Supplemental file for the manuscript:

Comparative Analysis of Six Correlation Metrics on Identifying DNA Co-Methylation Patterns

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Supplemental 8x8 tables

Table S1 includes a few 8x8 matrices comparing correlation scores computed for pairs using different sample sizes (10, 30, and 53). Pink cells have counts comprising 20%-40% of the row total. Blue cells have counts comprising 40%-70% of the row total. Green cells have counts comprising 70%-100% of the row totals.

The 8x8 matrices in Table S1 show a much stronger agreement (indicated by stronger diagonals) between the 30N and 53N sample correlations as opposed to the comparisons with the 10 sample correlations. This stronger diagonal is evident when comparing sample sizes for every metric and data type. Specifically, for all 30N x 53N 8x8 tables, at least 80% of the correlations generated with 30 samples fell within the corresponding diagonal cell of the 53N 8x8 table or one cell to the right or left. This indicated that most CG pairs consistently received similar correlation scores using 30 samples as compared to 53 samples. This strong agreement was not present when comparing 10N and 30N correlation scores, or when comparing 10N and 53N correlation scores. In general, there appeared to be a strong tendency for higher sample sizes (30N and 53N) to assign lower scores to many of the pairs that have high correlations scores when only 10 samples are provided. As a result, the 8x8 tables for 30N and 53N samples show a higher concentration of scores in the most crowded intervals for each metric ([-0.25, 0] and [0, 0.25] for Pearson, Spearman, and Kendall correlations; [0, 0.25] for Hoeffding; [0.25, 0.5] for Distance and MIC correlations).

Table S1: 8x8 matrices comparing correlation scores with different sample sizes

A. B.Pearson

B.Pearson.10kx10k.30N [-1,-0.75)	B.Pearson.10kx10k.30N [-0.75,-0.5)	B.Pearson.10kx10k.30N [-0.5,-0.25)	B.Pearson.10kx10k.30N [-0.25,0)	B.Pearson.10kx10k.30N [0,0.25)	B.Pearson.10kx10k.30N [0.25,0.5)	B.Pearson.10kx10k.30N [0.5,0.75)	B.Pearson.10kx10k.30N [0.75,1]	Total Count
B.pearson.10kx10k.10N [-1,-0.75)	6.4% 47761	21.075% 157261	24.157% 180255	21.569% 160947	15.746% 117496	7.891% 58885	2.525% 18844	0.632% 4718	746167
B.pearson.10kx10k.10N [-0.75,-0.5)	0.946% 36985	8.814% 344396	19.913% 778103	25.635% 1001662	23.407% 914601	14.287% 558259	5.441% 212602	1.554% 60741	3907349
B.pearson.10kx10k.10N [-0.5,-0.25)	0.444% 36585	4.619% 380619	15.01% 1236852	24.679% 2033482	26.21% 2159653	18.511% 1525292	8.034% 662055	2.489% 205148	8239686
B.pearson.10kx10k.10N [-0.25,0)	0.359% 40831	3.107% 353173	11.342% 1289317	22.184% 2521711	27.349% 3108847	21.914% 2490997	10.397% 1181892	3.344% 380212	11366980
B.pearson.10kx10k.10N [0,0.25)	0.315% 36543	2.317% 268808	8.702% 1009525	18.964% 2200044	27.36% 3173966	25.23% 2926931	13.011% 1509456	4.097% 475331	11600604
B.pearson.10kx10k.10N [0.25,0.5)	0.255% 22373	1.673% 146713	6.427% 563349	15.367% 1346943	25.63% 2246516	28.448% 2493488	17.195% 1507169	5.003% 438518	8765069
B.pearson.10kx10k.10N [0.5,0.75)	0.172% 7638	1.059% 46853	4.124% 182480	10.928% 483504	21.184% 937214	29.761% 1316691	25.365% 1122170	7.403% 327529	4424079
B.pearson.10kx10k.10N [0.75,1]	0.064% 611	0.441% 4169	1.784% 16863	5.374% 50788	12.512% 118252	23.334% 220523	35.579% 336248	20.909% 197612	945066

B.Pearson.10kx10k.53N [-1,-0.75)	B.Pearson.10kx10k.53N [-0.75,-0.5)	B.Pearson.10kx10k.53N [-0.5,-0.25)	B.Pearson.10kx10k.53N [-0.25,0)	B.Pearson.10kx10k.53N [0,0.25)	B.Pearson.10kx10k.53N [0.25,0.5)	B.Pearson.10kx10k.53N [0.5,0.75)	B.Pearson.10kx10k.53N [0.75,1]	Total Count
B.pearson.10kx10k.10N [-1,-0.75)	6.08% 45367	22.39% 167074	24.202% 180589	23.544% 175684	16.236% 121155	5.981% 44633	1.308% 9767	0.254% 1898	746167
B.pearson.10kx10k.10N [-0.75,-0.5)	0.511% 19985	7.169% 280119	19.644% 767590	30.614% 1196220	26.675% 1042320	11.71% 457556	2.998% 117148	0.675% 26411	3907349
B.pearson.10kx10k.10N [-0.5,-0.25)	0.152% 12564	2.874% 236884	13.821% 1138849	29.751% 2451421	31.505% 2595928	16.126% 1328789	4.657% 383748	1.11% 91503	8239686
B.pearson.10kx10k.10N [-0.25,0)	0.119% 13626	1.745% 198376	10.248% 1164898	26.532% 3015911	33.671% 3827427	19.937% 2266324	6.214% 706444	1.53% 173974	11366980
B.pearson.10kx10k.10N [0,0.25)	0.104% 12078	1.294% 150124	7.925% 919409	22.877% 2653982	34.059% 3951140	23.806% 2761660	8.037% 932426	1.894% 219785	11600604
B.pearson.10kx10k.10N [0.25,0.5)	0.083% 7309	0.937% 82197	5.92% 518957	18.624% 1632432	32.385% 2838634	28.236% 2474960	11.478% 1006138	2.332% 204442	8765069
B.pearson.10kx10k.10N [0.5,0.75)	0.054% 2419	0.582% 25757	3.747% 165801	13.199% 583959	26.834% 1187161	31.6% 1398011	20.072% 888002	3.909% 172969	4424079
B.pearson.10kx10k.10N [0.75,1]	0.023% 219	0.219% 2070	1.578% 14920	6.356% 60077	15.519% 146669	25.598% 241918	33.845% 319859	16.859% 159334	945066

