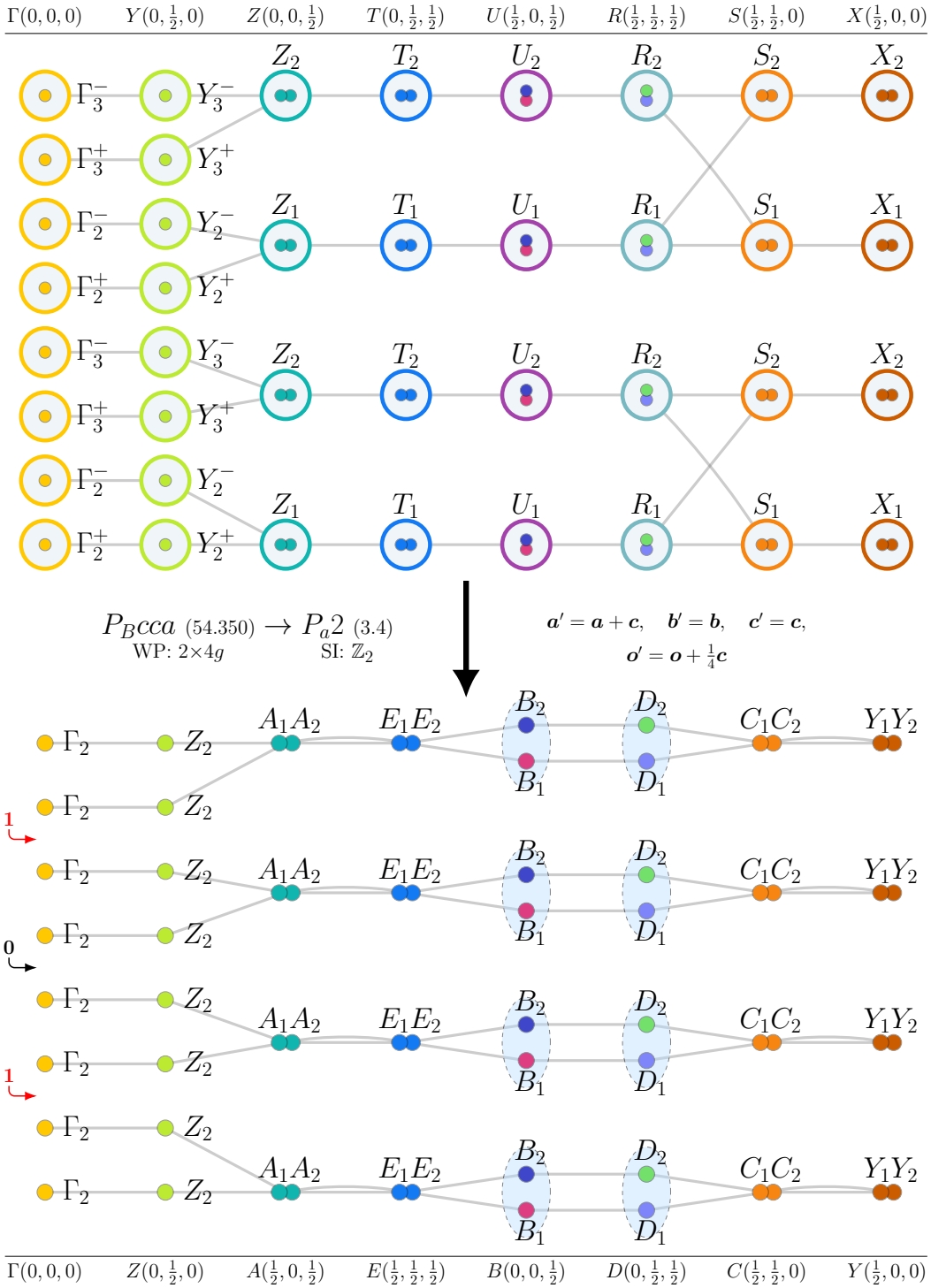
Supplementary Table 560: Possible SI values in each gap. Supergroup: P_Bcca (54.350), WP: $4g$+$4g$, Subgroup: $Pm'm'2$ (25.60).

Gap#	Possible SI Values
1	0, 1, G
2	1
3	0, 1, G
4	0
5	0, 1, G
6	1
7	0, 1, G
8	0

58.1.3 Topological bands in subgroup P_a2 (3.4)

Perturbation:

- E ∥ [010] and strain ⊥ [010].



Supplementary Figure 286: Topological magnon bands in subgroup P_a2 (3.4) for magnetic moments on Wyckoff positions $4g + 4g$ of supergroup P_Bcca (54.350).

Supplementary Table 561: Number of times a given SI can appear in the band structure. Supergroup: P_Bcca (54.350), WP: 4g+4g, Subgroup: P_a2 (3.4).

SI	Possible #Gaps
0	1
1	2
gapless	4
nontrivial	2

Supplementary Table 562: Possible SI values in each gap. Supergroup: P_Bcca (54.350), WP: $4g$+$4g$, Subgroup: P_a2 (3.4).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0
5	G
6	1
7	G
8	0

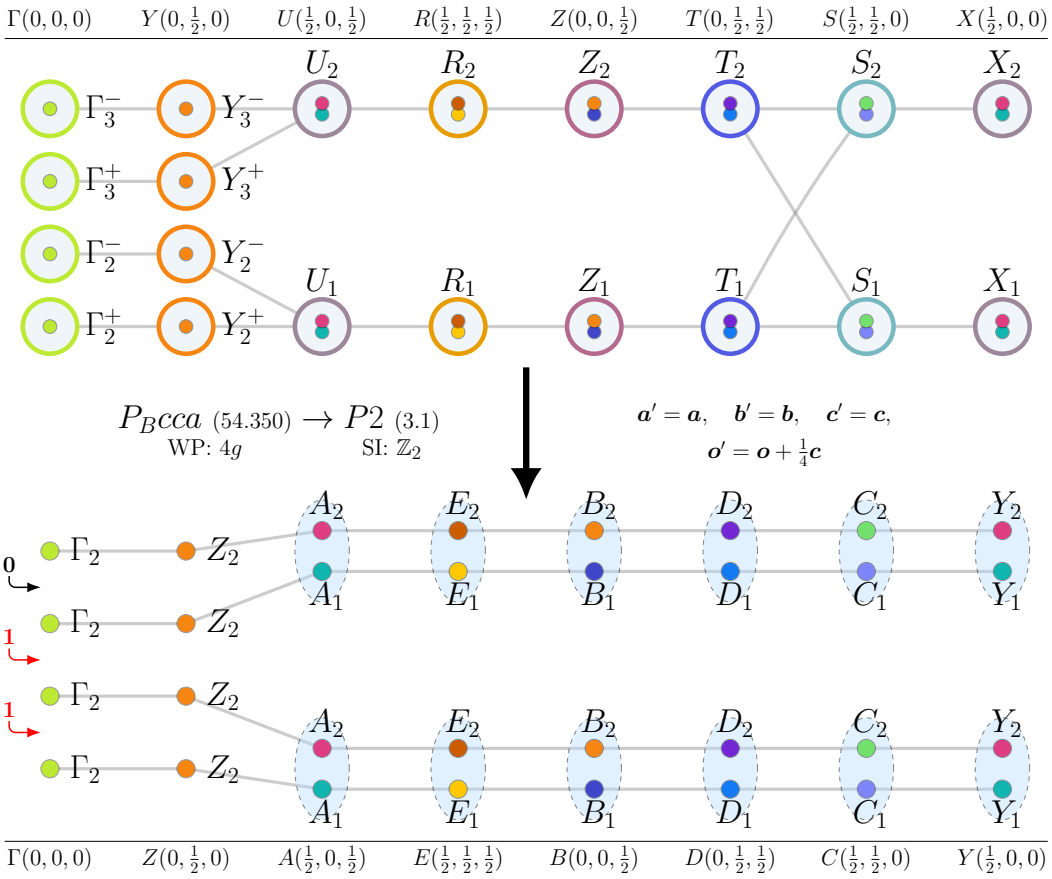
58.2 WP: $4g$

BCS Materials: $NdNiMg_{15}$ (9 K)[233].

58.2.1 Topological bands in subgroup $P2$ (3.1)

Perturbation:

- E ∥ [010] and B ∥ [010] and strain ⊥ [010].



Supplementary Figure 287: Topological magnon bands in subgroup $P2$ (3.1) for magnetic moments on Wyckoff position $4g$ of supergroup P_Bcca (54.350).

[233]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.457`

Supplementary Table 563: Number of times a given SI can appear in the band structure. Supergroup: P_Bcca (54.350), WP: $4g$, Subgroup: $P2$ (3.1).

SI	Possible #Gaps
0	0, 1, 2
1	1, 2, 3
gapless	0, 1, 2
nontrivial	1, 2, 3

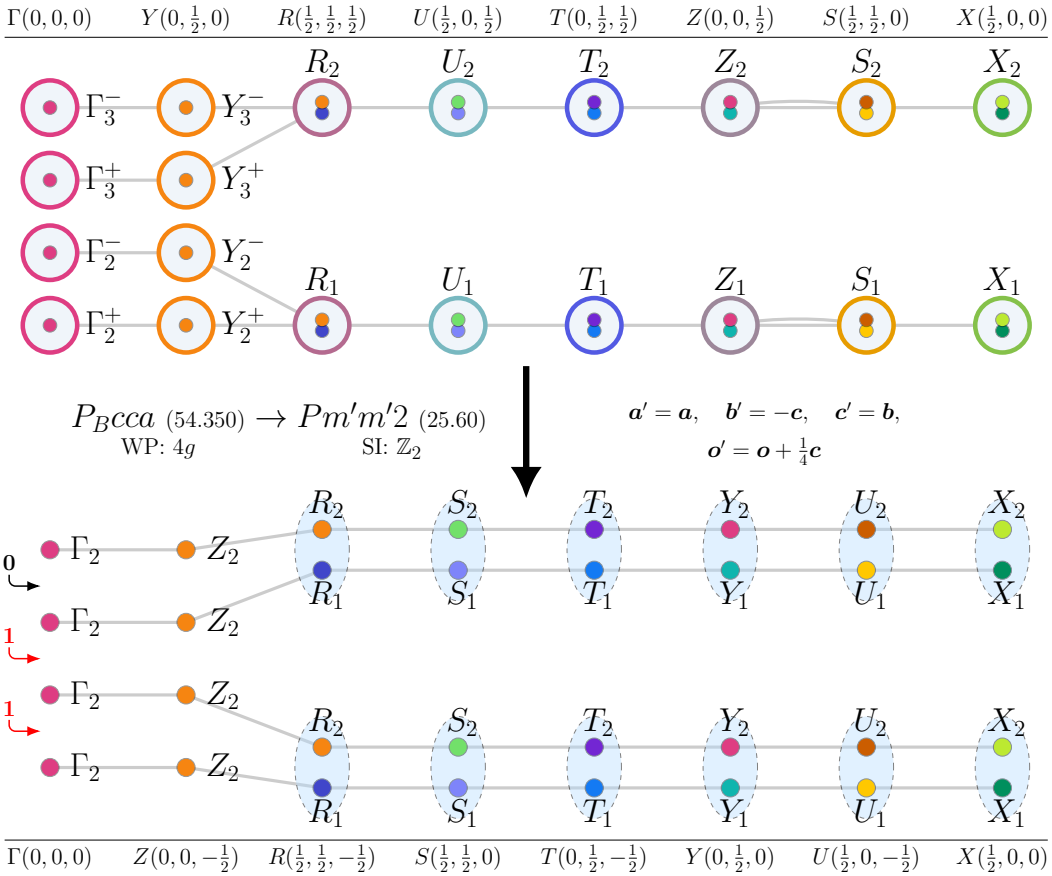
Supplementary Table 564: Possible SI values in each gap. Supergroup: P_Bcca (54.350), WP: $4g$, Subgroup: $P2$ (3.1).

Gap#	Possible SI Values
1	0, 1, G
2	1
3	0, 1, G
4	0

58.2.2 Topological bands in subgroup $Pm'm'2$ (25.60)

Perturbation:

- E ∥ [010] and B ∥ [010].



Supplementary Figure 288: Topological magnon bands in subgroup $Pm'm'2$ (25.60) for magnetic moments on Wyckoff position $4g$ of supergroup P_Bcca (54.350).

Supplementary Table 565: Number of times a given SI can appear in the band structure. Supergroup: P_Bcca (54.350), WP: $4g$, Subgroup: $Pm'm'2$ (25.60).

SI	Possible #Gaps
0	0, 1, 2
1	1, 2, 3
gapless	0, 1, 2
nontrivial	1, 2, 3

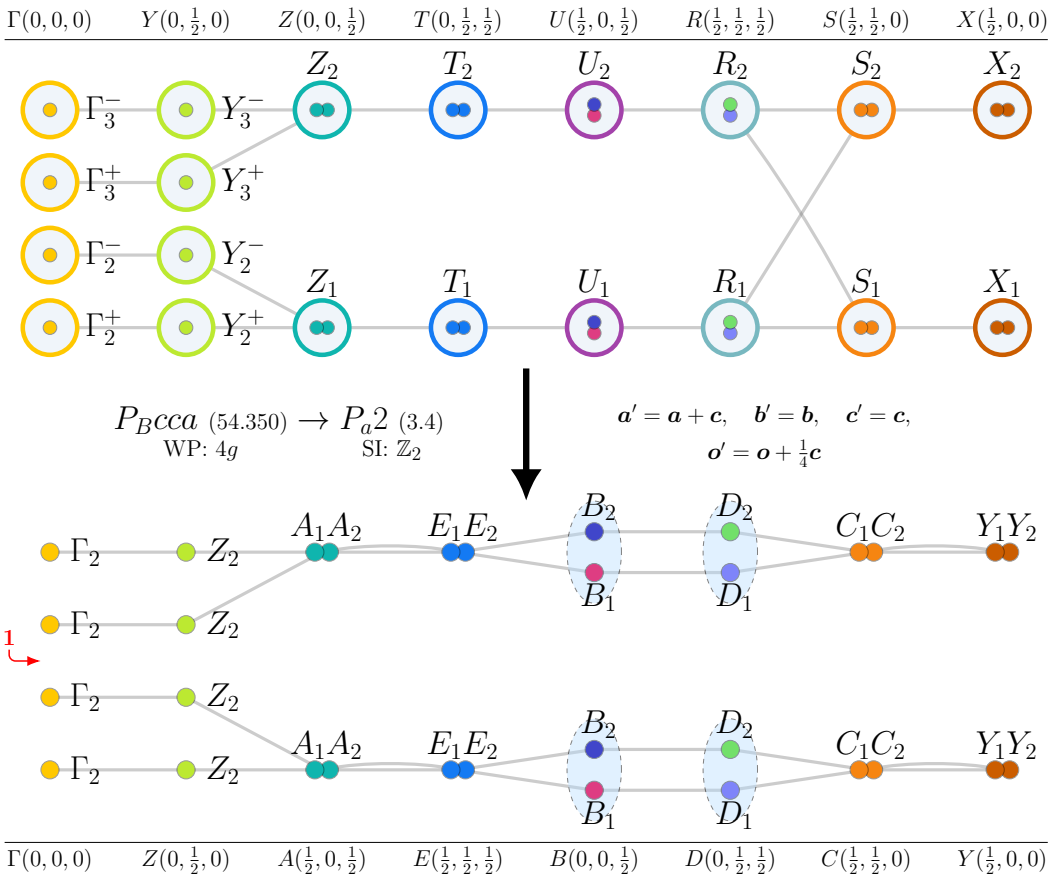
Supplementary Table 566: Possible SI values in each gap. Supergroup: P_Bcca (54.350), WP: $4g$, Subgroup: $Pm'm'2$ (25.60).

Gap#	Possible SI Values
1	0, 1, G
2	1
3	0, 1, G
4	0

58.2.3 Topological bands in subgroup P_a2 (3.4)

Perturbation:

- E ∥ [010] and strain ⊥ [010].



Supplementary Figure 289: Topological magnon bands in subgroup P_a2 (3.4) for magnetic moments on Wyckoff position $4g$ of supergroup P_Bcca (54.350).

Supplementary Table 567: Number of times a given SI can appear in the band structure. Supergroup: P_Bcca (54.350), WP: $4g$, Subgroup: P_a2 (3.4).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 568: Possible SI values in each gap. Supergroup: P_Bcca (54.350), WP: $4g$, Subgroup: P_a2 (3.4).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

59 MSG P_Icca (54.352)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'_1/c'$ (14.79), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Pb'a'2$ (32.138), P_Icc2 (27.86), $P2/c$ (13.65), $Pc'ca'$ (54.344), P_C2/c (13.74), $P2$ (3.1), $Pc'c'2$ (27.81), $P2/c$ (13.65), $Pc'c'a$ (54.342), P_C2/c (13.74), $P2_1/c$ (14.75), $Pb'c'a$ (61.436), P_C2_1/c (14.84).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2'_1$ (4.9), $P2'_1$ (4.9), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pc'a2'_1$ (29.101), $Pc'c2'$ (27.80), P_Cc (7.30), Pc (7.24), $Pca'2'_1$ (29.102), $Pb'a2'$ (32.137), P_Cc (7.30), Pc (7.24), $Pca'2'_1$ (29.102), $Pc'a2'_1$ (29.101), P_Cc (7.30), P_C2 (3.6), P_C2 (3.6), P_Iba2 (32.143), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_C2_1 (4.12), P_Ica2_1 (29.110).

59.1 WP: $8c$

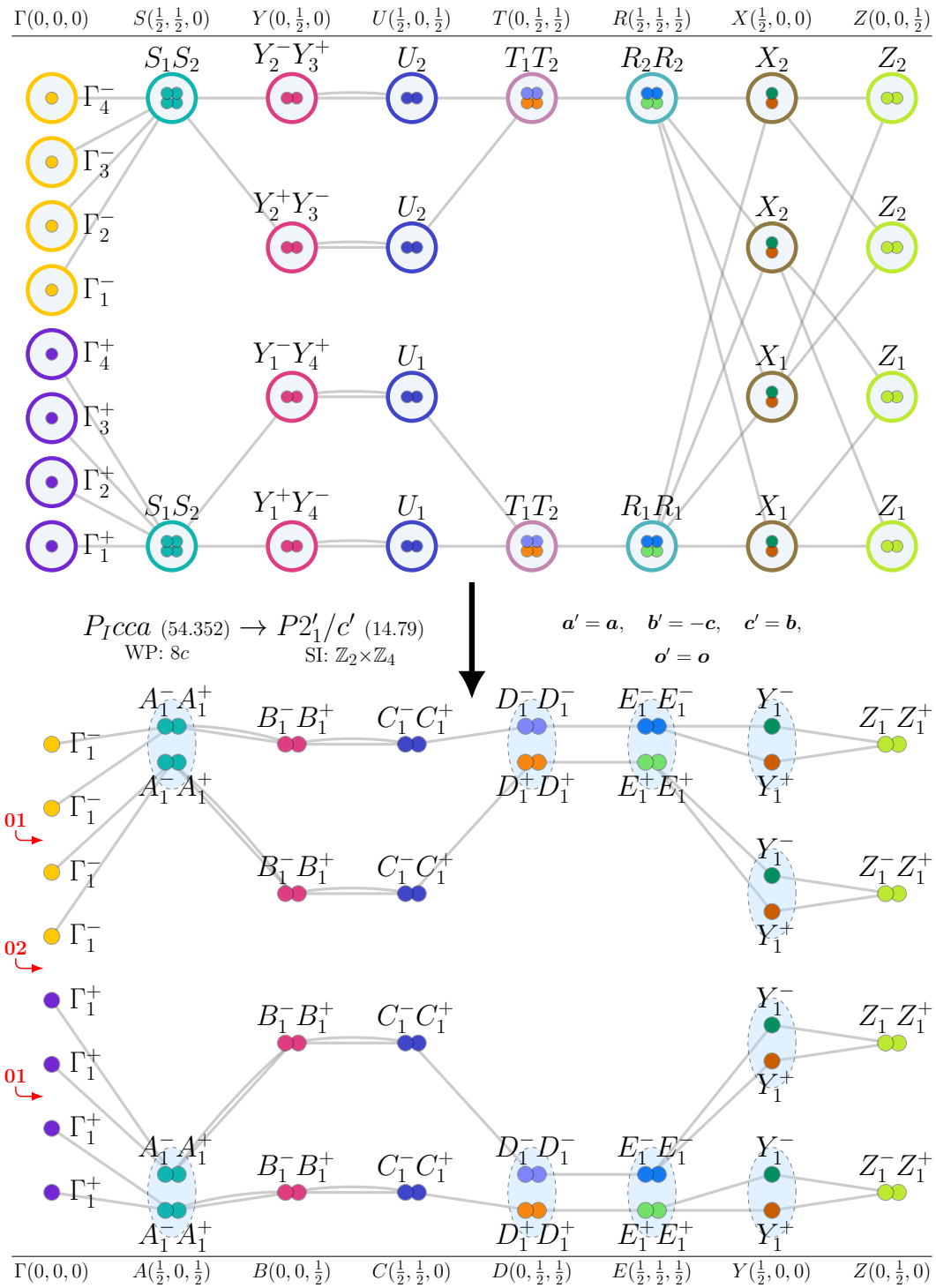
BCS Materials: Sr_2IrO_4 (240 K)[234], Sr_2IrO_4 (224 K)[235].

59.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[234]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.3`
[235]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.77`



Supplementary Figure 290: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8c$ of supergroup P_Icca (54.352).

Supplementary Table 569: Number of times a given SI can appear in the band structure. Supergroup: P_Icca (54.352), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

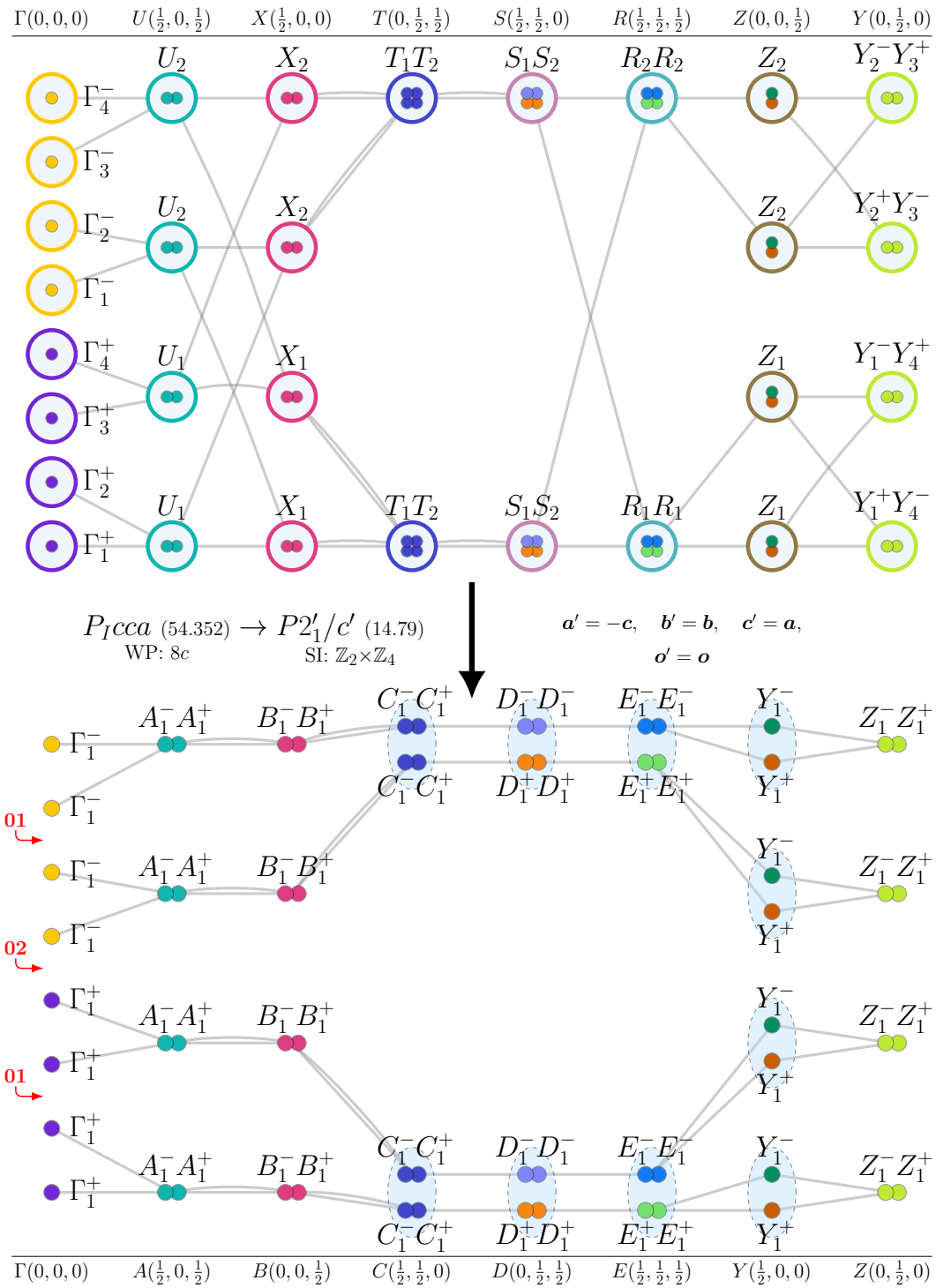
Supplementary Table 570: Possible SI values in each gap. Supergroup: P_Icca (54.352), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

59.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],
- B $\perp$ [010].



Supplementary Figure 291: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8c$ of supergroup P_Icca (54.352).

Supplementary Table 571: Number of times a given SI can appear in the band structure. Supergroup: P_Icca (54.352), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

continued on next page...

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 572: Possible SI values in each gap. Supergroup: P_Icca (54.352), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

59.2 WP: $16f$

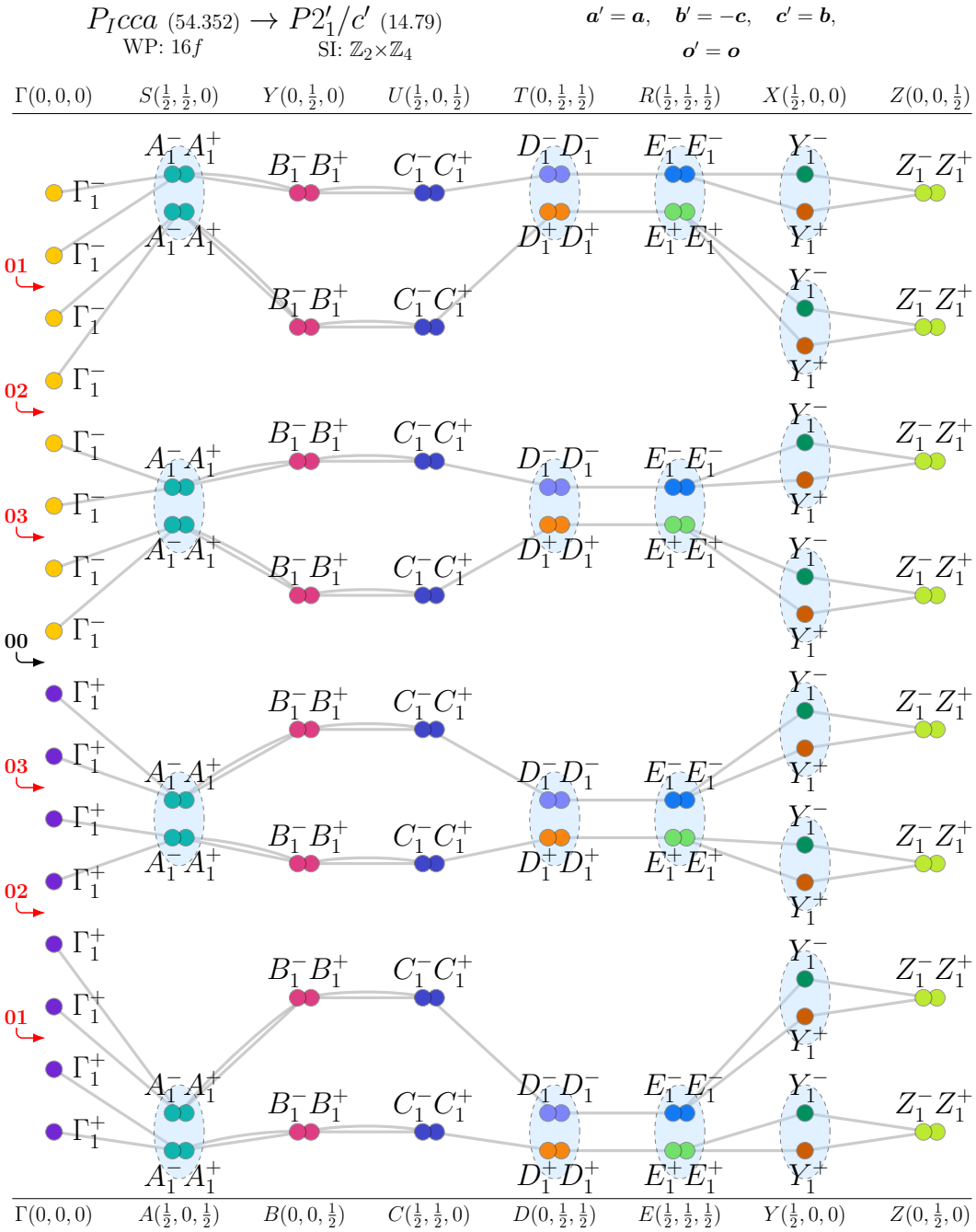
BCS Materials: $BaCo_2V_2O_8$ (5 K)[236].

59.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[236]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.30`



Supplementary Figure 292: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $16f$ of supergroup P_Icca (54.352).

Supplementary Table 573: Number of times a given SI can appear in the band structure. Supergroup: P_Icca (54.352), WP: $16f$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2, 3
01	0, 1, 2, 3, 4
02	0, 1, 2, 3
03	0, 1, 2, 3, 4
10	0, 1, 2, 3, 4
11	0, 1, 2, 3
12	0, 1, 2, 3, 4
13	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

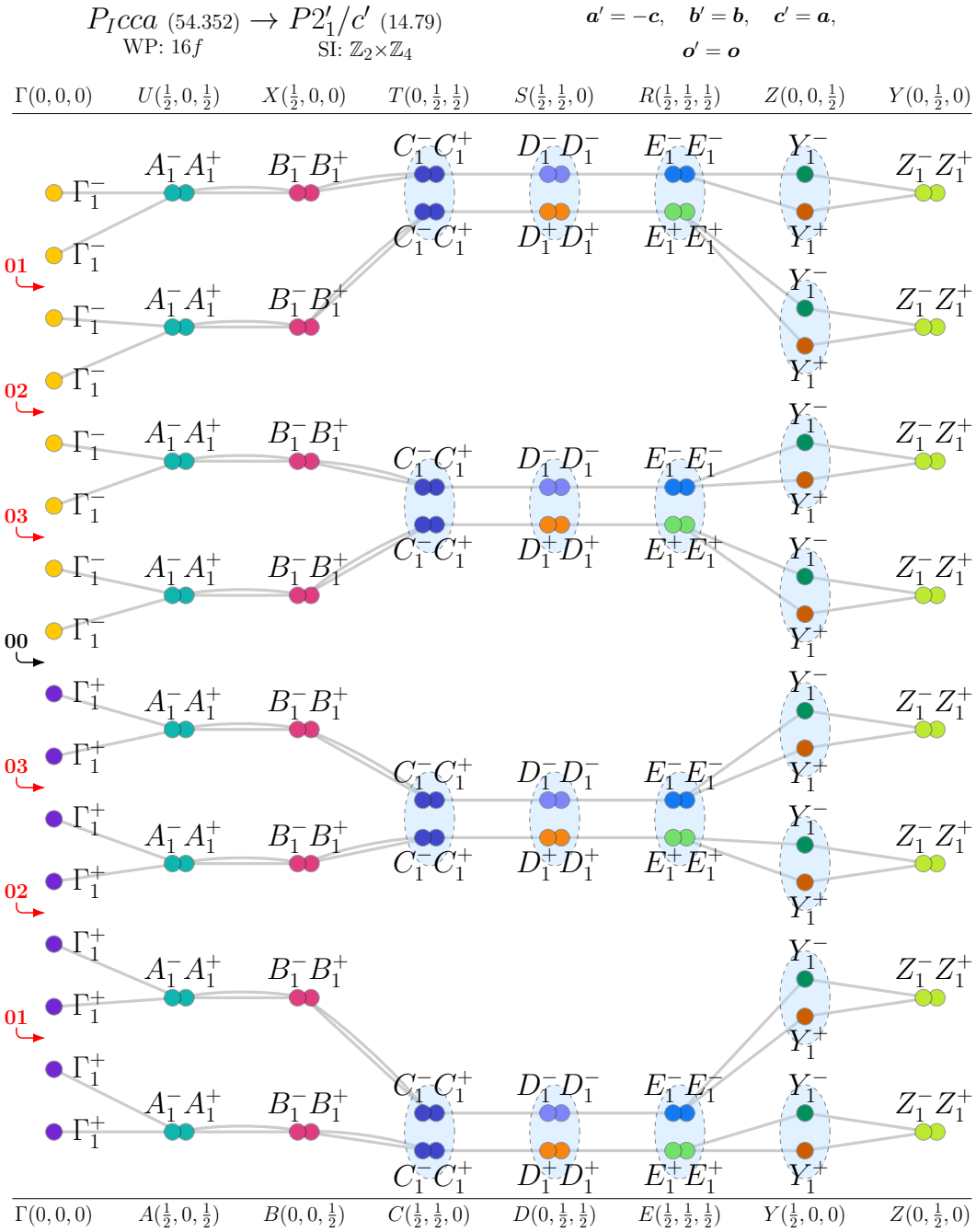
Supplementary Table 574: Possible SI values in each gap. Supergroup: P_Icca (54.352), WP: $16f$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00, 02, 11, 13
13	G
14	01, 03, 10, 12
15	G
16	00

59.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],
- B $\perp$ [010].



Supplementary Figure 293: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $16f$ of supergroup P_Icca (54.352).

Supplementary Table 575: Number of times a given SI can appear in the band structure. Supergroup: P_Icca (54.352), WP: $16f$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2, 3
01	0, 1, 2, 3, 4
02	0, 1, 2, 3
03	0, 1, 2, 3, 4
10	0, 1, 2, 3, 4
11	0, 1, 2, 3
12	0, 1, 2, 3, 4
13	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

Supplementary Table 576: Possible SI values in each gap. Supergroup: P_Icca (54.352), WP: $16f$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00, 02, 11, 13
13	G
14	01, 03, 10, 12
15	G
16	00

60 MSG $Pc'c'n$ (56.369)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P2_1'/c'$ (14.79), $P2$ (3.1), $Pc'c'2$ (27.81), $P2/c$ (13.65).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), $P2_1'$ (4.9), $P2_1'$ (4.9), Pc (7.24), $Pna'2_1'$ (33.147), $Pna'2_1'$ (33.147).

60.1 WP: $4a$

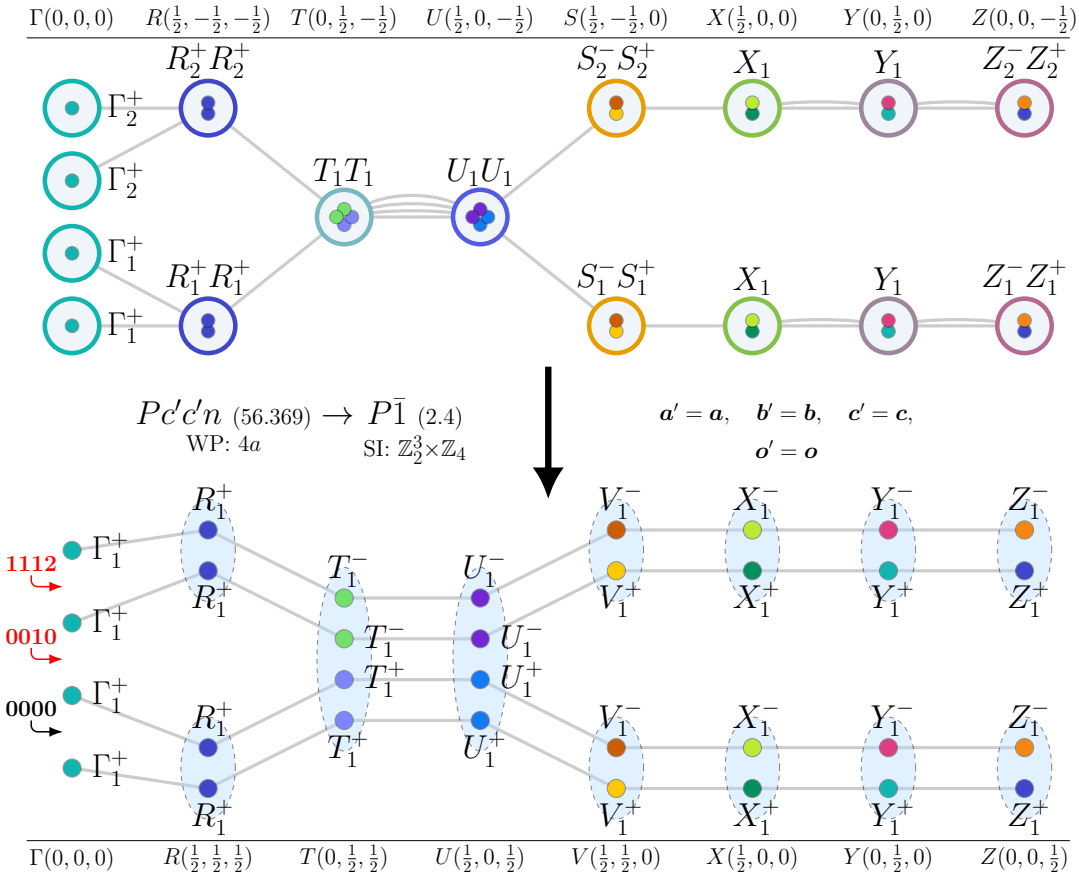
BCS Materials: La_2NiO_4 (80 K)[237].

60.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[237] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.45`



Supplementary Figure 294: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $Pc'c'n$ (56.369).

Supplementary Table 577: Number of times a given SI can appear in the band structure. Supergroup: $Pc'c'n$ (56.369), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2
0012	0, 1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1, 2
1111	0, 1
1112	0, 1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

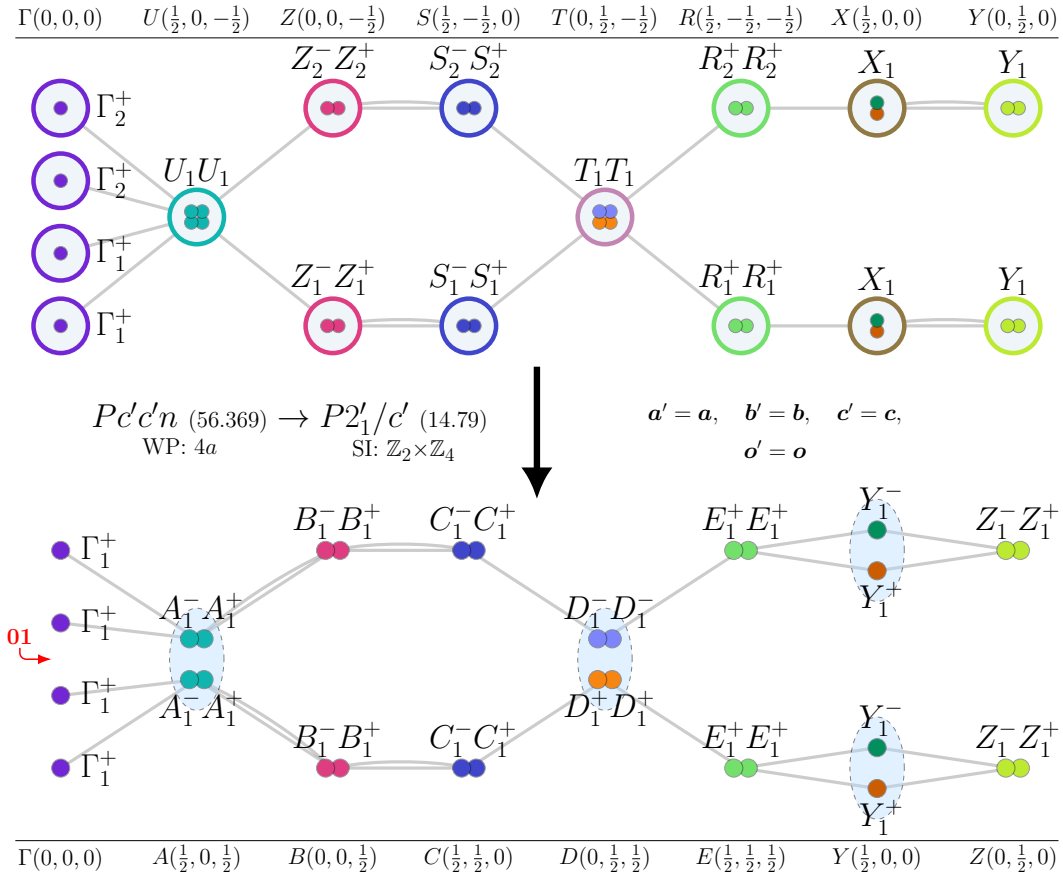
Supplementary Table 578: Possible SI values in each gap. Supergroup: $Pc'c'n$ (56.369), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	0010, 0012, 0111, 0113, 1011 1013, 1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

60.1.2 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 295: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup $Pc'c'n$ (56.369).

Supplementary Table 579: Number of times a given SI can appear in the band structure. Supergroup: $Pc'c'n$ (56.369), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

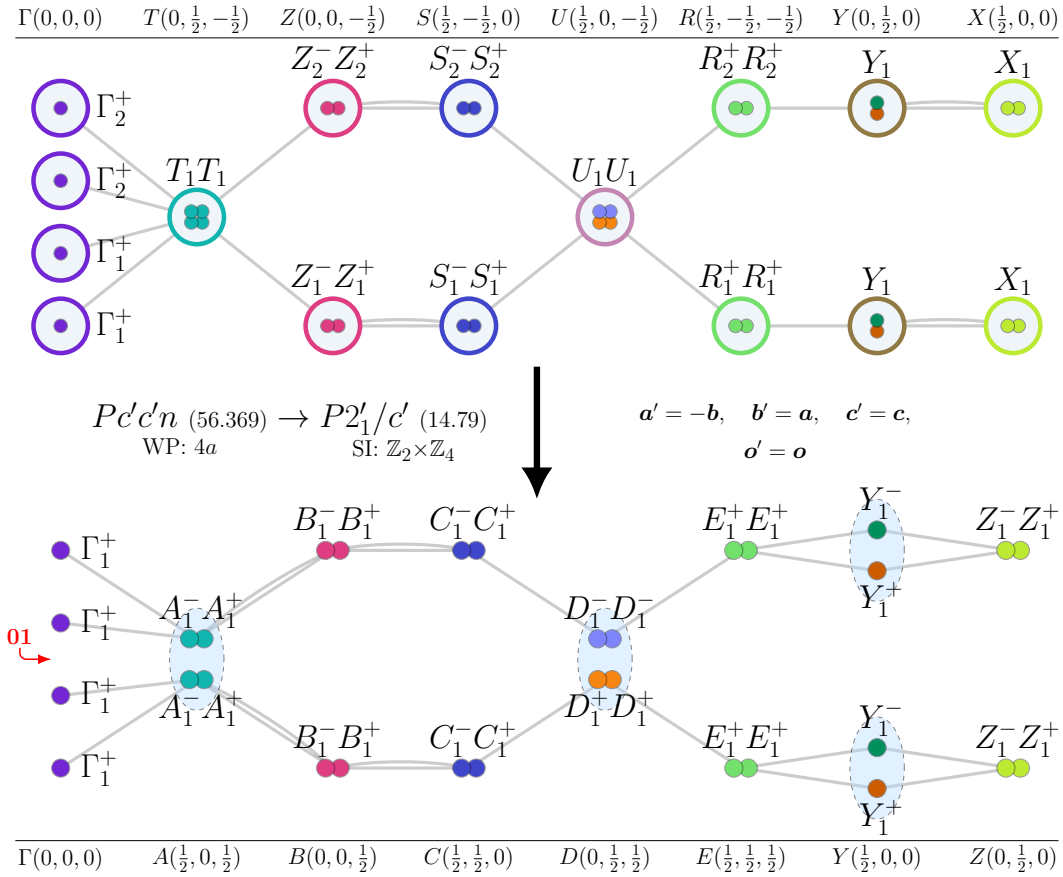
Supplementary Table 580: Possible SI values in each gap. Supergroup: $Pc'c'n$ (56.369), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

60.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain ⊥ [100],
- (B ∥ [010] or B ⊥ [100]).



Supplementary Figure 296: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup $Pc'c'n$ (56.369).

Supplementary Table 581: Number of times a given SI can appear in the band structure. Supergroup: $Pc'c'n$ (56.369), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 582: Possible SI values in each gap. Supergroup: $Pc'c'n$ (56.369), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

61 MSG P_bccn (56.372)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'/c'$ (13.69), $P2_1'/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2$ (3.1), P_a2 (3.4), P_acc2 (27.83), $P2/c$ (13.65), $Pnn'a'$ (52.311), P_a2/c (13.70), $P2_1/c$ (14.75), $Pbc'n'$ (60.423), P_b2_1/c (14.81), $P2_1/c$ (14.75), $Pcc'a'$ (54.343), P_a2_1/c (14.80).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2'$ (3.3), $P2_1'$ (4.9), P_S1 (1.3), Pc (7.24), $Pn'n2'$ (34.158), $Pna'2_1'$ (33.147), P_ac (7.27), Pc (7.24), $Pn'c2'$ (30.113), $Pc'a2_1'$ (29.101), P_bc (7.29), Pc (7.24), $Pc'c2'$ (27.80), $Pb'a2'$ (32.137), P_ac (7.27), $Pn'c'2$ (30.115), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_b2_1 (4.11), P_cna2_1 (33.151), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_a2_1 (4.10), P_bna2_1 (33.150).

61.1 WP: $8c$

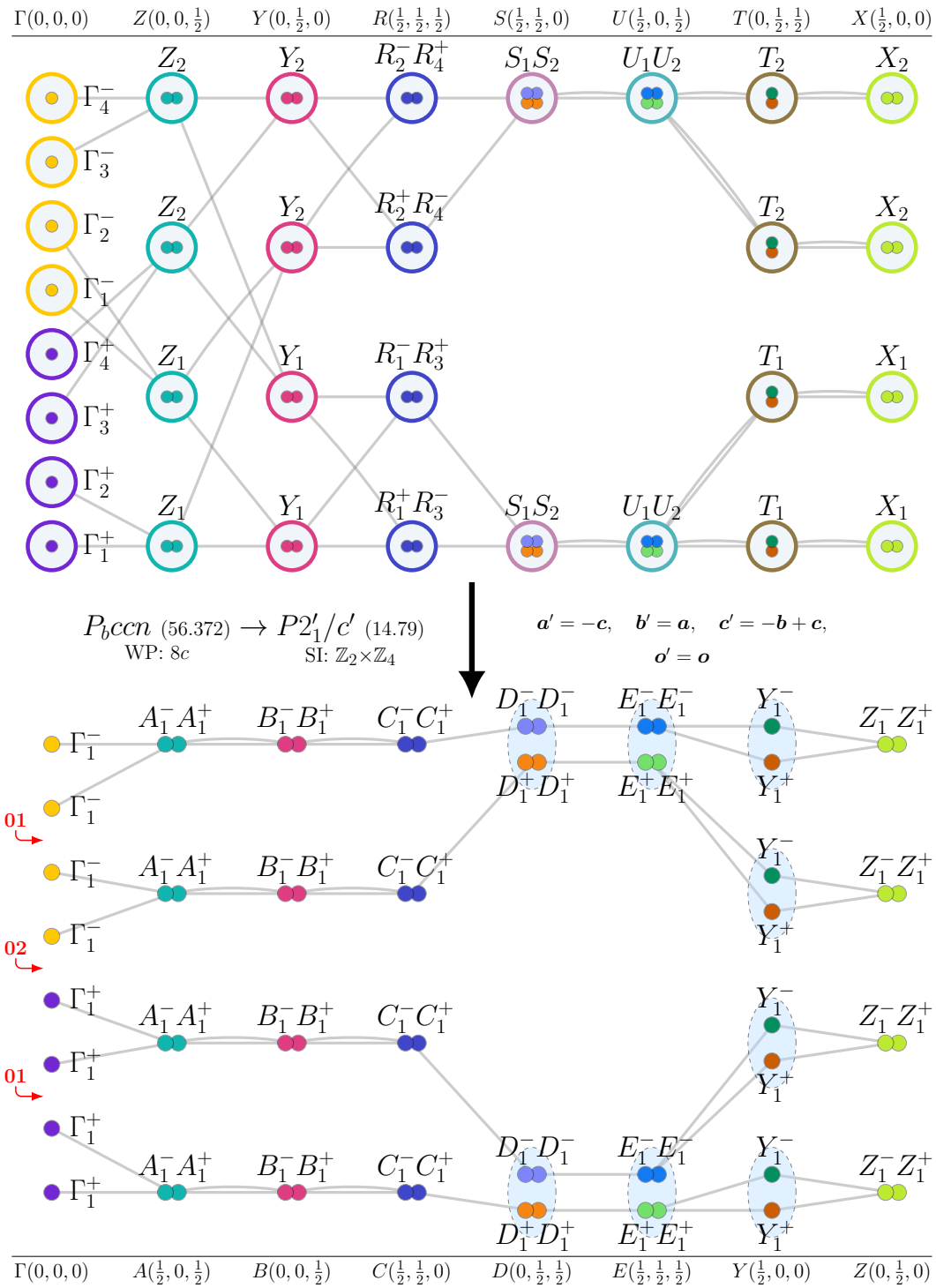
BCS Materials: $CsCoCl_3(D_2O)_2$ (3.4 K)[238].

61.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].

[238] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.99`



Supplementary Figure 297: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8c$ of supergroup P_bccn (56.372).

Supplementary Table 583: Number of times a given SI can appear in the band structure. Supergroup: P_bccn (56.372), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 584: Possible SI values in each gap. Supergroup: P_bccn (56.372), WP: $8c$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

62 MSG P_ccn (56.373)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'_1/m'$ (11.54), $P2'_1/m'$ (11.54), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Pm'm'2$ (25.60), P_b2 (3.5), P_cc2 (27.82), $P2/c$ (13.65), $Pm'm'n$ (59.409), P_b2/c (13.71), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_c2_1/c (14.82), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_c2_1/c (14.82).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), Pm' (6.20), $P2'_1$ (4.9), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), $Pm'n2'_1$ (31.125), $Pm'n2'_1$ (31.125), P_bc (7.29), Pc (7.24), $Pm'c2'_1$ (26.68), $Pn'a2'_1$ (33.146), P_cc (7.28), Pc (7.24), $Pm'c2'_1$ (26.68), $Pn'a2'_1$ (33.146), P_cc (7.28), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_ana2_1 (33.149), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_ana2_1 (33.149).

62.1 WP: $4a + 8c$

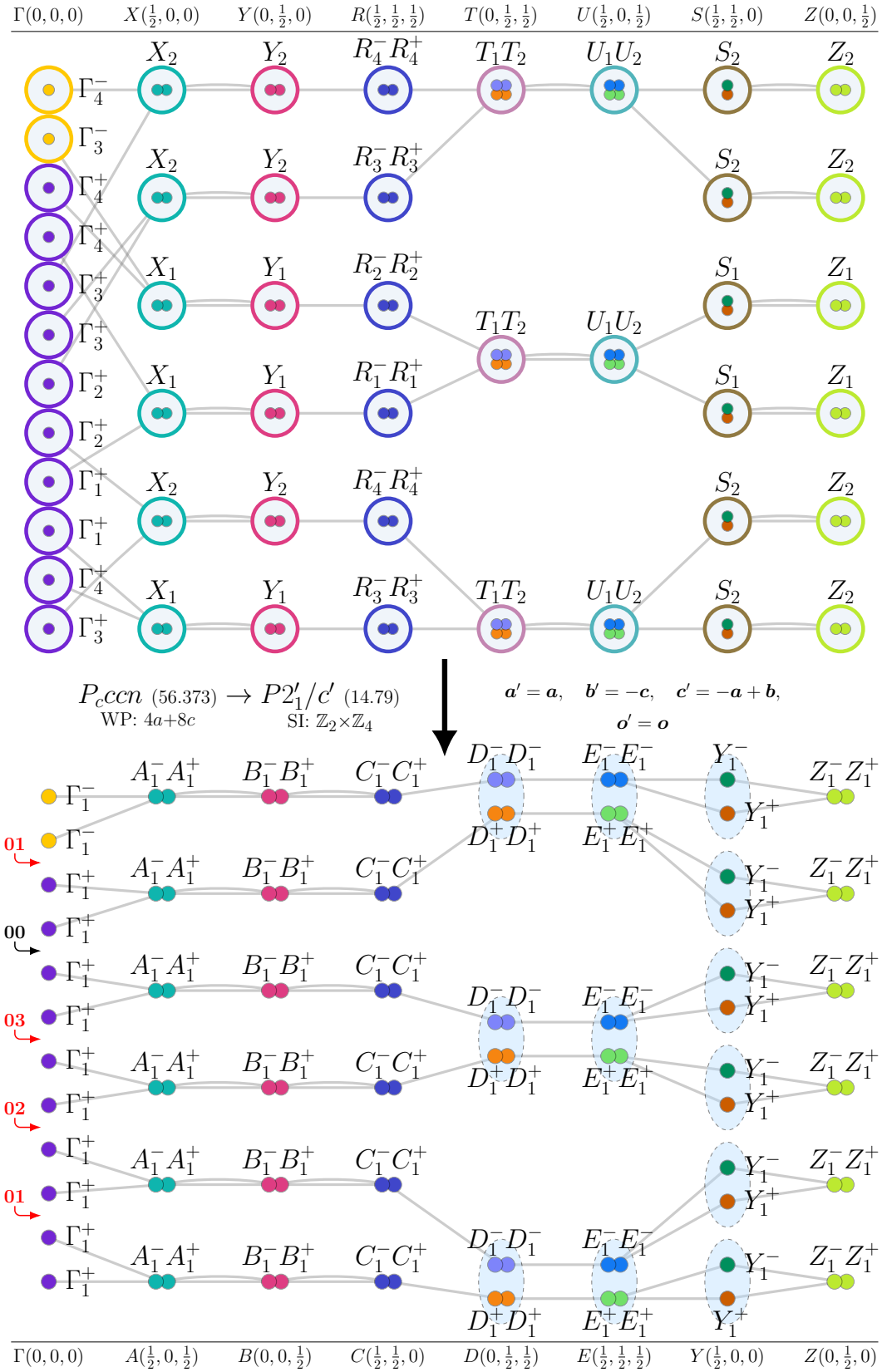
BCS Materials: $Cu_3Y(SeO_3)_2O_2Cl$ (35 K)[239], $Cu_3Bi(SeO_3)_2O_2Br$ (27.4 K)[240].

62.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[239]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.123`
[240]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.122`



Supplementary Figure 298: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a+8c$ of supergroup $P_{c}ccn$ (56.373).

Supplementary Table 585: Number of times a given SI can appear in the band structure. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1
12	0, 1, 2, 3
13	0, 1
gapless	6
nontrivial	4, 5

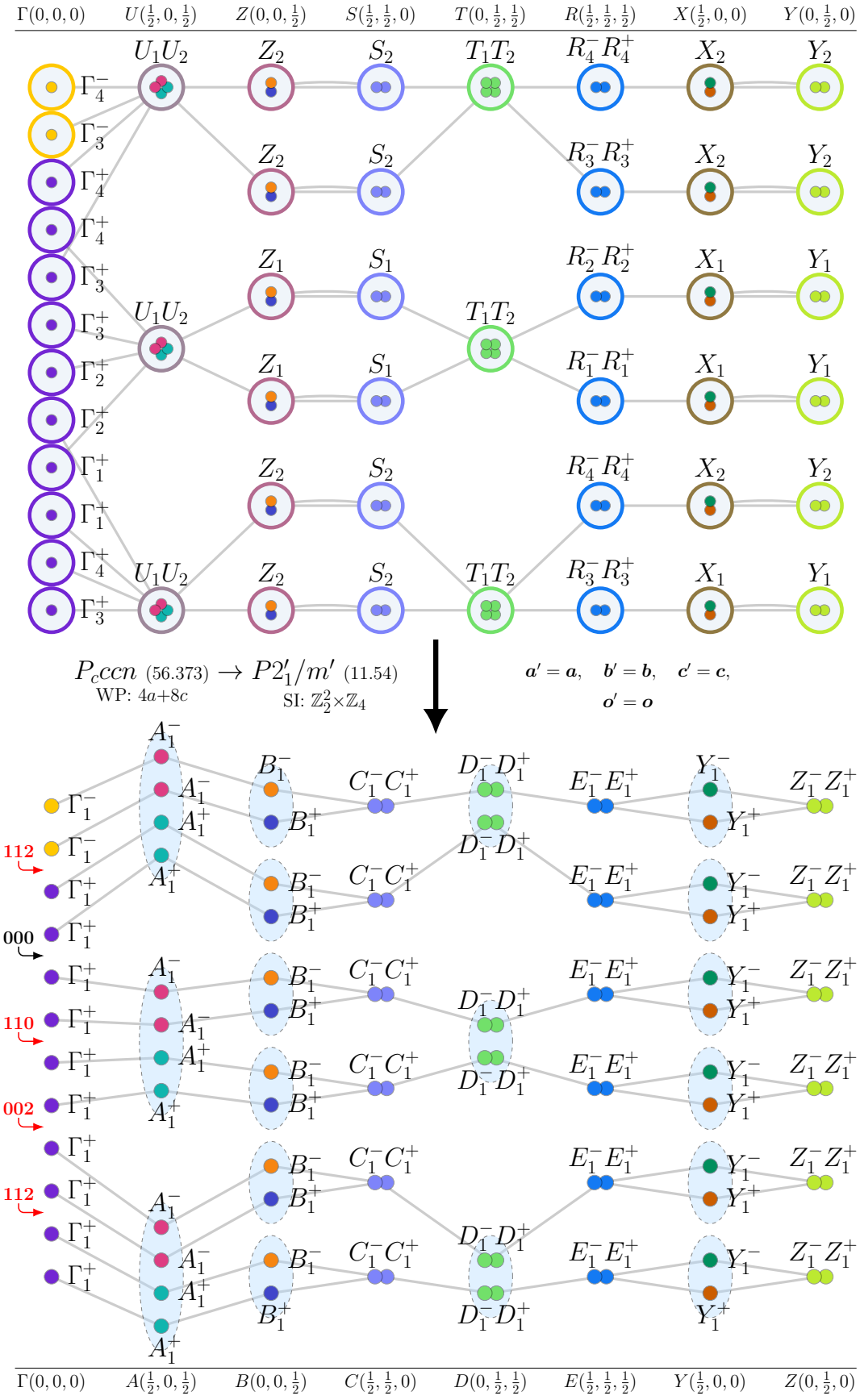
Supplementary Table 586: Possible SI values in each gap. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 13
9	G
10	01, 03, 10, 12
11	G
12	00

62.1.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 299: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4a + 8c$ of supergroup $P_c ccn$ (56.373).

Supplementary Table 587: Number of times a given SI can appear in the band structure. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3
113	0, 1, 2, 3
gapless	6
nontrivial	3, 4, 5

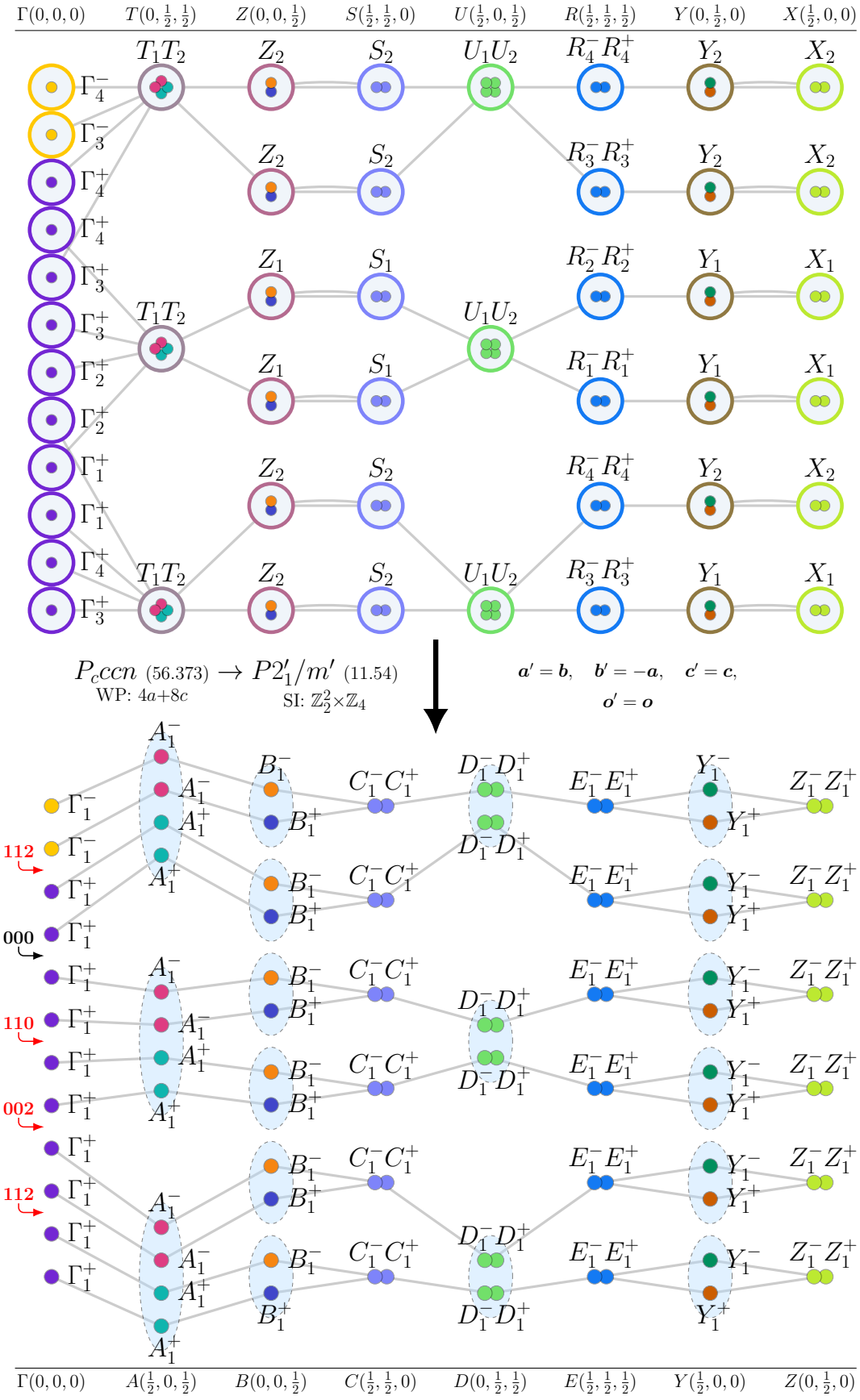
Supplementary Table 588: Possible SI values in each gap. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	000, 001, 003, 110, 111 112, 113
3	G
4	000, 002, 111
5	G
6	001, 002, 003, 110, 111 112, 113
7	G
8	000, 002, 113
9	G
10	000, 001, 003, 110, 111 112, 113
11	G
12	000

62.1.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 300: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4a + 8c$ of supergroup P_ccn (56.373).

Supplementary Table 589: Number of times a given SI can appear in the band structure. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3
113	0, 1, 2, 3
gapless	6
nontrivial	3, 4, 5

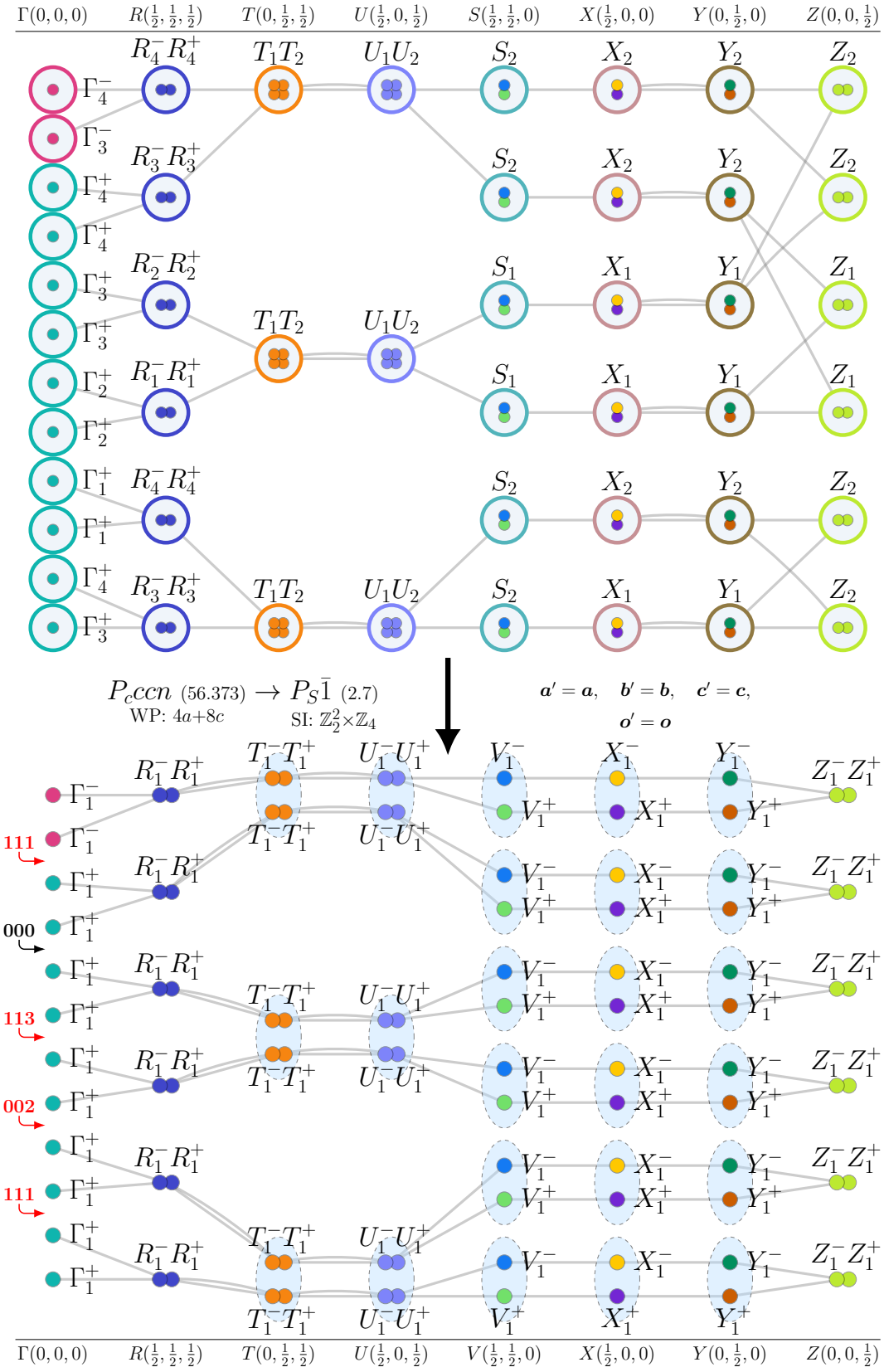
Supplementary Table 590: Possible SI values in each gap. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	000, 001, 003, 110, 111 112, 113
3	G
4	000, 002, 111
5	G
6	001, 002, 003, 110, 111 112, 113
7	G
8	000, 002, 113
9	G
10	000, 001, 003, 110, 111 112, 113
11	G
12	000

62.1.4 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 301: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4a+8c$ of supergroup $P_c ccn$ (56.373).

Supplementary Table 591: Number of times a given SI can appear in the band structure. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P_{S}\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
002	1, 2, 3
111	1, 2, 3
113	1, 2, 3
gapless	6
nontrivial	3, 4, 5

Supplementary Table 592: Possible SI values in each gap. Supergroup: $P_{c}ccn$ (56.373), WP: $4a$+$8c$, Subgroup: $P_{S}\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 111, 113
3	G
4	000, 002, 111
5	G
6	002, 111, 113
7	G
8	000, 002, 113
9	G
10	000, 111, 113
11	G
12	000

63 MSG $P_{A}ccn$ (56.374)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'/m'$ (10.46), $P2'_1/c'$ (14.79), $P_{S}\bar{1}$ (2.7), $P2$ (3.1), $Pm'a'2$ (28.91), $P_{A}cc2$ (27.85), $P2/c$ (13.65), $Pm'na'$ (53.328), $P_{C}2/c$ (13.74), $P2_1/c$ (14.75), $Pb'c'a$ (61.436), $P_{A}2_1/c$ (14.83), $P2_1/c$ (14.75), $Pb'am'$ (55.358), $P_{a}2_1/c$ (14.80).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), Pc' (7.26), $P2'_1$ (4.9), $P2'$ (3.3), $P2'_1$ (4.9), $P_{S}1$ (1.3), Pc (7.24), $Pnc'2'$ (30.114), $Pm'n2'_1$ (31.125), $P_{C}c$ (7.30), Pc (7.24), $Pca'2'_1$ (29.102), $Pc'a2'_1$ (29.101), $P_{A}c$ (7.31), Pc (7.24), $Pm'c2'_1$ (26.68), $Pb'a2'$ (32.137), $P_{a}c$ (7.27), $P_{C}2$ (3.6), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), $P_{C}2_1$ (4.12), $P_{B}na2_1$ (33.153), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), $P_{a}2_1$ (4.10), $P_{C}na2_1$ (33.154).

63.1 WP: $4a$

BCS Materials: La_2CuO_4 (316 K)[241], La_2CuO_4 (50 K)[242], Gd_2CuO_4 (18 K)[243].

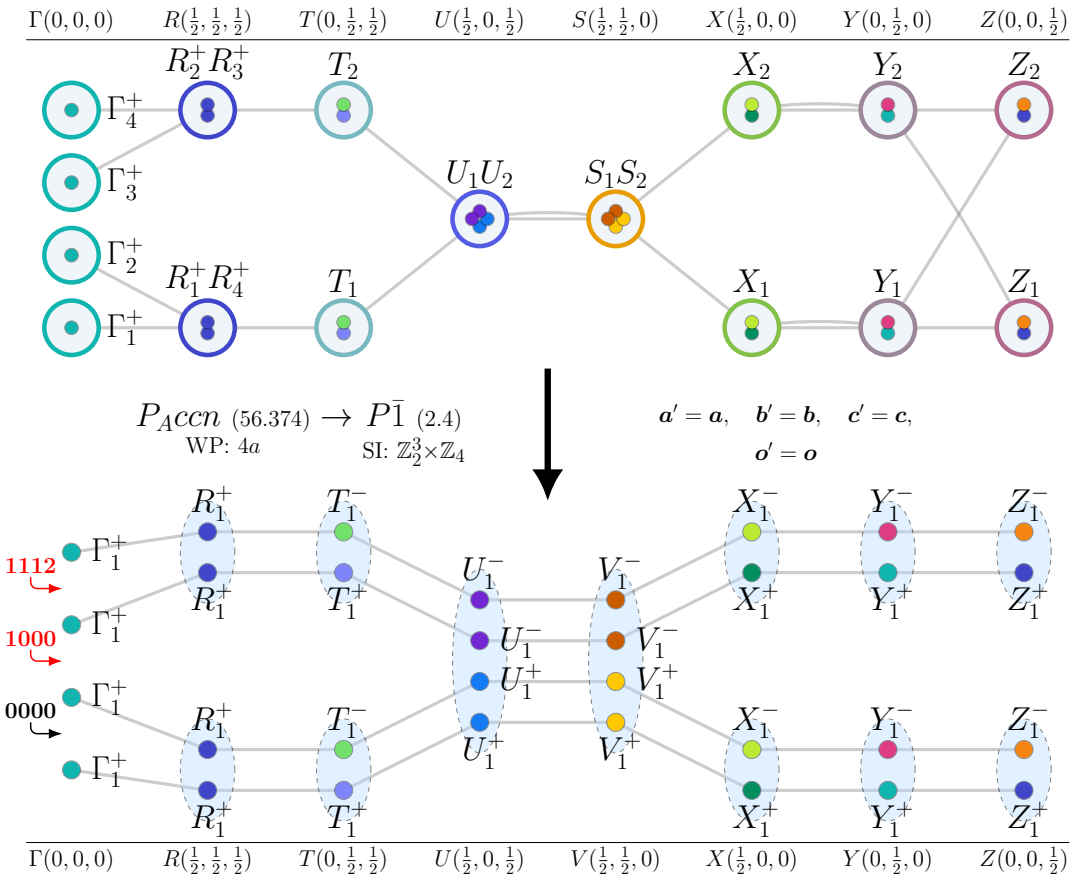
63.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,

[241]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.405`
[242]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.23`
[243]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.105`

- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],
- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 302: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_Accn (56.374).

Supplementary Table 593: Number of times a given SI can appear in the band structure. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2

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SI	Possible #Gaps
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2, 3
1001	0, 1, 2
1002	0, 1, 2, 3
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1111	0, 1
1112	0, 1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

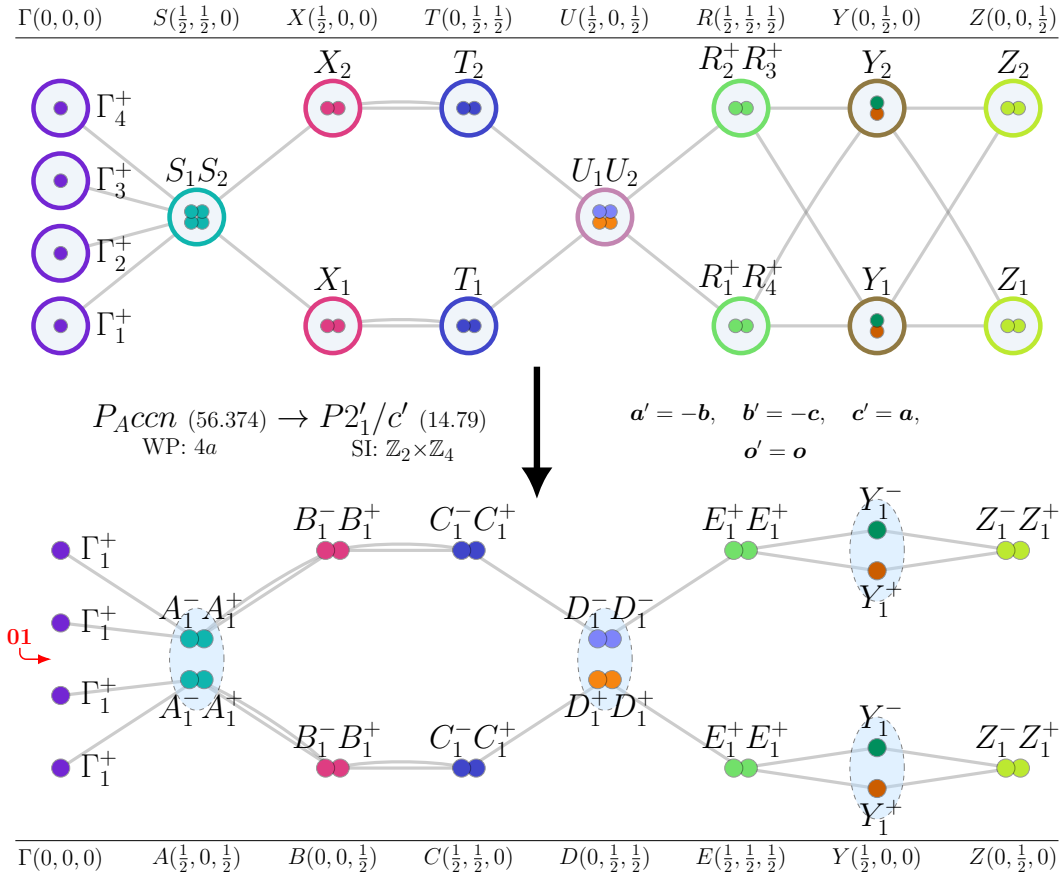
Supplementary Table 594: Possible SI values in each gap. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1000, 1002, 1011, 1013, 1101 1103, 1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

63.1.2 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 303: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Accn (56.374).

Supplementary Table 595: Number of times a given SI can appear in the band structure. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 596: Possible SI values in each gap. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

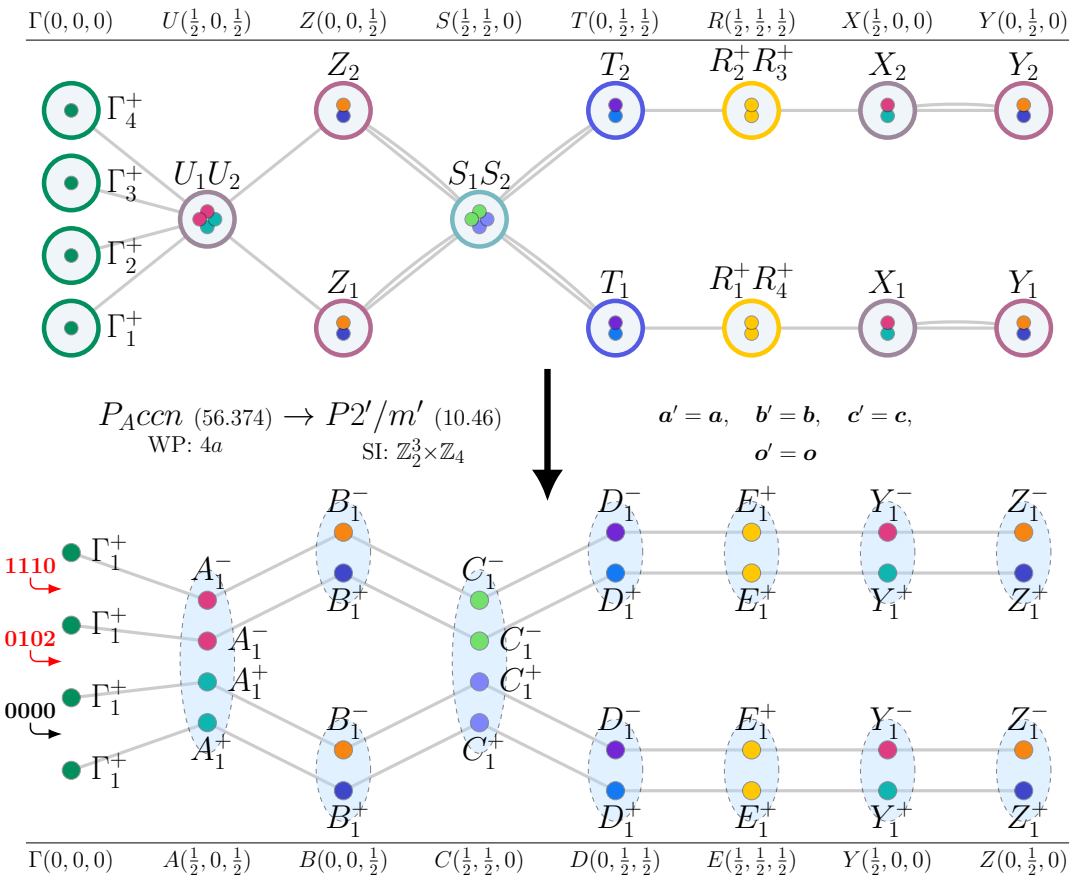
Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

63.1.3 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],

- B $\perp$ [010].



Supplementary Figure 304: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4a$ of supergroup P_Accn (56.374).

Supplementary Table 597: Number of times a given SI can appear in the band structure. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2
0102	0, 1, 2, 3
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2, 3
0112	0, 1, 2
0113	0, 1, 2, 3
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2

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SI	Possible #Gaps
1100	0, 1, 2
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1
1113	0, 1
gapless	0
nontrivial	1, 2, 3

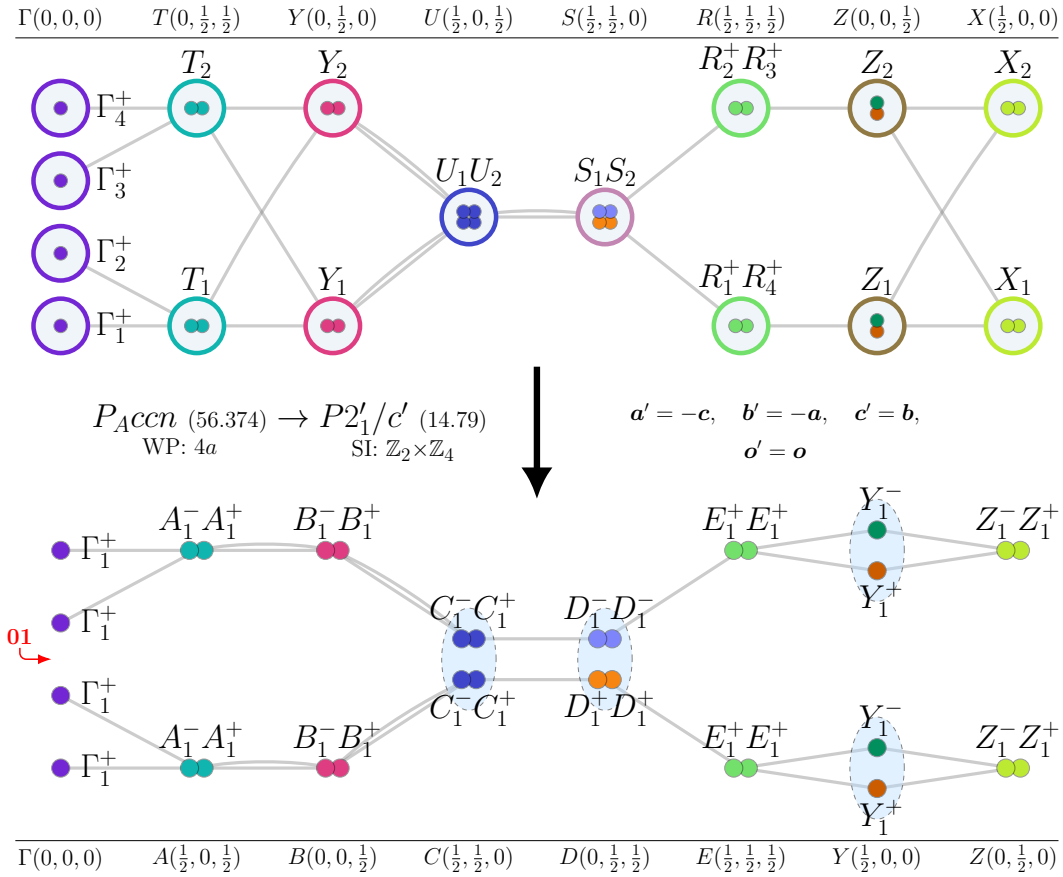
Supplementary Table 598: Possible SI values in each gap. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
2	0100, 0102, 0111, 0113, 1101 1103, 1110
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
4	0000

63.1.4 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 305: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Accn (56.374).

Supplementary Table 599: Number of times a given SI can appear in the band structure. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

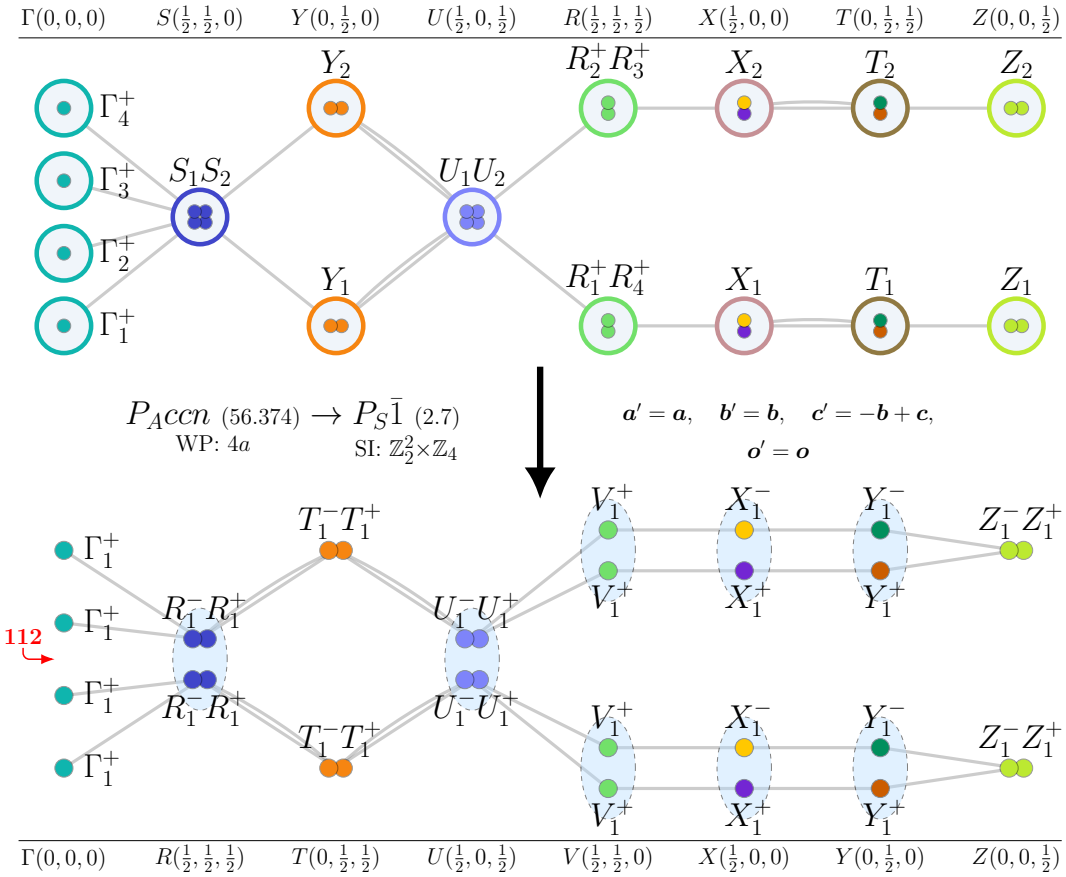
Supplementary Table 600: Possible SI values in each gap. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

63.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 306: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_Accn (56.374).

Supplementary Table 601: Number of times a given SI can appear in the band structure. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 602: Possible SI values in each gap. Supergroup: P_Accn (56.374), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

64 MSG P_abcm (57.386)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P2_1'/c'$ (14.79), $P2_1'/c'$ (14.79), $P_S\bar{1}$ (2.7), P_bca2_1 (29.105), $P2_1/m$ (11.50), $Pn'ma'$ (62.448), P_a2_1/m (11.55), $P2_1/c$ (14.75), $Pb'c'a$ (61.436), P_a2_1/c (14.80), $P2$ (3.1), P_b2 (3.5), $P2/c$ (13.65), $Pb'cn'$ (60.424), P_b2/c (13.71).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2_1'$ (4.9), $P2_1'$ (4.9), $P2_1'$ (4.9), P_S1 (1.3), Pm (6.18), $Pmc'2_1'$ (26.69), $Pmn'2_1'$ (31.126), P_am (6.21), Pc (7.24), $Pca'2_1'$ (29.102), $Pc'a2_1'$ (29.101), P_ac (7.27), Pc (7.24), $Pn'a2_1'$ (33.146), $Pca'2_1'$ (29.102), P_bc (7.29), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_a2_1 (4.10), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_a2_1 (4.10), P_bmc2_1 (26.72), $Pn'c'2$ (30.115), P_cma2 (28.94).

64.1 WP: $8d$

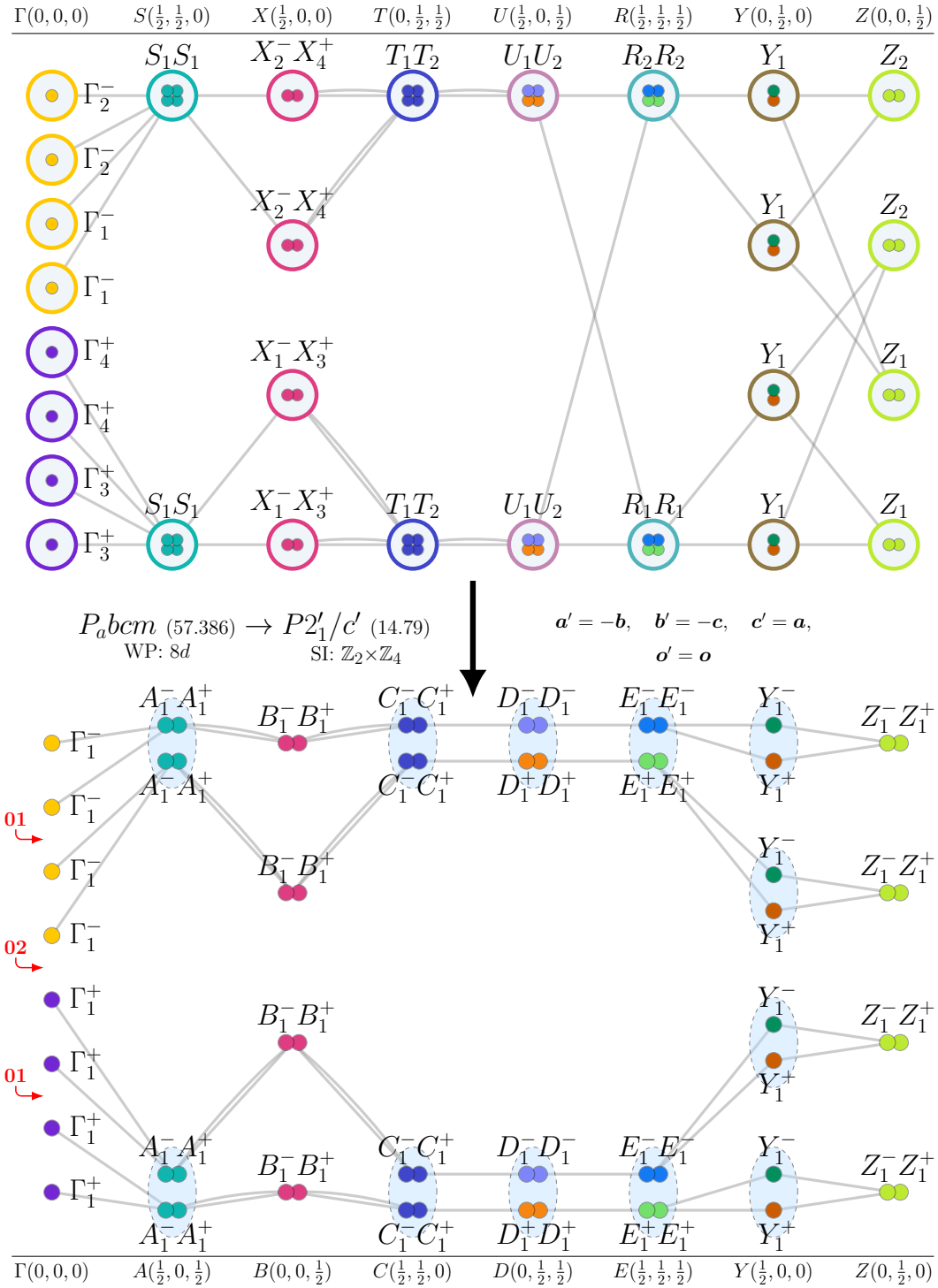
BCS Materials: BaCoSO (220 K)[244], BaCoSO[245].

64.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[244]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.593`
[245]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.594`



Supplementary Figure 307: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8d$ of supergroup P_abcm (57.386).

Supplementary Table 603: Number of times a given SI can appear in the band structure. Supergroup: P_abcm (57.386), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

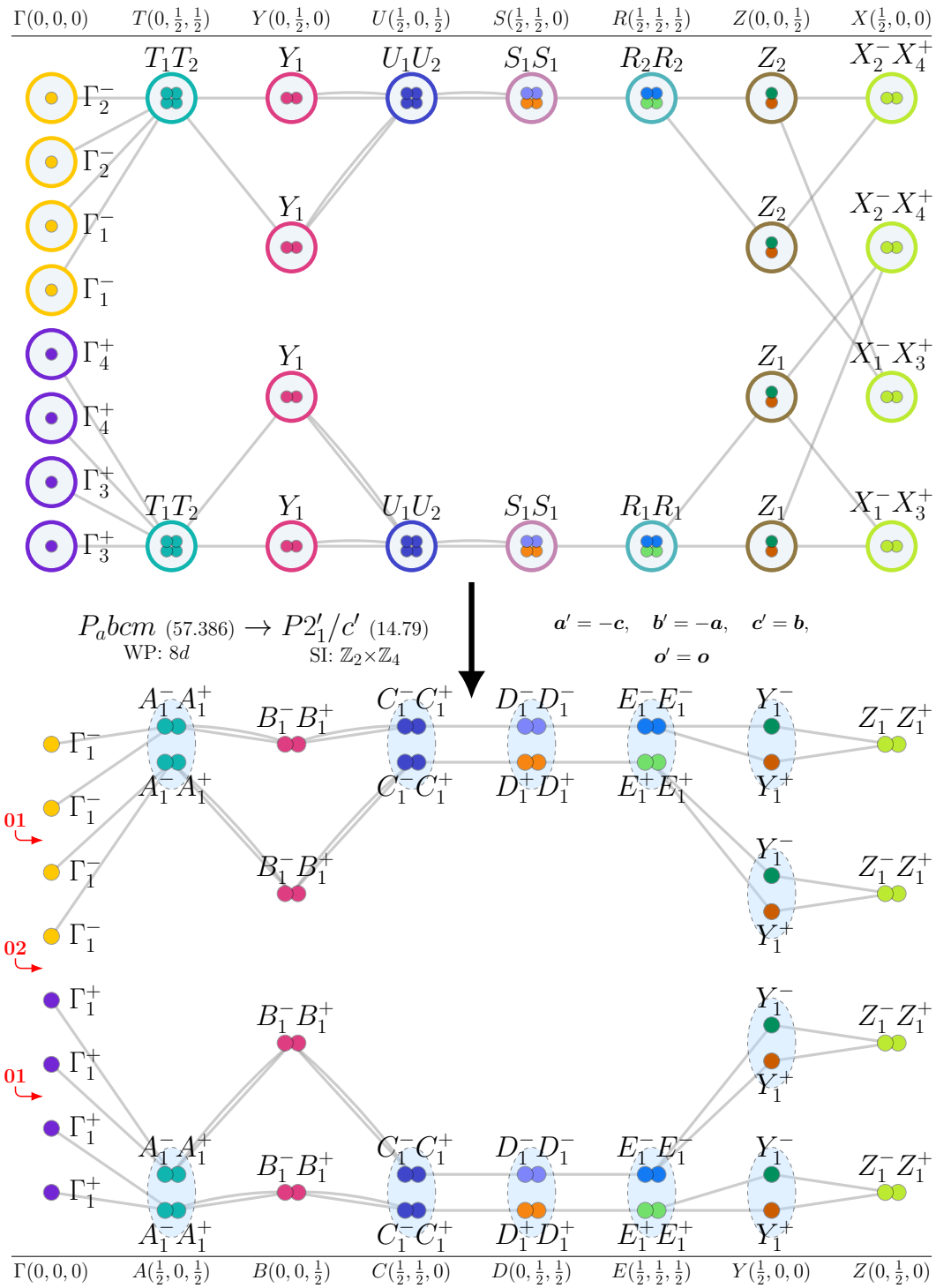
Supplementary Table 604: Possible SI values in each gap. Supergroup: P_abcm (57.386), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

64.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 308: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8d$ of supergroup P_abcm (57.386).

