

Supplementary Text S1

The regularised horseshoe (RHS) prior[?] defined as

$$\begin{aligned}\beta|\zeta_p, \varepsilon, c &\sim \text{Normal}(0, \varepsilon^2 \tilde{\zeta}_p^2), \quad \tilde{\zeta}_p^2 = \frac{c^2 \zeta_p^2}{c^2 + \varepsilon^2 \zeta_p^2} \\ \zeta_p &\sim \text{student-}t_{\nu_1}^+(0, 1), \quad p = 1, 2, \dots, P \\ c^2 &\sim \text{inverse-Gamma}(\nu_2, \nu_2 s^2 / 2) \\ \varepsilon &\sim \text{student-}t_{\nu_3}^+(0, \varepsilon_0),\end{aligned}$$

has the conditional mean $\mathbb{E}[\beta_p|\varepsilon, \tilde{\zeta}_p] = 0$ and variance $\mathbb{V}[\beta_p|\varepsilon, \tilde{\zeta}_p] = \varepsilon^2 \tilde{\zeta}_p^2$. To constrain the horseshoe prior to the positive real domain \mathbb{R}^+ , we can introduce an auxiliary random variable $z_p \sim \text{half-Normal}^+(0, \sigma^2)$ ($\mathbb{E}[z] = \sigma\sqrt{2/\pi}$, $\mathbb{V}[z] = \sigma^2(1 - 2/\pi)$) and parameterise the RHS variable as $\gamma_p = \varepsilon \tilde{\zeta}_p \times z_p$. When $\sigma^2 = 1$, we can easily verify that the random variable γ_p has conditional mean and variance,

$$\mathbb{E}[\gamma_p|\varepsilon, \tilde{\zeta}_p] = \sqrt{\frac{2}{\pi}} \varepsilon \tilde{\zeta}_p, \quad \mathbb{V}[\gamma_p|\varepsilon, \tilde{\zeta}_p] = \left(1 - \frac{2}{\pi}\right) \varepsilon^2 \tilde{\zeta}_p^2 \approx 0.36 \mathbb{V}[\beta_p|\varepsilon, \tilde{\zeta}_p].$$

Although the distribution has a non-zero mean, $\mathbb{E}[\gamma_p|\varepsilon, \tilde{\zeta}_p] \rightarrow 0$ as $\varepsilon \tilde{\zeta}_p \rightarrow 0$, *i.e.*, γ_p is pulled towards zero by the global shrinkage parameter ε . To ensure that the variance of the constrained parameters γ_p remain the same as the RHS prior, we can set the variance of the auxiliary random variable z to $\sigma^2 = (1 - 2/\pi)^{-1}$ which gives us,

$$\mathbb{V}[\gamma_p|\varepsilon, \tilde{\zeta}_p] = \left(1 - \frac{2}{\pi}\right)^{-1} \left(1 - \frac{2}{\pi}\right) \varepsilon^2 \tilde{\zeta}_p^2 = \varepsilon^2 \tilde{\zeta}_p^2 = \mathbb{V}[\beta_p|\varepsilon, \tilde{\zeta}_p].$$