B.Pearson.10kx10k.53N [-1,-0.75)	B.Pearson.10kx10k.53N [-0.75,-0.5)	B.Pearson.10kx10k.53N [-0.5,-0.25)	B.Pearson.10kx10k.53N [-0.25,0)	B.Pearson.10kx10k.53N [0,0.25)	B.Pearson.10kx10k.53N [0.25,0.5)	B.Pearson.10kx10k.53N [0.5,0.75)	B.Pearson.10kx10k.53N [0.75,1]	Total Count
B.Pearson.10kx10k.30N [-1,-0.75)	39.459% 90492	51.964% 119169	7.757% 17791	0.78% 1790	0.035% 81	0.001% 4	0% 0	0% 0	229327
B.Pearson.10kx10k.30N [-0.75,-0.5)	1.174% 19991	45.186% 769078	46.491% 791279	6.575% 111920	0.516% 8792	0.05% 863	0.003% 68	0% 1	1701992
B.Pearson.10kx10k.30N [-0.5,-0.25)	0.033% 1746	4.175% 219488	55.23% 2903346	37.047% 1947515	3.263% 171542	0.223% 11724	0.024% 1296	0.001% 87	5256744
B.Pearson.10kx10k.30N [-0.25,0)	0.009% 920	0.257% 25197	10.634% 1042123	65.116% 6380824	22.986% 2252473	0.908% 89008	0.082% 8083	0.004% 453	9799081
B.Pearson.10kx10k.30N [0,0.25)	0.002% 358	0.063% 8097	0.787% 100562	23.752% 3034696	65.214% 8332154	9.93% 1268824	0.239% 30613	0.009% 1241	12776545
B.Pearson.10kx10k.30N [0.25,0.5)	0% 57	0.012% 1479	0.129% 15002	2.385% 276521	39.865% 4620876	54.235% 6286522	3.342% 387455	0.027% 3154	11591066
B.Pearson.10kx10k.30N [0.5,0.75)	0% 3	0.001% 92	0.013% 906	0.243% 15963	4.803% 314629	49.295% 3229090	44.795% 2934329	0.846% 55424	6550436
B.Pearson.10kx10k.30N [0.75,1]	0% 0	0% 1	0% 4	0.021% 457	0.473% 9887	4.202% 87816	47.932% 1001688	47.37% 989956	2089809

B. Spearman

Spearman.10kx10k.30N [-1,-0.75)	Spearman.10kx10k.30N [-0.75,-0.5)	Spearman.10kx10k.30N [-0.5,-0.25)	Spearman.10kx10k.30N [-0.25,0)	Spearman.10kx10k.30N [0,0.25)	Spearman.10kx10k.30N [0.25,0.5)	Spearman.10kx10k.30N [0.5,0.75)	Spearman.10kx10k.30N [0.75,1]	Total Count
Spearman.10kx10k.10N [-1,-0.75)	7.452% 56832	22.348% 170424	26.049% 198647	24.052% 183418	14.734% 112359	4.839% 36902	0.519% 3958	0.003% 28	762568
Spearman.10kx10k.10N [-0.75,-0.5)	0.965% 36392	9.806% 369436	22.114% 833128	29.718% 1119605	25.345% 954865	10.661% 401659	1.374% 51801	0.012% 474	3767360
Spearman.10kx10k.10N [-0.5,-0.25)	0.192% 14973	4.668% 363417	16.823% 1309536	28.458% 2215242	30.6% 2381932	16.545% 1287886	2.676% 208364	0.034% 2690	7784040
Spearman.10kx10k.10N [-0.25,0)	0.064% 7712	2.44% 291257	12.308% 1468930	25.081% 2993235	32.481% 3876390	22.723% 2711901	4.81% 574036	0.089% 10710	11934171
Spearman.10kx10k.10N [0,0.25)	0.025% 3077	1.306% 159475	8.563% 1045101	20.821% 2541016	31.774% 3877691	28.764% 3510313	8.507% 1038293	0.235% 28771	12203737
Spearman.10kx10k.10N [0.25,0.5)	0.009% 812	0.699% 58488	5.584% 466619	16.12% 1346968	28.82% 2408133	33.422% 2792632	14.617% 1221336	0.725% 60591	8355579
Spearman.10kx10k.10N [0.5,0.75)	0.003% 150	0.35% 14965	3.246% 138801	10.976% 469282	23.267% 994761	34.456% 1473107	24.497% 1047351	3.201% 136866	4275283
Spearman.10kx10k.10N [0.75,1]	0% 9	0.13% 1186	1.351% 12333	5.317% 48505	13.805% 125941	27.044% 246719	35.113% 320328	17.236% 157241	912262