Supplementary Table 605: Number of times a given SI can appear in the band structure. Supergroup: P_abcm (57.386), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

continued on next page...

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 606: Possible SI values in each gap. Supergroup: P_abcm (57.386), WP: $8d$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

65 MSG P_Cbcm (57.391)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'/c'$ (13.69), $P2'_1/m'$ (11.54), $P_S\bar{1}$ (2.7), $P2_1/m$ (11.50), $Pmm'n'$ (59.410), P_a2_1/m (11.55), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_C2_1/c (14.84), $P2$ (3.1), $P2/c$ (13.65), $Pn'n'a$ (52.310), P_A2/c (13.73).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pm' (6.20), $P2'_1$ (4.9), $P2'$ (3.3), $P2'_1$ (4.9), P_S1 (1.3), Pm (6.18), $Pm'm2'$ (25.59), $Pmn'2'_1$ (31.126), P_am (6.21), Pc (7.24), $Pm'c2'_1$ (26.68), $Pn'a2'_1$ (33.146), P_Cc (7.30), Pc (7.24), $Pn'a2'_1$ (33.146), $Pn'c2'$ (30.113), P_Ac (7.31), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_Cca2_1 (29.109), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_C2_1 (4.12), P_Amc2_1 (26.74), $Pn'n'2$ (34.159), P_C2 (3.6), P_Ama2 (28.95).

65.1 WP: $4c + 8g$

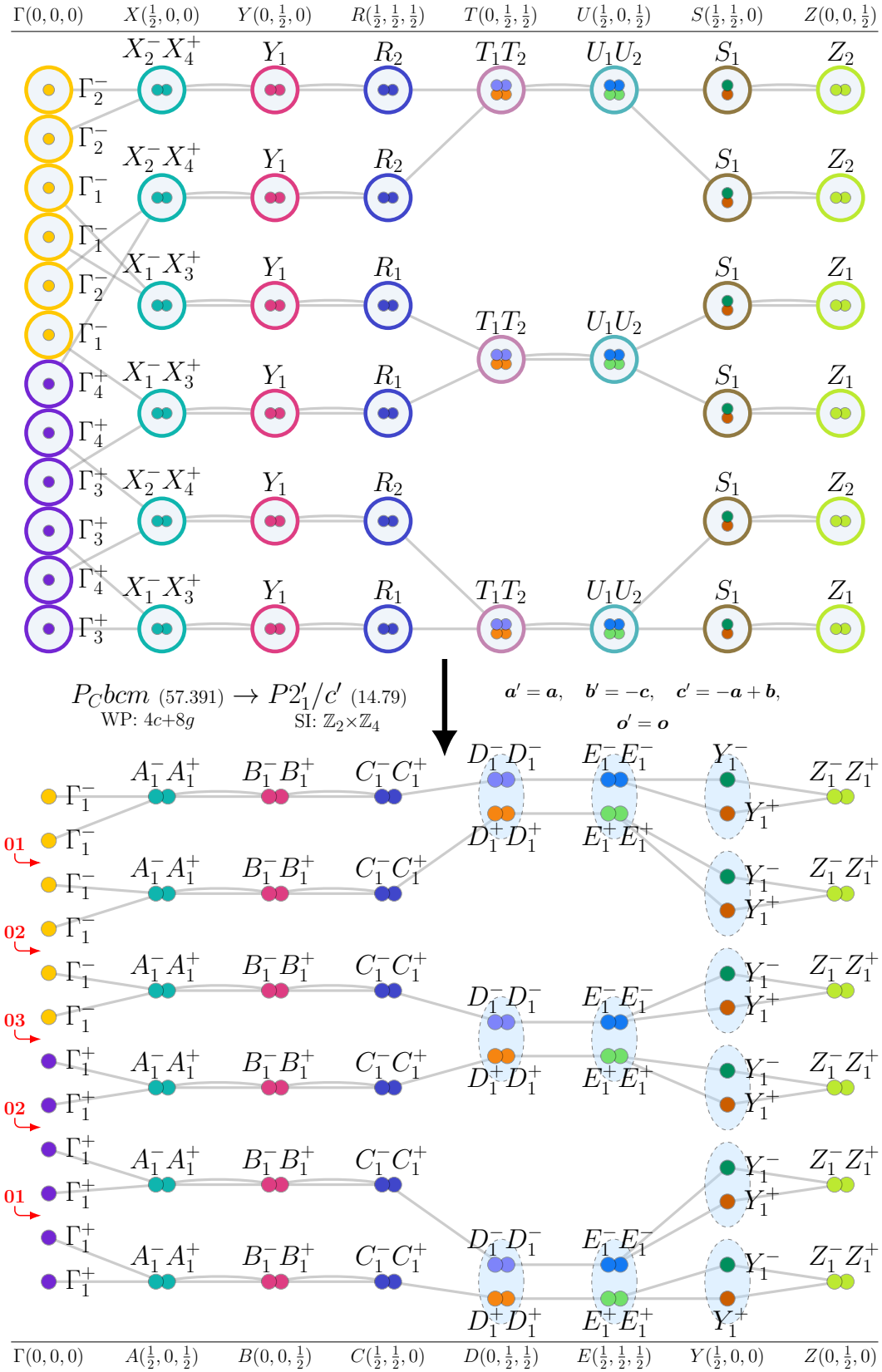
BCS Materials: $LaSr_3Fe_3O_9$ (350 K)[246].

65.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[246]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.481`



Supplementary Figure 309: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+8g$ of supergroup P_Cbcm (57.391).

Supplementary Table 607: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcm (57.391), WP: $4c$+$8g$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 608: Possible SI values in each gap. Supergroup: P_Cbcm (57.391), WP: $4c$+$8g$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

65.2 WP: $4c$

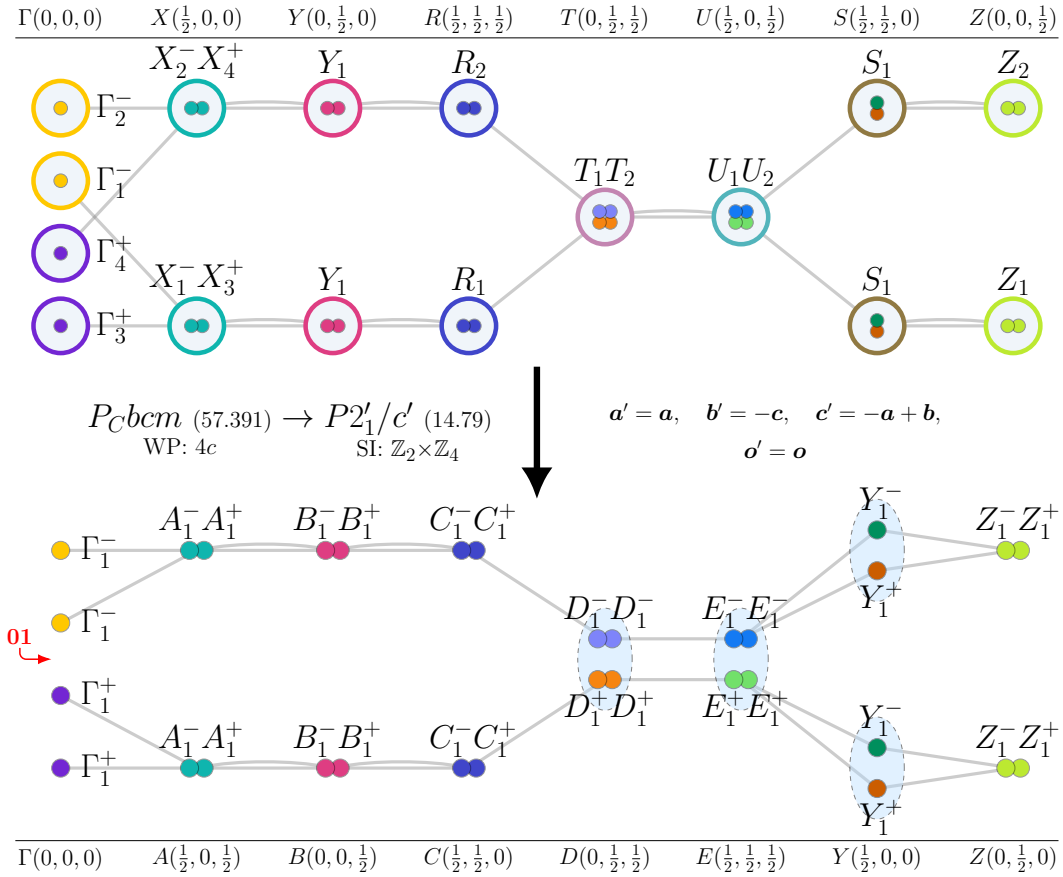
BCS Materials: $CeRu_2Al_{10}$ (27 K)[247].

65.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[247] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.8`



Supplementary Figure 310: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup P_Cbcm (57.391).

Supplementary Table 609: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcm (57.391), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

Supplementary Table 610: Possible SI values in each gap. Supergroup: P_Cbcm (57.391), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

65.3 WP: $4c + 8f$

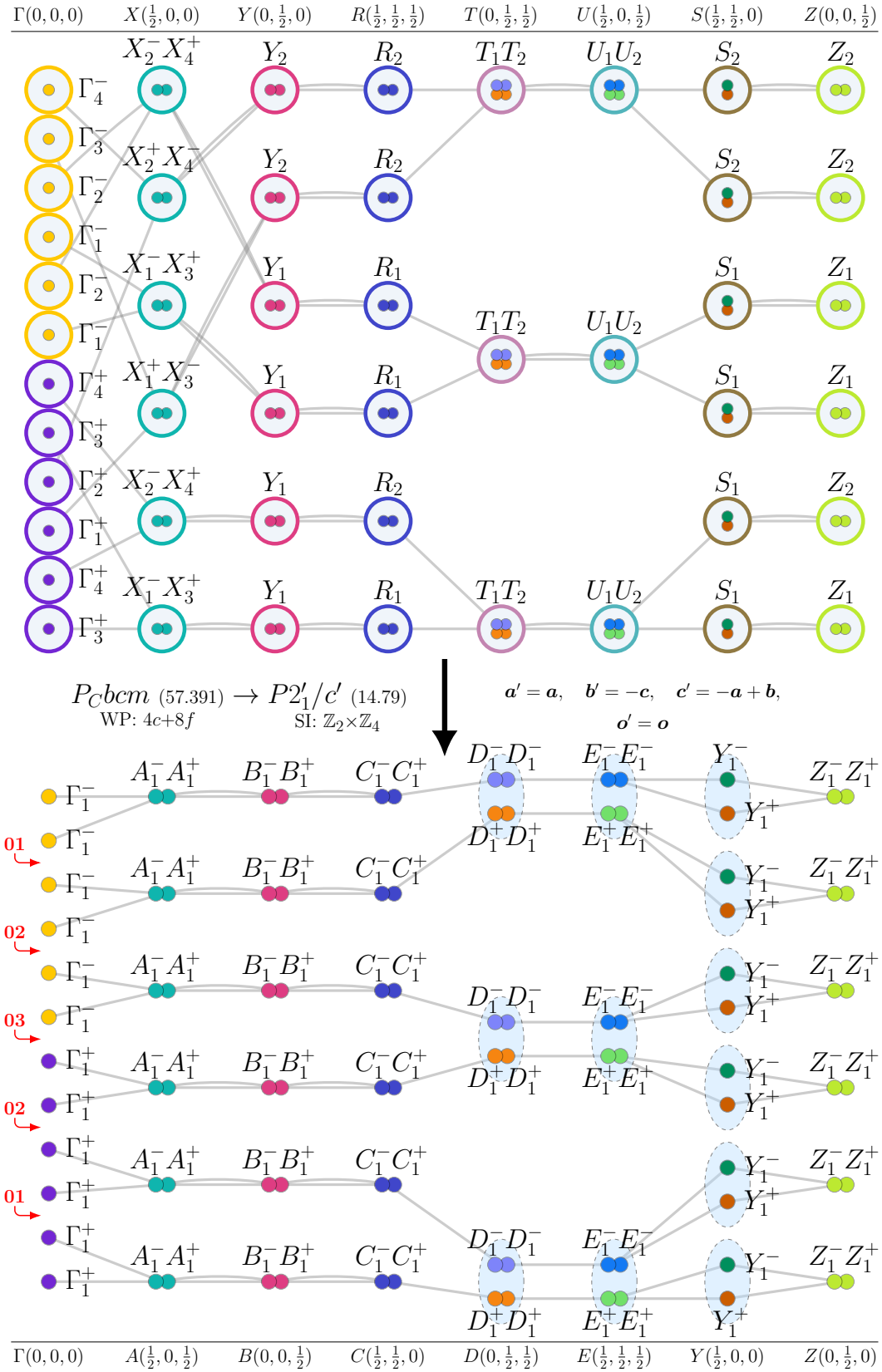
BCS Materials: Er_3Ge_4 (7.3 K)[248].

65.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[248] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.362`



Supplementary Figure 311: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+8f$ of supergroup P_Cbcm (57.391).

Supplementary Table 611: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcm (57.391), WP: $4c$+$8f$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 612: Possible SI values in each gap. Supergroup: P_Cbcm (57.391), WP: $4c$+$8f$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

66 MSG P_Bnnm (58.402)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P2_1'/m'$ (11.54), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Pm'a'2$ (28.91), $P2/m$ (10.42), $Pm'ma'$ (51.296), P_C2/m (10.49), $P2_1/c$ (14.75), $Pb'c'n$ (60.422), P_c2_1/c (14.82), $P2_1/c$ (14.75), $Pnm'a'$ (62.447), P_C2_1/c (14.84).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), Pc' (7.26), $P2_1'$ (4.9), $P2_1'$ (4.9), $P2'$ (3.3), P_S1 (1.3), Pm (6.18), $Pmc'2_1'$ (26.69), $Pm'm2'$ (25.59), P_Cm (6.23), Pc (7.24), $Pna'2_1'$ (33.147), $Pnc'2'$ (30.114), P_cc (7.28), Pc (7.24), $Pm'n2_1'$ (31.125), $Pna'2_1'$ (33.147), P_Cc (7.30), P_C2 (3.6), P_Ann2 (34.162), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_a2_1 (4.10), P_Cmn2_1 (31.133), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_C2_1 (4.12), P_Bmn2_1 (31.132).

66.1 WP: $4a$

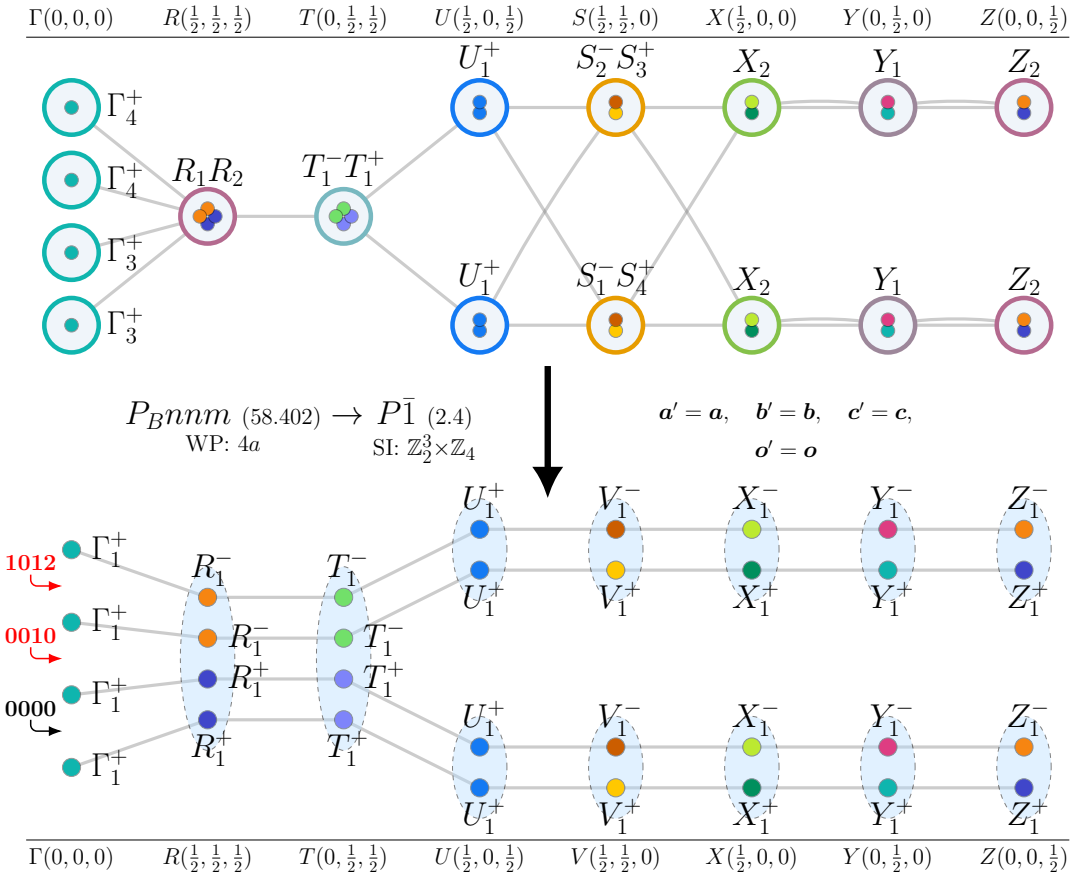
BCS Materials: $CsNiF_3$ (2.67 K)[249].

[249]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.527`

66.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],
- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 312: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_Bnnm (58.402).

Supplementary Table 613: Number of times a given SI can appear in the band structure. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
0003	0, 1, 2
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

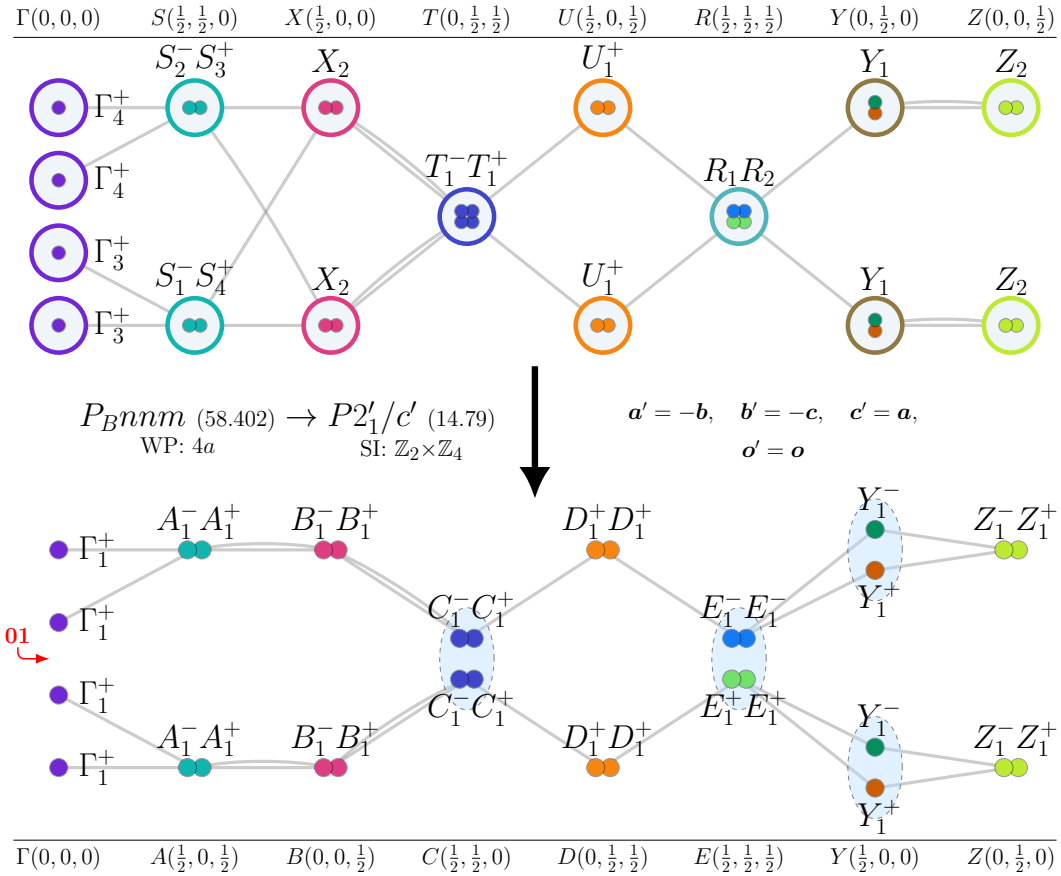
Supplementary Table 614: Possible SI values in each gap. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	0010, 0011, 0012, 0013, 1011 1012, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

66.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 313: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Bnnm (58.402).

Supplementary Table 615: Number of times a given SI can appear in the band structure. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

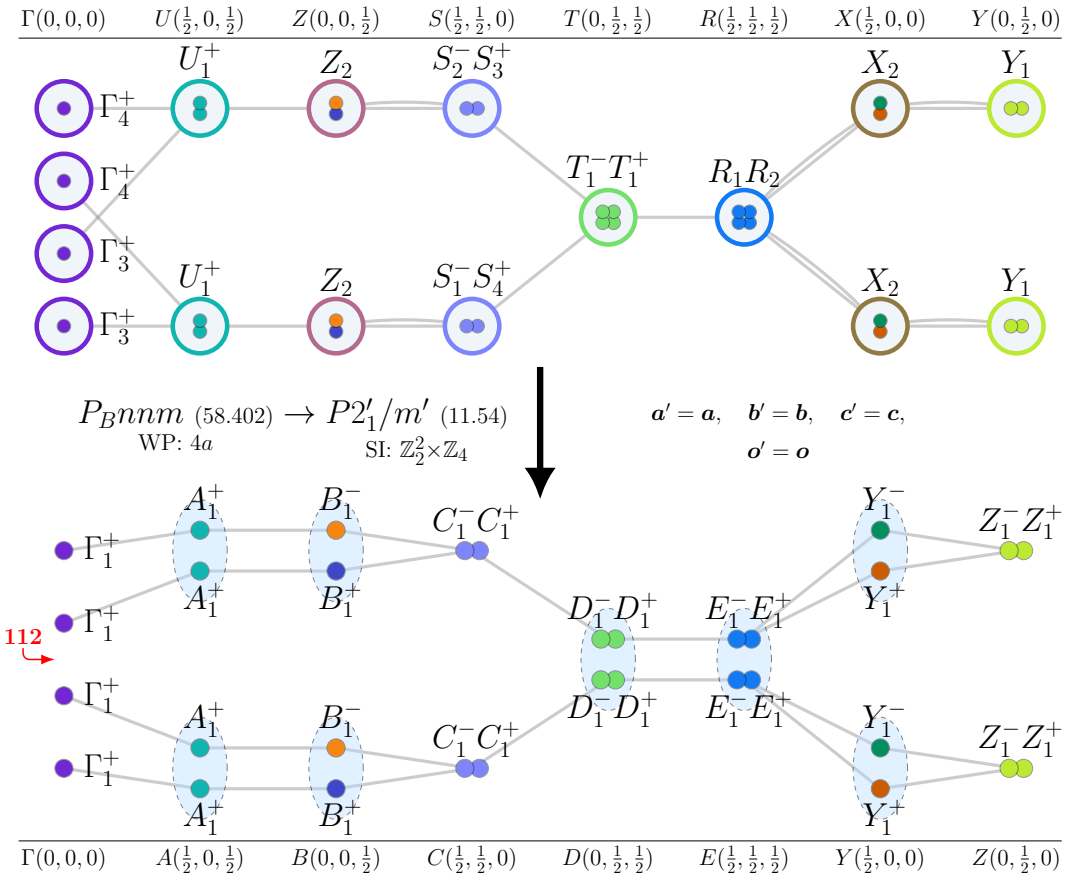
Supplementary Table 616: Possible SI values in each gap. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

66.1.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 314: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4a$ of supergroup P_Bnnm (58.402).

Supplementary Table 617: Number of times a given SI can appear in the band structure. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 618: Possible SI values in each gap. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	112
3	G

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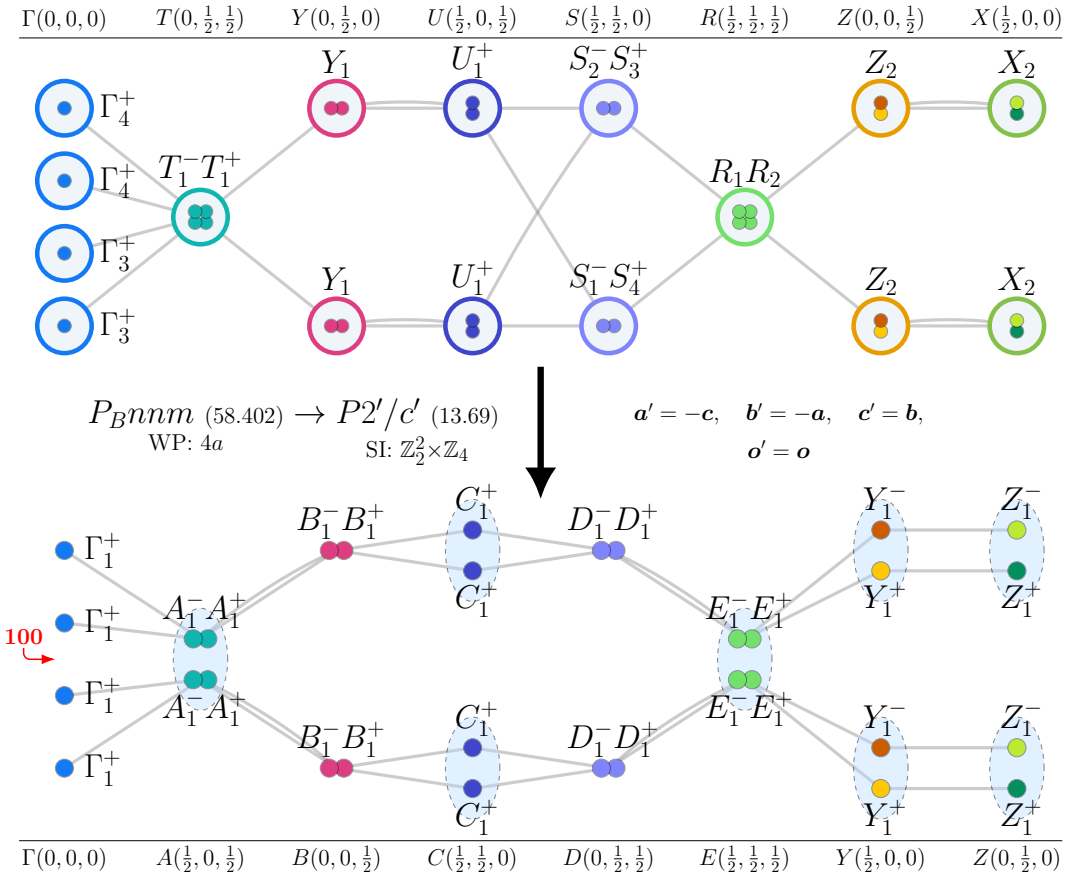
...continued

Gap#	Possible SI Values
4	000

66.1.4 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 315: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4a$ of supergroup P_Bnnm (58.402).

Supplementary Table 619: Number of times a given SI can appear in the band structure. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

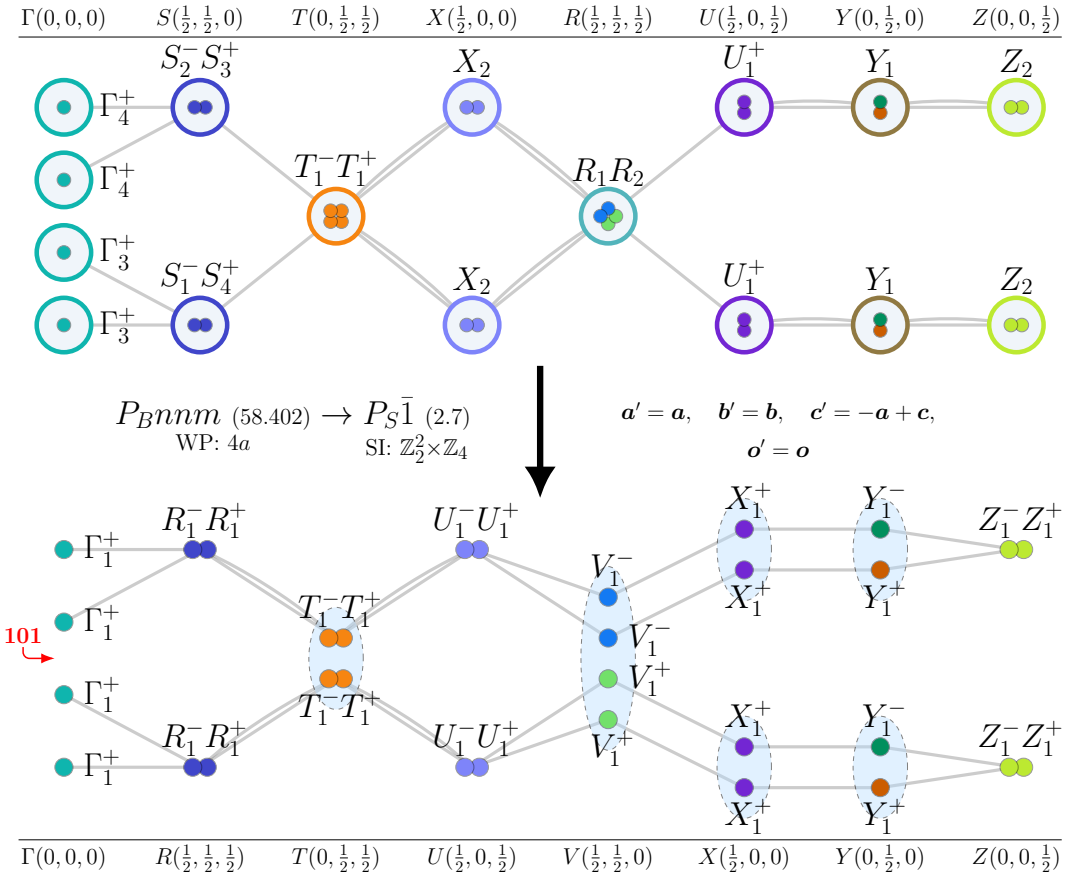
Supplementary Table 620: Possible SI values in each gap. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

66.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 316: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_Bnnm (58.402).

Supplementary Table 621: Number of times a given SI can appear in the band structure. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
100	0, 1
101	0, 1
103	0, 1
gapless	2
nontrivial	1

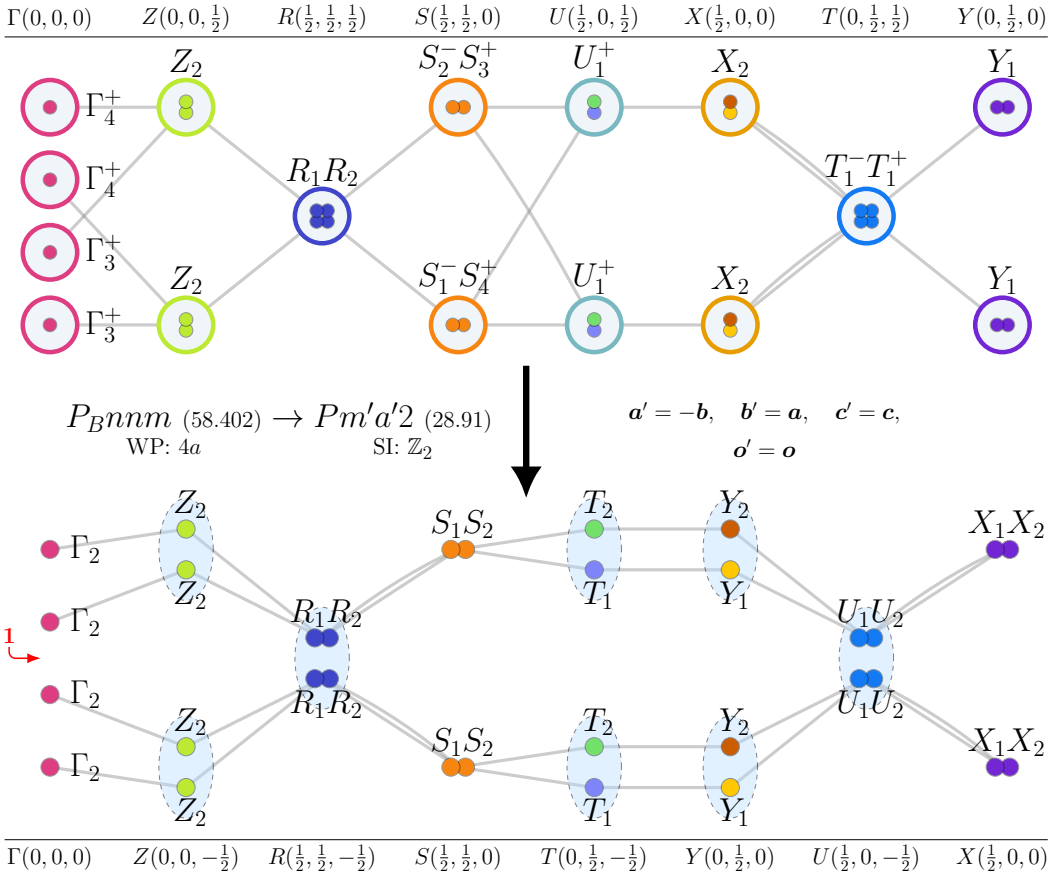
Supplementary Table 622: Possible SI values in each gap. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	100, 101, 103
3	G
4	000

66.1.6 Topological bands in subgroup $Pm'a'2$ (28.91)

Perturbation:

- E ∥ [001] and B ∥ [001].



Supplementary Figure 317: Topological magnon bands in subgroup $Pm'a'2$ (28.91) for magnetic moments on Wyckoff position $4a$ of supergroup P_Bnnm (58.402).

Supplementary Table 623: Number of times a given SI can appear in the band structure. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $Pm'a'2$ (28.91).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 624: Possible SI values in each gap. Supergroup: P_Bnnm (58.402), WP: $4a$, Subgroup: $Pm'a'2$ (28.91).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

67 MSG $Pm'm'n$ (59.409)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/m'$ (11.54), $P2'_1/m'$ (11.54), $P2$ (3.1), $Pm'm'2$ (25.60), $P2/c$ (13.65).

Trivial-SI Subgroups: Pm' (6.20), Pm' (6.20), $P2'_1$ (4.9), $P2'_1$ (4.9), Pc (7.24), $Pm'n2'_1$ (31.125), $Pm'n2'_1$ (31.125).

67.1 WP: $2b + 2b + 4c + 4d$

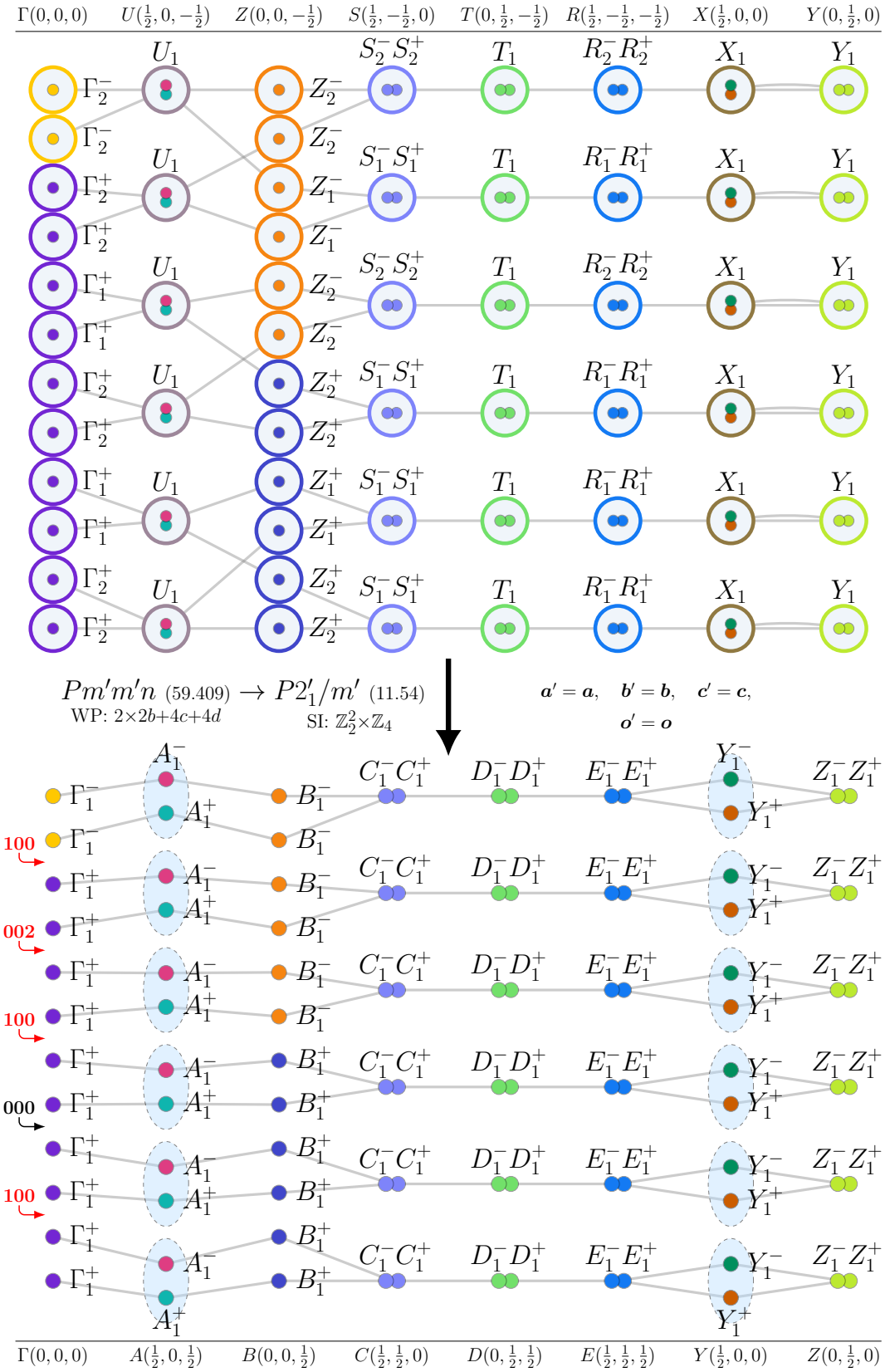
BCS Materials: $TmMn_3O_6$ (74 K)[250].

67.1.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).

[250]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.232`



Supplementary Figure 318: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $2b+2b+4c+4d$ of supergroup $Pm'm'n$ (59.409).

Supplementary Table 625: Number of times a given SI can appear in the band structure. Supergroup: $Pm'm'n$ (59.409), WP: 2b+2b+4c+4d, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2, 3
002	0, 1, 2, 3
011	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2, 3, 4
102	0, 1, 2, 3, 4
111	0, 1, 2, 3
113	0, 1, 2, 3
gapless	6
nontrivial	2, 3, 4, 5

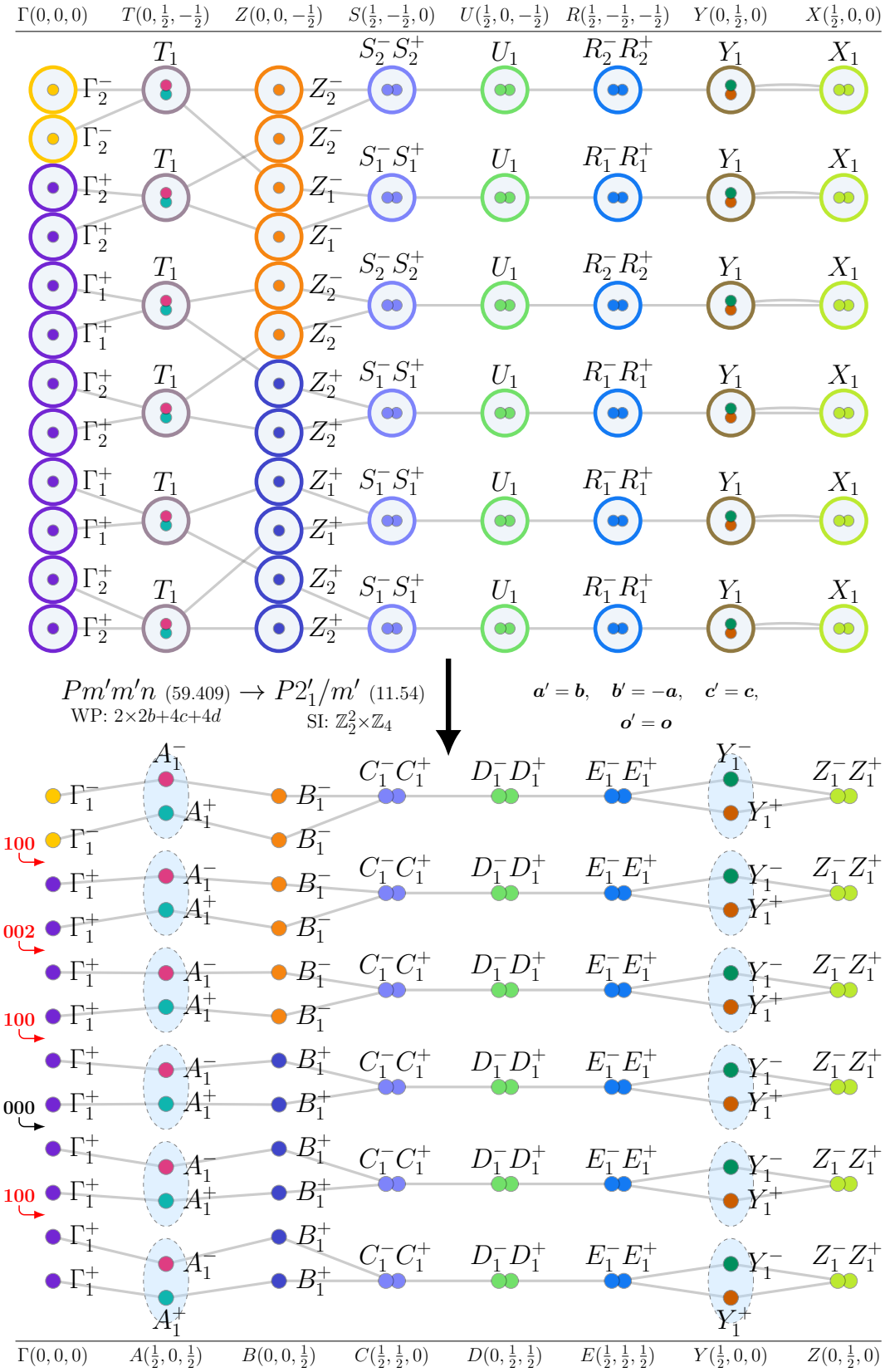
Supplementary Table 626: Possible SI values in each gap. Supergroup: $Pm'm'n$ (59.409), WP: 2b+2b+4c+4d, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	000, 011, 013, 100, 102 111, 113
3	G
4	000, 002, 011, 013, 100 102, 111, 113
5	G
6	000, 002, 011, 013, 100 102, 111, 113
7	G
8	000, 002, 011, 013, 100 102, 111, 113
9	G
10	000, 011, 013, 100, 102 111, 113
11	G
12	000

67.1.2 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 319: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff positions $2b + 2b + 4c + 4d$ of supergroup $Pm'm'n$ (59.409).

Supplementary Table 627: Number of times a given SI can appear in the band structure. Supergroup: $Pm'm'n$ (59.409), WP: 2b+2b+4c+4d, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2, 3
002	0, 1, 2, 3
011	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2, 3, 4
102	0, 1, 2, 3, 4
111	0, 1, 2, 3
113	0, 1, 2, 3
gapless	6
nontrivial	2, 3, 4, 5

Supplementary Table 628: Possible SI values in each gap. Supergroup: $Pm'm'n$ (59.409), WP: 2b+2b+4c+4d, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	000, 011, 013, 100, 102 111, 113
3	G
4	000, 002, 011, 013, 100 102, 111, 113
5	G
6	000, 002, 011, 013, 100 102, 111, 113
7	G
8	000, 002, 011, 013, 100 102, 111, 113
9	G
10	000, 011, 013, 100, 102 111, 113
11	G
12	000

68 MSG $Pmm'n'$ (59.410)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2_1'/m'$ (11.54), $P2_1/m$ (11.50).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), $P2'$ (3.3), $P2_1'$ (4.9), Pm (6.18), $Pm'm2'$ (25.59), $Pmn'2_1'$ (31.126), $P2_1$ (4.7), $Pm'n'2_1$ (31.127).

68.1 WP: $4c$

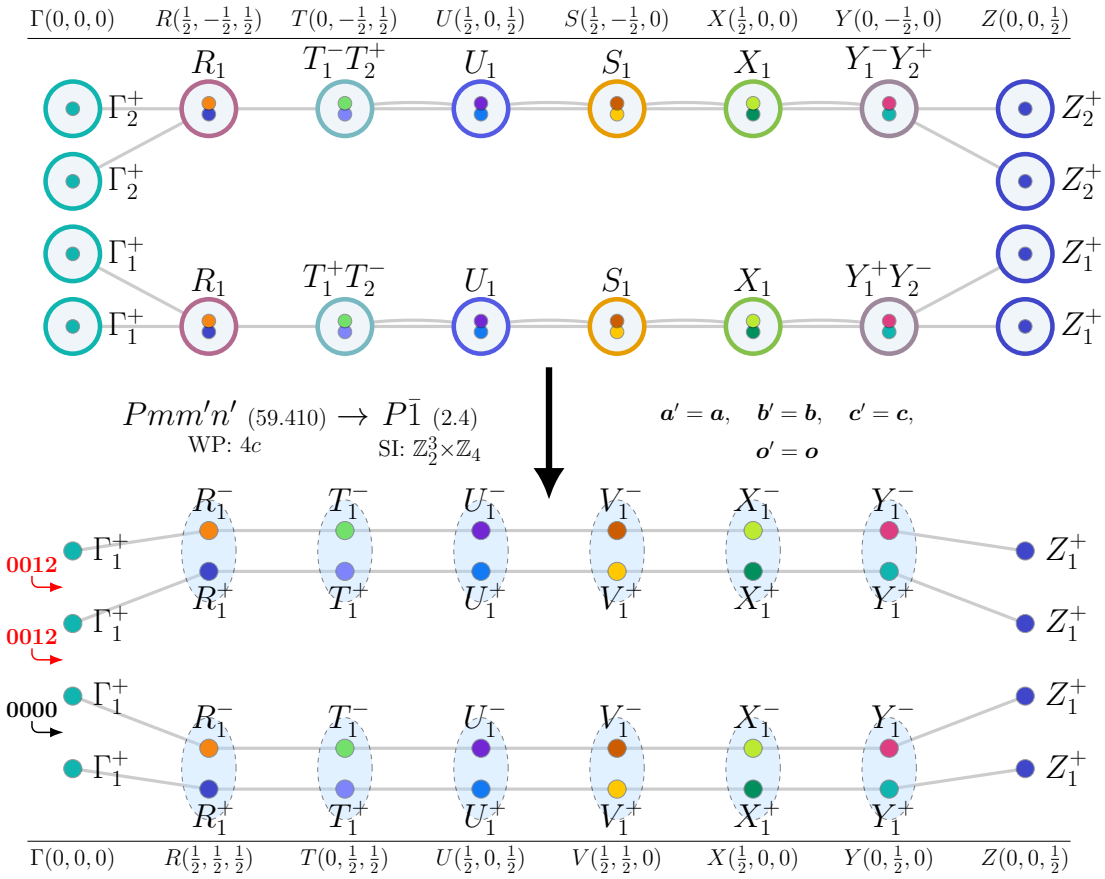
BCS Materials: $CsMnF_4$ (18.9 K)[251].

[251] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.327`

68.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B ∥ [010] or B ⊥ [001]) and strain ⊥ [100],
- (B ∥ [010] or B ⊥ [001]) and strain ⊥ [010],
- (B ∥ [001] or B ⊥ [010]) and strain ⊥ [100],
- (B ∥ [001] or B ⊥ [010]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 320: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $Pmm'n'$ (59.410).

Supplementary Table 629: Number of times a given SI can appear in the band structure. Supergroup: $Pmm'n'$ (59.410), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0011	0, 1, 2
0012	1, 2, 3
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2

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SI	Possible #Gaps
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1
1102	0, 1, 2
1103	0, 1
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

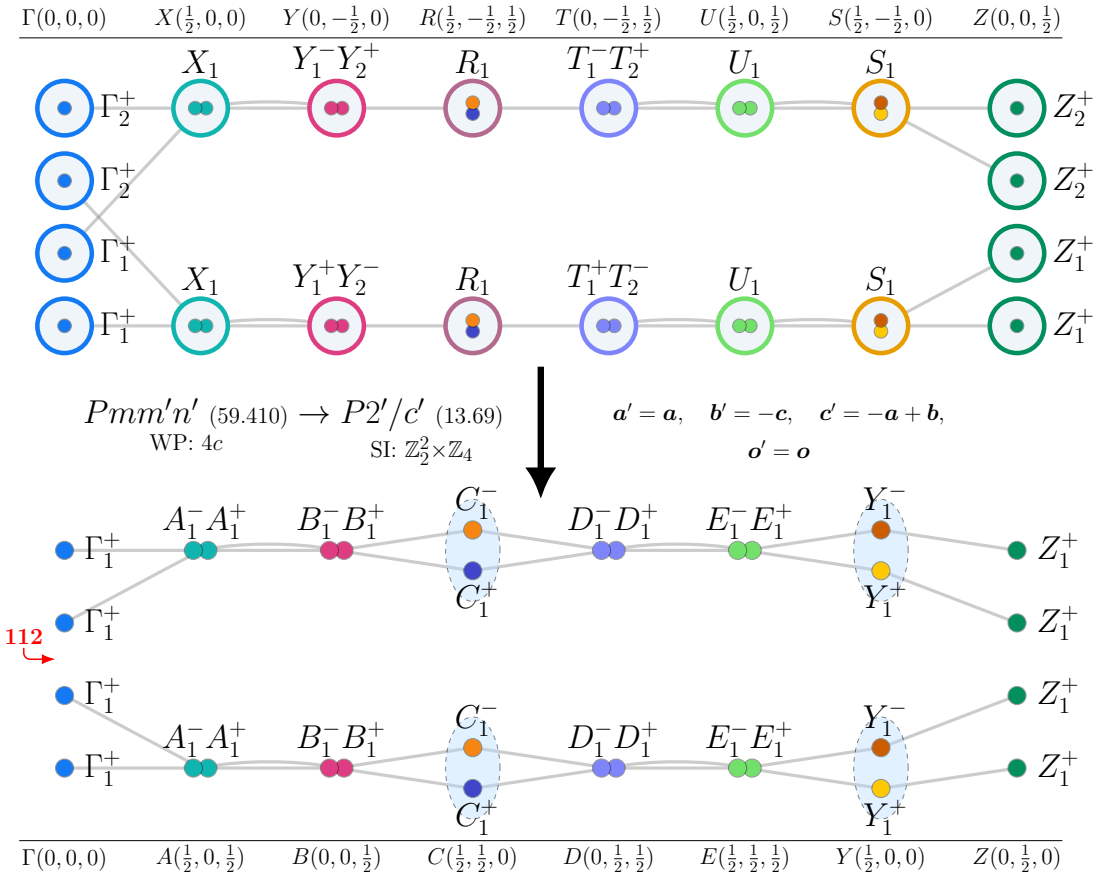
Supplementary Table 630: Possible SI values in each gap. Supergroup: $Pmm'n'$ (59.410), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1102, 1103 1110, 1112, 1113
2	0012
3	0000, 0001, 0003, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1110, 1111, 1112
4	0000

68.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).



Supplementary Figure 321: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4c$ of supergroup $Pmm'n'$ (59.410).

Supplementary Table 631: Number of times a given SI can appear in the band structure. Supergroup: $Pmm'n'$ (59.410), WP: $4c$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

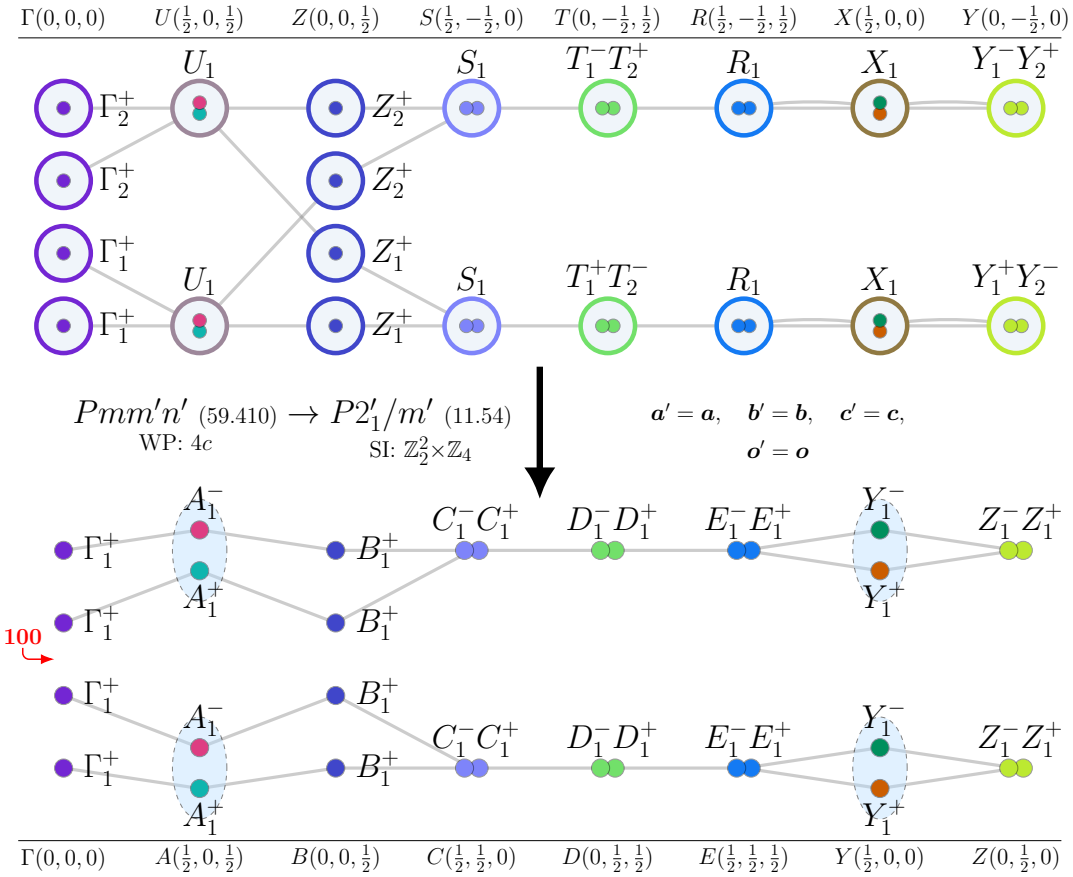
Supplementary Table 632: Possible SI values in each gap. Supergroup: $Pmm'n'$ (59.410), WP: $4c$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

68.1.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain ⊥ [010],
- (B ∥ [001] or B ⊥ [010]).



Supplementary Figure 322: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4c$ of supergroup $Pmm'n'$ (59.410).

Supplementary Table 633: Number of times a given SI can appear in the band structure. Supergroup: $Pmm'n'$ (59.410), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
100	1
gapless	2
nontrivial	1

Supplementary Table 634: Possible SI values in each gap. Supergroup: $Pmm'n'$ (59.410), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	100
3	G
4	000

69 MSG P_Cbcn (60.431)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/m'$ (11.54), $P2_1'/c'$ (14.79), $P2'/m'$ (10.46), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pnn'm'$ (58.398), P_c2_1/c (14.82), $P2$ (3.1), $Pm'm'2$ (25.60), $P2/c$ (13.65), $Pm'm'a$ (51.294), P_C2/c (13.74), $P2_1/c$ (14.75), $Pn'm'a$ (62.446), P_A2_1/c (14.83).

Trivial-SI Subgroups: Pm' (6.20), Pc' (7.26), Pm' (6.20), $P2'_1$ (4.9), $P2'_1$ (4.9), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pm'n2'_1$ (31.125), $Pn'n2'$ (34.158), P_cc (7.28), Pc (7.24), $Pm'c2'_1$ (26.68), $Pm'a2'$ (28.89), P_Cc (7.30), Pc (7.24), $Pn'a2'_1$ (33.146), $Pm'c2'_1$ (26.68), P_Ac (7.31), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_Cca2_1 (29.109), P_C2 (3.6), P_Anc2 (30.119), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_C2_1 (4.12), P_Ana2_1 (33.152).

69.1 WP: $8g$

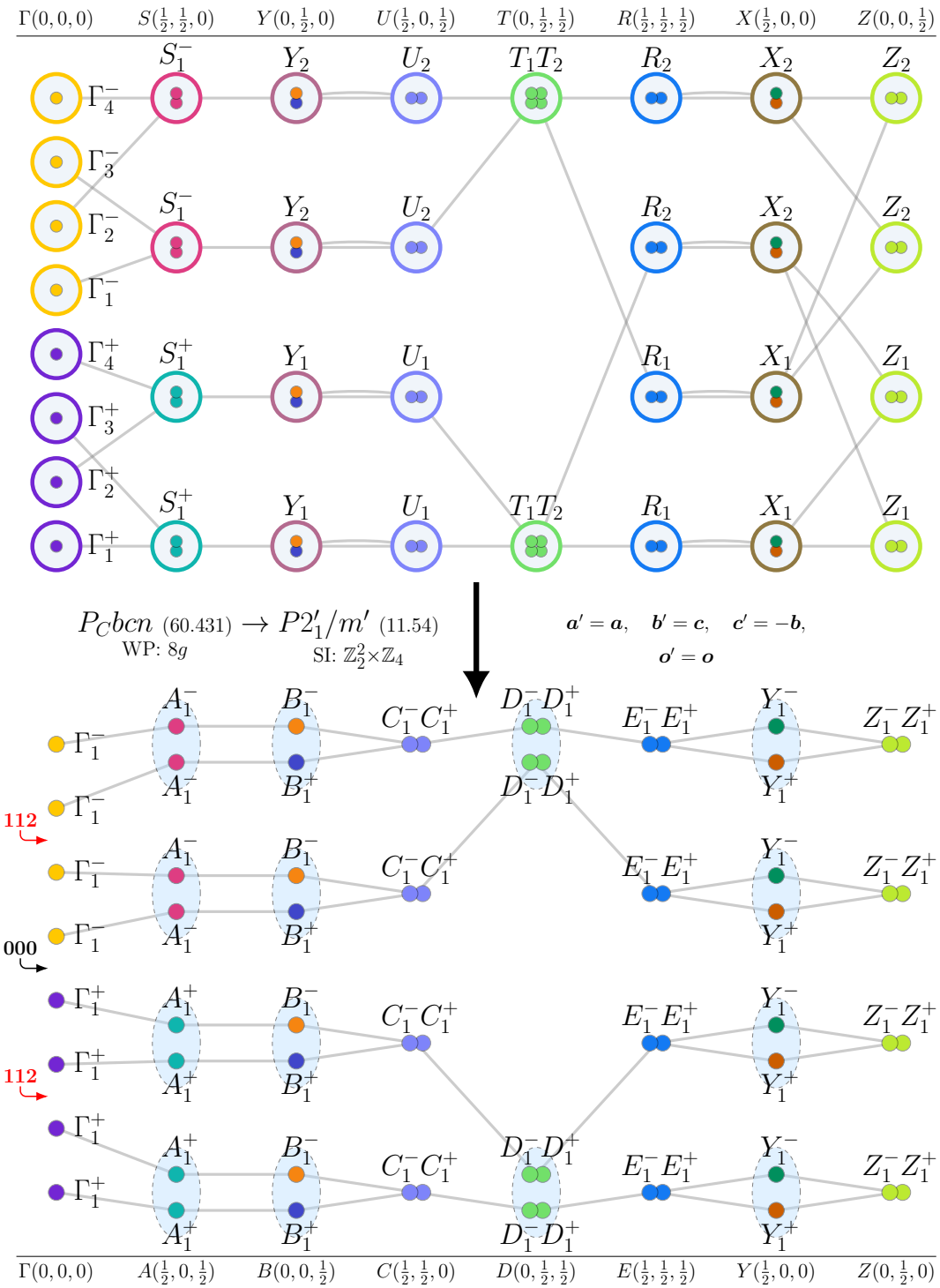
BCS Materials: Mn_5Si_3 (100 K)[252], Mn_5Si_3 (99 K)[253].

69.1.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[252]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.88`
[253]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.305`



Supplementary Figure 323: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $8g$ of supergroup P_Cbcn (60.431).

Supplementary Table 635: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $8g$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2

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SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

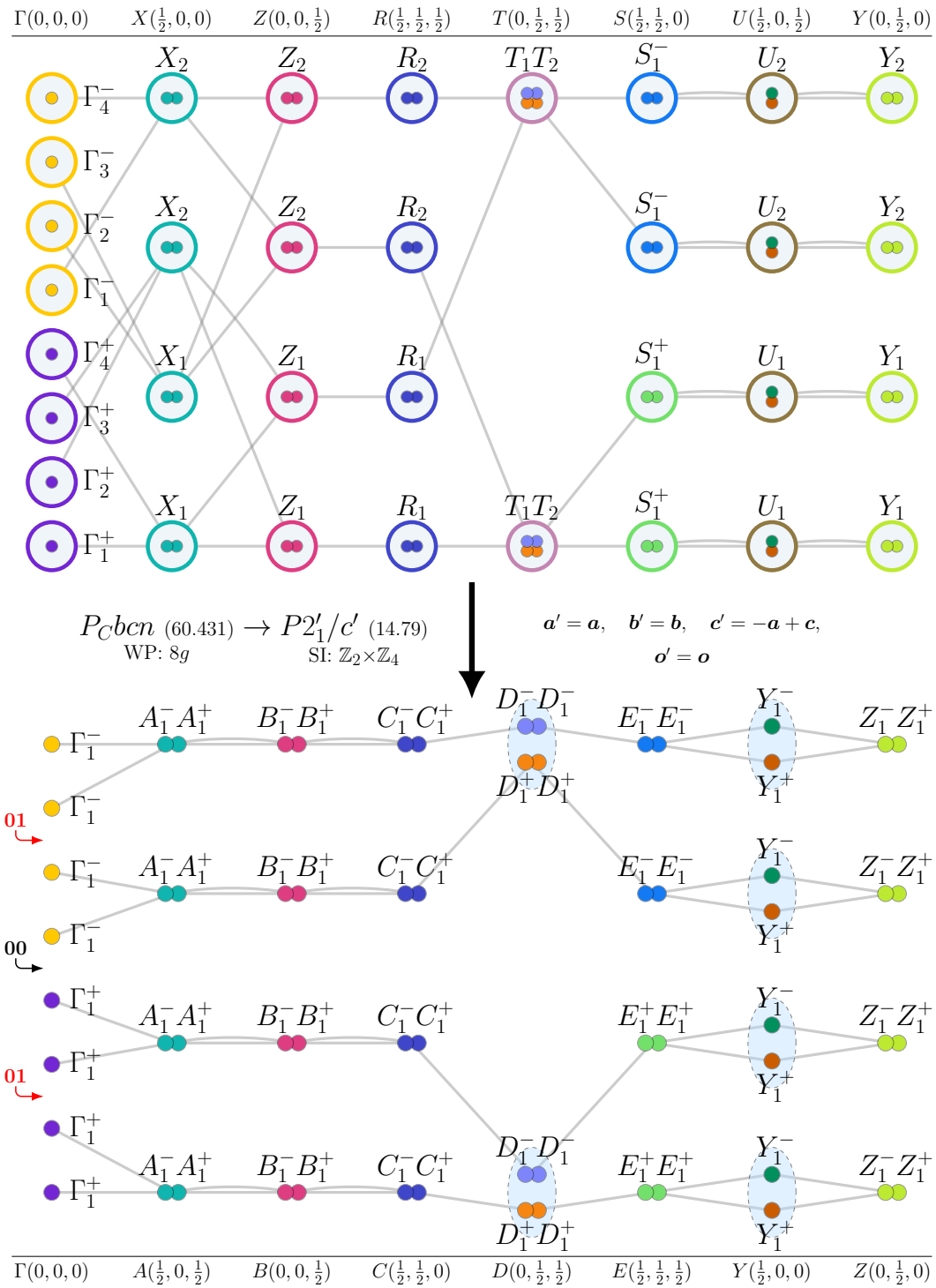
Supplementary Table 636: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $8g$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

69.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],
- B $\perp$ [010].



Supplementary Figure 324: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8g$ of supergroup P_Cbcn (60.431).

Supplementary Table 637: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

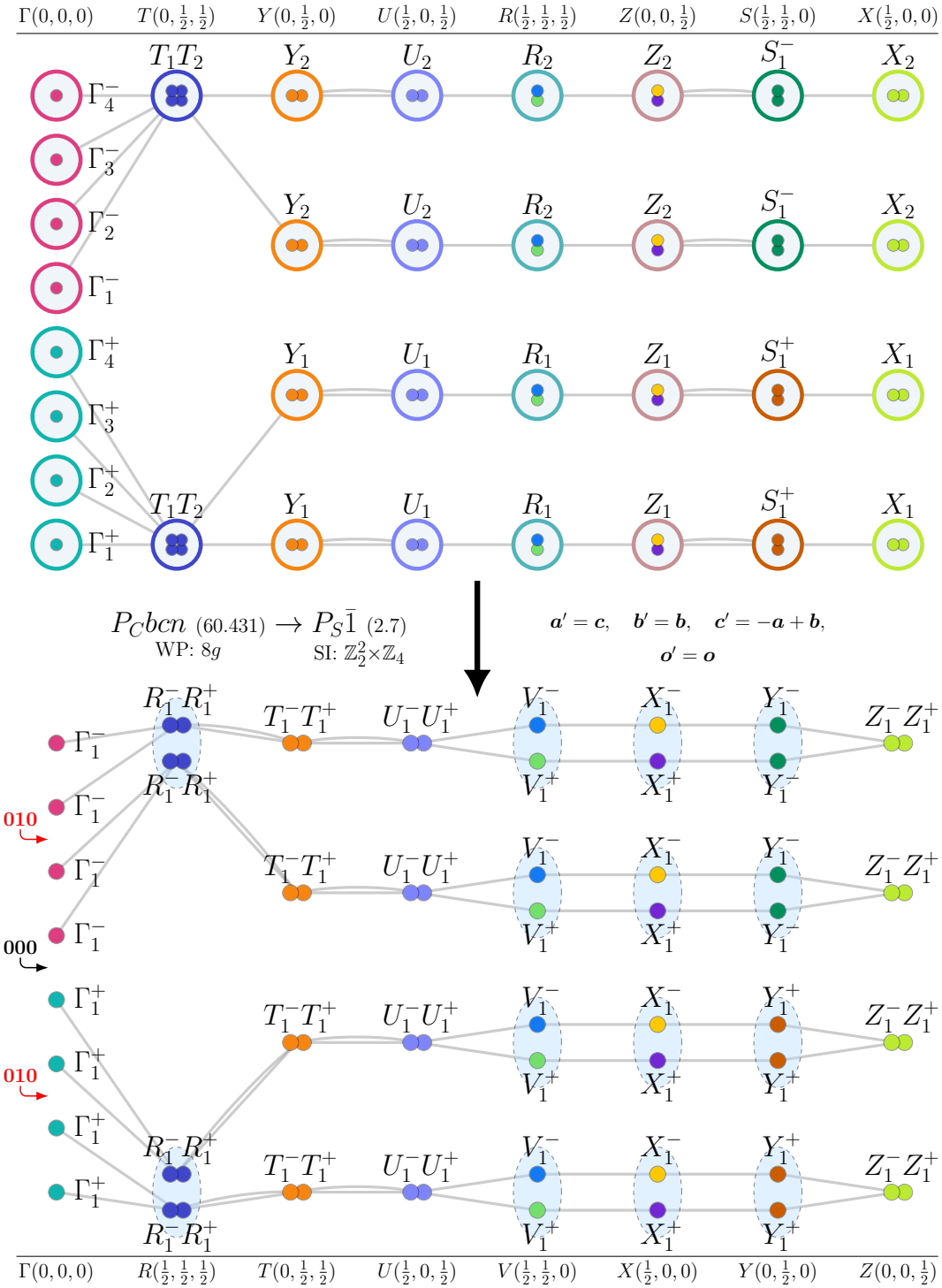
Supplementary Table 638: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

69.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 325: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8g$ of supergroup P_Cbcn (60.431).

Supplementary Table 639: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1
010	0, 1, 2
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
111	0, 1

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SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 640: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	010, 012, 101, 103
3	G
4	000, 002, 111, 113
5	G
6	010, 012, 101, 103
7	G
8	000

69.2 WP: $8e$

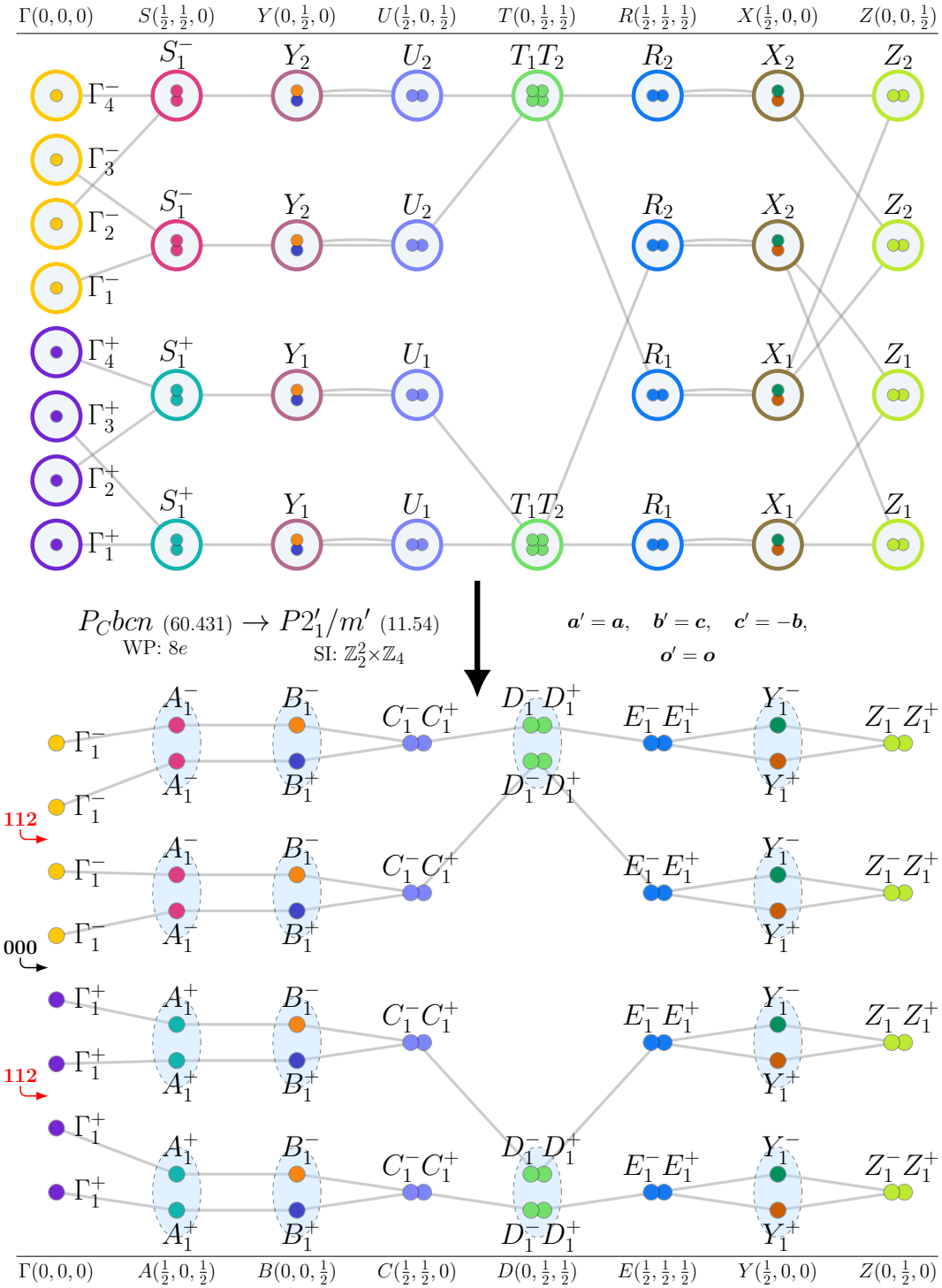
BCS Materials: $FeSn_2$ (93 K)[254].

69.2.1 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[254]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.67`



Supplementary Figure 326: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $8e$ of supergroup P_Cbcn (60.431).

Supplementary Table 641: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $8e$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2

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SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

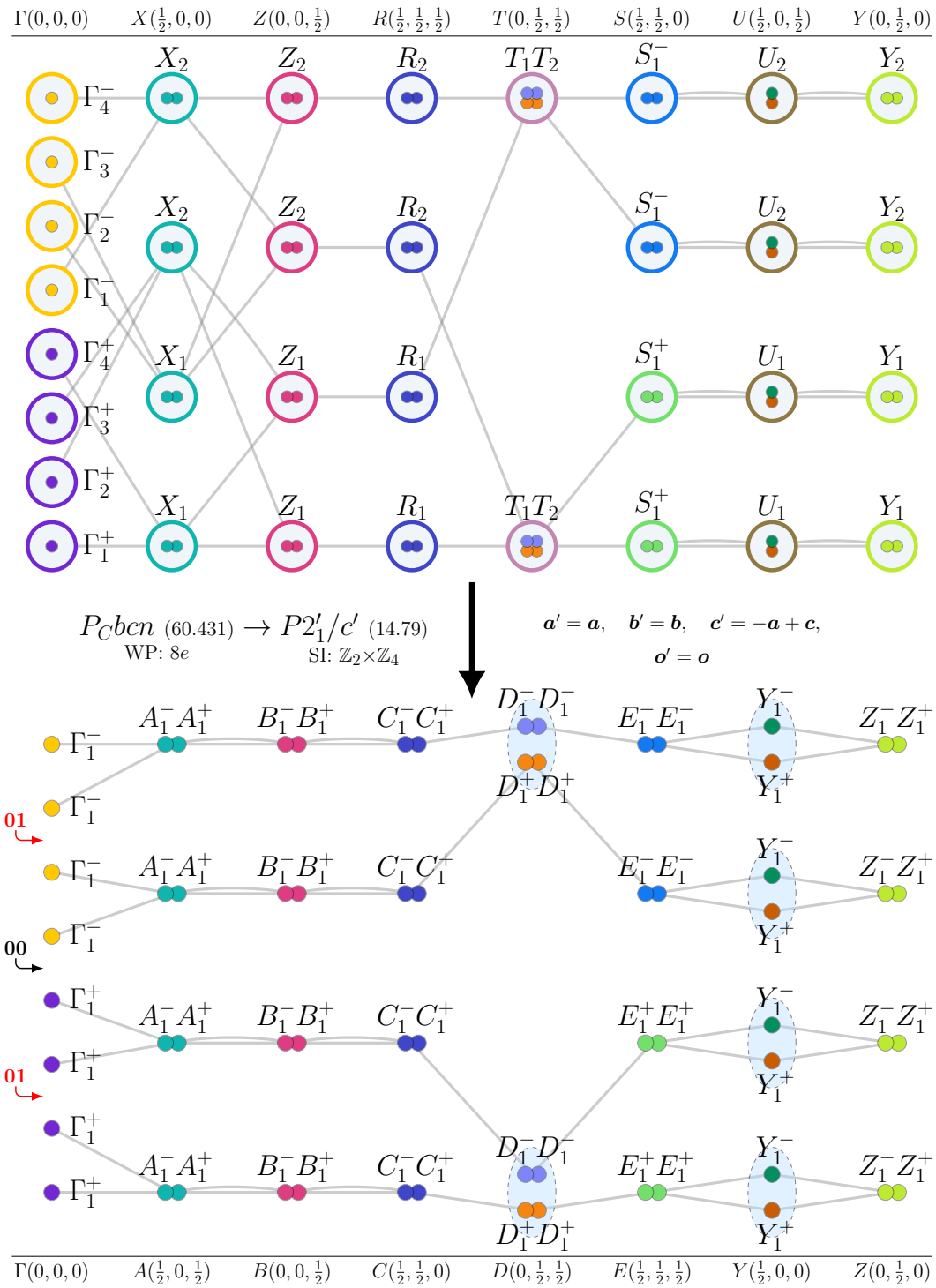
Supplementary Table 642: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $8e$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

69.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],
- B $\perp$ [010].



Supplementary Figure 327: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8e$ of supergroup P_Cbcn (60.431).

Supplementary Table 643: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $8e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

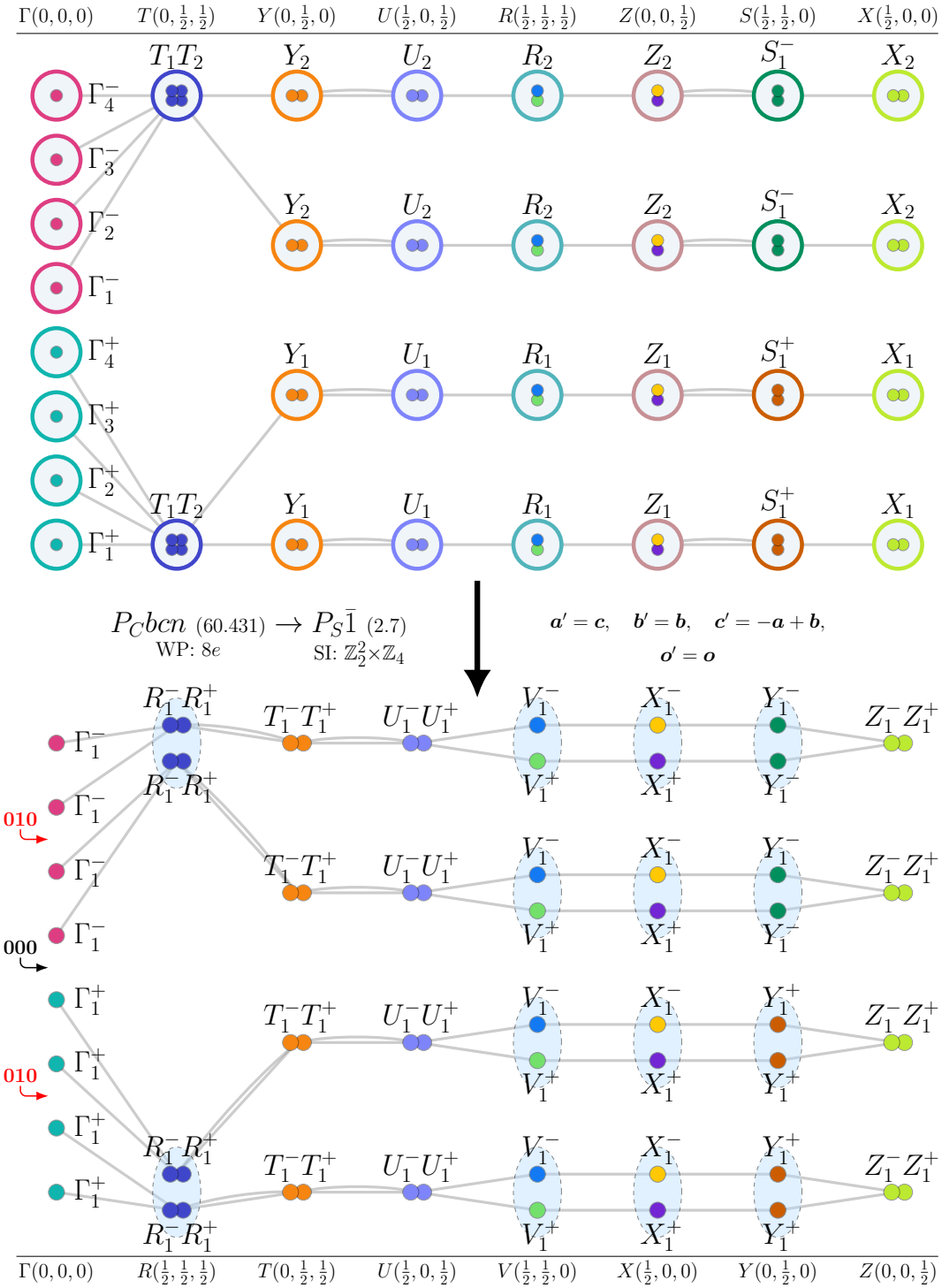
Supplementary Table 644: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $8e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

69.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 328: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8e$ of supergroup P_Cbcn (60.431).

Supplementary Table 645: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1
010	0, 1, 2
012	0, 1, 2
101	0, 1, 2
103	0, 1, 2
111	0, 1

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SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 646: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	010, 012, 101, 103
3	G
4	000, 002, 111, 113
5	G
6	010, 012, 101, 103
7	G
8	000

69.3 WP: $4a$

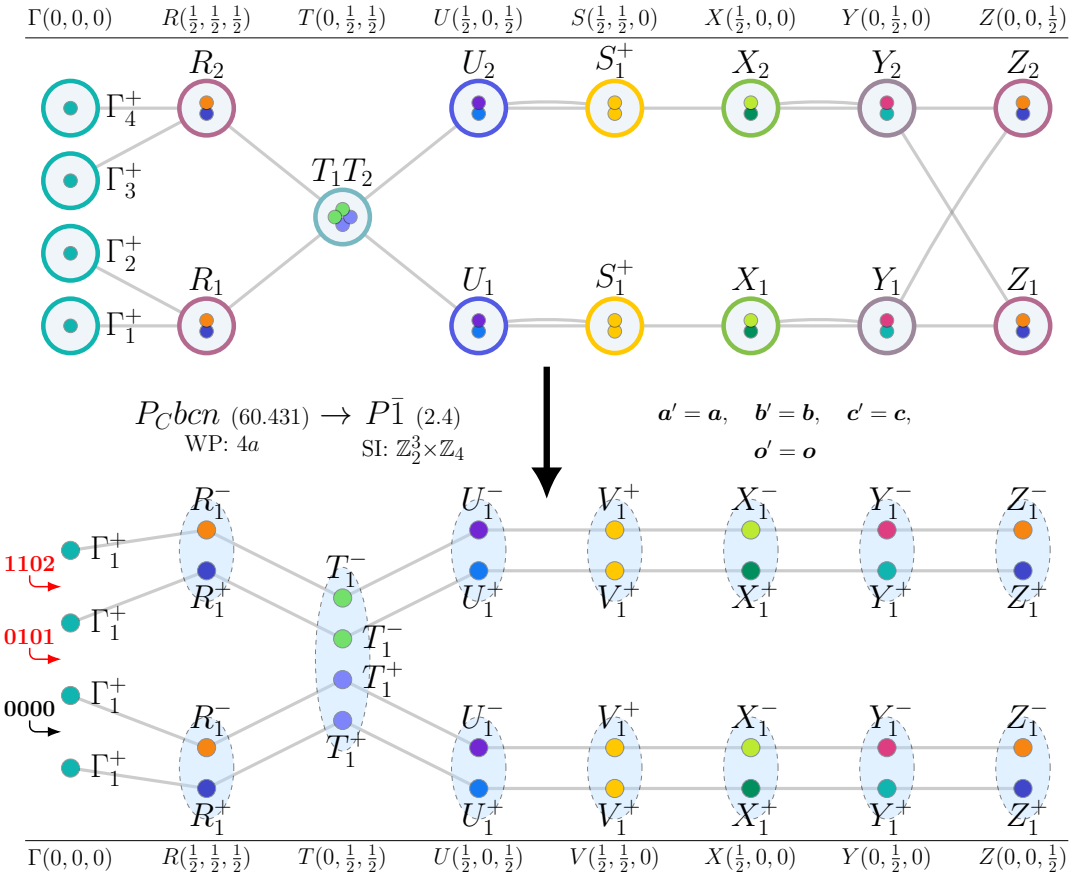
BCS Materials: $NiSO_4$ (37 K)[255], $FeSO_4$ (21 K)[256], $CoSO_4$ (15.5 K)[257], PrCuSi (5.1 K)[258].

69.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B $\parallel$ [100] and strain in generic direction,
- B $\parallel$ [010] and strain in generic direction,
- B $\parallel$ [001] and strain in generic direction,
- B $\perp$ [100] and strain $\perp$ [010],
- B $\perp$ [100] and strain $\perp$ [001],
- B $\perp$ [100] and strain in generic direction,
- B $\perp$ [010] and strain $\perp$ [100],
- B $\perp$ [010] and strain $\perp$ [001],
- B $\perp$ [010] and strain in generic direction,
- B $\perp$ [001] and strain $\perp$ [100],
- B $\perp$ [001] and strain $\perp$ [010],
- B $\perp$ [001] and strain in generic direction,
- B in generic direction.

[255]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.520`
[256]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.521`
[257]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.519`
[258]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.460`



Supplementary Figure 329: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cbcn (60.431).

Supplementary Table 647: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1
0012	0, 1, 2
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2, 3
1103	0, 1, 2

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SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

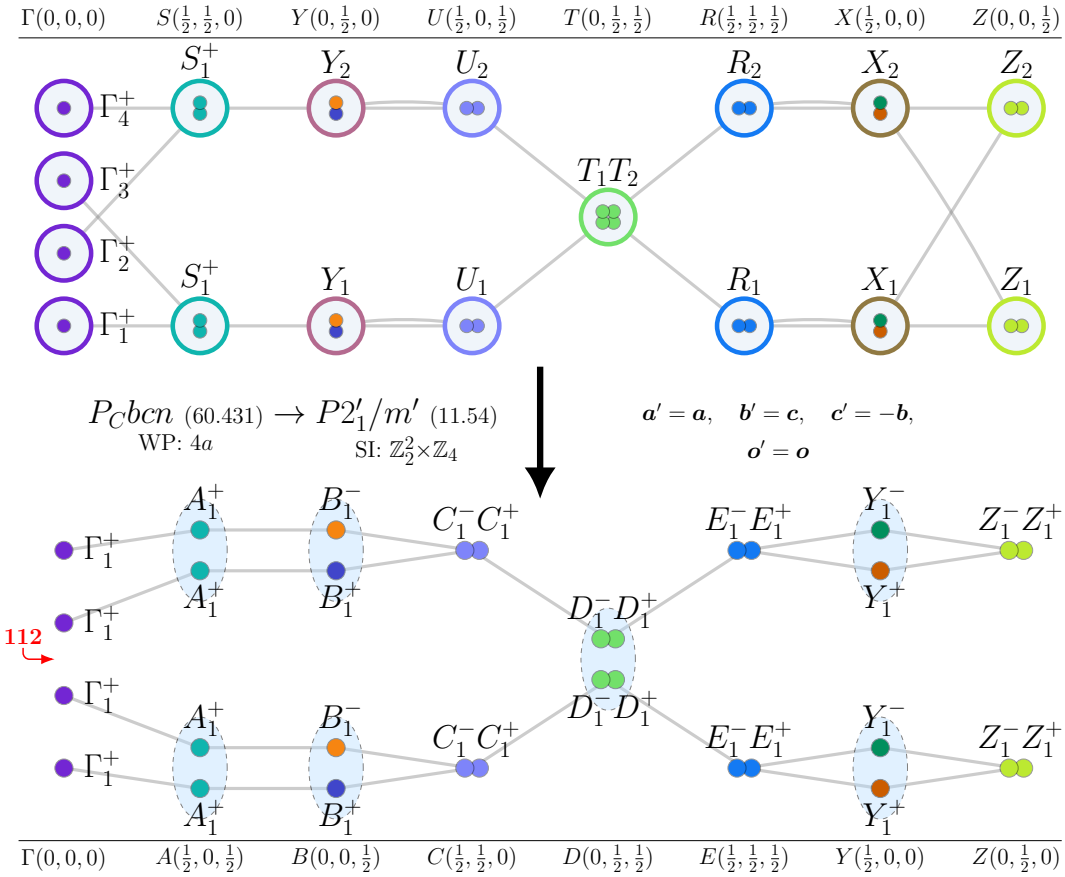
Supplementary Table 648: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1112, 1113
2	0101, 0103, 1102
3	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1111, 1112
4	0000

69.3.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 330: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cbcn (60.431).

Supplementary Table 649: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 650: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P2_1'/m'$ (11.54).

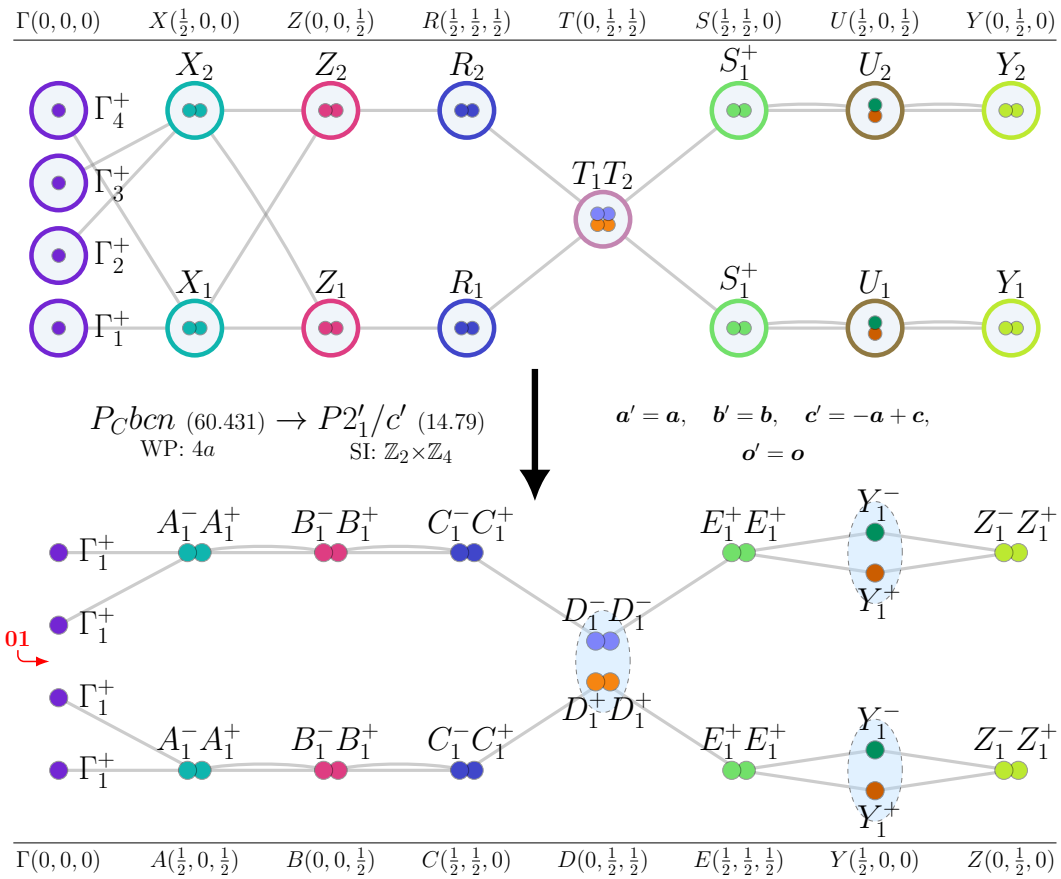
Gap#	Possible SI Values
1	G
2	112
3	G
4	000

69.3.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],

- B $\perp$ [010].



Supplementary Figure 331: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cbcn (60.431).

Supplementary Table 651: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

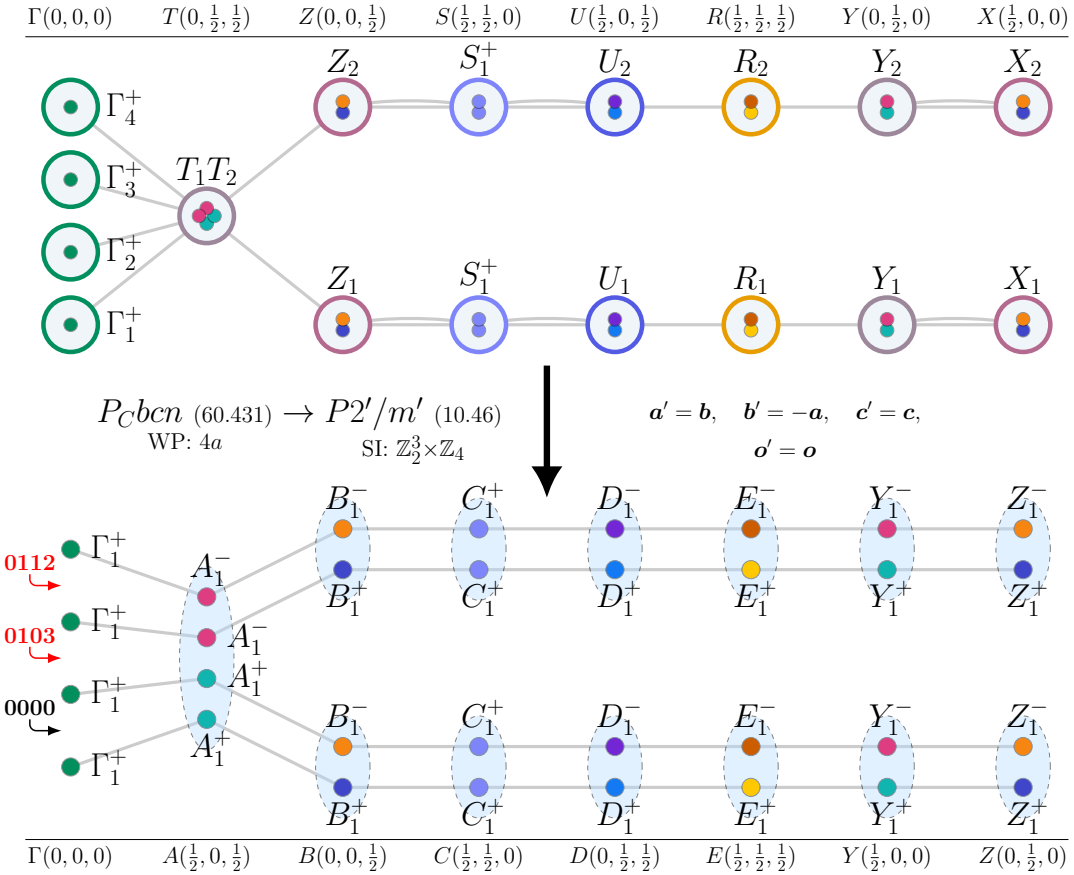
Supplementary Table 652: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

69.3.4 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 332: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cbcn (60.431).

