Supplementary information for
Contactless generation and trapping of hydrodynamic knots in sessile droplets by acoustic screw dislocations

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I. TRANSDUCER DESIGN AND CHARACTERIZATION

A. Laser vibrometry measurements

The scanning area and measured vibration signals are shown in Fig. 1.

B. Analytical, simulated and experimental swirling SAW

Accounting for the piezoelectric substrate anisotropy, the vertical displacement of the swirling SAW reads:

$$\mathcal{W}_\ell(\mathbf{r}) = \frac{i^\ell}{2\pi}\int_{-\pi}^{+\pi} a(\psi)e^{-i\ell\psi - i\mathbf{k}(\psi)\cdot\mathbf{r}}d\psi, \qquad (1)$$

with $\mathbf{k}(\psi)$ and $a(\psi)$ the wavevector and complex amplitude of a plane surface acoustic wave generated by a straight IDT [1, 2], and $\mathbf{r}$ the position vector relatively to the vortex center. This does not account for the finite aperture of the transducer. Therefore, we use the Green-function from Laude *et al.* [1]. The mismatch between the ideal vortex and the one obtained with the Green functions indicates that the inversion formula in [2] becomes less accurate for high topological order vortex. Finally, comparison of experimental data and Green function suggests that some additional effects such as bulk waves may interfere the experimental field.

II. ACOUSTIC STREAMING IN DROPLETS

A. Simulation

We consider the droplet made of a Newtonian fluid exposed to an isotropic swirling surface acoustic wave of amplitude a small enough to satisfy $\epsilon = \frac{a\omega}{c_0} \ll 1$. Mass conservation read:

$$\partial_t\rho + \nabla\cdot(\rho\mathbf{v}) = 0, \qquad (2)$$

and momentum conservation:

$$\partial_t(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) = -\nabla p + \mu\Delta\mathbf{v} + \left(\mu + \frac{\mu'}{3}\right)\nabla(\nabla\cdot\mathbf{v}). \qquad (3)$$

They are completed with the isentropic equation of state $\frac{1}{\rho_0 {c_0}^2} = \frac{1}{\rho}\left.\frac{\partial\rho}{\partial p}\right|_S$

To calculate the acoustic streaming, we resolve each physical quantity x into three contributions: hydrostatics x_0, acoustics $\tilde{x}$ and hydrodynamics $\bar{x}$. They represent respectively the system at rest (without acoustic field), the oscillating part of the perturbation induced by sound waves and the time averaged part of the perturbation over an acoustic period [3]. Thus, the fluid density ρ, the pressure p and the Eulerian velocity $\mathbf{v}$ read:

$$\rho = \rho_0 + \epsilon\tilde{\rho} + \epsilon^2\bar{\rho}, \qquad (4a)$$

$$p = p_0 + \epsilon\tilde{p} + \epsilon^2\bar{p}, \qquad (4b)$$

$$\mathbf{v} = \epsilon\tilde{\mathbf{v}} + \epsilon^2\bar{\mathbf{v}}. \qquad (4c)$$

Grouping the terms by successive powers of ϵ, and neglecting shock effects, we get (see for instance [3] for a detailed proof):

$$p_0 = Cst \qquad (5a)$$

$$\partial^2_{tt}\tilde{p} - {c_0}^2\Delta\tilde{p} - \nu b\partial_t\Delta\tilde{p} = 0, \qquad (5b)$$

$$\nu\Delta\bar{\mathbf{v}} + \mathbf{F} = \mathbf{0}, \qquad (5c)$$

where we took the time-average of the second order in ϵ to obtain the steady flow motion. We have $\mathbf{F} = \frac{\omega^2\nu b\langle\mathbf{\Pi}\rangle}{c_0^4}$. We note that because the second-order flow is nearly incompressible, Eq. (5c) yields the vorticity equation in the paper after using the identity $\Delta\bar{\mathbf{v}} = -\nabla\times\nabla\times\bar{\mathbf{v}} = -\nabla\times\bar{\mathbf{\Omega}}$.