Spearman.10kx10k.53N [-1,-0.75)	Spearman.10kx10k.53N [-0.75,-0.5)	Spearman.10kx10k.53N [-0.5,-0.25)	Spearman.10kx10k.53N [-0.25,0)	Spearman.10kx10k.53N [0,0.25)	Spearman.10kx10k.53N [0.25,0.5)	Spearman.10kx10k.53N [0.5,0.75)	Spearman.10kx10k.53N [0.75,1]	Total Count
Spearman.10kx10k.10N [-1,-0.75)	4.464% 34044	23.409% 178510	26.411% 201404	25.107% 191461	15.294% 116634	4.75% 36222	0.552% 4214	0.01% 79	762568
Spearman.10kx10k.10N [-0.75,-0.5)	0.389% 14673	7.761% 292421	21.63% 814901	33.083% 1246390	26.707% 1006165	9.287% 349883	1.117% 42089	0.022% 838	3767360
Spearman.10kx10k.10N [-0.5,-0.25)	0.071% 5577	3.162% 246164	15.745% 1225634	32.557% 2534263	33.175% 2582390	13.533% 1053484	1.713% 133375	0.04% 3153	7784040
Spearman.10kx10k.10N [-0.25,0)	0.03% 3677	1.693% 202084	11.624% 1387278	29.215% 3486670	36.522% 4358615	18.249% 2177923	2.6% 310290	0.063% 7634	11934171
Spearman.10kx10k.10N [0,0.25)	0.018% 2239	1.051% 128308	8.501% 1037561	24.801% 3026688	37.417% 4566378	23.802% 2904789	4.287% 523248	0.119% 14526	12203737
Spearman.10kx10k.10N [0.25,0.5)	0.011% 978	0.703% 58795	6.044% 505071	19.788% 1653421	35.693% 2982387	29.62% 2474967	7.801% 651823	0.336% 28137	8355579
Spearman.10kx10k.10N [0.5,0.75)	0.007% 333	0.44% 18837	3.922% 167701	14.11% 603245	30.029% 1283827	33.827% 1446224	15.914% 680381	1.748% 74735	4275283
Spearman.10kx10k.10N [0.75,1]	0.003% 30	0.21% 1923	1.869% 17055	7.243% 66083	18.416% 168009	29.424% 268430	30.329% 276680	12.502% 114052	912262

Spearman.10kx10k.53N [-1,-0.75)	Spearman.10kx10k.53N [-0.75,-0.5)	Spearman.10kx10k.53N [-0.5,-0.25)	Spearman.10kx10k.53N [-0.25,0)	Spearman.10kx10k.53N [0,0.25)	Spearman.10kx10k.53N [0.25,0.5)	Spearman.10kx10k.53N [0.5,0.75)	Spearman.10kx10k.53N [0.75,1]	Total Count
Spearman.10kx10k.30N [-1,-0.75)	36.237% 43470	61.099% 73293	2.632% 3158	0.029% 35	0% 1	0% 0	0% 0	0% 0	119957
Spearman.10kx10k.30N [-0.75,-0.5)	1.243% 17761	48.85% 697897	46.187% 659858	3.695% 52800	0.023% 330	0% 2	0% 0	0% 0	1428648
Spearman.10kx10k.30N [-0.5,-0.25)	0.005% 320	6.287% 344110	56.017% 3065906	36.039% 1972449	1.647% 90155	0.002% 155	0% 0	0% 0	5473095
Spearman.10kx10k.30N [-0.25,0)	0% 0	0.107% 11706	14.164% 1546378	62.063% 6775634	23.127% 2524938	0.536% 58598	0% 17	0% 0	10917271
Spearman.10kx10k.30N [0,0.25)	0% 0	0% 36	0.548% 80764	25.579% 3768421	61.665% 9084584	12.109% 1783974	0.097% 14293	0% 0	14732072
Spearman.10kx10k.30N [0.25,0.5)	0% 0	0% 0	0.004% 541	1.907% 237111	41.243% 5139436	52.102% 6492547	4.738% 590487	0.003% 397	12461119
Spearman.10kx10k.30N [0.5,0.75)	0% 0	0% 0	0% 0	0.026% 1171	5.033% 224776	52.823% 2358823	40.612% 1813535	1.504% 67162	4465467
Spearman.10kx10k.30N [0.75,1]	0% 0	0% 0	0% 0	0% 0	0.046% 185	4.485% 17823	51.279% 203768	44.189% 175595	397371

C. Kendall

Kendall.10kx10k.30N [-1,-0.75)	Kendall.10kx10k.30N [-0.75,-0.5)	Kendall.10kx10k.30N [-0.5,-0.25)	Kendall.10kx10k.30N [-0.25,0)	Kendall.10kx10k.30N [0,0.25)	Kendall.10kx10k.30N [0.25,0.5)	Kendall.10kx10k.30N [0.5,0.75)	Kendall.10kx10k.30N [0.75,1]	Total Count
Kendall.10kx10k.10N [-1,-0.75)	0.902% 866	28.573% 27412	38.216% 36663	23.916% 22944	7.641% 7331	0.74% 710	0.008% 8	0% 0	95934
Kendall.10kx10k.10N [-0.75,-0.5)	0.028% 493	6.036% 104848	27.785% 482594	39.887% 692784	23.05% 400351	3.18% 55249	0.03% 528	0% 0	1736847
Kendall.10kx10k.10N [-0.5,-0.25)	0% 63	1.221% 77427	15.574% 987357	40.216% 2549550	35.682% 2262124	7.196% 456202	0.108% 6879	0% 0	6339602
Kendall.10kx10k.10N [-0.25,0)	0% 11	0.315% 50702	8.438% 1356967	33.437% 5376969	43.514% 6997341	13.931% 2240249	0.362% 58327	0% 3	16080569
Kendall.10kx10k.10N [0,0.25)	0% 1	0.098% 16277	4.268% 708313	24.576% 4077987	45.546% 7557456	24.269% 4026919	1.24% 205760	0% 64	16592777
Kendall.10kx10k.10N [0.25,0.5)	0% 0	0.031% 2223	2.005% 140131	15.986% 1117211	41.093% 2871840	36.477% 2549229	4.4% 307510	0.005% 396	6988540
Kendall.10kx10k.10N [0.5,0.75)	0% 0	0.009% 193	0.819% 16741	8.457% 172777	29.55% 603681	44.748% 914163	16.175% 330459	0.239% 4891	2042905
Kendall.10kx10k.10N [0.75,1]	0% 0	0.003% 4	0.179% 211	2.413% 2844	11.894% 14015	33.636% 39633	45.31% 53387	6.562% 7732	117826