Supplementary Table 653: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1
1002	0, 1, 2
1003	0, 1
1010	0, 1, 2
1011	0, 1, 2

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SI	Possible #Gaps
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

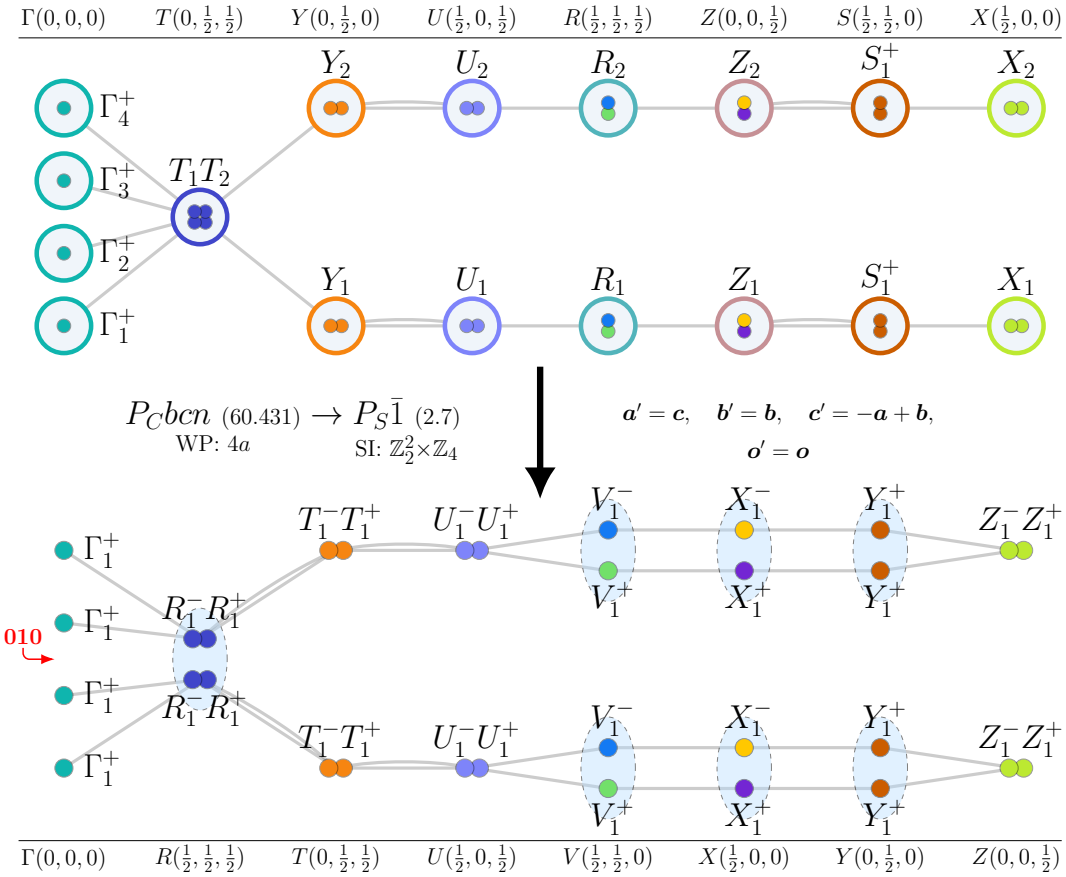
Supplementary Table 654: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0111, 0112, 0113, 1000 1001, 1002, 1010, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
2	0101, 0103, 0112
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0111, 0112, 0113, 1000 1002, 1003, 1010, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
4	0000

69.3.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 333: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_Cbcn (60.431).

Supplementary Table 655: Number of times a given SI can appear in the band structure. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
010	1
gapless	2
nontrivial	1

Supplementary Table 656: Possible SI values in each gap. Supergroup: P_Cbcn (60.431), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	010
3	G
4	000

70 MSG P_Ibcn (60.432)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/m'$ (10.46), $P2_1'/c'$ (14.79), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pb'c'n$ (60.422), P_A2_1/c (14.83), $P2$ (3.1), $Pm'a'2$ (28.91), $P2/c$ (13.65), $Pc'cm'$ (49.270), P_C2/c (13.74), $P2_1/c$ (14.75), $Pb'am'$ (55.358), P_C2_1/c (14.84).

Trivial-SI Subgroups: Pm' (6.20), Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2_1'$ (4.9), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pna'2_1'$ (33.147), $Pnc'2'$ (30.114), P_Ac (7.31), Pc (7.24), $Pc'c2'$ (27.80), $Pm'a2'$ (28.89), P_Cc (7.30), Pc (7.24), $Pb'a2'$ (32.137), $Pm'c2_1'$ (26.68), P_Cc (7.30), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_C2_1 (4.12), P_Ica2_1 (29.110), P_C2 (3.6), P_Inc2 (30.122), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_C2_1 (4.12), P_Ina2_1 (33.155).

70.1 WP: $4a$

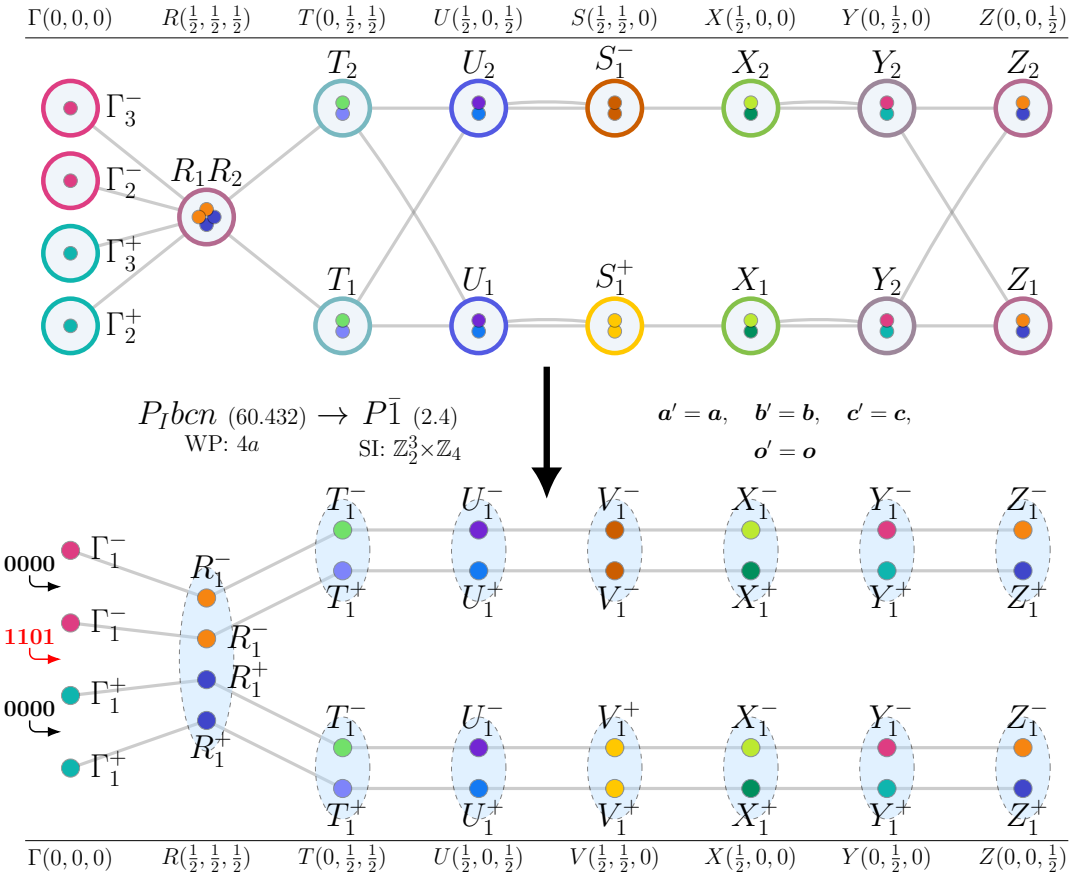
BCS Materials: $FeSn_2$ (378 K)[259], $FeGe_2$ (315 K)[260].

70.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],
- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.

[259]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.556`
[260]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.557`



Supplementary Figure 334: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup P_Ibcn (60.432).

Supplementary Table 657: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2, 3
1101	0, 1, 2, 3
1102	0, 1, 2, 3

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SI	Possible #Gaps
1103	0, 1, 2, 3
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	1, 2, 3

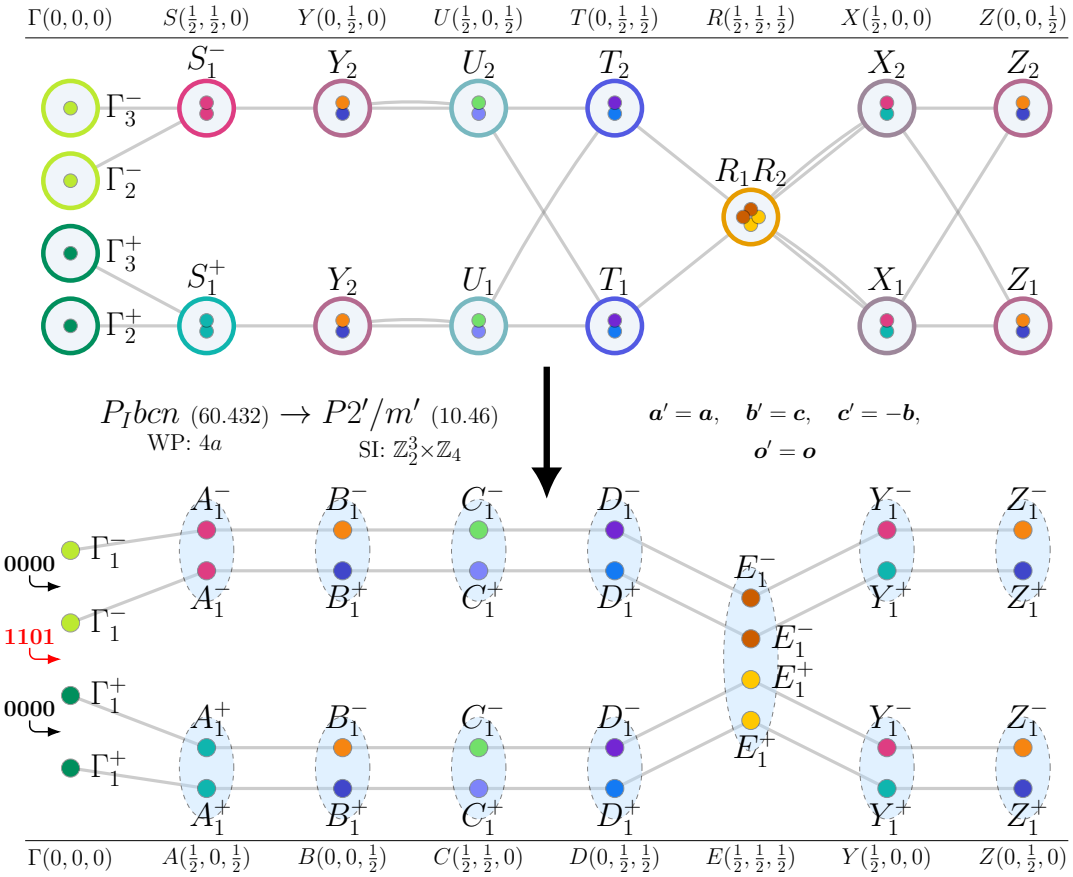
Supplementary Table 658: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0010, 0011, 0012, 0013, 1100 1101, 1102, 1103
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

70.1.2 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [001],
- B $\parallel$ [010] and strain $\perp$ [001],
- B $\perp$ [001].



Supplementary Figure 335: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4a$ of supergroup P_Ibcn (60.432).

Supplementary Table 659: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2, 3
1101	0, 1, 2, 3

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SI	Possible #Gaps
1102	0, 1, 2, 3
1103	0, 1, 2, 3
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	1, 2, 3

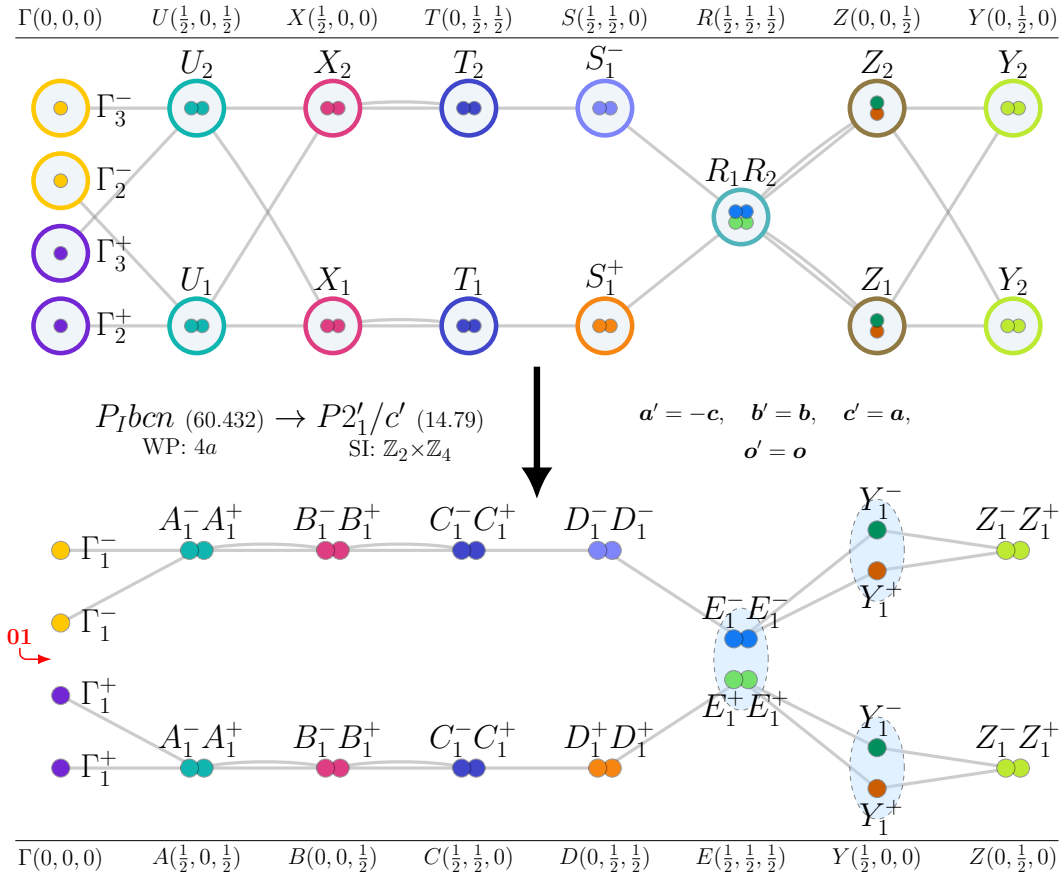
Supplementary Table 660: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0010, 0011, 0012, 0013, 1100 1101, 1102, 1103
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

70.1.3 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [010],
- B $\parallel$ [001] and strain $\perp$ [010],
- B $\perp$ [010].



Supplementary Figure 336: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_Ibcn (60.432).

Supplementary Table 661: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

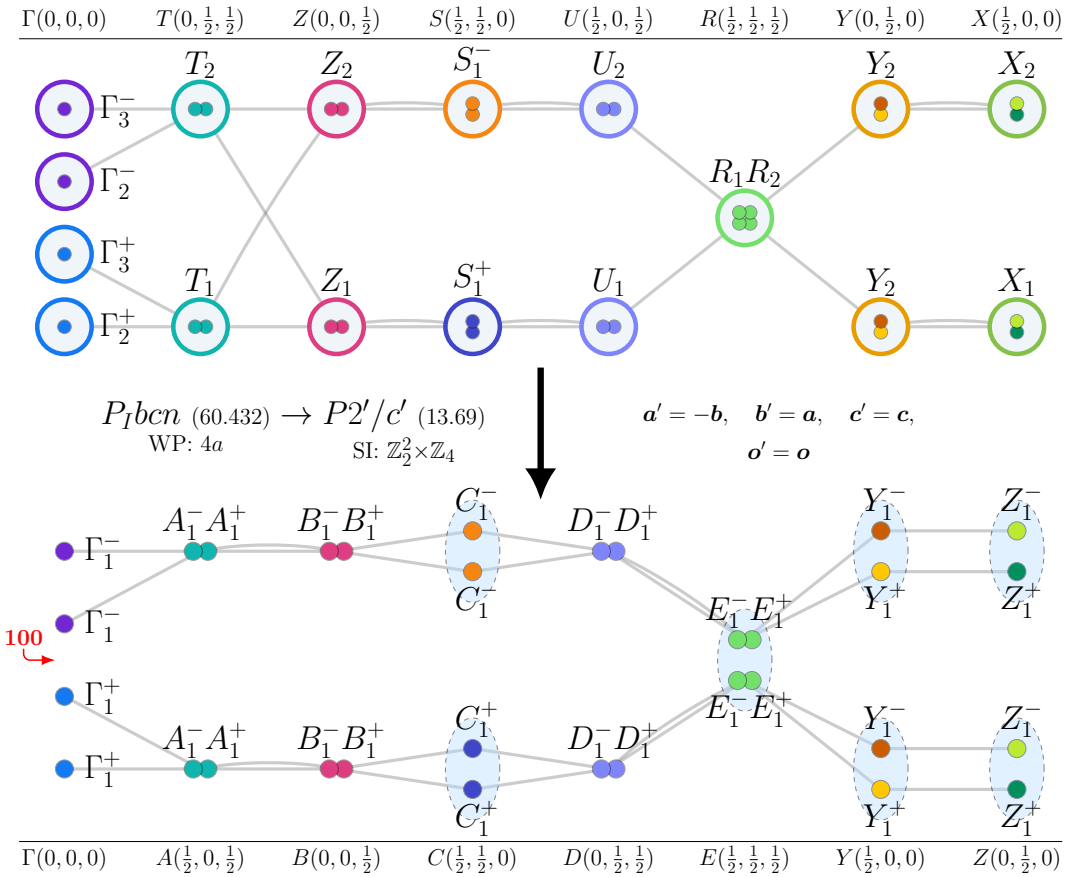
Supplementary Table 662: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

70.1.4 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 337: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $4a$ of supergroup P_Ibcn (60.432).

Supplementary Table 663: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
011	0, 1
013	0, 1
100	0, 1
102	0, 1
gapless	2
nontrivial	1

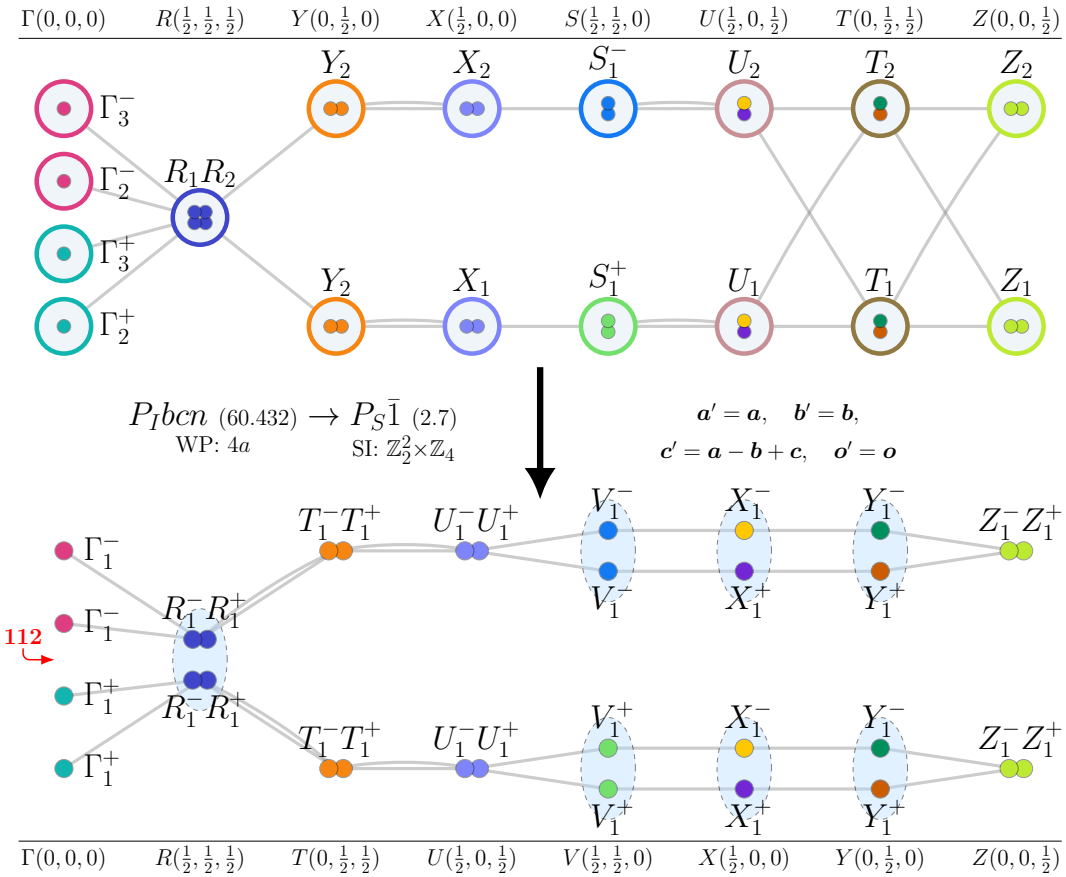
Supplementary Table 664: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	011, 013, 100, 102
3	G
4	000

70.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 338: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4a$ of supergroup P_Ibcn (60.432).

Supplementary Table 665: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
001	0, 1
003	0, 1
110	0, 1
112	0, 1
gapless	2
nontrivial	1

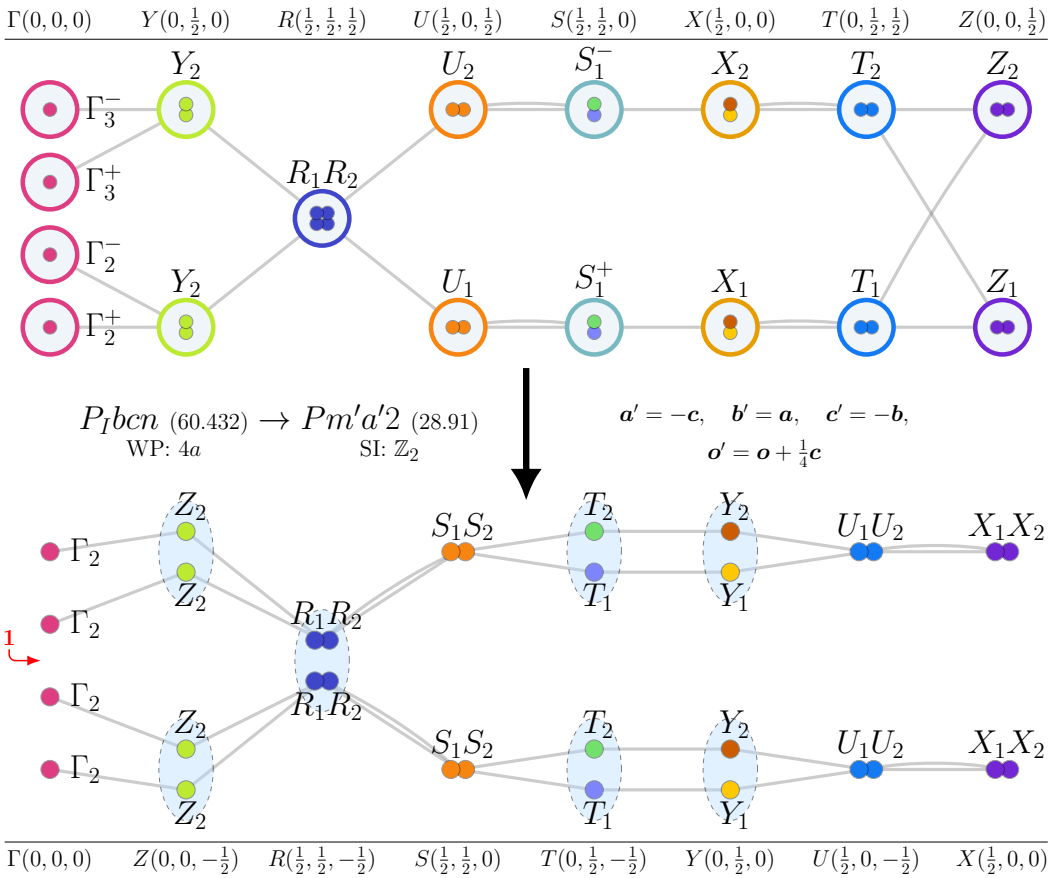
Supplementary Table 666: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000

70.1.6 Topological bands in subgroup $Pm'a'2$ (28.91)

Perturbation:

- E ∥ [010] and B ∥ [010].



Supplementary Figure 339: Topological magnon bands in subgroup $Pm'a'2$ (28.91) for magnetic moments on Wyckoff position $4a$ of supergroup P_Ibcn (60.432).

Supplementary Table 667: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $Pm'a'2$ (28.91).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 668: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4a$, Subgroup: $Pm'a'2$ (28.91).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

70.2 WP: $4b + 8j$

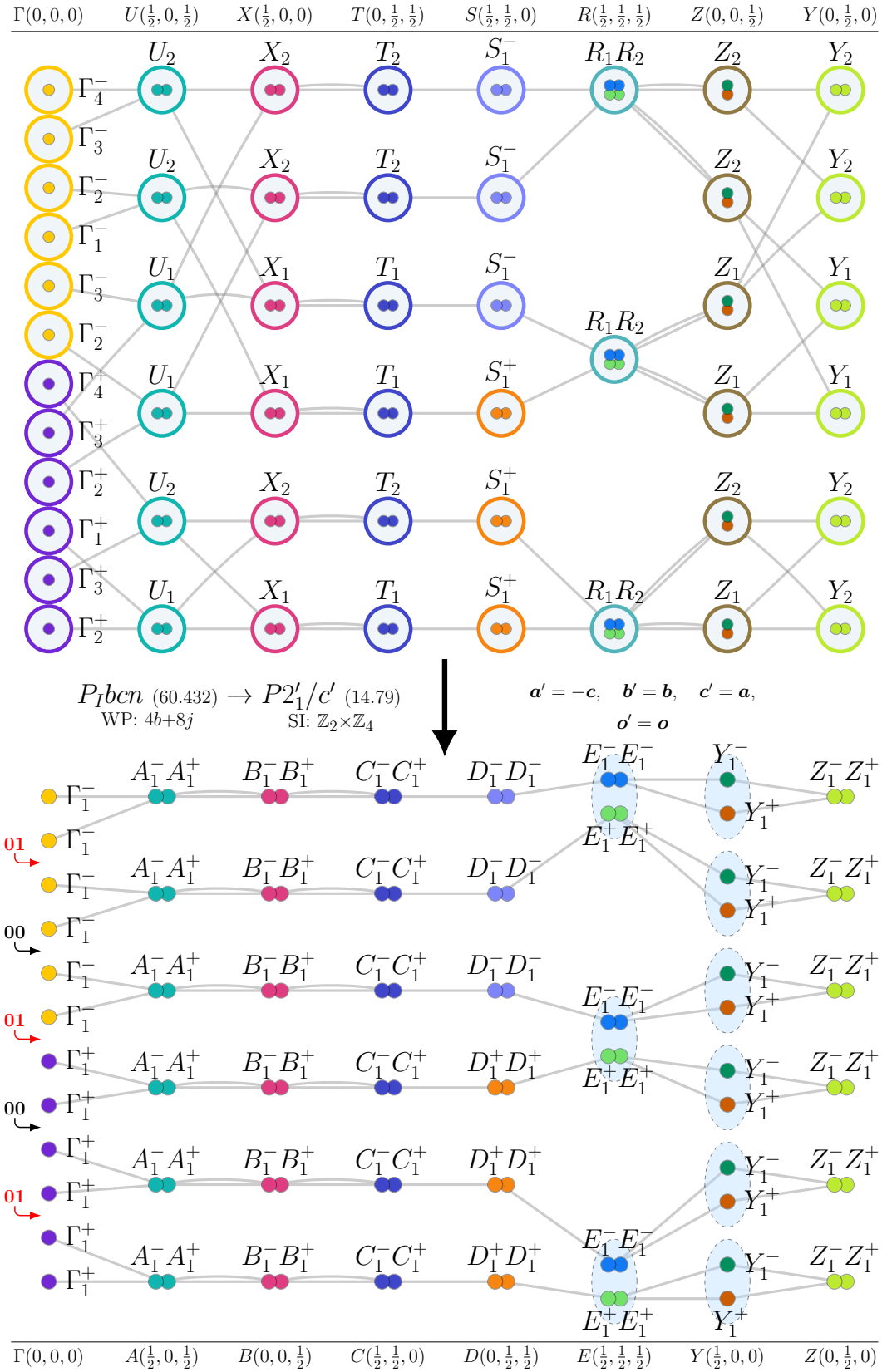
BCS Materials: Mn_3ZnN (140 K)[261].

70.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].

[261] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.31`



Supplementary Figure 340: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+8j$ of supergroup P_Ibcn (60.432).

Supplementary Table 669: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4b$+$8j$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

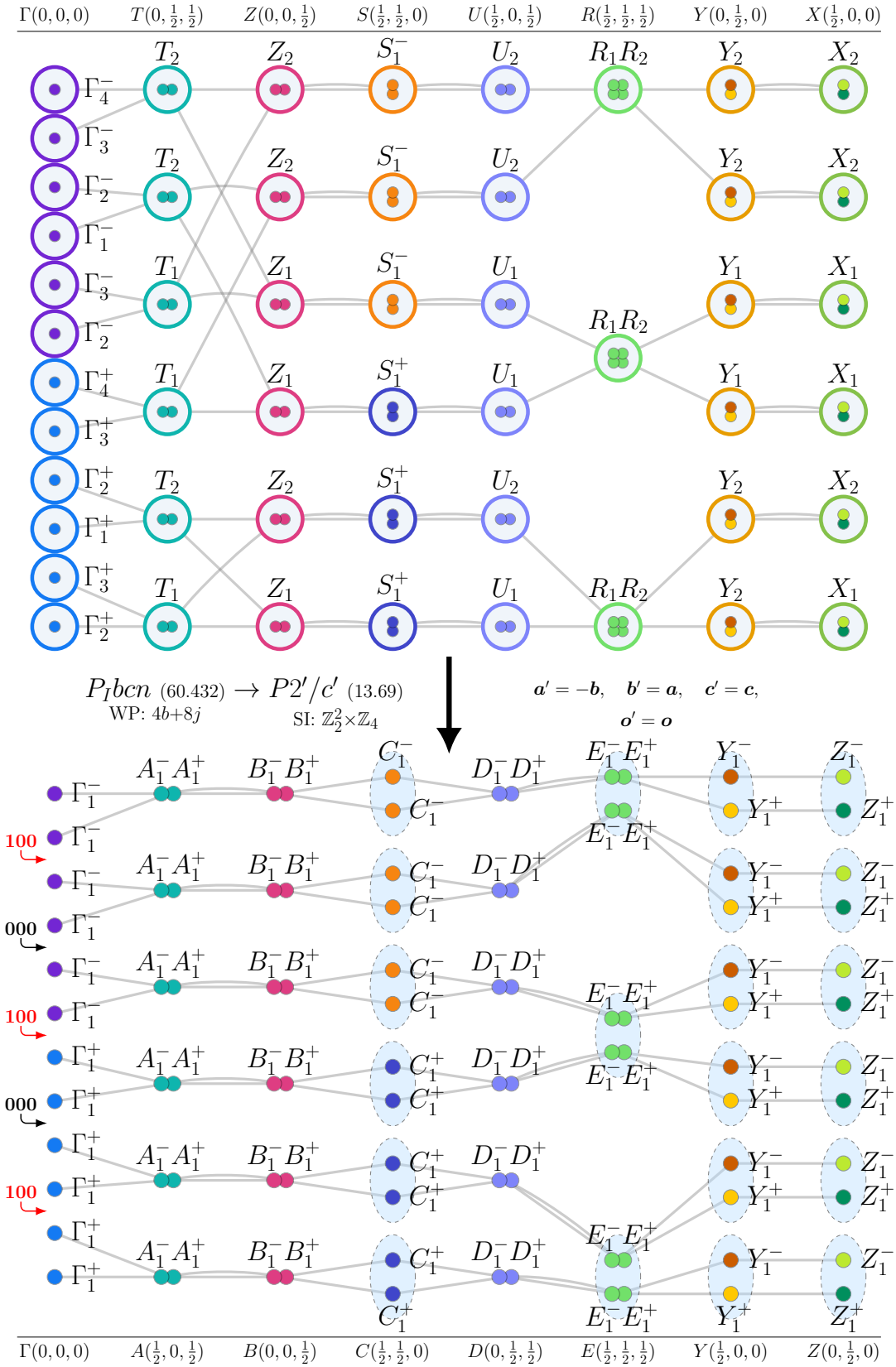
Supplementary Table 670: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4b$+$8j$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

70.2.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 341: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4b+8j$ of supergroup P_Ibcn (60.432).

Supplementary Table 671: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4b$+$8j$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2
002	0, 1, 2
011	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2, 3
102	0, 1, 2, 3
111	0, 1, 2
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

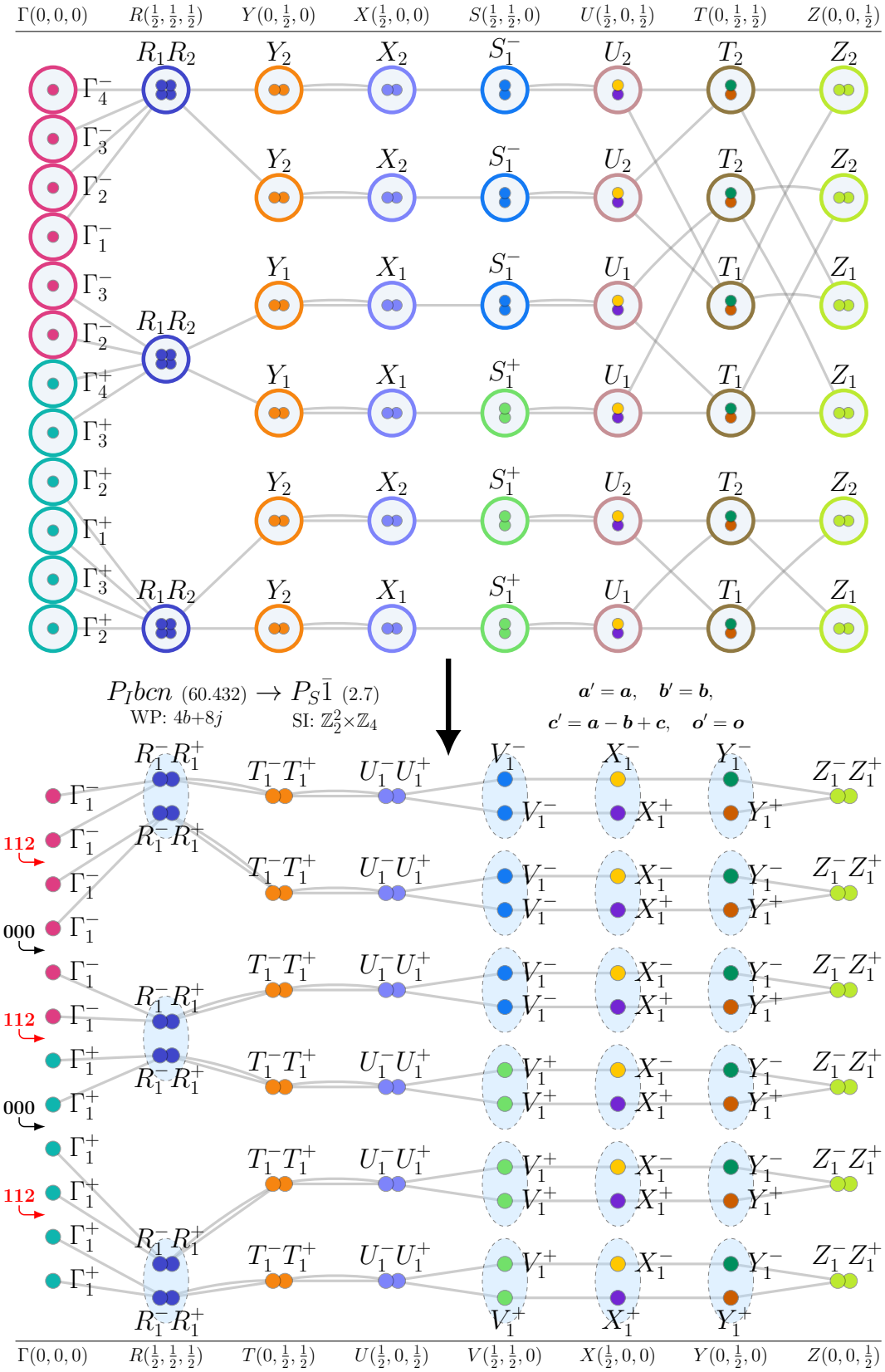
Supplementary Table 672: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4b$+$8j$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	011, 013, 100, 102
3	G
4	000, 002, 111, 113
5	G
6	011, 013, 100, 102
7	G
8	000, 002, 111, 113
9	G
10	011, 013, 100, 102
11	G
12	000

70.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 342: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4b+8j$ of supergroup P_Ibcn (60.432).

Supplementary Table 673: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $4b$+$8j$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 674: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $4b$+$8j$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

70.3 WP: $8j$

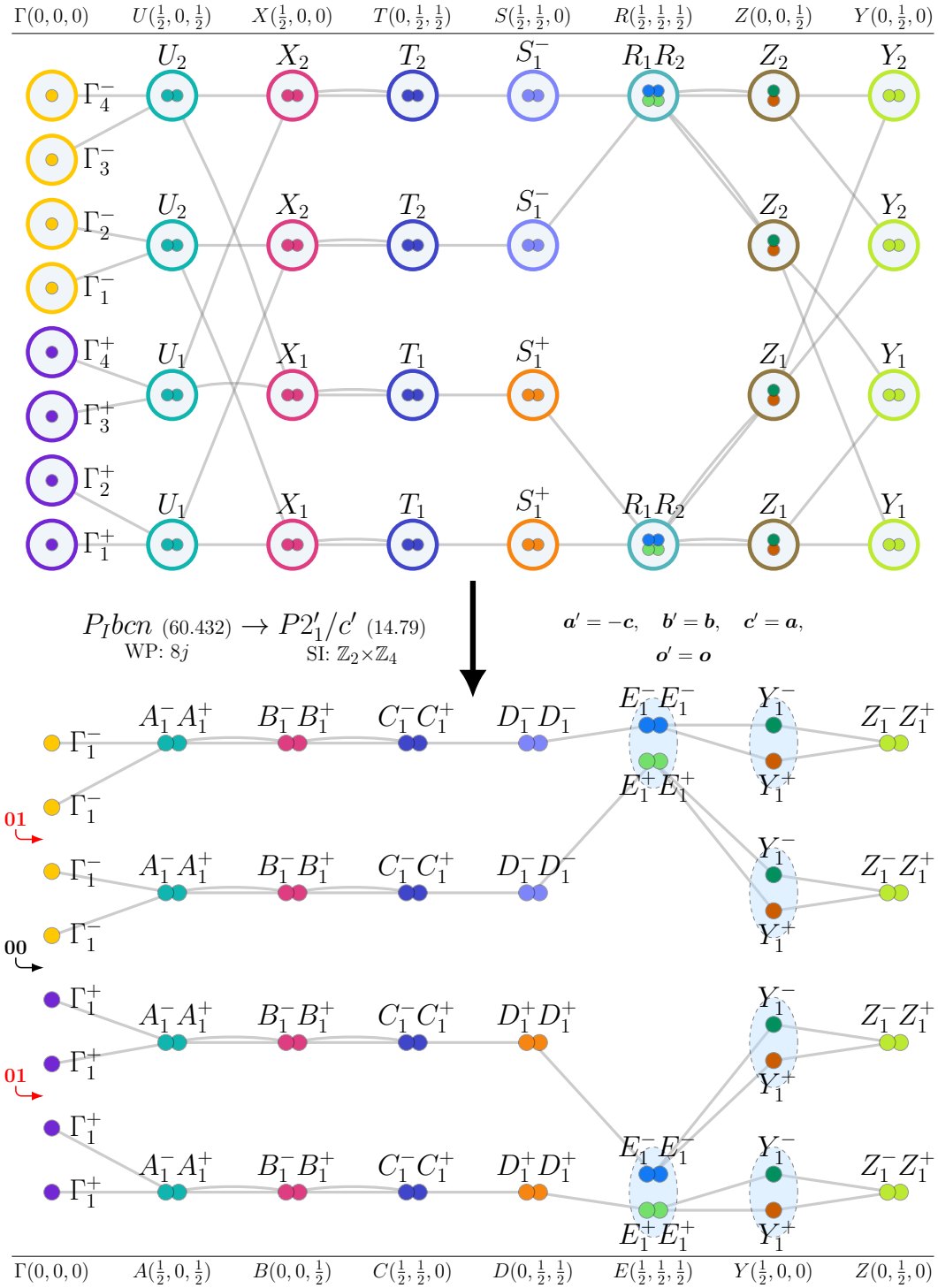
BCS Materials: $Ce_2Ni_3Ge_5$ (4.8 K)[262], $Ce_2Ni_3Ge_5$ (4.3 K)[263].

70.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].

[262]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.562`
[263]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.565`



Supplementary Figure 343: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8j$ of supergroup P_Ibcn (60.432).

Supplementary Table 675: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $8j$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

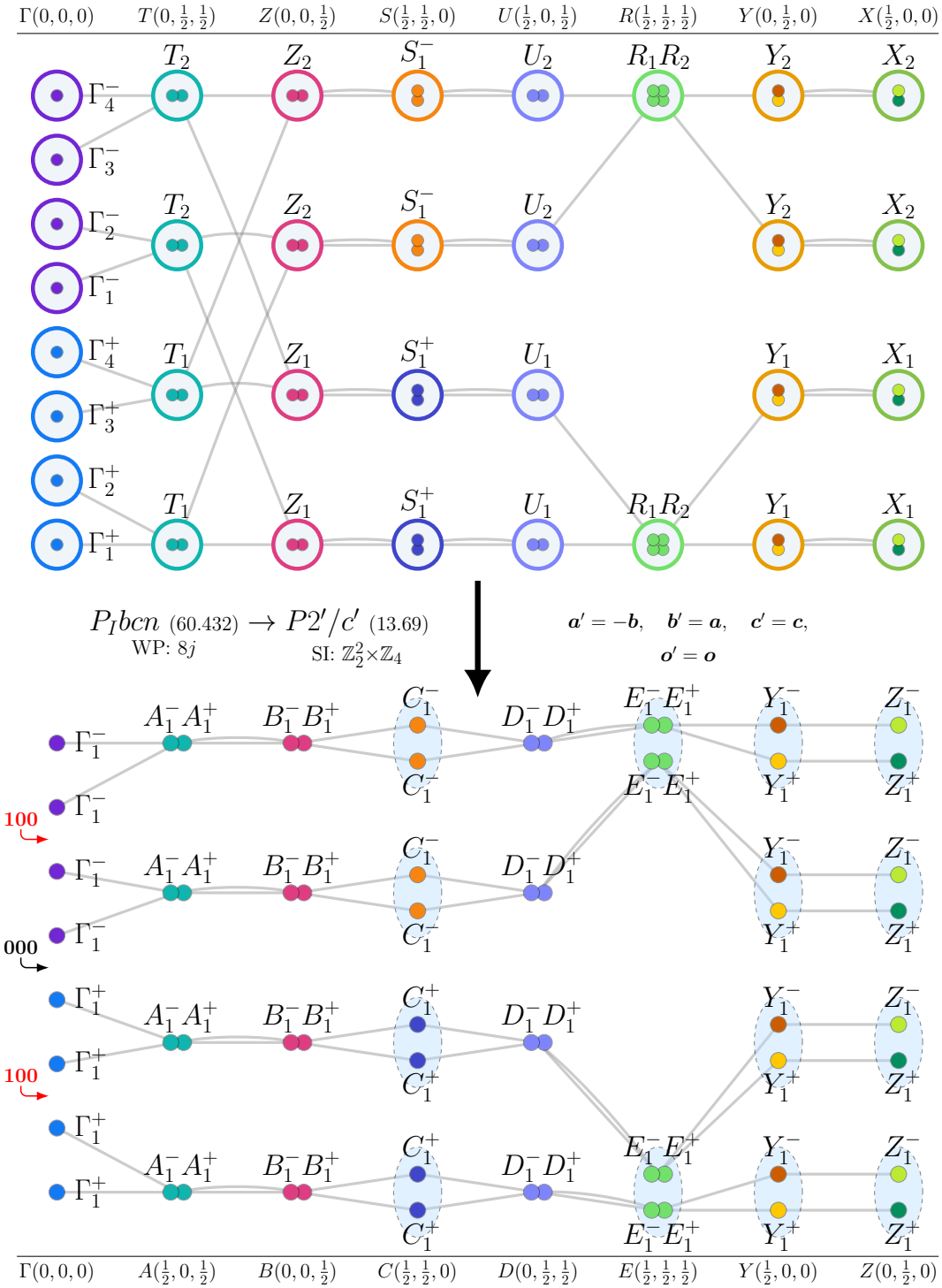
Supplementary Table 676: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $8j$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

70.3.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 344: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8j$ of supergroup P_Ibcn (60.432).

Supplementary Table 677: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $8j$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1
002	0, 1
011	0, 1, 2
013	0, 1, 2
100	0, 1, 2
102	0, 1, 2
111	0, 1

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SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

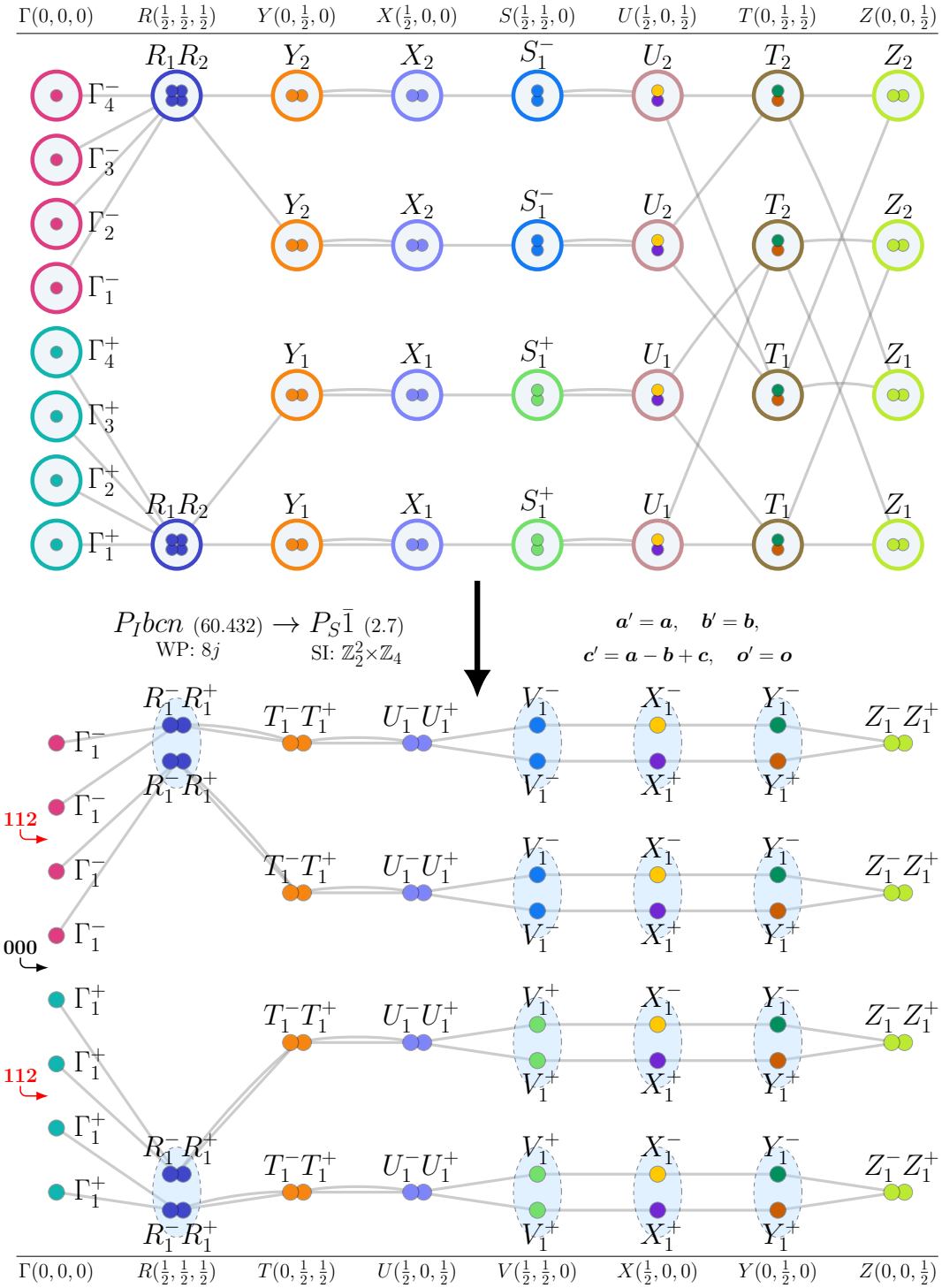
Supplementary Table 678: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $8j$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	011, 013, 100, 102
3	G
4	000, 002, 111, 113
5	G
6	011, 013, 100, 102
7	G
8	000

70.3.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 345: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8j$ of supergroup P_Ibcn (60.432).

Supplementary Table 679: Number of times a given SI can appear in the band structure. Supergroup: P_Ibcn (60.432), WP: $8j$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2

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SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 680: Possible SI values in each gap. Supergroup: P_Ibcn (60.432), WP: $8j$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

71 MSG $Pbca$ (61.433)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1/c$ (14.75), $P2_1/c$ (14.75), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pc (7.24), Pc (7.24), Pc (7.24), $P2_1$ (4.7), $Pca2_1$ (29.99), $P2_1$ (4.7), $Pca2_1$ (29.99), $P2_1$ (4.7), $Pca2_1$ (29.99).

71.1 WP: $4a$

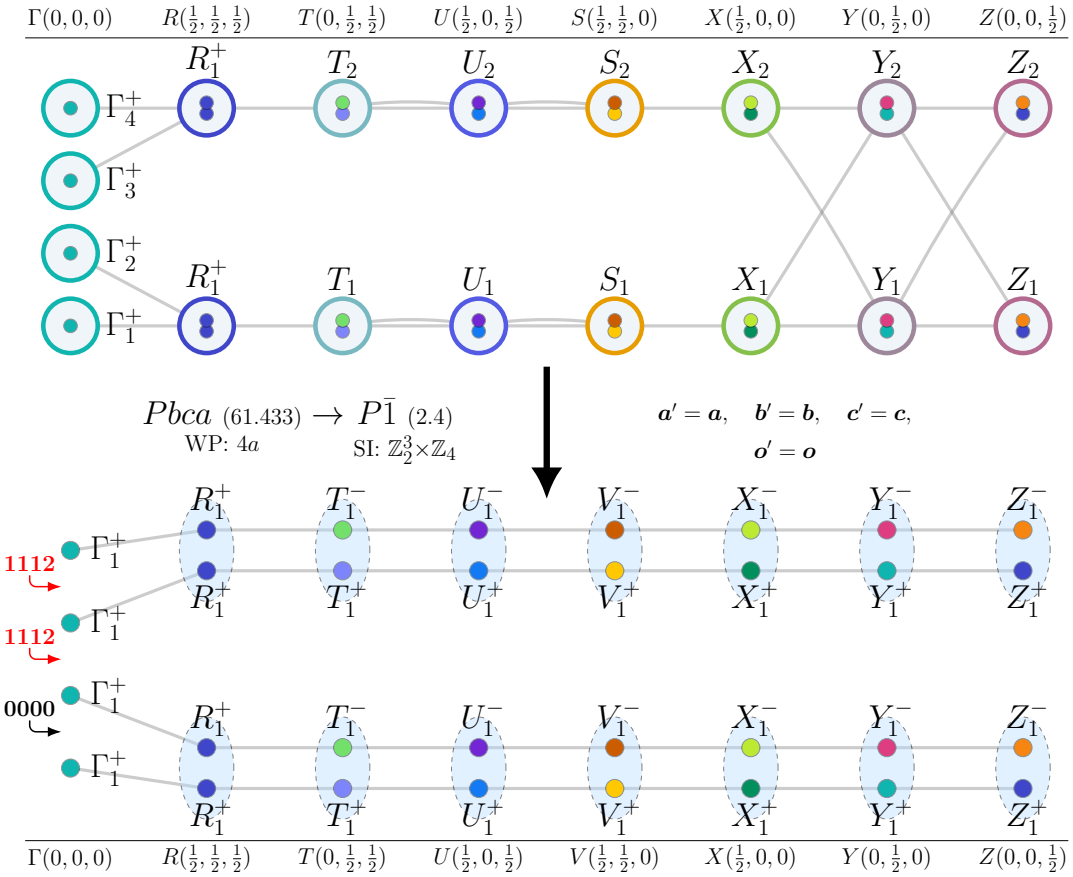
BCS Materials: Ca_2RuO_4 (110 K)[264].

71.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ⊥ [010],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [100],
- B ∥ [010] and strain ⊥ [001],
- B ∥ [001] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [010],
- B in generic direction.

[264]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.398`



Supplementary Figure 346: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $Pbca$ (61.433).

Supplementary Table 681: Number of times a given SI can appear in the band structure. Supergroup: $Pbca$ (61.433), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 682: Possible SI values in each gap. Supergroup: $Pbca$ (61.433), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
2	1112
3	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
4	0000

72 MSG P_Cbca (61.439)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'/c'$ (13.69), $P2'/m'$ (10.46), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pm'n'a$ (53.326), P_a2_1/c (14.80), $P2_1/c$ (14.75), $Pb'am'$ (55.358), P_C2_1/c (14.84), $P2_1/c$ (14.75), $Pc'cn'$ (56.370), P_A2_1/c (14.83).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pm' (6.20), $P2'_1$ (4.9), $P2'$ (3.3), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pm'a2'$ (28.89), $Pn'c2'$ (30.113), P_ac (7.27), Pc (7.24), $Pm'c2'_1$ (26.68), $Pb'a2'$ (32.137), P_Cc (7.30), Pc (7.24), $Pn'a2'_1$ (33.146), $Pc'c2'$ (27.80), P_Ac (7.31), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_Cca2_1 (29.109), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_C2_1 (4.12), P_Bca2_1 (29.108), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_C2_1 (4.12), P_Aca2_1 (29.107).

72.1 WP: $4a + 8e$

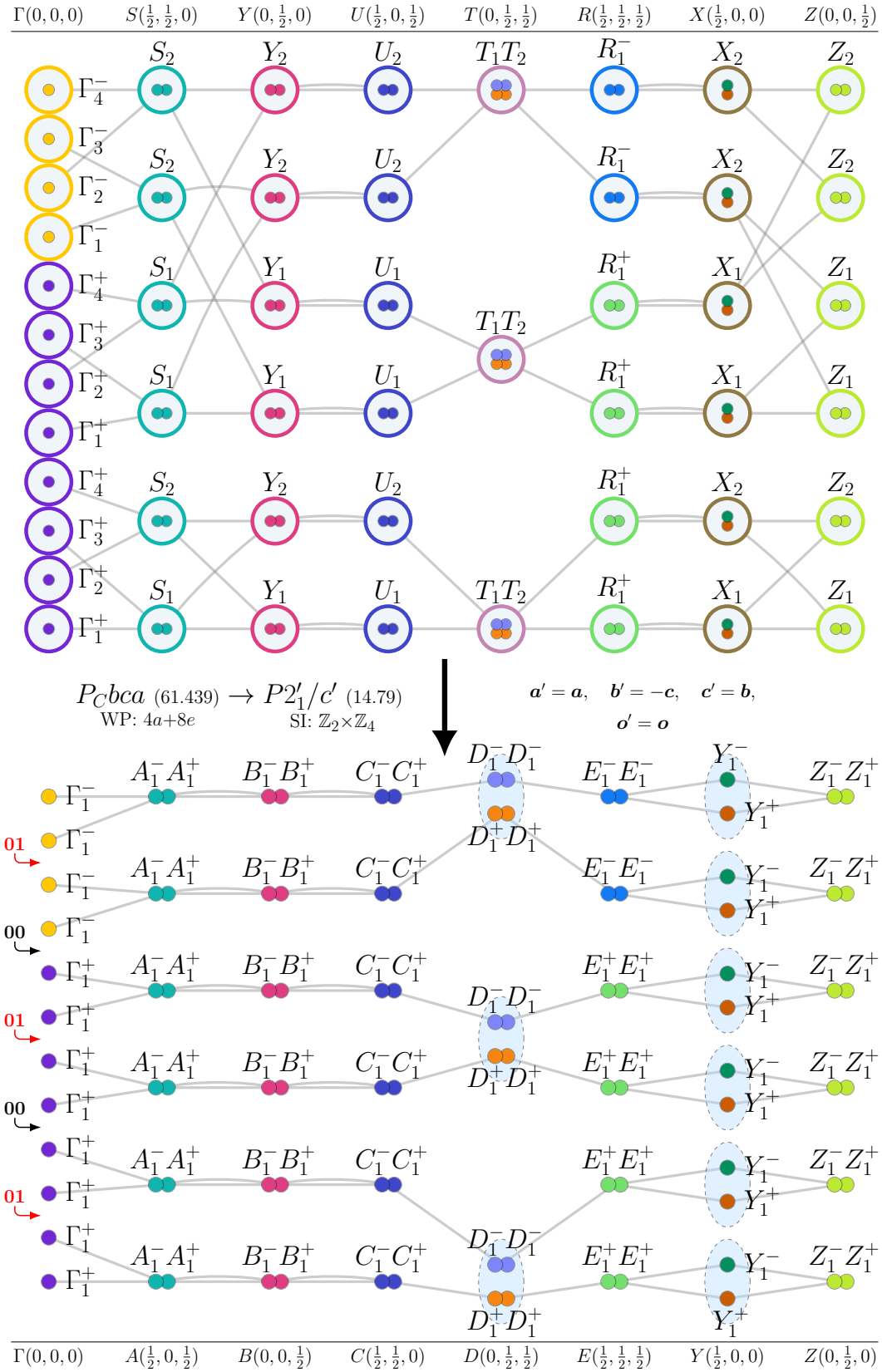
BCS Materials: $EuFe_2As_2$ (19 K)[265].

72.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[265] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.1`



Supplementary Figure 347: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a + 8e$ of supergroup P_Cbca (61.439).

Supplementary Table 683: Number of times a given SI can appear in the band structure. Supergroup: P_Cbca (61.439), WP: $4a$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

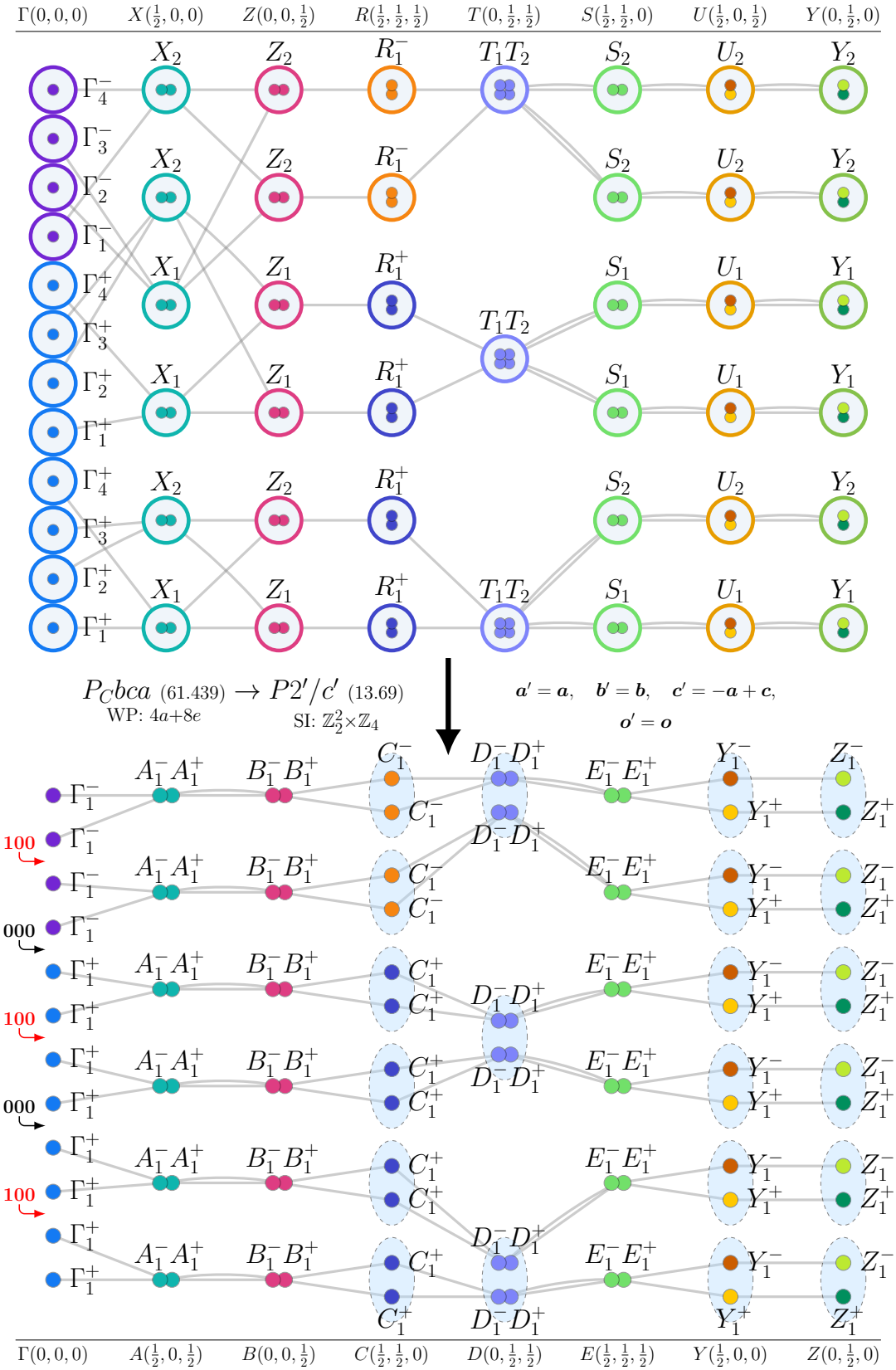
Supplementary Table 684: Possible SI values in each gap. Supergroup: P_Cbca (61.439), WP: $4a$+$8e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

72.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 348: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff positions $4a + 8e$ of supergroup P_Cbca (61.439).

Supplementary Table 685: Number of times a given SI can appear in the band structure. Supergroup: P_Cbca (61.439), WP: $4a$+$8e$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0, 1, 2
002	0, 1, 2
011	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2, 3
102	0, 1, 2, 3
111	0, 1, 2
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

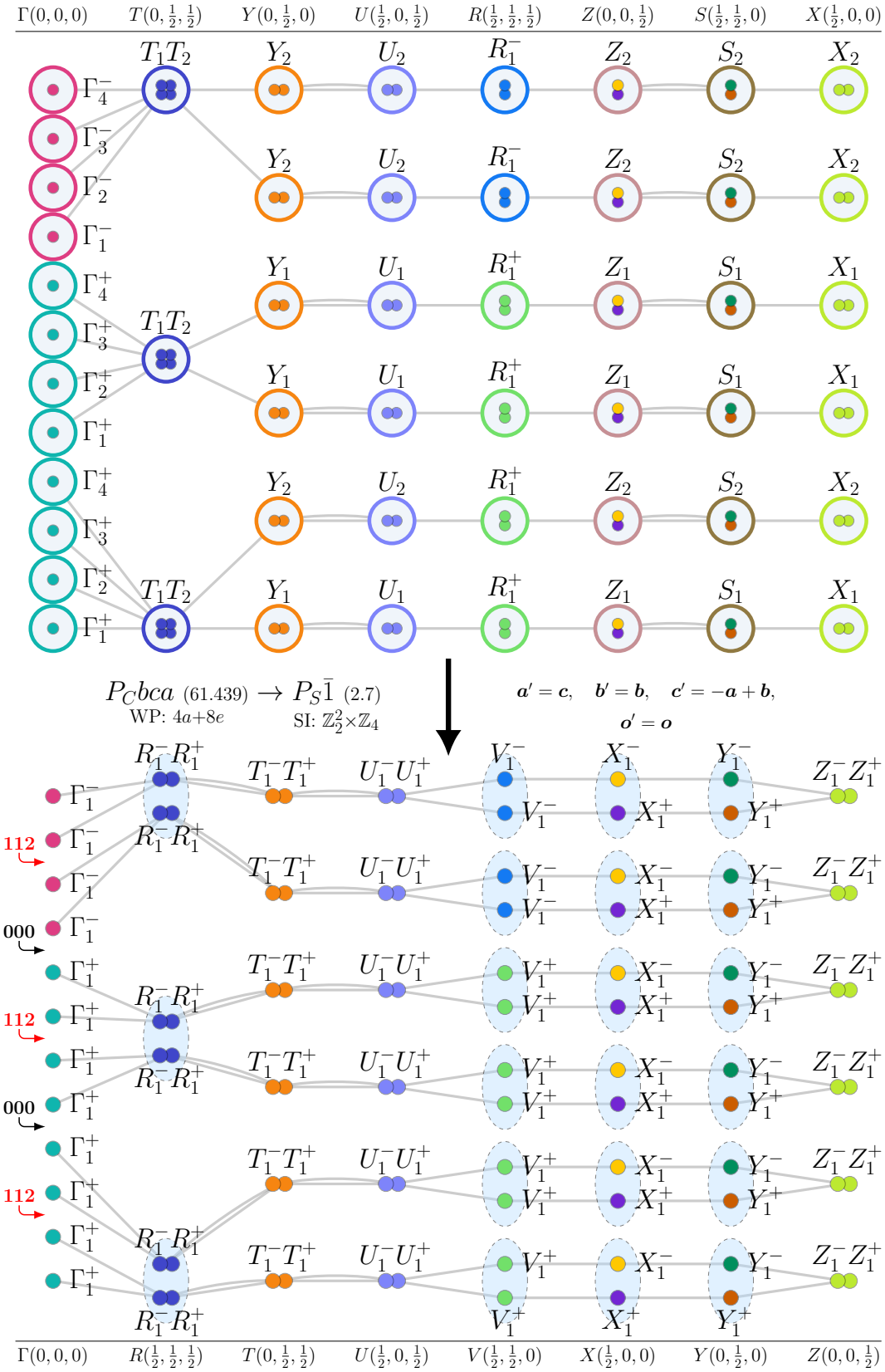
Supplementary Table 686: Possible SI values in each gap. Supergroup: P_Cbca (61.439), WP: $4a$+$8e$, Subgroup: $P2'/c'$ (13.69).

Gap#	Possible SI Values
1	G
2	011, 013, 100, 102
3	G
4	000, 002, 111, 113
5	G
6	011, 013, 100, 102
7	G
8	000, 002, 111, 113
9	G
10	011, 013, 100, 102
11	G
12	000

72.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 349: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4a+8e$ of supergroup P_Cbca (61.439).

Supplementary Table 687: Number of times a given SI can appear in the band structure. Supergroup: P_Cbca (61.439), WP: $4a$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 688: Possible SI values in each gap. Supergroup: P_Cbca (61.439), WP: $4a$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

73 MSG $Pnma$ (62.441)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1/c$ (14.75), $P2_1/m$ (11.50), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pc (7.24), Pm (6.18), Pc (7.24), $P2_1$ (4.7), $Pmn2_1$ (31.123), $P2_1$ (4.7), $Pna2_1$ (33.144), $P2_1$ (4.7), $Pmc2_1$ (26.66).

73.1 WP: $4b$

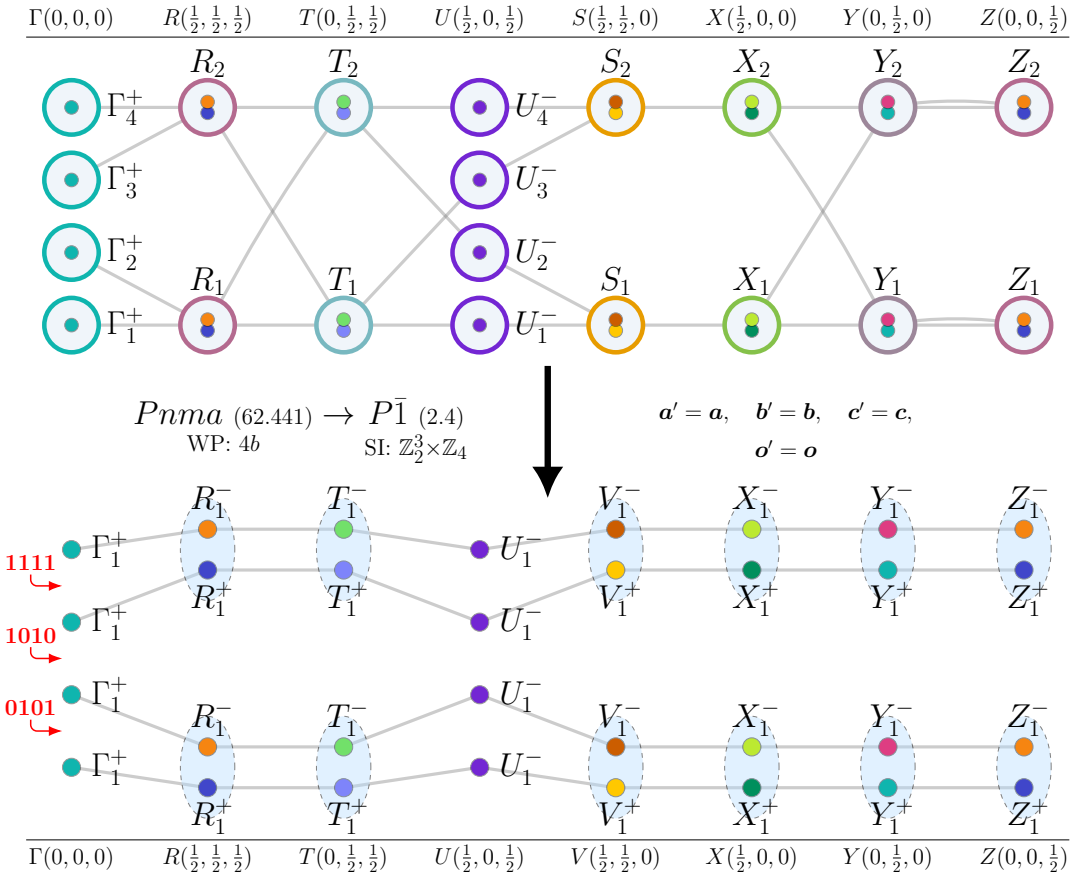
BCS Materials: $La_{0.75}Bi_{0.25}Fe_{0.5}Cr_{0.5}O_3$ (350 K)[266], $LaCrO_3$ (290 K)[267], $CeFeO_3$ (220 K)[268], $InCrO_3$ (93 K)[269], $TlCrO_3$ (89 K)[270], $ScCrO_3$ (73 K)[271].

[266]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.373`
[267]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.323`
[268]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.758`
[269]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.308`
[270]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.309`
[271]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.307`

73.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ⊥ [010],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [100],
- B ∥ [010] and strain ⊥ [001],
- B ∥ [001] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [010],
- B in generic direction.



Supplementary Figure 350: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $Pnma$ (62.441).

Supplementary Table 689: Number of times a given SI can appear in the band structure. Supergroup: $Pnma$ (62.441), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2

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...continued

SI	Possible #Gaps
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	1, 2, 3
1011	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 690: Possible SI values in each gap. Supergroup: $Pnma$ (62.441), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1010
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

73.2 WP: $4a + 4c$

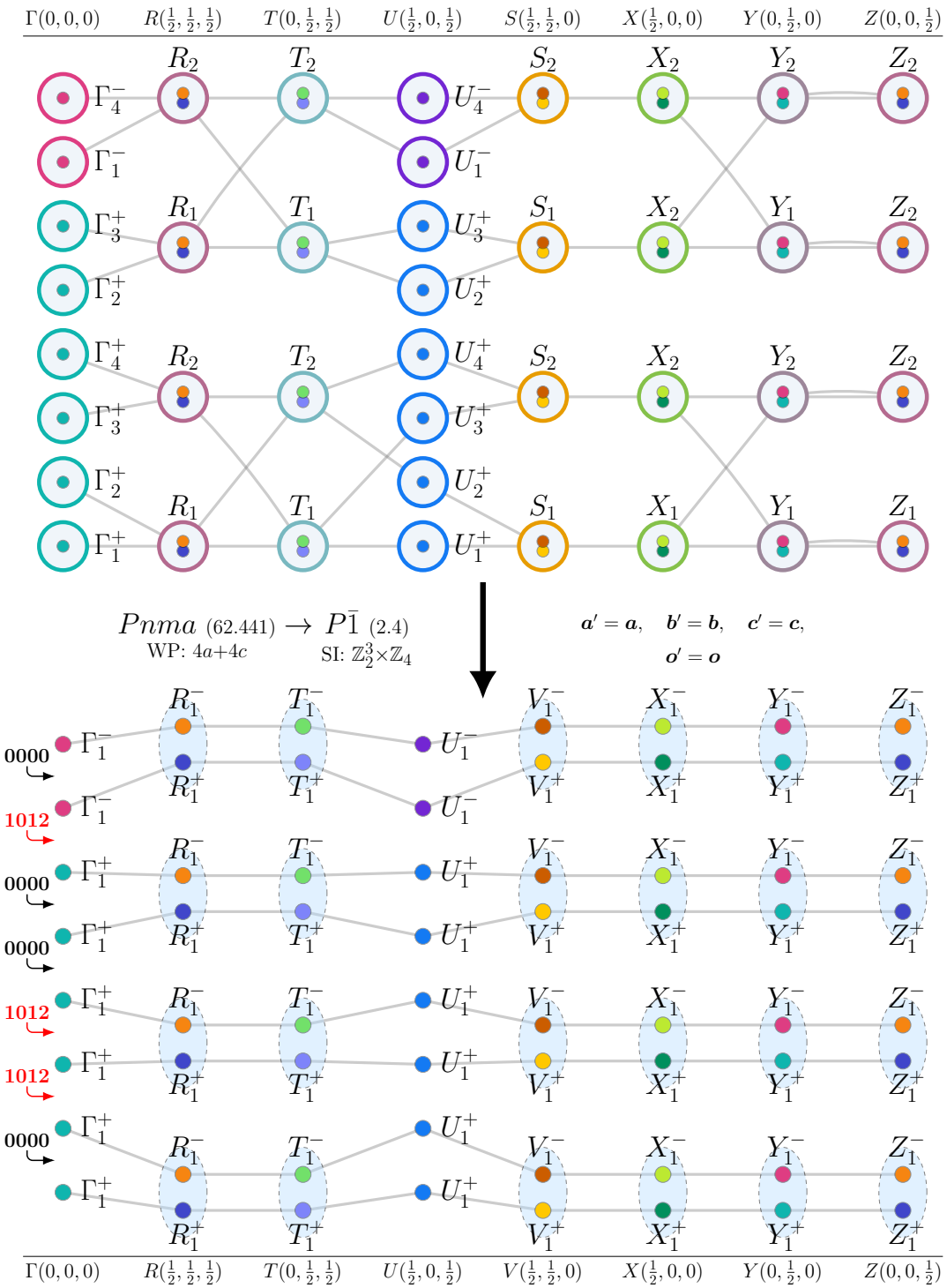
BCS Materials: Fe_2PO_5 (250 K)[272], $CuFePO_5$ (195 K)[273], $NiFePO_5$ (178 K)[274], Fe_2SiO_4 (65.3 K)[275], Co_2SiO_4 (49.5 K)[276], Co_2SiO_4 (49 K)[277], $NH_4Fe_2F_6$ (19 K)[278], Mn_2GeO_4 (17 K)[279], $RbFe_2F_6$ (16 K)[280].

73.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ⊥ [010],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [100],
- B ∥ [010] and strain ⊥ [001],
- B ∥ [001] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [010],
- B in generic direction.

[272]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.263
[273]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.260
[274]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.261
[275]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.221
[276]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.218
[277]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.219
[278]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.168
[279]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.102
[280]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.192



Supplementary Figure 351: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a + 4c$ of supergroup $Pnma$ (62.441).

Supplementary Table 691: Number of times a given SI can appear in the band structure. Supergroup: $Pnma$ (62.441), WP: $4a$+$4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2, 3, 4, 5, 6
0001	0, 1, 2, 3, 4
0002	0, 1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4

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SI	Possible #Gaps
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4, 5, 6
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4, 5, 6
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4, 5, 6
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4, 5, 6, 7
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	0, 1, 2, 3, 4, 5, 6
1112	0, 1, 2, 3, 4
1113	0, 1, 2, 3, 4, 5, 6
gapless	0
nontrivial	1, 2, 3, 4, 5, 6, 7

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Supplementary Table 692: Possible SI values in each gap. Supergroup: $Pnma$ (62.441), WP: $4a$+$4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0000, 0101, 0103, 1010, 1012 1111, 1113
3	0000, 0001, 0002, 0003, 0010 0011, 0012, 0013, 0100, 0101 0102, 0103, 0110, 0111, 0112 0113, 1000, 1001, 1002, 1003 1010, 1011, 1012, 1013, 1100 1101, 1102, 1103, 1110, 1111 1112, 1113
4	0000, 0002, 0101, 0103, 1012 1111, 1113

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Gap#	Possible SI Values
5	0000, 0001, 0002, 0003, 0010 0011, 0012, 0013, 0100, 0101 0102, 0103, 0110, 0111, 0112 0113, 1000, 1001, 1002, 1003 1010, 1011, 1012, 1013, 1100 1101, 1102, 1103, 1110, 1111 1112, 1113
6	0000, 0101, 0103, 1010, 1012 1111, 1113
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
8	0000

73.3 WP: $4b + 4c$

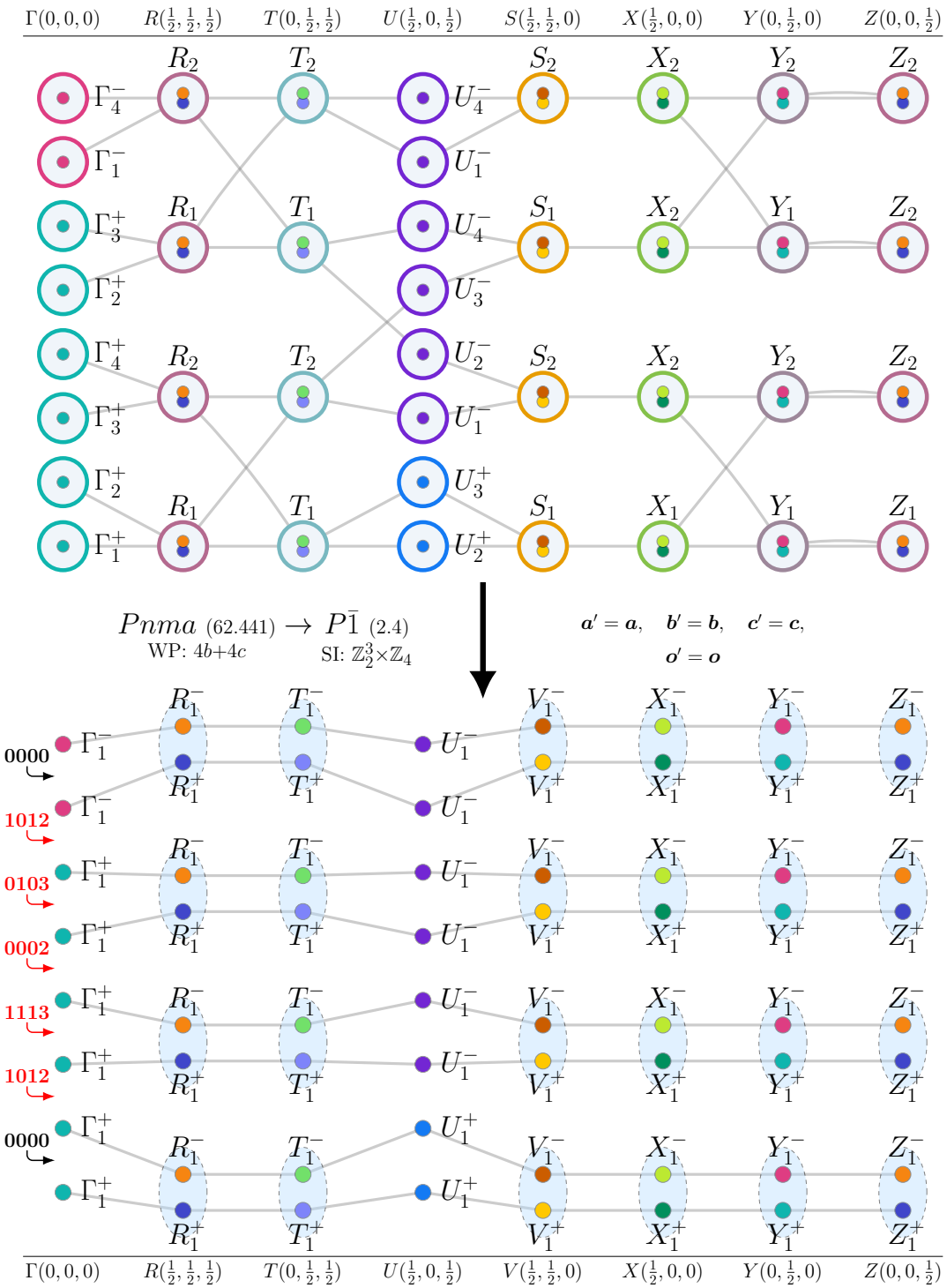
BCS Materials: $CeFeO_3$ (220 K)[281], $Rb_2Fe_2O(AsO_4)_2$ (25 K)[282], $SmCrO_3$ (10 K)[283], $Nd_{0.85}Sr_{0.15}CrO_3$[284], $Nd_{0.9}Sr_{0.1}CrO_3$[285], $Nd_{0.95}Sr_{0.05}CrO_3$[286].

73.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ⊥ [010],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [100],
- B ∥ [010] and strain ⊥ [001],
- B ∥ [001] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [010],
- B in generic direction.

[281] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.759
[282] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.90
[283] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.478
[284] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.678
[285] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.677
[286] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.676



Supplementary Figure 352: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b + 4c$ of supergroup $Pnma$ (62.441).

Supplementary Table 693: Number of times a given SI can appear in the band structure. Supergroup: $Pnma$ (62.441), WP: $4b$+$4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2, 3, 4, 5, 6
0001	0, 1, 2, 3, 4
0002	0, 1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4

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SI	Possible #Gaps
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4, 5, 6
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4, 5, 6
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4, 5, 6, 7
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4, 5, 6
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	0, 1, 2, 3, 4, 5, 6
1112	0, 1, 2, 3, 4
1113	0, 1, 2, 3, 4, 5, 6
gapless	0
nontrivial	1, 2, 3, 4, 5, 6, 7

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Supplementary Table 694: Possible SI values in each gap. Supergroup: $Pnma$ (62.441), WP: $4b$+$4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0000, 0101, 0103, 1010, 1012 1111, 1113
3	0000, 0001, 0002, 0003, 0010 0011, 0012, 0013, 0100, 0101 0102, 0103, 0110, 0111, 0112 0113, 1000, 1001, 1002, 1003 1010, 1011, 1012, 1013, 1100 1101, 1102, 1103, 1110, 1111 1112, 1113
4	0000, 0002, 0101, 0103, 1010 1111, 1113

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Gap#	Possible SI Values
5	0000, 0001, 0002, 0003, 0010 0011, 0012, 0013, 0100, 0101 0102, 0103, 0110, 0111, 0112 0113, 1000, 1001, 1002, 1003 1010, 1011, 1012, 1013, 1100 1101, 1102, 1103, 1110, 1111 1112, 1113
6	0000, 0101, 0103, 1010, 1012 1111, 1113
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
8	0000

73.4 WP: $4a$

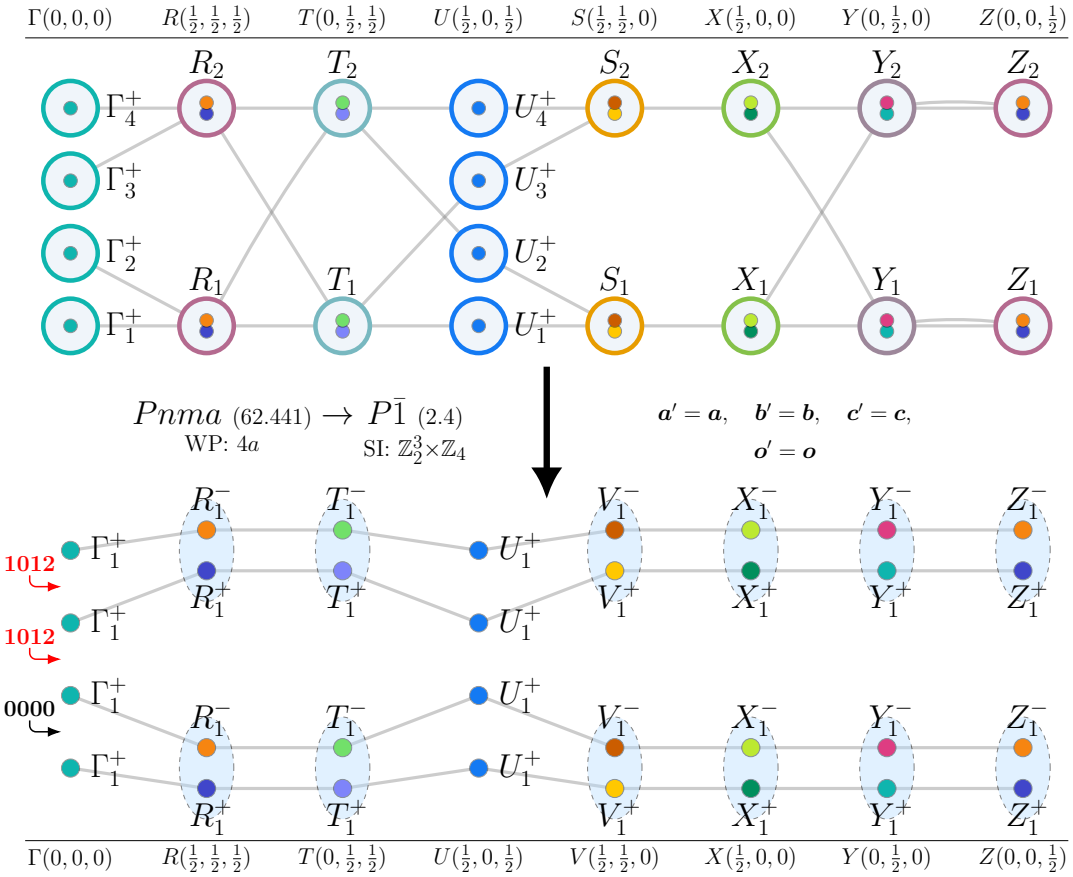
BCS Materials: $CoSO_4$ (12 K)[287], $CoSO_4$[288].

73.4.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B ∥ [100] and strain ⊥ [010],
- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [100],
- B ∥ [010] and strain ⊥ [001],
- B ∥ [001] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [010],
- B in generic direction.

[287] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.571`
[288] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.96`



Supplementary Figure 353: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $Pnma$ (62.441).

Supplementary Table 695: Number of times a given SI can appear in the band structure. Supergroup: $Pnma$ (62.441), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1011	0, 1, 2
1012	1, 2, 3
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2

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SI	Possible #Gaps
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

Supplementary Table 696: Possible SI values in each gap. Supergroup: $Pnma$ (62.441), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1012
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

74 MSG $Pn'm'a$ (62.446)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/m'$ (11.54), $P2'_1/c'$ (14.79), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pm' (6.20), Pc' (7.26), $P2'_1$ (4.9), $P2'_1$ (4.9), Pc (7.24), $Pn'a2'_1$ (33.146), $Pm'c2'_1$ (26.68), $P2_1$ (4.7), $Pm'n'2_1$ (31.127).

74.1 WP: $4b$

BCS Materials: $SmFeO_3$ (480 K)[289], $SrRuO_3$ (165 K)[290], $TbCrO_3$ (158 K)[291], $DyCrO_3$ (146 K)[292], $TeNiO_3$ (130 K)[293], $Ca_2PrCr_2TaO_9$ (130 K)[294], $Ca_2PrCr_2NbO_9$ (110 K)[295], YVO_3 (77 K)[296].

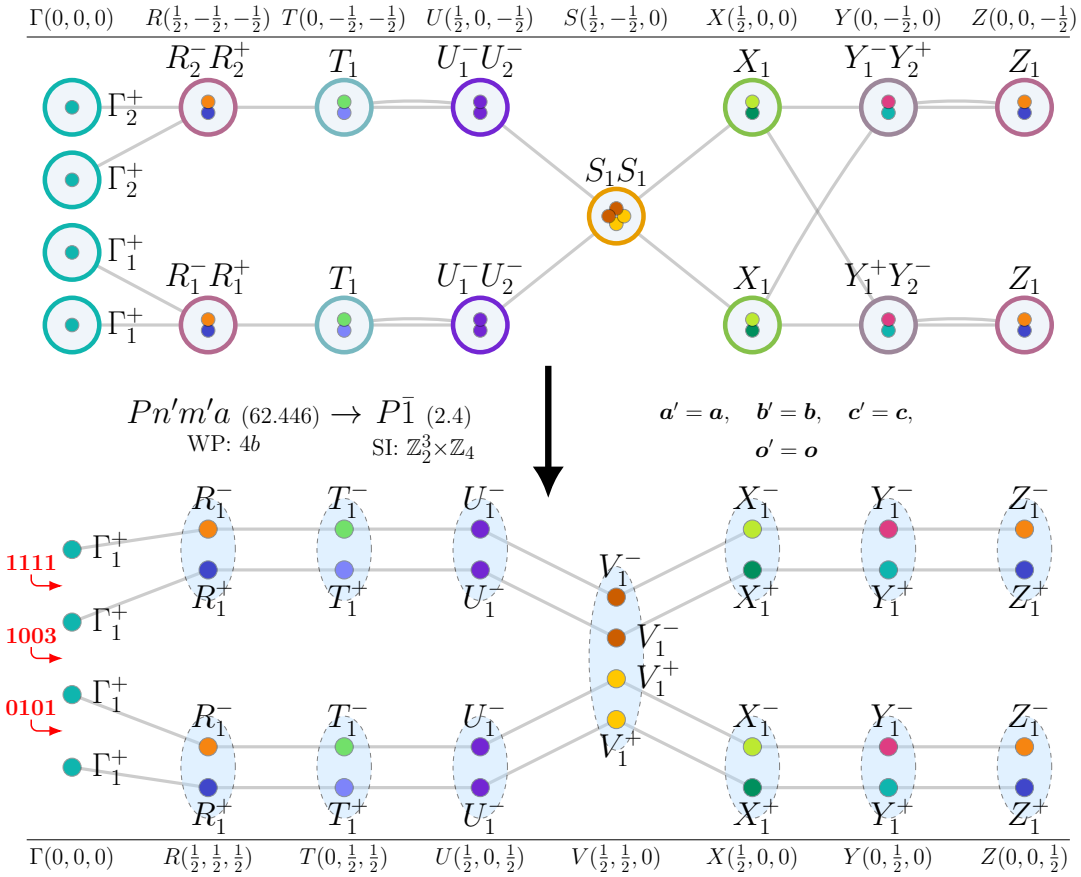
74.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,

[289]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.379
[290]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.732
[291]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.354
[292]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.592
[293]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.94
[294]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.202
[295]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.201
[296]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.787

- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [100],
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [001],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [010],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 354: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 697: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2, 3

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SI	Possible #Gaps
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

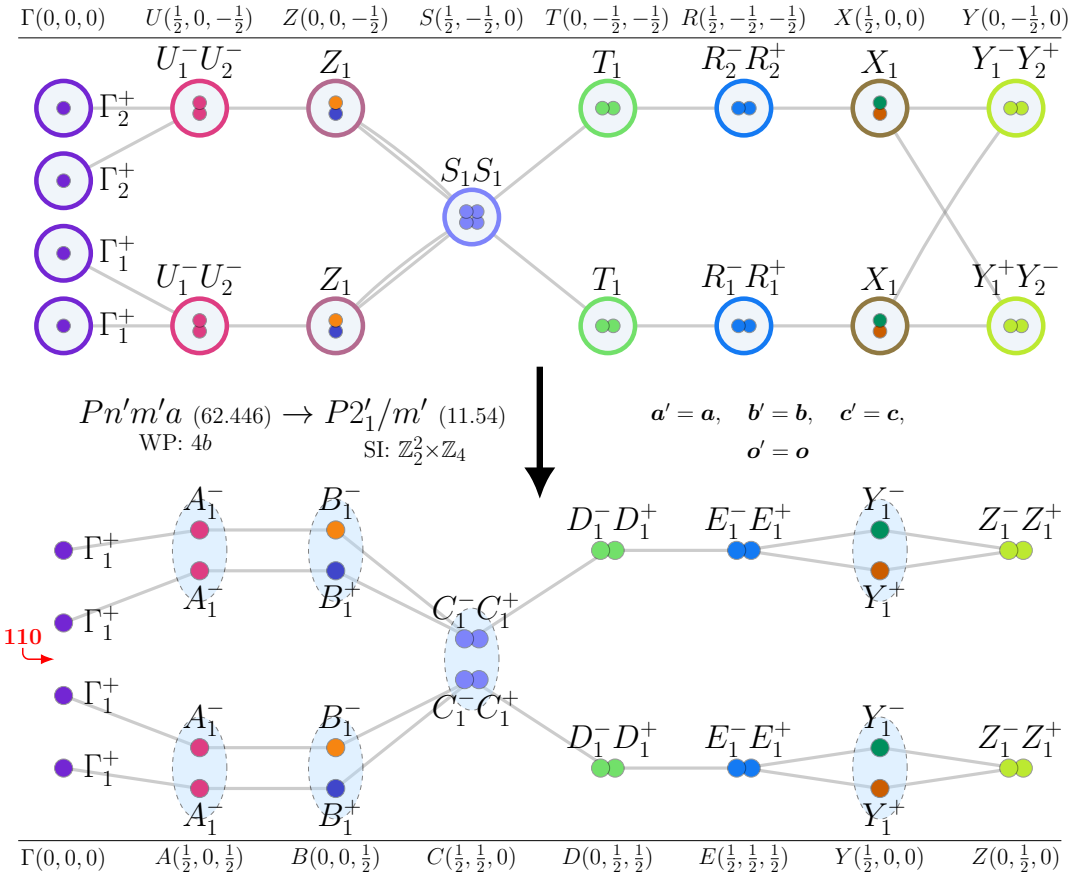
Supplementary Table 698: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1001, 1003, 1010
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

74.1.2 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 355: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 699: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

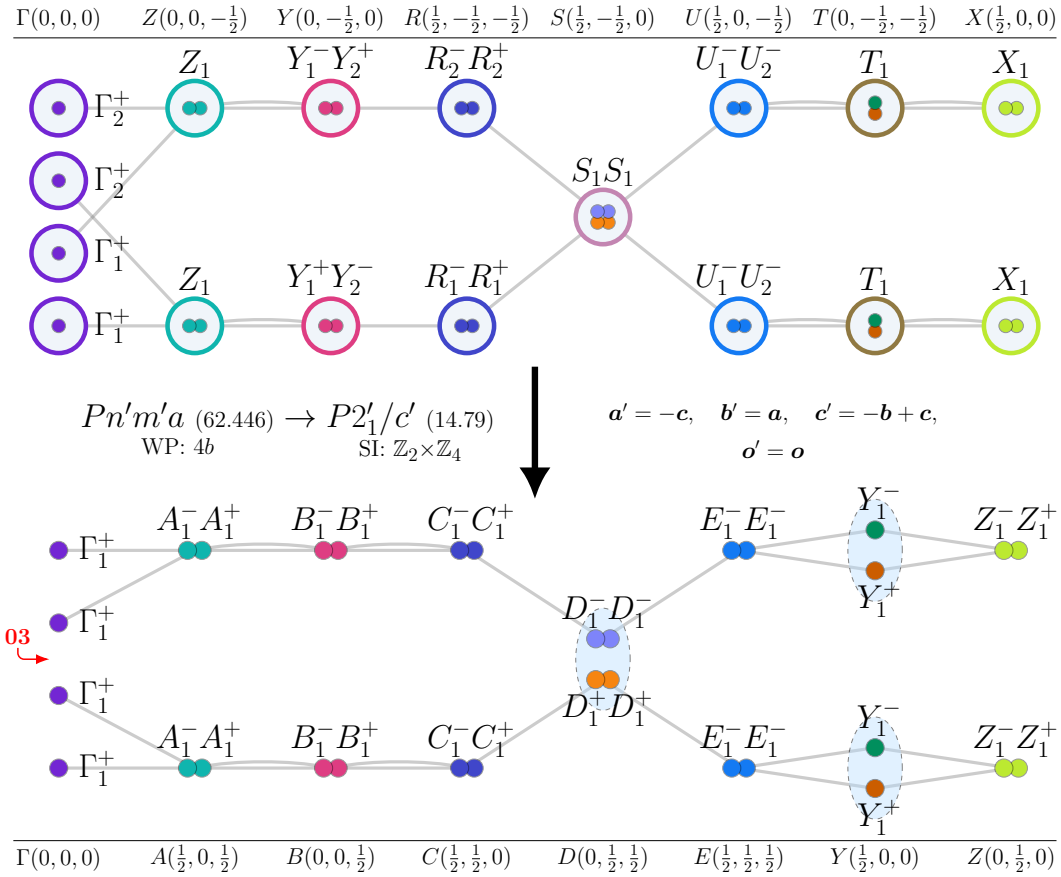
Supplementary Table 700: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

74.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain ⊥ [100],
- (B ∥ [010] or B ⊥ [100]).



