Supplementary Information for The asteroid 162173 Ryugu: a cometary origin

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May 31, 2021

S1 FORMULATION

Here, we describe the formulation of our model that a cometary nucleus transforms to an asteroid as a result of water ice sublimation. The outline is illustrated in Figure 1. We consider a spherically-symmetric highly-porous cometary nucleus with a two-layered structure consisting of the inner primitive region and the outer dust mantle, which are composed of water ice particles and rocky debris, respectively. Both the water ice particles and rocky debris are assumed to be spheres with diameters of $d_{\rm i}$ and $d_{\rm r}$, respectively. The internal temperature T is assumed to be uniform and to not vary with time. The physical quantities are uniform each in the primitive region and in the dust mantle, respectively.

S1.1 Definition of parameters

Initially, the cometary nucleus consists of only the primitive region, and its radius is R_0. As the water ice sublimates, the primitive region shrinks and the rocky debris left behind accumulates on its surface. The thickness Δ of the dust mantle increases with the decrease in the radius R of the primitive region. When the water ice has completely sublimated, R becomes zero and Δ gives the final radius R_∞ of the asteroid left behind. We denote the parameter in each region with a subscript (α), where $\alpha = {\rm p}$ for the primitive region and $\alpha = {\rm m}$ for the dust mantle. The macroporosity, volume fractions of water ice particles and rocky debris, and density of the region

α are denoted by $\epsilon_{(\alpha)}, \phi_{\mathrm{i}(\alpha)}, \phi_{\mathrm{r}(\alpha)}$, and $\rho_{(\alpha)}$, respectively. The relationship between the macroporosity and volume fractions is given by

$$\epsilon_{(\alpha)} = 1 - (\phi_{\mathrm{i}(\alpha)} + \phi_{\mathrm{r}(\alpha)}). \tag{S1}$$

We find $\phi_{\mathrm{i(m)}} = 0$ because there is no water ice in the dust mantle. The density $\rho_{(\alpha)}$ of each region is given by

$$\rho_{(\alpha)} = \varrho_{\mathrm{r}}\phi_{\mathrm{r}(\alpha)} + \varrho_{\mathrm{i}}\phi_{\mathrm{i}(\alpha)}, \tag{S2}$$

where ϱ_{i} and ϱ_{r} are the material densities of the water ice particles and rocky debris, respectively. The mass fraction f of water ice in the primitive region is given by

$$f = \frac{\varrho_{\mathrm{i}}\phi_{\mathrm{i(p)}}}{\varrho_{\mathrm{r}}\phi_{\mathrm{r(p)}} + \varrho_{\mathrm{i}}\phi_{\mathrm{i(p)}}} = \frac{\varrho_{\mathrm{i}}\phi_{\mathrm{i(p)}}}{\rho_{\mathrm{(p)}}}. \tag{S3}$$

The physical quantities defined above are not independent of each other. We choose $\epsilon_{\mathrm{(p)}}$, $\epsilon_{\mathrm{(m)}}$, and f as independent input parameters that are more relevant to observation. The other quantities are determined from these independent parameters as follows. From Eq. (S3), we obtain $\frac{f}{1-f} = \frac{\varrho_{\mathrm{i}}\phi_{\mathrm{i(p)}}}{\varrho_{\mathrm{r}}\phi_{\mathrm{r(p)}}}$. Solving this equation and Eq. (S1) for $\phi_{\mathrm{r(p)}}$ and $\phi_{\mathrm{i(p)}}$, respectively, we obtain

$$\phi_{\mathrm{r(p)}} = \frac{1 - \epsilon_{\mathrm{(p)}}}{1 + \frac{\varrho_{\mathrm{r}}}{\varrho_{\mathrm{i}}}\frac{f}{1-f}}, \quad \phi_{\mathrm{i(p)}} = \frac{1 - \epsilon_{\mathrm{(p)}}}{\frac{\varrho_{\mathrm{i}}}{\varrho_{\mathrm{r}}}\frac{1-f}{f} + 1} \tag{S4}$$