To solve the acoustic field, we use complex variables $\tilde{x} = \hat{x}e^{i\omega t} + c.c.$. Because the droplet is axisymmetric, symmetry conditions imposed by the incident wave $W_\ell = aJ_\ell(k_r r)e^{-i\ell\theta}$ ensure that all acoustic quantities $\tilde{x}$ can be expressed as $\hat{x} = \hat{x}^{(\ell)}e^{-i\ell\theta}$. The problem is then recast in cylindrical coordinates.

$$(k^2 - \frac{\ell^2}{r^2})\hat{p}^{(\ell)} = -\partial^2_{rr}\hat{p}^{(\ell)} - \partial^2_{zz}\hat{p}^{(\ell)} - \frac{1}{r}\partial_r\hat{p}^{(\ell)}, \qquad (6)$$

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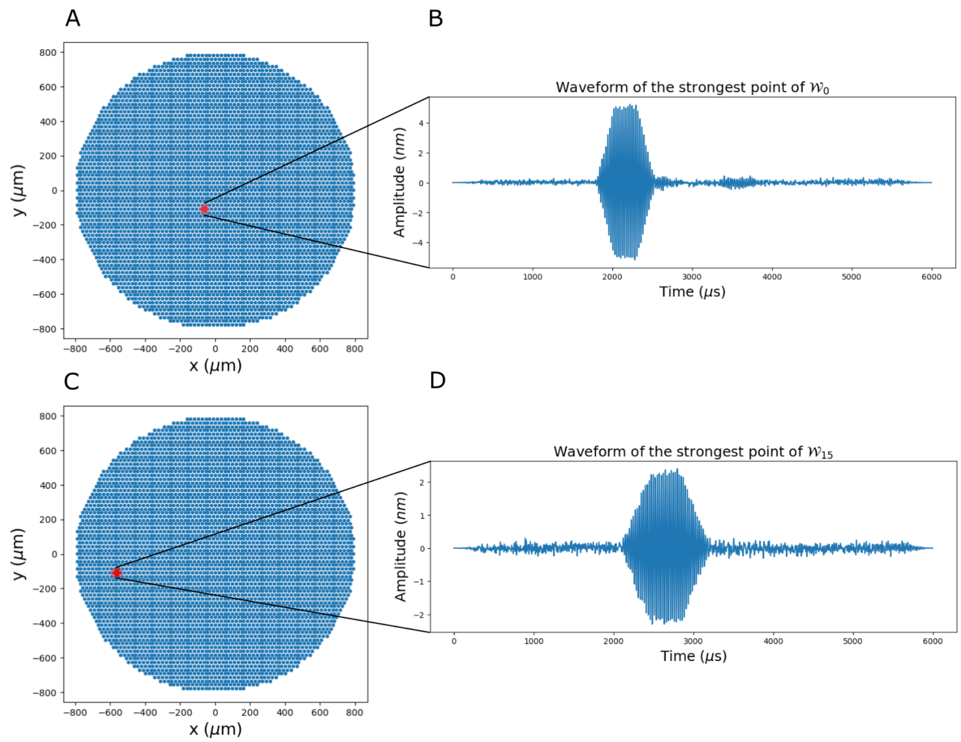

FIG. 1. A), C) LDV scanning area and B), D) signal waveforms for $\mathcal{W}_0$ and $\mathcal{W}_{15}$ at the most intense vibration points.

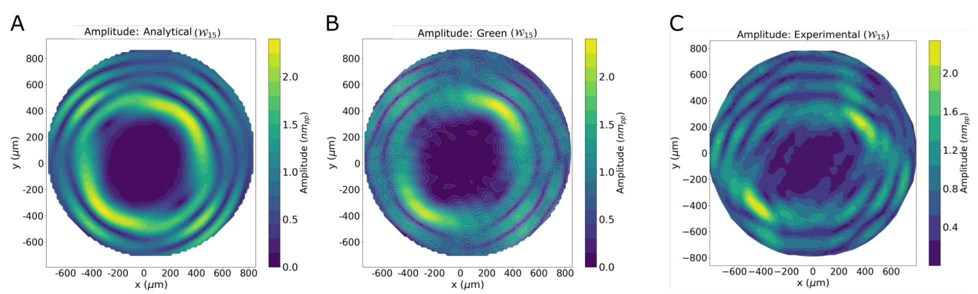

FIG. 2. Analytical magnitude (Eq. (1)), Green function predictions and experimental amplitude of the $\mathcal{W}_{15}$.

where $k^2 = \omega^2/({c_0}^2(1 + i/Re_{ac}))$ and the acoustic Reynolds number $Re_{ac} = \frac{{c_0}^2}{\omega\nu b}$. Equation Eq. (6) is solved with the PDE interface of the Comsol axisymmetric model. The acoustic streaming force is then computed by expressing the individual components of the average pointing vector $\langle\mathbf{\Pi}\rangle = \frac{1}{2}Re(\hat{p}^*\hat{\mathbf{v}})$ in cylindrical coordinates:

$$\langle\Pi_r\rangle = -\frac{1}{2\rho_0\omega}Re\left(\hat{p}^{(\ell)*}\partial_r\hat{p}^{(\ell)}\right), \tag{7a}$$

$$\langle\Pi_\theta\rangle = \frac{1}{2\rho_0\omega}Re\left(\hat{p}^{(\ell)*}\frac{i\ell}{r}\hat{p}^{(\ell)}\right), \tag{7b}$$

$$\langle\Pi_z\rangle = -\frac{1}{2\rho_0\omega}Re\left(\hat{p}^{(\ell)*}\partial_z\hat{p}^{(\ell)}\right) \tag{7c}$$

Comsol then solves the Stokes in axisymmetric coordinates with the streaming force as an external force. Code details are provided in a separate pdf.