Supplementary Figure 356: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 701: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 702: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

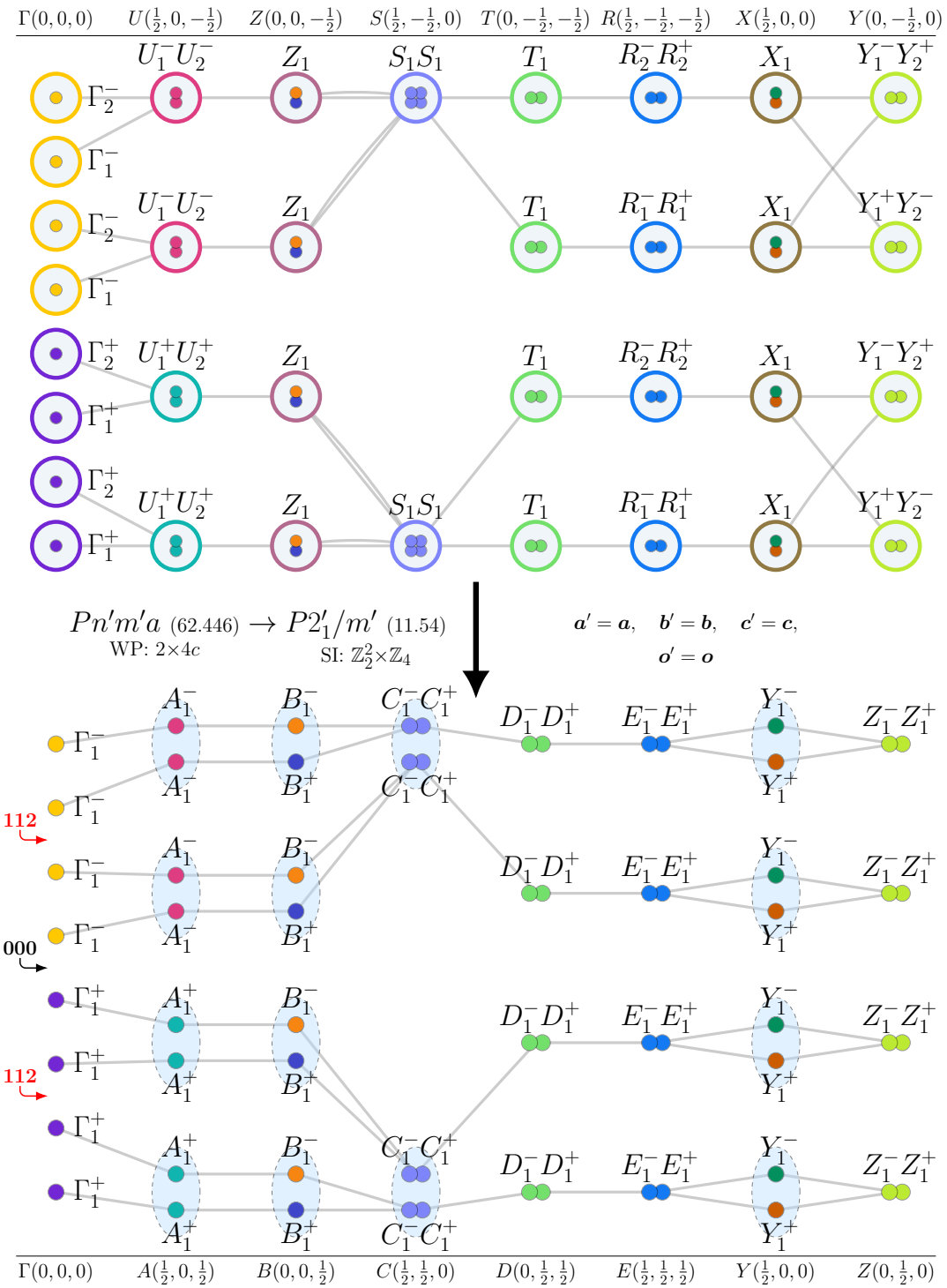
74.2 WP: $4c + 4c$

BCS Materials: MnCoGe (345 K)[297].

74.2.1 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 357: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4c + 4c$ of supergroup $Pn'm'a$ (62.446).

[297]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.445`

Supplementary Table 703: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4c$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

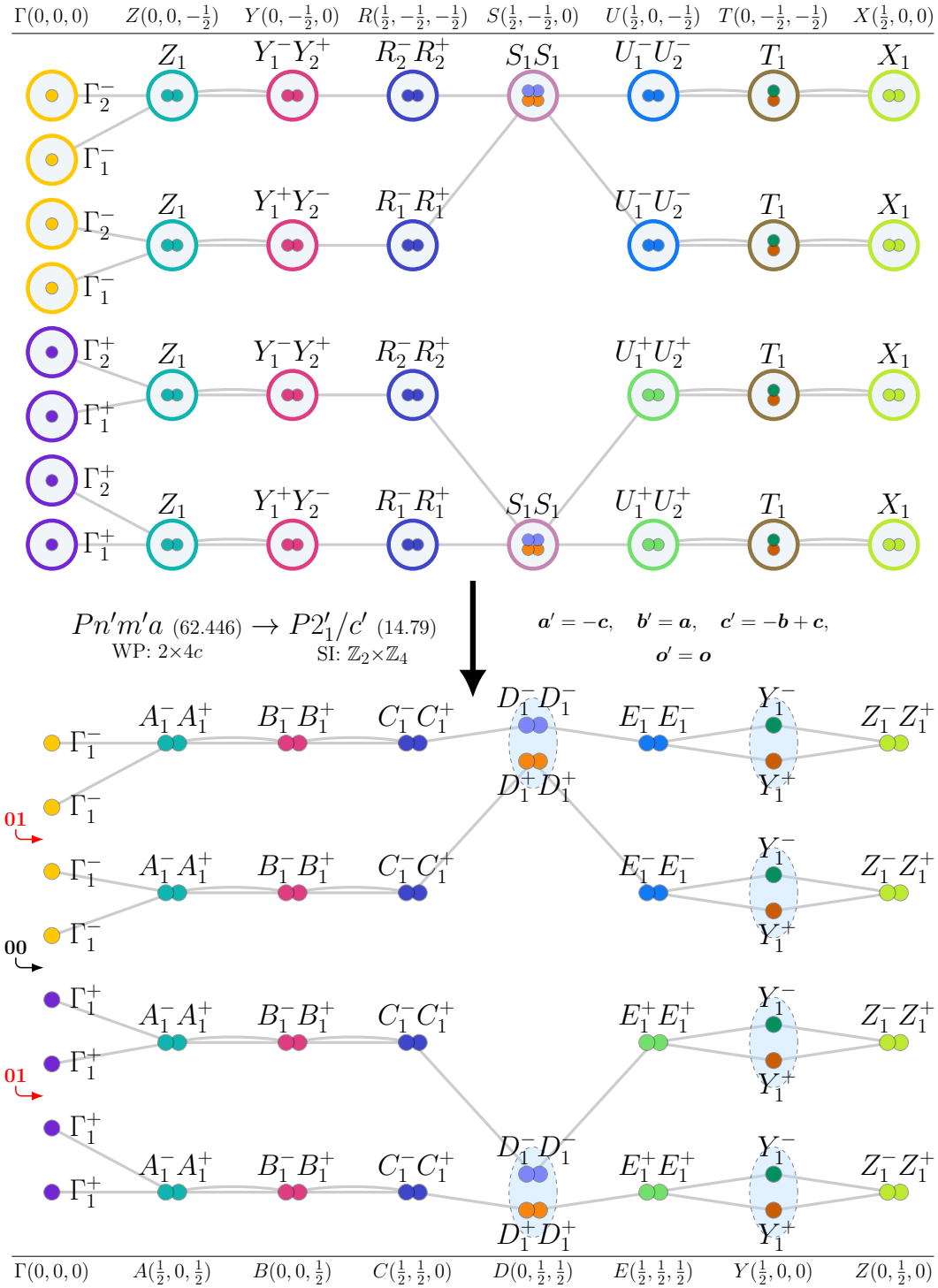
Supplementary Table 704: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4c$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

74.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 358: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 705: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

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...continued

SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 706: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

74.3 WP: $4b + 4b$

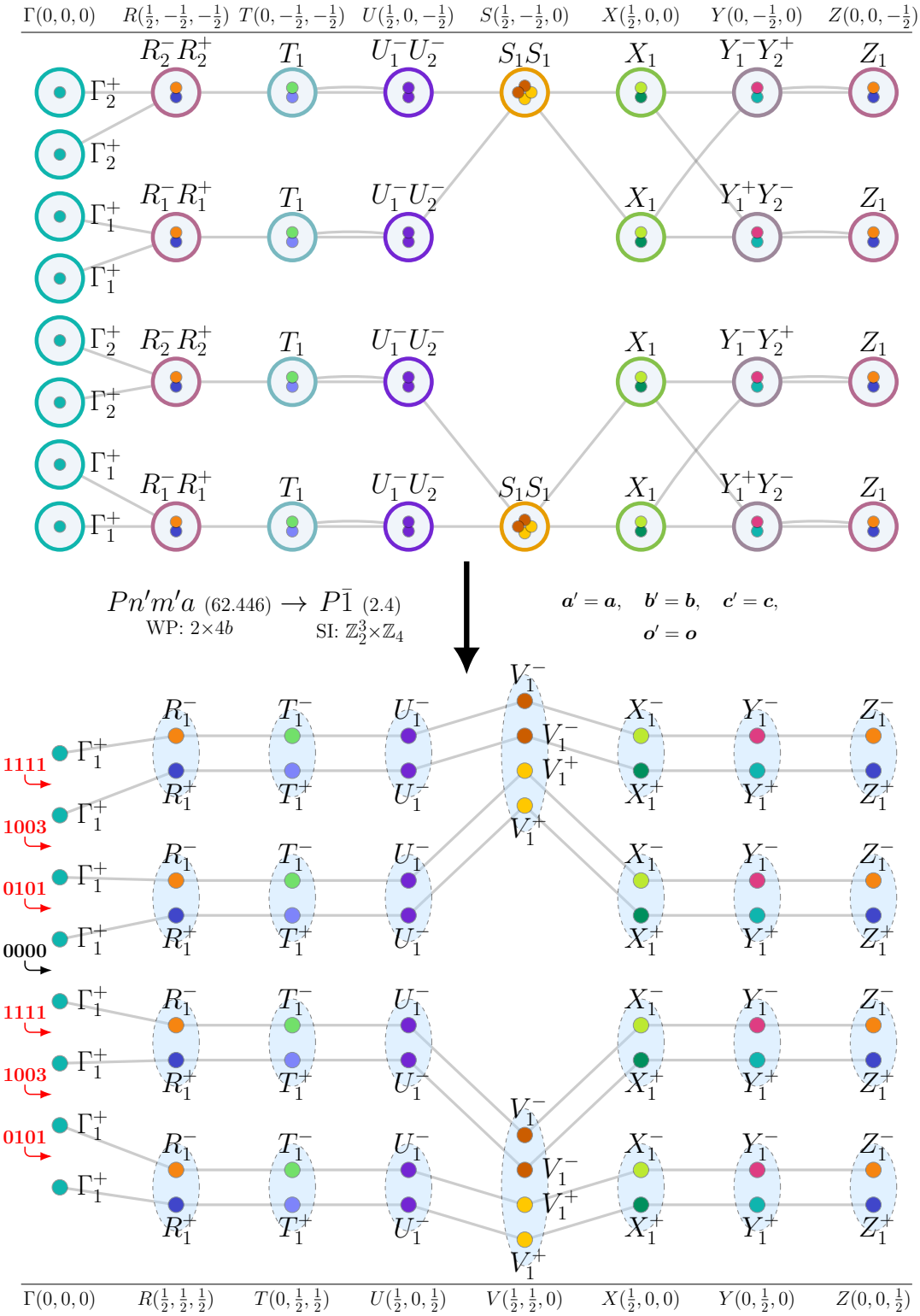
BCS Materials: $Bi_{0.85}Ca_{0.15}Fe_{0.55}Mn_{0.45}O_3$ (268 K)[298].

74.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[298] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.820`



Supplementary Figure 359: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b + 4b$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 707: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	1, 2, 3, 4, 5
0001	0, 1, 2, 3, 4
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4

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SI	Possible #Gaps
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2
0102	0, 1, 2, 3, 4
0103	0, 1, 2
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4
1001	0, 1, 2, 3, 4, 5, 6
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4, 5, 6
1010	0, 1, 2, 3, 4, 5, 6
1011	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	0, 1, 2
1112	0, 1, 2, 3, 4
1113	0, 1, 2
gapless	0
nontrivial	2, 3, 4, 5, 6

4441

Supplementary Table 708: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: 4*b*+4*b*, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1001, 1003, 1010
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000
5	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
6	1001, 1003, 1010

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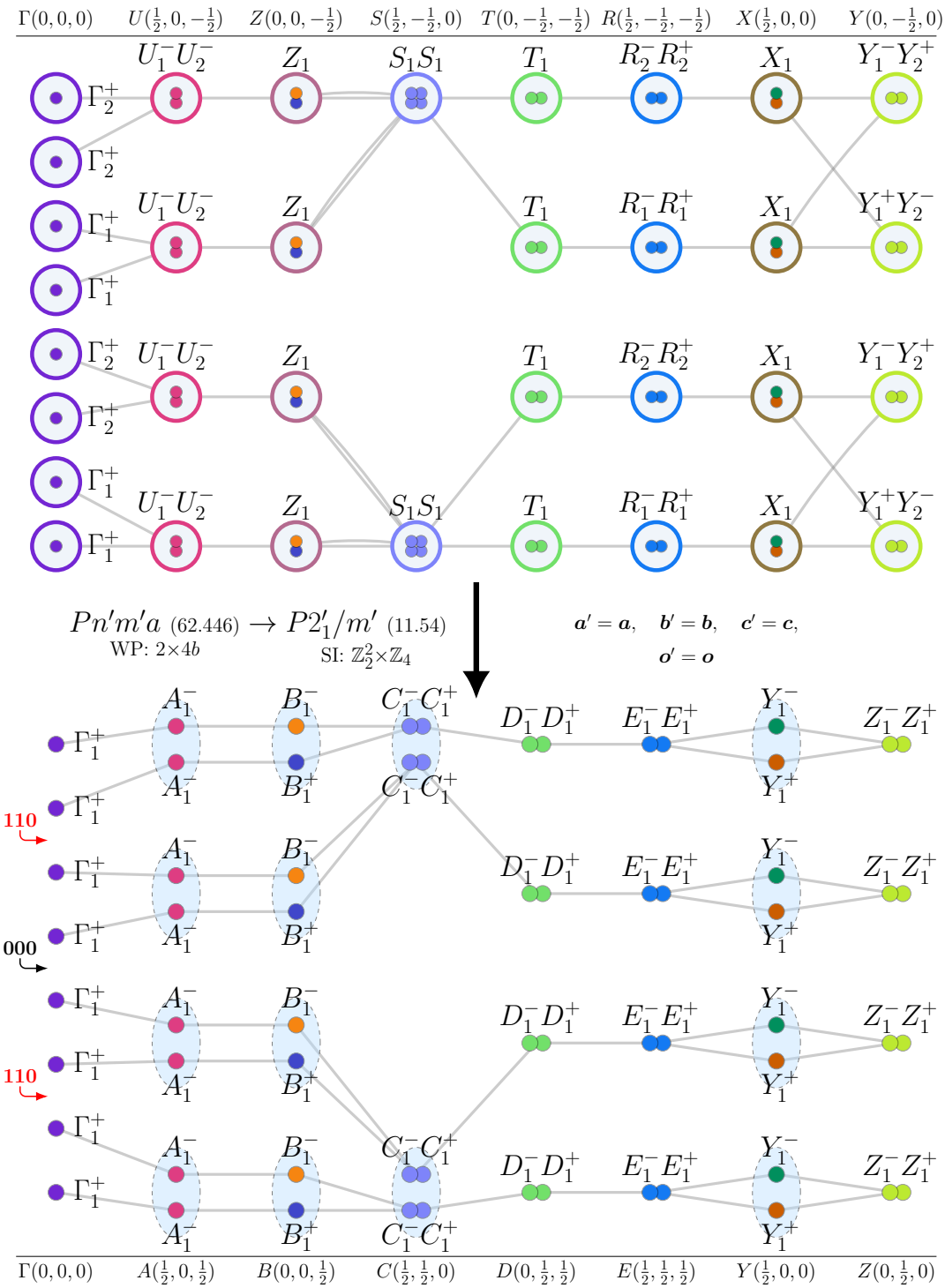
...continued

Gap#	Possible SI Values
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
8	0000

74.3.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 360: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4b+4b$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 709: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4b$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	1
110	2
gapless	4
nontrivial	2

4447

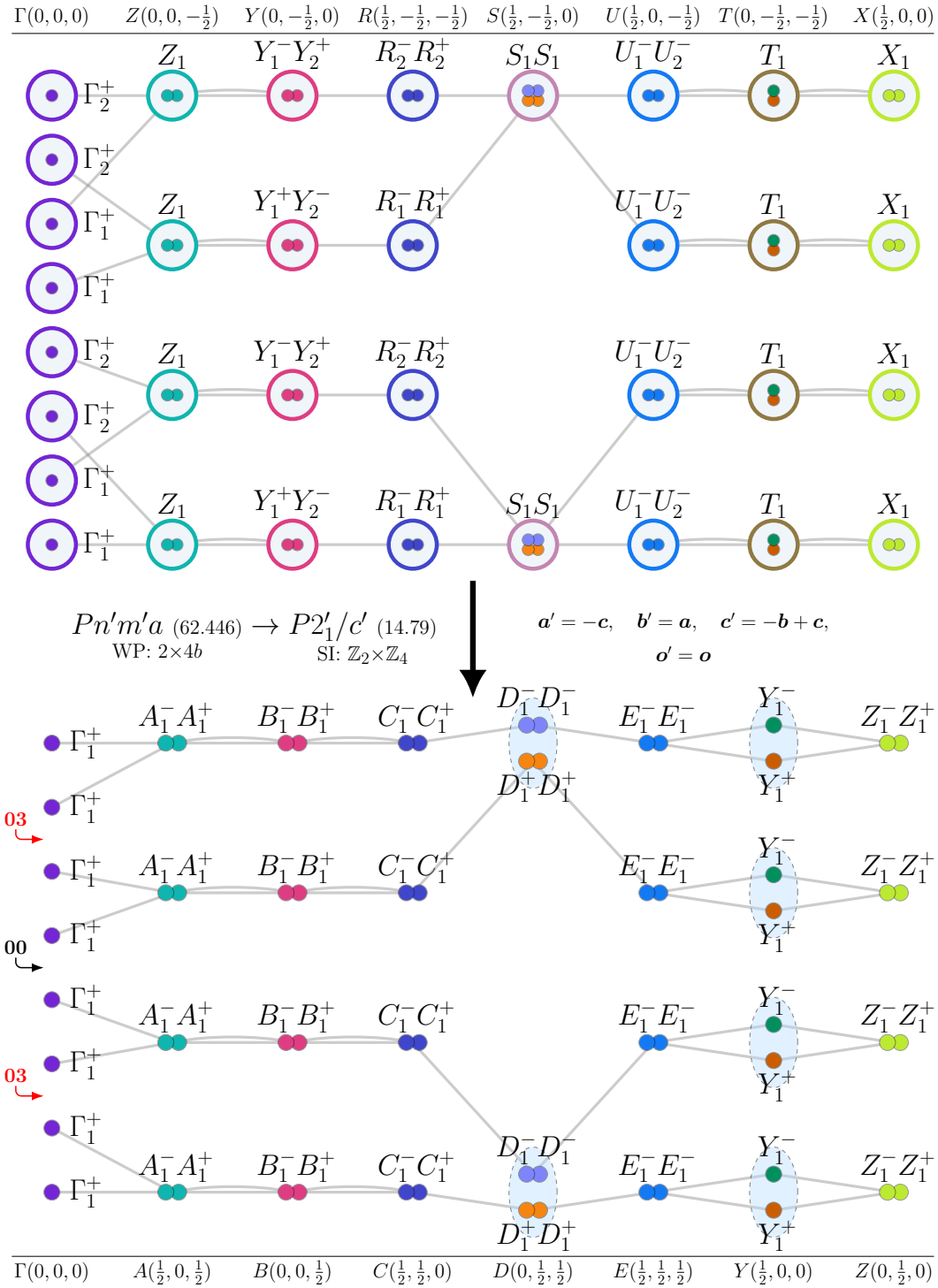
Supplementary Table 710: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4b$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000
5	G
6	110
7	G
8	000

74.3.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 361: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+4b$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 711: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	1
01	0, 1, 2
03	0, 1, 2
gapless	4
nontrivial	2

Supplementary Table 712: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00
5	G
6	01, 03
7	G
8	00

74.4 WP: $4c$

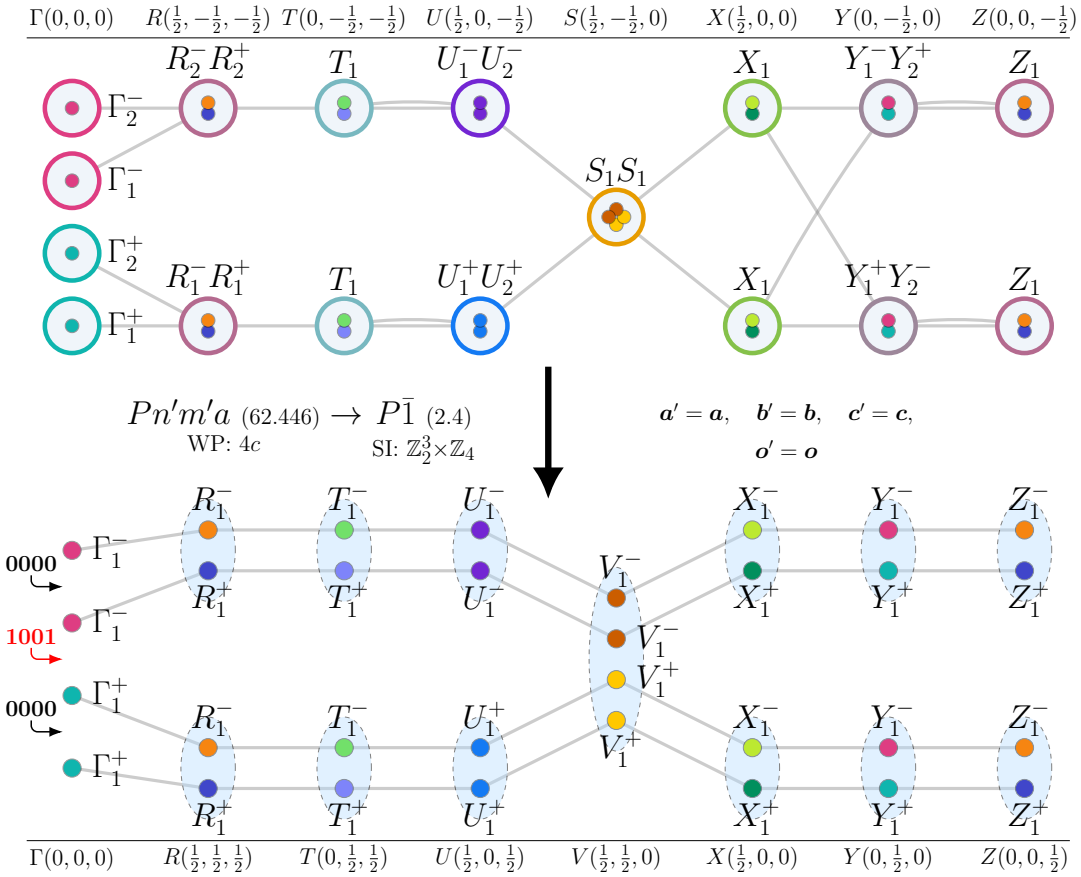
BCS Materials: TbPt (56 K)[299], $TbPt_{0.8}Cu_{0.2}$ (47 K)[300], NdSi (46 K)[301], DyPt (23 K)[302], TmNi (7 K)[303].

74.4.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[299]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.684`
[300]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.248`
[301]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.407`
[302]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.687`
[303]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.409`



Supplementary Figure 362: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 713: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2
0112	0, 1, 2, 3
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2, 3
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2
1012	0, 1, 2, 3
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2

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SI	Possible #Gaps
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	1, 2, 3

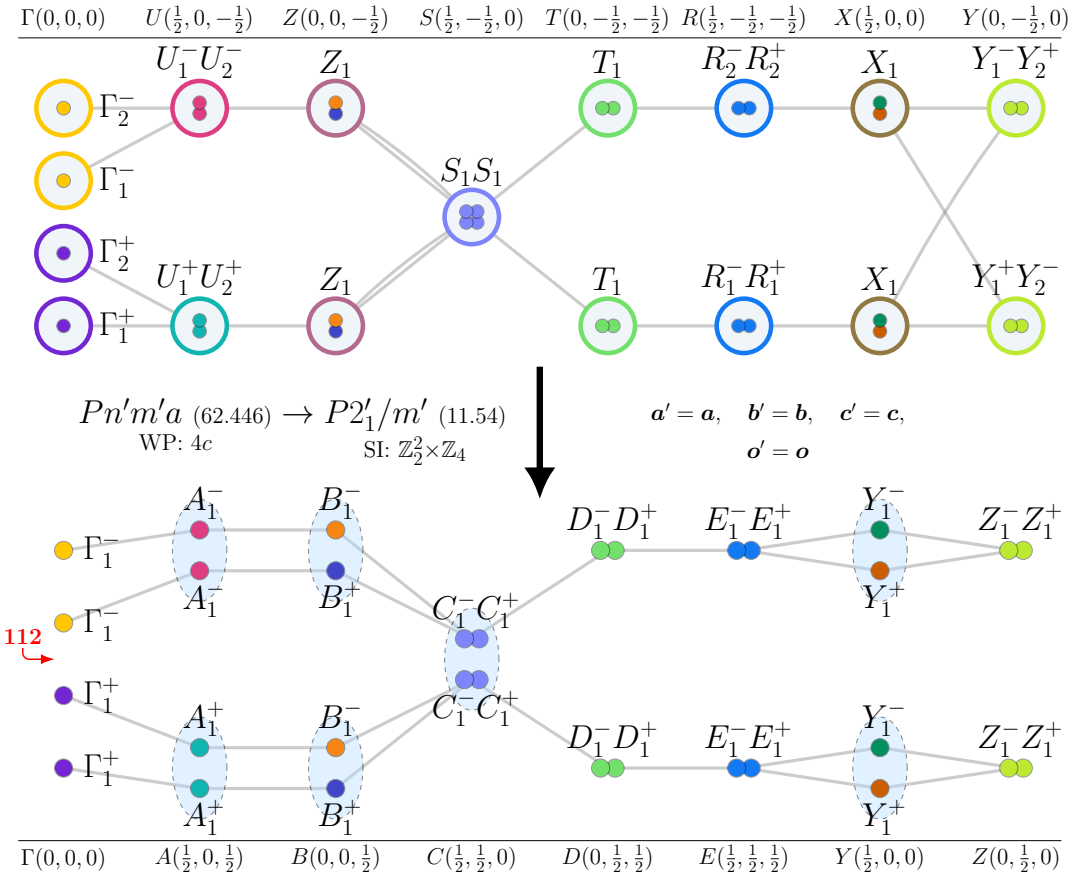
Supplementary Table 714: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0101, 0103, 0110, 0112, 1001 1003, 1010, 1012
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

74.4.2 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 363: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 715: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
001	0, 1
003	0, 1
110	0, 1
112	0, 1
gapless	2
nontrivial	1

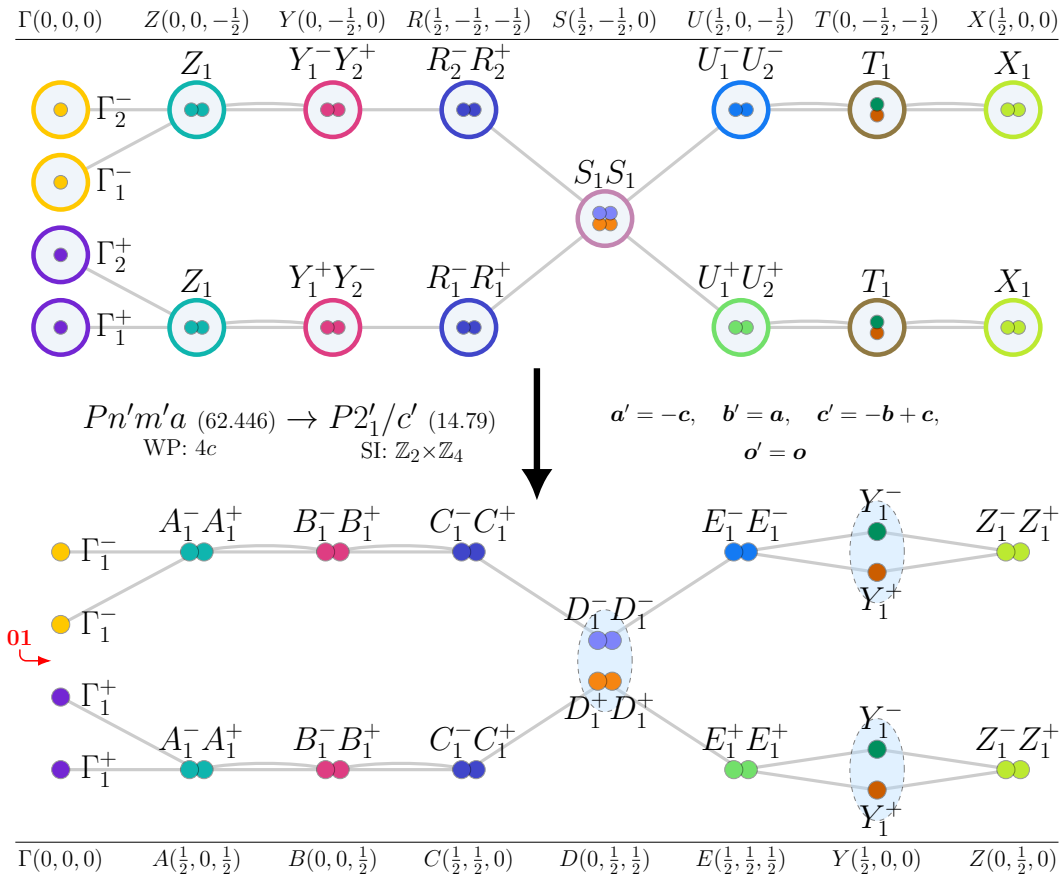
Supplementary Table 716: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000

74.4.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain ⊥ [100],
- (B ∥ [010] or B ⊥ [100]).



Supplementary Figure 364: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 717: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4c$, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

Supplementary Table 718: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4c$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

74.5 WP: $4a + 4c$

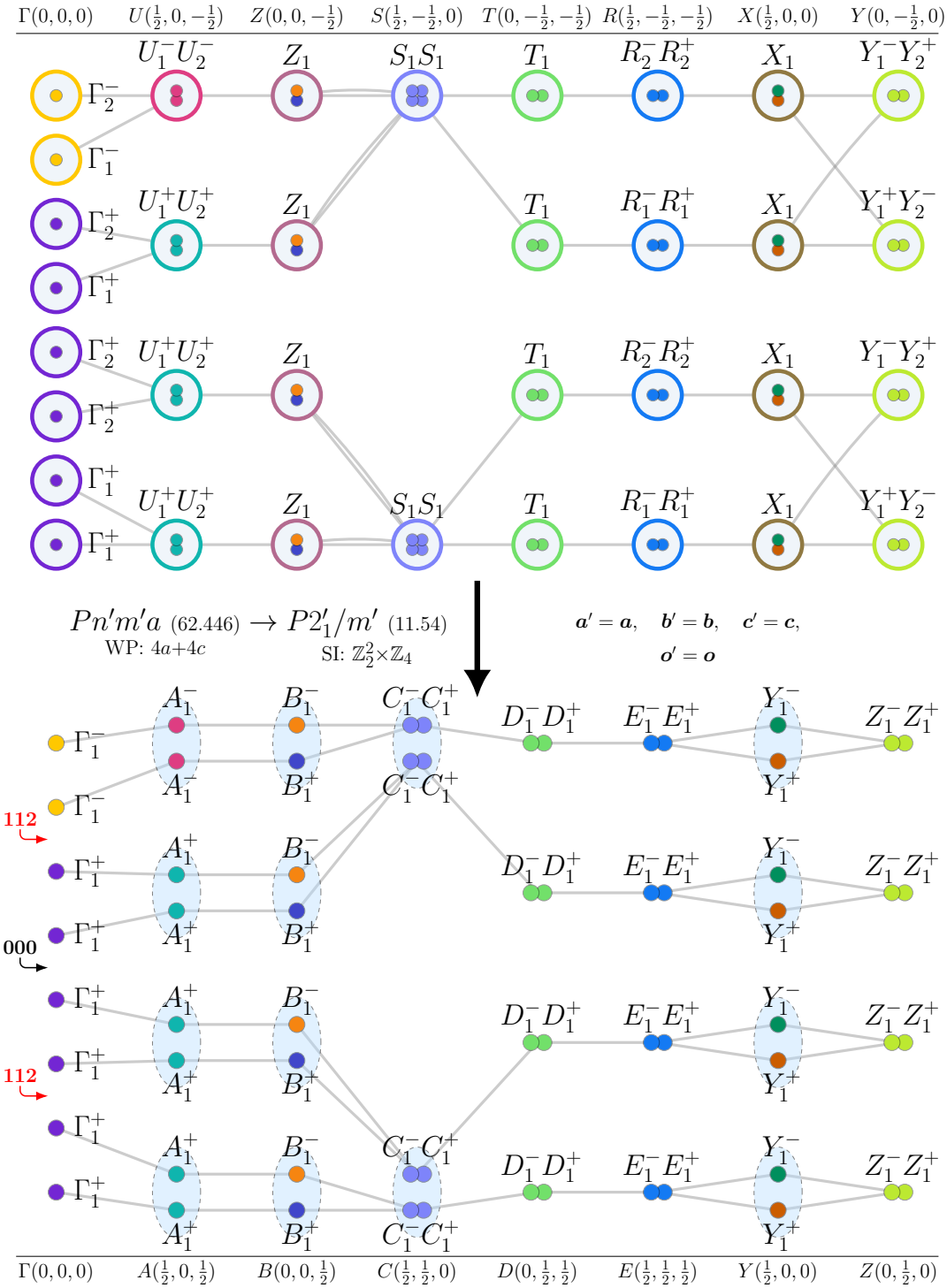
BCS Materials: Mn_2SiO_4 (47.1 K)[304], Mn_2GeO_4 (47 K)[305], $Ca_2Fe_2O_5$[306].

74.5.1 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).

[304]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.220`
[305]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.101`
[306]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.93`



Supplementary Figure 365: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4a+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 719: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1

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SI	Possible #Gaps
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

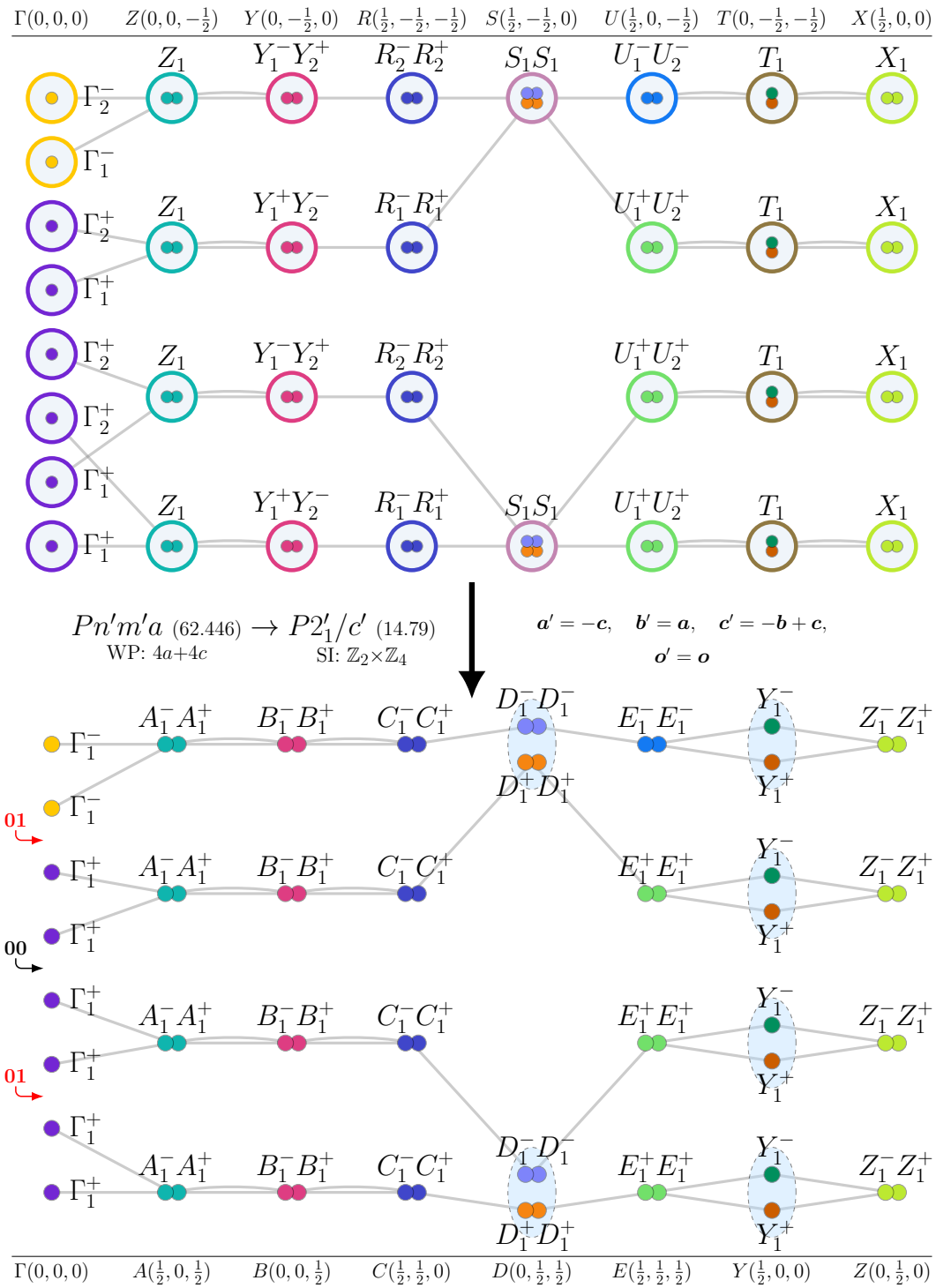
Supplementary Table 720: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

74.5.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 366: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 721: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

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SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 722: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

74.6 WP: $4b + 4c$

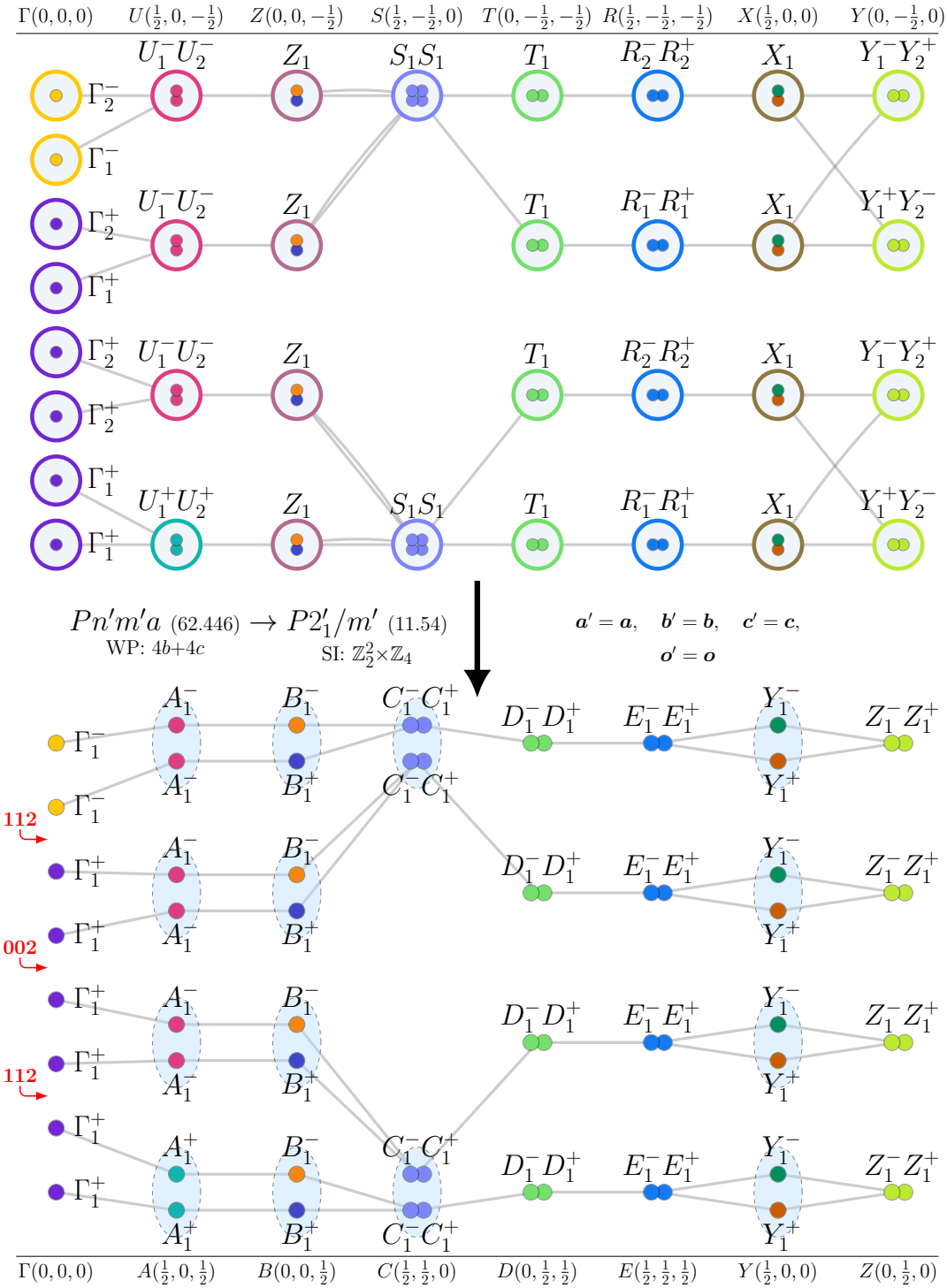
BCS Materials: $SmCrO_3$ (37 K)[307], $SmCrO_3$ (37 K)[308], $TbFeO_3$ (8.4 K)[309].

74.6.1 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).

[307]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.698`
[308]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.697`
[309]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.352`



Supplementary Figure 367: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 723: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1

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SI	Possible #Gaps
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

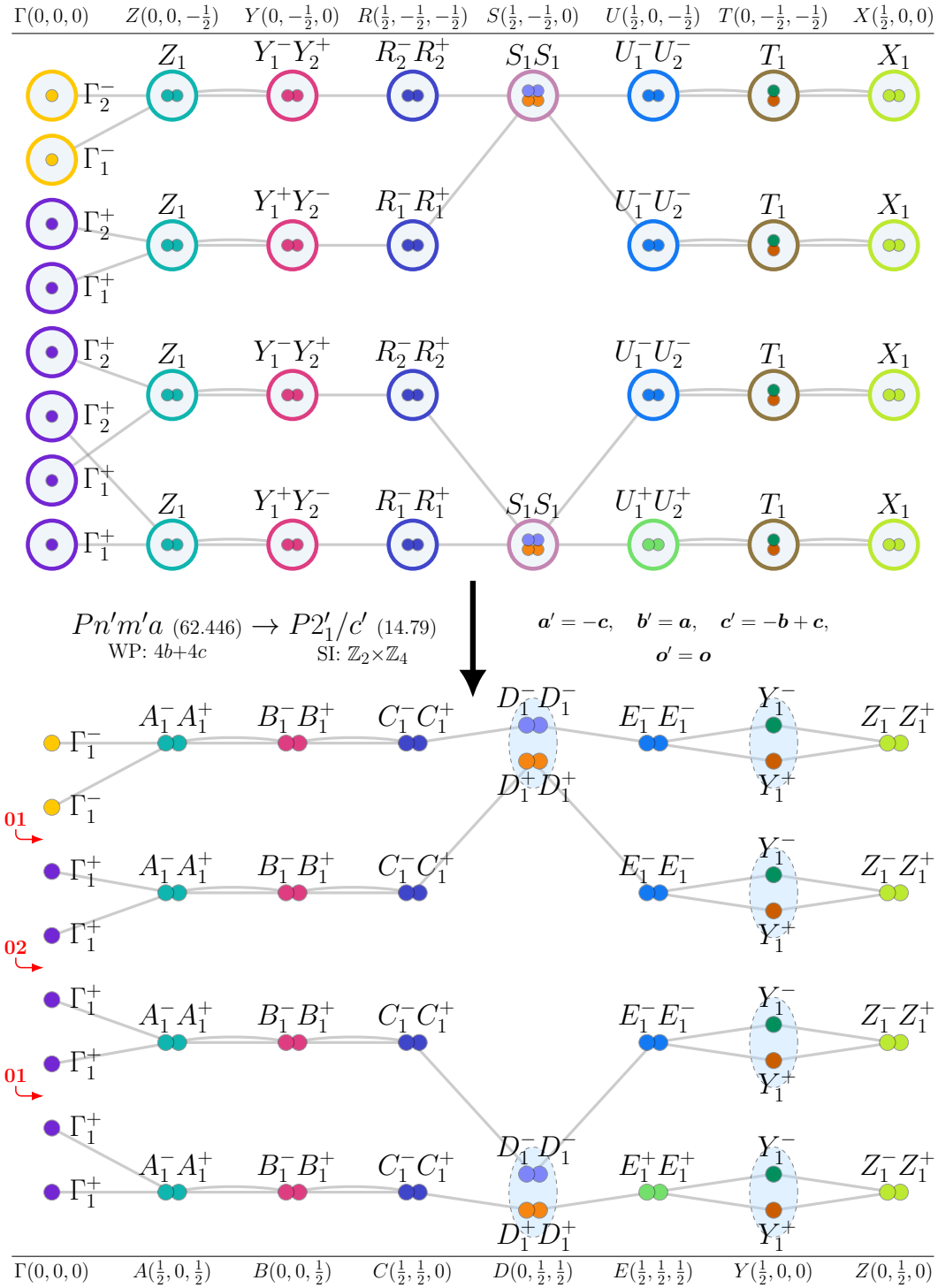
Supplementary Table 724: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

74.6.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 368: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 725: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

continued on next page...

...continued

SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 726: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

74.7 WP: $4a + 4a + 4c$

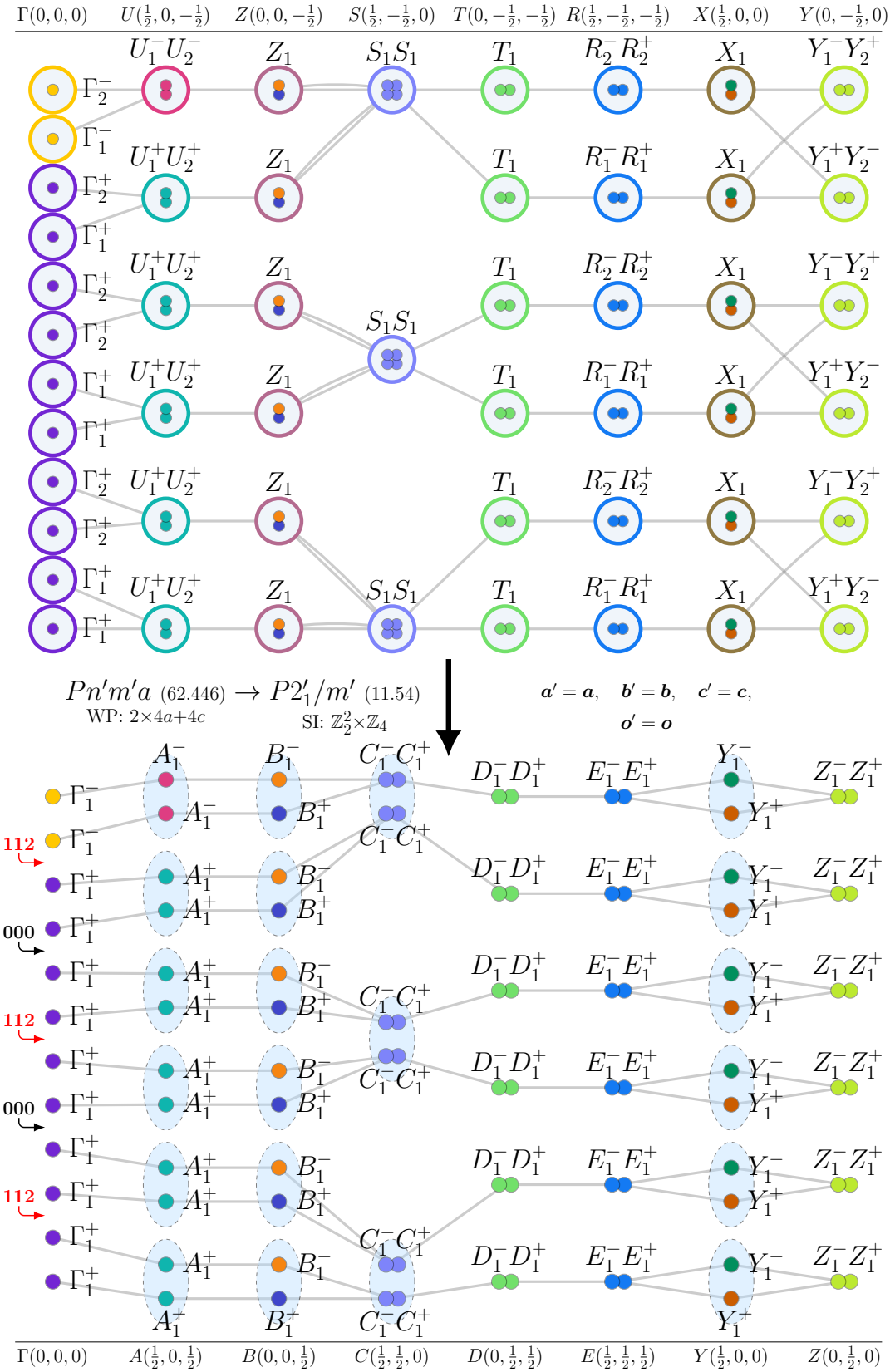
BCS Materials: $Ca_2Fe_{0.875}Cr_{0.125}GaO_5$[310].

74.7.1 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).

[310]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.206`



Supplementary Figure 369: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4a+4a+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 727: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4a$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

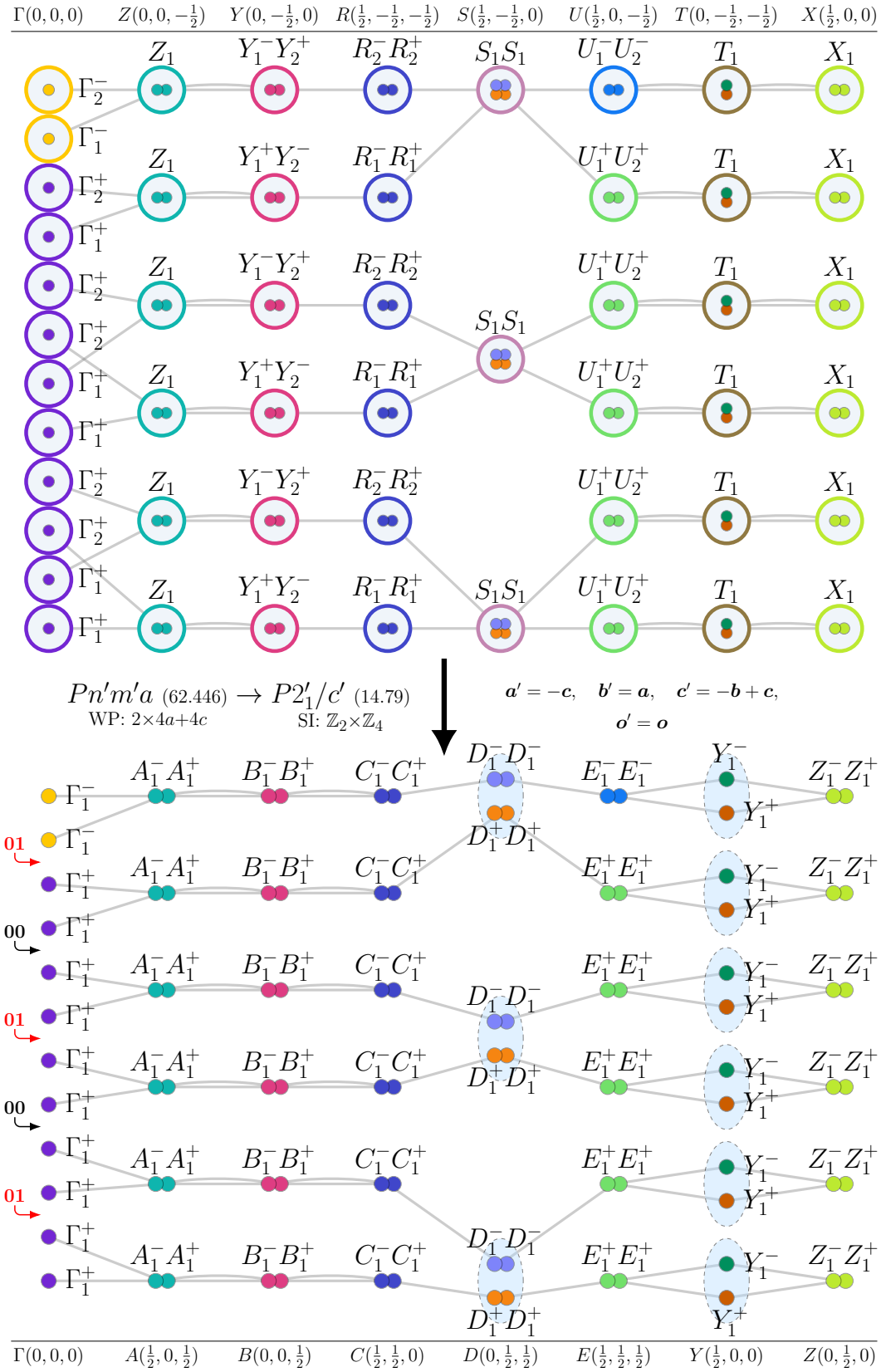
Supplementary Table 728: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4a$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

74.7.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 370: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a+4a+4c$ of supergroup $Pn'm'a$ (62.446).

Supplementary Table 729: Number of times a given SI can appear in the band structure. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4a$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 730: Possible SI values in each gap. Supergroup: $Pn'm'a$ (62.446), WP: $4a$+$4a$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

75 MSG $Pnm'a'$ (62.447)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P2_1'/m'$ (11.54), $P2_1/c$ (14.75).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), $P2_1'$ (4.9), $P2_1'$ (4.9), Pc (7.24), $Pm'n2_1'$ (31.125), $Pna'2_1'$ (33.147), $P2_1$ (4.7), $Pm'c'2_1$ (26.70).

75.1 WP: $4a + 4c$

BCS Materials: $CoFePO_5$ (175 K)[311].

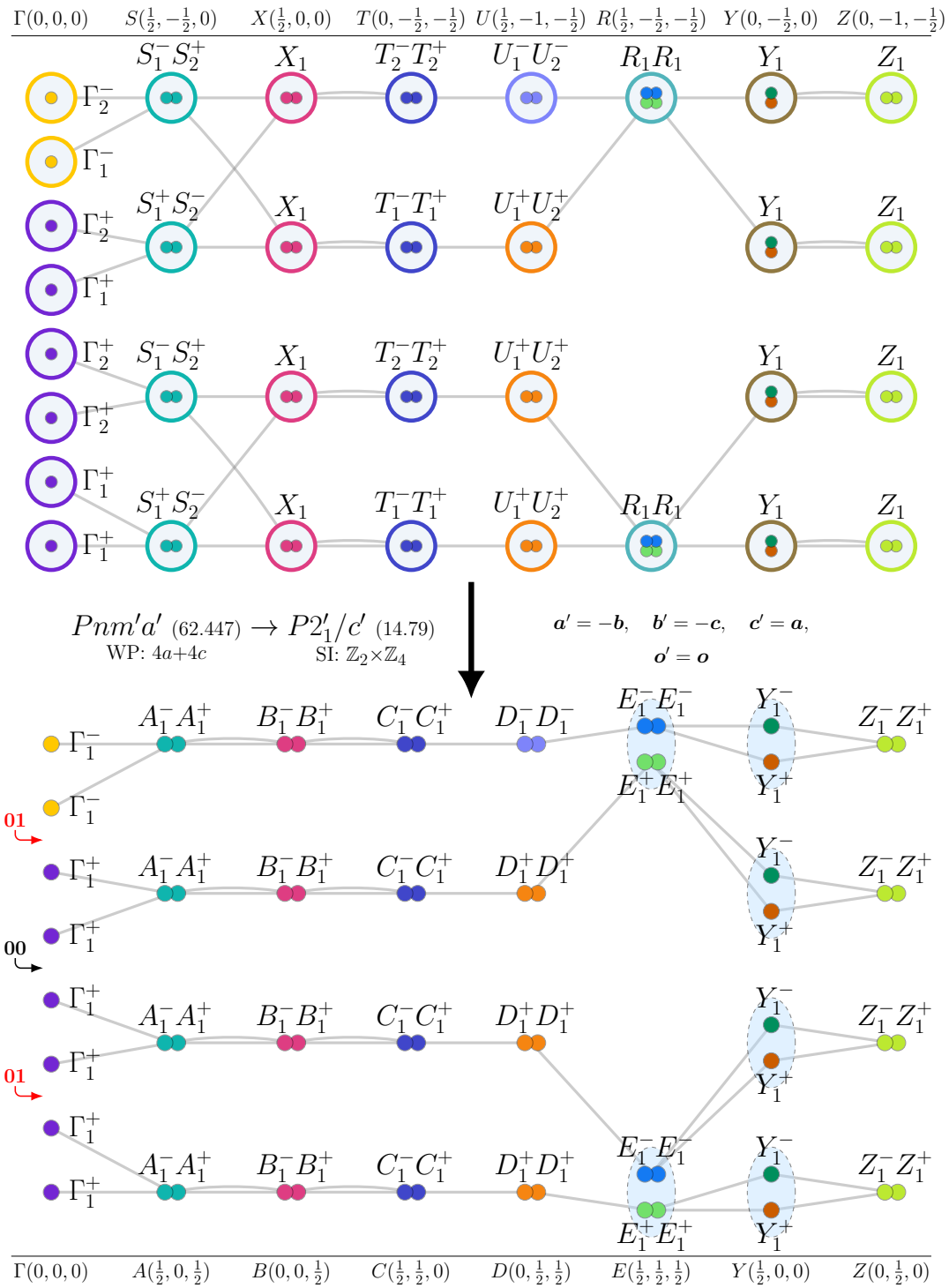
75.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],

[311] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.262`

- (B ∥ [010] or B ⊥ [001]).



Supplementary Figure 371: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4a+4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 731: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4a$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2

continued on next page...

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SI	Possible #Gaps
10	0, 1, 2
11	0, 1
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

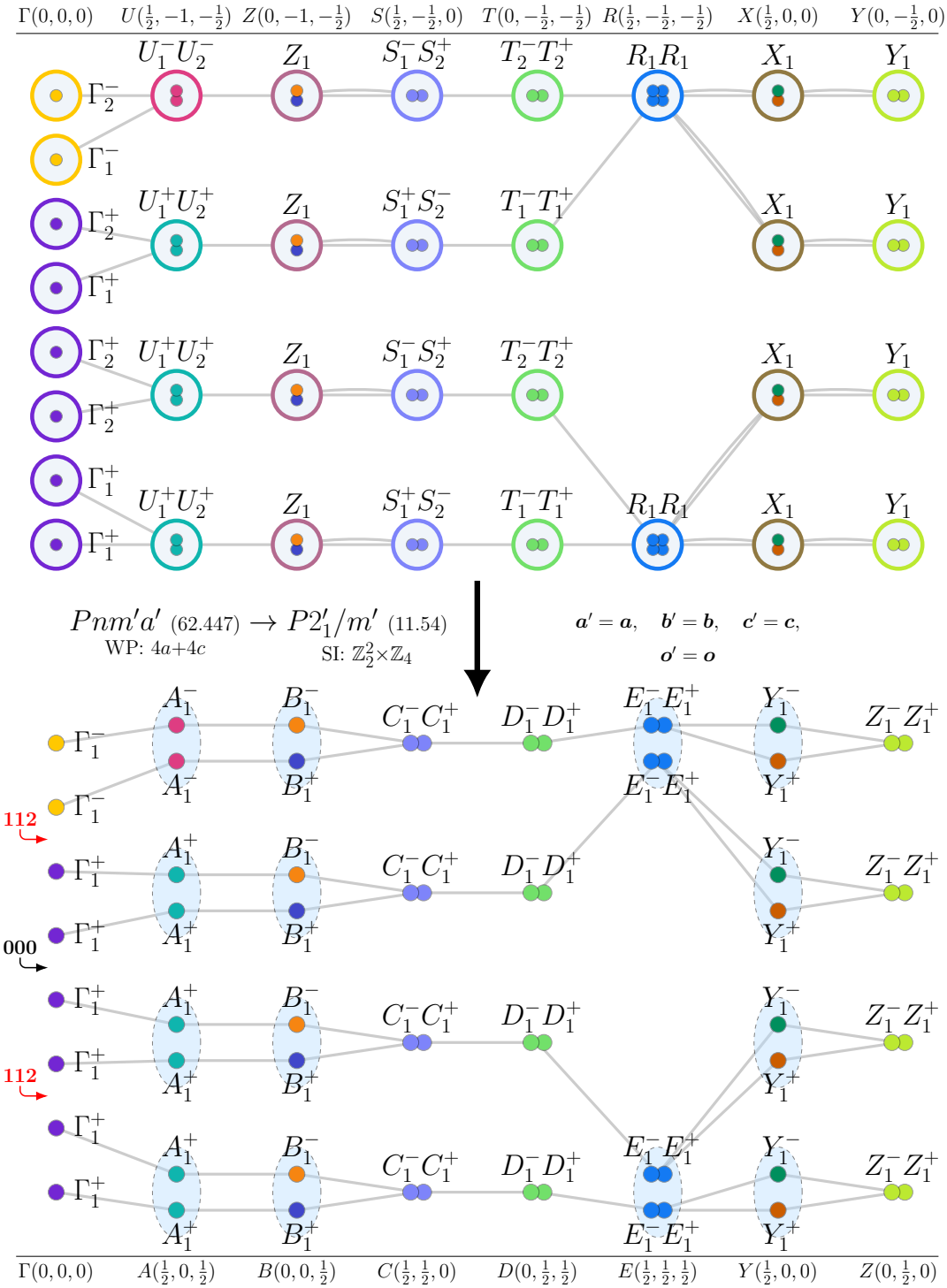
Supplementary Table 732: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4a$+$4c$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

75.1.2 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 372: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4a + 4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 733: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: 4a+4c, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1

continued on next page...

...continued

SI	Possible #Gaps
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 734: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4a$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

75.2 WP: $4a$

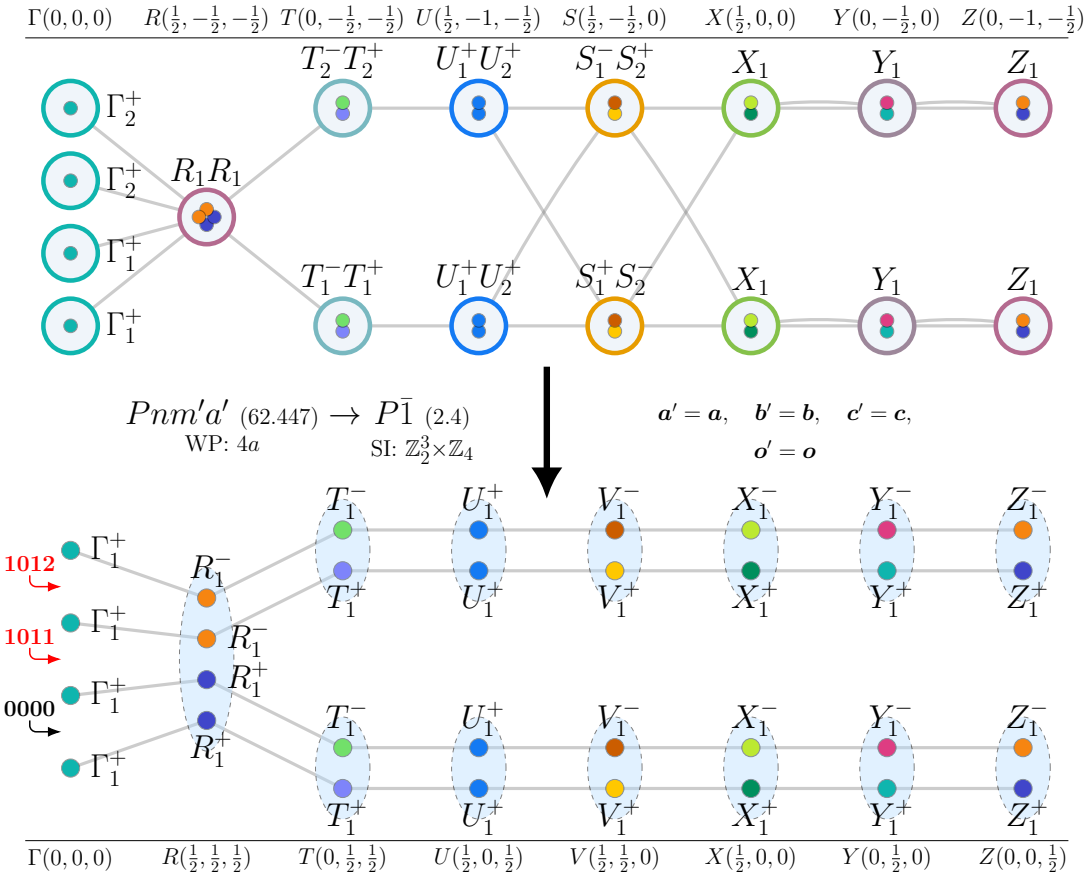
BCS Materials: Ca_2MnGaO_5 (160 K)[312].

75.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [001],
- B in generic direction.

[312]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.825`



Supplementary Figure 373: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 735: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

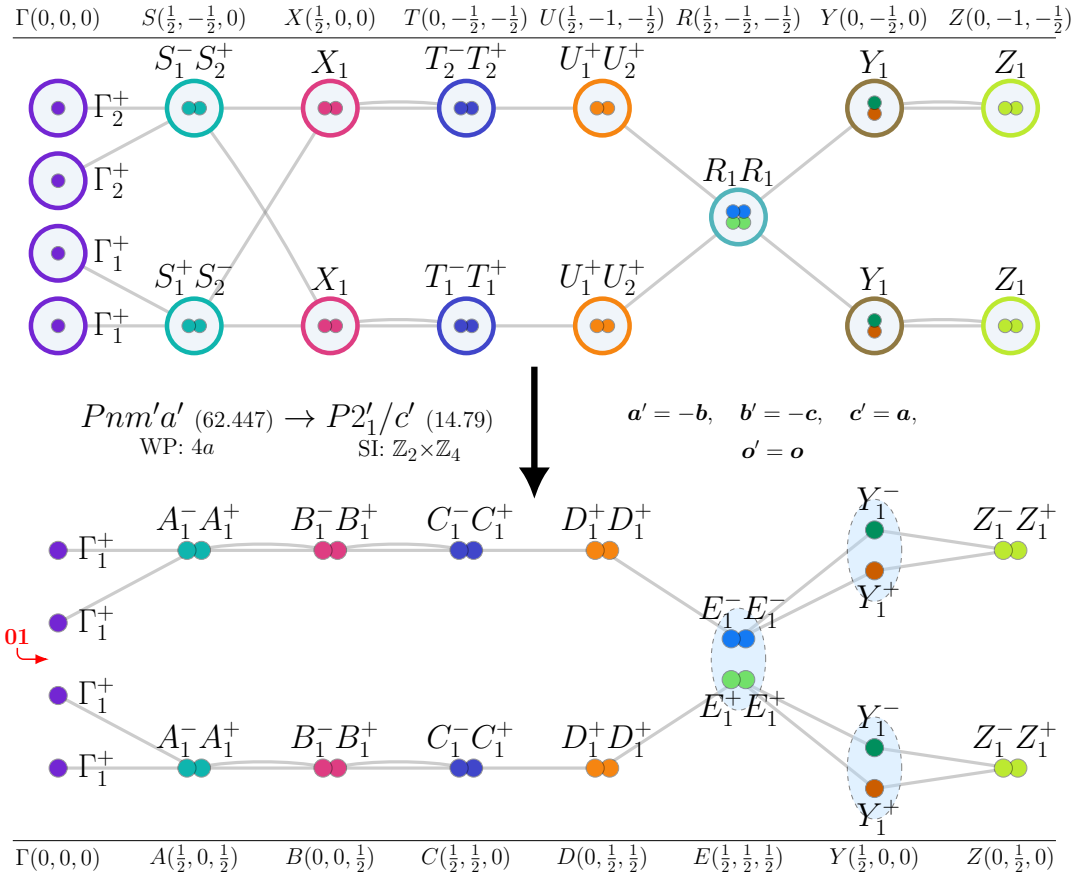
Supplementary Table 736: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1011, 1012, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

75.2.2 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).



Supplementary Figure 374: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 737: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 738: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

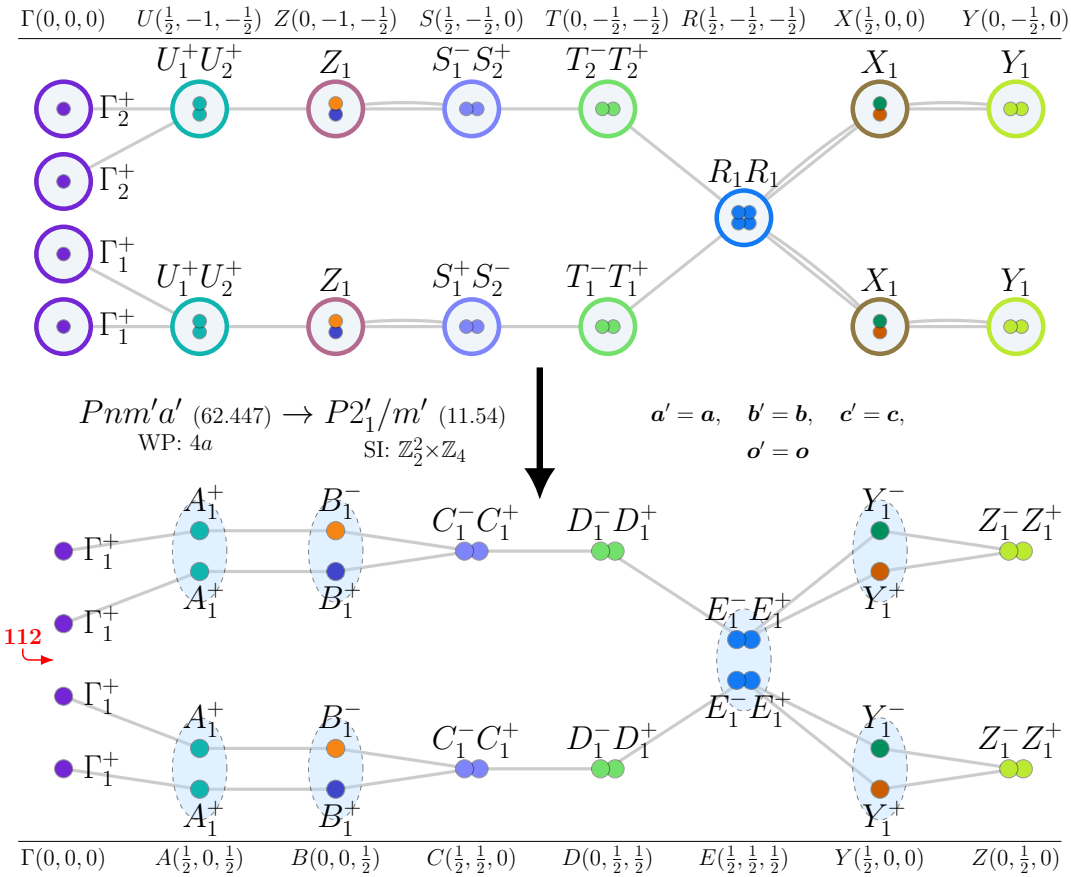
Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

75.2.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],

- (B ∥ [001] or B ⊥ [010]).



Supplementary Figure 375: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4a$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 739: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4a$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

Supplementary Table 740: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4a$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

75.3 WP: $4b + 4c + 4c$

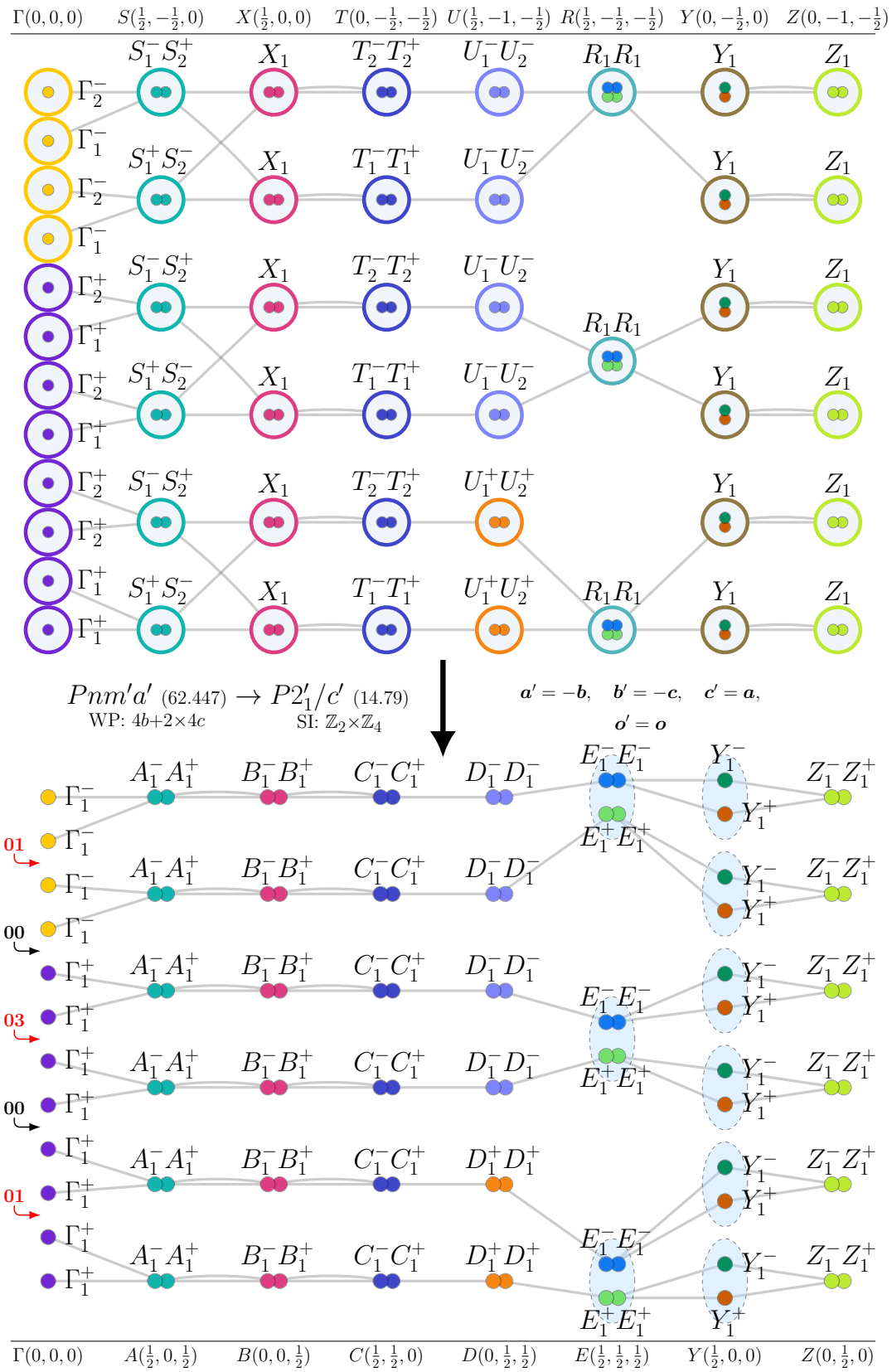
BCS Materials: $(Tm_{0.7}Mn_{0.3})MnO_3$ (104 K)[313], $(Ho_{0.8}Mn_{0.2})MnO_3$ (76 K)[314], $(Ho_{0.8}Mn_{0.2})MnO_3$ (40 K)[315].

75.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).

[313]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.293`
[314]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.648`
[315]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.649`



Supplementary Figure 376: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b + 4c + 4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 741: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4b$+$4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

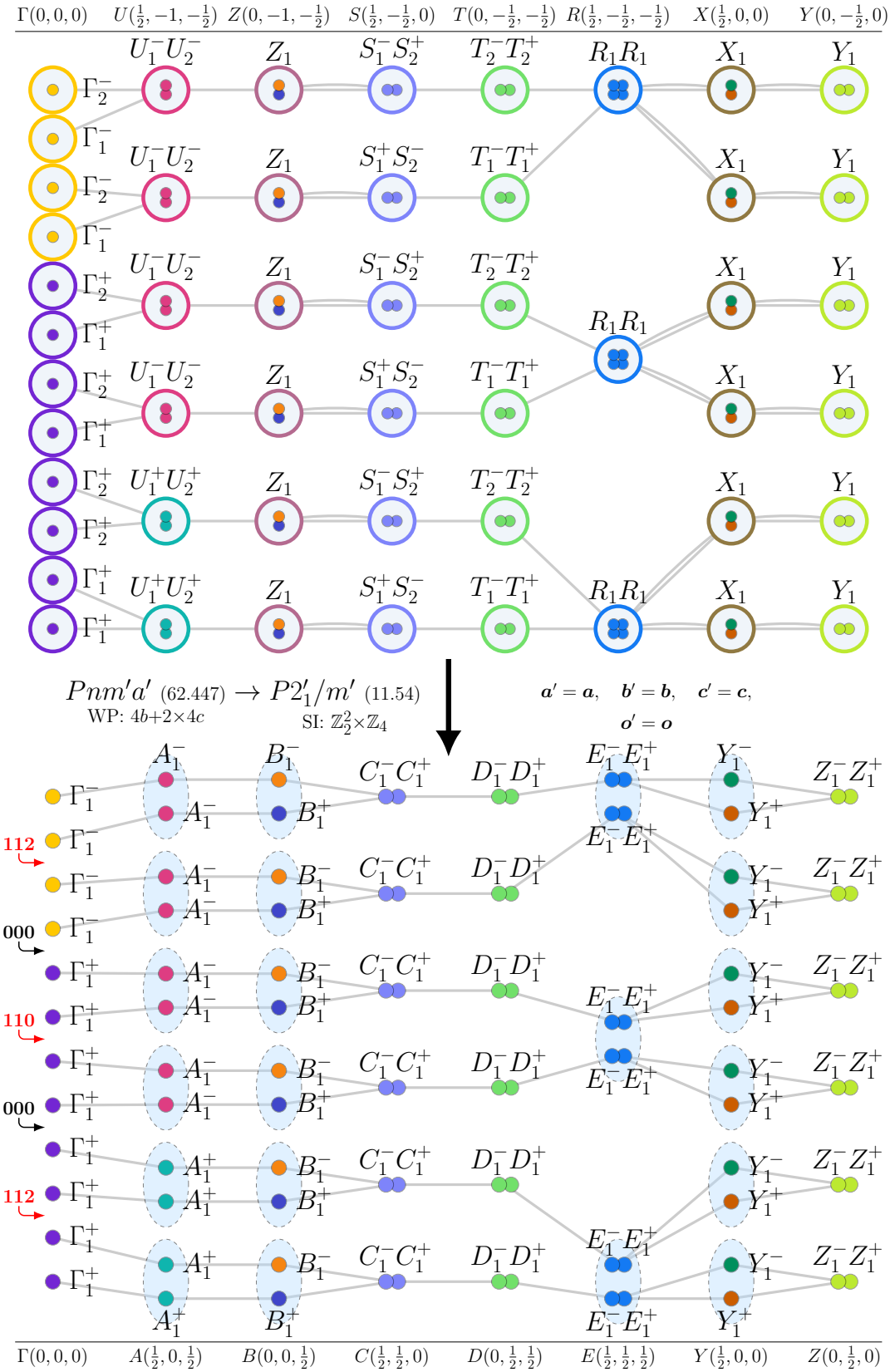
Supplementary Table 742: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4b$+$4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

75.3.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 377: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4b+4c+4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 743: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: 4*b*+4*c*+4*c*, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 744: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: 4*b*+4*c*+4*c*, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

75.4 WP: $4b + 4c$

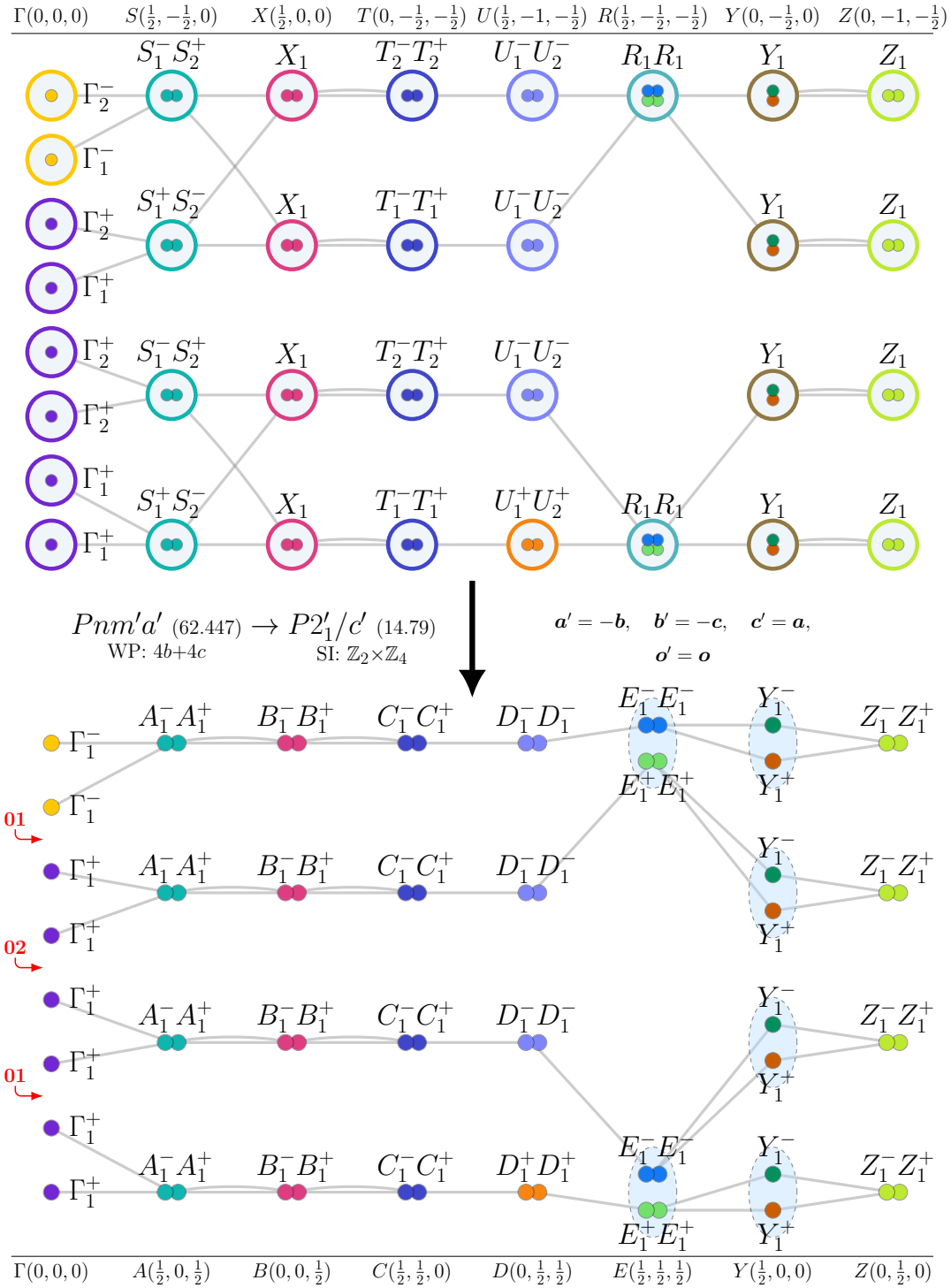
BCS Materials: $(Lu_{0.6}Mn_{0.4})MnO_3$ (67 K)[316], $CsMn_2F_6$ (24.1 K)[317], $Tb_{0.55}Sr_{0.45}MnO_3$ (8 K)[318].

75.4.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).

[316]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.661`
[317]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.726`
[318]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.534`



Supplementary Figure 378: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 745: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

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SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

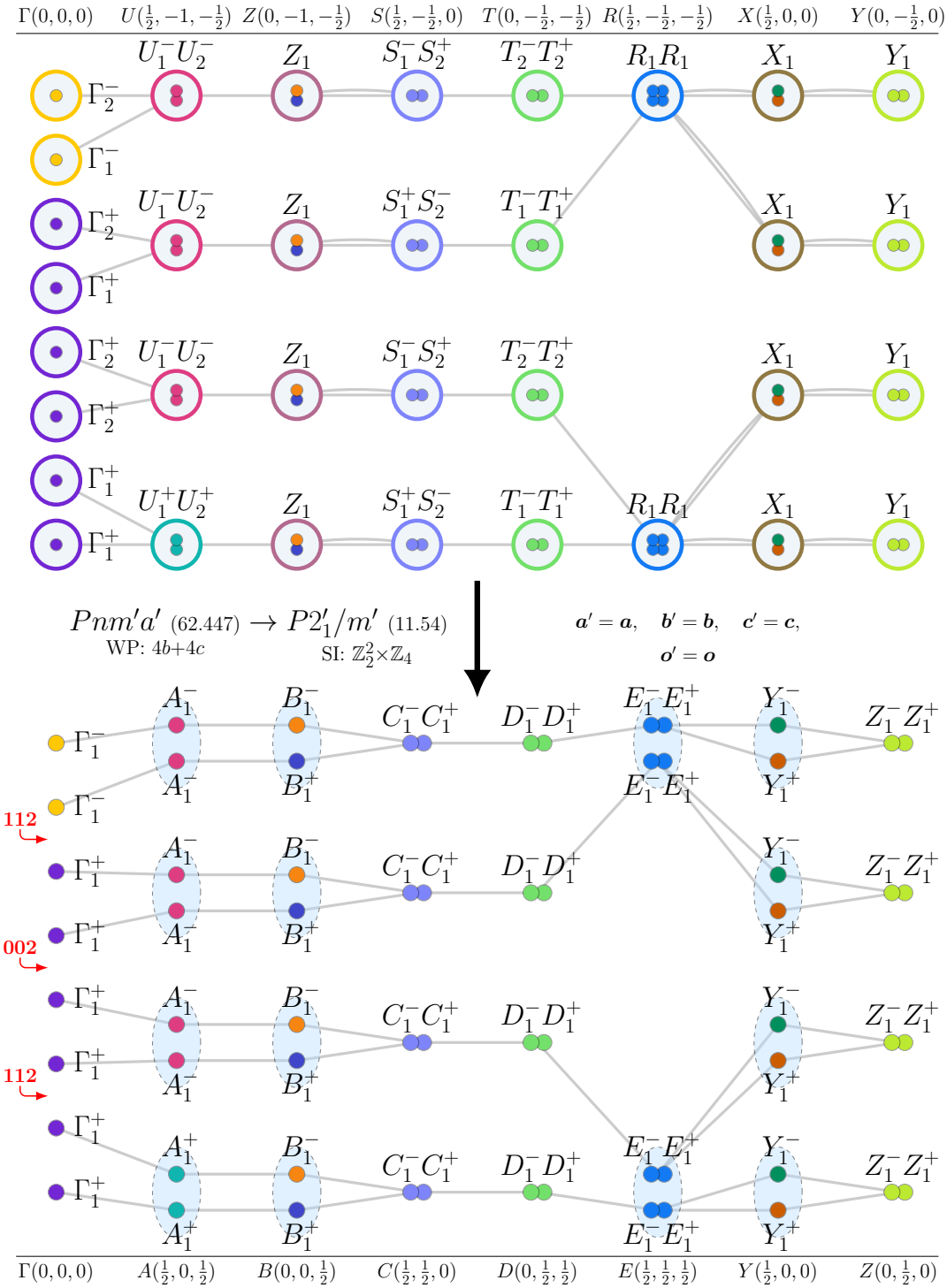
Supplementary Table 746: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4b$+$4c$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

75.4.2 Topological bands in subgroup $P2'_1/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 379: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 747: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4b$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1

continued on next page...

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SI	Possible #Gaps
112	0, 1, 2
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 748: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4b$+$4c$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

75.5 WP: $4b$

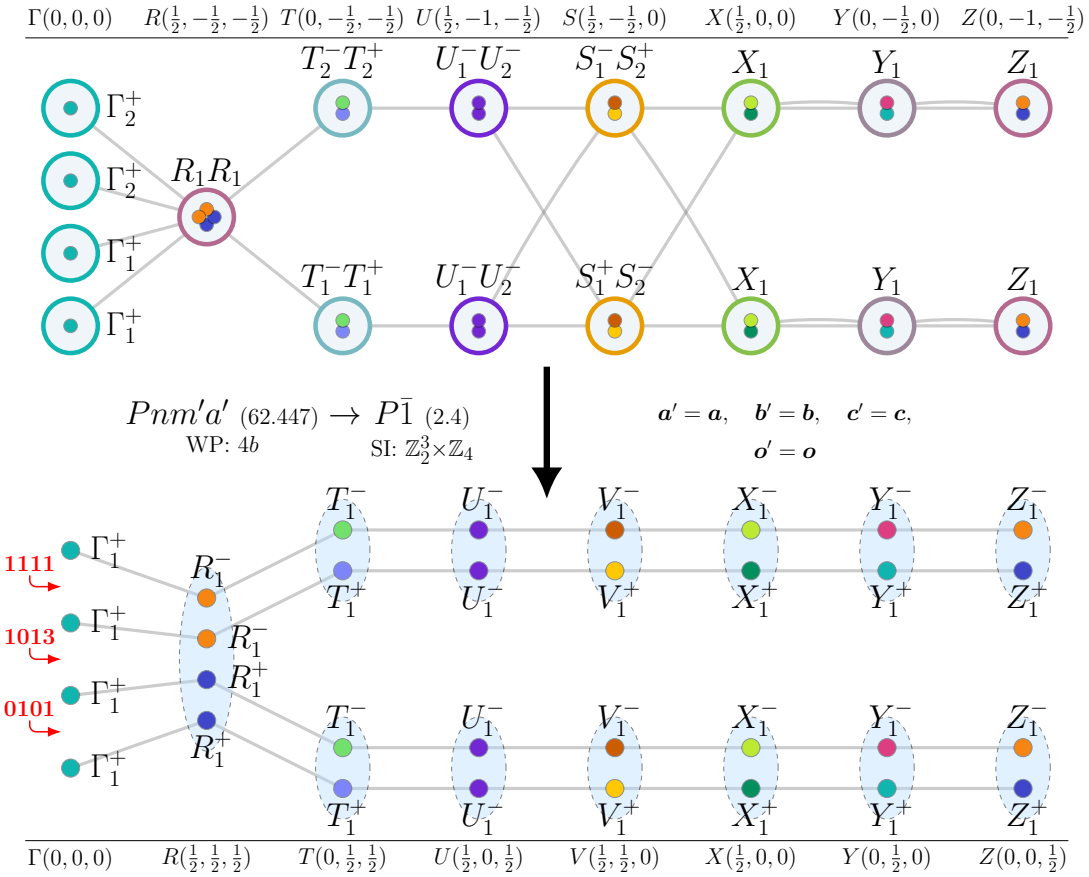
BCS Materials: $Tb_{0.55}Sr_{0.45}MnO_3$ (65 K)[319], $Tb_{0.55}Sr_{0.45}MnO_3$ (65 K)[320].

75.5.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [001],
- B in generic direction.