Kendall.10kx10k.53N [-1,-0.75)	Kendall.10kx10k.53N [-0.75,-0.5)	Kendall.10kx10k.53N [-0.5,-0.25)	Kendall.10kx10k.53N [-0.25,0)	Kendall.10kx10k.53N [0,0.25)	Kendall.10kx10k.53N [0.25,0.5)	Kendall.10kx10k.53N [0.5,0.75)	Kendall.10kx10k.53N [0.75,1]	Total Count
Kendall.10kx10k.10N [-1,-0.75)	0.092% 89	24.007% 23031	41.401% 39718	24.354% 23364	9.16% 8788	0.974% 935	0.009% 9	0% 0	95934
Kendall.10kx10k.10N [-0.75,-0.5)	0.002% 49	3.996% 69410	25.939% 450530	42.235% 733573	24.796% 430685	2.986% 51866	0.042% 734	0% 0	1736847
Kendall.10kx10k.10N [-0.5,-0.25)	0% 1	0.556% 35264	12.568% 796810	43.844% 2779566	37.581% 2382544	5.359% 339753	0.089% 5660	0% 4	6339602
Kendall.10kx10k.10N [-0.25,0)	0% 4	0.134% 21664	6.629% 1065990	37.509% 6031740	46.575% 7489569	8.975% 1443325	0.175% 28266	0% 11	16080569
Kendall.10kx10k.10N [0,0.25)	0% 1	0.059% 9871	3.786% 628274	28.921% 4798827	51.368% 8523485	15.387% 2553228	0.476% 79033	0% 58	16592777
Kendall.10kx10k.10N [0.25,0.5)	0% 0	0.033% 2369	2.203% 153960	20.237% 1414332	49.749% 3476739	25.72% 1797469	2.053% 143531	0.002% 140	6988540
Kendall.10kx10k.10N [0.5,0.75)	0% 0	0.015% 325	1.134% 23180	11.779% 240646	38.365% 783774	37.828% 772808	10.814% 220926	0.06% 1246	2042905
Kendall.10kx10k.10N [0.75,1]	0% 0	0.003% 4	0.357% 421	3.907% 4604	16.93% 19948	34.974% 41209	41.402% 48783	2.424% 2857	117826

Kendall.10kx10k.53N [-1,-0.75)	Kendall.10kx10k.53N [-0.75,-0.5)	Kendall.10kx10k.53N [-0.5,-0.25)	Kendall.10kx10k.53N [-0.25,0)	Kendall.10kx10k.53N [0,0.25)	Kendall.10kx10k.53N [0.25,0.5)	Kendall.10kx10k.53N [0.5,0.75)	Kendall.10kx10k.53N [0.75,1]	Total Count
Kendall.10kx10k.30N [-1,-0.75)	8.019% 115	91.143% 1307	0.836% 12	0% 0	0% 0	0% 0	0% 0	0% 0	1434
Kendall.10kx10k.30N [-0.75,-0.5)	0.01% 29	41.261% 115156	57.598% 160750	1.128% 3149	0% 2	0% 0	0% 0	0% 0	279086
Kendall.10kx10k.30N [-0.5,-0.25)	0% 0	1.213% 45237	56.767% 2116852	41.711% 1555410	0.307% 11478	0% 0	0% 0	0% 0	3728977
Kendall.10kx10k.30N [-0.25,0)	0% 0	0.001% 238	6.252% 876139	74.167% 10393199	19.546% 2738999	0.032% 4491	0% 0	0% 0	14013066
Kendall.10kx10k.30N [0,0.25)	0% 0	0% 0	0.024% 5129	19.489% 4037006	74.651% 15463513	5.833% 1208303	0% 188	0% 0	20714139
Kendall.10kx10k.30N [0.25,0.5)	0% 0	0% 0	0% 1	0.368% 37884	47.477% 4881820	50.937% 5237560	1.216% 125089	0% 0	10282354
Kendall.10kx10k.30N [0.5,0.75)	0% 0	0% 0	0% 0	0% 4	2.048% 19720	57.142% 550198	40.726% 392134	0.083% 802	962858
Kendall.10kx10k.30N [0.75,1]	0% 0	0% 0	0% 0	0% 0	0% 0	0.313% 41	72.833% 9531	26.853% 3514	13086