B. Unmasked GDPT results

The unmasked GDPT results are shown in Fig. 3.

[1] V. Laude, D. Gérard, N. Khelfaoui, C. F. Jerez-Hanckes, S. Benchabane, and A. Khelif, Subwavelength focusing of surface acoustic waves generated by an annular interdigital

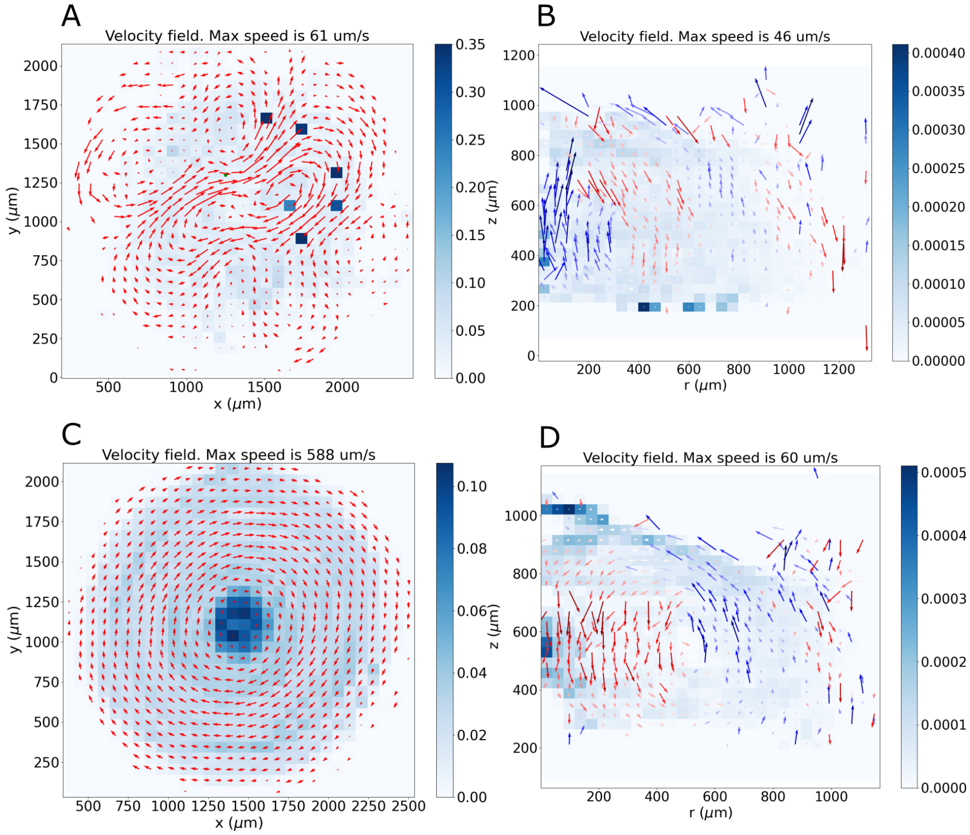

FIG. 3. Red arrows represent: A), C) acoustic streaming azimuthal direction and velocity, B), D) acoustic streaming axial direction and velocity for $\mathcal{W}_0$ and $\mathcal{W}_{15}$ in the sessile droplets calculated by GDPT, respectively. The blue blocks represent the probability of occurrence for red fluorescent plastic particles.

transducer, Applied Physics Letters **92**, 094104 (2008).

[2] A. Riaud, M. Baudoin, O. B. Matar, L. Becerra, and J.-L. Thomas, Selective manipulation of microscopic particles with precursor swirling rayleigh waves, Physical Review Applied **7**, 024007 (2017).

[3] A. Riaud, M. Baudoin, O. B. Matar, J.-L. Thomas, and P. Brunet, On the influence of viscosity and caustics on acoustic streaming in sessile droplets: an experimental and a numerical study with a cost-effective method, Journal of Fluid Mechanics **821**, 384 (2017).