[319]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.536`
[320]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.535`



Supplementary Figure 380: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 749: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

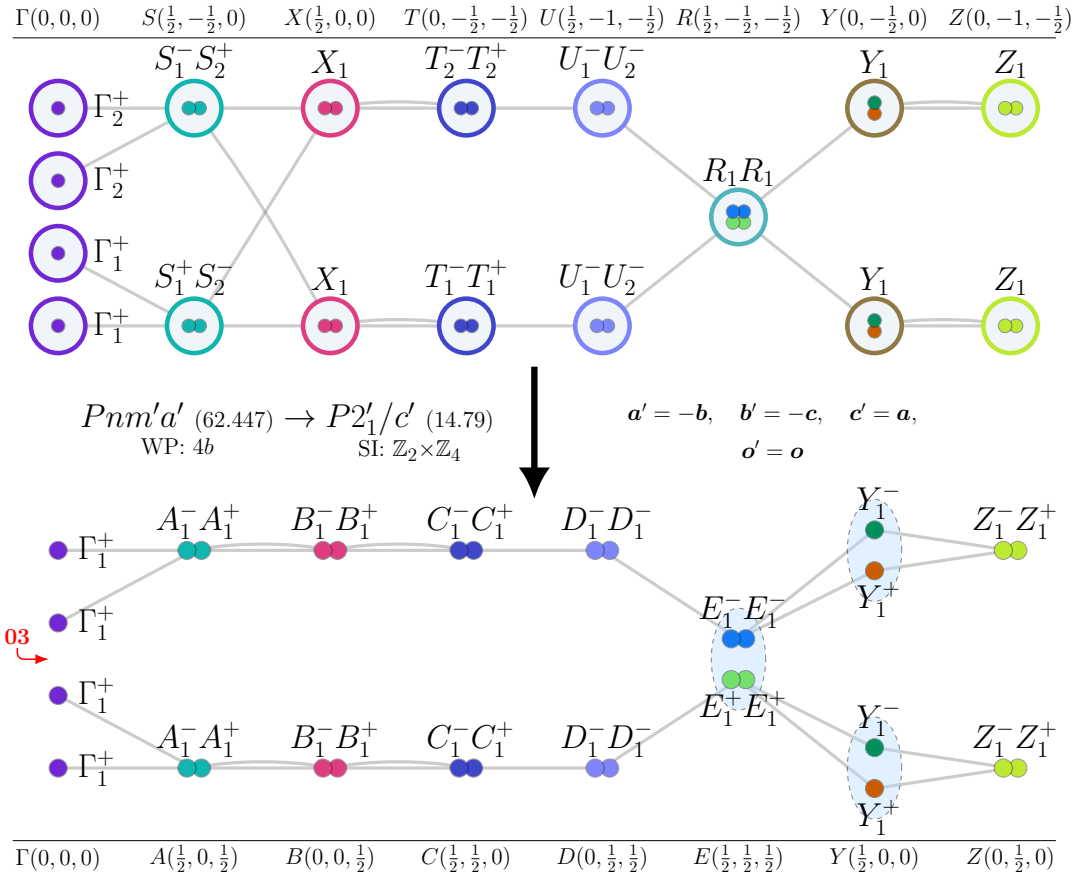
Supplementary Table 750: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1010, 1011, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

75.5.2 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).



Supplementary Figure 381: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 751: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 752: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

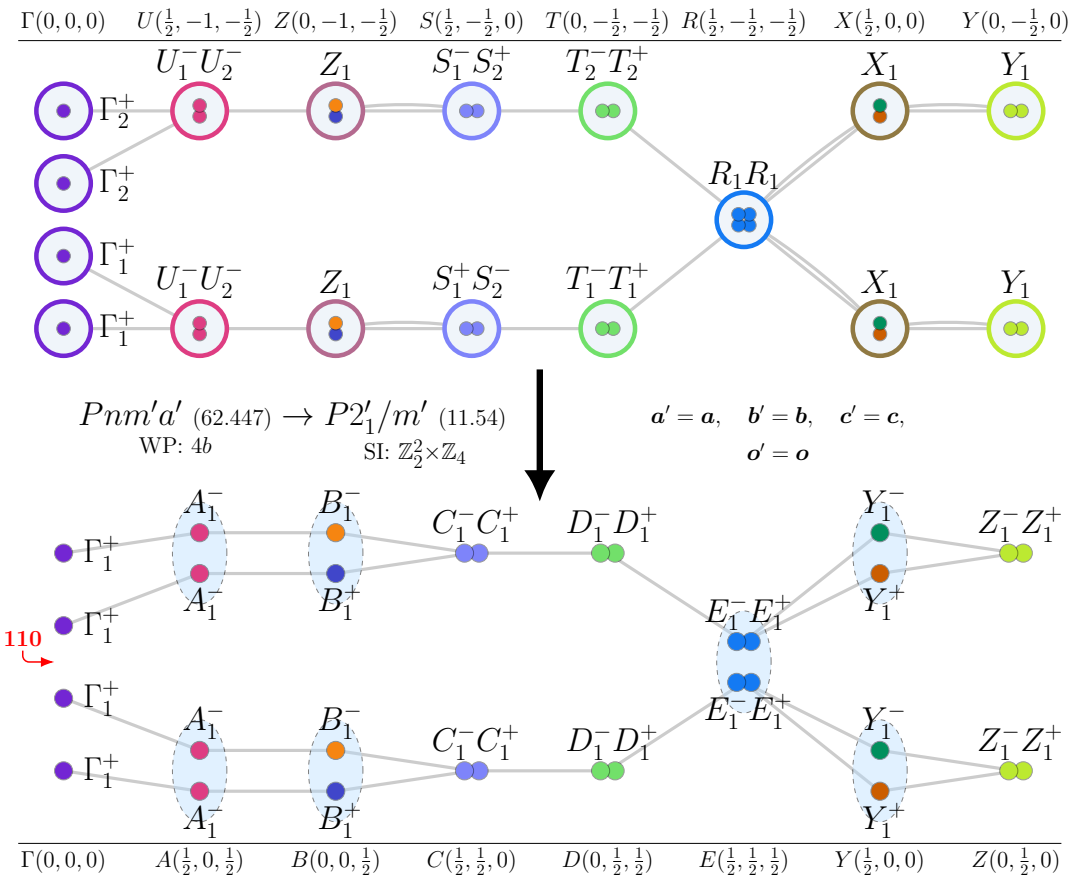
Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

75.5.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],

- (B ∥ [001] or B ⊥ [010]).



Supplementary Figure 382: Topological magnon bands in subgroup $P2'_1/m'$ (11.54) for magnetic moments on Wyckoff position $4b$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 753: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4b$, Subgroup: $P2'_1/m'$ (11.54).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

Supplementary Table 754: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4b$, Subgroup: $P2'_1/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

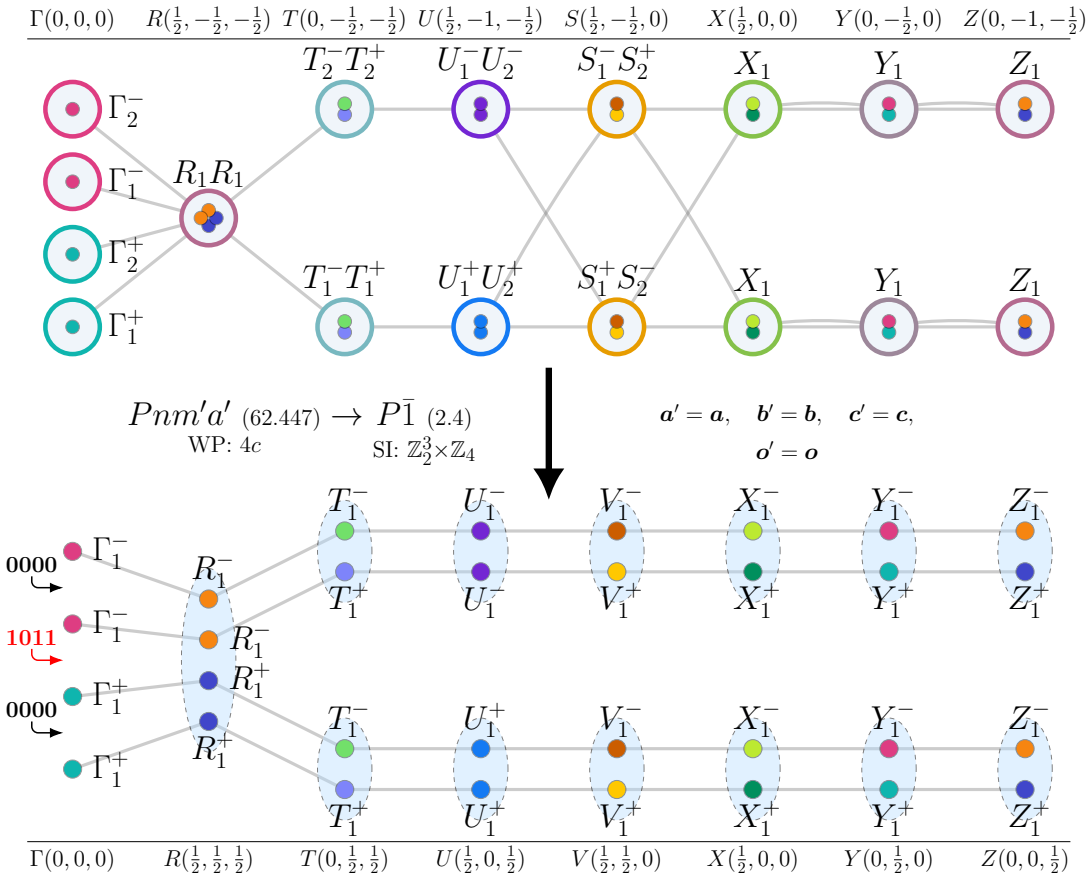
75.6 WP: $4c$

BCS Materials: PrSi (54 K)[321], $NdNi_{0.6}Cu_{0.4}$ (37.5 K)[322], HoNi (35.5 K)[323], HoPt (16 K)[324], ErPt (16 K)[325], TmPt (6 K)[326].

75.6.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B ∥ [010] or B ⊥ [001]) and strain ⊥ [100],
- (B ∥ [010] or B ⊥ [001]) and strain ⊥ [010],
- (B ∥ [001] or B ⊥ [010]) and strain ⊥ [100],
- (B ∥ [001] or B ⊥ [010]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 383: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 755: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2

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[321]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.408
[322]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.249
[323]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.480
[324]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.686
[325]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.685
[326]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.688

...continued

SI	Possible #Gaps
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2, 3
0102	0, 1, 2, 3
0103	0, 1, 2, 3
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	1, 2, 3

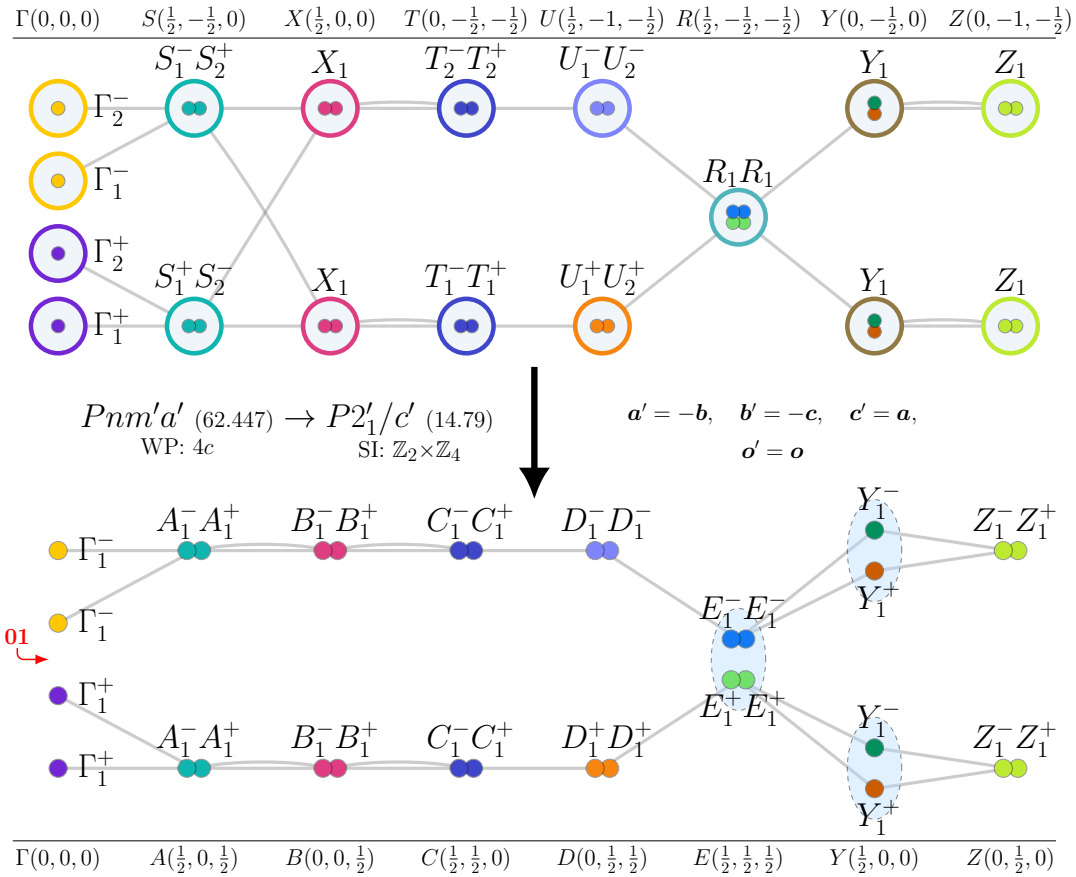
Supplementary Table 756: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0100, 0101, 0102, 0103, 1010 1011, 1012, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

75.6.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).



Supplementary Figure 384: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 757: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

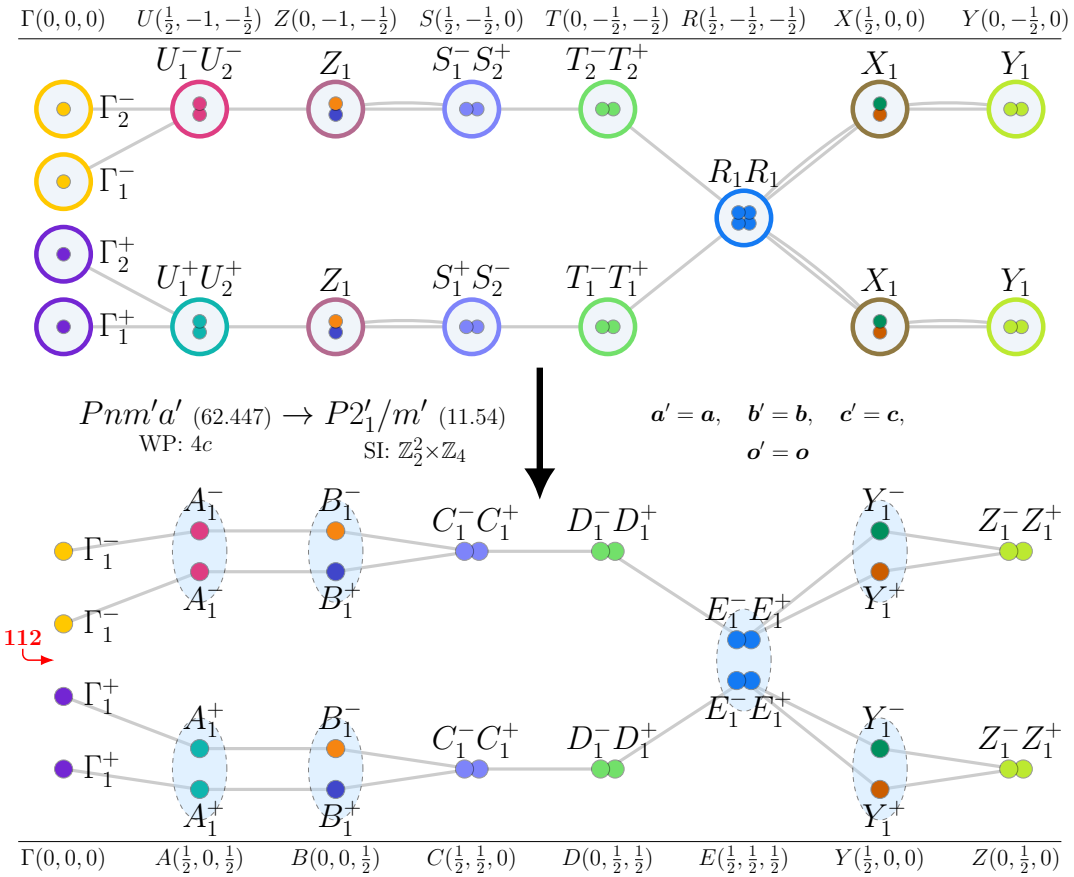
Supplementary Table 758: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

75.6.3 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain ⊥ [010],
- (B ∥ [001] or B ⊥ [010]).



Supplementary Figure 385: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff position $4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 759: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0
001	0, 1
003	0, 1
110	0, 1
112	0, 1
gapless	2
nontrivial	1

Supplementary Table 760: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112

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...continued

Gap#	Possible SI Values
3	G
4	000

75.7 WP: $4c + 4c + 4c$

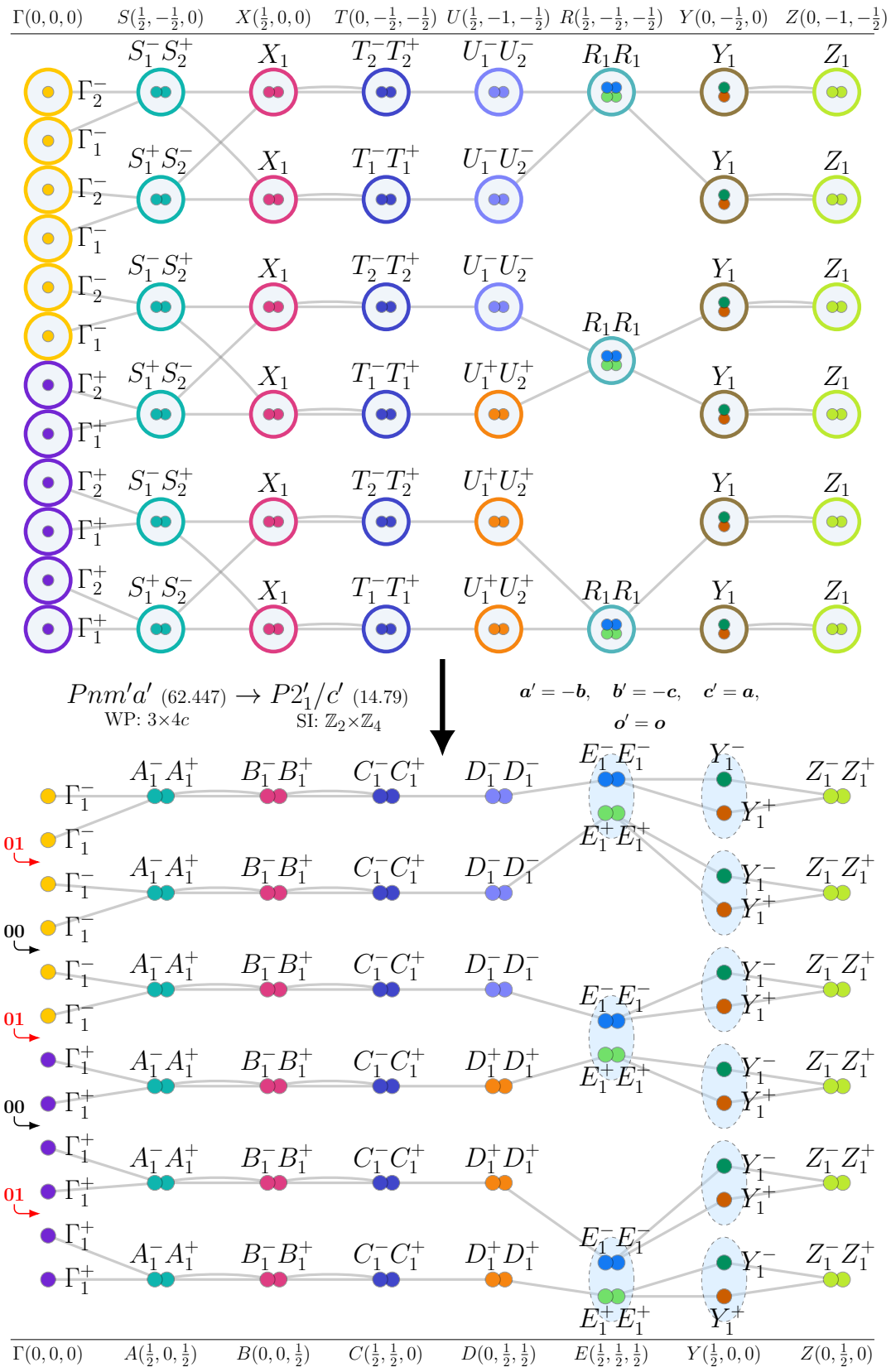
BCS Materials: Ho_3NiGe_2 (43 K)[327], Pr_3CoGe_2 (28 K)[328].

75.7.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [001]).

[327]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.437`
[328]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.438`



Supplementary Figure 386: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+4c+4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 761: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4c$+$4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

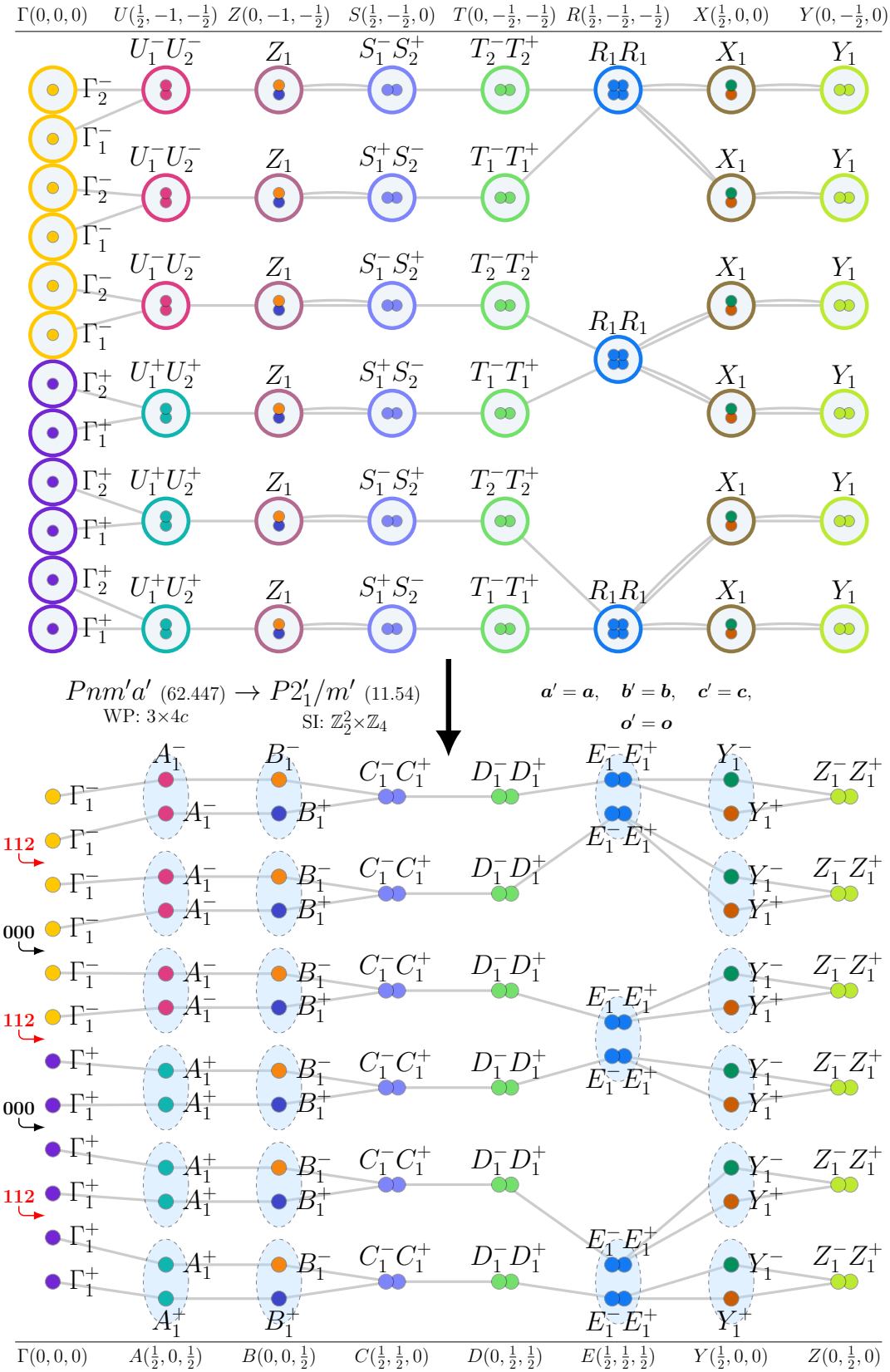
Supplementary Table 762: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4c$+$4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

75.7.2 Topological bands in subgroup $P2_1'/m'$ (11.54)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 387: Topological magnon bands in subgroup $P2_1'/m'$ (11.54) for magnetic moments on Wyckoff positions $4c+4c+4c$ of supergroup $Pnm'a'$ (62.447).

Supplementary Table 763: Number of times a given SI can appear in the band structure. Supergroup: $Pnm'a'$ (62.447), WP: $4c$+$4c$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 764: Possible SI values in each gap. Supergroup: $Pnm'a'$ (62.447), WP: $4c$+$4c$+$4c$, Subgroup: $P2_1'/m'$ (11.54).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

76 MSG $Pn'ma'$ (62.448)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $P2_1'/c'$ (14.79), $P2_1/m$ (11.50).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), $P2_1'$ (4.9), $P2_1'$ (4.9), Pm (6.18), $Pmn'2_1'$ (31.126), $Pmc'2_1'$ (26.69), $P2_1$ (4.7), $Pn'a'2_1$ (33.148).

76.1 WP: $4b$

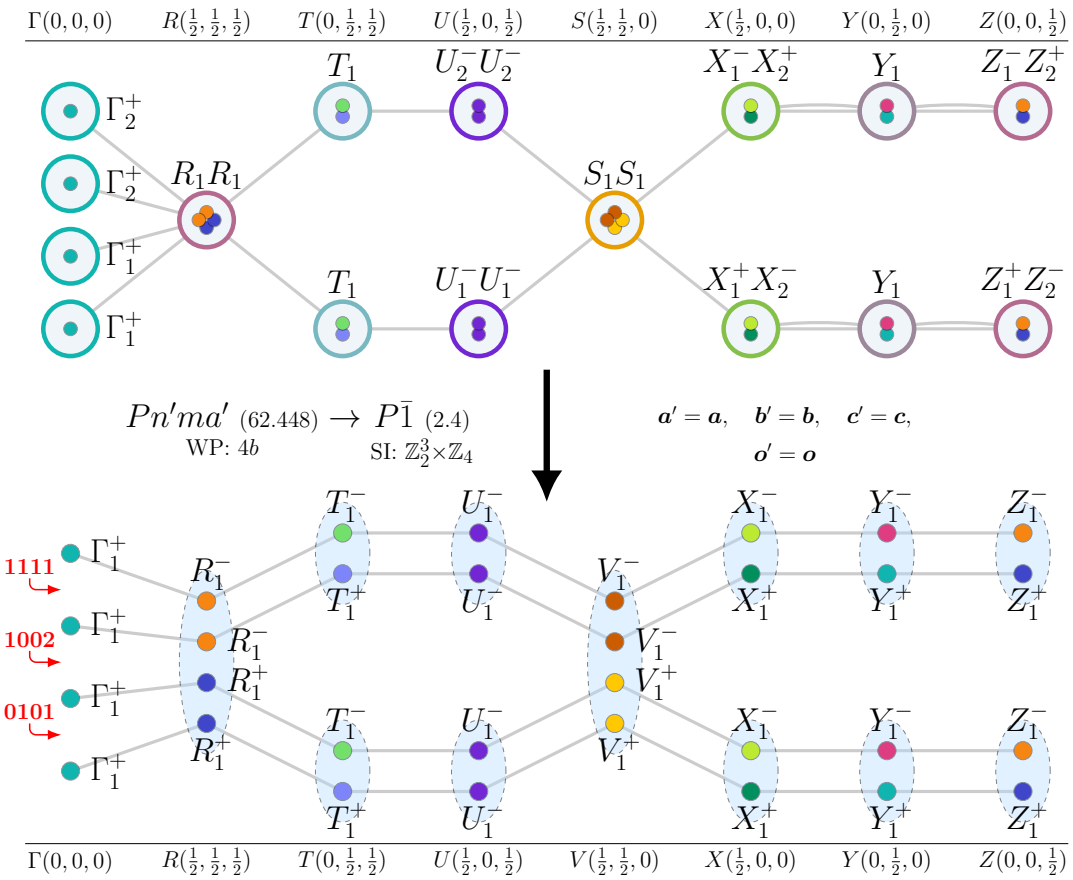
BCS Materials: $NdFeO_3$ (760 K)[329], $CeFeO_3$ (720 K)[330], $TbFeO_3$ (681 K)[331], $SmFeO_3$ (670 K)[332], $NaOsO_3$ (410 K)[333], $LaCrO_3$ (310 K)[334], $YCrO_3$ (141 K)[335], $LaMnO_3$ (139.5 K)[336], $ErCrO_3$ (133 K)[337], $TmCrO_3$ (124 K)[338], $YRuO_3$ (97 K)[339], $PrMnO_3$ (91 K)[340], $Pr_{0.95}K_{0.05}MnO_3$ (85 K)[341], $NdMnO_3$ (78 K)[342], $KMnF_3$ (77 K)[343], $NdMnO_3$ (73 K)[344], $(CH_3NH_3)(Co(COOH)_3$ (15.9 K)[345], $Ho_{0.2}Bi_{0.8}FeO_3$[346], $Ho_{0.15}Bi_{0.85}FeO_3$[347].

76.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[329]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.336
[330]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.757
[331]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.351
[332]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.380
[333]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.25
[334]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.417
[335]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.586
[336]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.1
[337]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.591
[338]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.587
[339]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.513
[340]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.608
[341]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.610
[342]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.288
[343]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.432
[344]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.370
[345]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.368
[346]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.560
[347]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.559



Supplementary Figure 388: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 765: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2

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SI	Possible #Gaps
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

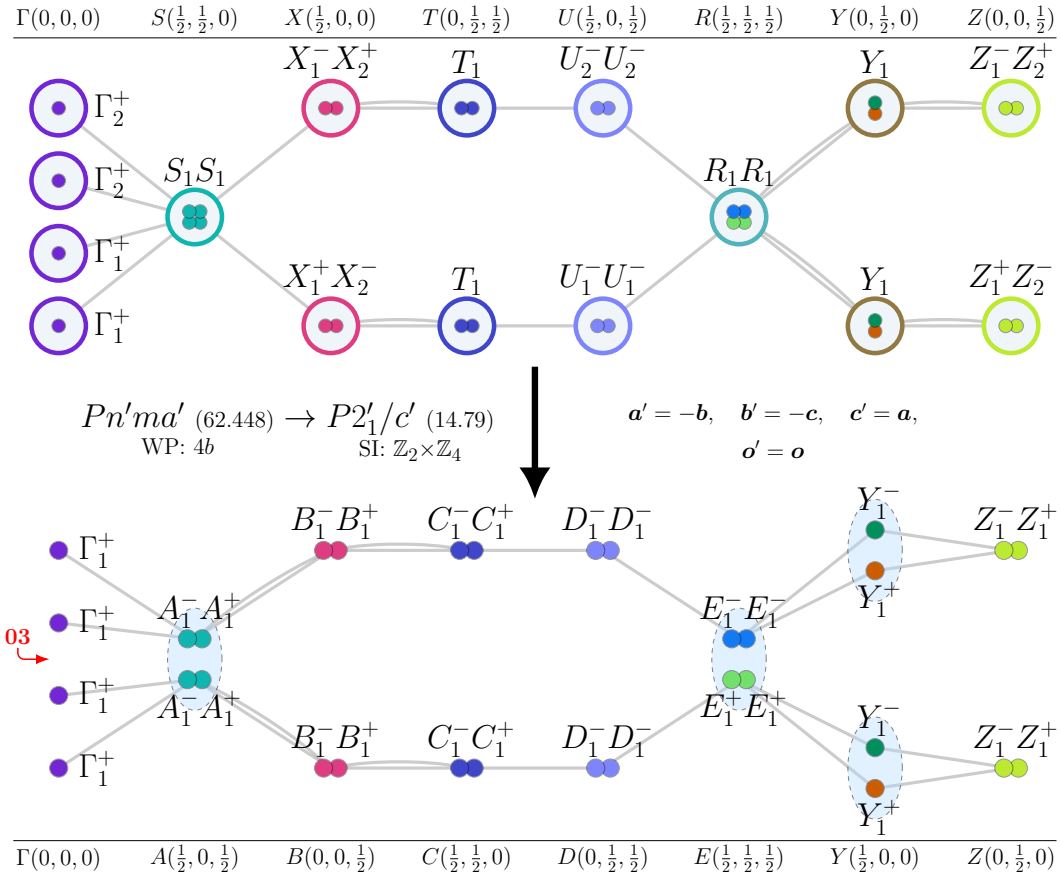
Supplementary Table 766: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1000, 1001, 1002, 1003, 1010 1011, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

76.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).



Supplementary Figure 389: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 767: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 768: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

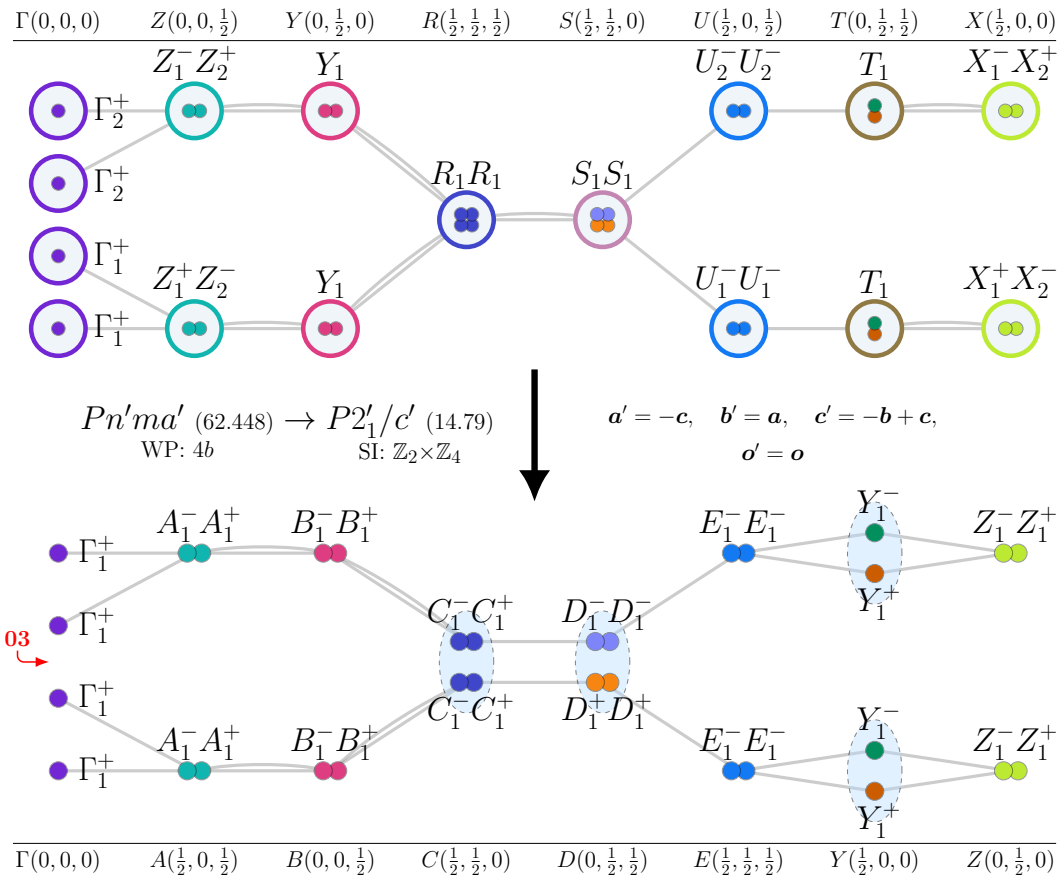
Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

76.1.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],

- (B ∥ [001] or B ⊥ [100]).



Supplementary Figure 390: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 769: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 770: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

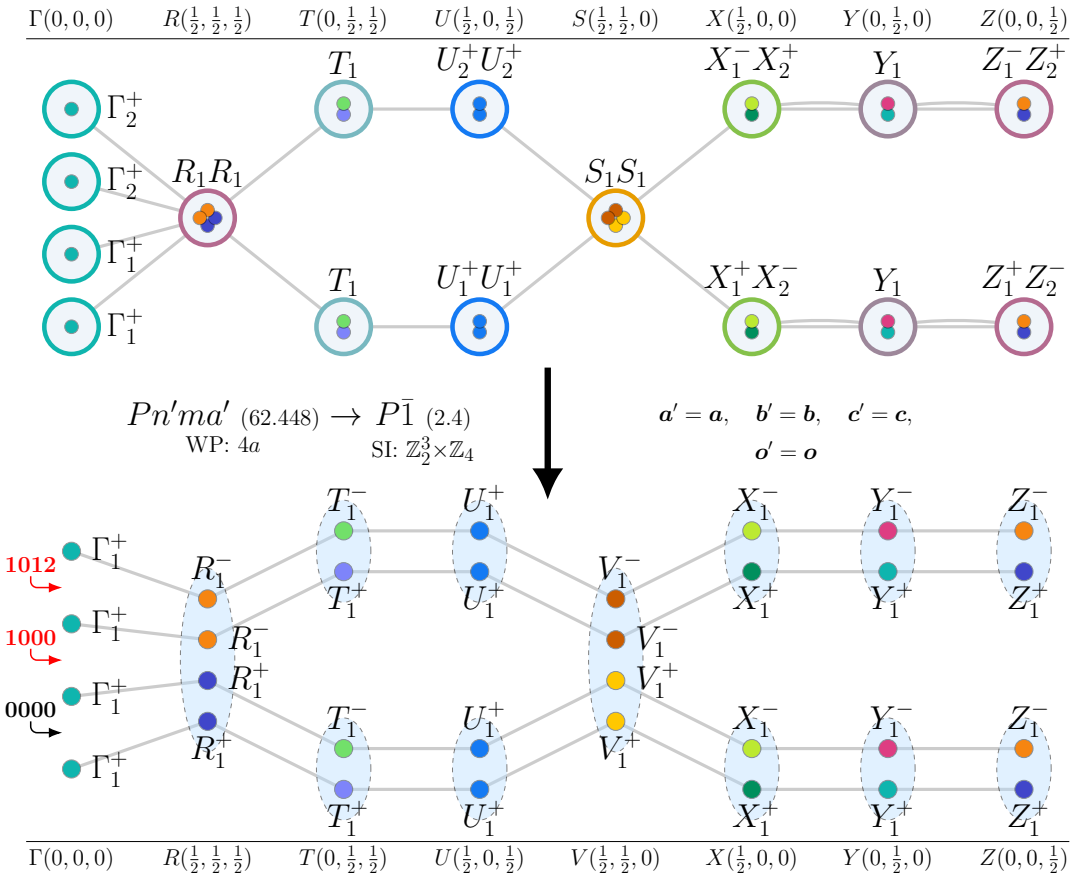
76.2 WP: $4a$

BCS Materials: $La_{0.5}Sr_{0.5}FeO_{2.5}F_{0.5}$ (700 K)[348], $La_{0.875}Ba_{0.125}Mn_{0.875}Ti_{0.125}O_3$[349], $La_{0.90}Ba_{0.10}Mn_{0.90}Ti_{0.10}O_3$[350], $La_{0.95}Ba_{0.05}Mn_{0.95}Ti_{0.05}O_3$[351], $La_{0.95}Ba_{0.05}MnO_3$[352], $La_{0.95}Ca_{0.05}MnO_3$[353], $LaMnO_3$[354], $YCr_{0.5}Mn_{0.5}O_3$[355].

76.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B ∥ [100] or B ⊥ [001]) and strain ⊥ [100],
- (B ∥ [100] or B ⊥ [001]) and strain ⊥ [010],
- (B ∥ [001] or B ⊥ [100]) and strain ⊥ [010],
- (B ∥ [001] or B ⊥ [100]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 391: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4a$ of supergroup $Pn'ma'$ (62.448).

[348] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.34
[349] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.647
[350] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.646
[351] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.645
[352] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.644
[353] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.643
[354] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.642
[355] BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.100

Supplementary Table 771: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1
0102	0, 1, 2
0103	0, 1
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

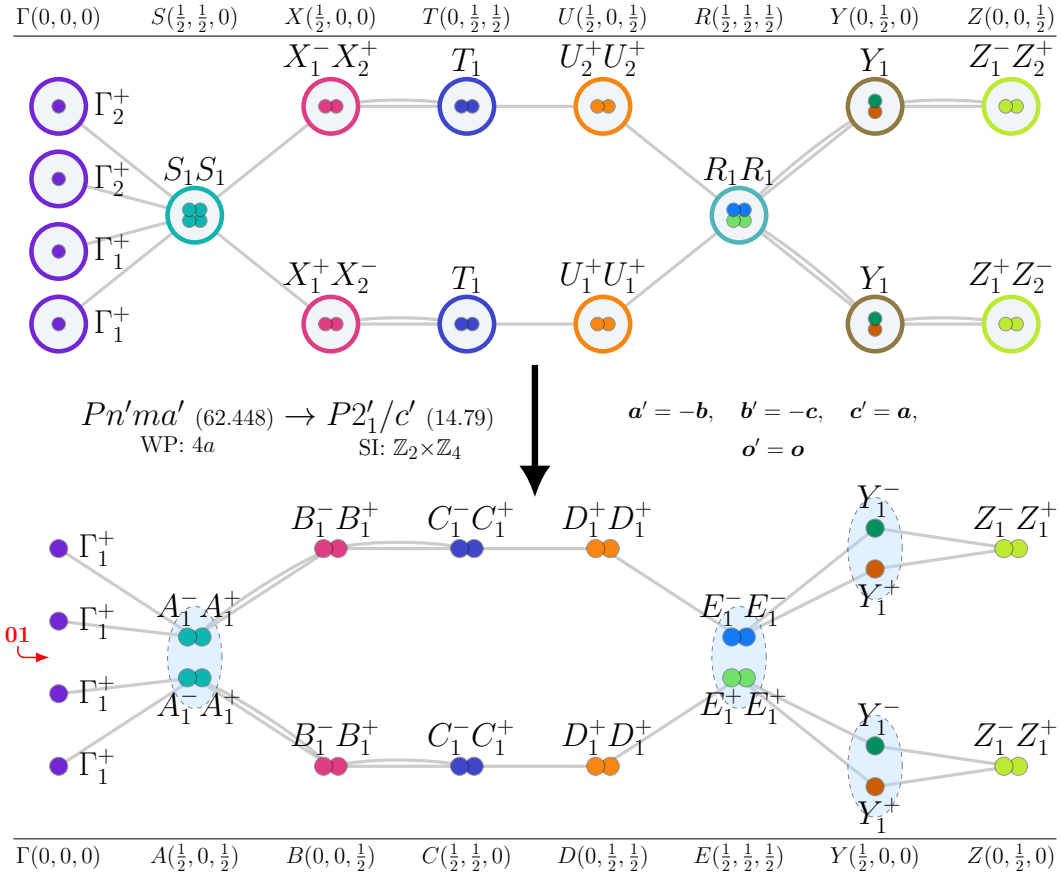
Supplementary Table 772: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4a$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1000, 1001, 1002, 1003, 1011 1012, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1011, 1012 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000

76.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).



Supplementary Figure 392: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 773: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

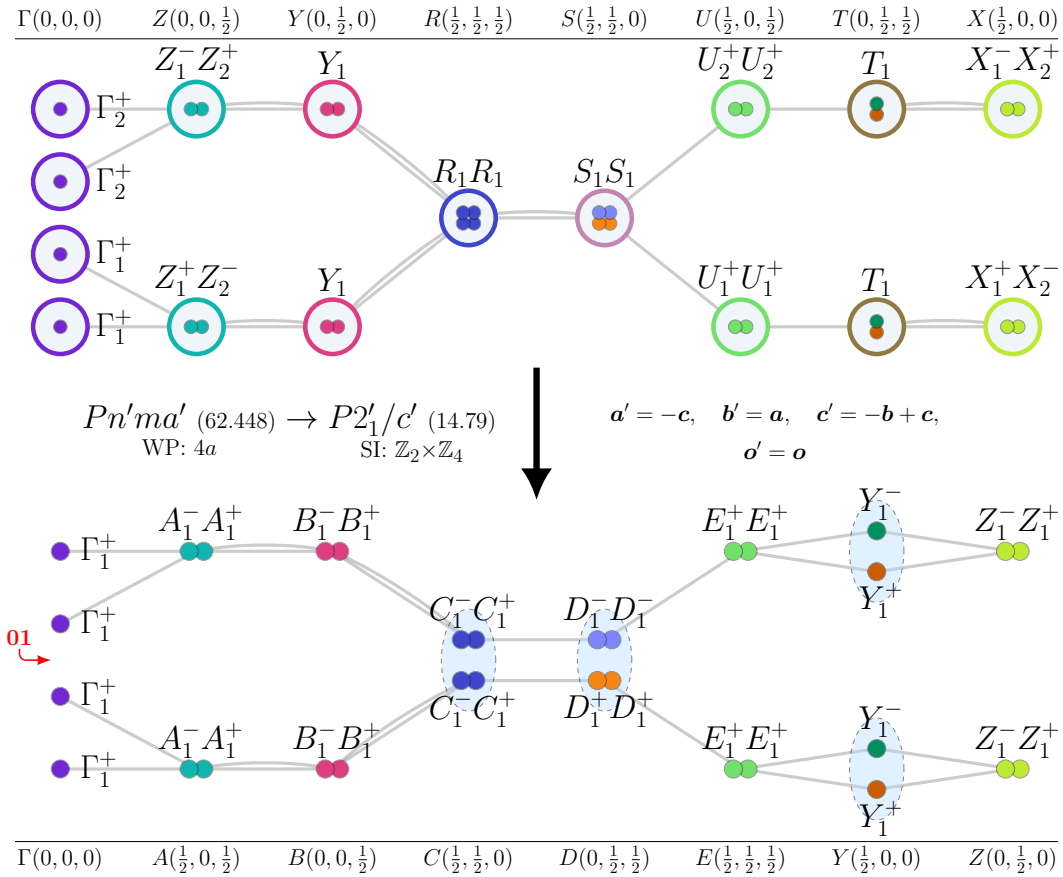
Supplementary Table 774: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

76.2.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain ⊥ [100],
- (B ∥ [001] or B ⊥ [100]).



Supplementary Figure 393: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 775: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
gapless	2
nontrivial	1

Supplementary Table 776: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00

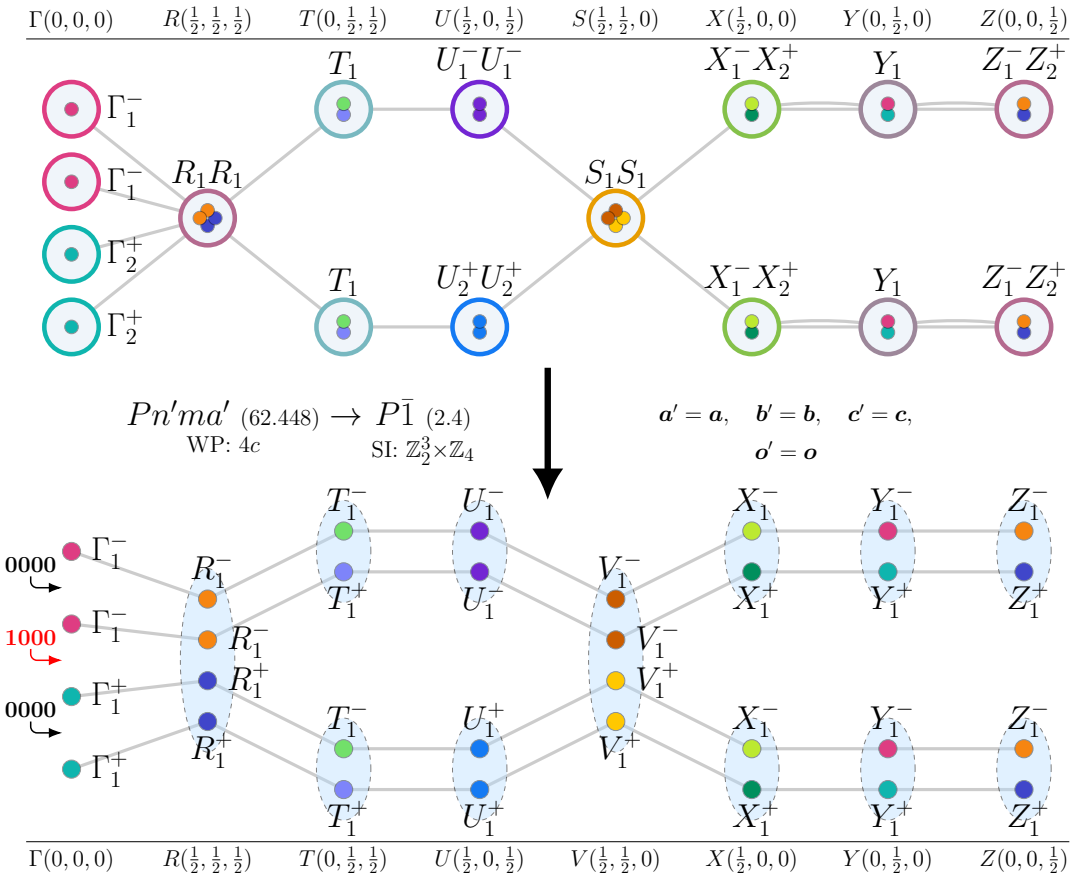
76.3 WP: $4c$

BCS Materials: $SrMnSb_2$ (565 K)[356].

76.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.



Supplementary Figure 394: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 777: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2

continued on next page...

[356]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.767`

...continued

SI	Possible #Gaps
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2, 3
0101	0, 1, 2, 3
0102	0, 1, 2, 3
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	1, 2, 3

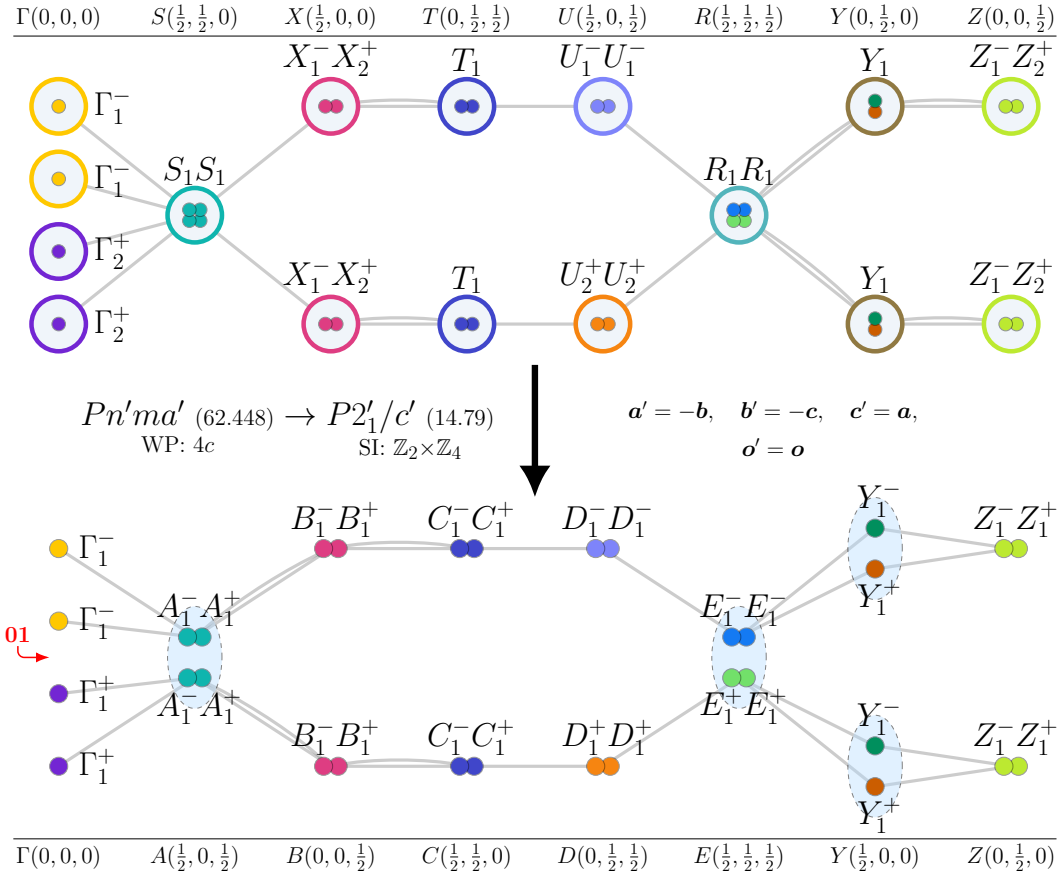
Supplementary Table 778: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0100, 0101, 0102, 0103, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1010, 1011, 1012 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000

76.3.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).



Supplementary Figure 395: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 779: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

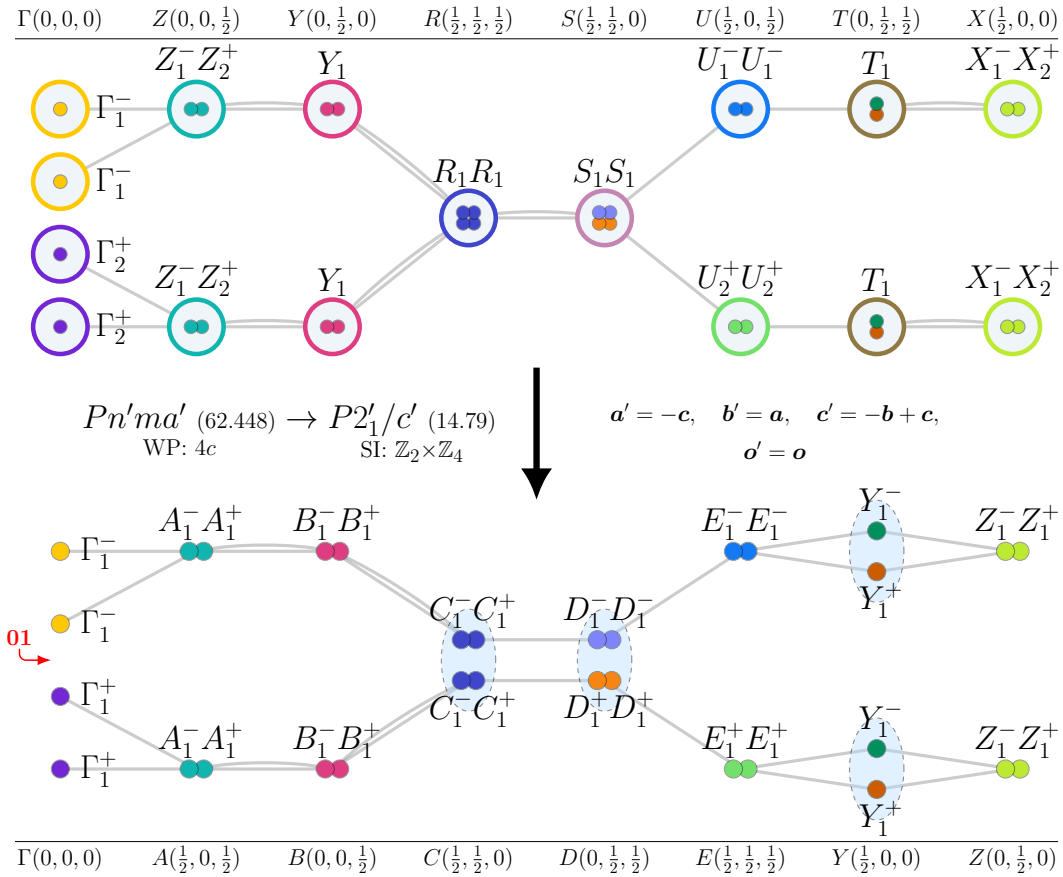
Supplementary Table 780: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

76.3.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [100]).



Supplementary Figure 396: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4c$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 781: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

Supplementary Table 782: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12

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Gap#	Possible SI Values
3	G
4	00

76.4 WP: $4c + 8d$

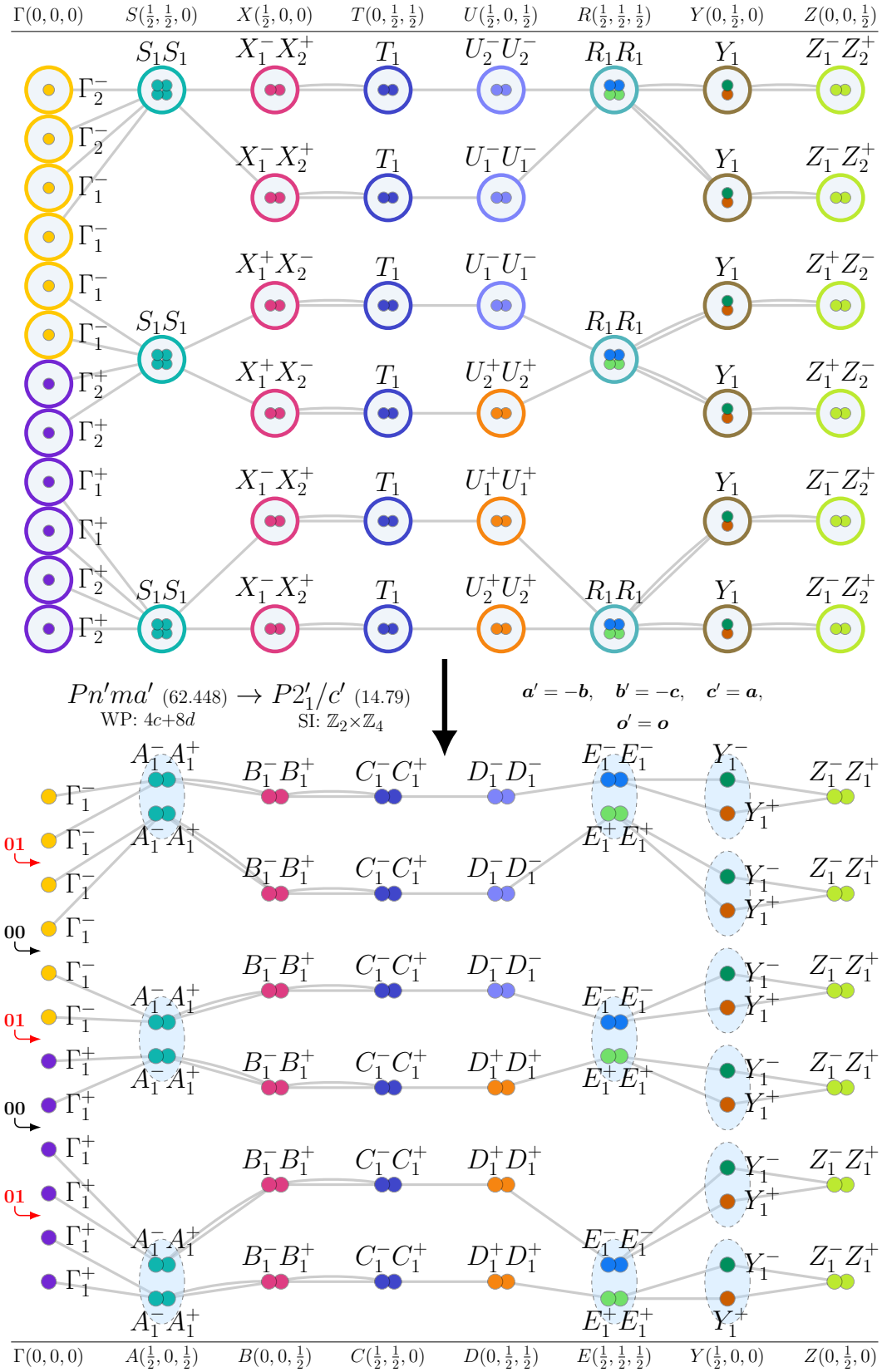
BCS Materials: Mn_3Sn_2 (262 K)[357], Mn_3Sn_2 (227 K)[358].

76.4.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).

[357]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.662`
[358]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.663`



Supplementary Figure 397: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4c+8d$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 783: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4c$+$8d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

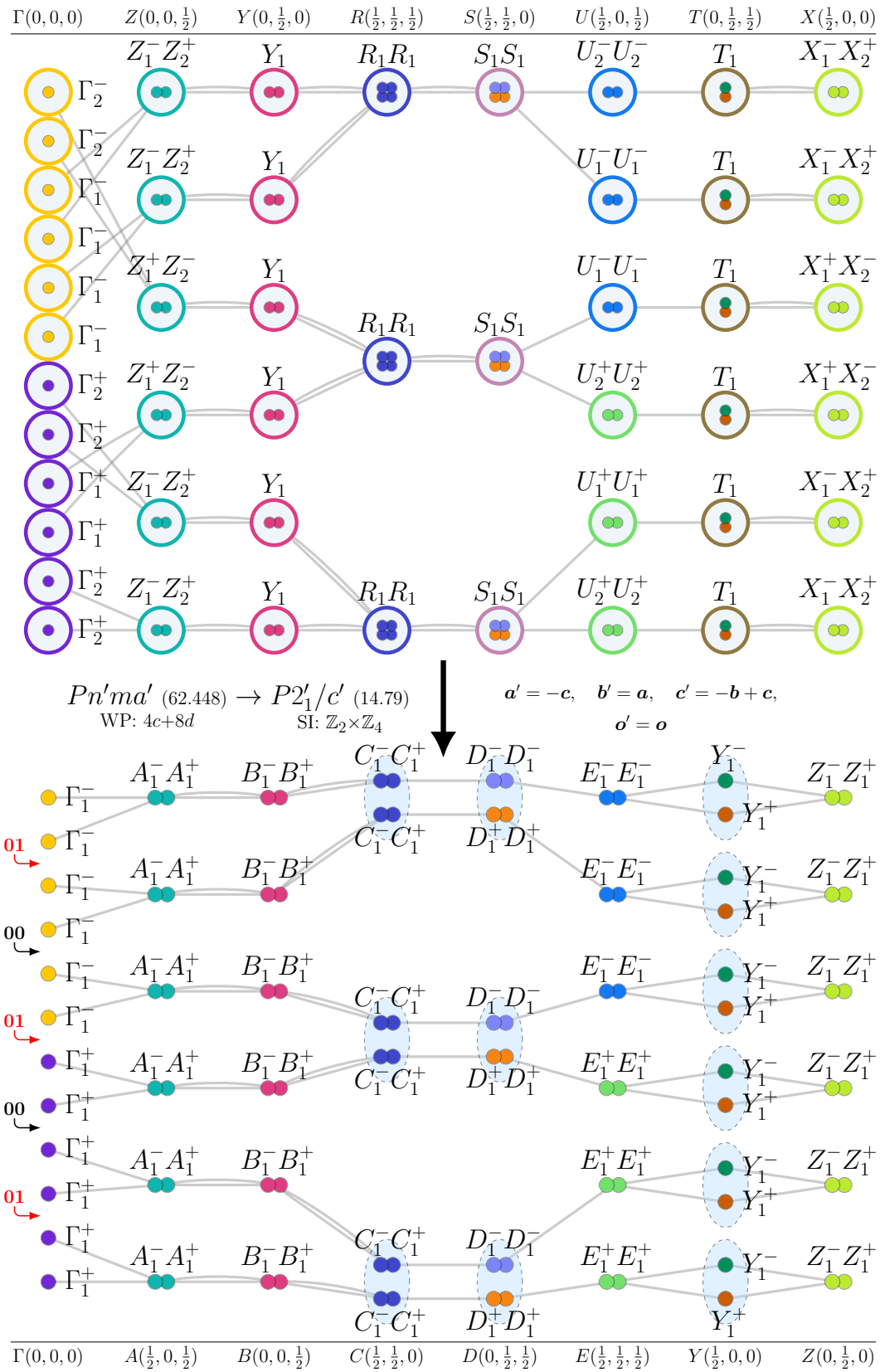
Supplementary Table 784: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4c$+$8d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

76.4.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [100]).



Supplementary Figure 398: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4c + 8d$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 785: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4c$+$8d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 786: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4c$+$8d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

76.5 WP: $4b + 4c$

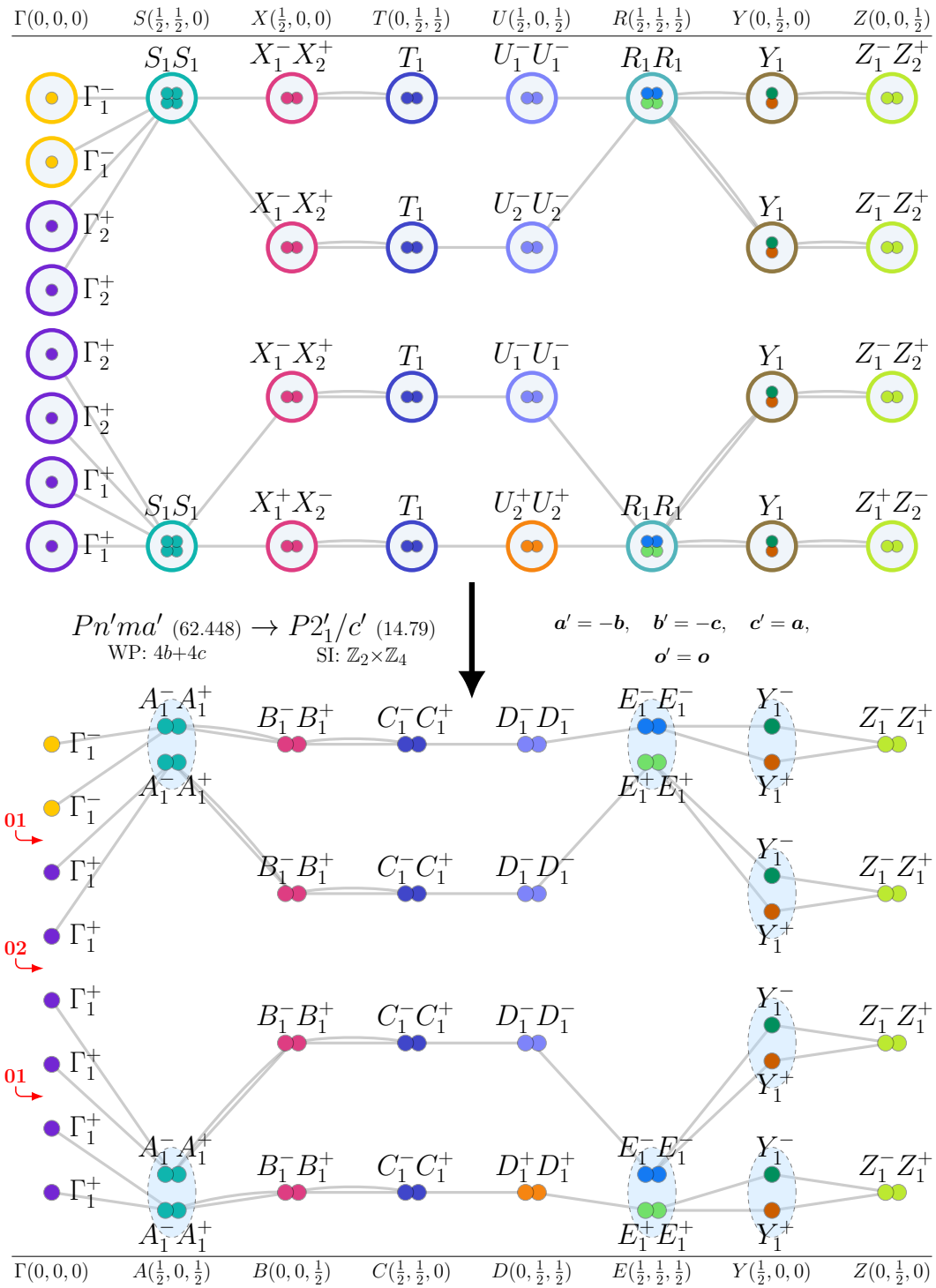
BCS Materials: $PrCrO_3$ (239 K)[359], $SmCrO_3$ (192 K)[360], $SmCrO_3$ (191 K)[361], $NdMn_{0.8}Fe_{0.2}O_3$ (59 K)[362], $Rb_2Fe_2O(AsO_4)_2$ (25 K)[363], $NdMnO_3$ (15 K)[364], $NdMn_{0.8}Fe_{0.2}O_3$ (13 K)[365], $NdMnO_3$ (13 K)[366].

76.5.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).

[359]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.588`
[360]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.696`
[361]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.479`
[362]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.660`
[363]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.91`
[364]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.371`
[365]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.659`
[366]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.289`



Supplementary Figure 399: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 787: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

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SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

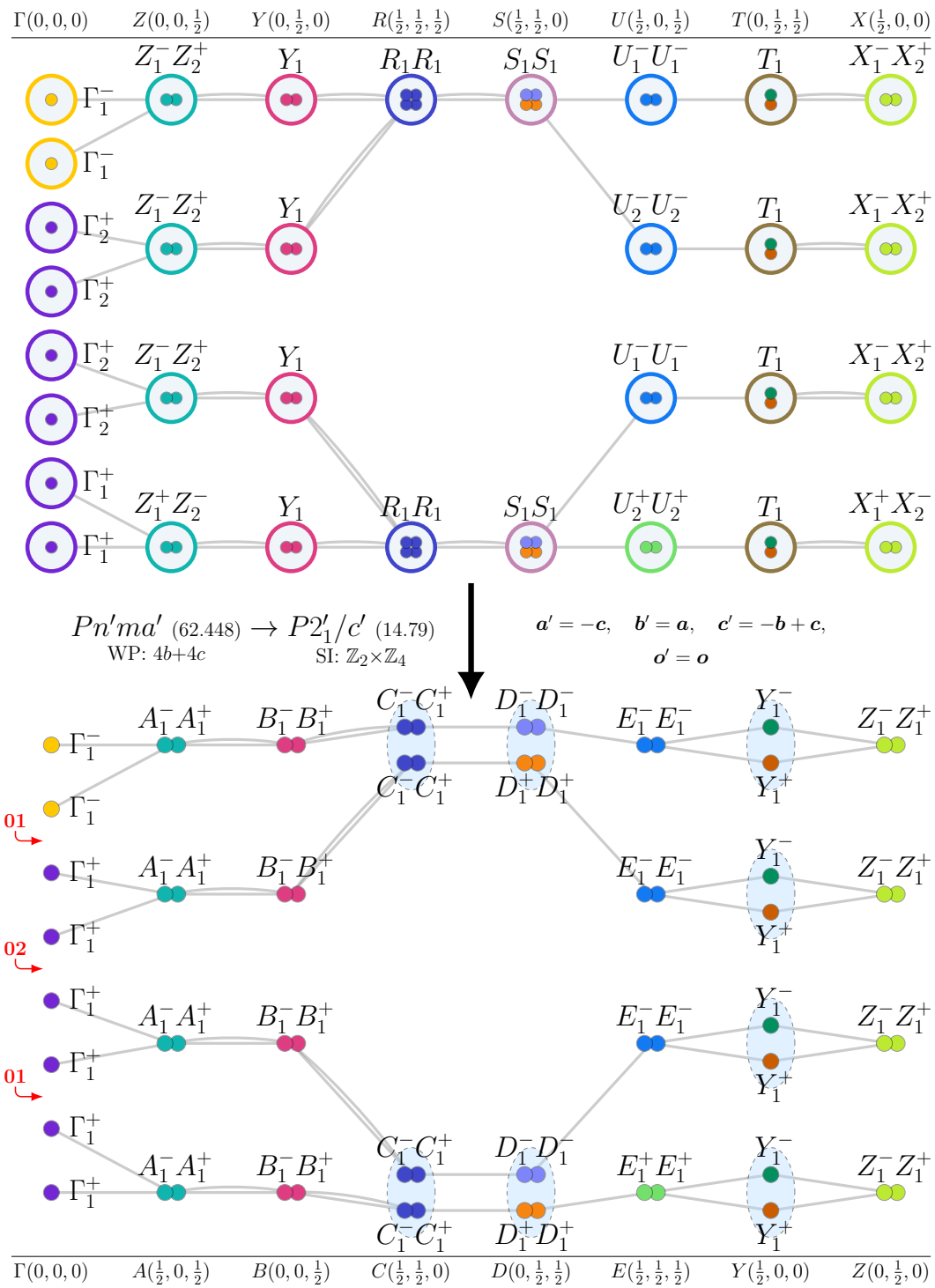
Supplementary Table 788: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

76.5.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [100]).



Supplementary Figure 400: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 789: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

continued on next page...

...continued

SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 790: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4b$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

76.6 WP: $4b + 4b$

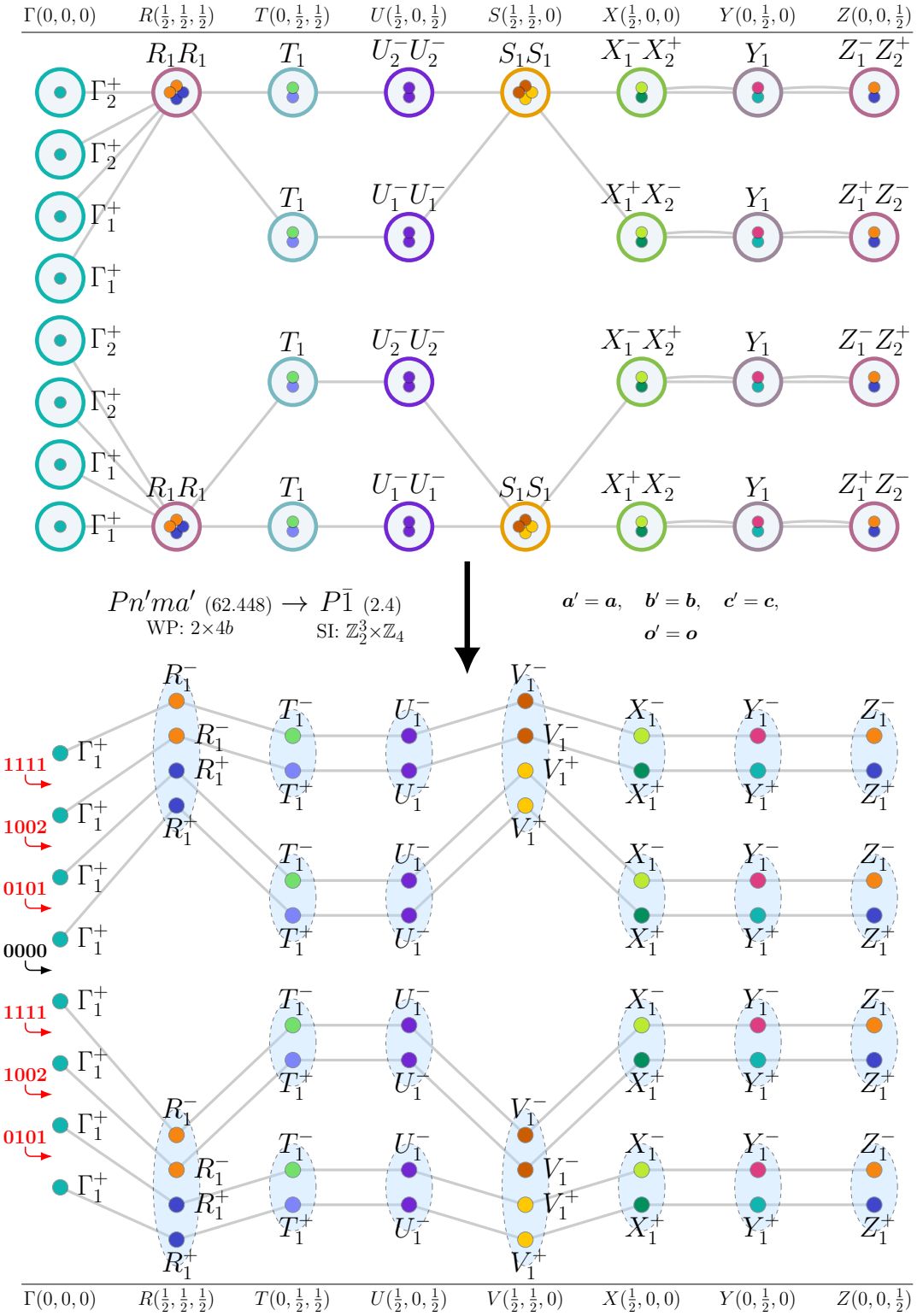
BCS Materials: $TbCr_{0.5}Mn_{0.5}O_3$ (84 K)[367].

76.6.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[367] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.679`



Supplementary Figure 401: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b+4b$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 791: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $4b$+$4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	1, 2, 3, 4, 5
0001	0, 1, 2, 3, 4
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4

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SI	Possible #Gaps
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2
0102	0, 1, 2, 3, 4
0103	0, 1, 2
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4, 5, 6
1001	0, 1, 2, 3, 4, 5, 6
1002	0, 1, 2, 3, 4, 5, 6
1003	0, 1, 2, 3, 4, 5, 6
1010	0, 1, 2, 3, 4, 5, 6
1011	0, 1, 2, 3, 4, 5, 6
1013	0, 1, 2, 3, 4, 5, 6
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	0, 1, 2
1112	0, 1, 2, 3, 4
1113	0, 1, 2
gapless	0
nontrivial	2, 3, 4, 5, 6

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Supplementary Table 792: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: 4*b*+4*b*, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
2	1000, 1001, 1002, 1003, 1010 1011, 1013
3	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
4	0000
5	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1112, 1113
6	1000, 1001, 1002, 1003, 1010 1011, 1013

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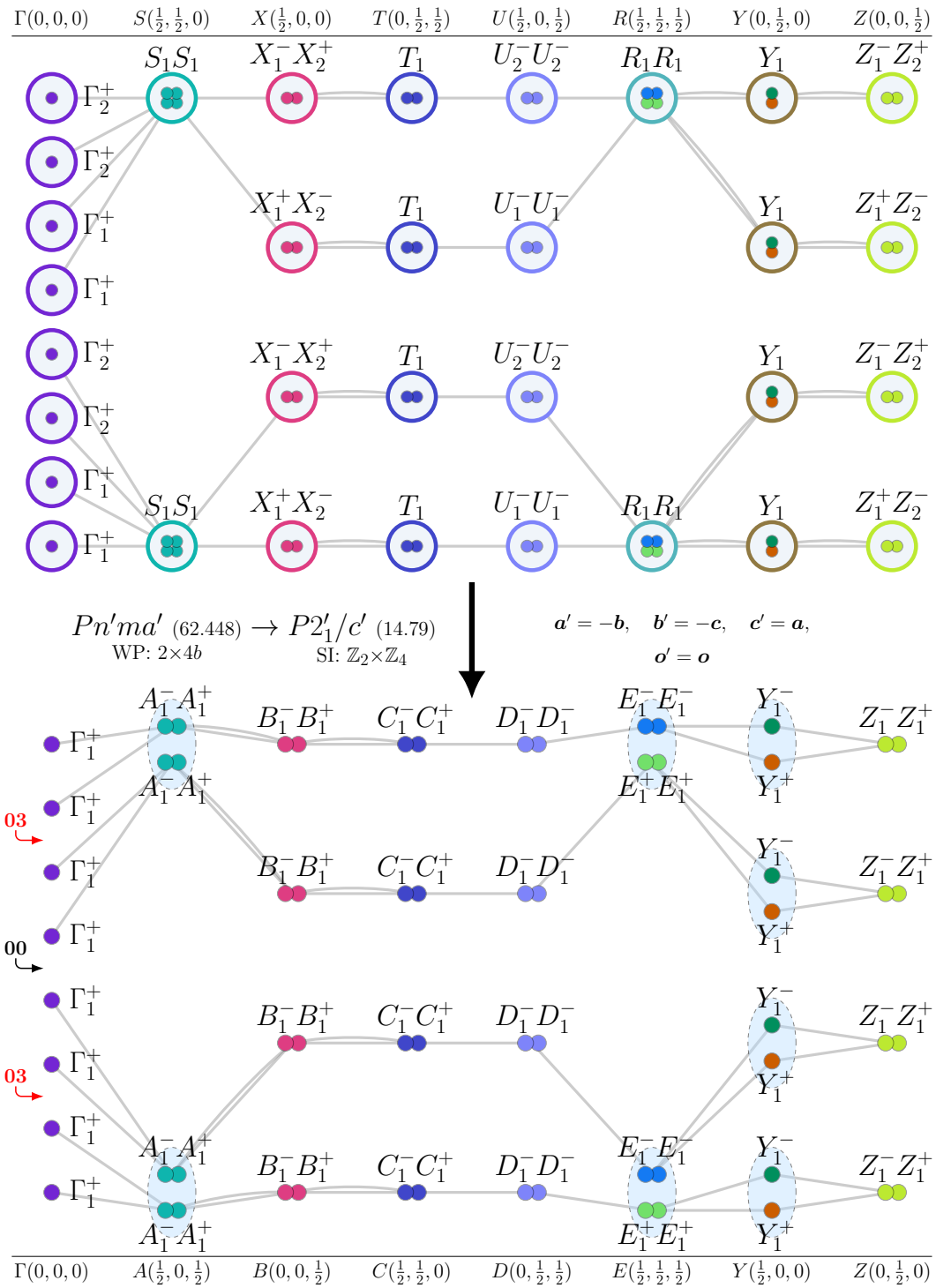
...continued

Gap#	Possible SI Values
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1013, 1100, 1101, 1102, 1103 1110, 1111, 1112
8	0000

76.6.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).



Supplementary Figure 402: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b+4b$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 793: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: 4*b*+4*b*, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	1
01	0, 1, 2
03	0, 1, 2
gapless	4
nontrivial	2

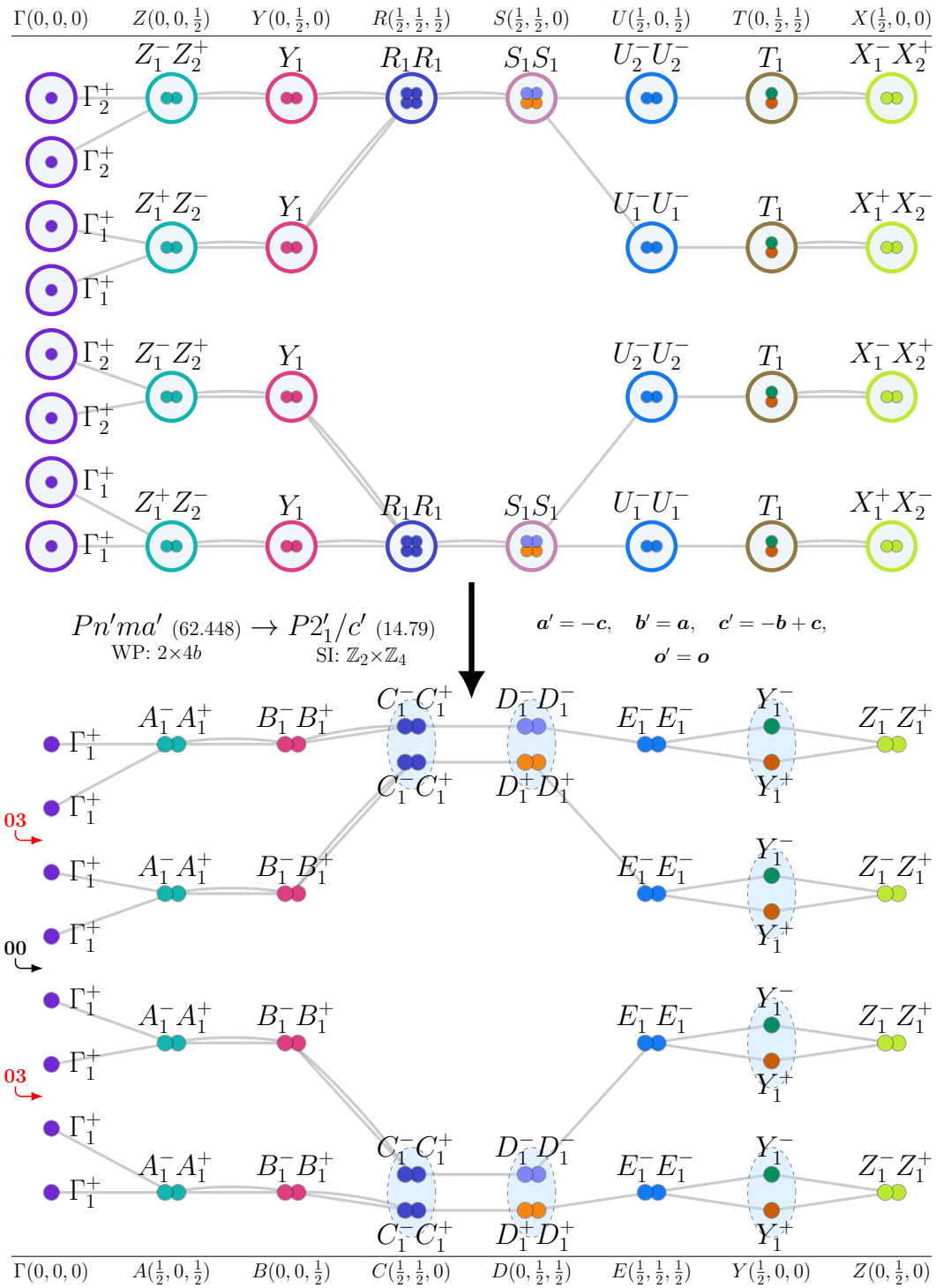
Supplementary Table 794: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $4b$+$4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00
5	G
6	01, 03
7	G
8	00

76.6.3 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [100]).



Supplementary Figure 403: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4b + 4b$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 795: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: 4*b*+4*b*, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	1
01	0, 1, 2
03	0, 1, 2
gapless	4
nontrivial	2

Supplementary Table 796: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: 4*b*+4*b*, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00
5	G
6	01, 03
7	G
8	00

76.7 WP: $8d$

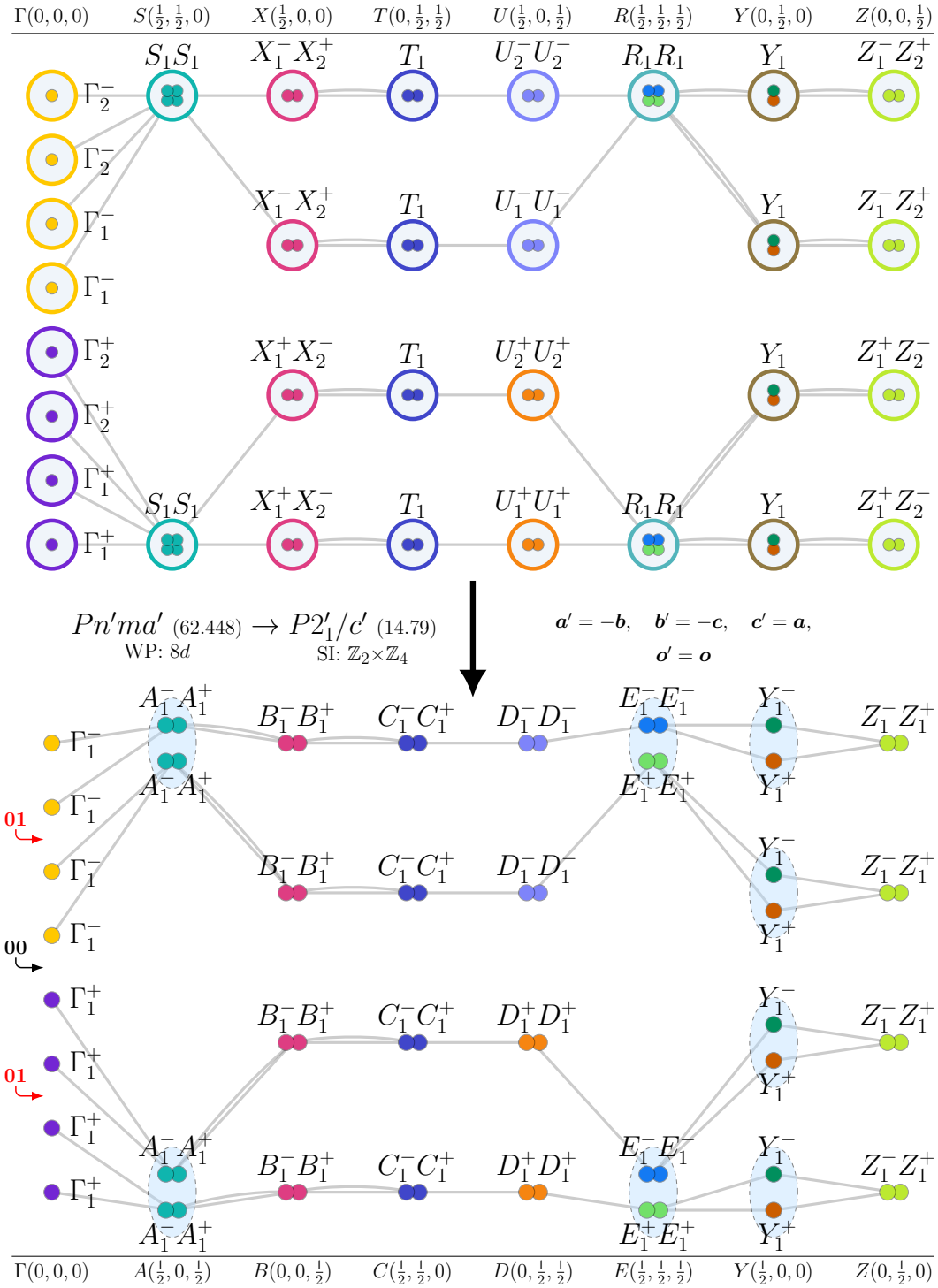
BCS Materials: $Mn_2SeO_3F_2$ (26 K)[368].

76.7.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [001],
- (B $\parallel$ [100] or B $\perp$ [001]).

[368] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.755`



Supplementary Figure 404: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8d$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 797: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

continued on next page...

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SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

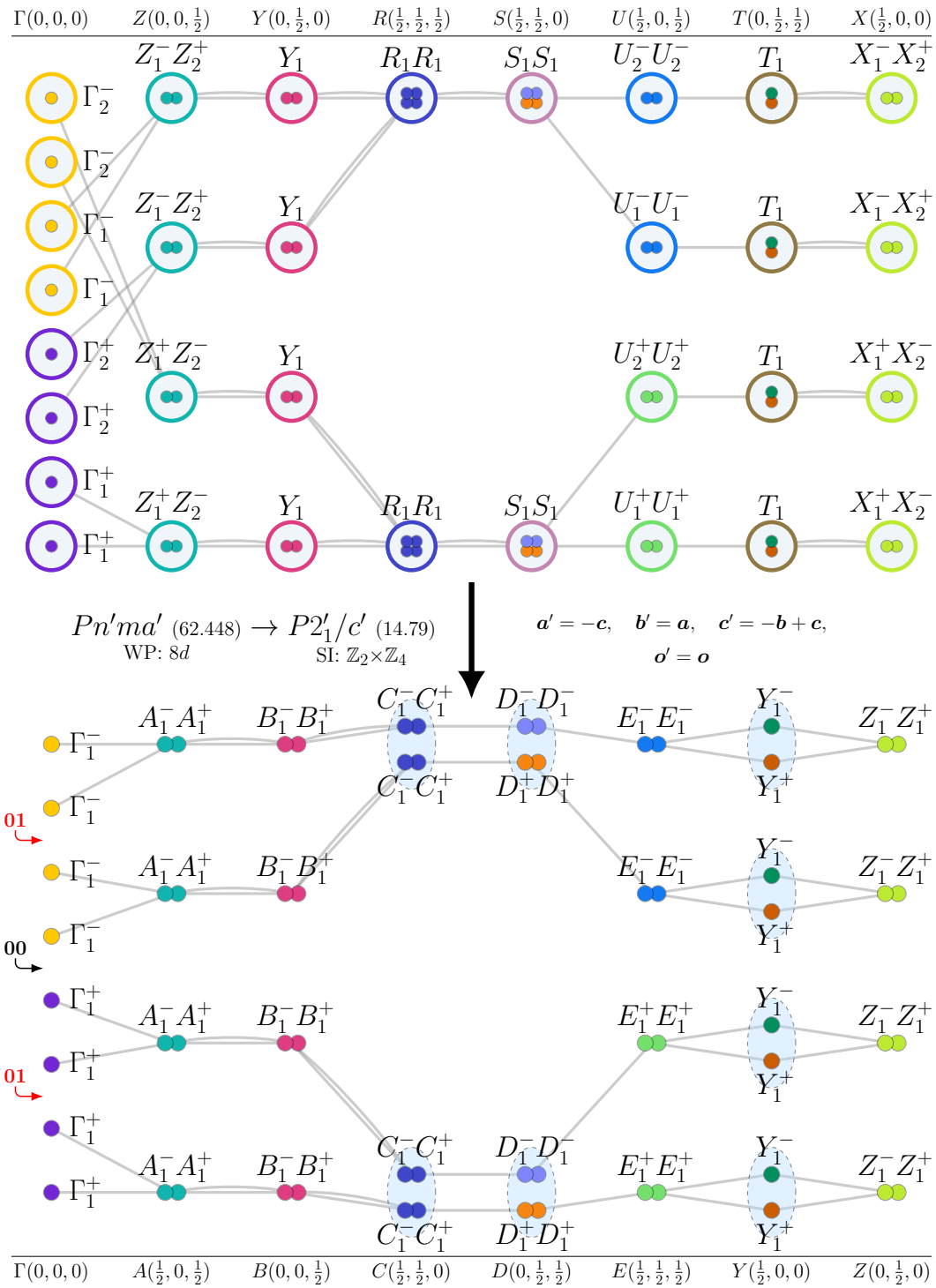
Supplementary Table 798: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

76.7.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [100]).



Supplementary Figure 405: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8d$ of supergroup $Pn'ma'$ (62.448).

Supplementary Table 799: Number of times a given SI can appear in the band structure. Supergroup: $Pn'ma'$ (62.448), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

continued on next page...

...continued

SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 800: Possible SI values in each gap. Supergroup: $Pn'ma'$ (62.448), WP: $8d$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

77 MSG P_anma (62.450)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/m'$ (11.54), $P2_1'/c'$ (14.79), $P2'/c'$ (13.69), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pc'cn'$ (56.370), P_c2_1/c (14.82), $P2_1/m$ (11.50), $Pmm'n'$ (59.410), P_a2_1/m (11.55), $P2_1/c$ (14.75), $Pnm'a'$ (62.447), P_b2_1/c (14.81).

Trivial-SI Subgroups: Pm' (6.20), Pc' (7.26), Pc' (7.26), $P2_1'$ (4.9), $P2_1'$ (4.9), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), $Pn'a2_1'$ (33.146), $Pc'c2'$ (27.80), P_cc (7.28), Pm (6.18), $Pmn'2_1'$ (31.126), $Pm'm2'$ (25.59), P_am (6.21), Pc (7.24), $Pna'2_1'$ (33.147), $Pm'n2_1'$ (31.125), P_bc (7.29), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_a2_1 (4.10), P_bmn2_1 (31.129), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_ana2_1 (33.149), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_b2_1 (4.11), P_cmc2_1 (26.73).