for the primitive region, and

$$\phi_{\mathrm{r(m)}} = 1 - \epsilon_{\mathrm{(m)}} \tag{S5}$$

for the dust mantle. The ratio $p = \rho_{\mathrm{(m)}}/\rho_{\mathrm{(p)}}$ of densities between the primitive region and the dust mantle is obtained as

$$p = \frac{\varrho_{\mathrm{r}}\phi_{\mathrm{r(m)}}}{\varrho_{\mathrm{r}}\phi_{\mathrm{r(p)}} + \varrho_{\mathrm{i}}\phi_{\mathrm{i(p)}}} = \frac{1 - \epsilon_{\mathrm{(m)}}}{1 - \epsilon_{\mathrm{(p)}}}\left(1 + \frac{\varrho_{\mathrm{r}} - \varrho_{\mathrm{i}}}{\varrho_{\mathrm{i}}}f\right). \tag{S6}$$

Supplementary Table 1 shows the default values of input parameters. Unless otherwise noted, the values in this table are used in calculations.

Supplementary Table 1: Default values of input parameters used in calculations.

Quantity	Notation	Value
Independent parameters:		
Initial radius of cometary nucleus	R_0	3 km
Temperature of cometary nucleus	T	200 K
Diameter of water ice particles	d_{i}	1 μm
Diameter of rocky debris	d_{r}	1 cm
Initial mass fraction of water ice	f	0.99
Macroporosity in primitive region	$\epsilon_{(\mathrm{p})}$	0.6
Macroporosity in dust mantle	$\epsilon_{(\mathrm{m})}$	0.6
Dependent parameters determined by $f, \epsilon_1, \epsilon_2$*:*		
Volume fraction of rocky debris in primitive region	$\phi_{\mathrm{r(p)}}$	0.00134
Volume fraction of water ice particles in primitive region	$\phi_{\mathrm{i(p)}}$	0.39866
Volume fraction of rocky debris in dust mantle	$\phi_{\mathrm{r(m)}}$	0.4
Density of primitive region	$\rho_{(\mathrm{p})}$	0.403 g/cm^3
Density of dust mantle	$\rho_{(\mathrm{m})}$	1.200 g/cm^3
Ratio in densities of dust mantle to primitive region	p	2.98
Material constants:		
Material density of water ice particles	ϱ_{i}	1.0 g/cm^3
Material density of rocky debris	ϱ_{r}	3.0 g/cm^3

S1.2 Distribution of vapor

S1.2.1 Vapor flow in pores

The pores inside the cometary nucleus are filled with water vapor generated by the sublimation of water ice particles. The production rate $q_{(\mathrm{p})}$ of the water vapor per unit volume of the primitive region is given by [22]

$$q_{(\mathrm{p})} = \phi_{\mathrm{i(p)}} S \left(\frac{m}{2\pi k_{\mathrm{B}} T} \right)^{1/2} (P_{\mathrm{e}} - P), \tag{S7}$$

where $S = 6/d_{\mathrm{i}}$ is the surface-to-volume ratio of water ice particles, P_{e} is the equilibrium vapor pressure of water ice, P is the pressure of water vapor filling the pores, k_{B} is the Boltzmann constant, and m is the mass of a water molecule. The equilibrium vapor pressure is given by [22]

$$P_{\mathrm{e}} = 3.56 \times 10^{12} \exp\left(-\frac{6141.667}{T} \right) \ \mathrm{Pa}. \tag{S8}$$

On the other hand, the dust mantle does not contain water ice particles, so the production rate $q_{(\mathrm{m})}$ is naturally zero.