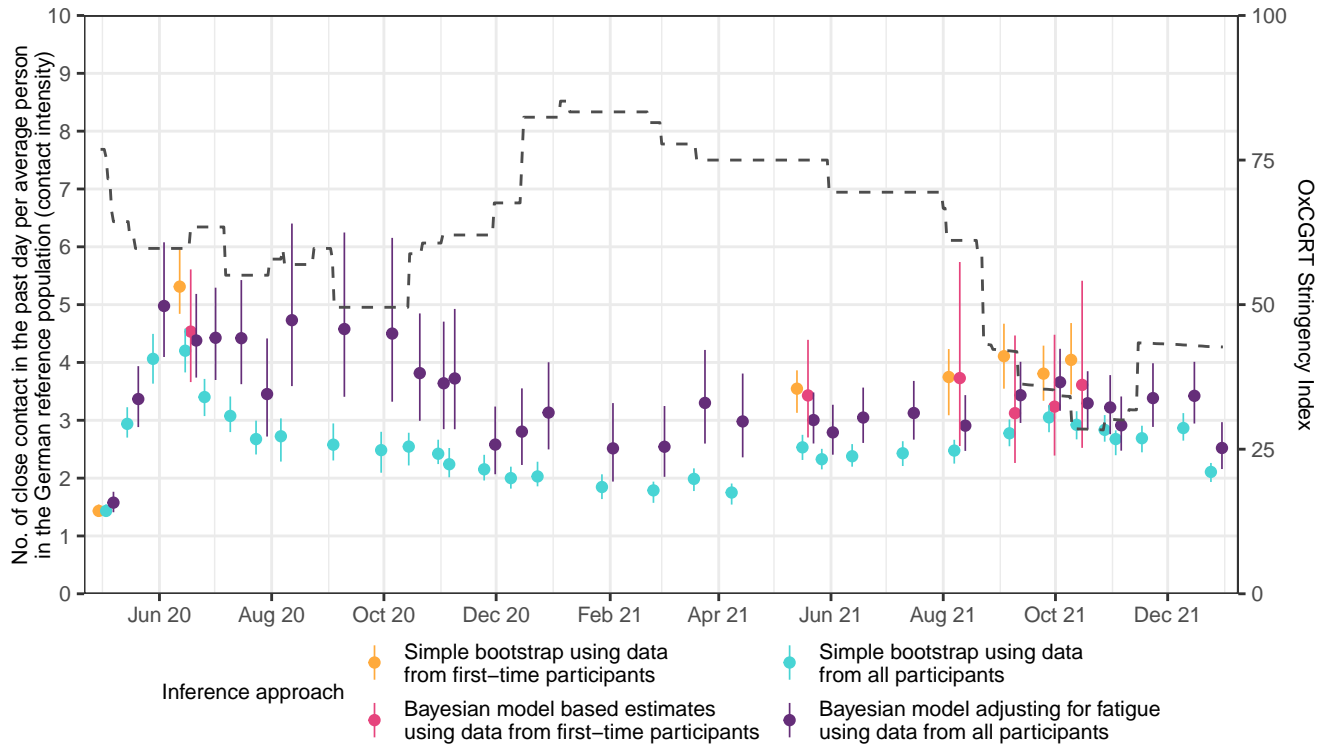


Figure S1. Comparison between simple bootstrap based and Bayesian model based estimates for longitudinal contact intensity during the COVID-19 pandemic in Germany., national-level contact intensity estimates (point: simple bootstrap mean or posterior median estimate, linerange: 95% bootstrap confidence or 95% credible intervals) are shown according to different estimation approaches: Simple bootstrap² using data from first-time participants only, for waves with more than 300 first-time participants (orange); Simple bootstrap² using data from all participants and not adjusting for reporting fatigue (blue); Bayesian model using data from first-time participants only, for waves with more than 300 first-time participants (pink); Bayesian model using data from all participants and adjusting for reporting fatigue (purple). The dashed line represents the OxCGRT Stringency Index with higher values indicating a higher degree of contact restrictions (min: 0, max: 100).