77.1 WP: $4a + 4b$

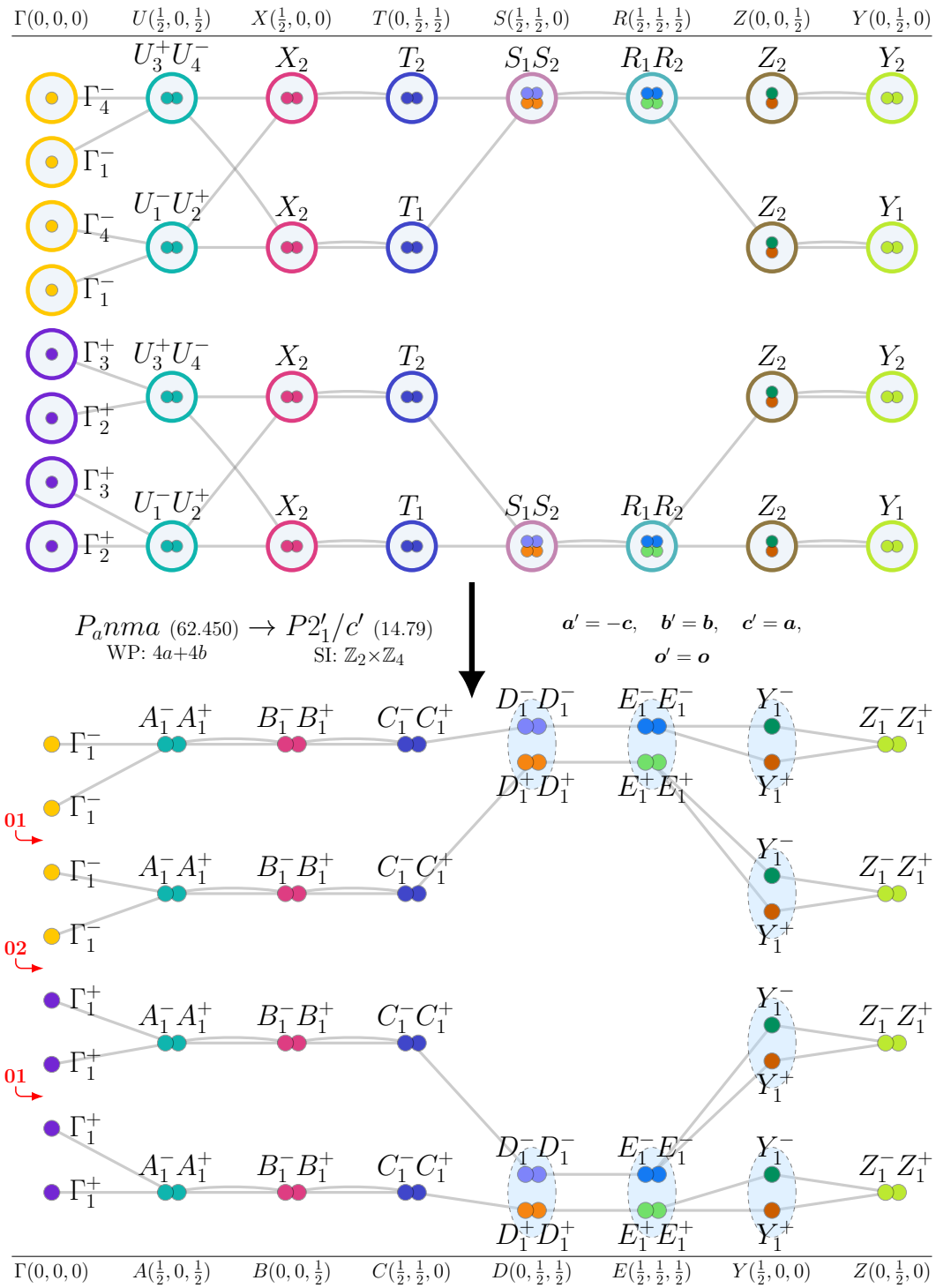
BCS Materials: Mn_2As (573 K)[369], Cr_2As (393 K)[370], Fe_2As (353 K)[371], NdCoAsO (14 K)[372].

77.1.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].

[369]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.132`
[370]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.130`
[371]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.131`
[372]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.179`



Supplementary Figure 406: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4a + 4b$ of supergroup P_anma (62.450).

Supplementary Table 801: Number of times a given SI can appear in the band structure. Supergroup: P_anma (62.450), WP: $4a$+$4b$, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1

continued on next page...

...continued

SI	Possible #Gaps
12	0, 1, 2
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 802: Possible SI values in each gap. Supergroup: P_anma (62.450), WP: $4a$+$4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

77.2 WP: $4b$

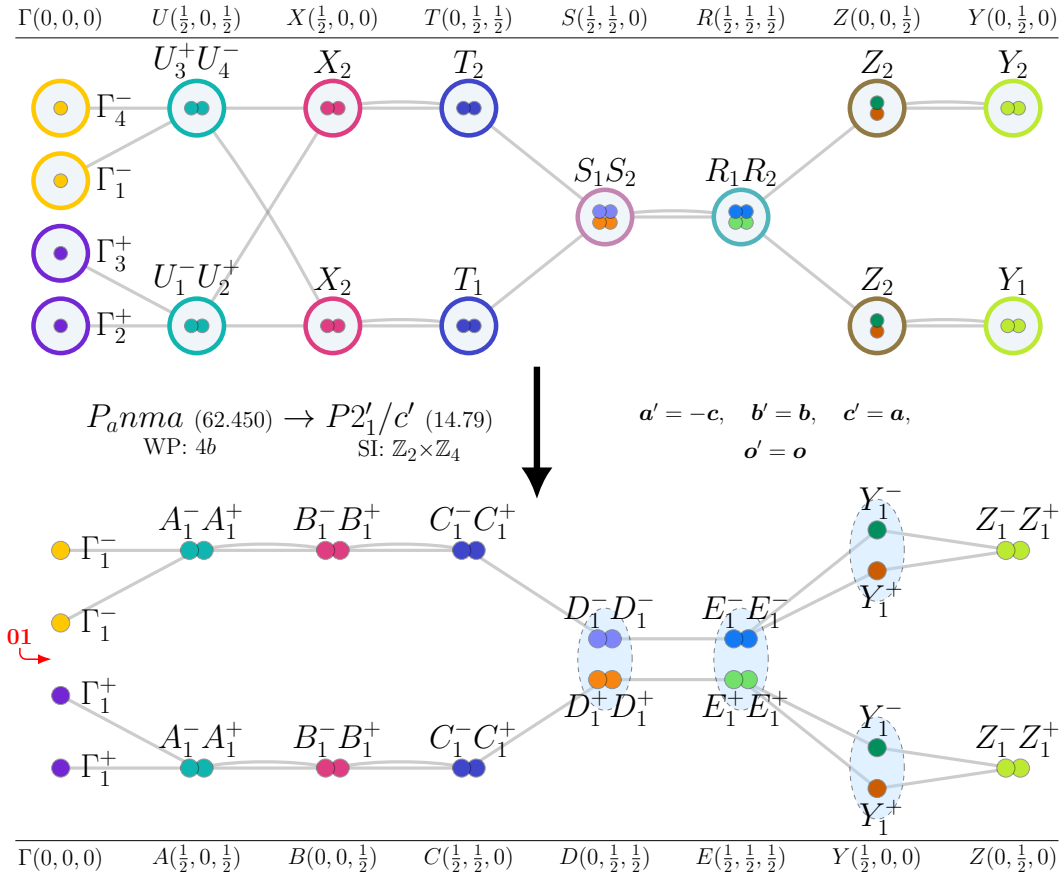
BCS Materials: CrN (273 K)[373], DyOCl (10 K)[374], $ErFe_2Si_2$ (2.6 K)[375].

77.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].

[373]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.28
[374]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.643
[375]BCS web page: http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.635



Supplementary Figure 407: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4b$ of supergroup P_anma (62.450).

Supplementary Table 803: Number of times a given SI can appear in the band structure. Supergroup: P_anma (62.450), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

Supplementary Table 804: Possible SI values in each gap. Supergroup: P_anma (62.450), WP: $4b$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

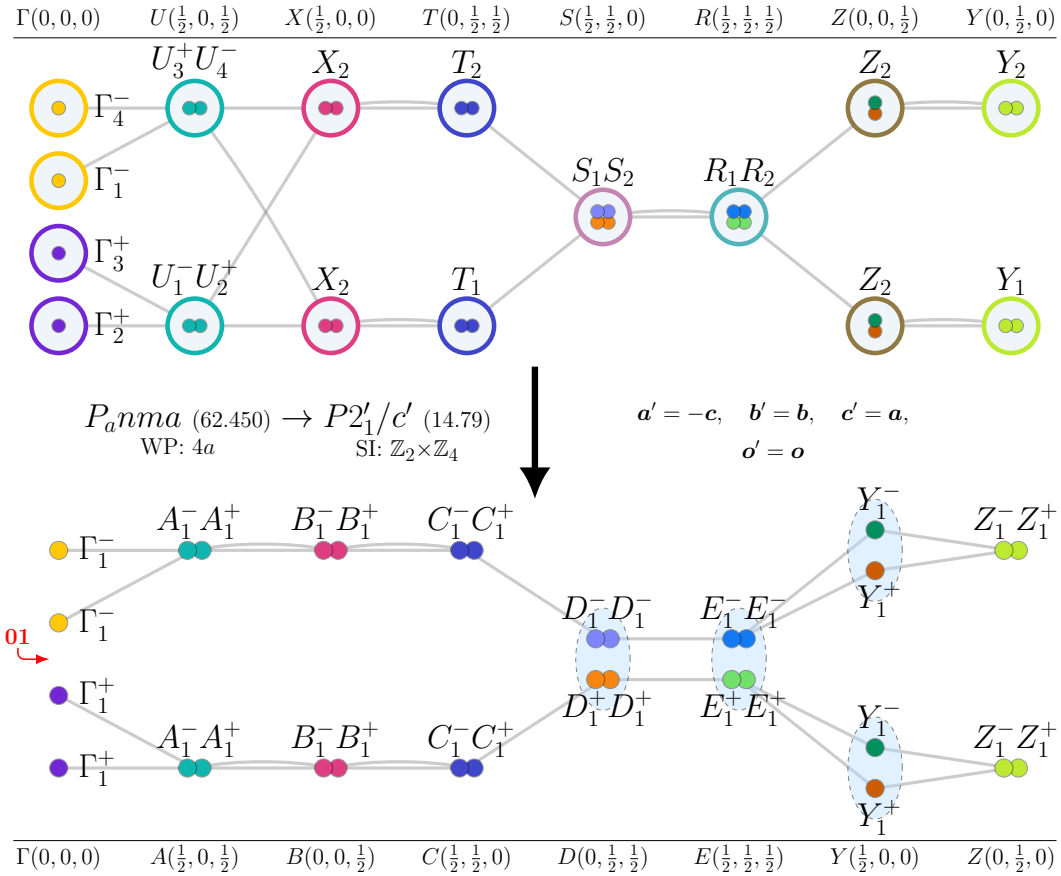
77.3 WP: $4a$

BCS Materials: $NdPd_5Al_2$ (1.3 K)[376].

77.3.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 408: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $4a$ of supergroup P_anma (62.450).

Supplementary Table 805: Number of times a given SI can appear in the band structure. Supergroup: P_anma (62.450), WP: $4a$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0
01	0, 1
03	0, 1
10	0, 1
12	0, 1
gapless	2
nontrivial	1

[376]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.507`

Supplementary Table 806: Possible SI values in each gap. Supergroup: P_anma (62.450), WP: $4a$, Subgroup: $P2'_1/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00

78 MSG P_bnma (62.451)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P2'/m'$ (10.46), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pb'am'$ (55.358), P_a2_1/c (14.80), $P2_1/m$ (11.50), $Pn'ma'$ (62.448), P_b2_1/m (11.56), $P2_1/c$ (14.75), $Pnn'm'$ (58.398), P_a2_1/c (14.80).

Trivial-SI Subgroups: Pc' (7.26), Pm' (6.20), Pc' (7.26), $P2'_1$ (4.9), $P2'$ (3.3), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), $Pb'a2'$ (32.137), $Pm'c2'_1$ (26.68), P_ac (7.27), Pm (6.18), $Pmc'2'_1$ (26.69), $Pmn'2'_1$ (31.126), P_bm (6.22), Pc (7.24), $Pm'n2'_1$ (31.125), $Pn'n2'$ (34.158), P_ac (7.27), $P2_1$ (4.7), $Pm'c'2_1$ (26.70), P_a2_1 (4.10), P_amn2_1 (31.128), $P2_1$ (4.7), $Pn'a'2_1$ (33.148), P_b2_1 (4.11), P_cna2_1 (33.151), $P2_1$ (4.7), $Pm'n'2_1$ (31.127), P_a2_1 (4.10), P_amc2_1 (26.71).

78.1 WP: $4d + 8h$

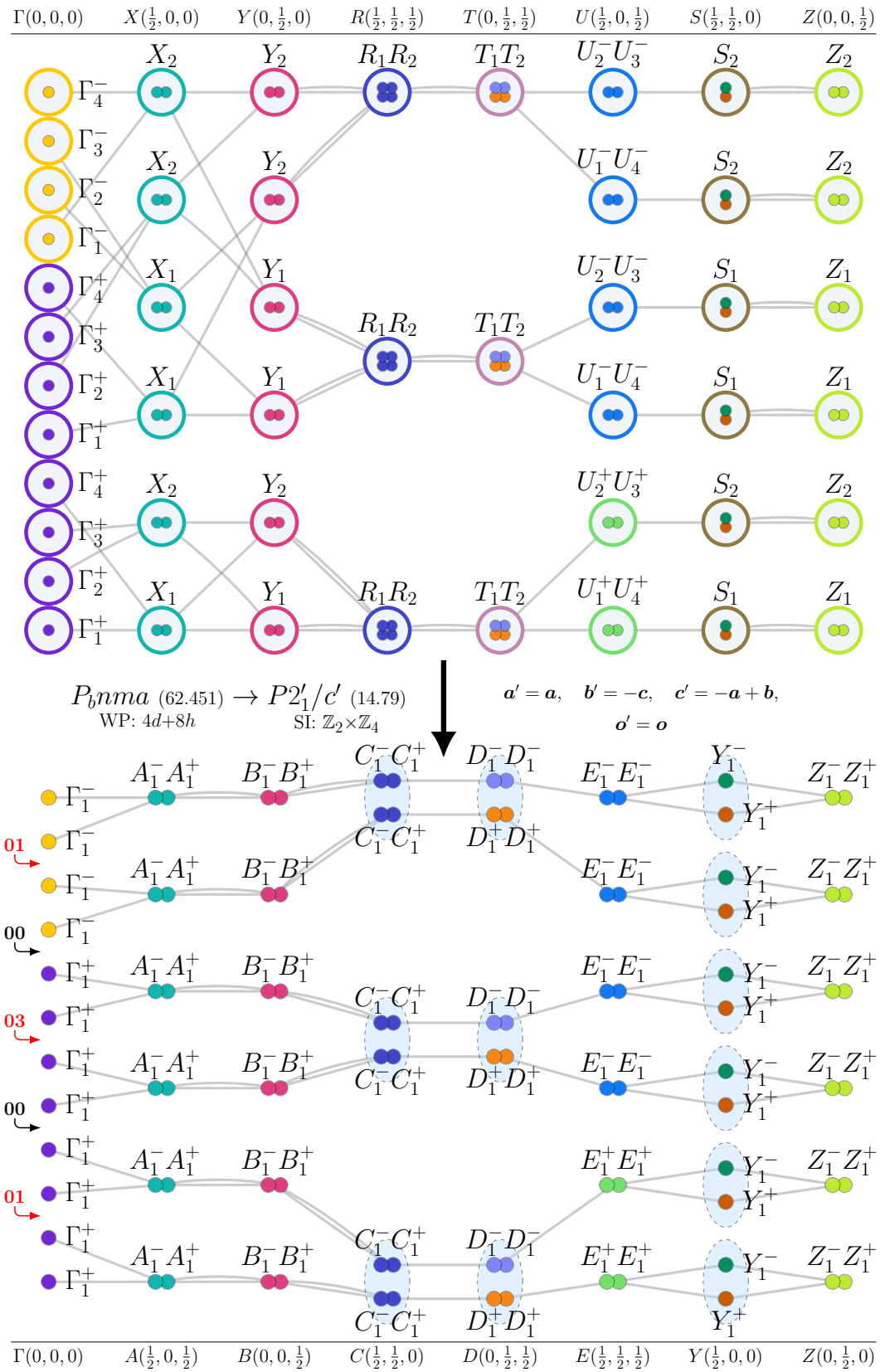
BCS Materials: Fe_2MnBO_5 (90 K)[377].

78.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[377] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.391`



Supplementary Figure 409: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4d + 8h$ of supergroup P_bnma (62.451).

Supplementary Table 807: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $4d$+$8h$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

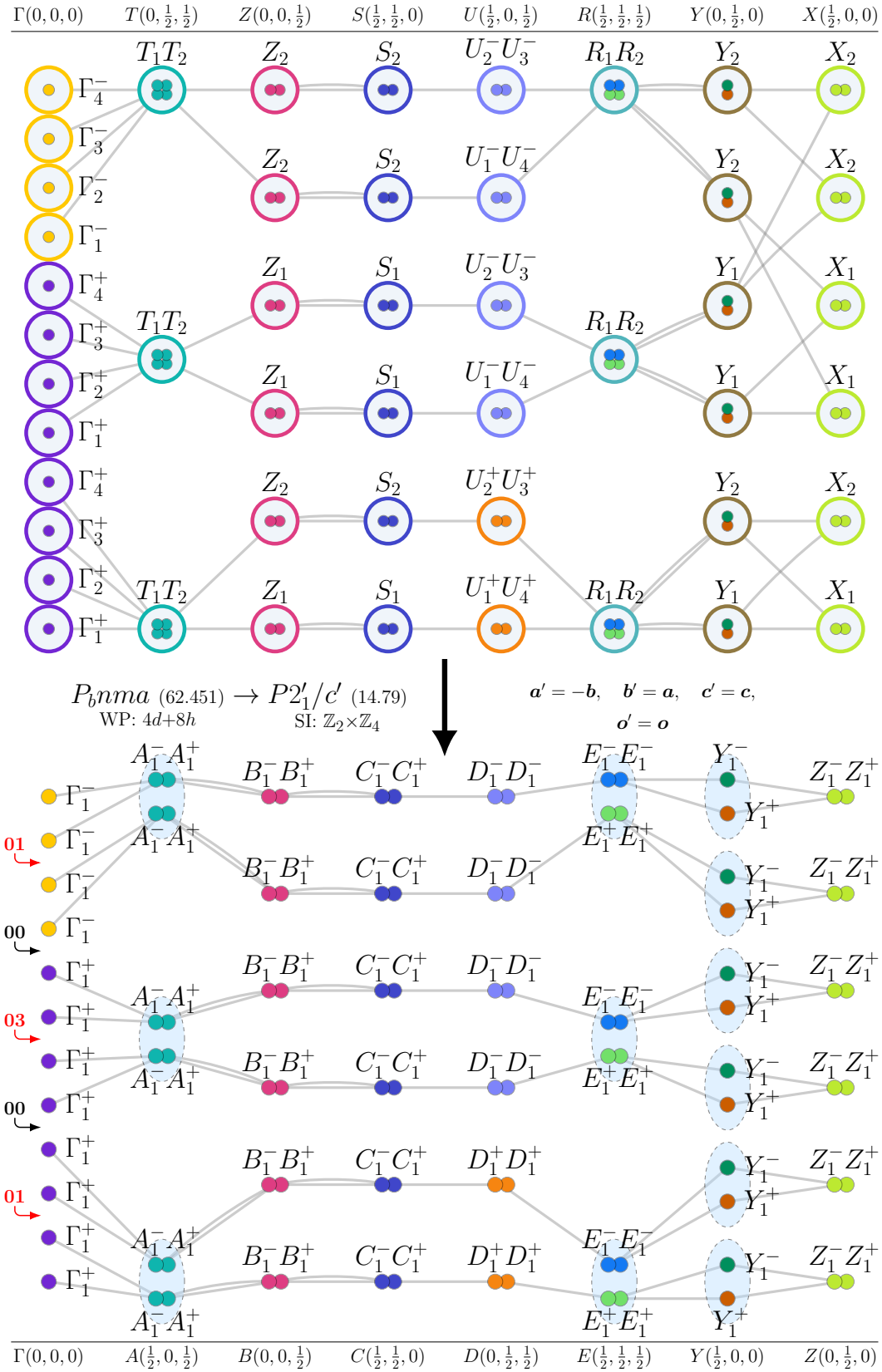
Supplementary Table 808: Possible SI values in each gap. Supergroup: P_bnma (62.451), WP: $4d$+$8h$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

78.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 410: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff positions $4d+8h$ of supergroup P_bnma (62.451).

Supplementary Table 809: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $4d$+$8h$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2
01	0, 1, 2, 3
02	0, 1, 2
03	0, 1, 2, 3
10	0, 1, 2, 3
11	0, 1, 2
12	0, 1, 2, 3
13	0, 1, 2
gapless	6
nontrivial	3, 4, 5

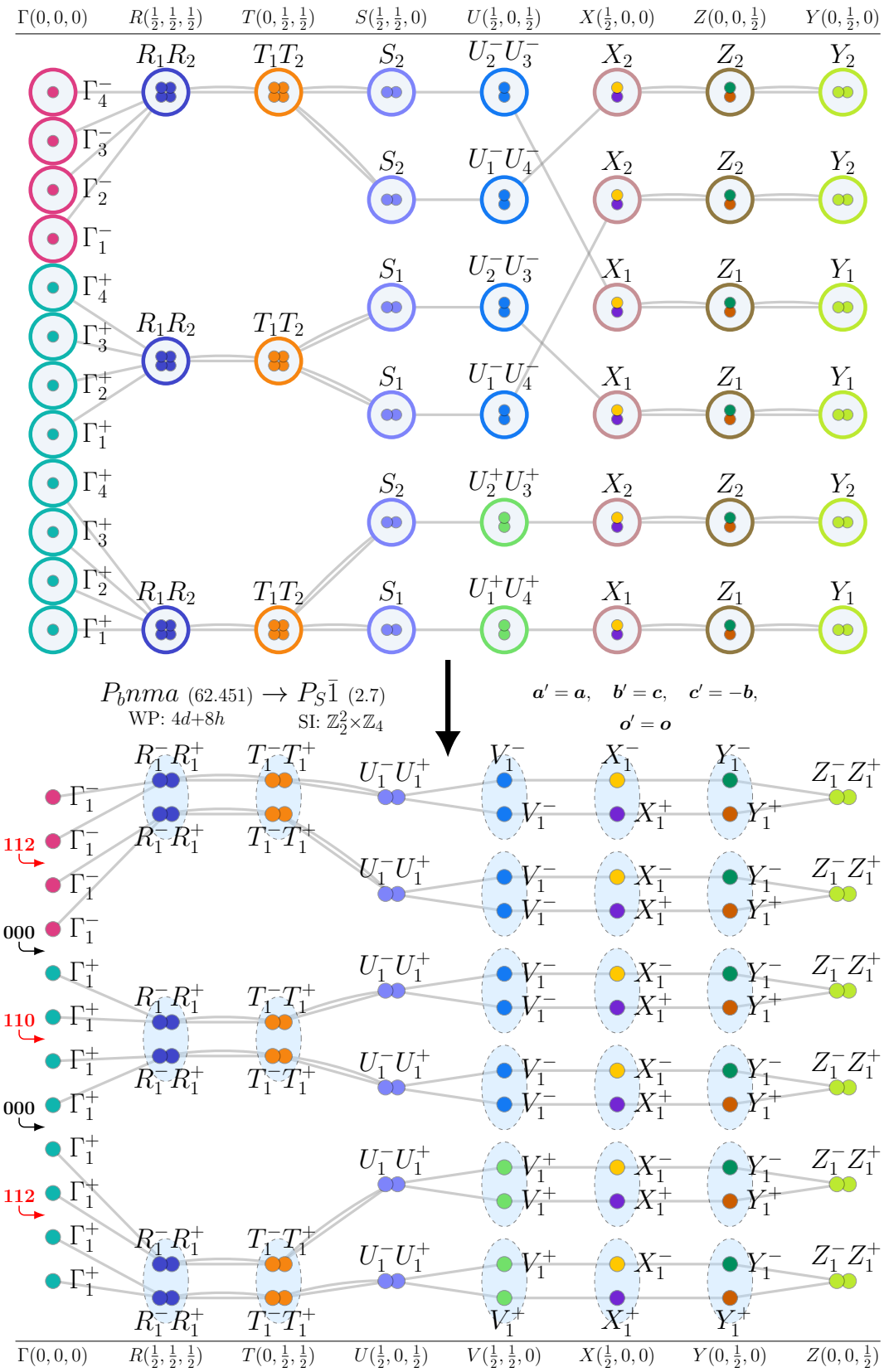
Supplementary Table 810: Possible SI values in each gap. Supergroup: P_bnma (62.451), WP: $4d$+$8h$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00

78.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 411: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4d + 8h$ of supergroup P_bnma (62.451).

Supplementary Table 811: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $4d$+$8h$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3
113	0, 1, 2
gapless	6
nontrivial	3, 4, 5

Supplementary Table 812: Possible SI values in each gap. Supergroup: P_bnma (62.451), WP: $4d$+$8h$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000, 002, 111, 113
9	G
10	001, 003, 110, 112
11	G
12	000

78.2 WP: $8g$

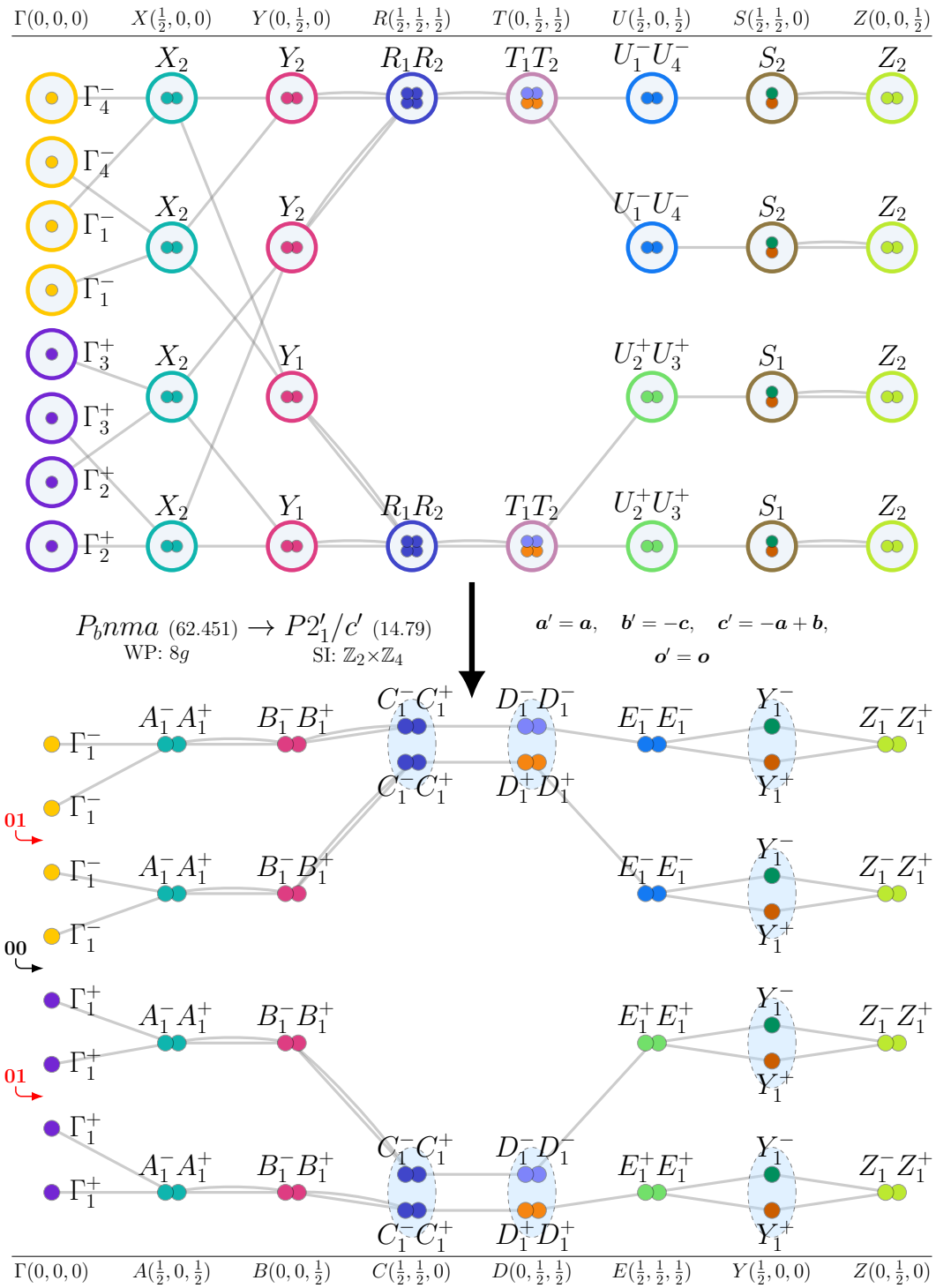
BCS Materials: $Sr_2Fe_3S_2O_3$ (76 K)[378], Pr_2Pd_2In (5 K)[379].

78.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[378] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.625`
[379] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.334`



Supplementary Figure 412: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8g$ of supergroup P_bnma (62.451).

Supplementary Table 813: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

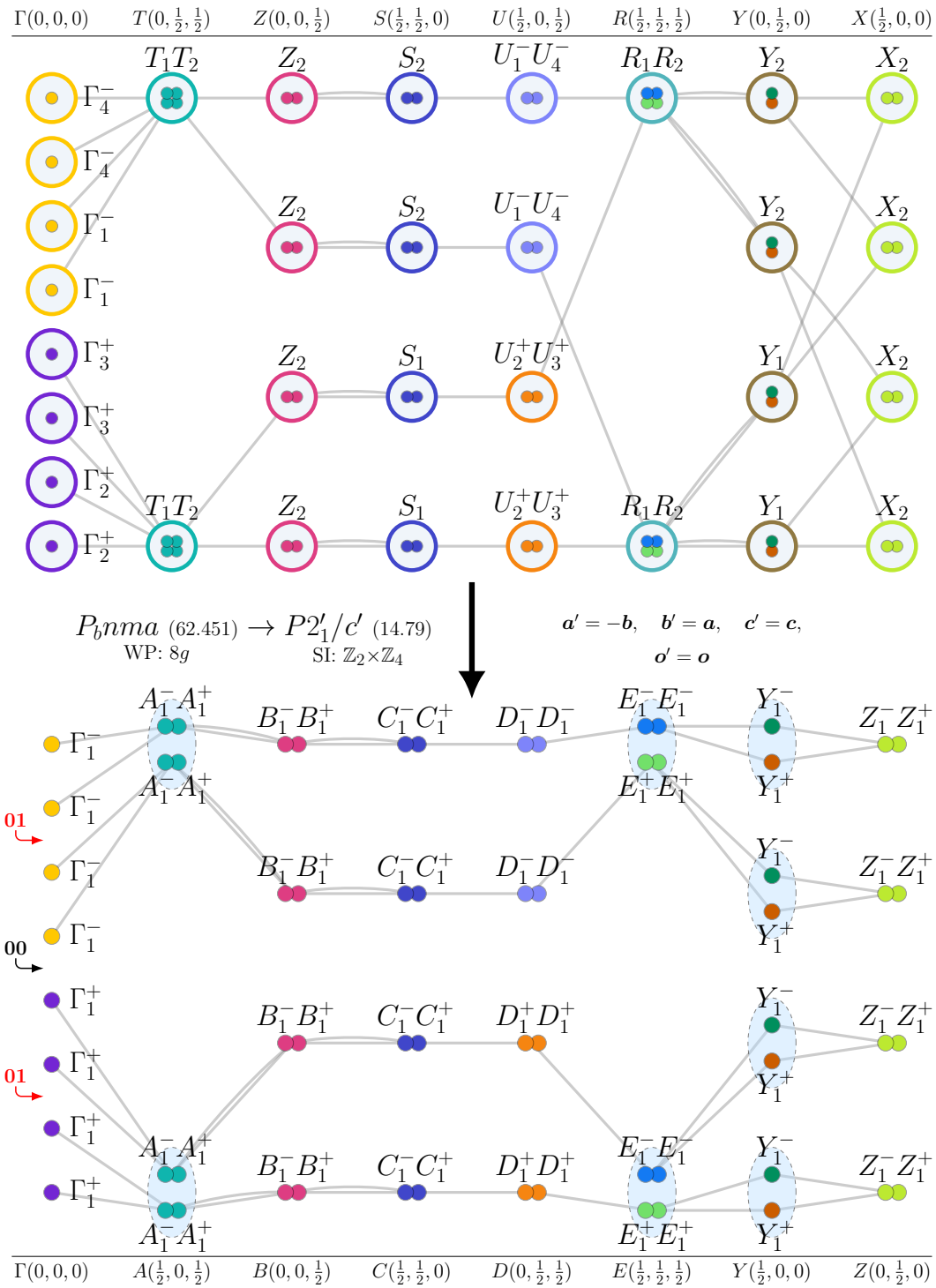
Supplementary Table 814: Possible SI values in each gap. Supergroup: $P_{b}nma$ (62.451), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

78.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 413: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8g$ of supergroup P_bnma (62.451).

Supplementary Table 815: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

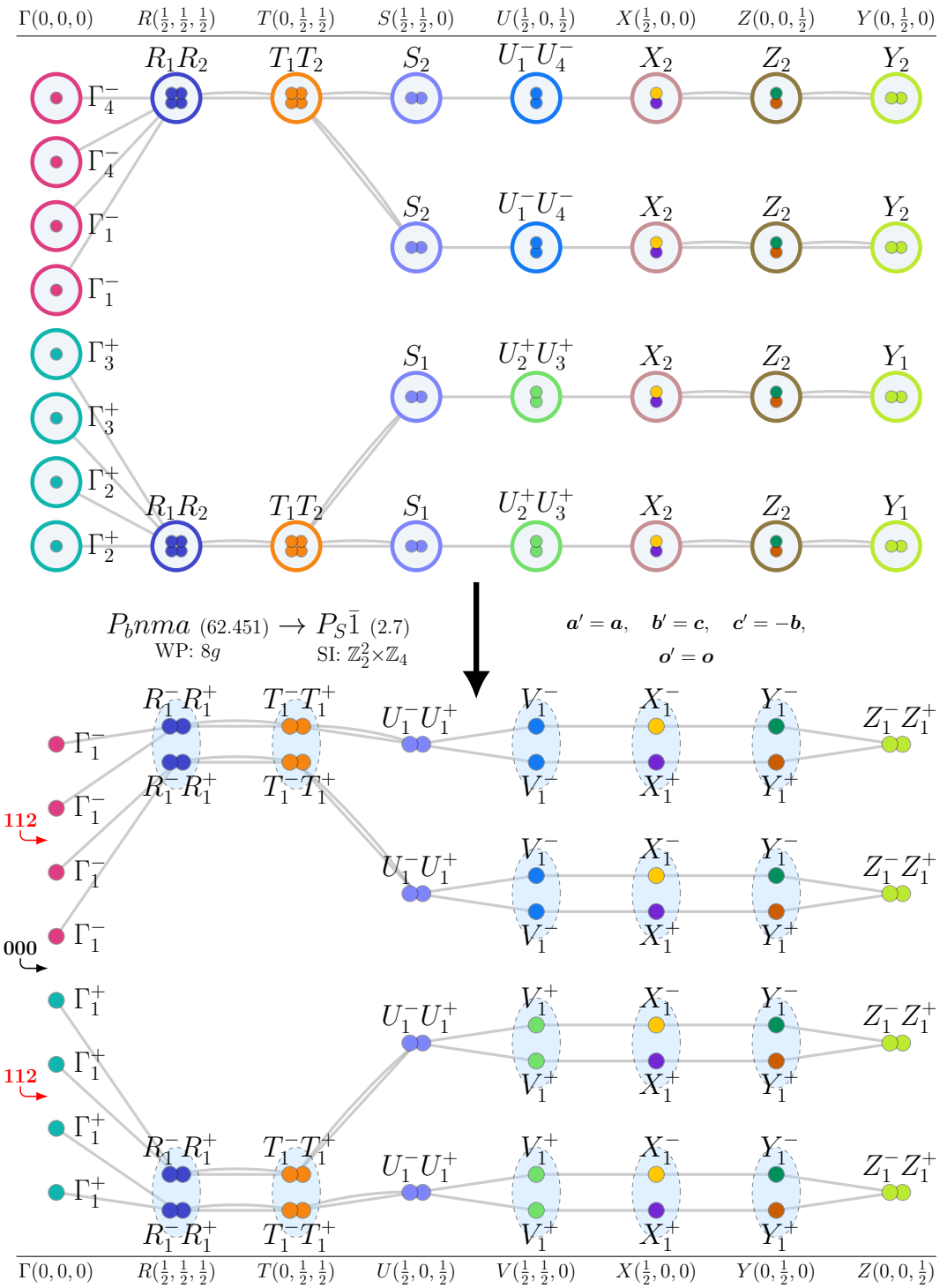
Supplementary Table 816: Possible SI values in each gap. Supergroup: $P_{b}nma$ (62.451), WP: $8g$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

78.2.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 414: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8g$ of supergroup P_bnma (62.451).

Supplementary Table 817: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 818: Possible SI values in each gap. Supergroup: P_bnma (62.451), WP: $8g$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

78.3 WP: $8h$

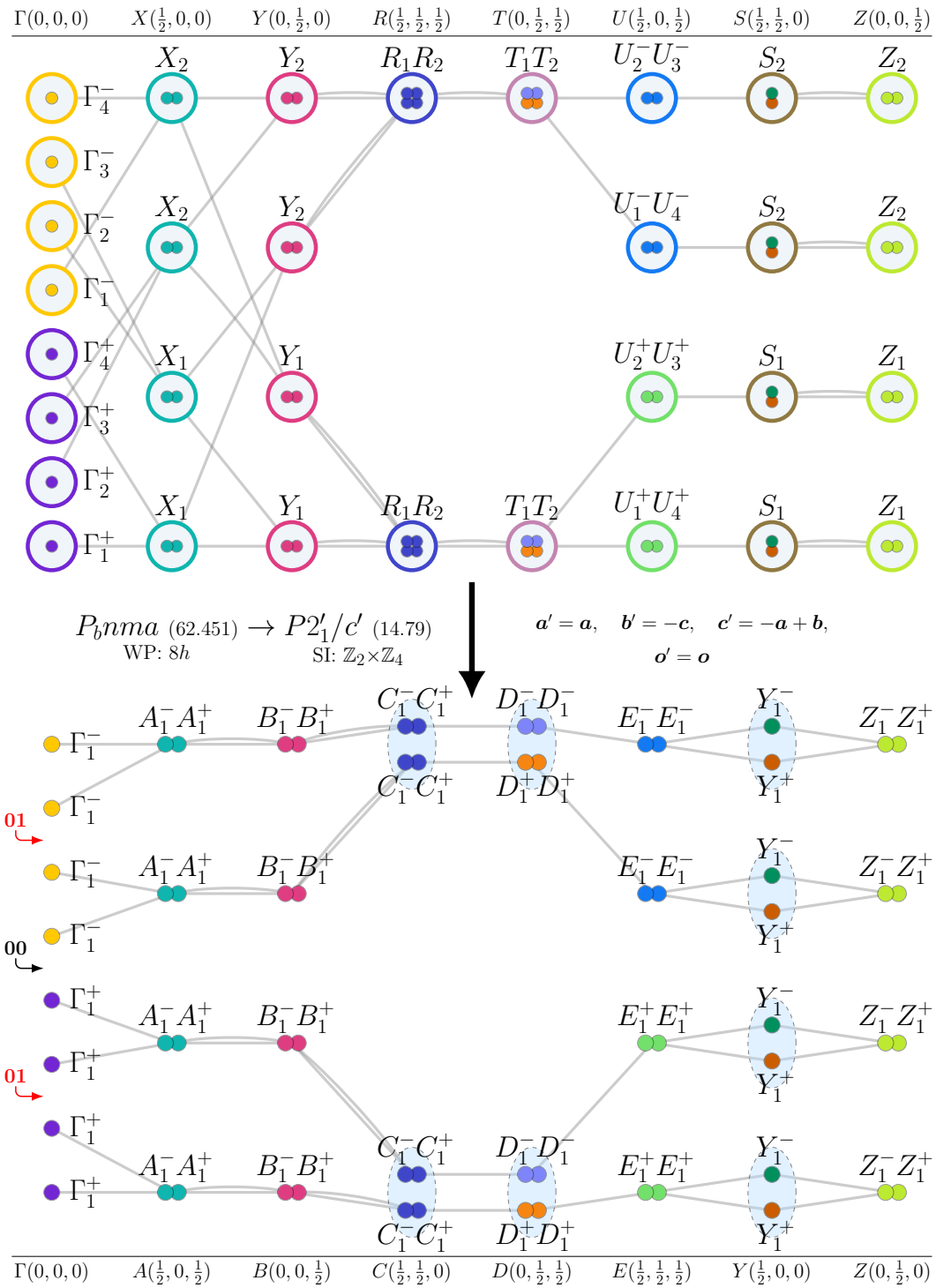
BCS Materials: $Tb_2Pd_{2.05}Sn_{0.95}$ (20.8 K)[380].

78.3.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].

[380]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.336`



Supplementary Figure 415: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8h$ of supergroup P_bnma (62.451).

Supplementary Table 819: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $8h$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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...continued

SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

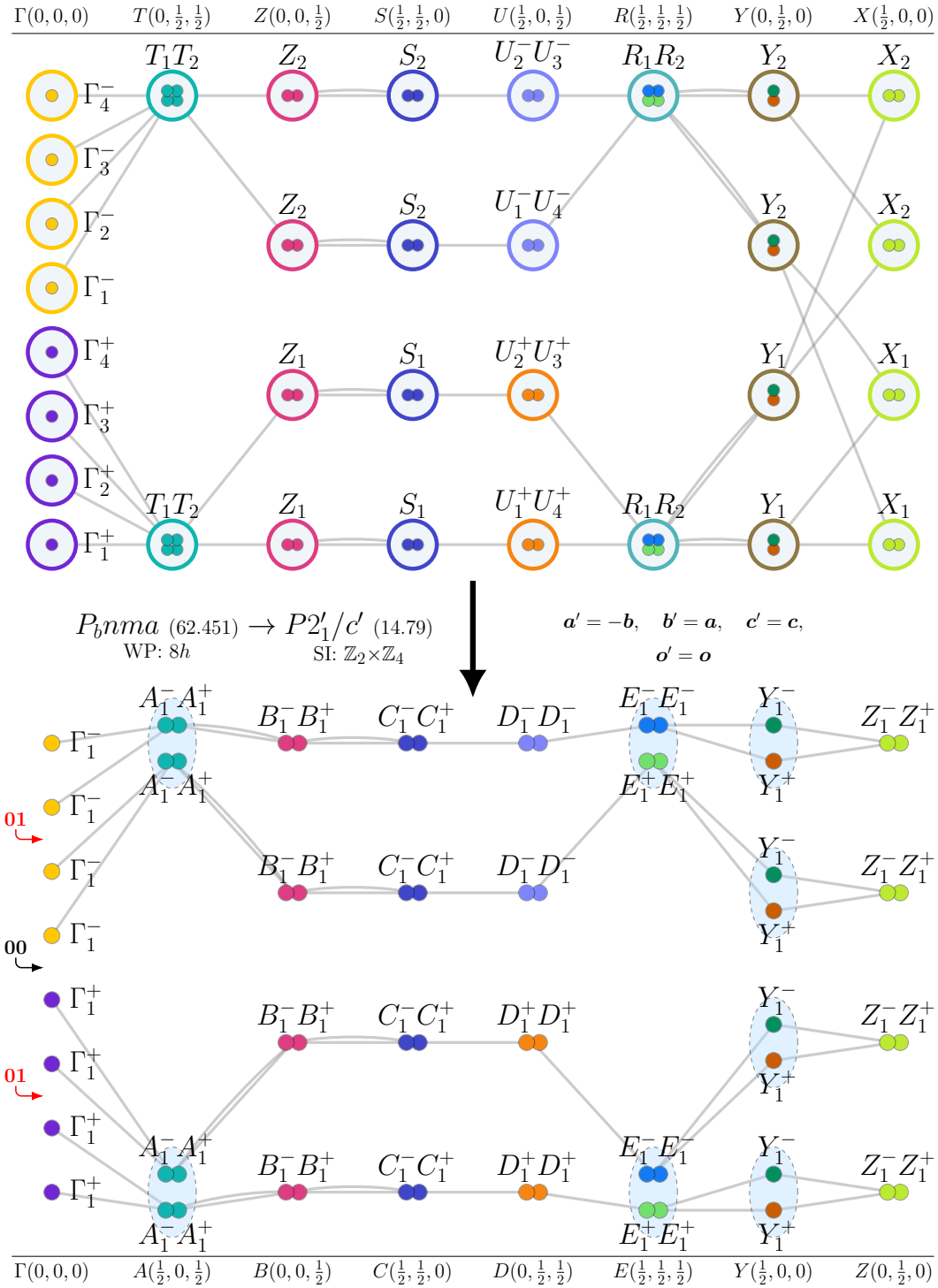
Supplementary Table 820: Possible SI values in each gap. Supergroup: P_bnma (62.451), WP: $8h$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

78.3.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 416: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8h$ of supergroup P_bnma (62.451).

Supplementary Table 821: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $8h$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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...continued

SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

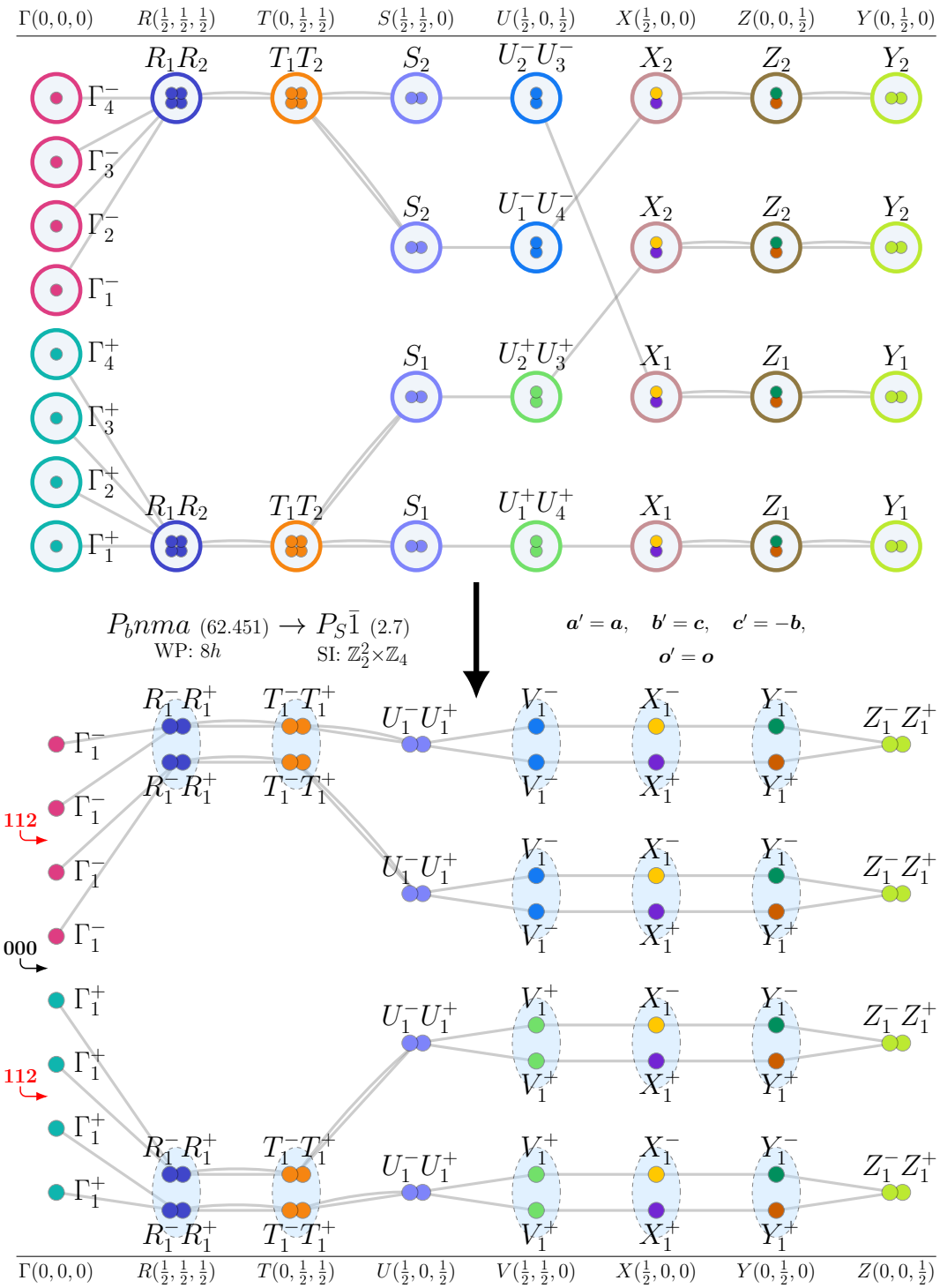
Supplementary Table 822: Possible SI values in each gap. Supergroup: $P_{b}nma$ (62.451), WP: $8h$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

78.3.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 417: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8h$ of supergroup P_bnma (62.451).

Supplementary Table 823: Number of times a given SI can appear in the band structure. Supergroup: P_bnma (62.451), WP: $8h$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
113	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 824: Possible SI values in each gap. Supergroup: P_bnma (62.451), WP: $8h$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	001, 003, 110, 112
3	G
4	000, 002, 111, 113
5	G
6	001, 003, 110, 112
7	G
8	000

79 MSG P_cnma (62.452)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $P2'_1/c'$ (14.79), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2_1/c$ (14.75), $Pb'c'a$ (61.436), P_b2_1/c (14.81), $P2_1/m$ (11.50), $Pb'c'm$ (57.382), P_a2_1/m (11.55), $P2_1/c$ (14.75), $Pb'c'n$ (60.422), P_a2_1/c (14.80).

Trivial-SI Subgroups: Pc' (7.26), Pc' (7.26), Pc' (7.26), $P2'$ (3.3), $P2'_1$ (4.9), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), $Pc'a2'_1$ (29.101), $Pca'2'_1$ (29.102), P_bc (7.29), Pm (6.18), $Pma'2'$ (28.90), $Pmc'2'_1$ (26.69), P_am (6.21), Pc (7.24), $Pnc'2'$ (30.114), $Pna'2'_1$ (33.147), P_ac (7.27), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_b2_1 (4.11), P_cmn2_1 (31.130), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_a2_1 (4.10), P_bna2_1 (33.150), $P2_1$ (4.7), $Pc'a'2_1$ (29.103), P_a2_1 (4.10), P_bmc2_1 (26.72).

79.1 WP: $16e$

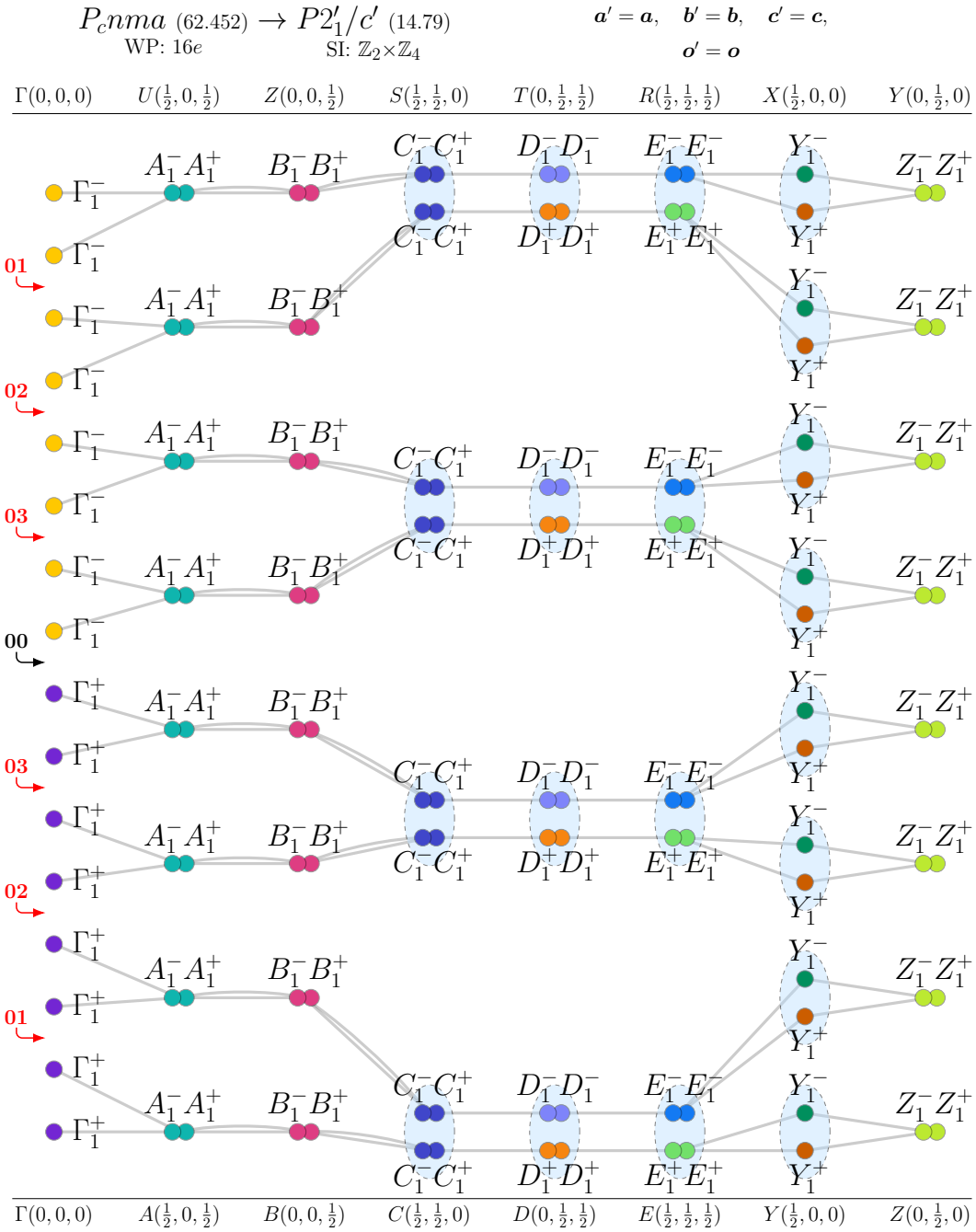
BCS Materials: Mn_3O_4 (210 K)[381].

79.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].

[381] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.1`



Supplementary Figure 418: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $16e$ of supergroup P_cnma (62.452).

Supplementary Table 825: Number of times a given SI can appear in the band structure. Supergroup: P_cnma (62.452), WP: $16e$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2, 3
01	0, 1, 2, 3, 4
02	0, 1, 2, 3
03	0, 1, 2, 3, 4
10	0, 1, 2, 3, 4
11	0, 1, 2, 3
12	0, 1, 2, 3, 4
13	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

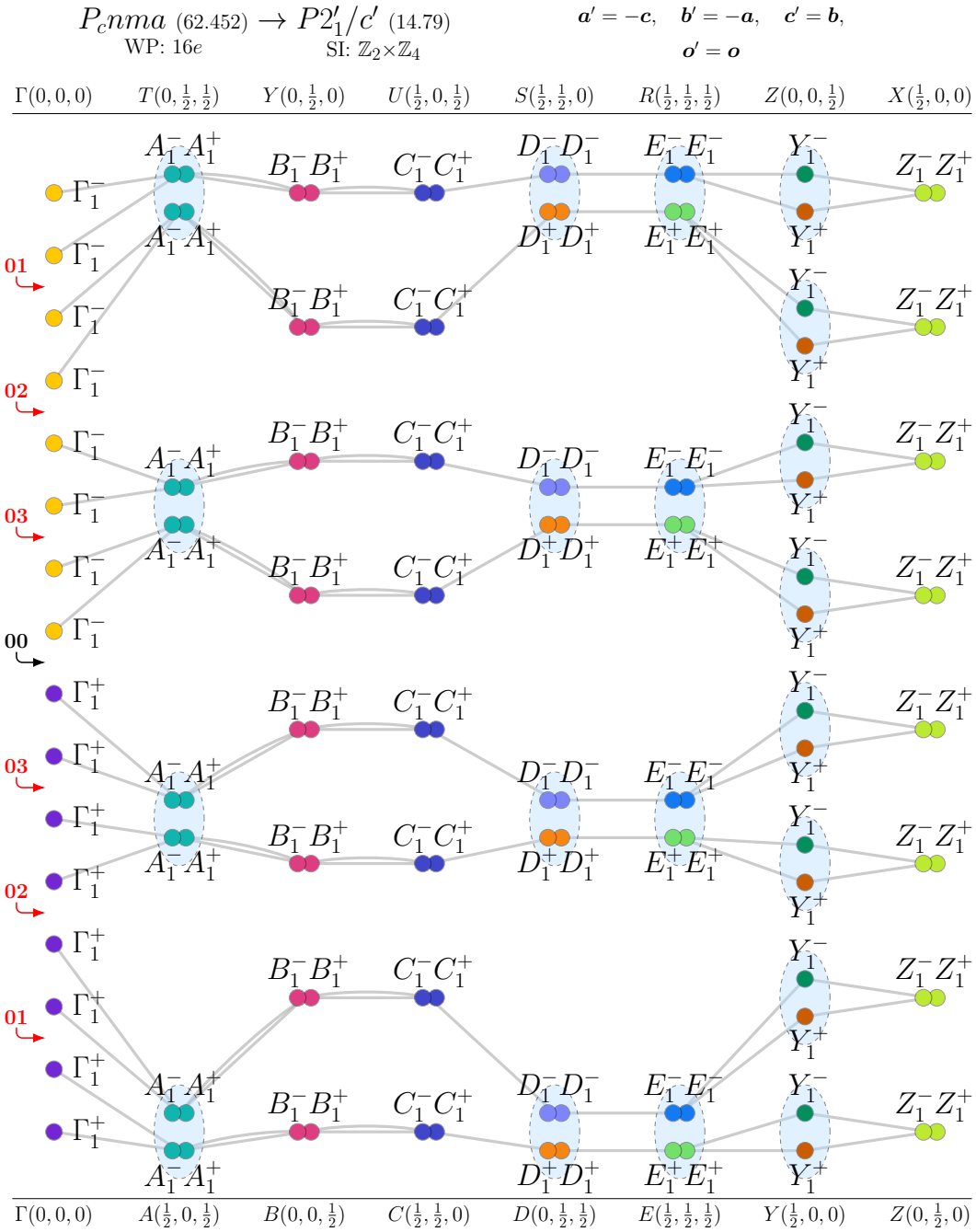
Supplementary Table 826: Possible SI values in each gap. Supergroup: P_cnma (62.452), WP: $16e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00, 02, 11, 13
13	G
14	01, 03, 10, 12
15	G
16	00

79.1.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 419: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position 16e of supergroup P_cnma (62.452).

Supplementary Table 827: Number of times a given SI can appear in the band structure. Supergroup: P_cnma (62.452), WP: 16e, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1, 2, 3
01	0, 1, 2, 3, 4
02	0, 1, 2, 3
03	0, 1, 2, 3, 4
10	0, 1, 2, 3, 4
11	0, 1, 2, 3
12	0, 1, 2, 3, 4
13	0, 1, 2, 3
gapless	8
nontrivial	4, 5, 6, 7

Supplementary Table 828: Possible SI values in each gap. Supergroup: P_cnma (62.452), WP: $16e$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00, 02, 11, 13
9	G
10	01, 03, 10, 12
11	G
12	00, 02, 11, 13
13	G
14	01, 03, 10, 12
15	G
16	00

79.2 WP: $8c$

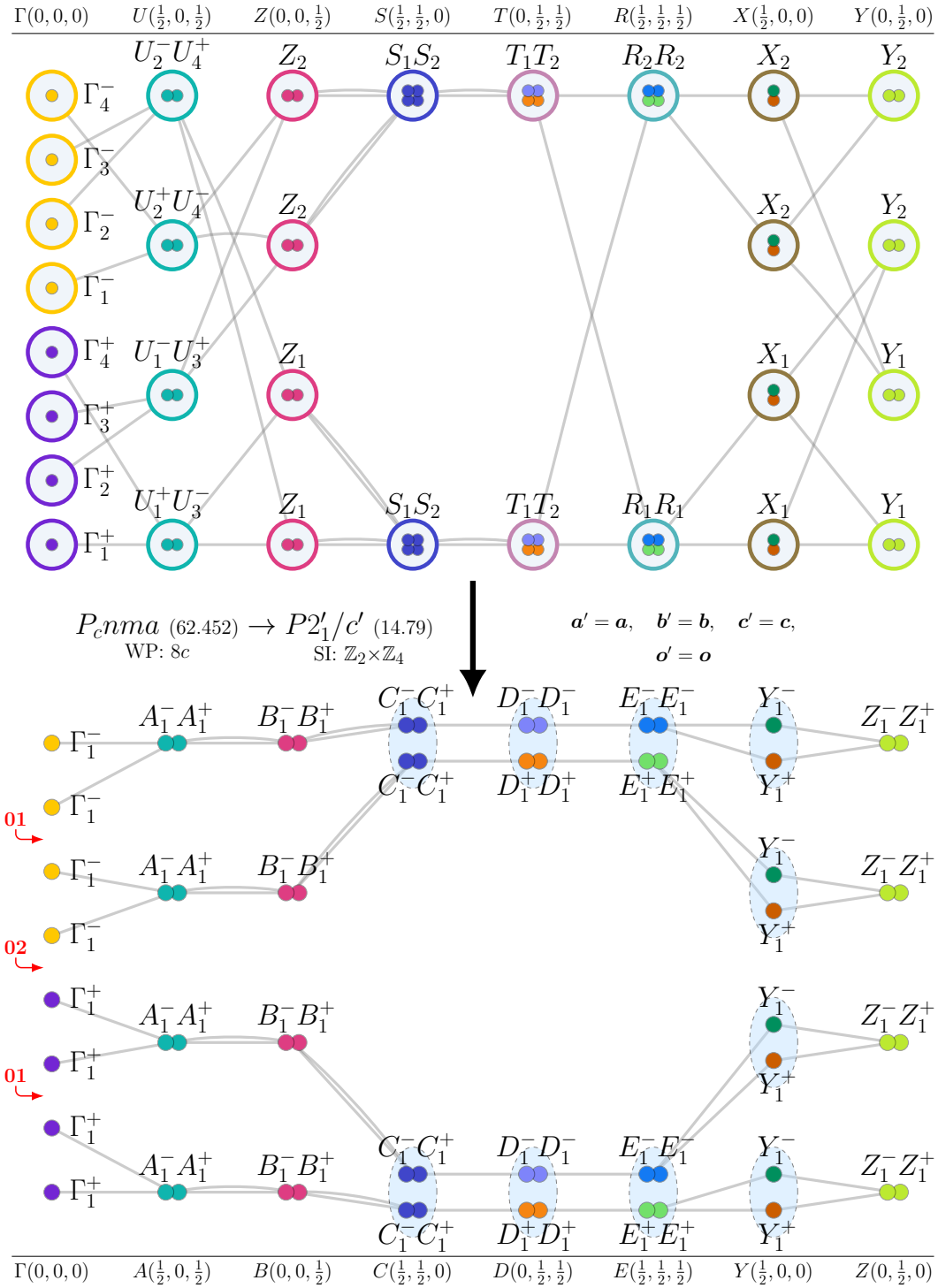
BCS Materials: VPO_4 (10.3 K)[382].

79.2.1 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].

[382]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.523`



Supplementary Figure 420: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position $8c$ of supergroup P_cnma (62.452).

Supplementary Table 829: Number of times a given SI can appear in the band structure. Supergroup: P_cnma (62.452), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

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SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

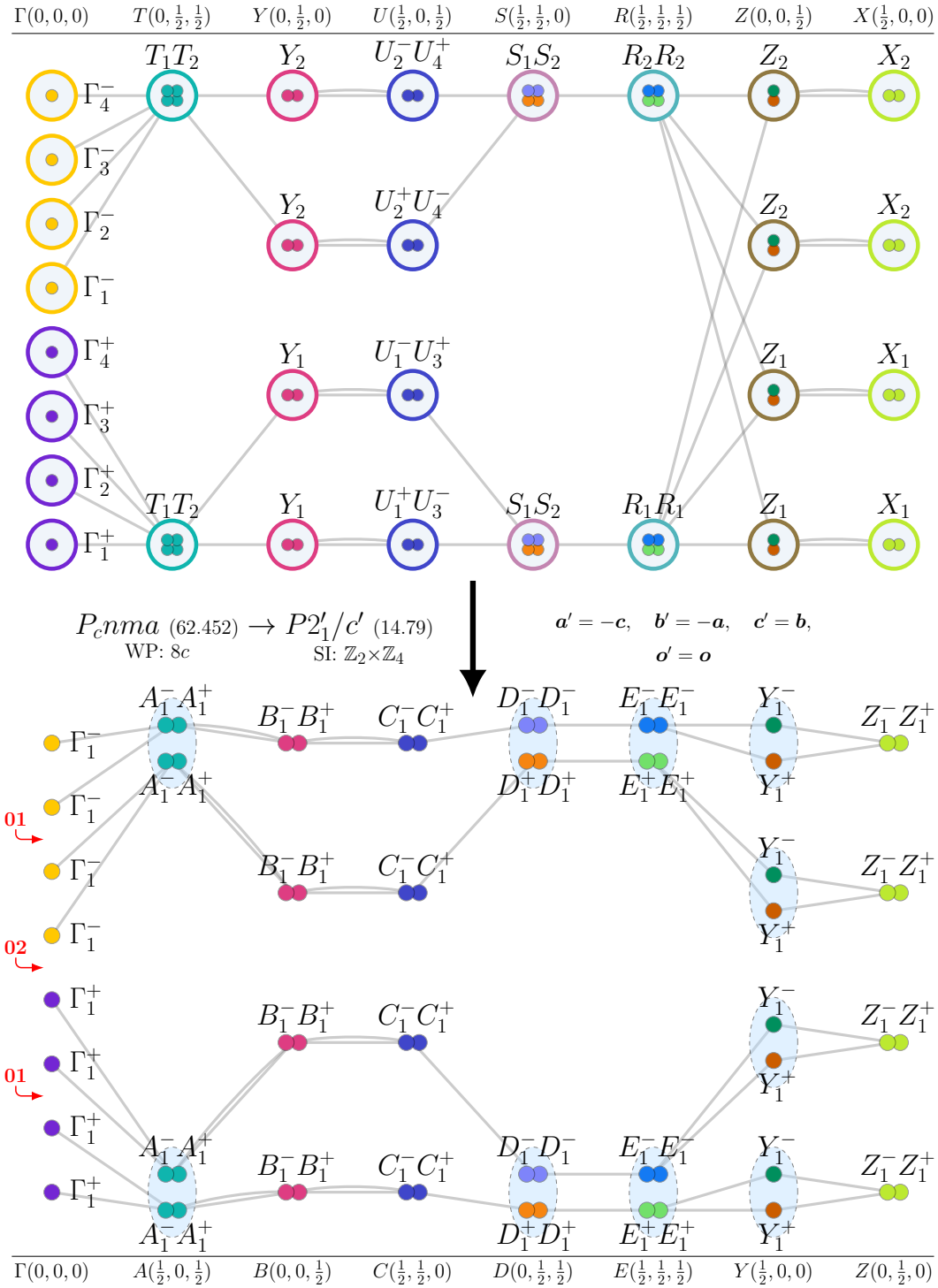
Supplementary Table 830: Possible SI values in each gap. Supergroup: $Pcnma$ (62.452), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

79.2.2 Topological bands in subgroup $P2_1'/c'$ (14.79)

Perturbations:

- B $\parallel$ [010] and strain $\perp$ [100],
- B $\parallel$ [001] and strain $\perp$ [100],
- B $\perp$ [100].



Supplementary Figure 421: Topological magnon bands in subgroup $P2_1'/c'$ (14.79) for magnetic moments on Wyckoff position 8c of supergroup P_cnma (62.452).

Supplementary Table 831: Number of times a given SI can appear in the band structure. Supergroup: P_cnma (62.452), WP: 8c, Subgroup: $P2_1'/c'$ (14.79).

SI	Possible #Gaps
00	0, 1
01	0, 1, 2
02	0, 1
03	0, 1, 2
10	0, 1, 2
11	0, 1
12	0, 1, 2

continued on next page...

...continued

SI	Possible #Gaps
13	0, 1
gapless	4
nontrivial	2, 3

Supplementary Table 832: Possible SI values in each gap. Supergroup: P_cnma (62.452), WP: $8c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03, 10, 12
3	G
4	00, 02, 11, 13
5	G
6	01, 03, 10, 12
7	G
8	00

80 MSG C_cmcm (63.466)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/m'$ (10.46), $C2'/m'$ (12.62), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), $P2_1/m$ (11.50), $Cm'c'm$ (63.462), P_b2_1/m (11.56), $C2/c$ (15.85), $Ccc'm'$ (66.496), C_c2/c (15.90), $C2/m$ (12.58), $Cmm'm'$ (65.486), C_c2/m (12.63).

Trivial-SI Subgroups: Pm' (6.20), Cm' (8.34), Cc' (9.39), $P2'$ (3.3), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Pm (6.18), $Ama'2'$ (40.206), $Amm'2'$ (38.190), P_bm (6.22), Cc (9.37), $Cc'c2'$ (37.182), $Am'a2'$ (40.205), C_cc (9.40), Cm (8.32), $Cm'm2'$ (35.167), $Am'm2'$ (38.189), C_cm (8.35), $P2_1$ (4.7), $Cm'c'2_1$ (36.176), P_b2_1 (4.11), C_cmc2_1 (36.177), $C2$ (5.13), $Am'a'2$ (40.207), C_c2 (5.16), A_amm2 (38.192), $C2$ (5.13), $Am'm'2$ (38.191), C_c2 (5.16), A_ama2 (40.208).

80.1 WP: $4b + 8e$

BCS Materials: FeSn (373 K)[383].

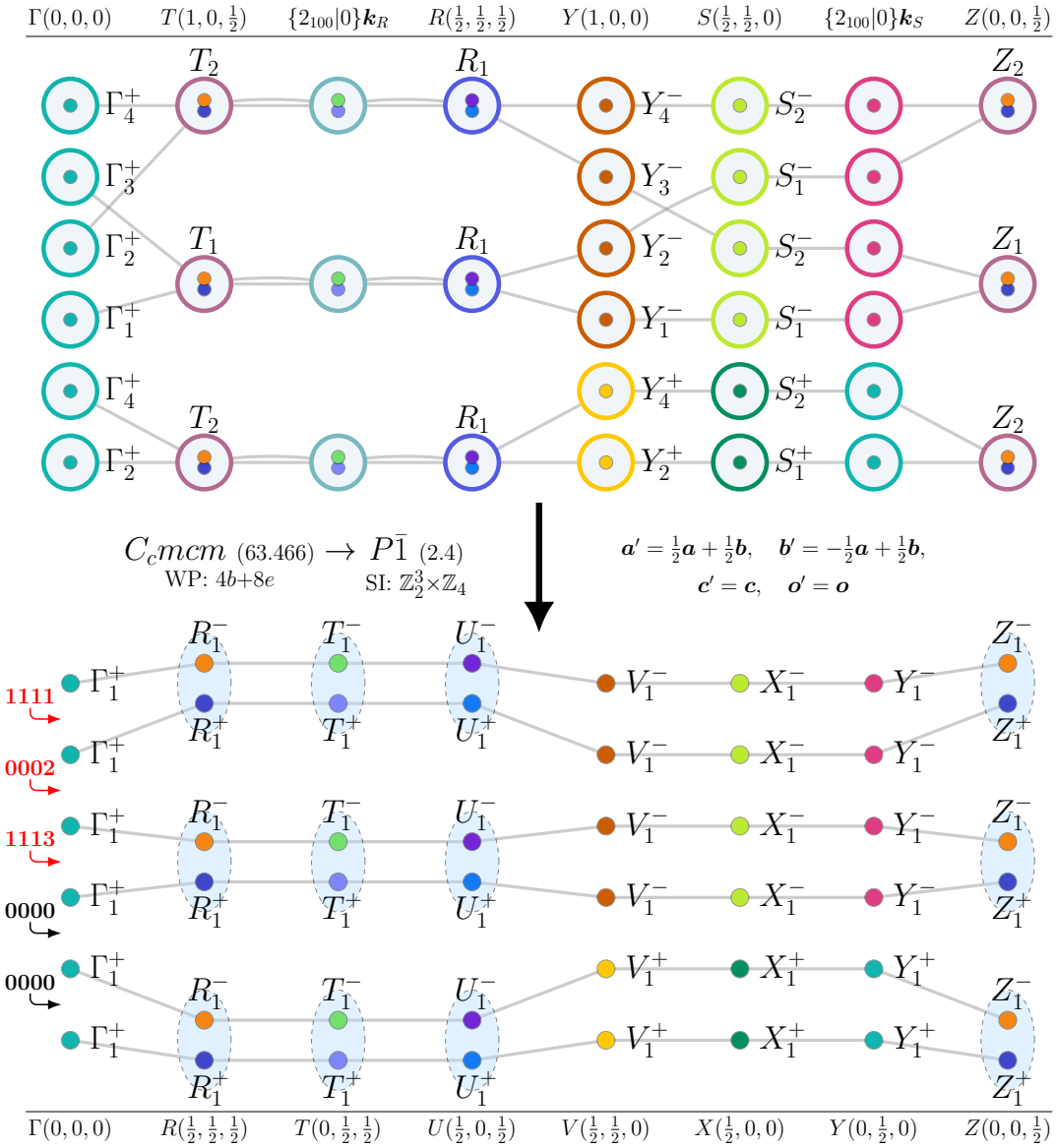
80.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,
- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],

[383] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.452`

- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 422: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b + 8e$ of supergroup C_cmcm (63.466).

Supplementary Table 833: Number of times a given SI can appear in the band structure. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3

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SI	Possible #Gaps
0100	0, 1, 2, 3
0101	0, 1, 2, 3
0102	0, 1, 2
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

5005

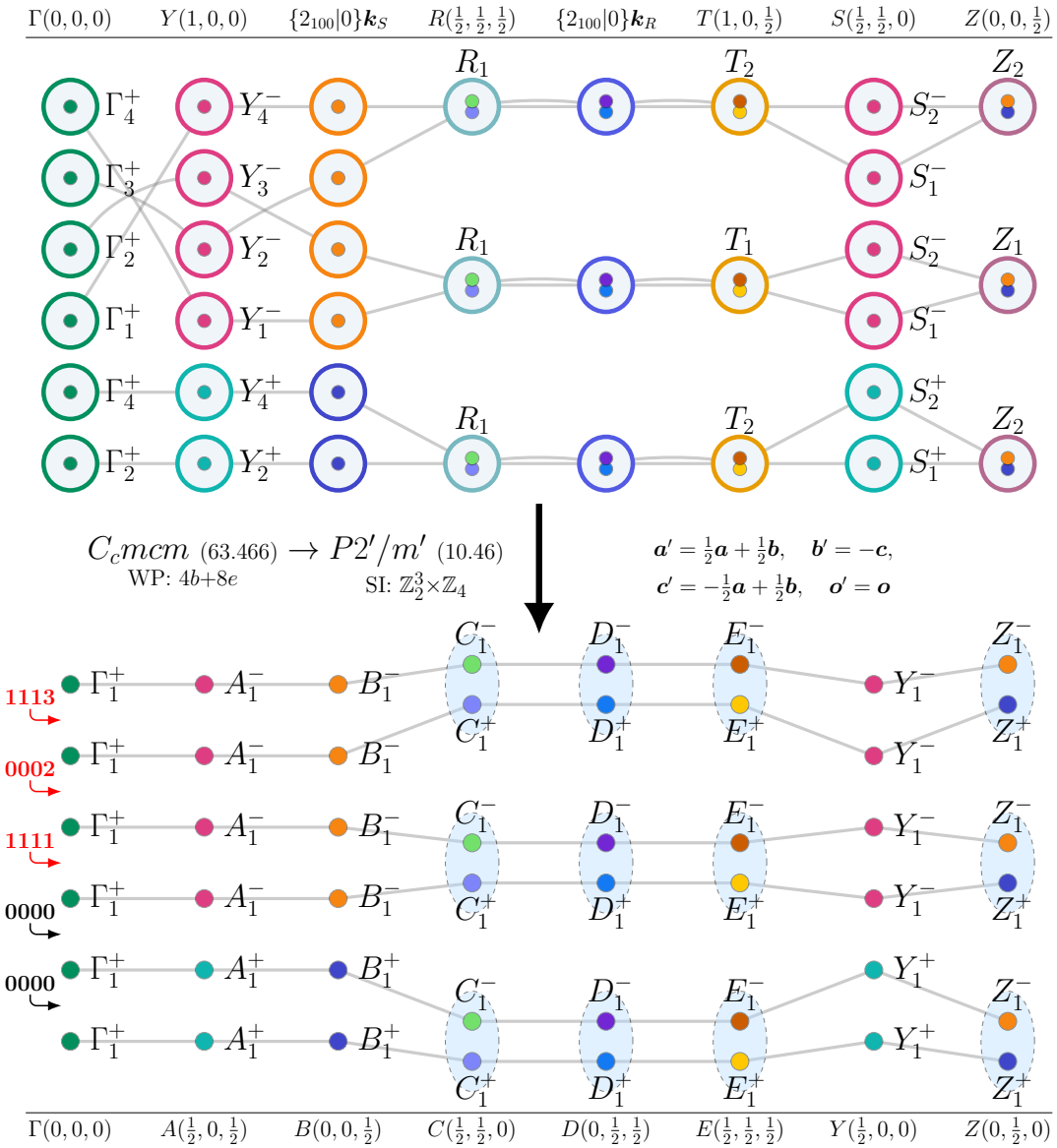
Supplementary Table 834: Possible SI values in each gap. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1112, 1113
2	0000, 0002, 0011, 1100, 1102 1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1103, 1110, 1111, 1112, 1113
4	0000, 0002, 0013, 1100, 1102 1111
5	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1111, 1112
6	0000

80.1.2 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbations:

- B ∥ [100] and strain ⊥ [001],
- B ∥ [010] and strain ⊥ [001],
- B ⊥ [001].



Supplementary Figure 423: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff positions $4b+8e$ of supergroup C_cmcm (63.466).

Supplementary Table 835: Number of times a given SI can appear in the band structure. Supergroup: C_cmcm (63.466), WP: 4b+8e, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3
0002	0, 1, 2, 3
0003	0, 1, 2, 3
0010	0, 1, 2, 3

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SI	Possible #Gaps
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0100	0, 1, 2
0101	0, 1, 2, 3
0102	0, 1, 2, 3
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3
1102	0, 1, 2
1103	0, 1, 2, 3
1110	0, 1, 2, 3
1111	0, 1, 2, 3
1112	0, 1, 2, 3
1113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

5012

Supplementary Table 836: Possible SI values in each gap. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1111, 1112
2	0000, 0002, 0013, 1100, 1102 1111
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1103, 1110, 1111, 1112, 1113
4	0000, 0002, 0011, 1100, 1102 1113

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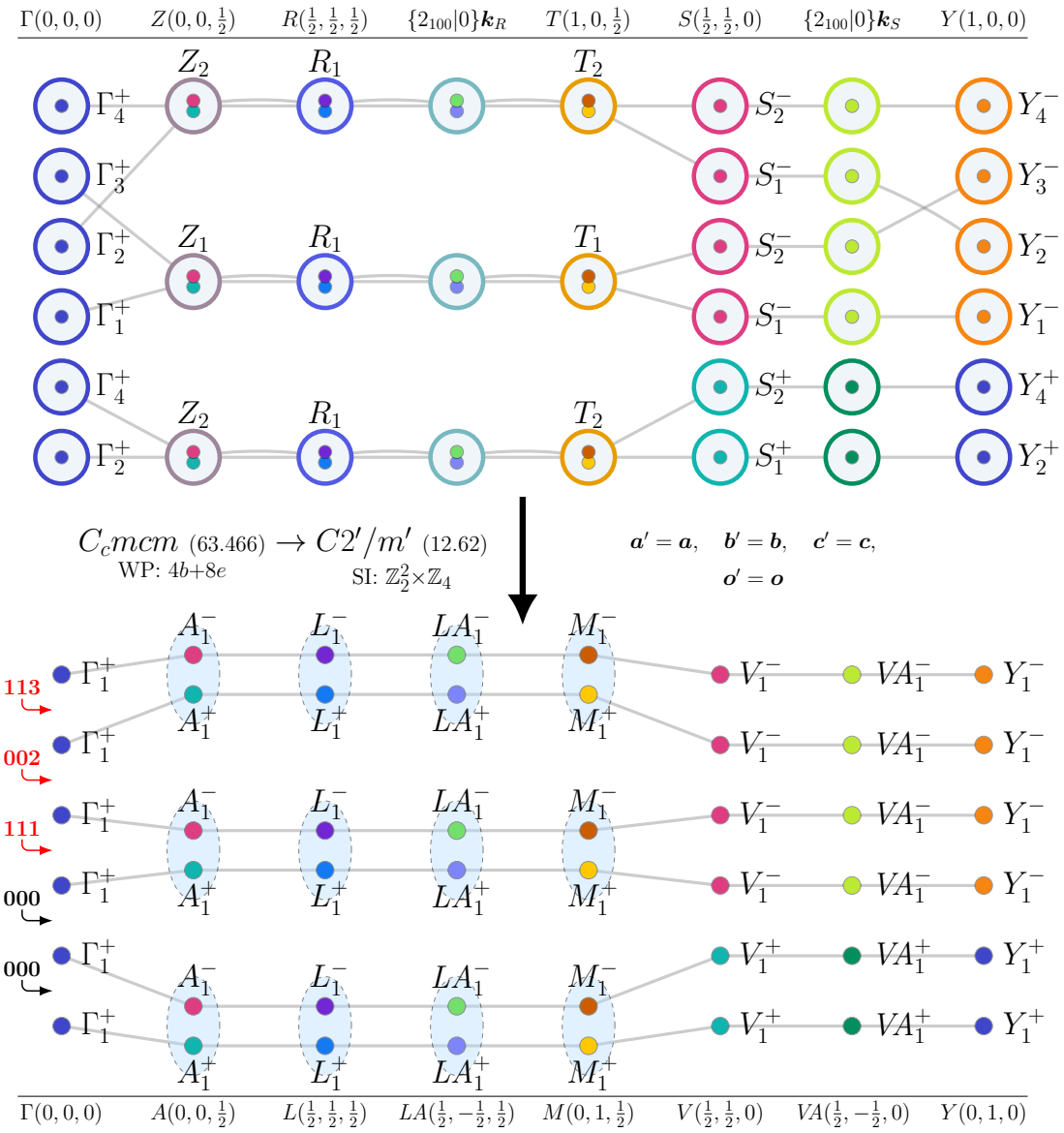
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Gap#	Possible SI Values
5	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1112, 1113
6	0000

80.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 424: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b+8e$ of supergroup C_cmcm (63.466).

Supplementary Table 837: Number of times a given SI can appear in the band structure. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2, 3
003	0, 1, 2, 3
010	0, 1, 2, 3
011	0, 1, 2, 3
012	0, 1, 2, 3
013	0, 1, 2, 3
100	0, 1, 2
101	0, 1, 2, 3
102	0, 1, 2, 3
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2, 3
112	0, 1, 2, 3
113	0, 1, 2, 3
gapless	0
nontrivial	3, 4, 5

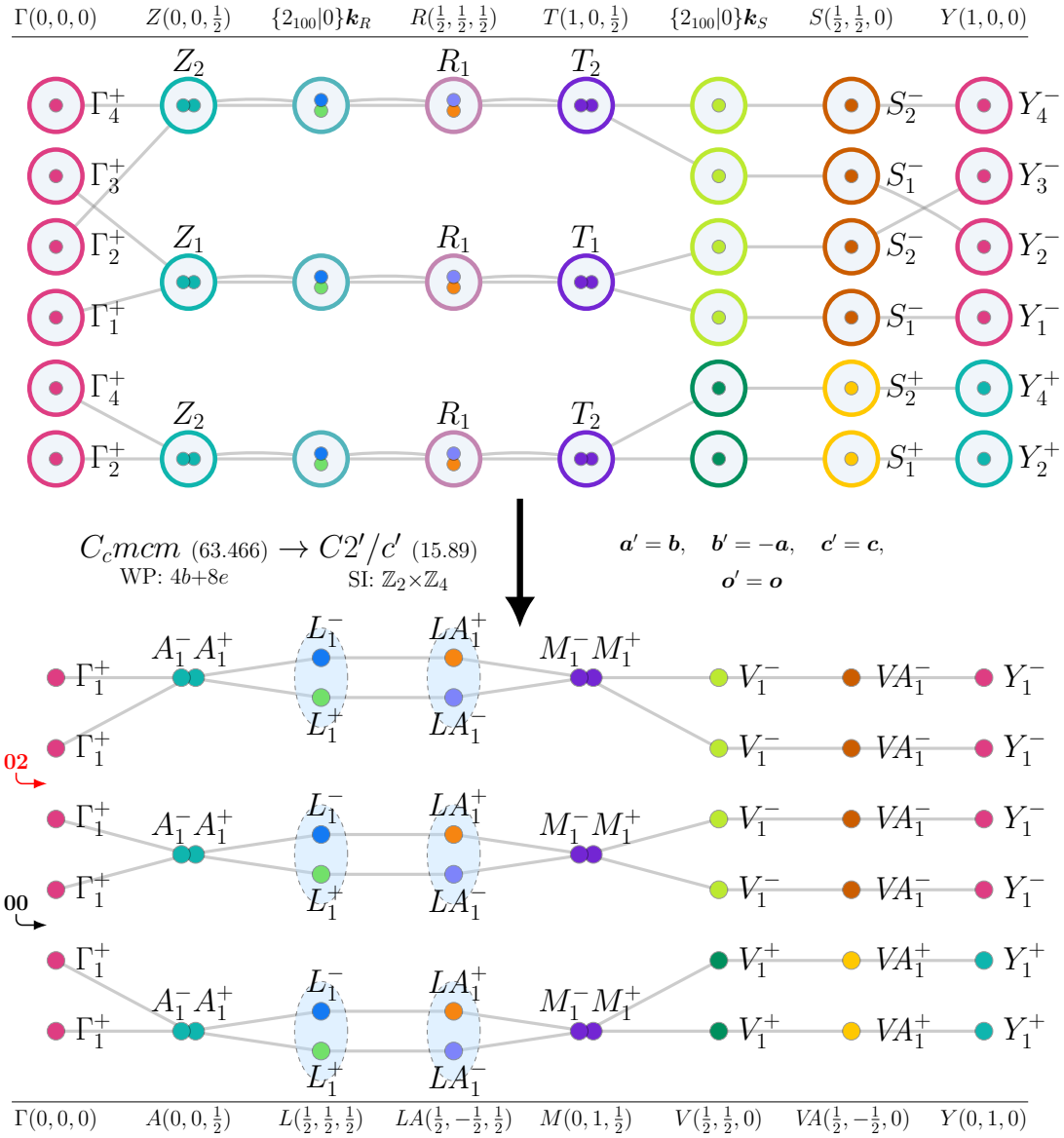
Supplementary Table 838: Possible SI values in each gap. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	000, 001, 003, 010, 011 012, 100, 101, 103, 110 111, 112
2	000, 002, 011, 100, 102 111
3	001, 002, 003, 010, 011 012, 013, 101, 102, 103 110, 111, 112, 113
4	000, 002, 013, 100, 102 113
5	000, 001, 003, 010, 012 013, 100, 101, 103, 110 112, 113
6	000

80.1.4 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 425: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $4b+8e$ of supergroup C_cmcm (63.466).

Supplementary Table 839: Number of times a given SI can appear in the band structure. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
01	0, 1
02	0, 1, 2
03	0, 1
10	0, 1, 2
11	0, 1
12	0, 1
13	0, 1
gapless	3
nontrivial	1, 2

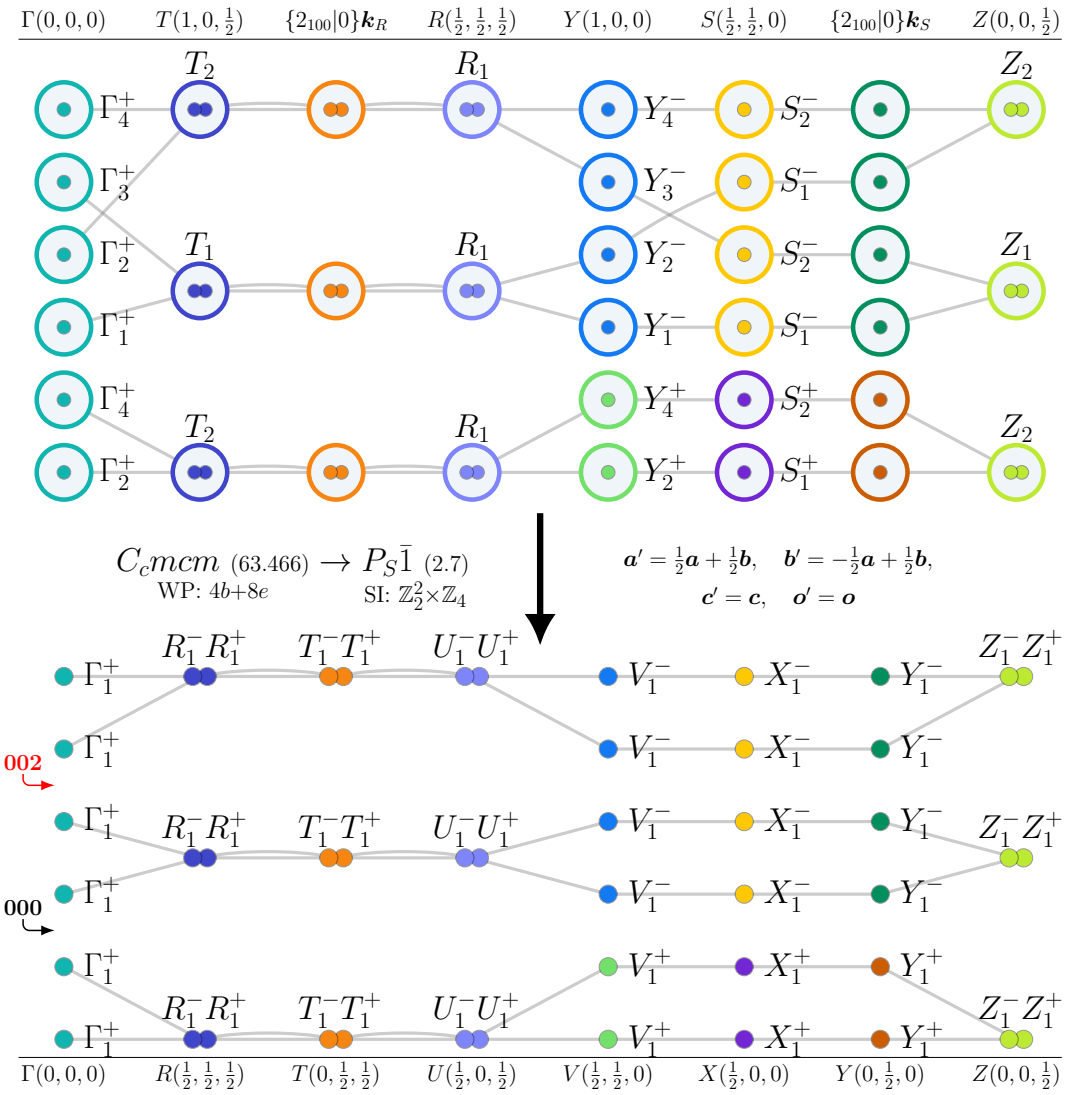
Supplementary Table 840: Possible SI values in each gap. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 01, 02, 10, 12 13
3	G
4	00, 02, 03, 10, 11 12
5	G
6	00

80.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 426: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4b + 8e$ of supergroup C_cmcm (63.466).

Supplementary Table 841: Number of times a given SI can appear in the band structure. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
001	0, 1
002	0, 1, 2
003	0, 1
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	3
nontrivial	1, 2

Supplementary Table 842: Possible SI values in each gap. Supergroup: C_cmcm (63.466), WP: $4b$+$8e$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	000, 002, 003, 110, 111 112
3	G
4	000, 001, 002, 110, 112 113
5	G
6	000

81 MSG C_Amcm (63.468)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/c'$ (13.69), $C2'/m'$ (12.62), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), $P2_1/m$ (11.50), $Cm'c'm$ (63.462), P_C2_1/m (11.57), $Ab'a'2$ (41.215), $C2/c$ (15.85), $Ccc'a'$ (68.516), C_c2/c (15.90), $Ab'm'2$ (39.199), $C2/m$ (12.58), $Cmm'a'$ (67.506), C_c2/m (12.63).

Trivial-SI Subgroups: Pc' (7.26), Cm' (8.34), Cc' (9.39), $P2'$ (3.3), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Pm (6.18), $Ama'2'$ (40.206), $Amm'2'$ (38.190), P_Cm (6.23), Cc (9.37), $Cc'c2'$ (37.182), $Ab'a2'$ (41.213), C_cc (9.40), Cm (8.32), $Cm'm2'$ (35.167), $Ab'm2'$ (39.197), C_cm (8.35), $P2_1$ (4.7), $Cm'c'2_1$ (36.176), P_C2_1 (4.12), C_Amc2_1 (36.179), $C2$ (5.13), C_c2 (5.16), A_Bmm2 (38.194), $C2$ (5.13), C_c2 (5.16), A_Bma2 (40.210).

81.1 WP: $8d$

BCS Materials: $Pr_{0.5}Sr_{0.5}MnO_3$ (140 K)[384].

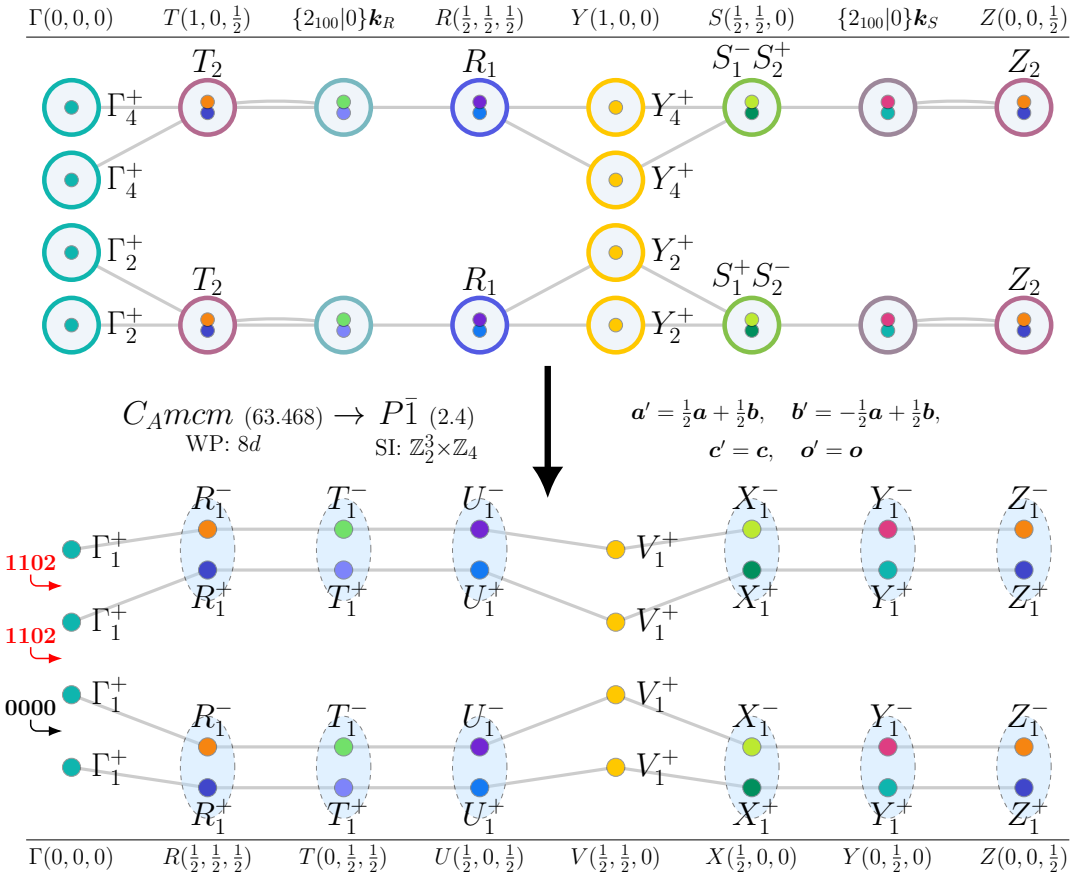
81.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [100] and strain in generic direction,
- B ∥ [010] and strain in generic direction,
- B ∥ [001] and strain in generic direction,

[384]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.273`

- B ⊥ [100] and strain ⊥ [010],
- B ⊥ [100] and strain ⊥ [001],
- B ⊥ [100] and strain in generic direction,
- B ⊥ [010] and strain ⊥ [100],
- B ⊥ [010] and strain ⊥ [001],
- B ⊥ [010] and strain in generic direction,
- B ⊥ [001] and strain ⊥ [100],
- B ⊥ [001] and strain ⊥ [010],
- B ⊥ [001] and strain in generic direction,
- B in generic direction.