The flow of water vapor in the porous cometary nucleus is driven by the pressure gradient. The cometary nucleus is cold, the equilibrium vapor pressure is low, and the water vapor filling the pores is dilute. The mean free path is a few centimeters at 200 K, which is much longer than the typical size of pores [22]. Therefore, the flow can be regarded as a free molecular flow. Assuming that the region α is randomly packed with spherical particles of diameter $d_{(\alpha)}$, the flux $\boldsymbol{J}_{(\alpha)}$ of water vapor is given by [22]

$$\boldsymbol{J}_{(\alpha)} = -\frac{16}{3} \left(\frac{m}{2\pi k_{\mathrm{B}}} \right)^{1/2} \frac{\epsilon_{(\alpha)}^{3/2}}{(1-\epsilon_{(\alpha)})^{1/3}} d_{(\alpha)} \boldsymbol{\nabla} \left(\frac{P}{\sqrt{T}} \right). \tag{S9}$$

The primitive region contains both of water ice particles and rocky debris. Since the flux is controlled by smaller particles, the particle diameter $d_{(\mathrm{p})}$ in the primitive region can be assumed to be equal to the diameter d_{i} of water ice particles. On the other hand, since only rocky debris exists in the dust mantle, the particle diameter $d_{(\mathrm{m})}$ is equal to the diameter d_{r} of rocky debris. Although the diameter of the rocky debris assumed in this study is about the same as the mean free path of water vapor, we use the equation for a free molecular flow, because it makes the model simpler.

Let us assume that the vapor flow inside the cometary nucleus reaches steady state on the evolution timescale of the cometary nucleus. The steady flow satisfies the following continuity equation in each region:

$$\boldsymbol{\nabla} \cdot \boldsymbol{J}_{(\alpha)} = q_{(\alpha)}. \tag{S10}$$

Substituting Eqs. (S7) and (S9) into Eq. (S10) yields an equation for the pressure distribution $P(r)$. When the temperature T is uniform, the equation becomes a Poisson equation in the primitive region and a Laplace equation in the dust mantle, respectively. These equations can be solved analytically under appropriate boundary conditions.

S1.2.2 Boundary condition

We denote the pressure distributions in the primitive region and in the dust mantle as $P_{(\mathrm{p})}(r)$ and $P_{(\mathrm{m})}(r)$, respectively. These two distributions are connected so as to satisfy the following two boundary conditions at $r = R$ (contact boundary). The first boundary condition is that the pressure is continuous; namely, $P_{(\mathrm{p})}(R) = P_{(\mathrm{m})}(R)$ (boundary condition i). The second boundary condition is that the flux is continuous; namely, $J_{(\mathrm{p})}(R) = J_{(\mathrm{m})}(R)$ (boundary condition ii). In addition, we consider a zero-flux condition at the center of the cometary nucleus ($J_{(\mathrm{p})}(0) = 0$, boundary condition iii) and zero pressure at the mantle surface ($P_{(\mathrm{m})}(R + \Delta) = 0$, boundary condition iv).

Using Eq. (S9), the boundary condition (ii) is rewritten as

$$\frac{dP_{(\mathrm{p})}}{dr} = \chi \frac{dP_{(\mathrm{m})}}{dr}, \qquad (\text{at } r = R) \tag{S11}$$

where χ is a dimensionless quantity defined by

$$\chi \equiv \left(\frac{\epsilon_{(\mathrm{m})}}{\epsilon_{(\mathrm{p})}} \right)^{3/2} \left(\frac{1 - \epsilon_{(\mathrm{m})}}{1 - \epsilon_{(\mathrm{p})}} \right)^{-1/3} \frac{d_{(\mathrm{m})}}{d_{(\mathrm{p})}}. \tag{S12}$$

In this paper, we assume $d_{(\mathrm{p})} \ll d_{(\mathrm{m})}$, so $\chi \gg 1$ is valid unless the macroporosities of the primitive region and dust mantle are very different. Therefore, at the contact boundary, the magnitude of the pressure gradient in the primitive region is much larger than that in the dust mantle.