D. Hoeffding

Hoeffding.10kx10k.30N [-1,-0.75)	Hoeffding.10kx10k.30N [-0.75,-0.5)	Hoeffding.10kx10k.30N [-0.5,-0.25)	Hoeffding.10kx10k.30N [-0.25,0)	Hoeffding.10kx10k.30N [0,0.25)	Hoeffding.10kx10k.30N [0.25,0.5)	Hoeffding.10kx10k.30N [0.5,0.75)	Hoeffding.10kx10k.30N [0.75,1]	Total Count
Hoeffding.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.10N [-0.25,0)	0% 0	0% 0	0% 0	86.536% 10275079	63.234% 17783205	0.228% 64296	0% 83	0% 0	28122663
Hoeffding.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	28.943% 5815838	70.098% 14085163	0.953% 191629	0.004% 899	0% 0	20093529
Hoeffding.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	13.6% 209335	78.023% 1200895	8.156% 125546	0.218% 3362	0% 0	1539138
Hoeffding.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	5.541% 12140	67.791% 148524	24.023% 52633	2.635% 5774	0.007% 17	219088
Hoeffding.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	1.972% 406	50.204% 10333	37.401% 7698	10.222% 2104	0.199% 41	20582

Hoeffding.10kx10k.53N [-1,-0.75)	Hoeffding.10kx10k.53N [-0.75,-0.5)	Hoeffding.10kx10k.53N [-0.5,-0.25)	Hoeffding.10kx10k.53N [-0.25,0)	Hoeffding.10kx10k.53N [0,0.25)	Hoeffding.10kx10k.53N [0.25,0.5)	Hoeffding.10kx10k.53N [0.5,0.75)	Hoeffding.10kx10k.53N [0.75,1]	Total Count
Hoeffding.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.10N [-0.25,0)	0% 0	0% 0	0% 0	31.832% 8952122	68.047% 19136735	0.119% 33687	0% 119	0% 0	28122663
Hoeffding.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	27.222% 5469943	72.285% 14524691	0.49% 98644	0.001% 251	0% 0	20093529
Hoeffding.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	15.054% 231716	79.786% 1228031	5.114% 78715	0.043% 676	0% 0	1539138
Hoeffding.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	7.387% 16186	73.506% 161043	18.402% 40317	0.703% 1541	0% 1	219088
Hoeffding.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	3.128% 644	59.309% 12207	34.117% 7022	3.444% 709	0% 0	20582

Hoeffding.10kx10k.30N [-1,-0.75)	Hoeffding.10kx10k.30N [-0.75,-0.5)	Hoeffding.10kx10k.30N [-0.5,-0.25)	Hoeffding.10kx10k.30N [-0.25,0)	Hoeffding.10kx10k.30N [0,0.25)	Hoeffding.10kx10k.30N [0.25,0.5)	Hoeffding.10kx10k.30N [0.5,0.75)	Hoeffding.10kx10k.30N [0.75,1]	Total Count
Hoeffding.10kx10k.30N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.30N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.30N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
Hoeffding.10kx10k.30N [-0.25,0)	0% 0	0% 0	0% 0	58.071% 9473158	41.928% 6839640	0% 0	0% 0	0% 0	16312798
Hoeffding.10kx10k.30N [0,0.25)	0% 0	0% 0	0% 0	15.641% 5197453	84.143% 27959313	0.214% 71328	0% 26	0% 0	33228120
Hoeffding.10kx10k.30N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	59.671% 263630	40.172% 177485	0.155% 687	0% 0	441802
Hoeffding.10kx10k.30N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	1.014% 124	78.309% 9571	20.675% 2527	0% 0	12222
Hoeffding.10kx10k.30N [0.75,1]	0% 0	0% 0	0% 0	0% 0	0% 0	1.724% 1	96.551% 56	1.724% 1	58

E. B.Distance

B.Distance.10kx10k.30N [-1,-0.75)	10k.30N [-0.75,-0.5)	10k.30N [-0.5,-0.25)	kx10k.30N [-0.25,0)	0kx10k.30N [0,0.25)	x10k.30N [0.25,0.5)	x10k.30N [0.5,0.75)	0kx10k.30N [0.75,1]	Total Count
B.distance.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	0% 0	32.674% 2545	52.869% 4118	11.747% 915	2.708% 211	7789
B.distance.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	8.507% 2152737	67.162% 16994078	20.384% 5157898	3.945% 998227	25302940
B.distance.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	5.454% 1205477	65.184% 14405303	24.863% 5494511	4.497% 993841	22099132
B.distance.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	0% 0	2.069% 53492	46.884% 1212032	37.583% 971580	13.462% 348035	2585139

B.Distance.10kx10k.53N [-1,-0.75)	10k.53N [-0.75,-0.5)	10k.53N [-0.5,-0.25)	kx10k.53N [-0.25,0)	0kx10k.53N [0,0.25)	x10k.53N [0.25,0.5)	x10k.53N [0.5,0.75)	0kx10k.53N [0.75,1]	Total Count
B.distance.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.distance.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	0% 0	49.248% 3836	43.112% 3358	6.663% 519	0.975% 76	7789
B.distance.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	28.122% 7115933	59.862% 15146890	10.784% 2728784	1.23% 311333	25302940
B.distance.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	23.016% 5086502	60.796% 13435420	14.634% 3234186	1.552% 343024	22099132
B.distance.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	0% 0	11.949% 308917	47.378% 1224795	30.939% 799840	9.732% 251587	2585139