Supplementary Figure 427: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position 8d of supergroup C_Amcm (63.468).

Supplementary Table 843: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: 8d, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2
0011	0, 1
0012	0, 1, 2
0013	0, 1
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2

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SI	Possible #Gaps
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1102	1, 2, 3
1103	0, 1, 2
1110	0, 1, 2
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	1, 2, 3

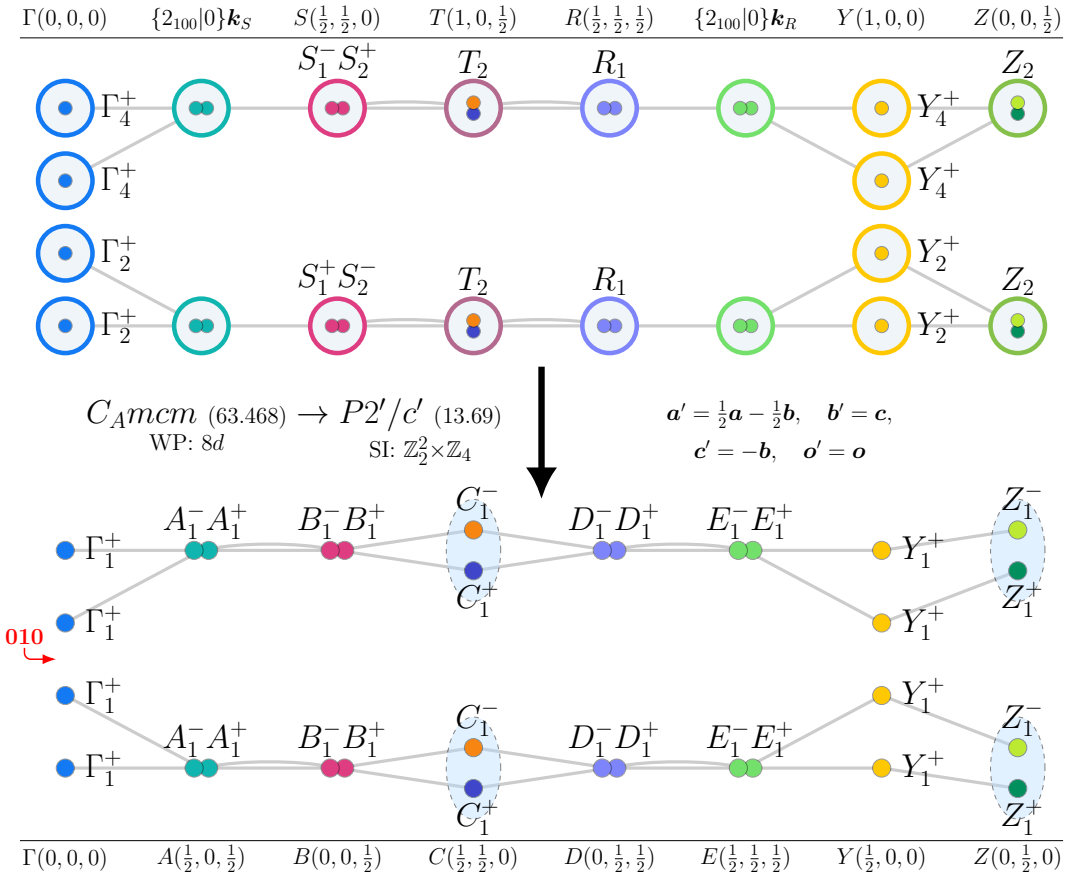
Supplementary Table 844: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1112, 1113
2	1102
3	0000, 0001, 0003, 0010, 0011 0012, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1101, 1102, 1103 1110, 1111, 1112
4	0000

81.1.2 Topological bands in subgroup $P2'/c'$ (13.69)

Perturbations:

- B $\parallel$ [100] and strain $\perp$ [001],
- B $\parallel$ [010] and strain $\perp$ [001],
- B $\perp$ [001].



Supplementary Figure 428: Topological magnon bands in subgroup $P2'/c'$ (13.69) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 845: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $P2'/c'$ (13.69).

SI	Possible #Gaps
000	0
010	1
gapless	2
nontrivial	1

Supplementary Table 846: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $P2'/c'$ (13.69).

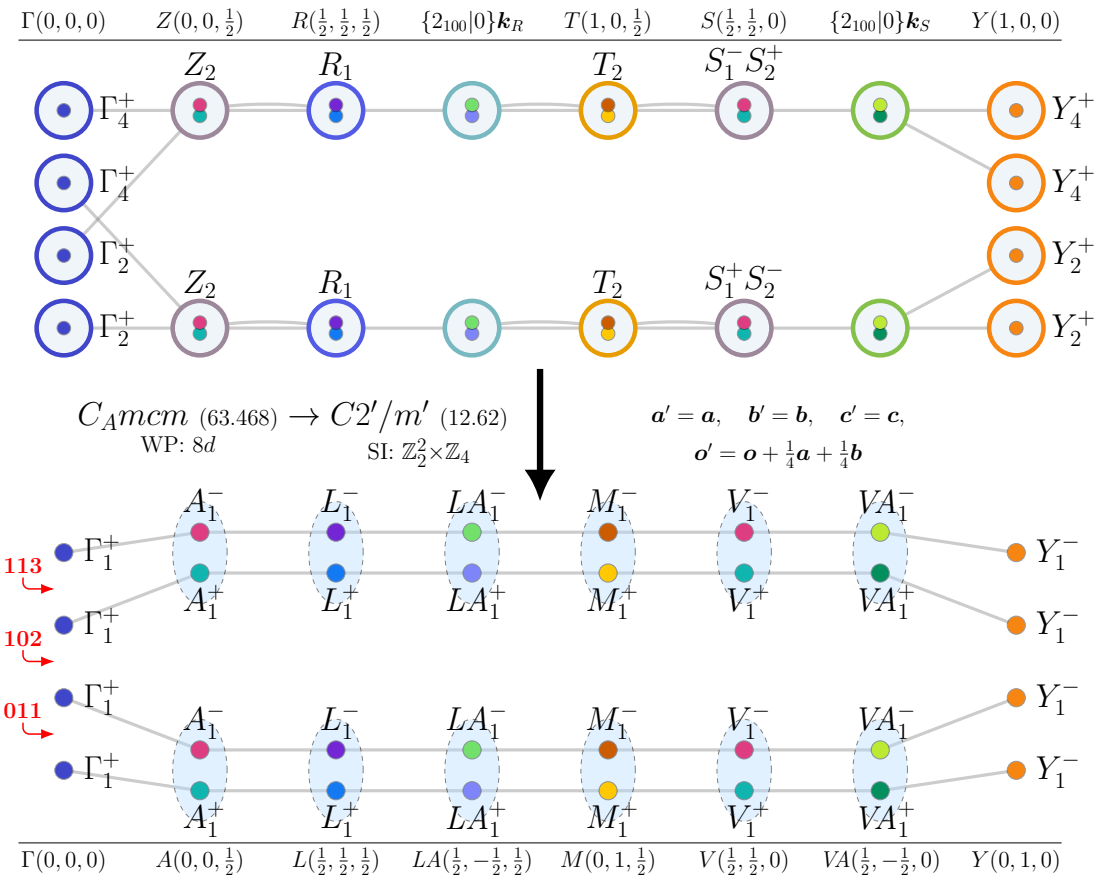
Gap#	Possible SI Values
1	G
2	010
3	G
4	000

81.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],

- B $\perp$ [010].



Supplementary Figure 429: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 847: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
010	0, 1, 2
011	0, 1
012	0, 1, 2
013	0, 1
102	1
110	0, 1, 2
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	3

Supplementary Table 848: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	010, 011, 012, 110, 111 112
2	102

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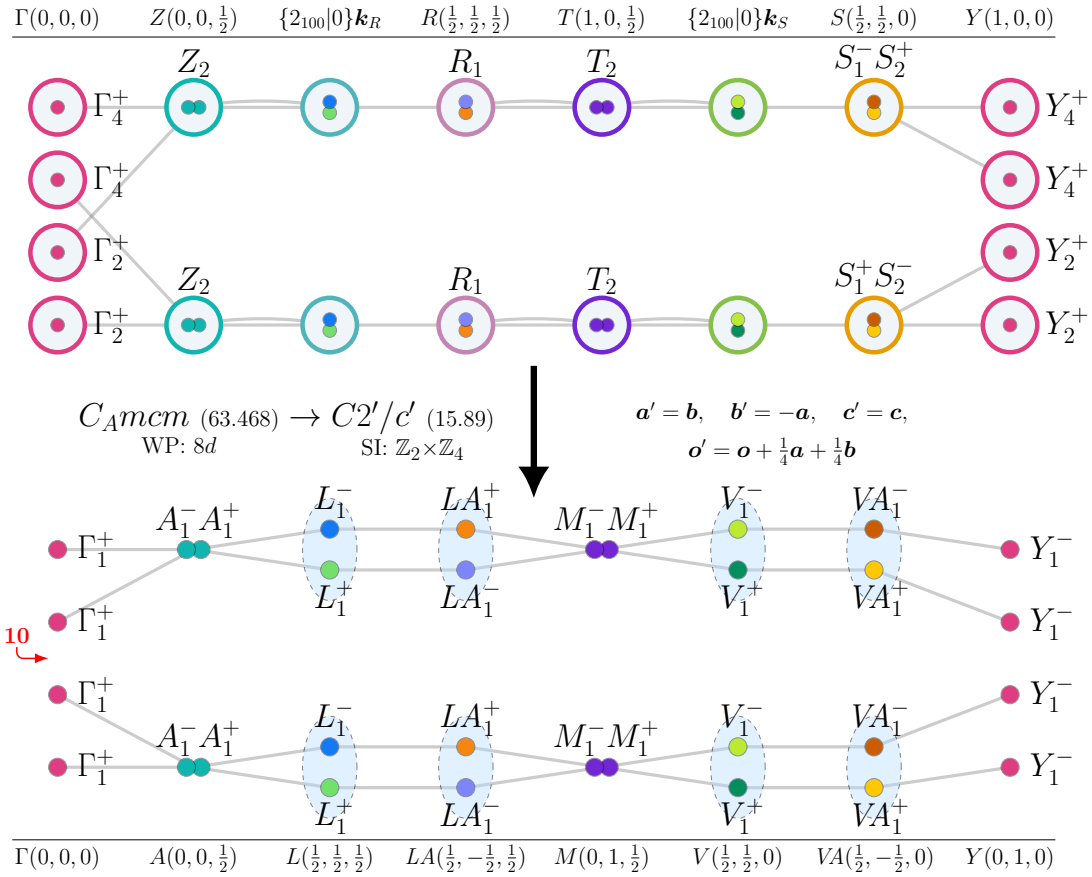
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Gap#	Possible SI Values
3	010, 012, 013, 110, 112 113
4	000

81.1.4 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 430: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 849: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
10	1
gapless	2
nontrivial	1

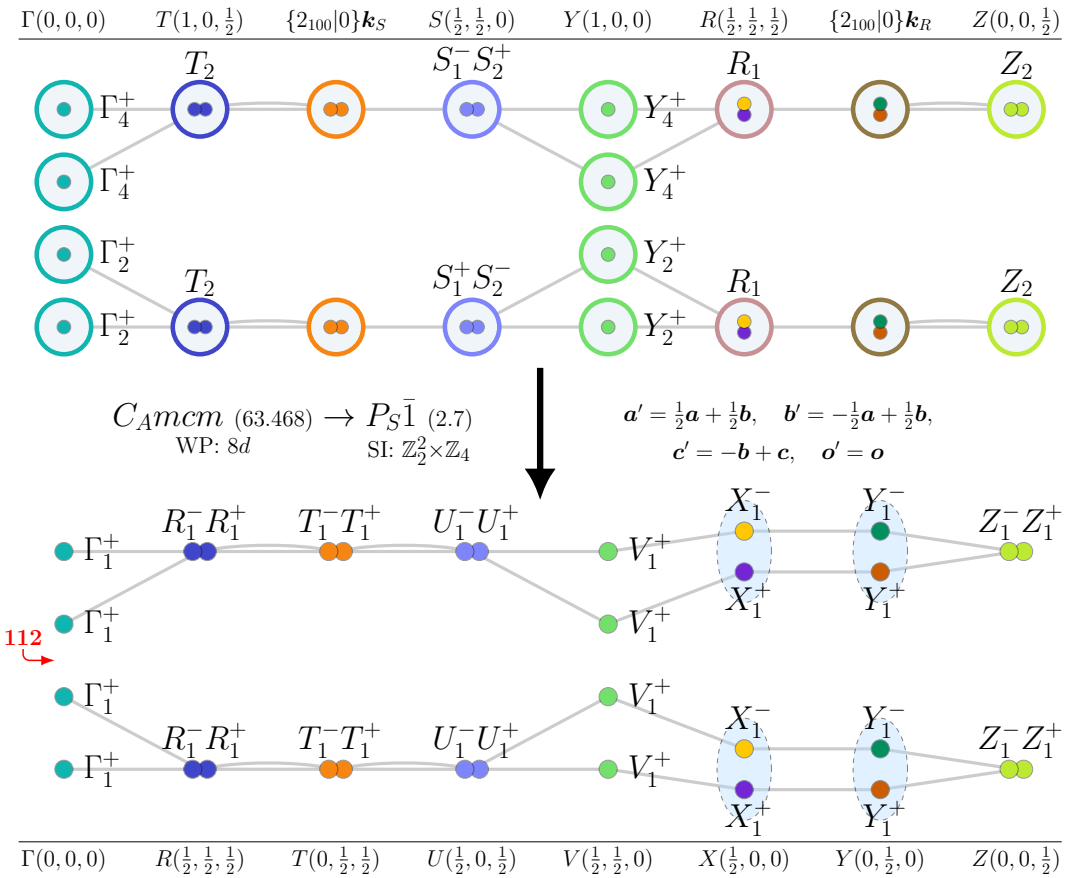
Supplementary Table 850: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	10
3	G
4	00

81.1.5 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 431: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 851: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
112	1
gapless	2
nontrivial	1

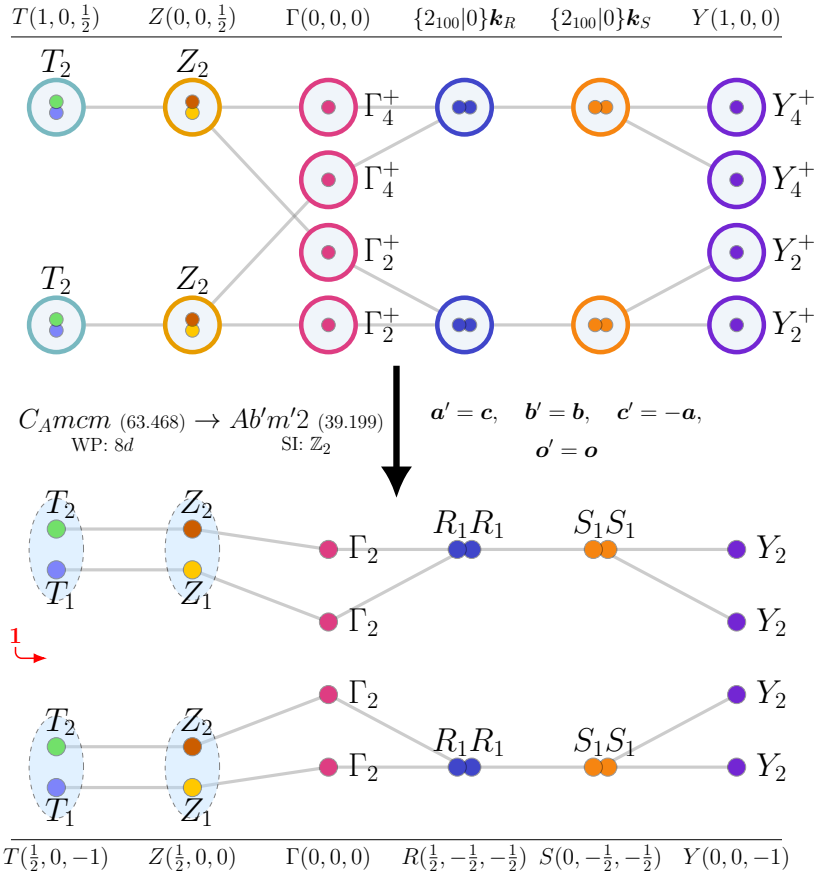
Supplementary Table 852: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	112
3	G
4	000

81.1.6 Topological bands in subgroup $Ab'm'2$ (39.199)

Perturbation:

- E ∥ [100] and B ∥ [100].



Supplementary Figure 432: Topological magnon bands in subgroup $Ab'm'2$ (39.199) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 853: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $Ab'm'2$ (39.199).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

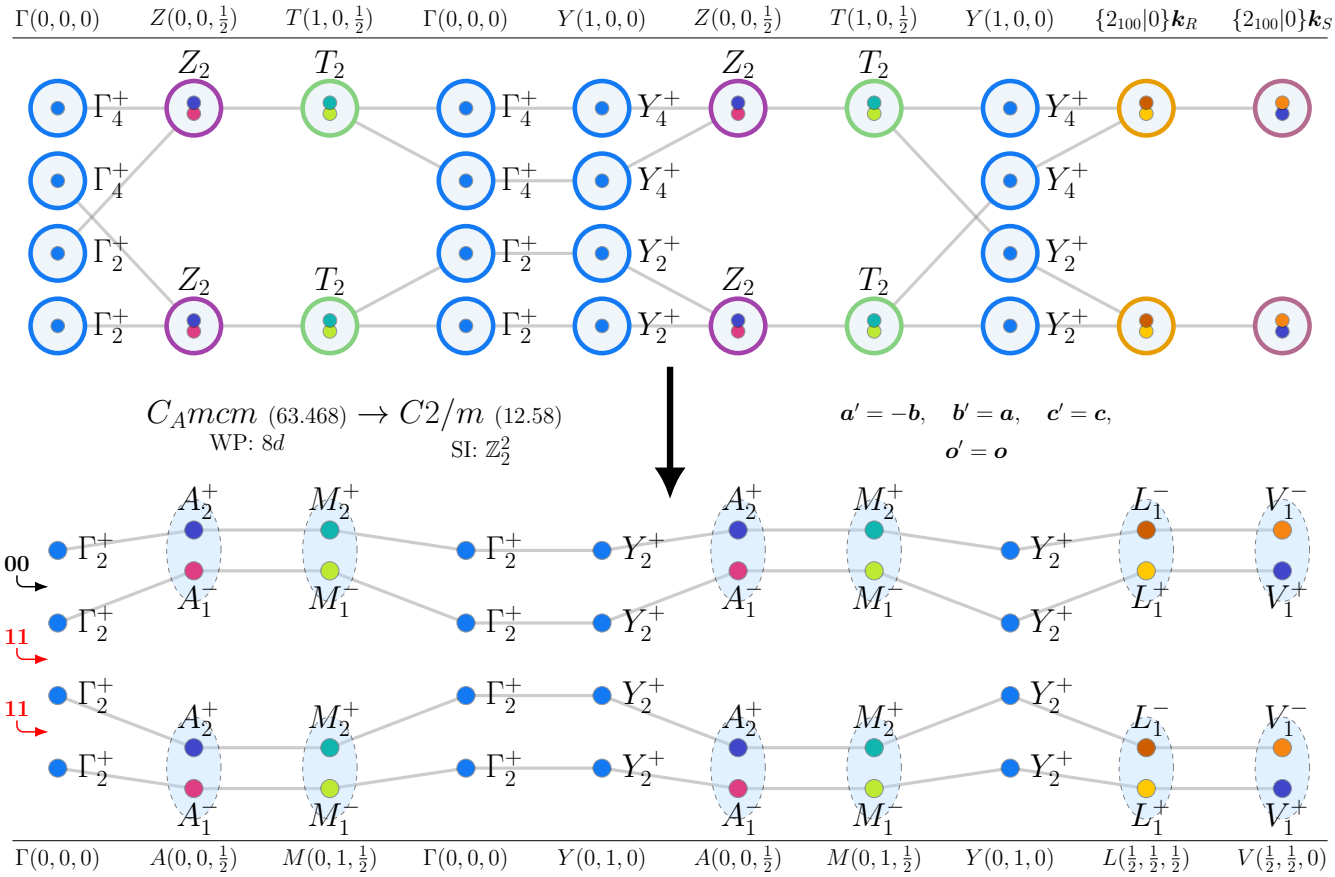
Supplementary Table 854: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $Ab'm'2$ (39.199).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

81.1.7 Topological bands in subgroup $C2/m$ (12.58)

Perturbation:

- B ∥ [100] and strain ⊥ [100].



Supplementary Figure 433: Topological magnon bands in subgroup $C2/m$ (12.58) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 855: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $C2/m$ (12.58).

SI	Possible #Gaps
00	0, 1, 2
11	1, 2, 3
gapless	0, 1, 2
nontrivial	1, 2, 3

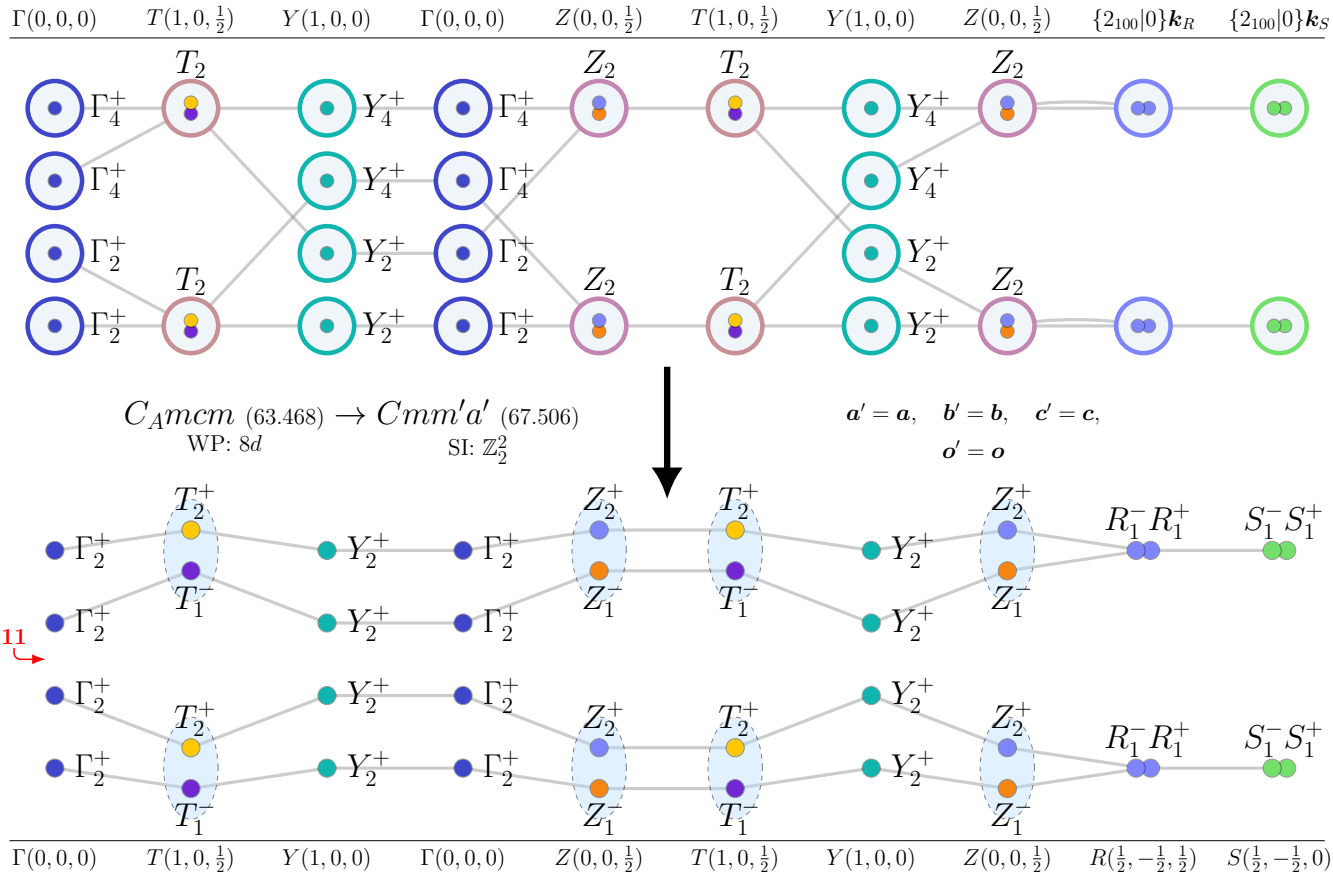
Supplementary Table 856: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $C2/m$ (12.58).

Gap#	Possible SI Values
1	00, 11, G
2	11
3	00, 11, G
4	00

81.1.8 Topological bands in subgroup $Cmm'a'$ (67.506)

Perturbation:

- B ∥ [100].



Supplementary Figure 434: Topological magnon bands in subgroup $Cmm'a'$ (67.506) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 857: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $Cmm'a'$ (67.506).

SI	Possible #Gaps
00	0
11	1
gapless	2
nontrivial	1

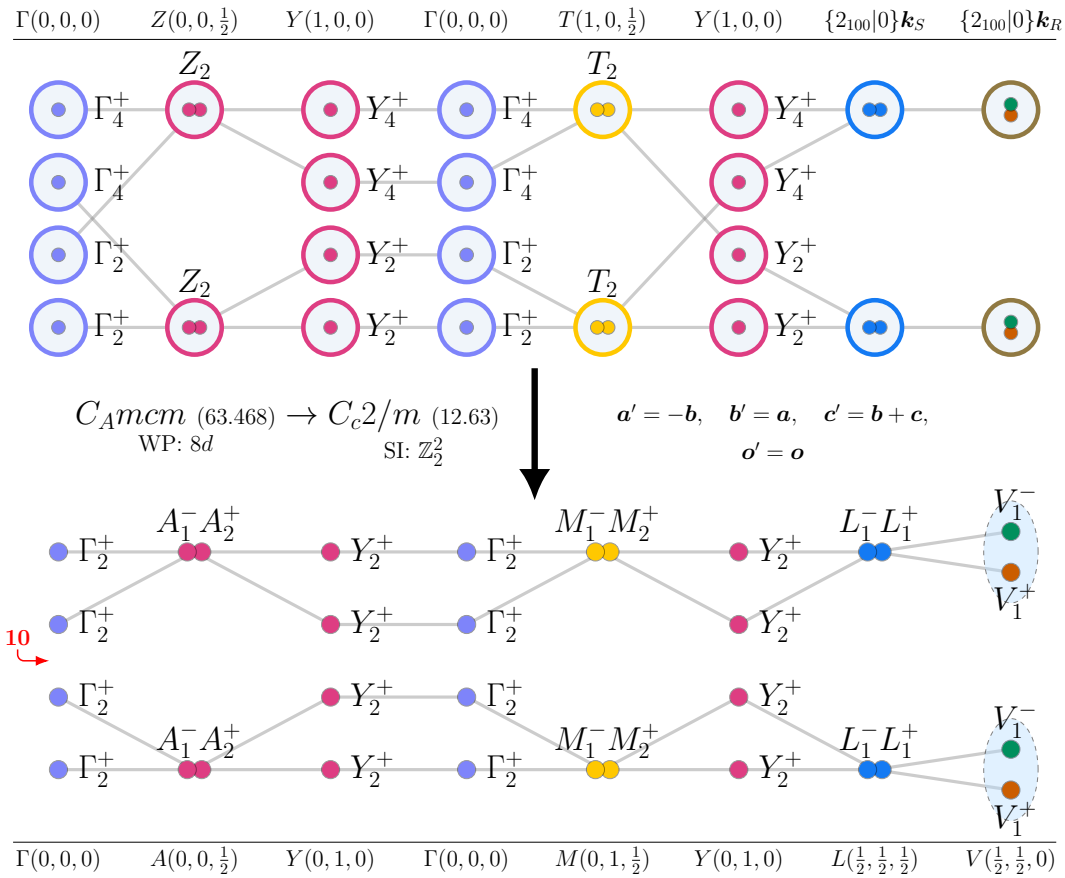
Supplementary Table 858: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: $Cmm'a'$ (67.506).

Gap#	Possible SI Values
1	G
2	11
3	G
4	00

81.1.9 Topological bands in subgroup C_c2/m (12.63)

Perturbation:

- strain $\perp$ [100].



Supplementary Figure 435: Topological magnon bands in subgroup C_c2/m (12.63) for magnetic moments on Wyckoff position $8d$ of supergroup C_Amcm (63.468).

Supplementary Table 859: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: C_c2/m (12.63).

SI	Possible #Gaps
00	0
10	1
gapless	2
nontrivial	1

Supplementary Table 860: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8d$, Subgroup: C_c2/m (12.63).

Gap#	Possible SI Values
1	G
2	10
3	G
4	00

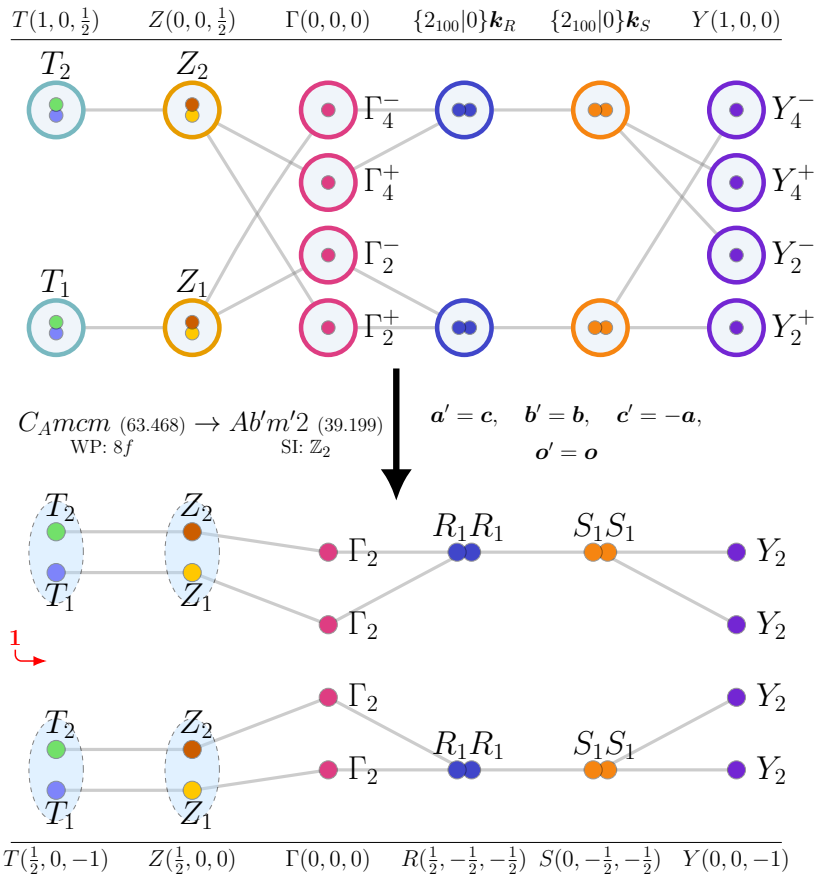
81.2 WP: $8f$

BCS Materials: $CsCo_2Se_2$ (78 K)[385].

81.2.1 Topological bands in subgroup $Ab'm'2$ (39.199)

Perturbation:

- E ∥ [100] and B ∥ [100].



Supplementary Figure 436: Topological magnon bands in subgroup $Ab'm'2$ (39.199) for magnetic moments on Wyckoff position $8f$ of supergroup C_Amcm (63.468).

[385]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.458`

Supplementary Table 861: Number of times a given SI can appear in the band structure. Supergroup: C_Amcm (63.468), WP: $8f$, Subgroup: $Ab'm'2$ (39.199).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 862: Possible SI values in each gap. Supergroup: C_Amcm (63.468), WP: $8f$, Subgroup: $Ab'm'2$ (39.199).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

82 MSG C_Ammm (65.490)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2_1'/c'$ (14.79), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Cc'c'2$ (37.183), $P2/m$ (10.42), $Cc'c'm$ (66.495), P_C2/m (10.49), $Ab'a'2$ (41.215), $C2/m$ (12.58), $Cmc'a'$ (64.475), C_c2/m (12.63), $Ab'a'2$ (41.215), $C2/m$ (12.58), $Cmc'a'$ (64.475), C_c2/m (12.63).

Trivial-SI Subgroups: Pc' (7.26), Cc' (9.39), Cc' (9.39), $P2_1'$ (4.9), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Pm (6.18), $Ama'2'$ (40.206), $Ama'2'$ (40.206), P_Cm (6.23), Cm (8.32), $Cmc'2_1'$ (36.175), $Ab'm2'$ (39.197), C_cm (8.35), Cm (8.32), $Cmc'2_1'$ (36.175), $Ab'm2'$ (39.197), C_cm (8.35), P_C2 (3.6), C_Amm2 (35.171), $C2$ (5.13), C_c2 (5.16), A_Bmm2 (38.194), $C2$ (5.13), C_c2 (5.16), A_Bmm2 (38.194).

82.1 WP: $8f$

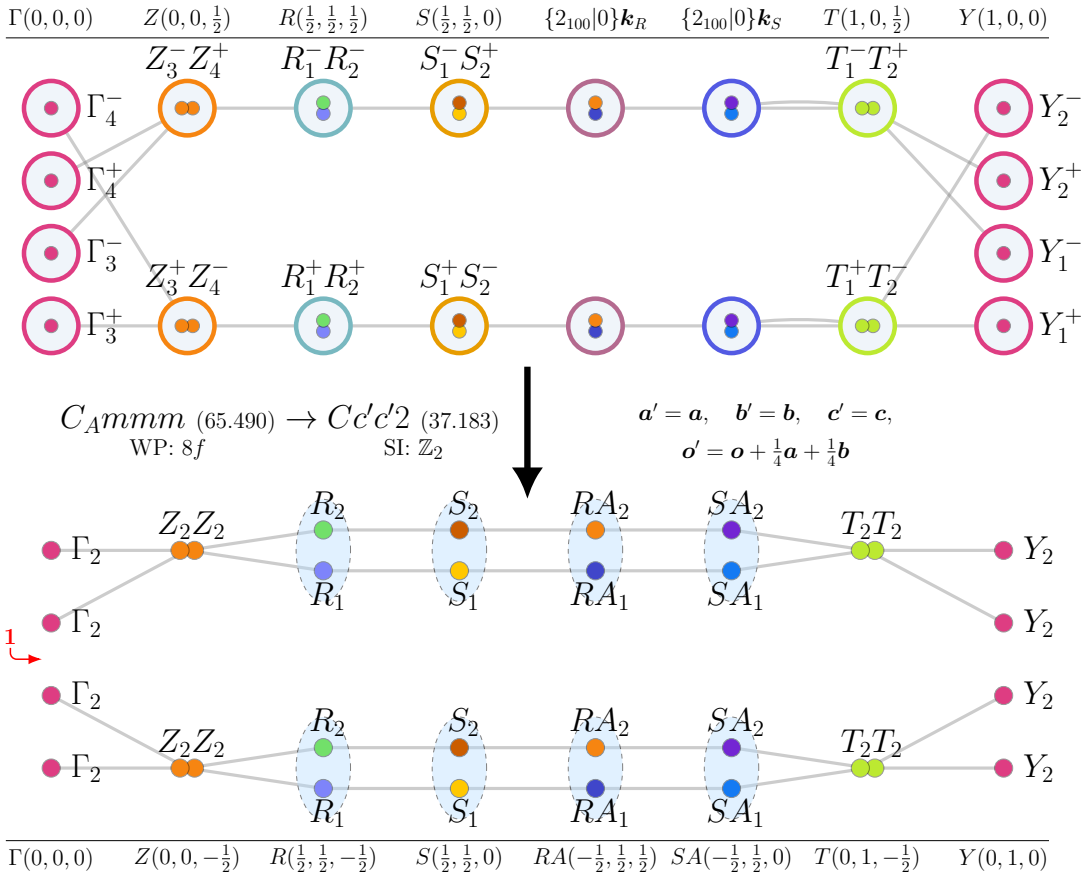
BCS Materials: $KCuMnS_2$ (160 K)[386].

82.1.1 Topological bands in subgroup $Cc'c'2$ (37.183)

Perturbation:

- E ∥ [001] and B ∥ [001].

[386]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.392`



Supplementary Figure 437: Topological magnon bands in subgroup $Cc'c'2$ (37.183) for magnetic moments on Wyckoff position $8f$ of supergroup C_Ammm (65.490).

Supplementary Table 863: Number of times a given SI can appear in the band structure. Supergroup: C_Ammm (65.490), WP: $8f$, Subgroup: $Cc'c'2$ (37.183).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 864: Possible SI values in each gap. Supergroup: C_Ammm (65.490), WP: $8f$, Subgroup: $Cc'c'2$ (37.183).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

83 MSG C_amma (67.509)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/m'$ (10.46), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P_S\bar{1}$ (2.7), $P2$ (3.1), $Cm'm'2$ (35.168), P_a2 (3.4), $P2/c$ (13.65), $Cm'm'a$ (67.505), P_c2/c (13.72), C_a2 (5.17), $C2/m$ (12.58), $Cmm'm'$ (65.486), C_a2/m (12.64), C_a2 (5.17), $C2/m$ (12.58), $Cmm'm'$ (65.486), C_a2/m (12.64).

Trivial-SI Subgroups: Pm' (6.20), Cm' (8.34), Cm' (8.34), $P2'$ (3.3), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Pc (7.24), $Abm'2'$ (39.198), $Abm'2'$ (39.198), P_cc (7.28), Cm (8.32), $Cm'm2'$ (35.167), $Am'm2'$ (38.189), C_am (8.36), Cm (8.32), $Cm'm2'$ (35.167), $Am'm2'$ (38.189), C_am (8.36), C_amm2 (35.170), $C2$ (5.13), $Am'm'2$ (38.191), A_bbm2 (39.201), $C2$ (5.13), $Am'm'2$ (38.191), A_bbm2 (39.201).

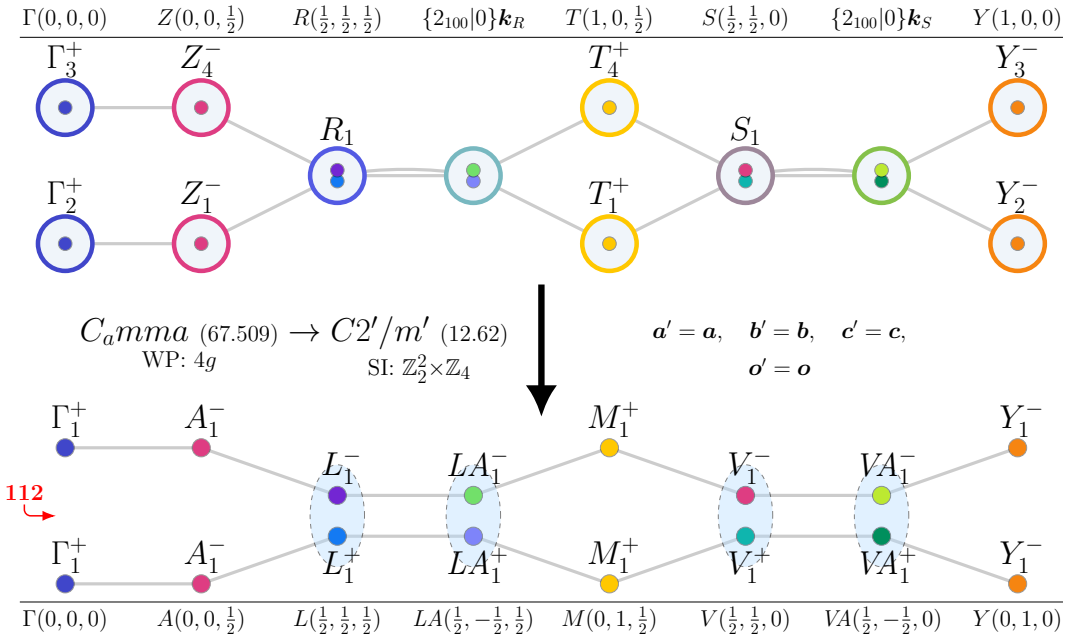
83.1 WP: $4g$

BCS Materials: $MnPt_{0.5}Pd_{0.5}$ (866 K)[387].

83.1.1 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 438: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4g$ of supergroup C_amma (67.509).

Supplementary Table 865: Number of times a given SI can appear in the band structure. Supergroup: C_amma (67.509), WP: $4g$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
012	0, 1
112	0, 1
gapless	0
nontrivial	1

[387] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.466`

Supplementary Table 866: Possible SI values in each gap. Supergroup: C_amma (67.509), WP: $4g$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	012, 112
2	000

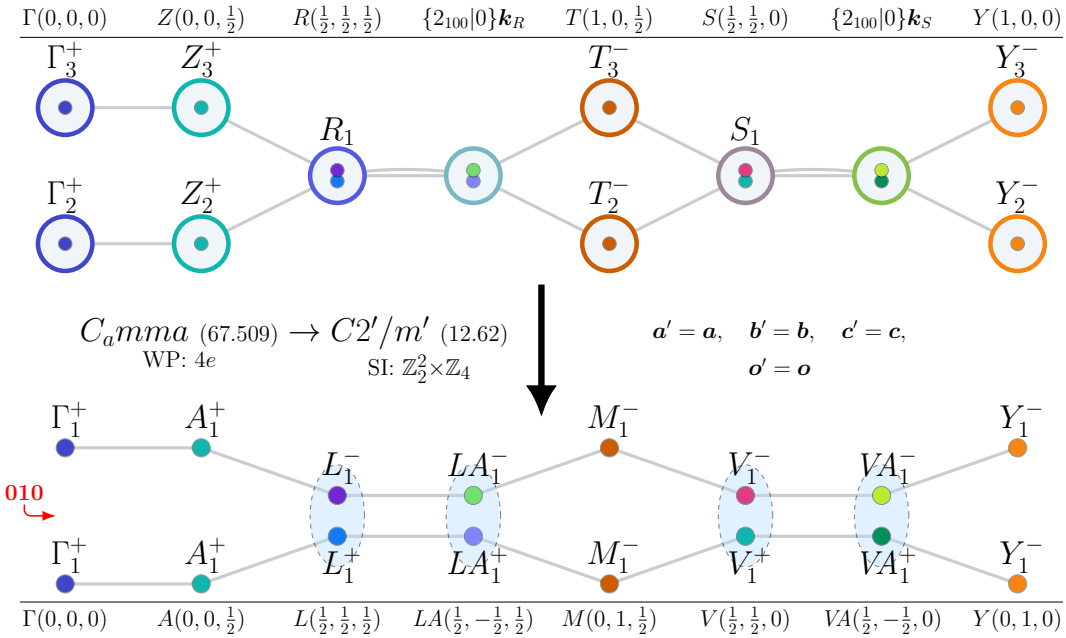
83.2 WP: $4e$

BCS Materials: $Tb_{0.6}Y_{0.4}RhIn_5$ (37.6 K)[388].

83.2.1 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [100] and strain ⊥ [010],
- B ∥ [001] and strain ⊥ [010],
- B ⊥ [010].



Supplementary Figure 439: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4e$ of supergroup C_amma (67.509).

Supplementary Table 867: Number of times a given SI can appear in the band structure. Supergroup: C_amma (67.509), WP: $4e$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
010	0, 1
110	0, 1
gapless	0
nontrivial	1

[388] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.467`

Supplementary Table 868: Possible SI values in each gap. Supergroup: C_amma (67.509), WP: $4e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	010, 110
2	000

84 MSG F_Smmm (69.526)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P_S\bar{1}$ (2.7), $Fm'm'2$ (42.222), C_a2 (5.17), $C2/m$ (12.58), $Fm'm'm$ (69.524), C_a2/m (12.64), $Fm'm'2$ (42.222), C_a2 (5.17), $C2/m$ (12.58), $Fm'm'm$ (69.524), C_a2/m (12.64), $Fm'm'2$ (42.222), C_a2 (5.17), $C2/m$ (12.58), $Fm'm'm$ (69.524), C_a2/m (12.64).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Cm (8.32), $Fm'm2'$ (42.221), $Fm'm2'$ (42.221), C_am (8.36), Cm (8.32), $Fm'm2'$ (42.221), $Fm'm2'$ (42.221), C_am (8.36), Cm (8.32), $Fm'm2'$ (42.221), $Fm'm2'$ (42.221), C_am (8.36), $C2$ (5.13), F_Smm2 (42.223), $C2$ (5.13), F_Smm2 (42.223), $C2$ (5.13), F_Smm2 (42.223).

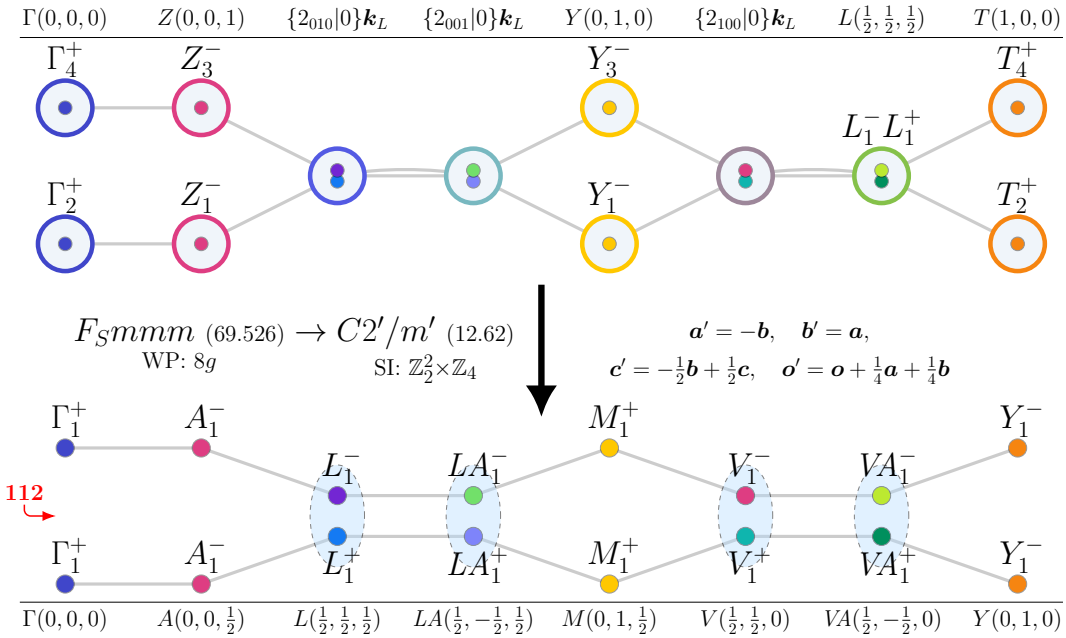
84.1 WP: $8g$

BCS Materials: $SrFeO_2$ (473 K)[389].

84.1.1 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- B ∥ [010] and strain ⊥ [100],
- B ∥ [001] and strain ⊥ [100],
- B ⊥ [100].



Supplementary Figure 440: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $8g$ of supergroup F_Smmm (69.526).

[389]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.65`

Supplementary Table 869: Number of times a given SI can appear in the band structure. Supergroup: F_Smmm (69.526), WP: $8g$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
012	0, 1
112	0, 1
gapless	0
nontrivial	1

Supplementary Table 870: Possible SI values in each gap. Supergroup: F_Smmm (69.526), WP: $8g$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	012, 112
2	000

85 MSG $Fd'd'd$ (70.530)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/c'$ (15.89), $C2/c$ (15.85).

Trivial-SI Subgroups: Cc' (9.39), Cc' (9.39), $C2'$ (5.15), $C2'$ (5.15), Cc (9.37), $Fd'd2'$ (43.226), $Fd'd2'$ (43.226), $C2$ (5.13), $Fd'd'2$ (43.227).

85.1 WP: $16c + 8b$

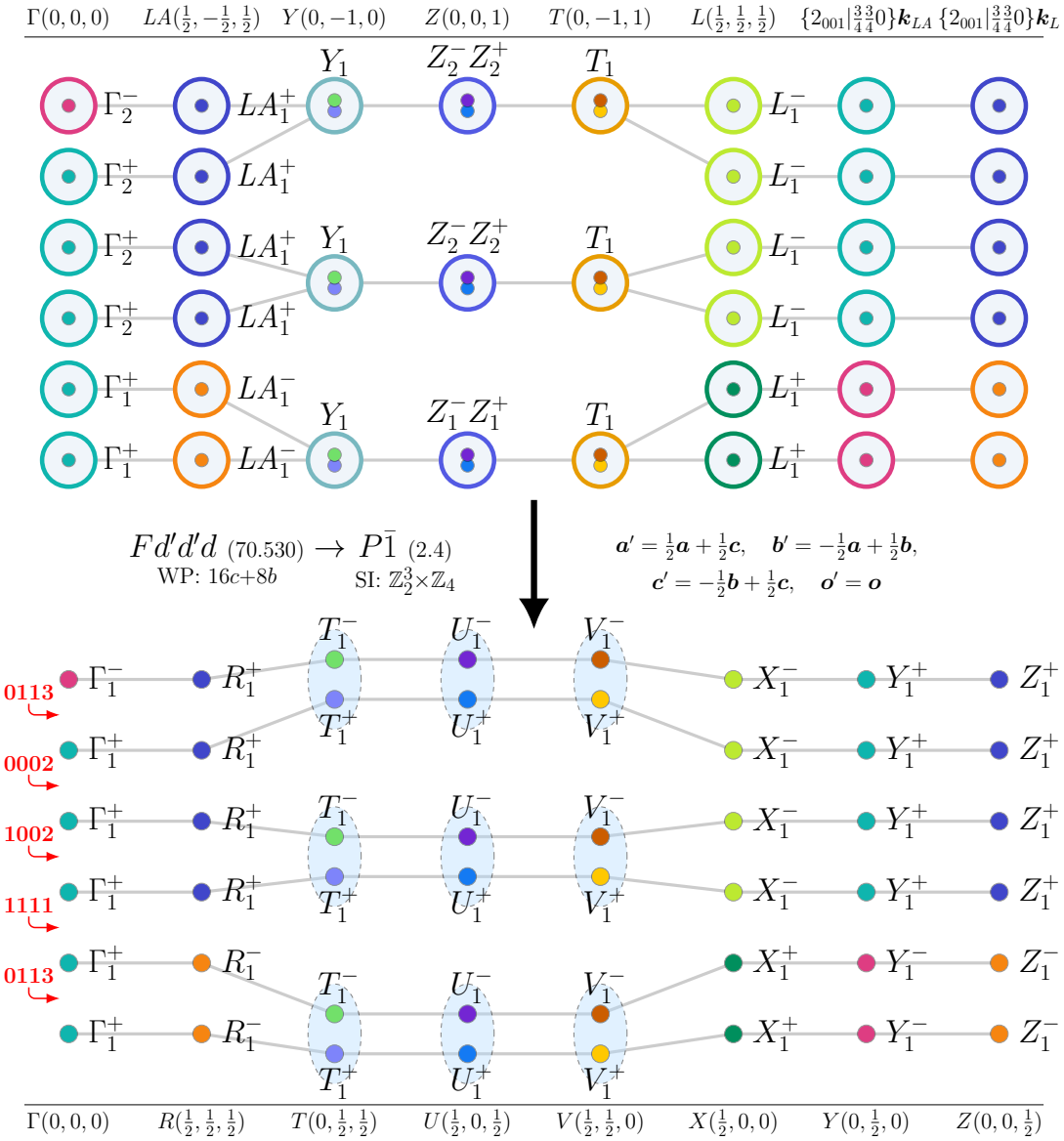
BCS Materials: $NiCr_2O_4$ (74 K)[390].

85.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[390]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.4`



Supplementary Figure 441: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $16c+8b$ of supergroup $Fd'd'd$ (70.530).

Supplementary Table 871: Number of times a given SI can appear in the band structure. Supergroup: $Fd'd'd$ (70.530), WP: 16c+8b, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0001	0, 1, 2, 3
0002	0, 1
0003	0, 1, 2, 3
0010	0, 1, 2, 3
0012	0, 1, 2, 3
0100	0, 1, 2, 3
0102	0, 1, 2, 3
0111	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1002	0, 1, 2, 3
1011	0, 1, 2, 3
1013	0, 1, 2, 3
1101	0, 1, 2, 3
1103	0, 1, 2, 3

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SI	Possible #Gaps
1110	0, 1, 2, 3
1111	0, 1, 2
1112	0, 1, 2, 3
1113	0, 1, 2
gapless	0
nontrivial	4, 5

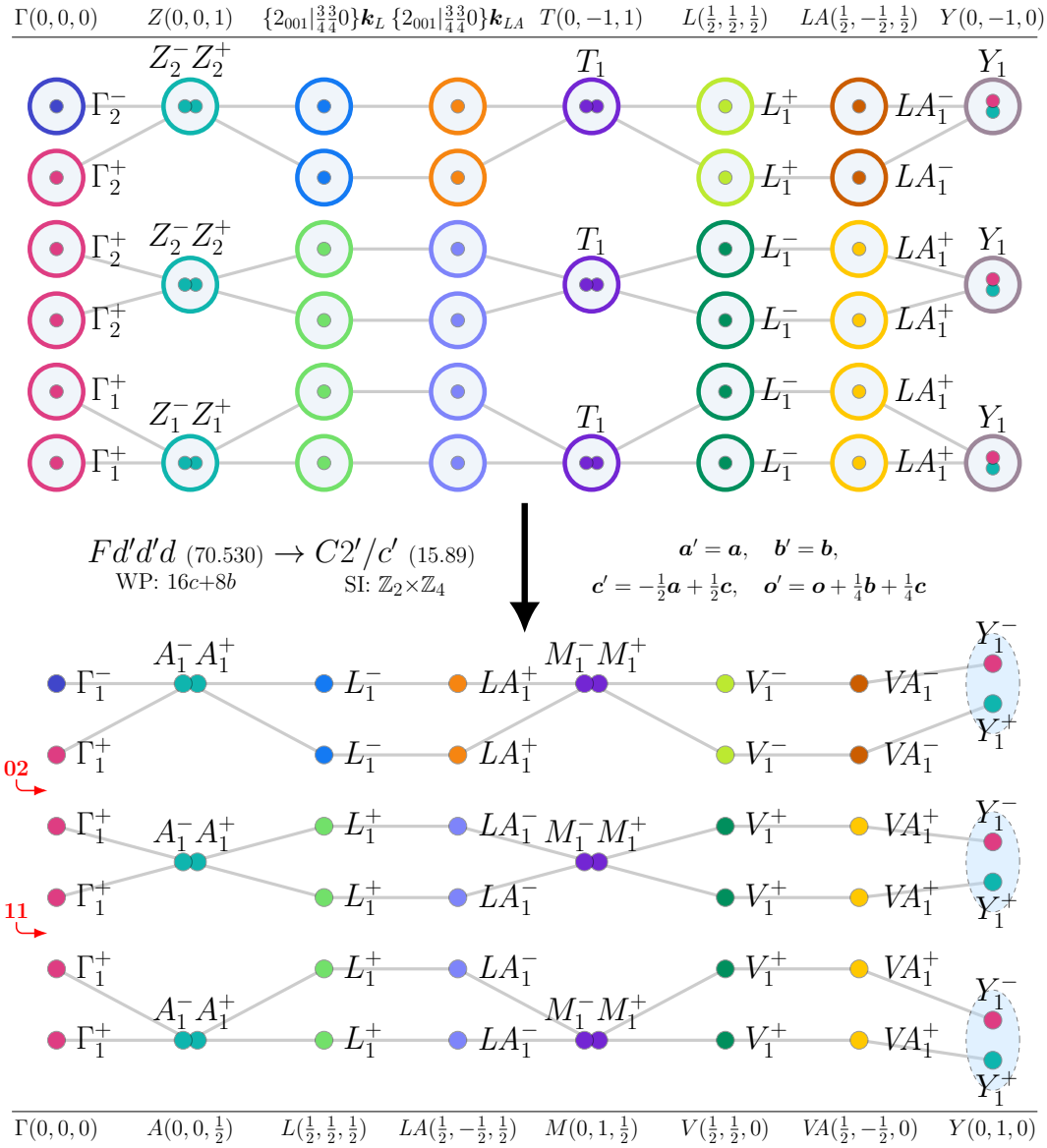
Supplementary Table 872: Possible SI values in each gap. Supergroup: $Fd'd'd$ (70.530), WP: 16c+8b, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
2	0000, 0002, 1111, 1113
3	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
4	0000, 0002, 1111, 1113
5	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
6	0000

85.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 442: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $16c+8b$ of supergroup $Fd'd'd$ (70.530).

Supplementary Table 873: Number of times a given SI can appear in the band structure. Supergroup: $Fd'd'd$ (70.530), WP: $16c$+$8b$, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	3
nontrivial	1, 2

Supplementary Table 874: Possible SI values in each gap. Supergroup: $Fd'd'd$ (70.530), WP: $16c$+$8b$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G

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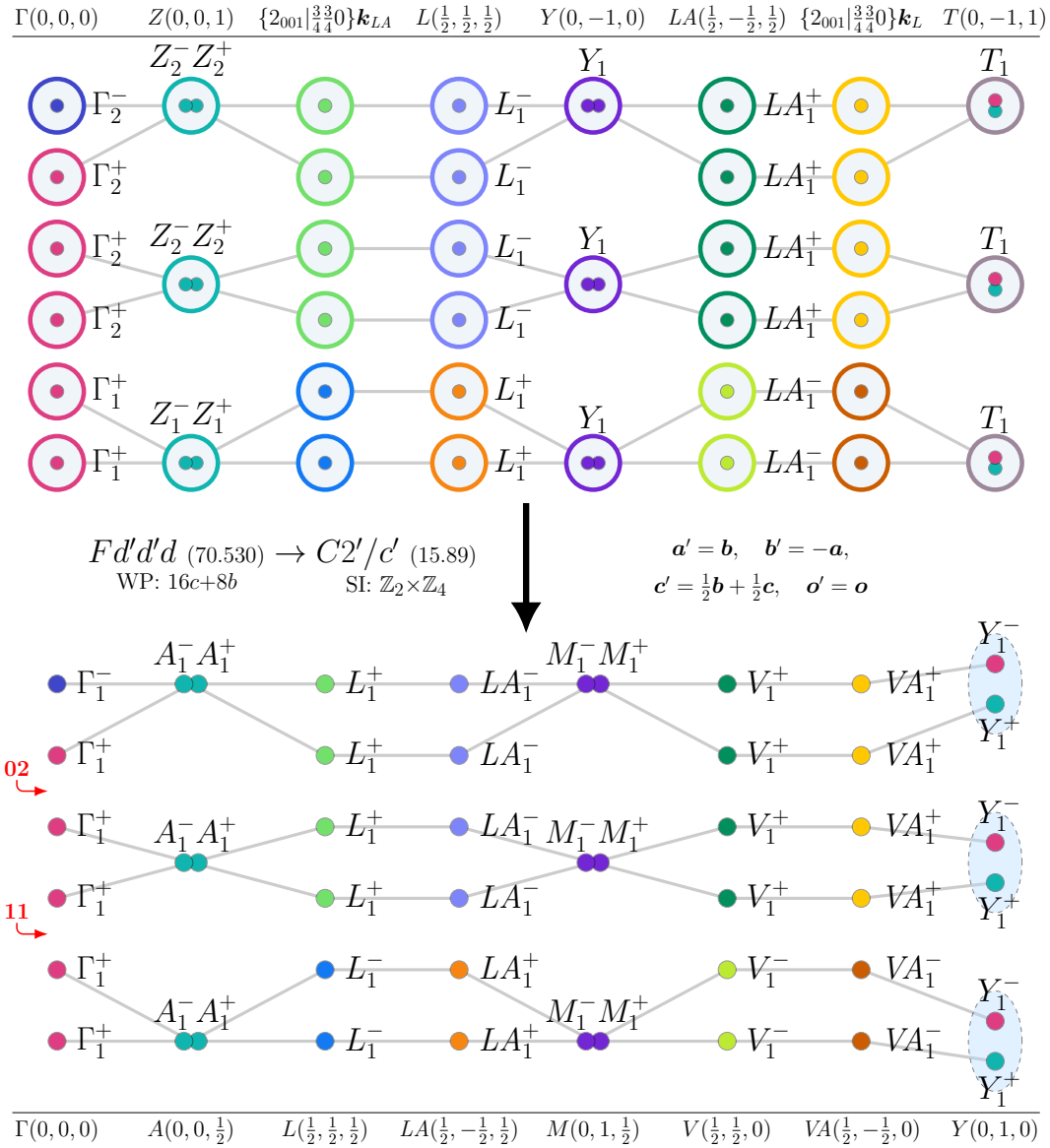
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Gap#	Possible SI Values
4	00, 02, 11, 13
5	G
6	00

85.1.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 443: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff positions $16c+8b$ of supergroup $Fd'd'd$ (70.530).

Supplementary Table 875: Number of times a given SI can appear in the band structure. Supergroup: $Fd'd'd$ (70.530), WP: 16c+8b, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0, 1
02	0, 1
11	0, 1, 2
13	0, 1, 2
gapless	3
nontrivial	1, 2

Supplementary Table 876: Possible SI values in each gap. Supergroup: $Fd'd'd$ (70.530), WP: 16c+8b, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	00, 02, 11, 13
3	G
4	00, 02, 11, 13
5	G
6	00

85.2 WP: 16c

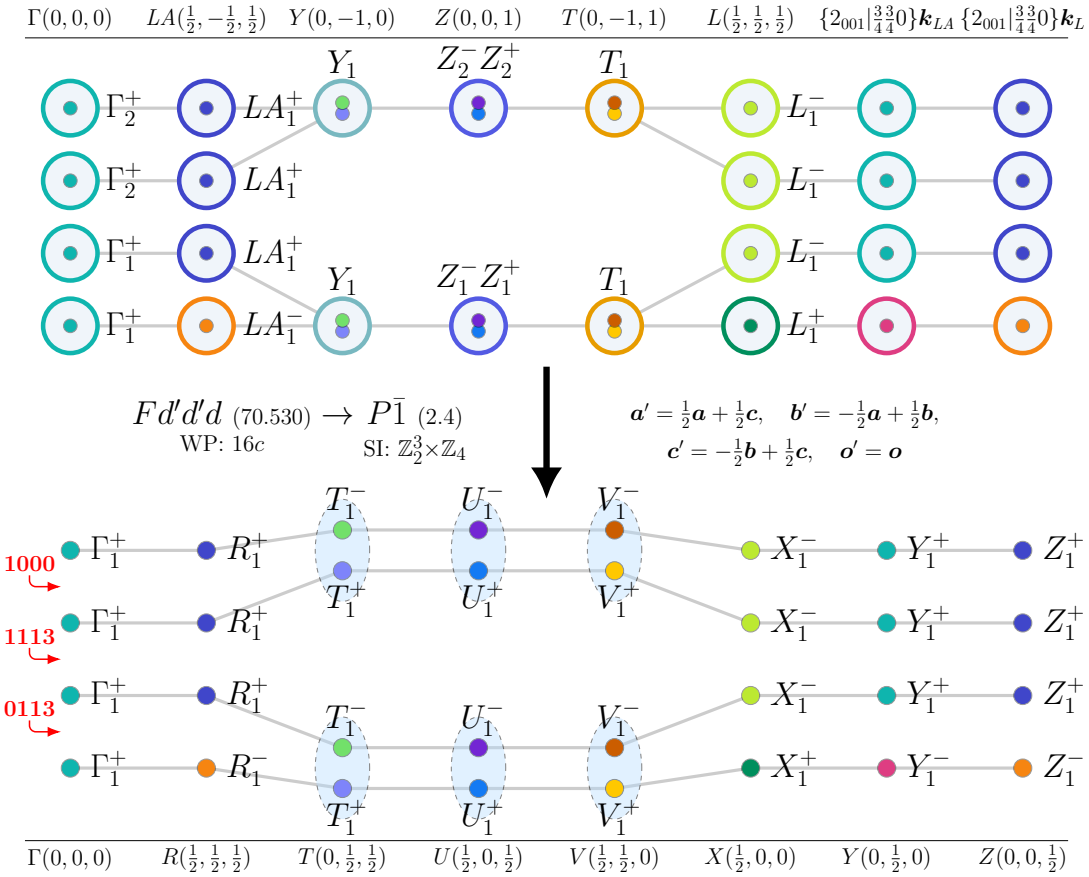
BCS Materials: Bi_2RuMnO_7 (20 K)[391].

85.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [100] or B $\perp$ [010]) and strain $\perp$ [001],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [010],
- (B $\parallel$ [010] or B $\perp$ [100]) and strain $\perp$ [001],
- B in generic direction.

[391] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.153`



Supplementary Figure 444: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $16c$ of supergroup $Fd'd'd$ (70.530).

Supplementary Table 877: Number of times a given SI can appear in the band structure. Supergroup: $Fd'd'd$ (70.530), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0001	0, 1, 2
0003	0, 1, 2
0010	0, 1
0012	0, 1, 2
0100	0, 1
0102	0, 1, 2
0111	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1002	0, 1
1011	0, 1, 2
1013	0, 1, 2
1101	0, 1, 2
1103	0, 1, 2
1110	0, 1
1111	0, 1
1112	0, 1, 2
1113	0, 1
gapless	0
nontrivial	3

5224

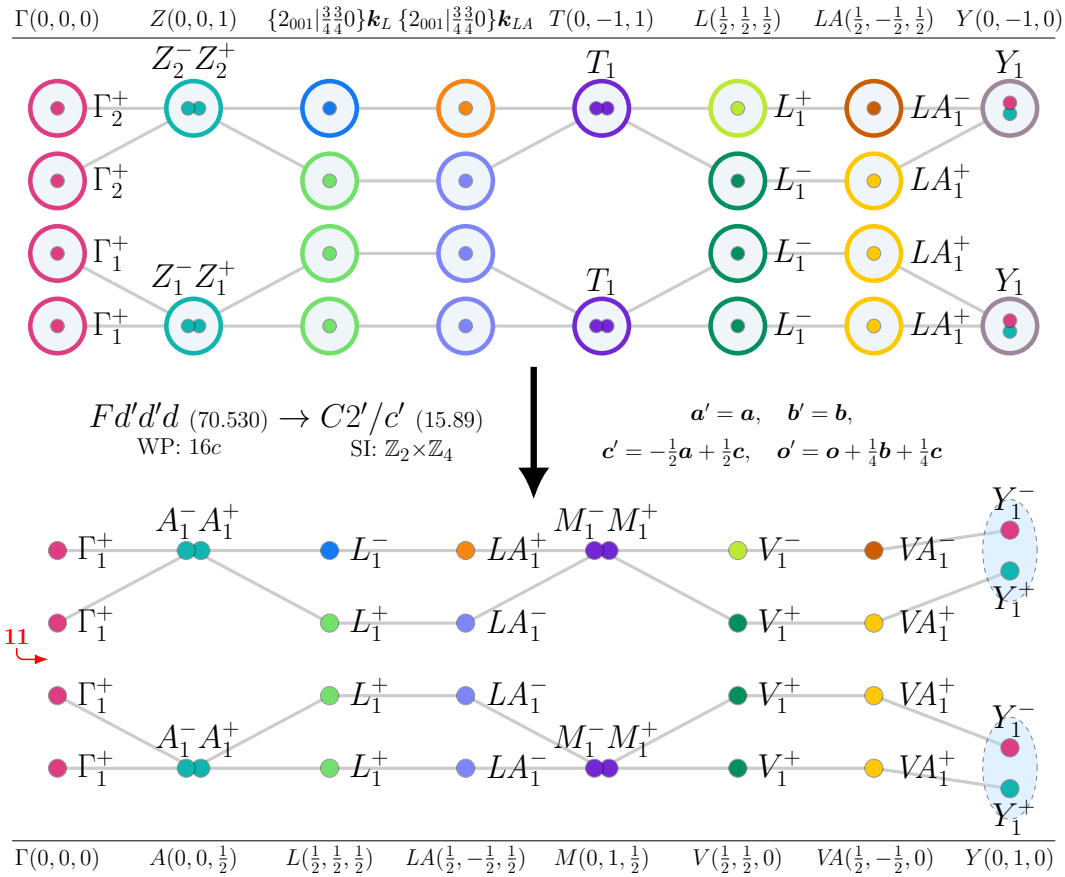
Supplementary Table 878: Possible SI values in each gap. Supergroup: $Fd'd'd$ (70.530), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
2	1111, 1113
3	0001, 0003, 0010, 0012, 0100 0102, 0111, 0113, 1000, 1002 1011, 1013, 1101, 1103, 1110 1112
4	0000

85.2.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 445: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position $16c$ of supergroup $Fd'd'd$ (70.530).

Supplementary Table 879: Number of times a given SI can appear in the band structure. Supergroup: $Fd'd'd$ (70.530), WP: 16c, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

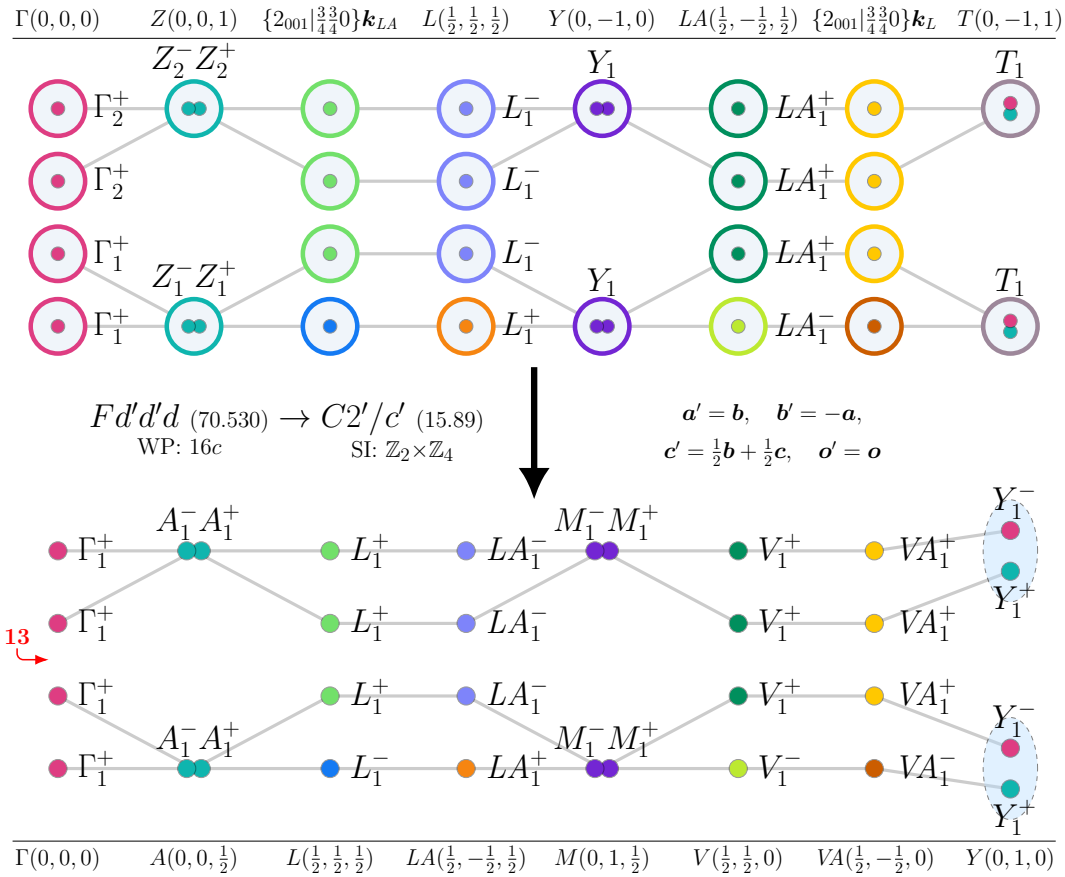
Supplementary Table 880: Possible SI values in each gap. Supergroup: $Fd'd'd$ (70.530), WP: 16c, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

85.2.3 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 446: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 16c of supergroup $Fd'd'd$ (70.530).

Supplementary Table 881: Number of times a given SI can appear in the band structure. Supergroup: $Fd'd'd$ (70.530), WP: 16c, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
11	0, 1
13	0, 1
gapless	2
nontrivial	1

Supplementary Table 882: Possible SI values in each gap. Supergroup: $Fd'd'd$ (70.530), WP: 16c, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	11, 13
3	G
4	00

86 MSG I_cbca (73.553)

Nontrivial-SI Subgroups: C_ac (9.41), $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $P_S\bar{1}$ (2.7), C_a2 (5.17), I_cba2 (45.239), $C2/c$ (15.85), $Im'm'a$ (74.558), C_a2/c (15.91), I_aba2 (45.240), $C2/c$ (15.85), $Iba'm'$ (72.544), C_c2/c (15.90), I_aba2 (45.240), $C2/c$ (15.85), $Iba'm'$ (72.544), C_c2/c (15.90).

Trivial-SI Subgroups: Cc' (9.39), Cm' (8.34), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), $C2'$ (5.15), P_S1 (1.3), Cc (9.37), $Im'a2'$ (46.243), $Im'a2'$ (46.243), Cc (9.37), $Im'a2'$ (46.243), $Ib'a2'$ (45.237), C_cc (9.40), Cc (9.37), $Im'a2'$ (46.243), $Ib'a2'$ (45.237), C_cc (9.40), $C2$ (5.13), $Im'm'2$ (44.232), $C2$ (5.13), $Im'a'2$ (46.245), C_c2 (5.16), $C2$ (5.13), $Im'a'2$ (46.245), C_c2 (5.16).

86.1 WP: $8g$

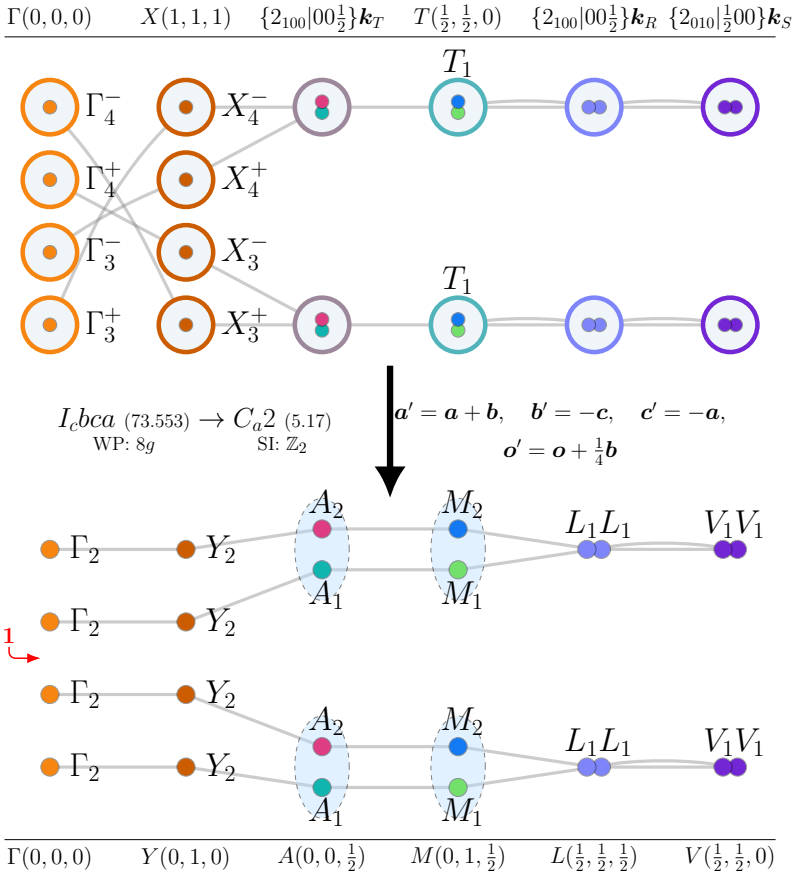
BCS Materials: $YbCo_2Si_2$ (0.9 K)[392].

86.1.1 Topological bands in subgroup C_a2 (5.17)

Perturbation:

- E ∥ [001] and strain ⊥ [001].

[392]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.176`



Supplementary Figure 447: Topological magnon bands in subgroup C_a2 (5.17) for magnetic moments on Wyckoff position $8g$ of supergroup I_cbca (73.553).

Supplementary Table 883: Number of times a given SI can appear in the band structure. Supergroup: I_cbca (73.553), WP: $8g$, Subgroup: C_a2 (5.17).

SI	Possible #Gaps
0	0
1	1
gapless	2
nontrivial	1

Supplementary Table 884: Possible SI values in each gap. Supergroup: I_cbca (73.553), WP: $8g$, Subgroup: C_a2 (5.17).

Gap#	Possible SI Values
1	G
2	1
3	G
4	0

87 MSG $Im'm'a$ (74.558)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/m'$ (12.62), $C2'/m'$ (12.62), $C2/c$ (15.85).

Trivial-SI Subgroups: Cm' (8.34), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), Cc (9.37), $Im'a2'$ (46.243), $Im'a2'$ (46.243), $C2$ (5.13),

$Im'm'2$ (44.232).

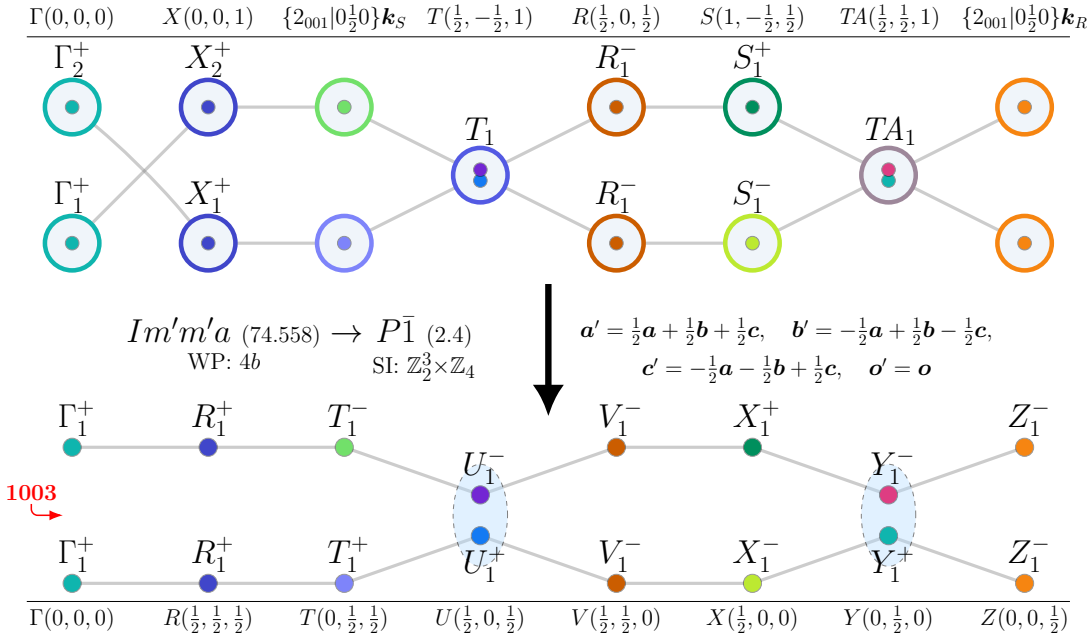
87.1 WP: $4b$

BCS Materials: $Pr_{0.5}Sr_{0.5}CoO_3$ (230 K)[393], $Pr_{0.5}Sr_{0.4}Ba_{0.1}CoO_3$ (226 K)[394].

87.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [100],
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [001],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [010],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 448: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4b$ of supergroup $Im'm'a$ (74.558).

Supplementary Table 885: Number of times a given SI can appear in the band structure. Supergroup: $Im'm'a$ (74.558), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0010	0, 1
0111	0, 1
0113	0, 1
1001	0, 1
1003	0, 1
1100	0, 1
gapless	0
nontrivial	1

[393] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.304`
[394] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.717`

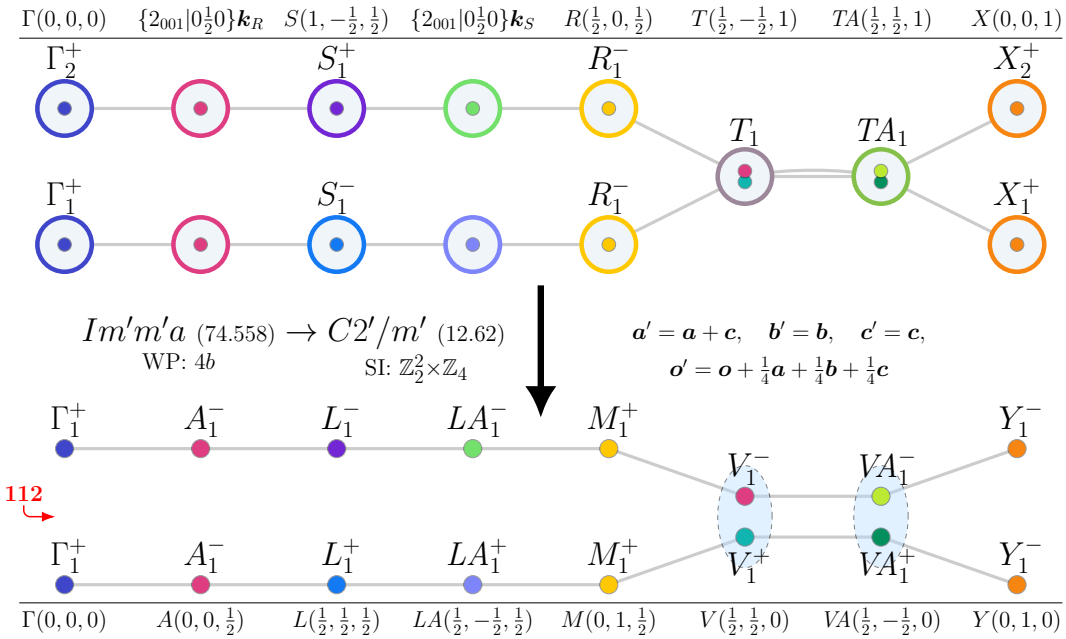
Supplementary Table 886: Possible SI values in each gap. Supergroup: $Im'm'a$ (74.558), WP: $4b$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0111, 0113, 1001, 1003 1100
2	0000

87.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 449: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4b$ of supergroup $Im'm'a$ (74.558).

Supplementary Table 887: Number of times a given SI can appear in the band structure. Supergroup: $Im'm'a$ (74.558), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
012	0, 1
112	0, 1
gapless	0
nontrivial	1

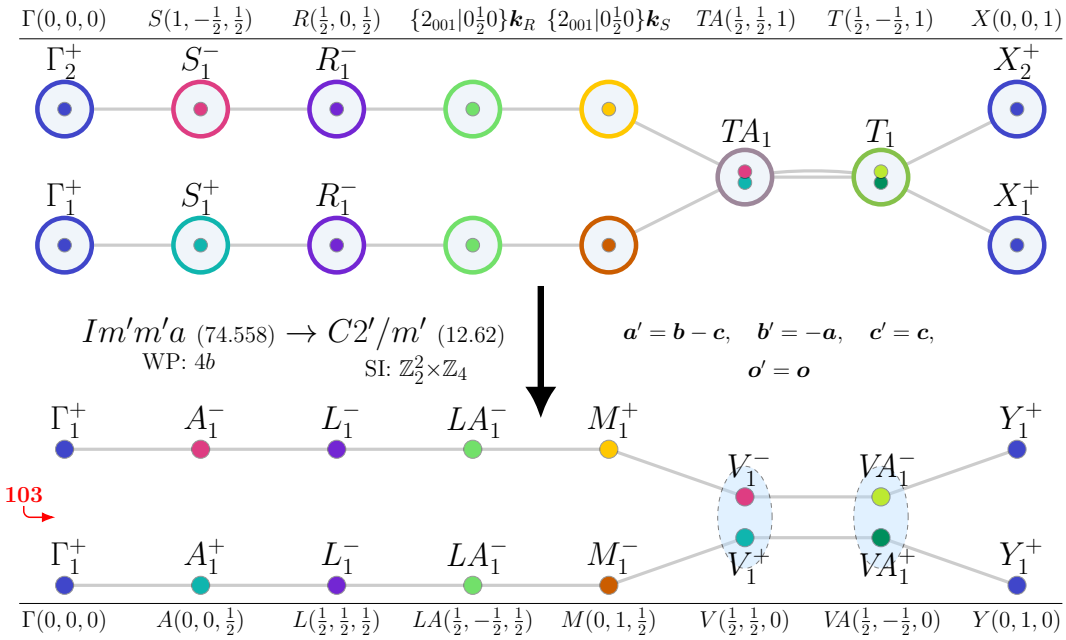
Supplementary Table 888: Possible SI values in each gap. Supergroup: $Im'm'a$ (74.558), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	012, 112
2	000

87.1.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 450: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4b$ of supergroup $Im'm'a$ (74.558).

Supplementary Table 889: Number of times a given SI can appear in the band structure. Supergroup: $Im'm'a$ (74.558), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1
003	0, 1
101	0, 1
103	0, 1
gapless	0
nontrivial	1

Supplementary Table 890: Possible SI values in each gap. Supergroup: $Im'm'a$ (74.558), WP: $4b$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 101, 103
2	000

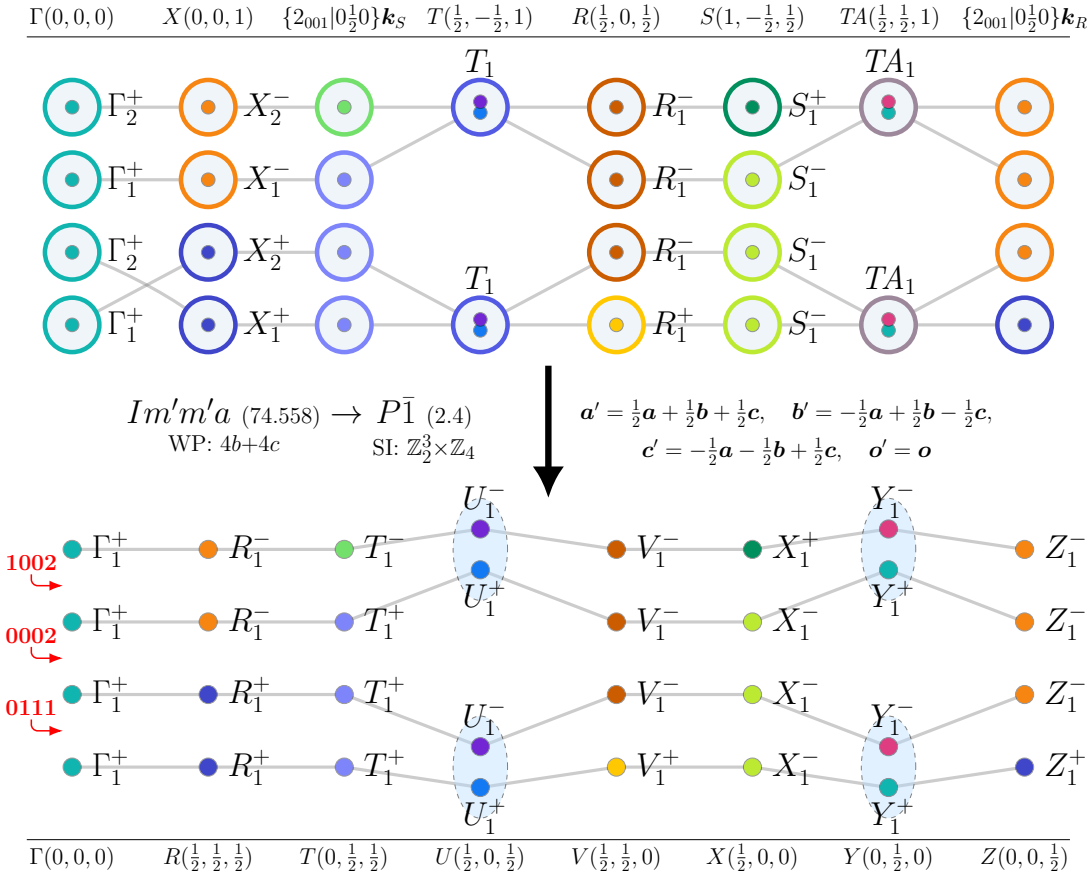
87.2 WP: $4b + 4c$

BCS Materials: Na_2NiCrF_7 (4 K)[395], Na_2NiCrF_7 (4 K)[396].

87.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [100],
- (B ∥ [100] or B ⊥ [010]) and strain ⊥ [001],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [010],
- (B ∥ [010] or B ⊥ [100]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 451: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b + 4c$ of supergroup $Im'm'a$ (74.558).

Supplementary Table 891: Number of times a given SI can appear in the band structure. Supergroup: $Im'm'a$ (74.558), WP: $4b$+$4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0001	0, 1
0002	0, 1
0003	0, 1
0010	0, 1, 2

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[395]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.573`
[396]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.572`

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SI	Possible #Gaps
0011	0, 1, 2
0012	0, 1
0013	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1
1003	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1
1103	0, 1, 2
1110	0, 1
1111	0, 1
1112	0, 1
1113	0, 1
gapless	0
nontrivial	2, 3

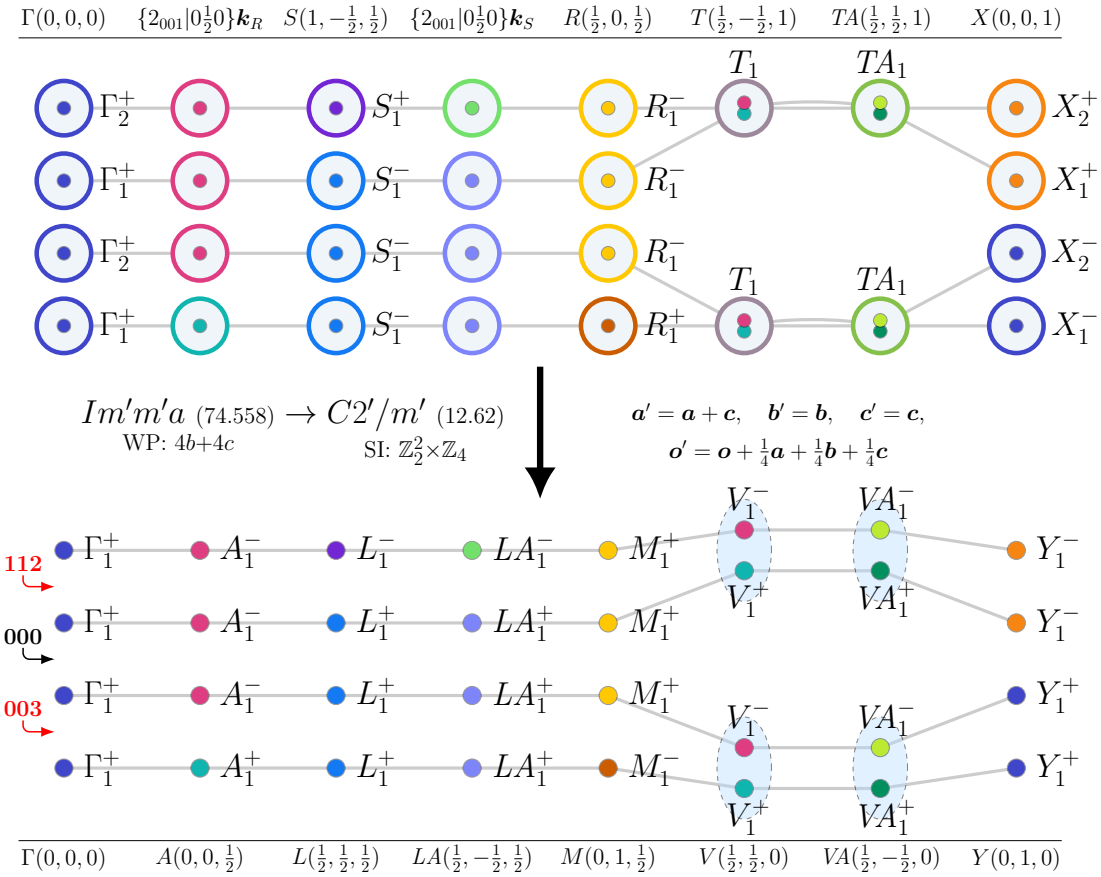
Supplementary Table 892: Possible SI values in each gap. Supergroup: $Im'm'a$ (74.558), WP: $4b$+$4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0001, 0002, 0003, 1110 1111, 1112, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

87.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [100] or B $\perp$ [010]).



Supplementary Figure 452: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Im'm'a$ (74.558).