S1.2.3 Analytic solution

Solving equations for $P_{(\mathrm{p})}(r)$ and $P_{(\mathrm{m})}(r)$ together with the boundary conditions (i)-(iv), we obtain the analytical solution as follows:

$$P_{(\mathrm{p})}(r) = \left[1 - g_{R,\Delta}\frac{\sinh(r/h)}{\sinh(R/h)}\frac{R}{r}\right]P_{\mathrm{e}}, \quad (\text{for } 0 \le r \le R) \tag{S13}$$

$$P_{(\mathrm{m})}(r) = (1 - g_{R,\Delta})\frac{R}{\Delta}\left(\frac{R+\Delta}{r} - 1\right)P_{\mathrm{e}}, \quad (\text{for } R < r \le R+\Delta) \tag{S14}$$

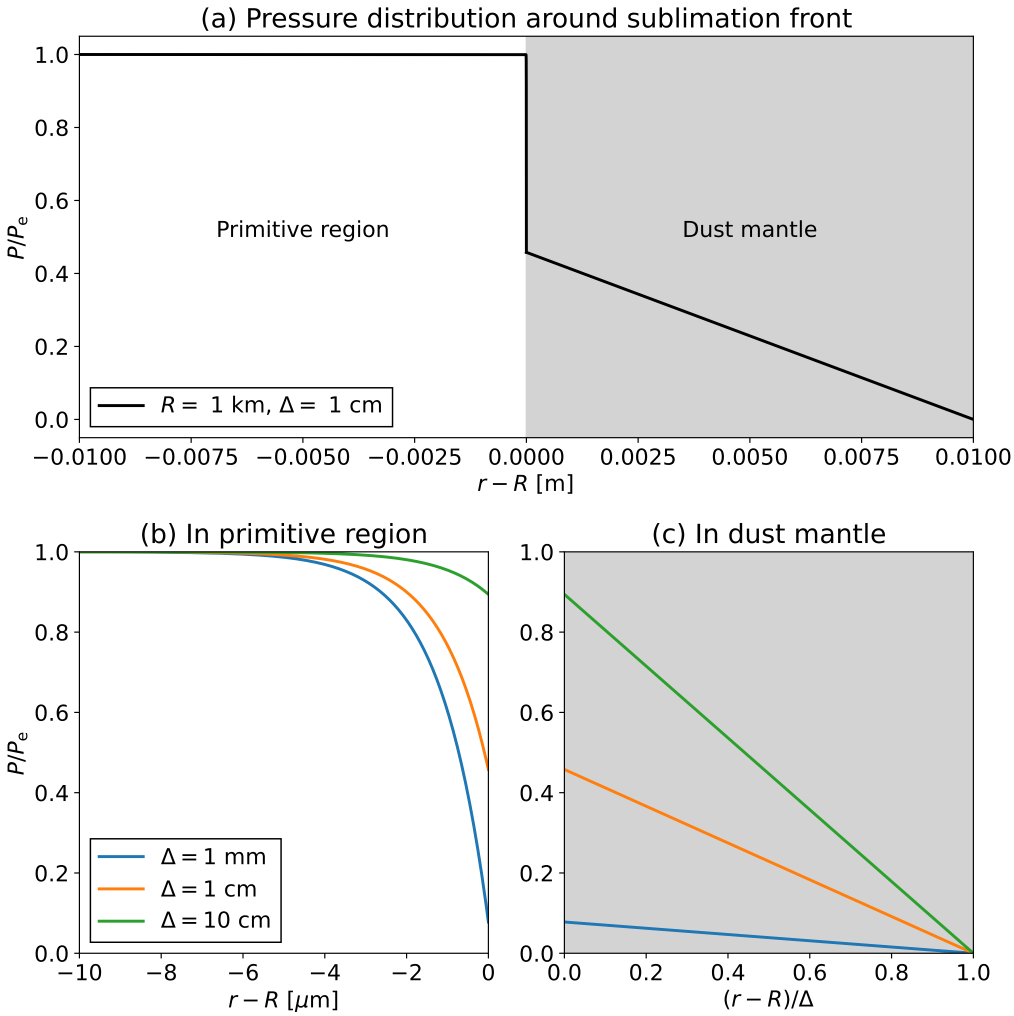
where $g_{R,\Delta}$ and h are constants defined by

$$g_{R,\Delta} \equiv \frac{\chi(1+\Delta/R)\tanh(R/h)}{(\Delta/h) + [\chi + (\chi-1)\Delta/R]\tanh(R/h)}, \tag{S15}$$

$$h \equiv \frac{2\sqrt{2}}{3}\frac{\epsilon_{(\mathrm{p})}^{3/4}}{(1-\epsilon_{(\mathrm{p})})^{1/6}}\frac{d_{(\mathrm{p})}}{\phi_{\mathrm{i(p)}}^{1/2}}. \tag{S16}$$

Substituting the values listed in Supplementary Table 1, we obtain $h = 1.61\ \mu\mathrm{m}$.