B.Distance.10kx10k.53N [-1,-0.75)	10k.53N [-0.75,-0.5)	10k.53N [-0.5,-0.25)	kx10k.53N [-0.25,0)	0kx10k.53N [0,0.25)	x10k.53N [0.25,0.5)	x10k.53N [0.5,0.75)	0kx10k.53N [0.75,1]	Total Count
B.Distance.10kx10k.30N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.Distance.10kx10k.30N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.Distance.10kx10k.30N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.Distance.10kx10k.30N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
B.Distance.10kx10k.30N [0,0.25)	0% 0	0% 0	0% 0	0% 0	80.602% 2751958	19.287% 658536	0.107% 3676	0.002% 81	3414251
B.Distance.10kx10k.30N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	29.819% 9725782	68.297% 22275545	1.88% 613249	0.002% 955	32615531
B.Distance.10kx10k.30N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	0.322% 37444	58.75% 6829671	40.36% 4691902	0.566% 65887	11624904
B.Distance.10kx10k.30N [0.75,1]	0% 0	0% 0	0% 0	0% 0	0% 4	1.995% 46711	62.149% 1454502	35.854% 839097	2340314

F. M.Distance

M.Distance.10kx10k.30N [-1,-0.75)	10k.30N [-0.75,-0.5)	10k.30N [-0.5,-0.25)	kx10k.30N [-0.25,0)	0kx10k.30N [0,0.25)	x10k.30N [0.25,0.5)	x10k.30N [0.5,0.75)	0kx10k.30N [0.75,1]	Total Count
M.distance.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	0% 0	11.526% 481	65.588% 2737	19.865% 829	3.019% 126	4173
M.distance.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	6.674% 1642488	67.289% 16559006	23.336% 5742765	2.699% 664239	24608498
M.distance.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	4.276% 976047	64.909% 14815728	27.316% 6235070	3.497% 798359	22825204
M.distance.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	0% 0	1.489% 38086	46.316% 1184366	38.887% 994414	13.306% 340259	2557125

M.Distance.10kx10k.53N [-1,-0.75)	10k.53N [-0.75,-0.5)	10k.53N [-0.5,-0.25)	kx10k.53N [-0.25,0)	0kx10k.53N [0,0.25)	x10k.53N [0.25,0.5)	x10k.53N [0.5,0.75)	0kx10k.53N [0.75,1]	Total Count
M.distance.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.distance.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	0% 0	29.283% 1222	60.148% 2510	10.256% 428	0.311% 13	4173
M.distance.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	25.414% 6254020	63.145% 15539131	11.155% 2745188	0.285% 70159	24608498
M.distance.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	21.057% 4806508	63.23% 14432482	15.047% 3434519	0.664% 151695	22825204
M.distance.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	0% 0	10.686% 273266	48.618% 1243228	31.484% 805099	9.21% 235532	2557125

M.Distance.10kx10k.53N [-1,-0.75)	10k.53N [-0.75,-0.5)	10k.53N [-0.5,-0.25)	kx10k.53N [-0.25,0)	0kx10k.53N [0,0.25)	x10k.53N [0.25,0.5)	x10k.53N [0.5,0.75)	0kx10k.53N [0.75,1]	Total Count
M.Distance.10kx10k.30N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.Distance.10kx10k.30N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.Distance.10kx10k.30N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.Distance.10kx10k.30N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.Distance.10kx10k.30N [0,0.25)	0% 0	0% 0	0% 0	0% 0	79.267% 2106222	20.716% 550454	0.015% 425	0% 1	2657102
M.Distance.10kx10k.30N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	28.249% 9198414	69.962% 22781062	1.788% 582359	0% 2	32561837
M.Distance.10kx10k.30N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	0.234% 30380	60.486% 7846939	38.801% 5033741	0.478% 62018	12973078
M.Distance.10kx10k.30N [0.75,1]	0% 0	0% 0	0% 0	0% 0	0% 0	2.157% 38896	75.913% 1368709	21.929% 395378	1802983

G. MIC

MIC.10kx10k.30N [-1,-0.75)	10k.30N [-0.75,-0.5)	10k.30N [-0.5,-0.25)	kx10k.30N [-0.25,0)	0kx10k.30N [0,0.25)	x10k.30N [0.25,0.5)	x10k.30N [0.5,0.75)	0kx10k.30N [0.75,1]	Total Count
M.MIC.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	0% 0	34.722% 8416498	61.07% 14803177	4.047% 981085	0.16% 38829	24239589
M.MIC.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	29.34% 5261988	64.094% 11494913	6.184% 1109099	0.38% 68298	17934298
M.MIC.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	21.943% 1561916	65.062% 4631052	11.432% 813756	1.561% 111110	7117834
M.MIC.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	0% 0	12.491% 87850	58.813% 413624	22.051% 155087	6.642% 46718	703279

MIC.10kx10k.53N [-1,-0.75)	10k.53N [-0.75,-0.5)	10k.53N [-0.5,-0.25)	kx10k.53N [-0.25,0)	0kx10k.53N [0,0.25)	x10k.53N [0.25,0.5)	x10k.53N [0.5,0.75)	0kx10k.53N [0.75,1]	Total Count
M.MIC.10kx10k.10N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
M.MIC.10kx10k.10N [0,0.25)	0% 0	0% 0	0% 0	0% 0	14.618% 3543423	82.052% 19889274	3.238% 785031	0.09% 21861	24239589
M.MIC.10kx10k.10N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	12.799% 2295437	82.009% 14707783	4.972% 891751	0.219% 39327	17934298
M.MIC.10kx10k.10N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	10.181% 724710	79.347% 5647788	9.522% 677821	0.948% 67515	7117834
M.MIC.10kx10k.10N [0.75,1]	0% 0	0% 0	0% 0	0% 0	6.42% 45157	69.816% 491005	19.692% 138496	4.069% 28621	703279