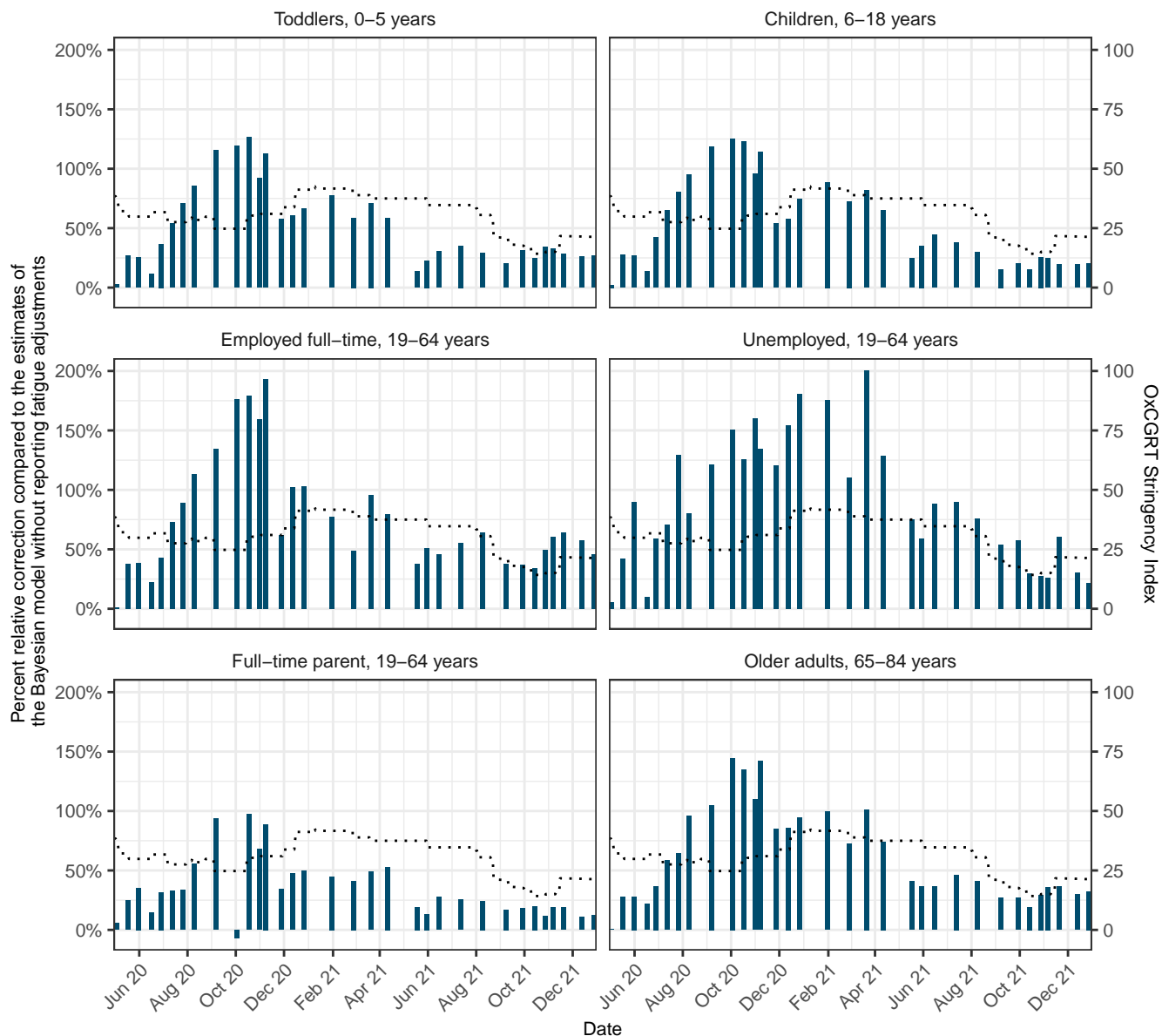


Figure S2. Percent relative correction of contact intensity estimates from the Hill model against the Bayesian model without reporting fatigue adjustments. Blue bars represent percent change in median contact intensity estimates from the Hill model relative to estimates from the Bayesian fatigue un-adjusted model. The dotted lines represents the OxCGRt Stringency Index with higher values indicating a higher degree of contact restrictions (min: 0, max: 100).

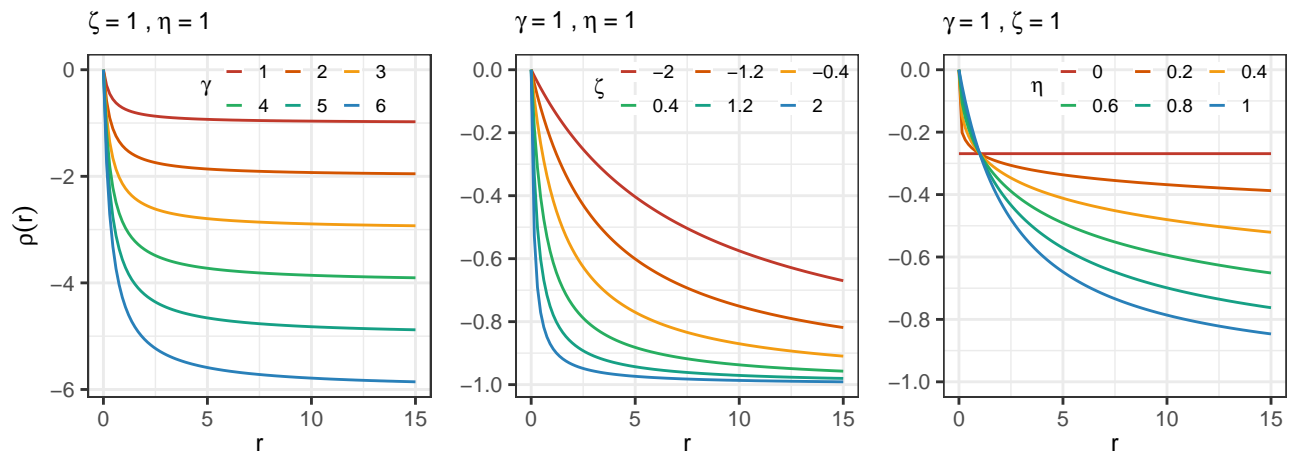


Figure S3. Functional form of the Hill function under different parameter values. Left: varying the scale parameter γ from 1 to 6 while fixing shape parameters ζ and η at 1. Centre: varying the shape parameter ζ from -2 to 2 while fixing the scale parameter γ and the second shape parameter η at 1. Right: varying the second shape parameter η from 0 to 1 while fixing the scale parameter γ and the first shape parameter ζ at 1.