Supplementary Figure 1 shows the analytic solutions of $P_{(\mathrm{p})}(r)$ and $P_{(\mathrm{m})}(r)$. Panel (a) shows $P_{(\mathrm{p})}(r)$ and $P_{(\mathrm{m})}(r)$ near the contact boundary. The horizontal axis is the distance from the contact boundary. Here, we use $R = 1$ km and $\Delta = 1$ cm. Throughout almost the entire area of the primitive region, $P_{(\mathrm{p})}(r)$ is equal to P_{e}, indicating that a solid-vapor equilibrium has been established. However, $P_{(\mathrm{p})}(r)$ decreases rapidly in a very narrow region near the contact boundary and is connected to the pressure $P_{(\mathrm{m})}(R)$ in the dust mantle. In the dust mantle, $P_{(\mathrm{m})}(r)$ decreases slowly toward the outside and becomes zero at the surface. Panels (b) and (c) respectively show the dependences of $P_{(\mathrm{p})}(r)$ and $P_{(\mathrm{m})}(r)$ on Δ. In panel (b), the horizontal axis is magnified around the contact boundary. In panel (c), the horizontal axis is normalized by Δ. The thicker the dust mantle, the closer the water vapor pressure at the contact boundary is to the equilibrium vapor pressure. This trend can be understood by considering that the dust mantle acts as a lid to prevent the leakage of the water vapor. However, for any mantle thicknesses, the pressure is almost equal to P_{e} as one dives deeper than a few times h from the contact boundary into the primitive region. This suggests that the water ice sublimates only at the very vicinity of the contact boundary. Therefore, we refer to the contact boundary as a sublimation front in the current study.



Supplementary Figure 1: Analytic solution of pressure distribution $P(r)$ of water vapor in cometary nucleus. Panel (a) shows $P(r)$ near the contact boundary between the primitive region and the dust mantle in the case with $R = 1$ km and $\Delta = 1$ cm. Panels (b) and (c) show the dependence on Δ. Panel (b) is a magnified view of $P(r)$ in the primitive region, and panel (c) is in the dust mantle. The horizontal axis indicates the distance from the contact boundary, where negative values indicate the primitive region side and positive values indicate the dust mantle side. Note that the horizontal axis in panel (c) is normalized by Δ. The region corresponding to the dust mantle is filled in gray. The pressure in the vertical axis is normalized by the equilibrium vapor pressure P_{e}.

S1.3 Shrinkage of nucleus and dust mantle formation

As can be seen in Supplementary Figure 1, $P_{(\mathrm{p})}(r)$ is not uniform in the very neighborhood of the sublimation front. The fact that the water vapor pressure varies from place-to-place means that the sublimation rate of water ice varies from place to place (see Eq. S7). In other words, water ice particles closer to the sublimation front sublimate faster, so physical quantities such as the volume fraction of water ice particles cannot be strictly uniform. However, the width of such inhomogeneous region is at most a few times larger than h, which is much smaller than the size of the entire cometary nucleus. Therefore, we can assume that the physical quantities in the primitive region are uniform and that water ice sublimates only from the surface of the primitive region. In this case, the time variation of the radius R of the primitive region is given by

$$\frac{dR}{dt} = -\frac{J_{(\mathrm{p,sf})}}{\varrho_{\mathrm{i}} \phi_{\mathrm{i(p)}}}, \tag{S17}$$

where $J_{(\mathrm{p,sf})}$ is the value of $J_{(\mathrm{p})}$ at the sublimation front and is given by

$$J_{(\mathrm{p,sf})} = 4 \left(\frac{m}{\pi k_{\mathrm{B}} T} \right)^{1/2} \frac{\epsilon_{(\mathrm{p})}^{3/4}}{(1-\epsilon_{(\mathrm{p})})^{1/6}} g_{R,\Delta} \left[\frac{1}{\tanh(R/h)} - \frac{1}{R/h} \right] P_{\mathrm{e}}, \tag{S18}$$

where we used Eq. (S13).