MIC.10kx10k.53N [-1,-0.75)	10k.53N [-0.75,-0.5)	10k.53N [-0.5,-0.25)	kx10k.53N [-0.25,0)	0kx10k.53N [0,0.25)	x10k.53N [0.25,0.5)	x10k.53N [0.5,0.75)	0kx10k.53N [0.75,1]	Total Count
MIC.10kx10k.30N [-1,-0.75)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
MIC.10kx10k.30N [-0.75,-0.5)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
MIC.10kx10k.30N [-0.5,-0.25)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
MIC.10kx10k.30N [-0.25,0)	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	NaN% 0	0
MIC.10kx10k.30N [0,0.25)	0% 0	0% 0	0% 0	0% 0	24.527% 3759666	75.261% 11536292	0.21% 32293	0% 1	15328252
MIC.10kx10k.30N [0.25,0.5)	0% 0	0% 0	0% 0	0% 0	9.016% 2825915	86.996% 27267207	3.949% 1237873	0.037% 11771	31342766
MIC.10kx10k.30N [0.5,0.75)	0% 0	0% 0	0% 0	0% 0	0.755% 23099	62.049% 1898104	34.81% 1064856	2.385% 72968	3059027
MIC.10kx10k.30N [0.75,1]	0% 0	0% 0	0% 0	0% 0	0.017% 47	12.925% 34247	59.661% 158077	27.394% 72584	264955

Supplemental bin plots

Figure S1 provides additional comparisons between Hoeffding's D statistic and other correlation methods. Along with Figure 3 C and D, Figure S1 A displays a noisy relationship with wide tails. This indicates a large range of Hoeffding scores corresponding to high correlations with the opposing metric. In contrast, Figure S1 B, like Figure 2 D, shows narrower tails indicating a more robust relationship between the two metrics.

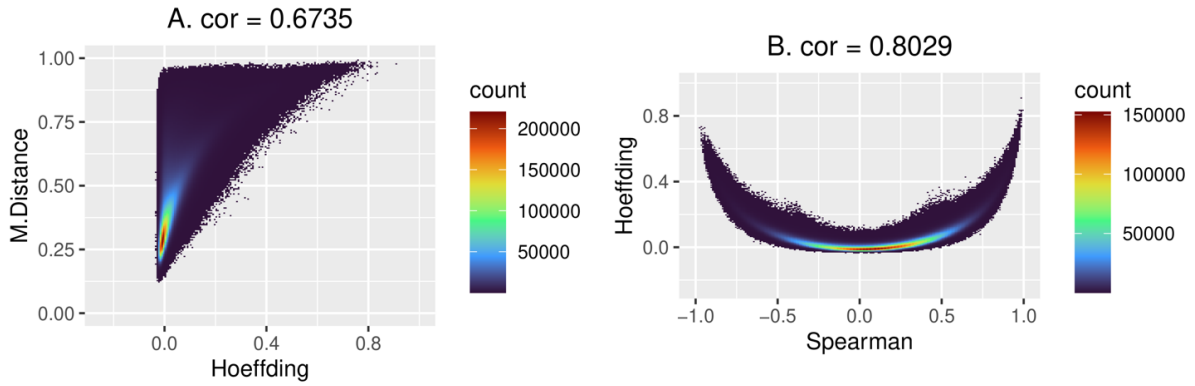

Figure S1. Additional scatterplot comparisons with Hoeffding's D Statistic
(A) Hoeffding vs. M.Distance, (B) Spearman vs. Hoeffding

Figure S2, together with Figure 2 F, shows all four comparisons of M.Pearson and B.Pearson plotted against Spearman and Kendall coefficients. The shape exhibited by the following graphs matches that of Figure 2 F. All four of these bin plots have a clear diagonal, with the slope of the diagonal being smaller when comparing to Kendall rather than Spearman. When comparing M.Pearson and B.Pearson to both Kendall and Spearman, strong Pearson correlations are assigned a wide range of scores by the opposing metric. In all four graphs, the size of the range of Kendall or Spearman values assigned to a fixed Pearson value remains relatively constant. In contrast, different values of Kendall or Spearman coefficients on the y-axis may obtain drastically different ranges of corresponding Pearson scores. Strong Kendall and Spearman scores correspond to a limited range of Pearson scores, while Kendall and Spearman scores near 0 are matched by Pearson scores ranging from strongly negative to strongly positive in all three bin plots in Figure S2, as well as in Figure 2 F.