Supplementary Table 893: Number of times a given SI can appear in the band structure. Supergroup: $Im'm'a$ (74.558), WP: $4b$+$4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
010	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1
101	0, 1, 2
102	0, 1
103	0, 1, 2
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	2, 3

5303

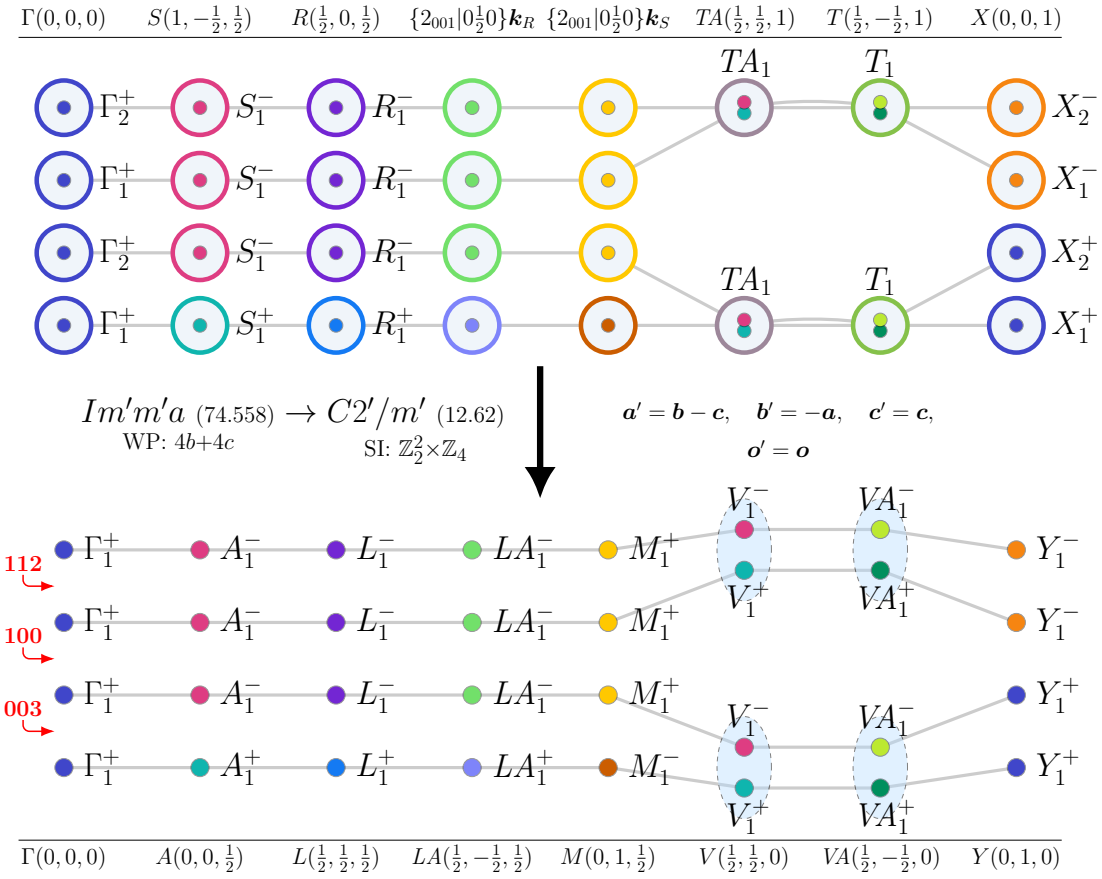
Supplementary Table 894: Possible SI values in each gap. Supergroup: $Im'm'a$ (74.558), WP: $4b$+$4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 011, 013, 100 102, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

87.2.3 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [100]).



Supplementary Figure 453: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Im'm'a$ (74.558).

Supplementary Table 895: Number of times a given SI can appear in the band structure. Supergroup: $Im'm'a$ (74.558), WP: $4b$+$4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1

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SI	Possible #Gaps
003	0, 1, 2
010	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1
101	0, 1, 2
102	0, 1
103	0, 1, 2
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 896: Possible SI values in each gap. Supergroup: $Im'm'a$ (74.558), WP: $4b$+$4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 011, 013, 100 102, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

88 MSG $Imm'a'$ (74.559)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $C2'/m'$ (12.62), $C2/m$ (12.58).

Trivial-SI Subgroups: Cc' (9.39), Cm' (8.34), $C2'$ (5.15), $C2'$ (5.15), Cm (8.32), $Im'm2'$ (44.231), $Ima'2'$ (46.244), $C2$ (5.13), $Im'a'2$ (46.245).

88.1 WP: $4c + 4c$

BCS Materials: $Bi_{0.8}La_{0.2}Fe_{0.5}Mn_{0.5}O_3$ (240 K)[397].

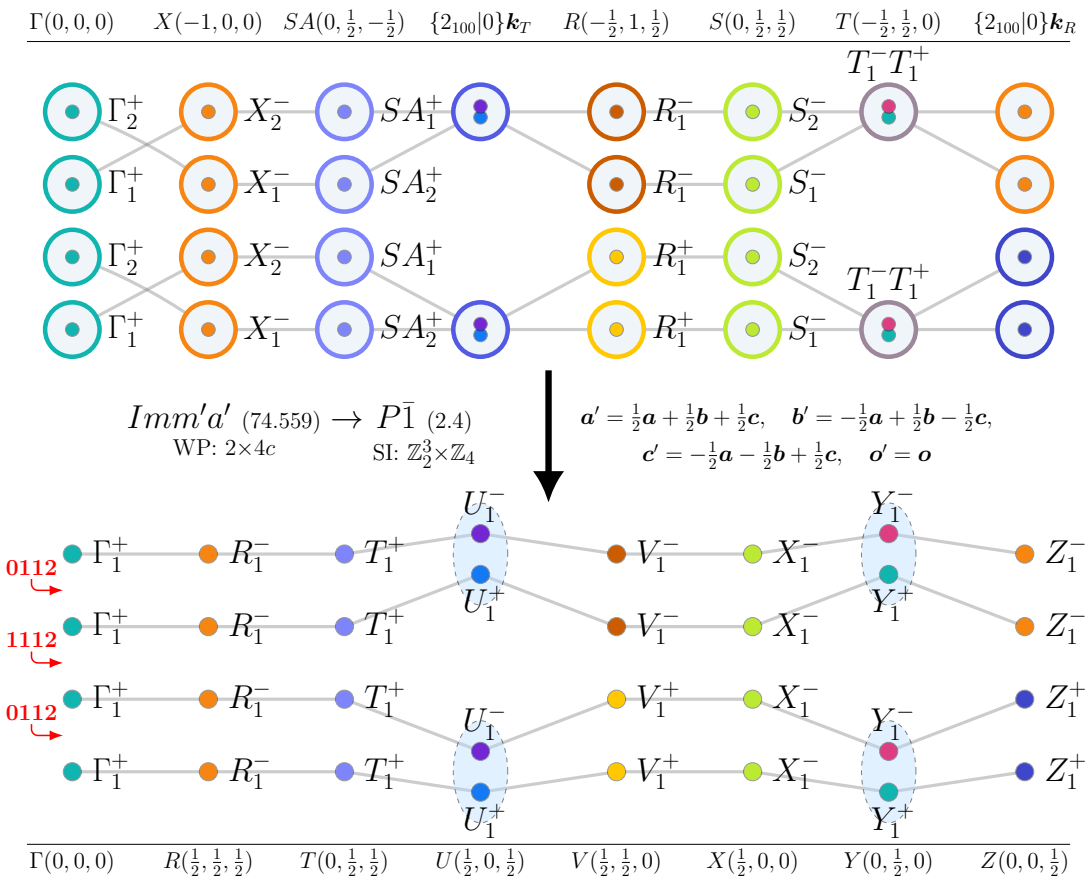
88.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],

[397] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.680`

- (B ∥ [001] or B ⊥ [010]) and strain ⊥ [001],
- B in generic direction.



Supplementary Figure 454: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 897: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: 4c+4c, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0011	0, 1, 2
0013	0, 1, 2
0112	0, 1, 2
1000	0, 1, 2
1101	0, 1, 2
1103	0, 1, 2
1112	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 898: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: 4c+4c, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0011, 0013, 0112, 1000, 1101 1103
2	0000, 1112
3	0011, 0013, 0112, 1000, 1101 1103

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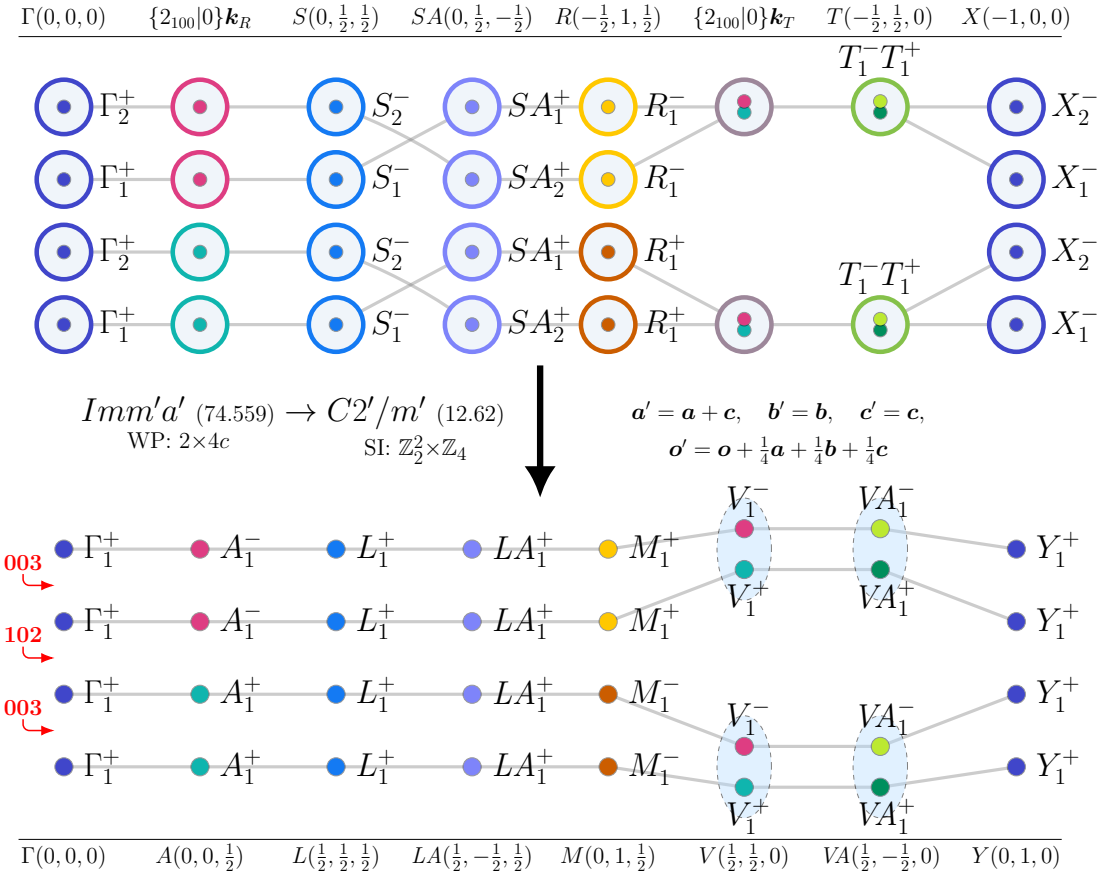
...continued

Gap#	Possible SI Values
4	0000

88.1.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 455: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4c+4c$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 899: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: 4c+4c, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
003	0, 1, 2
101	0, 1, 2
102	0, 1
103	0, 1, 2
gapless	0
nontrivial	2, 3

Supplementary Table 900: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4c$+$4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 101, 103
2	000, 102
3	001, 003, 101, 103
4	000

88.2 WP: $4b + 4c$

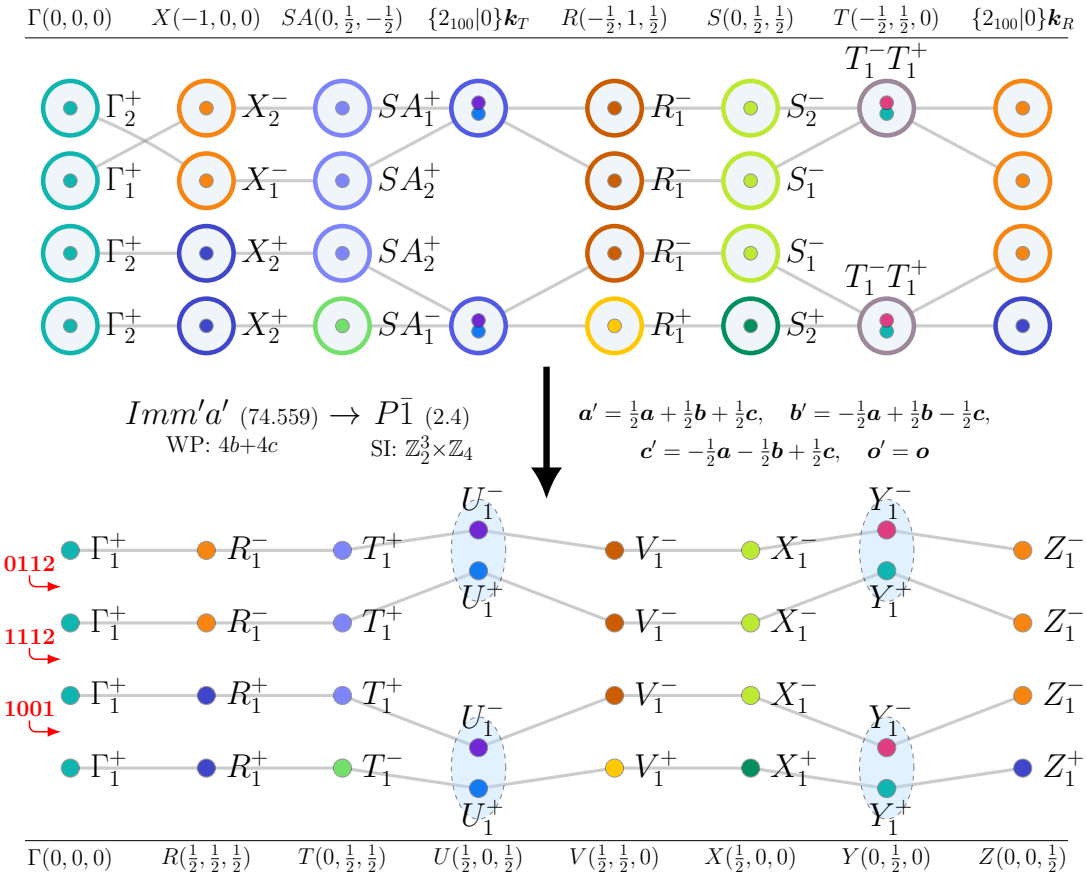
BCS Materials: Na_2NiFeF_7 (88 K)[398], Na_2NiFeF_7 (88 K)[399].

88.2.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [001],
- B in generic direction.

[398] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.580`
[399] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.579`



Supplementary Figure 456: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b+4c$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 901: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: $4b$+$4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0001	0, 1
0002	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1
0013	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1
1003	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1
1103	0, 1, 2
1110	0, 1
1111	0, 1
1112	0, 1
1113	0, 1
gapless	0

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SI	Possible #Gaps
nontrivial	2, 3

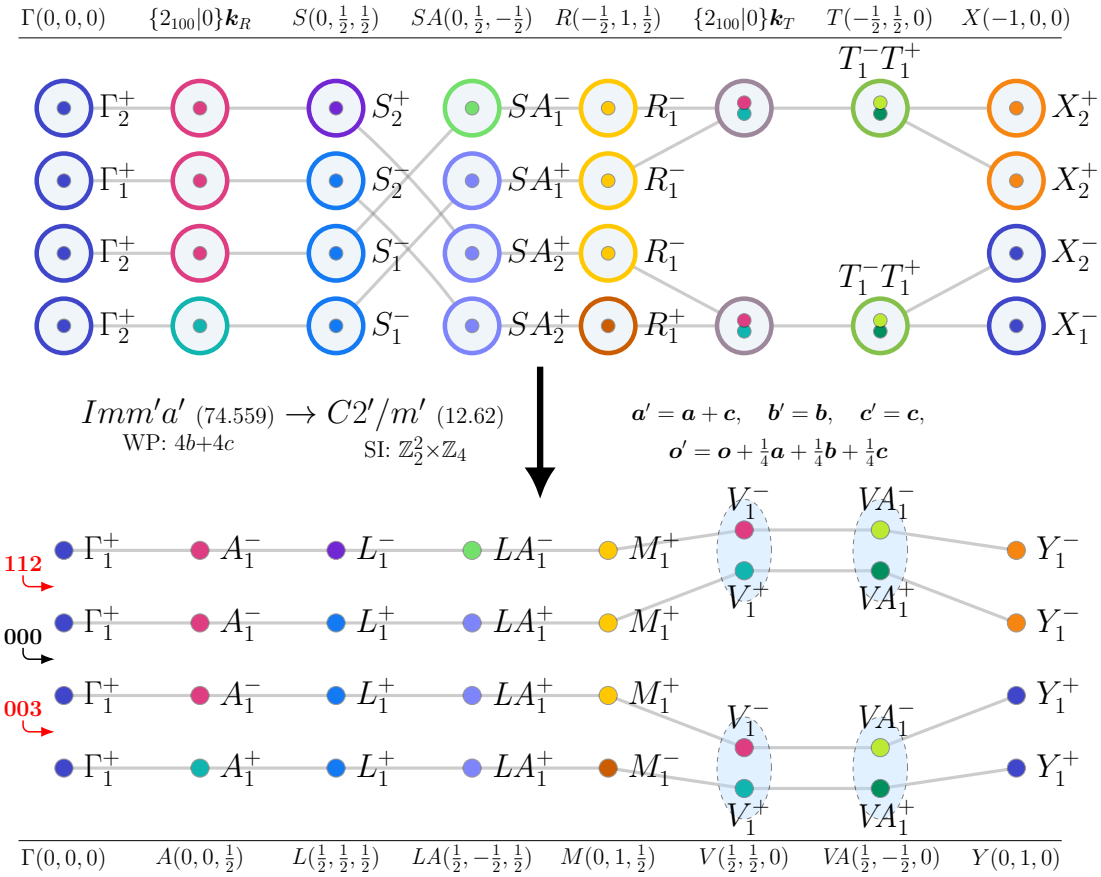
Supplementary Table 902: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4b$+$4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0001, 0002, 0003, 1110 1111, 1112, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

88.2.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 457: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b + 4c$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 903: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: $4b$+$4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
010	0, 1
011	0, 1
012	0, 1, 2
013	0, 1
100	0, 1
101	0, 1, 2
102	0, 1
103	0, 1, 2
110	0, 1
111	0, 1
112	0, 1, 2
113	0, 1
gapless	0
nontrivial	2, 3

Supplementary Table 904: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4b$+$4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 011, 013, 100 102, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

88.3 WP: $4a + 4c$

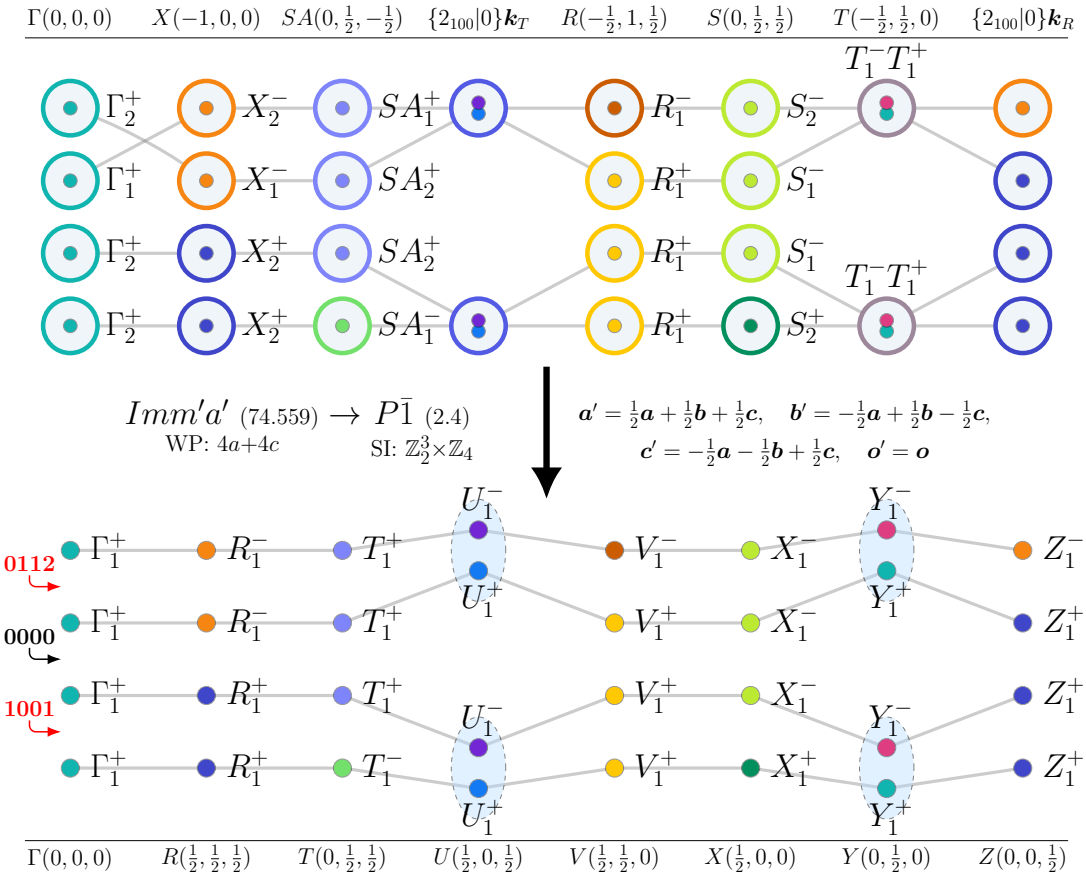
BCS Materials: $Fe_2F_5(H_2O)_2$ (48 K)[400].

88.3.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [001],
- B in generic direction.

[400]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.583`



Supplementary Figure 458: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a + 4c$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 905: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: $4a$+$4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0001	0, 1
0002	0, 1
0003	0, 1
0010	0, 1
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0110	0, 1
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1
1003	0, 1, 2
1100	0, 1
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	0, 1
1111	0, 1
1112	0, 1
1113	0, 1
gapless	0

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SI	Possible #Gaps
nontrivial	2, 3

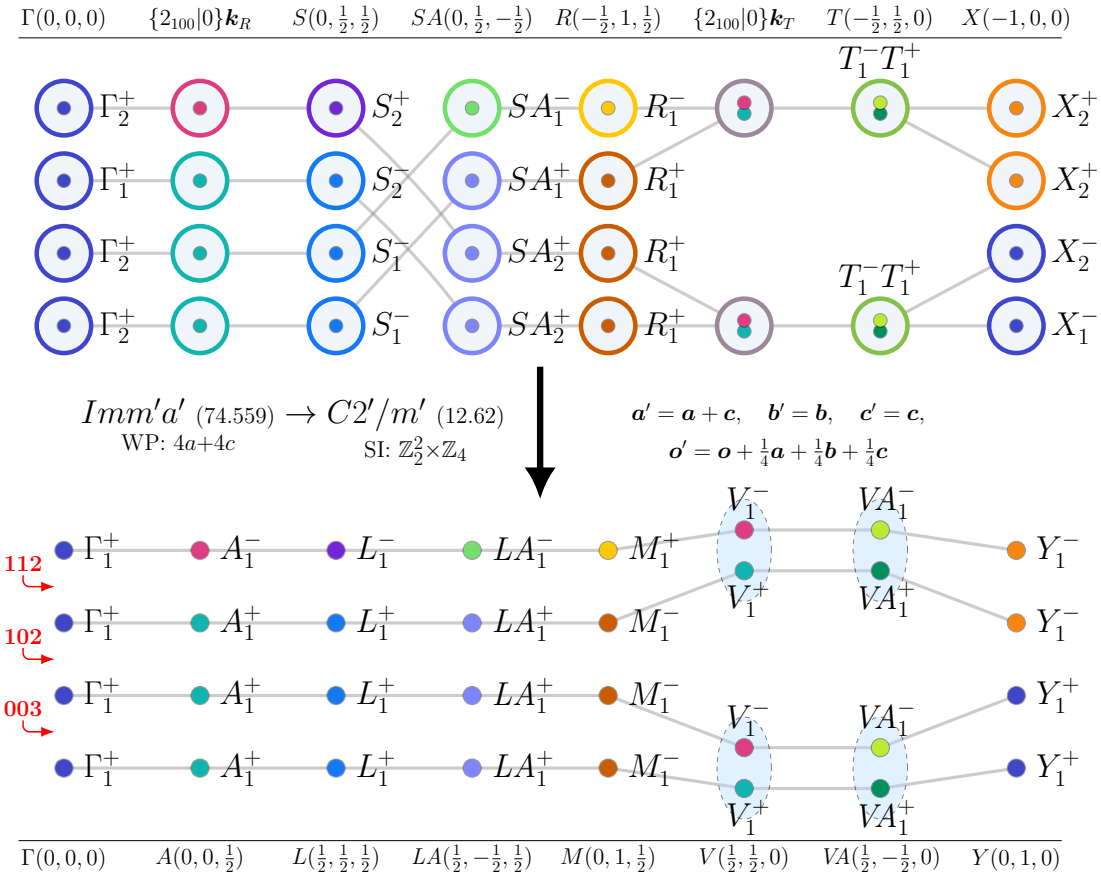
Supplementary Table 906: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4a$+$4c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0001, 0002, 0003, 1110 1111, 1112, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000

88.3.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 459: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4a+4c$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 907: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: $4a$+$4c$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1
001	0, 1, 2
002	0, 1
003	0, 1, 2
010	0, 1, 2
011	0, 1
012	0, 1
013	0, 1
100	0, 1
101	0, 1, 2
102	0, 1
103	0, 1, 2
110	0, 1, 2
111	0, 1
112	0, 1
113	0, 1
gapless	0
nontrivial	2, 3

5372

Supplementary Table 908: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4a$+$4c$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 011, 013, 100 102, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000

88.4 WP: $4b + 4c + 4e$

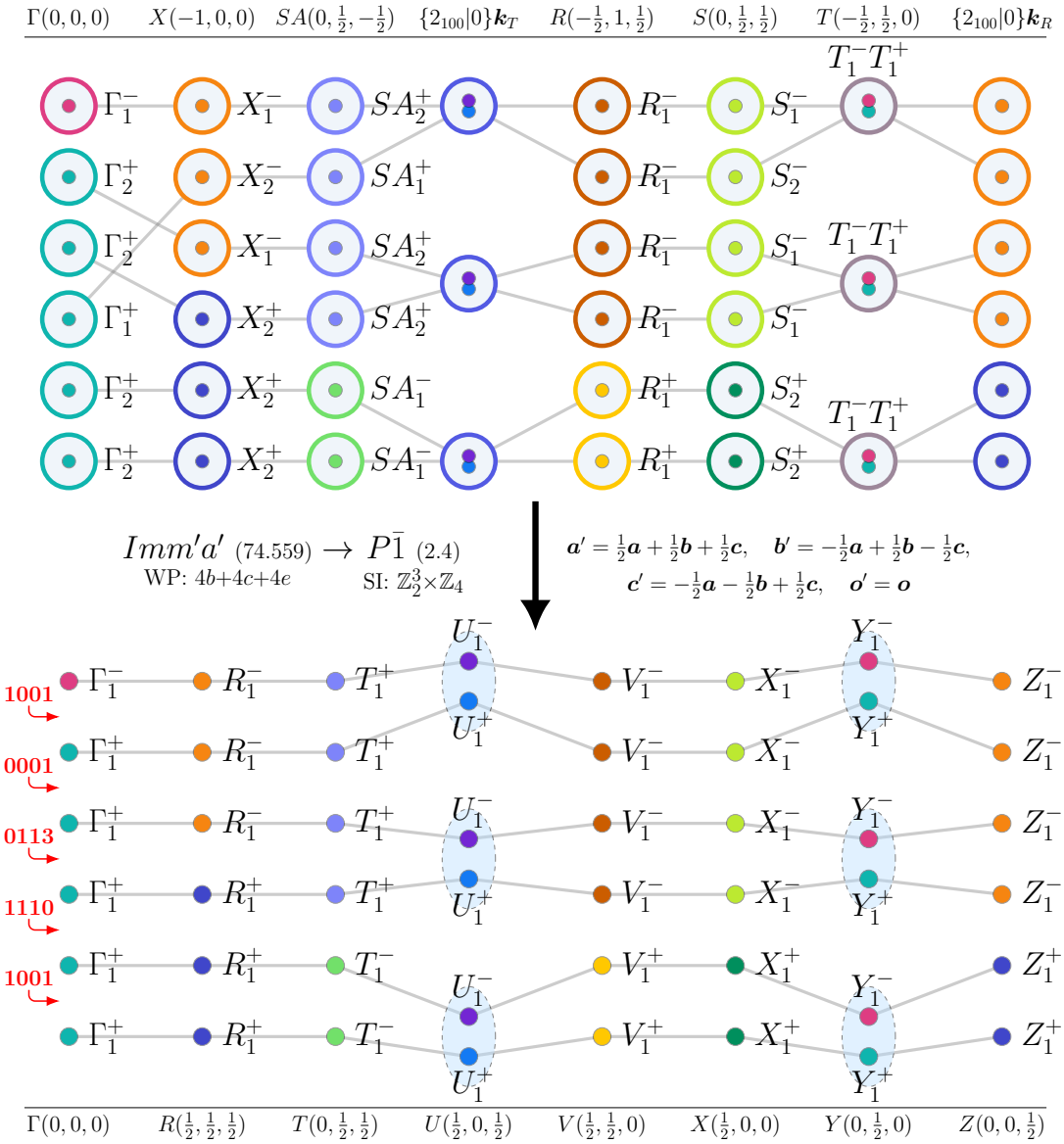
BCS Materials: $NdCo_2$ (42 K)[401].

88.4.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [001],
- B in generic direction.

[401] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.403`



Supplementary Figure 460: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4b+4c+4e$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 909: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: 4b+4c+4e, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2
0002	0, 1, 2
0003	0, 1, 2
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3

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SI	Possible #Gaps
1003	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3
1102	0, 1, 2, 3
1103	0, 1, 2, 3
1110	0, 1, 2
1111	0, 1, 2
1112	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	3, 4, 5

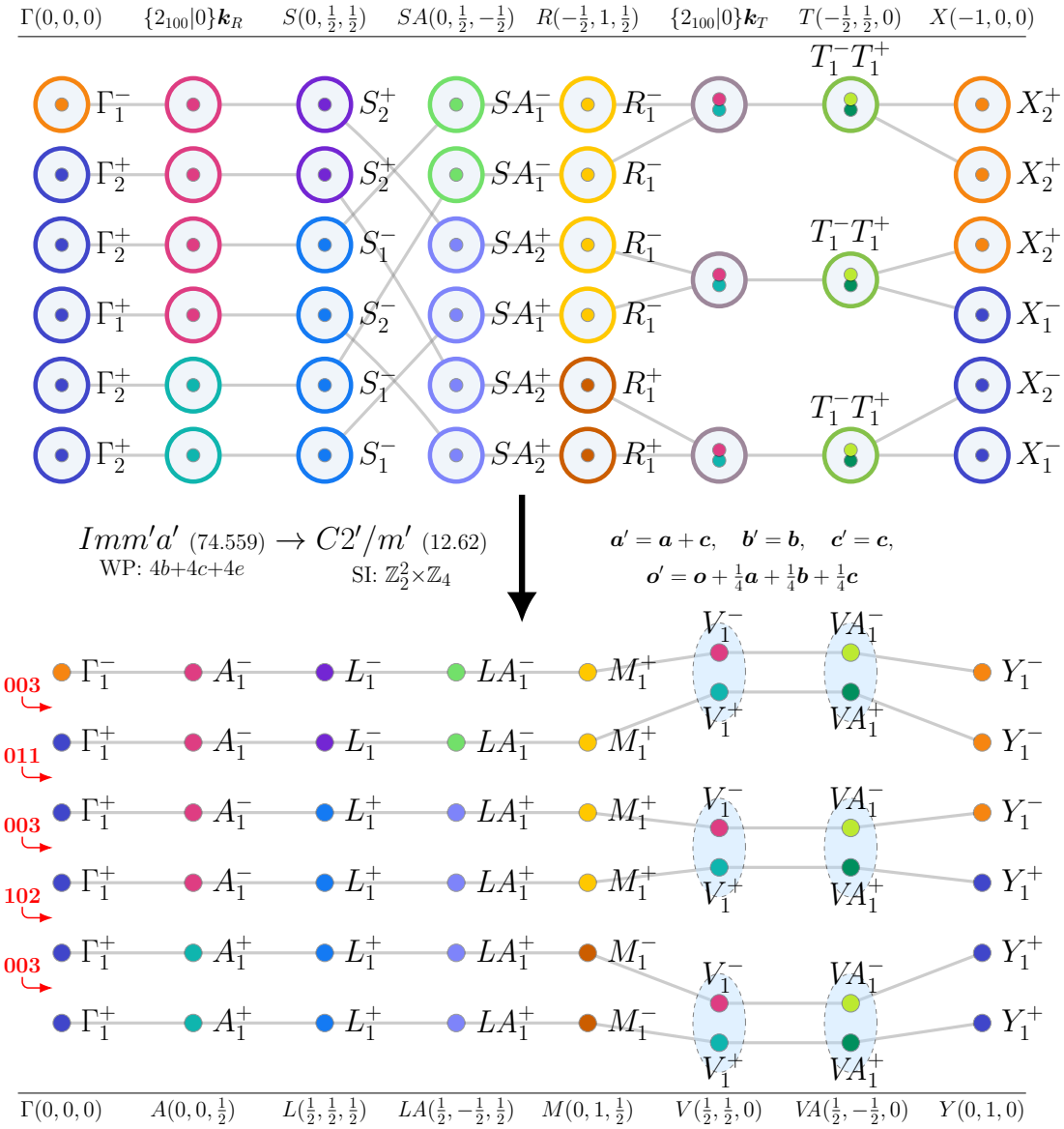
Supplementary Table 910: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4b$+$4c$+$4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0001, 0002, 0003, 1110 1111, 1112, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0001, 0002, 0003, 1110 1111, 1112, 1113
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	0000

88.4.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 461: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff positions $4b+4c+4e$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 911: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: 4b+4c+4e, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0, 1, 2
001	0, 1, 2, 3
002	0, 1, 2
003	0, 1, 2, 3
010	0, 1, 2, 3
011	0, 1, 2
012	0, 1, 2, 3
013	0, 1, 2
100	0, 1, 2
101	0, 1, 2, 3
102	0, 1, 2
103	0, 1, 2, 3
110	0, 1, 2, 3
111	0, 1, 2
112	0, 1, 2, 3

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SI	Possible #Gaps
113	0, 1, 2
gapless	0
nontrivial	3, 4, 5

Supplementary Table 912: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4b$+$4c$+$4e$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 010, 012, 101 103, 110, 112
2	000, 002, 011, 013, 100 102, 111, 113
3	001, 003, 010, 012, 101 103, 110, 112
4	000, 002, 011, 013, 100 102, 111, 113
5	001, 003, 010, 012, 101 103, 110, 112
6	000

88.5 WP: $4d$

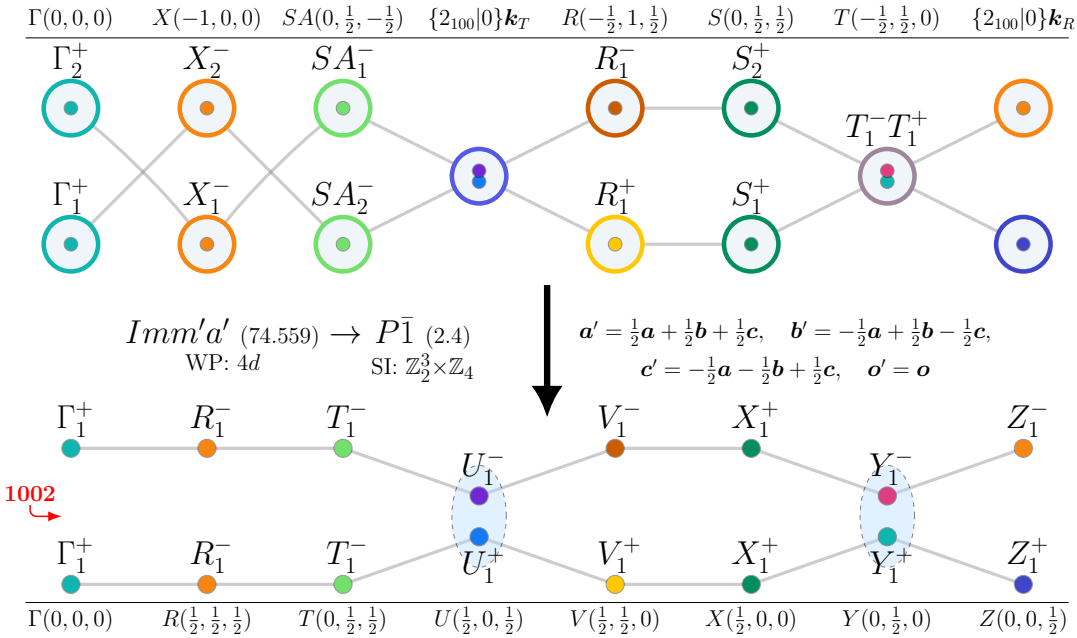
BCS Materials: $NpNiGa_5$ (18 K)[402].

88.5.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [100],
- (B $\parallel$ [010] or B $\perp$ [001]) and strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [100],
- (B $\parallel$ [001] or B $\perp$ [010]) and strain $\perp$ [001],
- B in generic direction.

[402] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.28`



Supplementary Figure 462: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4d$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 913: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0
0011	0, 1
0013	0, 1
0110	0, 1
1002	0, 1
1101	0, 1
1103	0, 1
gapless	0
nontrivial	1

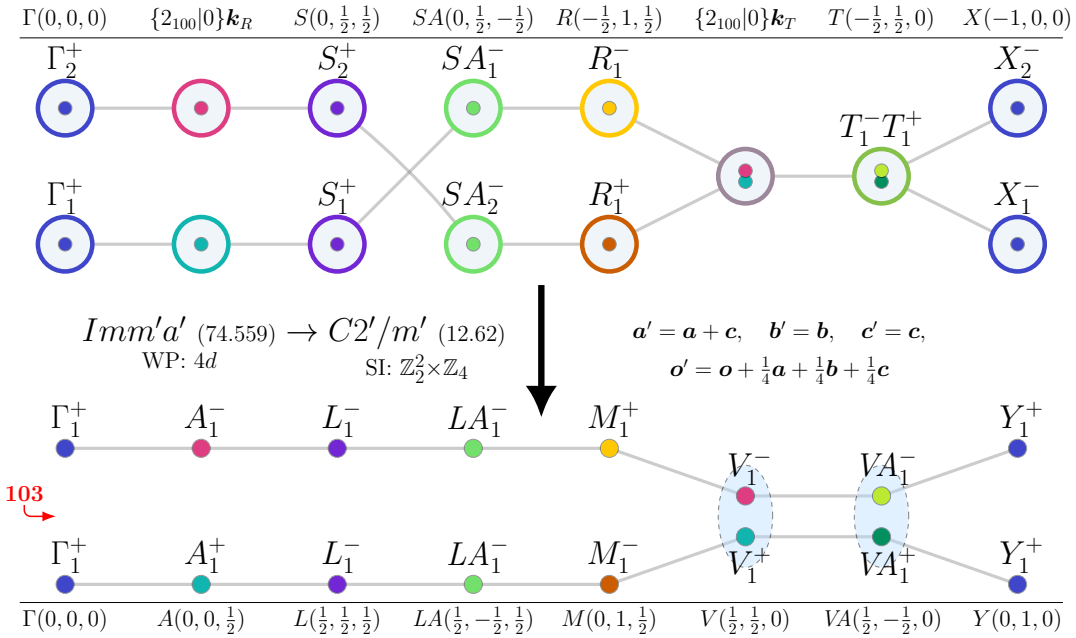
Supplementary Table 914: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0011, 0013, 0110, 1002, 1101 1103
2	0000

88.5.2 Topological bands in subgroup $C2'/m'$ (12.62)

Perturbations:

- strain $\perp$ [010],
- (B $\parallel$ [001] or B $\perp$ [010]).



Supplementary Figure 463: Topological magnon bands in subgroup $C2'/m'$ (12.62) for magnetic moments on Wyckoff position $4d$ of supergroup $Imm'a'$ (74.559).

Supplementary Table 915: Number of times a given SI can appear in the band structure. Supergroup: $Imm'a'$ (74.559), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

SI	Possible #Gaps
000	0
001	0, 1
003	0, 1
101	0, 1
103	0, 1
gapless	0
nontrivial	1

Supplementary Table 916: Possible SI values in each gap. Supergroup: $Imm'a'$ (74.559), WP: $4d$, Subgroup: $C2'/m'$ (12.62).

Gap#	Possible SI Values
1	001, 003, 101, 103
2	000

89 MSG $P4/n$ (85.59)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2$ (3.1), $P2/c$ (13.65), $P4$ (75.1).

Trivial-SI Subgroups: Pc (7.24).

89.1 WP: $2c+4e$

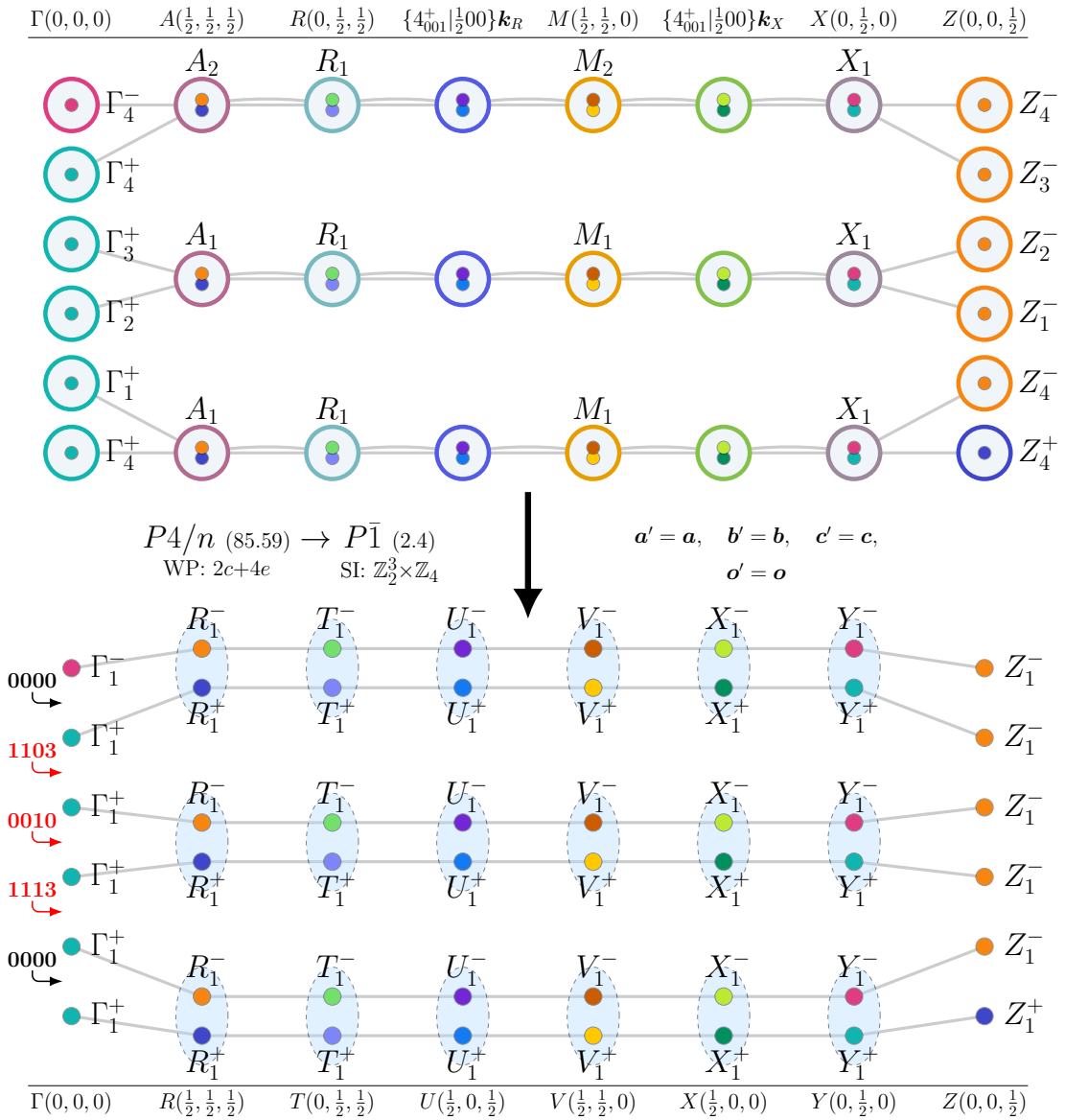
BCS Materials: Mn_3CuN (143 K)[403].

[403] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.5`

89.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.



Supplementary Figure 464: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $2c + 4e$ of supergroup $P4/n$ (85.59).

Supplementary Table 917: Number of times a given SI can appear in the band structure. Supergroup: $P4/n$ (85.59), WP: 2c+4e, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2, 3, 4
0001	0, 1, 2, 3
0002	0, 1
0003	0, 1, 2, 3
0010	0, 1, 2, 3, 4, 5
0011	0, 1, 2, 3
0012	0, 1, 2
0013	0, 1, 2, 3
0100	0, 1, 2, 3
0101	0, 1, 2, 3

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SI	Possible #Gaps
0102	0, 1, 2, 3
0103	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1010	0, 1, 2, 3
1011	0, 1, 2, 3
1012	0, 1, 2, 3
1013	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3
1111	0, 1, 2, 3, 4
1112	0, 1, 2, 3
1113	0, 1, 2, 3, 4
gapless	0
nontrivial	1, 2, 3, 4, 5

5424

Supplementary Table 918: Possible SI values in each gap. Supergroup: $P4/n$ (85.59), WP: $2c$+$4e$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
2	0000, 0010, 1101, 1113
3	0000, 0001, 0002, 0003, 0010 0011, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
4	0000, 0010, 1103, 1111
5	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112 1113
6	0000

90 MSG P_c4/n (85.64)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'_1/c'$ (14.79), $P_S\bar{1}$ (2.7), $P2$ (3.1), P_b2 (3.5), $P2/c$ (13.65), P_b2/c (13.71), $P4$ (75.1), P_c4 (75.4), $P4/n$ (85.59).

Trivial-SI Subgroups: Pc' (7.26), $P2'_1$ (4.9), P_S1 (1.3), Pc (7.24), P_bc (7.29).

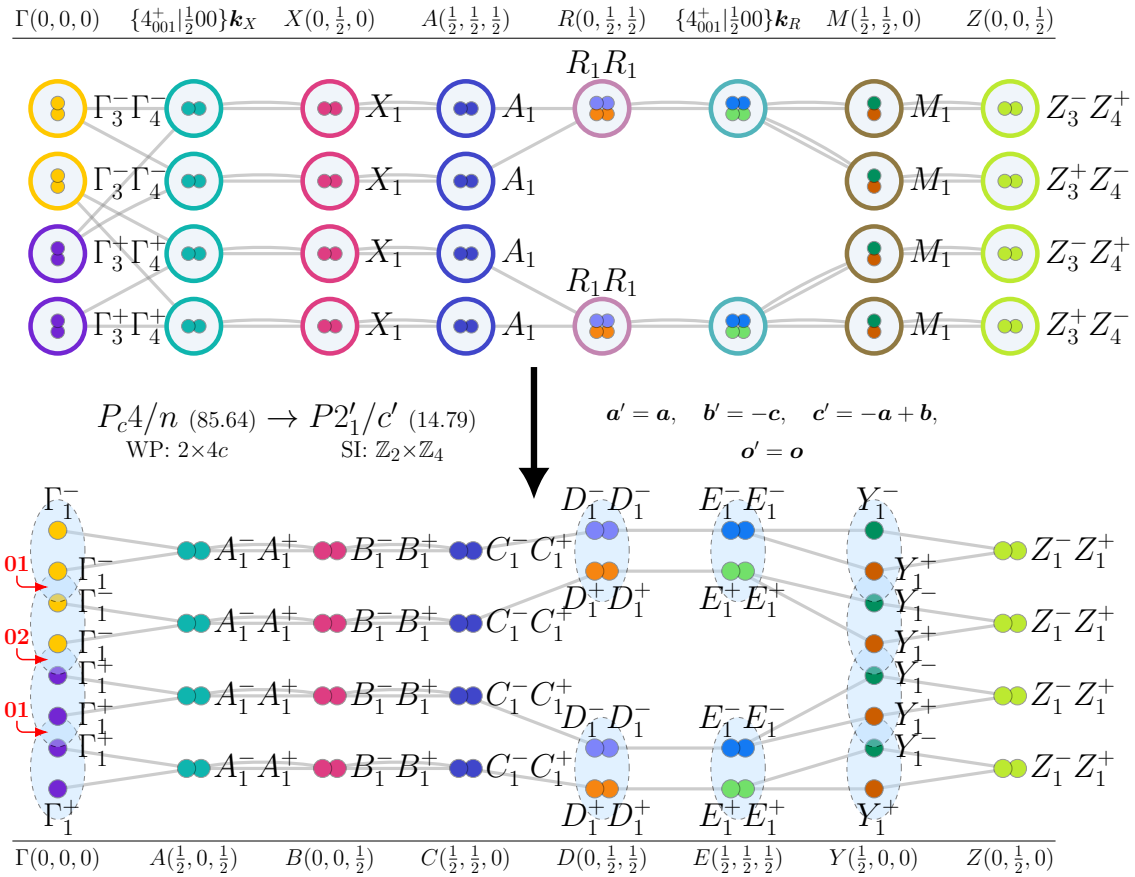
90.1 WP: $4c + 4c$

BCS Materials: Sr_2FeOsO_6 (67 K)[404].

90.1.1 Topological bands in subgroup $P2'_1/c'$ (14.79)

Perturbation:

- (B ∥ [100] or B ∥ [110] or B ⊥ [001]).



Supplementary Figure 465: Topological magnon bands in subgroup $P2'_1/c'$ (14.79) for magnetic moments on Wyckoff positions $4c + 4c$ of supergroup P_c4/n (85.64).

Supplementary Table 919: Number of times a given SI can appear in the band structure. Supergroup: P_c4/n (85.64), WP: 4c+4c, Subgroup: $P2'_1/c'$ (14.79).

SI	Possible #Gaps
00	0, 1

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[404]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=1.46`

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SI	Possible #Gaps
01	0, 1, 2
02	0, 1
03	0, 1, 2
gapless	4
nontrivial	2, 3

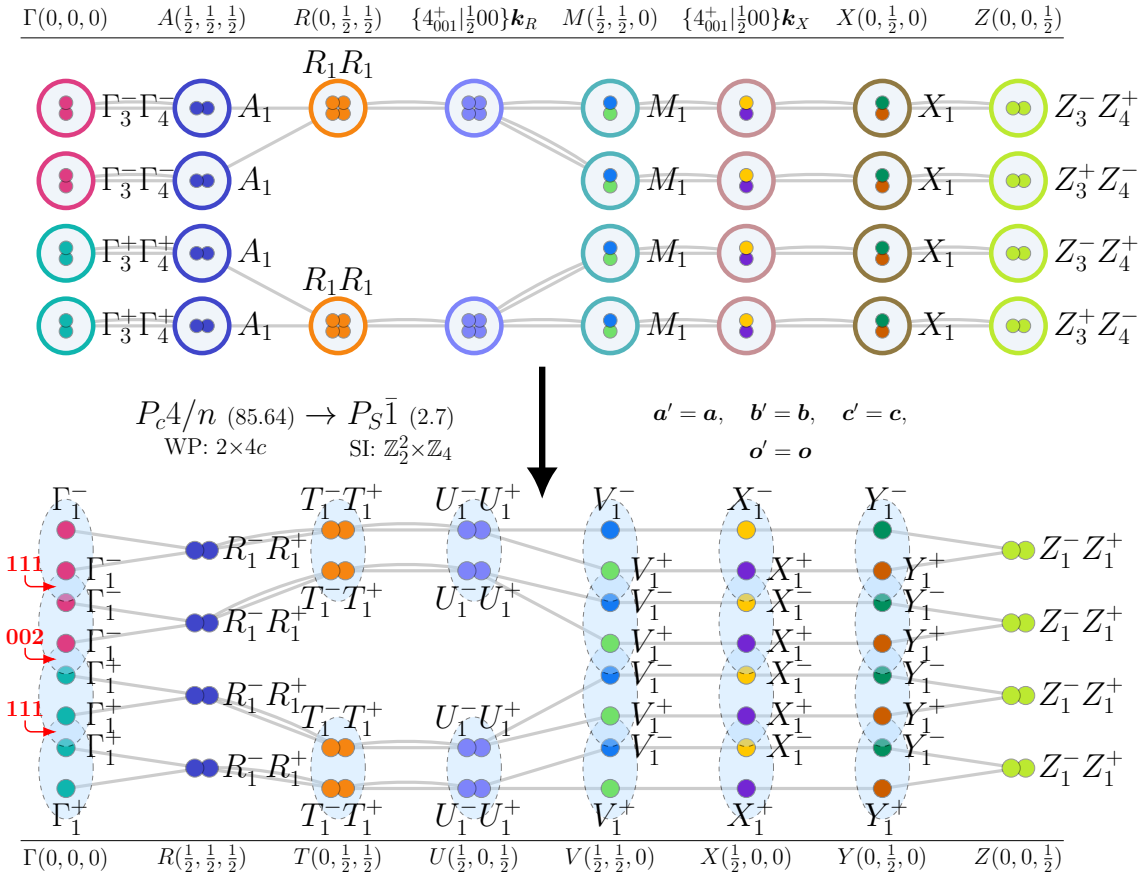
Supplementary Table 920: Possible SI values in each gap. Supergroup: P_c4/n (85.64), WP: $4c$+$4c$, Subgroup: $P2_1'/c'$ (14.79).

Gap#	Possible SI Values
1	G
2	01, 03
3	G
4	00, 02
5	G
6	01, 03
7	G
8	00

90.1.2 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 466: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff positions $4c + 4c$ of supergroup P_c4/n (85.64).

Supplementary Table 921: Number of times a given SI can appear in the band structure. Supergroup: P_c4/n (85.64), WP: 4c+4c, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0, 1
002	0, 1
111	0, 1, 2
113	0, 1, 2
gapless	4
nontrivial	2, 3

Supplementary Table 922: Possible SI values in each gap. Supergroup: P_c4/n (85.64), WP: 4c+4c, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111, 113
3	G
4	000, 002
5	G
6	111, 113
7	G
8	000

91 MSG P_C4_2/n (86.73)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $P2'/m'$ (10.46), $P_S\bar{1}$ (2.7), $P2$ (3.1), P_a2 (3.4), $P2/c$ (13.65), P_c2/c (13.72), $P4_2$ (77.13), P_C4_2 (77.17), $P4_2/n$ (86.67).

Trivial-SI Subgroups: Pm' (6.20), $P2'$ (3.3), P_S1 (1.3), Pc (7.24), P_cc (7.28).

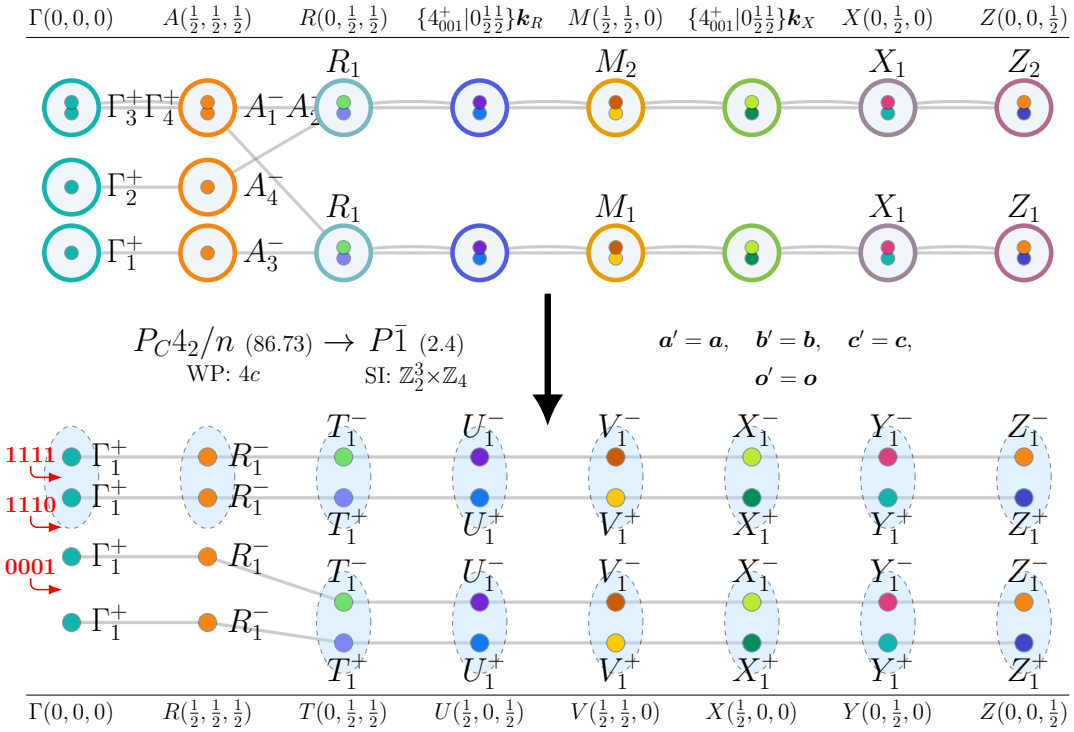
91.1 WP: $4c$

BCS Materials: $Sr_2CoO_2Ag_2Se_2$ (190 K)[405], $Ba_2CoO_2Ag_2Se_2$ (180 K)[406].

91.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- (B ∥ [100] or B ∥ [110] or B ⊥ [001]) and strain in generic direction,
- B in generic direction.



Supplementary Figure 467: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/n (86.73).

Supplementary Table 923: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/n (86.73), WP: $4c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2

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[405] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.23`
[406] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.24`

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SI	Possible #Gaps
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1110	1, 2, 3
1111	0, 1
1113	0, 1
gapless	0
nontrivial	1, 2, 3

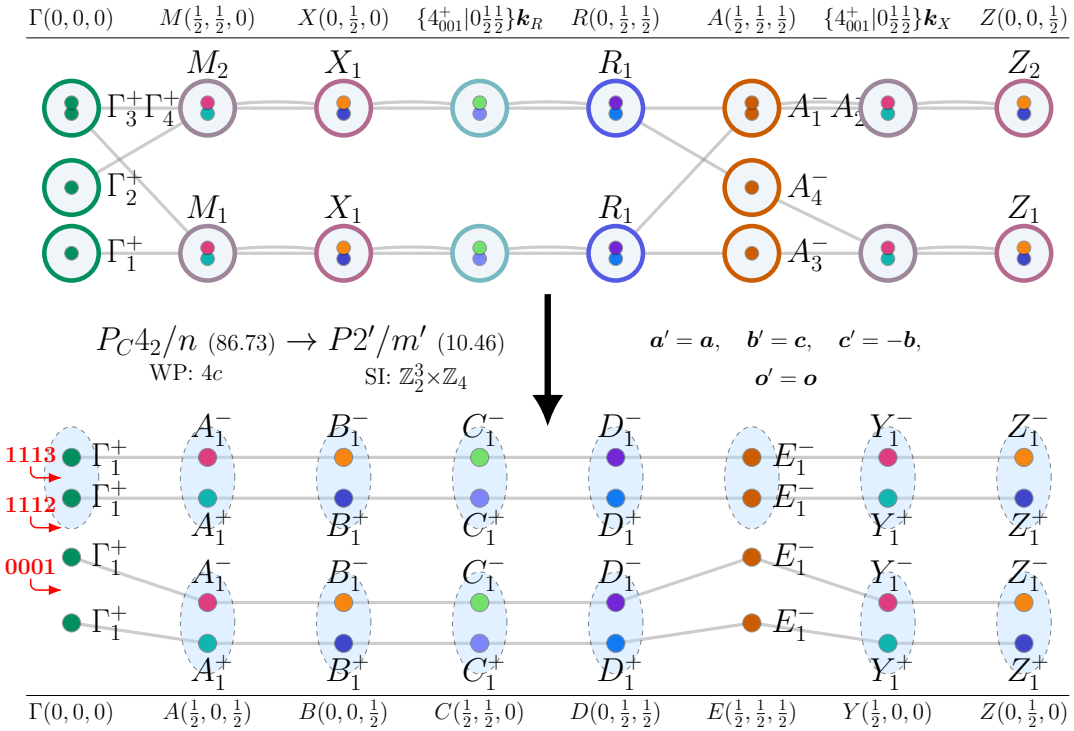
Supplementary Table 924: Possible SI values in each gap. Supergroup: $P_C 4_2/n$ (86.73), WP: 4*c*, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1113
2	1110
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1110, 1111
4	0000

91.1.2 Topological bands in subgroup $P2'/m'$ (10.46)

Perturbation:

- (B $\parallel$ [100] or B $\parallel$ [110] or B $\perp$ [001]).



Supplementary Figure 468: Topological magnon bands in subgroup $P2'/m'$ (10.46) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/n (86.73).

Supplementary Table 925: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/n (86.73), WP: $4c$, Subgroup: $P2'/m'$ (10.46).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1
0003	0, 1
0010	0, 1, 2
0011	0, 1, 2
0012	0, 1, 2
0013	0, 1, 2
0100	0, 1, 2
0101	0, 1, 2
0102	0, 1, 2
0103	0, 1, 2
0110	0, 1, 2
0111	0, 1, 2
0112	0, 1, 2
0113	0, 1, 2
1000	0, 1, 2
1001	0, 1, 2
1002	0, 1, 2
1003	0, 1, 2
1010	0, 1, 2
1011	0, 1, 2
1012	0, 1, 2
1013	0, 1, 2
1100	0, 1, 2
1101	0, 1, 2
1102	0, 1, 2
1103	0, 1, 2
1111	0, 1
1112	1, 2, 3
1113	0, 1

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SI	Possible #Gaps
gapless	0
nontrivial	1, 2, 3

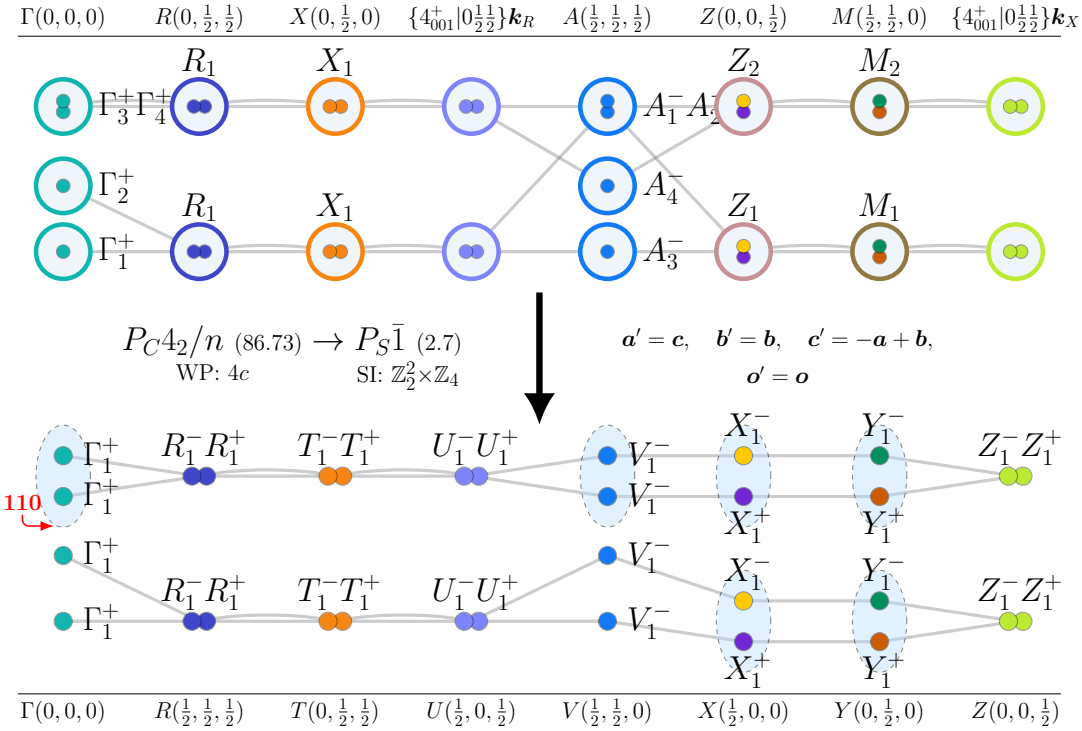
Supplementary Table 926: Possible SI values in each gap. Supergroup: P_C4_2/n (86.73), WP: $4c$, Subgroup: $P2'/m'$ (10.46).

Gap#	Possible SI Values
1	0000, 0001, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1111, 1112
2	1112
3	0000, 0003, 0010, 0011, 0012 0013, 0100, 0101, 0102, 0103 0110, 0111, 0112, 0113, 1000 1001, 1002, 1003, 1010, 1011 1012, 1013, 1100, 1101, 1102 1103, 1112, 1113
4	0000

91.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 469: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position $4c$ of supergroup P_C4_2/n (86.73).

Supplementary Table 927: Number of times a given SI can appear in the band structure. Supergroup: P_C4_2/n (86.73), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
110	1
gapless	2
nontrivial	1

Supplementary Table 928: Possible SI values in each gap. Supergroup: P_C4_2/n (86.73), WP: $4c$, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	110
3	G
4	000

92 MSG $I4_1/a$ (88.81)

Nontrivial-SI Subgroups: $P\bar{1}$ (2.4), $C2/c$ (15.85).

Trivial-SI Subgroups: Cc (9.37), $C2$ (5.13), $I4_1$ (80.29).

92.1 WP: $4a + 8d$

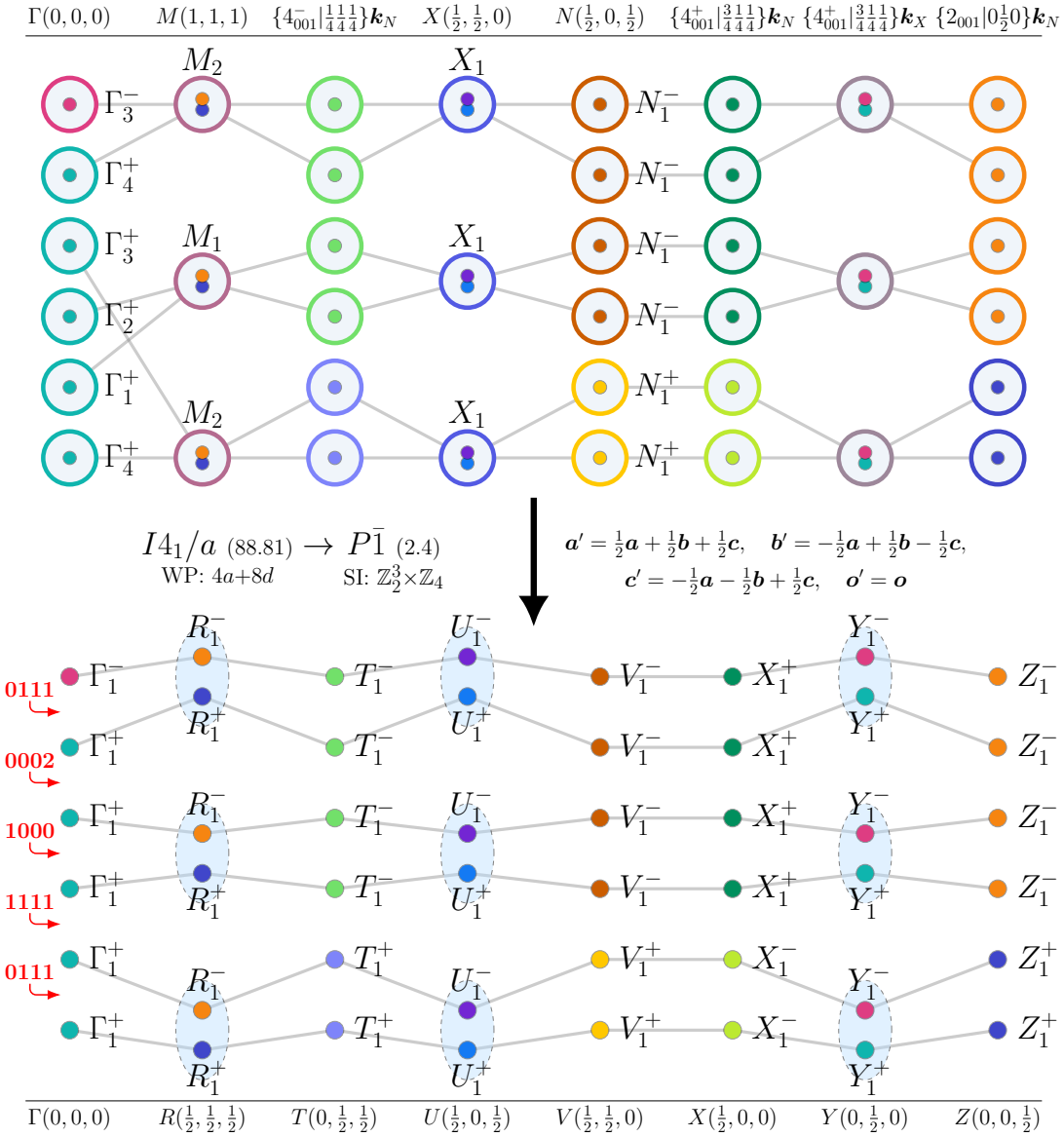
BCS Materials: MnV_2O_4 (53 K)[407].

92.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- strain in generic direction,
- B in generic direction.

[407] BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=0.64`



Supplementary Figure 470: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff positions $4a+8d$ of supergroup $I4_1/a$ (88.81).

Supplementary Table 929: Number of times a given SI can appear in the band structure. Supergroup: $I4_1/a$ (88.81), WP: $4a$+$8d$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1
0002	0, 1
0010	0, 1, 2, 3
0011	0, 1, 2, 3
0012	0, 1, 2, 3
0013	0, 1, 2, 3
0110	0, 1, 2, 3
0111	0, 1, 2, 3
0112	0, 1, 2, 3
0113	0, 1, 2, 3
1000	0, 1, 2, 3
1001	0, 1, 2, 3
1002	0, 1, 2, 3
1003	0, 1, 2, 3
1100	0, 1, 2, 3
1101	0, 1, 2, 3

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SI	Possible #Gaps
1102	0, 1, 2, 3
1103	0, 1, 2, 3
1111	0, 1, 2
1113	0, 1, 2
gapless	0
nontrivial	4, 5

Supplementary Table 930: Possible SI values in each gap. Supergroup: $I4_1/a$ (88.81), WP: $4a$+$8d$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
2	0000, 0002, 1111, 1113
3	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
4	0000, 0002, 1111, 1113
5	0010, 0011, 0012, 0013, 0110 0111, 0112, 0113, 1000, 1001 1002, 1003, 1100, 1101, 1102 1103
6	0000

93 MSG I_c4_1/a (88.86)

Nontrivial-SI Subgroups: C_ac (9.41), $P\bar{1}$ (2.4), $C2'/c'$ (15.89), $P_S\bar{1}$ (2.7), C_a2 (5.17), $C2/c$ (15.85), C_a2/c (15.91), I_c4_1 (80.32), $I4_1/a$ (88.81).

Trivial-SI Subgroups: Cc' (9.39), $C2'$ (5.15), P_S1 (1.3), Cc (9.37), $C2$ (5.13), $I4_1$ (80.29).

93.1 WP: $16c$

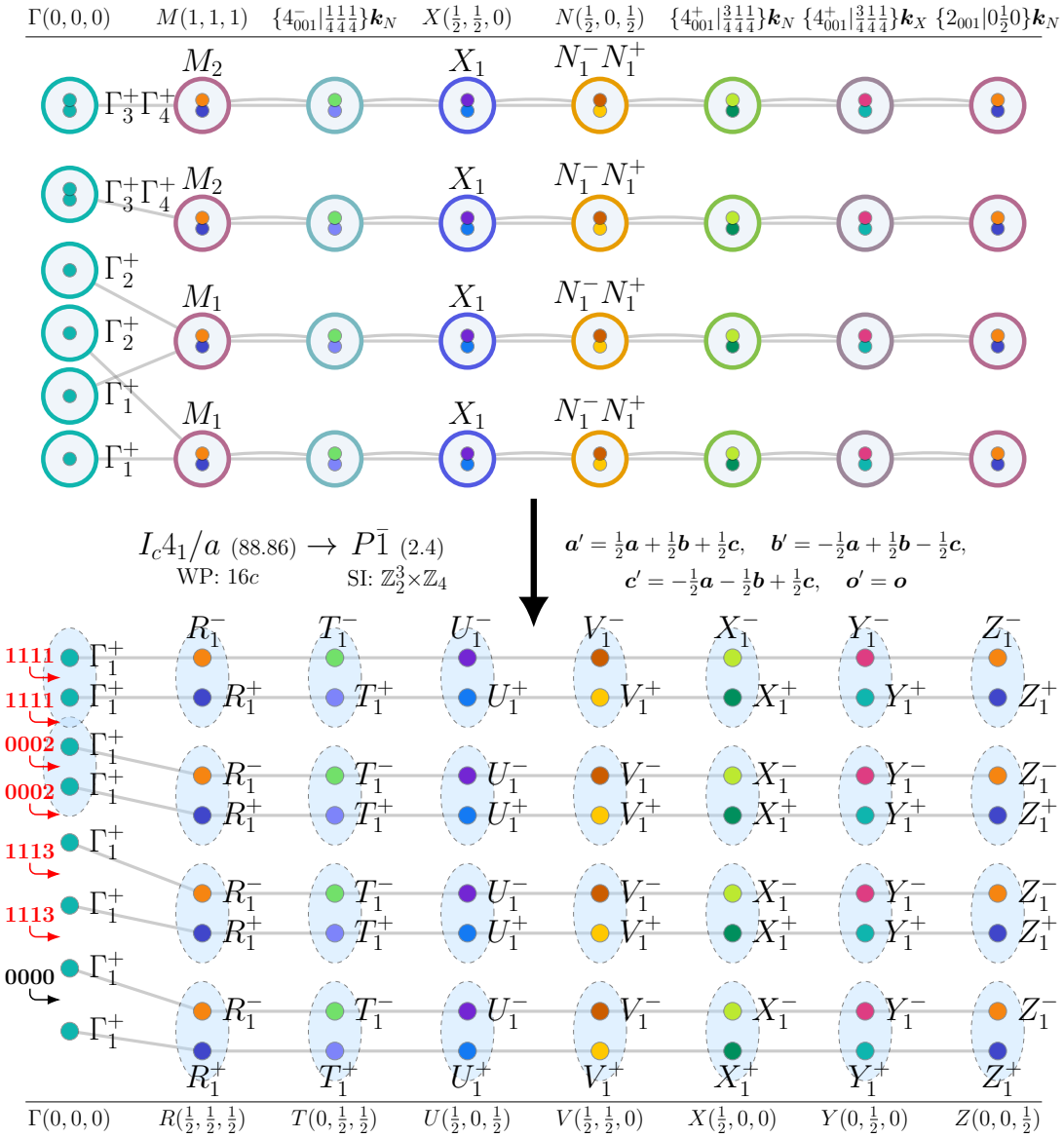
BCS Materials: $FeTa_2O_6$ (13 K)[408], $FeTa_2O_6$ (8.5 K)[409].

93.1.1 Topological bands in subgroup $P\bar{1}$ (2.4)

Perturbations:

- B ∥ [001] and strain in generic direction,
- (B ∥ [100] or B ∥ [110] or B ⊥ [001]) and strain in generic direction,
- B in generic direction.

[408]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.86`
[409]BCS web page: `http://webbdcrista1.ehu.es/magndata/index.php?this_label=2.22`



Supplementary Figure 471: Topological magnon bands in subgroup $P\bar{1}$ (2.4) for magnetic moments on Wyckoff position $16c$ of supergroup I_c4_1/a (88.86).

Supplementary Table 931: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/a (88.86), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

SI	Possible #Gaps
0000	0, 1, 2
0001	0, 1, 2, 3, 4
0002	1, 2, 3
0003	0, 1, 2, 3, 4
0010	0, 1, 2, 3, 4
0011	0, 1, 2, 3, 4
0012	0, 1, 2, 3, 4
0013	0, 1, 2, 3, 4
0100	0, 1, 2, 3, 4
0101	0, 1, 2, 3, 4
0102	0, 1, 2, 3, 4
0103	0, 1, 2, 3, 4
0110	0, 1, 2, 3, 4
0111	0, 1, 2, 3, 4
0112	0, 1, 2, 3, 4
0113	0, 1, 2, 3, 4
1000	0, 1, 2, 3, 4

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SI	Possible #Gaps
1001	0, 1, 2, 3, 4
1002	0, 1, 2, 3, 4
1003	0, 1, 2, 3, 4
1010	0, 1, 2, 3, 4
1011	0, 1, 2, 3, 4
1012	0, 1, 2, 3, 4
1013	0, 1, 2, 3, 4
1100	0, 1, 2, 3, 4
1101	0, 1, 2, 3, 4
1102	0, 1, 2, 3, 4
1103	0, 1, 2, 3, 4
1110	0, 1, 2, 3, 4
1111	1, 2, 3
1112	0, 1, 2, 3, 4
1113	1, 2, 3
gapless	0
nontrivial	5, 6, 7

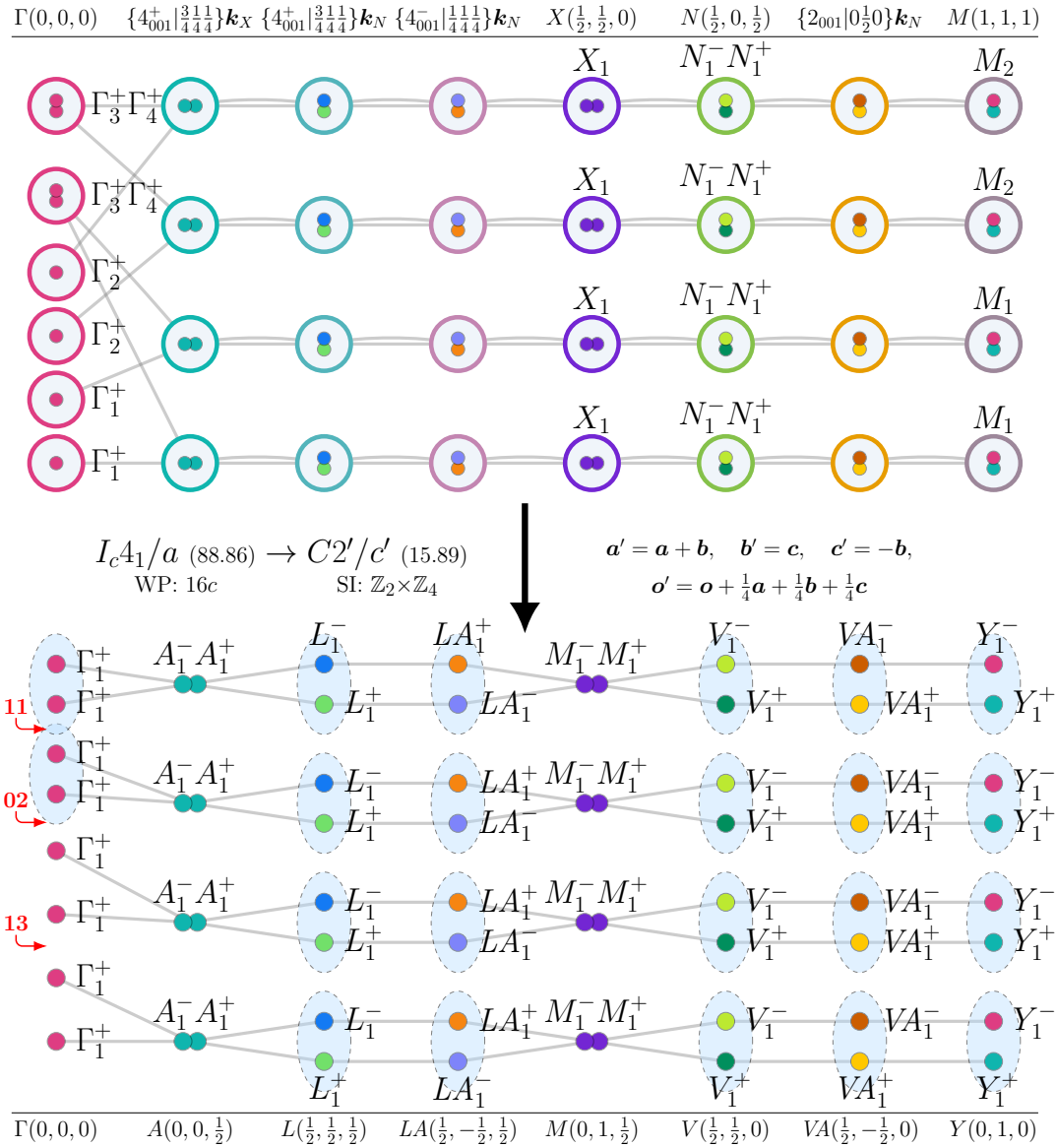
Supplementary Table 932: Possible SI values in each gap. Supergroup: I_c4_1/a (88.86), WP: $16c$, Subgroup: $P\bar{1}$ (2.4).

Gap#	Possible SI Values
1	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
2	1113
3	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1112, 1113
4	0002
5	0001, 0002, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
6	1111
7	0000, 0001, 0003, 0010, 0011 0012, 0013, 0100, 0101, 0102 0103, 0110, 0111, 0112, 0113 1000, 1001, 1002, 1003, 1010 1011, 1012, 1013, 1100, 1101 1102, 1103, 1110, 1111, 1112
8	0000

93.1.2 Topological bands in subgroup $C2'/c'$ (15.89)

Perturbation:

- (B ∥ [100] or B ∥ [110] or B ⊥ [001]).



Supplementary Figure 472: Topological magnon bands in subgroup $C2'/c'$ (15.89) for magnetic moments on Wyckoff position 16c of supergroup I_c4_1/a (88.86).

Supplementary Table 933: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/a (88.86), WP: 16c, Subgroup: $C2'/c'$ (15.89).

SI	Possible #Gaps
00	0
02	1
11	1
13	1
gapless	4
nontrivial	3

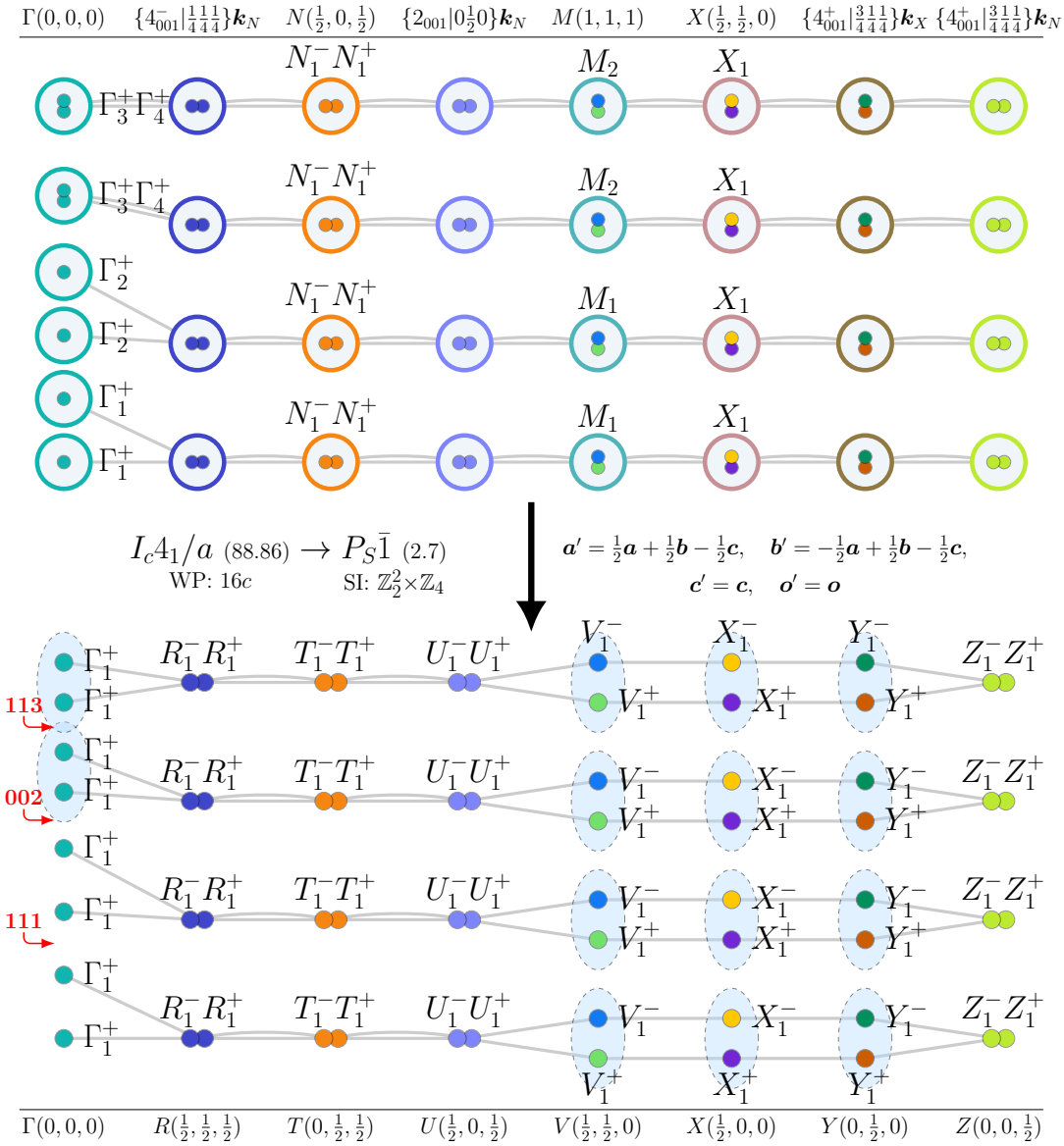
Supplementary Table 934: Possible SI values in each gap. Supergroup: I_c4_1/a (88.86), WP: $16c$, Subgroup: $C2'/c'$ (15.89).

Gap#	Possible SI Values
1	G
2	13
3	G
4	02
5	G
6	11
7	G
8	00

93.1.3 Topological bands in subgroup $P_S\bar{1}$ (2.7)

Perturbation:

- strain in generic direction.



Supplementary Figure 473: Topological magnon bands in subgroup $P_S\bar{1}$ (2.7) for magnetic moments on Wyckoff position 16c of supergroup I_c4_1/a (88.86).

Supplementary Table 935: Number of times a given SI can appear in the band structure. Supergroup: I_c4_1/a (88.86), WP: 16c, Subgroup: $P_S\bar{1}$ (2.7).

SI	Possible #Gaps
000	0
002	1
111	1
113	1
gapless	4
nontrivial	3

5509

Supplementary Table 936: Possible SI values in each gap. Supergroup: I_c4_1/a (88.86), WP: 16c, Subgroup: $P_S\bar{1}$ (2.7).

Gap#	Possible SI Values
1	G
2	111
3	G

continued on next page...

...continued

Gap#	Possible SI Values
4	002
5	G
6	113
7	G
8	000

94 SI formulas for subgroups

94.1 Subgroup $P2'/m'$ (10.46)

$$z_1 = n^{C_1^+} + n^{\Gamma_1^+} + n^{Y_1^+} + n^{Z_1^+} \bmod 2 \tag{23}$$
$$z_2 = n^{A_1^+} + n^{A_1^-} + n^{B_1^-} + n^{D_1^+} + n^{\Gamma_1^+} + n^{Z_1^+} \bmod 2 \tag{24}$$
$$z_3 = n^{A_1^-} + n^{B_1^-} + n^{\Gamma_1^+} + n^{Y_1^+} \bmod 2 \tag{25}$$
$$z_4 = 3n^{A_1^-} + n^{B_1^-} + n^{C_1^+} + n^{D_1^+} + 3n^{E_1^+} + n^{\Gamma_1^+} + 3n^{Y_1^+} + 3n^{Z_1^+} \bmod 4 \tag{26}$$

94.2 Subgroup P_I4nc (104.210)

$$z_1 = n^{A_5} + n^{\Gamma_1} + n^{\Gamma_2} \bmod 2 \tag{27}$$

94.3 Subgroup $P2_1'/m'$ (11.54)

$$z_1 = n^{\Gamma_1^+} + n^{Y_1^+} \bmod 2 \tag{28}$$
$$z_2 = n^{B_1^+} + n^{\Gamma_1^+} \bmod 2 \tag{29}$$
$$z_3 = n^{A_1^+} + 3n^{B_1^+} + n^{\Gamma_1^+} + 3n^{Y_1^+} \bmod 4 \tag{30}$$

94.4 Subgroup $C2/m$ (12.58)

$$z_1 = n^{A_2^+} + n^{\Gamma_2^+} + n^{L_1^+} + n^{M_1^+} + n^{V_1^+} + n^{Y_1^+} \bmod 2 \tag{31}$$
$$z_2 = n^{A_2^+} + n^{A_2^-} + n^{\Gamma_2^+} + n^{\Gamma_2^-} + n^{L_1^+} + n^{V_1^+} \bmod 2 \tag{32}$$

94.5 Subgroup $C2'/m'$ (12.62)

$$z_1 = n^{A_1^+} + n^{\Gamma_1^+} + n^{LA_1^+} + n^{VA_1^+} \bmod 2 \tag{33}$$
$$z_2 = n^{\Gamma_1^+} + n^{Y_1^+} \bmod 2 \tag{34}$$
$$z_3 = 3n^{A_1^+} + n^{\Gamma_1^+} + n^{M_1^+} + 3n^{Y_1^+} \bmod 4 \tag{35}$$

94.6 Subgroup C_c2/m (12.63)

$$z_1 = n^{\Gamma_1^-} + n^{V_1^+} + n^{Y_1^+} \bmod 2 \tag{36}$$
$$z_2 = n^{\Gamma_1^+} + n^{Y_1^+} \bmod 2 \tag{37}$$

94.7 Subgroup $I\bar{4}2'd'$ (122.337)

$$z_1 = n^{\Gamma_3} + n^{M_3M_4} + n^{PA_1} + n^{PA_3} + n^{X_1X_2} \bmod 2 \tag{38}$$
$$z_2 = n^{\Gamma_2} + n^{\Gamma_3} + n^{\Gamma_4} + n^{M_3M_4} + n^{PA_1} + n^{PA_3} + n^{X_1X_2} \bmod 2 \tag{39}$$

94.8 Subgroup $P2'/c'$ (13.69)

$$z_1 = n^{\Gamma_1^+} + n^{Y_1^+} \bmod 2 \tag{40}$$
$$z_2 = n^{C_1^+} + n^{\Gamma_1^+} \bmod 2 \tag{41}$$
$$z_3 = 3n^{C_1^+} + n^{\Gamma_1^+} + 3n^{Y_1^+} + n^{Z_1^+} \bmod 4 \tag{42}$$

94.9 Subgroup $P2_1'/c'$ (14.79)

$$z_1 = n^{\Gamma_1^+} \bmod 2 \tag{43}$$
$$z_2 = 2n^{D_1^+ D_1^+} + 2n^{E_1^+ E_1^+} + n^{\Gamma_1^+} + 3n^{Y_1^+} \bmod 4 \tag{44}$$

94.10 Subgroup $C2'/c'$ (15.89)

$$z_1 = n^{A_1^- A_1^+} + n^{\Gamma_1^+} + n^{LA_1^+} + n^{VA_1^+} \bmod 2 \tag{45}$$
$$z_2 = 3n^{\Gamma_1^+} + 2n^{VA_1^+} + 3n^{Y_1^+} \bmod 4 \tag{46}$$

94.11 Subgroup $P22'2_1'$ (17.10)

$$z_1 = n^{\Gamma_1} + n^{S_1} \bmod 2 \tag{47}$$

94.12 Subgroup $P\bar{1}$ (2.4)

$$z_1 = n^{\Gamma_1^-} + n^{T_1^-} + n^{Y_1^-} + n^{Z_1^-} \bmod 2 \tag{48}$$
$$z_2 = n^{\Gamma_1^-} + n^{U_1^-} + n^{X_1^-} + n^{Z_1^-} \bmod 2 \tag{49}$$
$$z_3 = n^{\Gamma_1^-} + n^{V_1^-} + n^{X_1^-} + n^{Y_1^-} \bmod 2 \tag{50}$$
$$z_4 = n^{\Gamma_1^-} + n^{R_1^-} + n^{T_1^-} + n^{U_1^-} + n^{V_1^-} + n^{X_1^-} + n^{Y_1^-} + n^{Z_1^-} \bmod 4 \tag{51}$$

94.13 Subgroup $P_S\bar{1}$ (2.7)

$$z_1 = n^{\Gamma_1^+} + n^{Y_1^+} \bmod 2 \tag{52}$$
$$z_2 = n^{\Gamma_1^+} + n^{X_1^+} \bmod 2 \tag{53}$$
$$z_3 = n^{\Gamma_1^+} + n^{V_1^+} + 3n^{X_1^+} + 3n^{Y_1^+} \bmod 4 \tag{54}$$

94.14 Subgroup $Pm'm'2$ (25.60)

$$z_1 = n^{\Gamma_1} + n^{R_1} + n^{T_1} + n^{U_1} \bmod 2 \tag{55}$$

94.15 Subgroup $Pm'a'2$ (28.91)

$$z_1 = n^{\Gamma_1} + n^{T_1} \bmod 2 \tag{56}$$

94.16 Subgroup $P2$ (3.1)

$$z_1 = n^{A_2} + n^{B_2} + n^{C_1} + n^{\Gamma_1} \bmod 2 \tag{57}$$

94.17 Subgroup P_a2 (3.4)

$$z_1 = n^{B_1} + n^{\Gamma_1} \bmod 2 \tag{58}$$

94.18 Subgroup $Cc'c'2$ (37.183)

$$z_1 = n^{RA_1} + n^{T_1T_1} + n^{Z_1Z_1} \bmod 2 \tag{59}$$

94.19 Subgroup C_Acc2 (37.186)

$$z_1 = n^{\Gamma_3} + n^{T_1T_2} \bmod 2 \tag{60}$$

94.20 Subgroup $Ab'm'2$ (39.199)

$$z_1 = n^{\Gamma_1} + n^{T_1} \bmod 2 \tag{61}$$

94.21 Subgroup $Ab'a'2$ (41.215)

$$z_1 = n^{\Gamma_2} \bmod 2 \tag{62}$$

94.22 Subgroup C_a2 (5.17)

$$z_1 = n^{A_1} + n^{\Gamma_1} \bmod 2 \tag{63}$$

94.23 Subgroup $Cmm'a'$ (67.506)

$$z_1 = n^{\Gamma_1^+} + n^{T_1^+} + n^{Y_1^-} + n^{Z_1^-} \bmod 2 \tag{64}$$
$$z_2 = n^{\Gamma_1^+} + n^{\Gamma_2^+} + n^{T_1^+} + n^{T_2^+} \bmod 2 \tag{65}$$

References

[1] Karaki, M. J. *et al.* An efficient material search for room-temperature topological magnons. *Science Advances* **9**, eade7731 (2023). URL https://www.science.org/doi/abs/10.1126/sciadv.ade7731. https://www.science.org/doi/pdf/10.1126/sciadv.ade7731.

[2] Muñoz, A., Alonso, J. A., Martínez-Lope, M. J., García-Muñoz, J. L. & Fernández-Díaz, M. T. Magnetic structure evolution of ndmno3 derived from neutron diffraction data. *Journal of Physics: Condensed Matter* **12**, 1361 (2000). URL https://dx.doi.org/10.1088/0953-8984/12/7/319.

[3] Porter, D. G. *et al.* Magnetic anisotropy and orbital ordering in $\mathrm{ca_2ruo_4}$. *Phys. Rev. B* **98**, 125142 (2018). URL https://link.aps.org/doi/10.1103/PhysRevB.98.125142.

[4] Kunkemöller, S. *et al.* Highly anisotropic magnon dispersion in $\mathrm{ca_2ruo_4}$: Evidence for strong spin orbit coupling. *Phys. Rev. Lett.* **115**, 247201 (2015). URL https://link.aps.org/doi/10.1103/PhysRevLett.115.247201.

[5] Braden, M., André, G., Nakatsuji, S. & Maeno, Y. Crystal and magnetic structure of $\mathrm{ca_2ruo_4}$: magnetoelastic coupling and the metal-insulator transition. *Phys. Rev. B* **58**, 847–861 (1998). URL https://link.aps.org/doi/10.1103/PhysRevB.58.847.