Rocky debris contained outside the primitive region accumulates on the surface of the primitive region and forms the dust mantle. From the mass conservation for the rocky debris, we obtain the following relationship between R and Δ [15]:

$$\frac{4\pi}{3}(R_0^3 - R^3)(1-f) = \frac{4\pi}{3}[(R+\Delta)^3 - R^3]p. \tag{S19}$$

Solving Eq. (S19) for Δ, we obtain the normalized mantle thickness $k = \Delta/R_0$ as follows:

$$k = \left[x^3 + \frac{1-f}{p}(1-x^3) \right]^{1/3} - x, \tag{S20}$$

where $x = R/R_0$. The value of k at $x = 0$, $k_\infty = (\frac{1-f}{p})^{1/3}$, gives the normalized final radius R_∞/R_0 when the cometary nucleus has transformed to an asteroid.

S1.4 Spin-up

Since assuming the spherical symmetry, the water vapor does not exert any reaction torque on the cometary nucleus when ejected. Therefore, the nucleus never starts spinning if not rotating initially. However, if the nucleus is initially rotating, the moment of inertia will change as it contracts, and its spin rate may also change. Watanabe [15] formulated the spin-up by taking into account the angular momentum loss due to the ice sublimation and the decrease in the moment of inertia due to the contraction of the cometary nucleus. However, he assumed the case where the cometary nucleus shrinks only slightly, so his model cannot be directly applied to the drastic change where the cometary nucleus loses almost all of water ice. Here, we modified the Watanabe's formulation to apply to the case where the radius of the cometary nucleus changes significantly.

The angular momentum of the cometary nucleus is $L = I\omega$, where I is the moment of inertia of the cometary nucleus and ω is its angular velocity. Differentiating L by R, we obtain

$$\frac{1}{\omega}\frac{d\omega}{dR} = \frac{1}{I\omega}\frac{dL}{dR} - \frac{1}{I}\frac{dI}{dR}. \tag{S21}$$

The angular momentum is reduced by the amount associated with the water vapor leaking from the mantle surface. Therefore, the time variation of L is given by[1]

$$\frac{dL}{dt} = -\frac{8\pi}{3}(R+\Delta)^4 J_{(\mathrm{m,s})}\omega, \tag{S22}$$

where $J_{(\mathrm{m,s})}$ is the value of $J_{(\mathrm{m})}$ at the mantle surface ($r = R + \Delta$). From Eq. (S22), we obtain

$$\frac{dL}{dR} = \frac{dL}{dt}\frac{dt}{dR} = \frac{8\pi}{3} f\rho_{(\mathrm{p})}(R+\Delta)^2 R^2\omega, \tag{S23}$$

where we used the continuity of the water vapor flowing in the pores given by $R^2 J_{(\mathrm{p,sf})} = (R+\Delta)^2 J_{(\mathrm{m,s})}$. The moment of inertia I of the cometary nucleus including the dust mantle is given by

$$I = \frac{8\pi}{15}\left[\rho_{(\mathrm{m})}(R+\Delta)^5 - (\rho_{(\mathrm{m})} - \rho_{(\mathrm{p})})R^5\right]. \tag{S24}$$

[1] We used the fact that the moment of inertia of a thin spherical shell with the mass M and radius R is given by $\frac{2}{3}MR^2$.