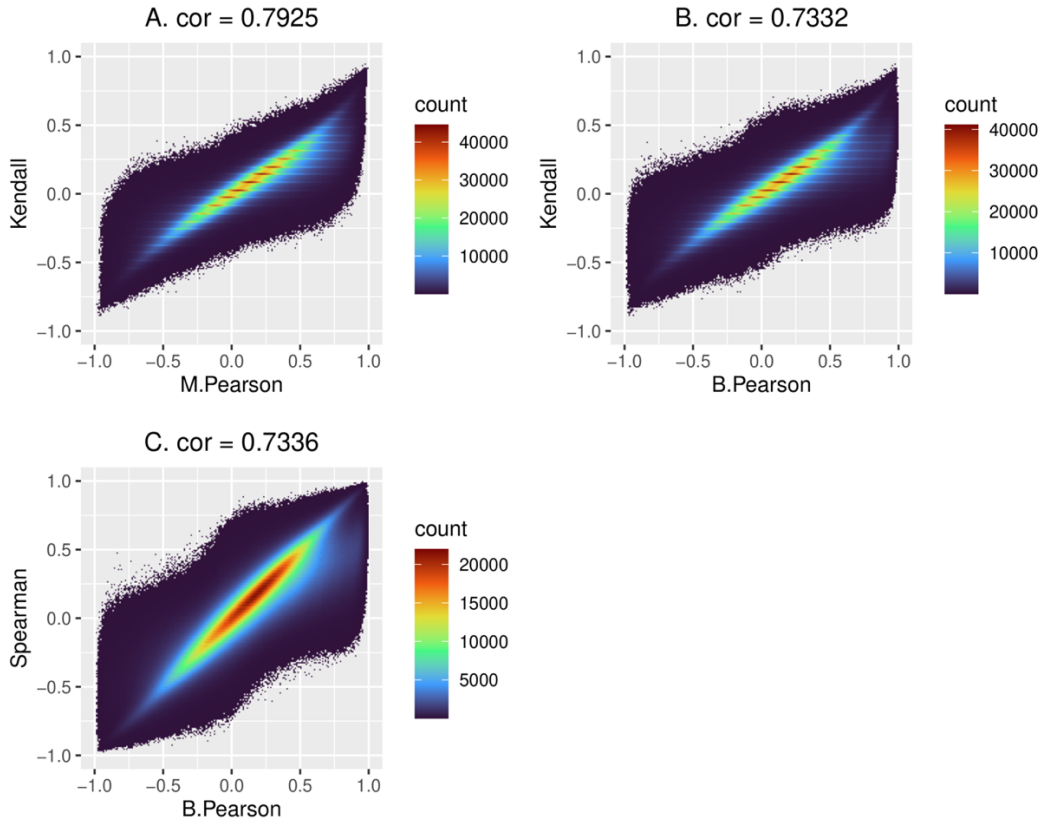

Figure S2. Scatterplots comparing Pearson correlations to Kendall/Spearman correlations
(A) M.Pearson vs. Kendall, (B) B.Pearson vs. Kendall, (C) B.Pearson vs. Spearman

Figure S3 provides additional comparisons between MIC and other correlation methods. As in Figure 3 B, Figure S3 A and B exhibit a loose chevron shape with significant noise, which indicated a large amount of disagreement between MIC and the opposing metrics as to whether the data were highly correlated or not. In Figure S3 C, the chevron is not apparent because Distance correlation only produces positive scores; however, the lower boundary of the plot is similar to that of Figure 3 B, with a large concentration of points having strong Distance coefficients and low MIC coefficients. Out of all the bin plot comparisons made with MIC, Figure S3 A and B most closely match the plot produced by Reshef et al. comparing Pearson and MIC.

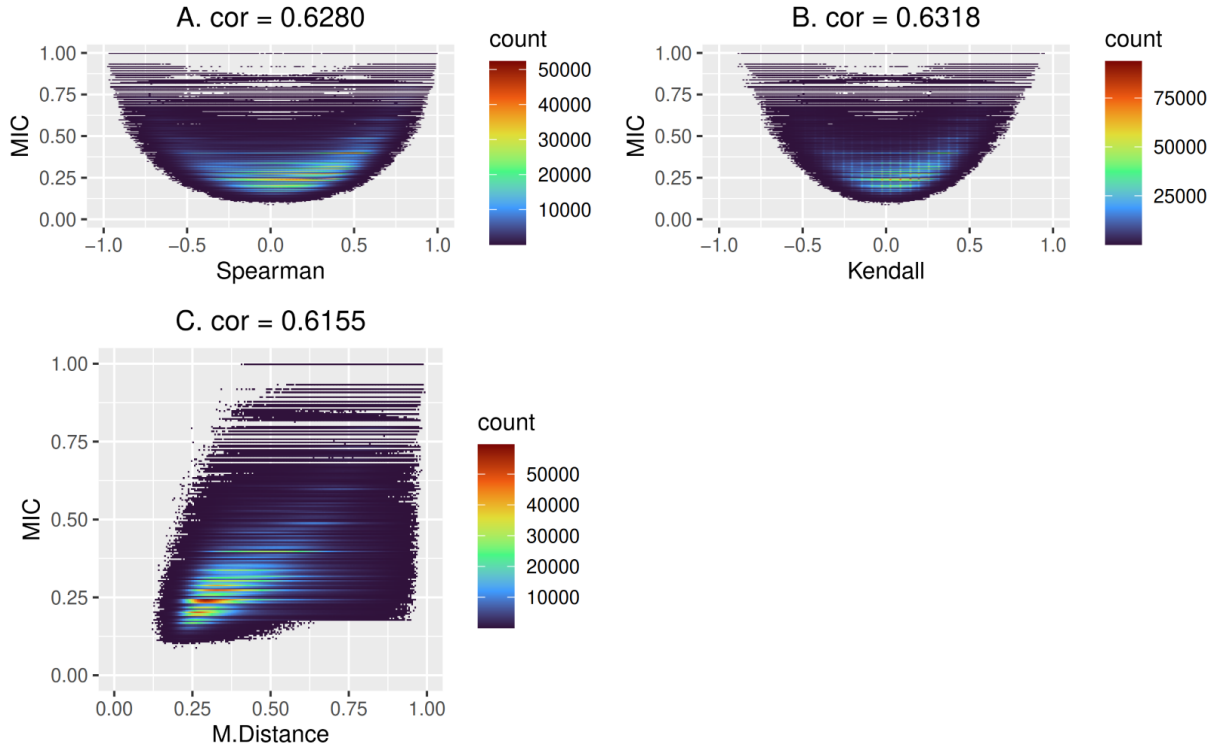

Figure S3. Additional scatterplot comparisons with MIC
(A) M.Distance vs. MIC, (B) Kendall vs. MIC, (C) Spearman vs. MIC