Substituting Eqs. (S23) and (S24) into Eq. (S21), and integrating for R from R_0 to R, we obtain the angular velocity $\omega(x)$ when the radius of the primitive region becomes $R = xR_0$, as the ratio to the initial value ω_0, as follows:

$$\frac{\omega(x)}{\omega_0} = \frac{\exp\left[D(x)\right]}{p(x+k)^5 - x^5(p-1)}, \tag{S25}$$

where $D(x)$ is a function defined by

$$D(x) \equiv \int_1^x \frac{5fx^2(x+k)^2}{p(x+k)^5 - x^5(p-1)} dx. \tag{S26}$$

When $x = 0$, the equation (S25) gives the final spin-up rate after the water ice sublimates completely. This final spin-up rate depends only on the values of f and p, and not on the process in the middle.

Eq. (S25) has the same form as the Watanabe's model, but the definition of the function $D(x)$ given by Eq. (S26) differs in two respects. The first respect is the difference in the relationship between x and k (see Eq. S20). The Watanabe's model uses the approximation $k = (1-x)(1-f)/p$, which is valid only when the contraction of the cometary nucleus is sufficiently small ($x \simeq 1$ and $k \ll 1$). The second respect is that in the Watanabe's model the numerator of the integrand was not $5fx^2(x+k)^2$ but $5fx^4$; namely, $(k/x)^2$ was ignored as sufficiently small for 1. In the Watanabe's model, the angular momentum is assumed to be carried away when the water vapor is released outside the primitive region. However, the water vapor ejected from the surface of the primitive region passes through the dust mantle before being ejected from the cometary nucleus, and slows down its rotation. The Watanabe's model is a good approximation when the contraction of the cometary nucleus is sufficiently small, but it cannot be applied to the situation where almost all the water ice sublimates, as in this study.

S1.5 Numerical scheme

Eq. (S17) was integrated numerically using the fourth-order accurate Runge–Kutta method. The time step Δt is variable and is taken to be smaller as the rate of change in R is larger. Specifically, Δt was given to satisfy the following:

$$\frac{R_0/N}{\Delta t} = \left|\frac{dR}{dt}\right|, \tag{S27}$$

where N is an integer and we set $N = 10^3$ in this study. If R becomes negative, we calculate the sublimation time at which R becomes just zero by linear interpolation with the value of R at the previous time step.

The increase in the angular velocity of rotation with the shrinking of the cometary nucleus was calculated using Eq. (S25). The integration of $D(x)$ given by Eq. (S26) was performed numerically using a package `integrate.quad()` in the Python library `SciPy`.

S1.6 Parameter dependence

The time it takes for the water ice to sublimate completely is called the sublimation time. The parameter dependence is revealed by normalizing Eq. (S17). Substituting Eq. (S18) into Eq. (S17), we obtain

$$\frac{dx}{d(t/\tau_{\rm sub})} \simeq -\left(\frac{1}{k} + \frac{1}{x}\right). \tag{S28}$$

Here, for $J_{\rm (p,sf)}$, we approximated $\tanh(R/h) \to 1$ because $R \gg h$, and ignored the term h/R as sufficiently small for 1. For $g_{R,\Delta}$, we used $\chi \ll 1$, and also approximated $\tanh(R/h) \to 1$ and ignored the term $\chi/(\Delta/h)$ as sufficiently small. This approximation is valid because $\chi/(\Delta/h) \sim (d_{\rm (m)}/d_{\rm (p)})/(\Delta/h) \sim d_{\rm (m)}/\Delta$, and the dust mantle is much thicker than the diameter of the rocky debris except in the very early stage of cometary nucleus evolution. From Eq. (S28), we can see that the time variation of R can be scaled by a timescale $\tau_{\rm sub}$, which is defined by

$$\tau_{\rm sub} \equiv \frac{3\sqrt{2}}{16} \frac{(1-\epsilon_{\rm (m)})^{1/3}}{\epsilon_{\rm (m)}^{3/2}} \left(\frac{\pi k_{\rm B} T}{m}\right)^{1/2} \frac{\varrho_{\rm i}\phi_{\rm i(p)}}{d_{\rm (m)}} \frac{R_0^2}{P_{\rm e}}. \tag{S29}$$

This means that the sublimation time is proportional to R_0^2, and inversely proportional to $d_{\rm (m)}$ and $P_{\rm e}(T)/\sqrt{T}$.