Optimization of PID and optimal discrete control for non linear systems via gradient descent applied to a fish meat dehydration process

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Abstract

In this paper, an optimization method for the gains of a PID controller, and the computation of a control sequence for a linear neutral time delay system is presented. Optimization of the gains of a PID controller is proposed as a tuning method for industrial PID controllers. The computation of the control sequence is presented as a contribution to the control of linear neutral time delay systems. To exemplify the result effectiveness, experimental results are presented by applying the control sequence to a dehydrator machine using LabVIEW software.

Keywords: gradient, PID, time delay systems, neutral time delay systems, linear time delay system

1 Introduction

On the industrial applications, the most widely used control algorithm is the Proportional-Integral-Derivative (PID). This algorithm is widely used in various industrial applications because of their simple structure, ease of implementation, and robustness on uncertain conditions [7]. The diversity of the industrial control applications includes the control of variables such as humidity, temperature, pressure, flow, level, velocity, among others [16]. However, the performance of a closed loop system with a PID controller strongly depends on how well the controller is tuned, in other words, one of the main reasons for poor performance is bad tuning practice [7], that can lead to a system output with oscillations, overshoots, and other undesirable effects. Therefore, it is of the great interest to develop strategies to tune PID controllers in a relatively easy way and without spending too much time. Among the enormous quantity of methods available for PID tuning, there are some that are preferred by the control engineers, such as manual adjustment of the PID parameters using trial-and-error, Ziegler-Nichols [9], or Cohen-Coon [12]. However, it is proved in practice that these methods may not always result in an acceptable system performance [4]. To address this issue, researchers have reported interesting ways to tune the PID gains in order to obtain a better closed loop system performance, for example, methods such as a refinement of the Ziegler-Nichols tuning formulas [10], or gain and phase method [11]. In addition, the problem of tuning the controller can also be seen as an optimization problem, since in practice, it is generally required to obtain the best possible performance of the system; keeping in mind energy expenditure. This is why in recent years, some works about the optimization of the gains of PID control have been reported. Methods like Particle Swarm Optimization [13, 20, 21], Ant Colony Optimization [14], Genetic Algorithm [22] or Hill Climbing [5] are an illustrative example of PID optimization through tuning of the controller. Another example is [4] where an optimal PI controller was tuned for multi variable and scalar cases of input delayed plants applying the linear quadratic regulator (LQR) approach; in this reference an optimal PID is also proposed, obtained by applying a pole exact cancellation in a second-order plant, and setting the derivative gain to the value of this canceled pole. Another alternative to optimally tune the parameters of a PID controller is the use of the gradient descent method. In this matter previous works have been reported, such as [17] where an adaptive PID using stochastic gradient descent with momentum method was proposed to be applied to a proton exchange membrane fuel cell power system, only simulation results were given. Another example is [8] where the first order gradient descent was used to obtain optimal gain values for a PD controller and compared with the second-order gradient in order to be applied to a quadrotor system, simulation results were given. In [6] the gradient descent method was used in order to achieve the tuning of the gains of a PID controller applied on a piezoelectric actuator model and simulation results were presented.

It is worth to mention that in the above mentioned works, the cost function to be minimized depends only on the error without taking into account the energy consumption. It is an important issue in industrial processes, such as the dehydration task. In fact, according to [15], the dehydration procedure is a major energy consumer used in many industries, including agriculture, biotechnology, food, textiles, minerals, pharmaceuticals, pulp and paper, polymers, wood, and others. Consequently, the energy optimization problem is an interesting issue when a PID control is considered for this kind of systems.

In this work, the proposed methodology consists in the use of the gradient descent algorithm to minimize a cost function (which penalizes the error and the control signal) by adjusting the parameters of the PID controller. A nonlinear mathematical model of the plant is addressed. Moreover, an extra control sequence is added algebraically to the PID control signal, which optimizes even more the performance index. As is well known, the PID controller does not reach a global optimum in the LQR sense [4], so the additional signal improves the optimization of the closed loop scheme. These control laws are applied to a food dehydration process, modeled as a discrete time delayed nonlinear system. In addition to the aforementioned, following the ideas presented in [23] the food dehydration system in closed loop with a PID controler can be seen as a discrete neutral time delay system for which the gradient descent method is proposed to obtain a suboptimal control sequence.

To achieve PID tuning, it is necessary to calculate the gradient of the cost function with respect to the control parameters (gains K_p, K_i and K_d). Moreover, an initial control sequence must be known in advance. The proposal consists of two stages. The first one, the optimization of the PID gains, and then the closed loop is calculated. After that, using the closed loop of the plant, the additional control sequence in open loop is obtained. Finally, the sum of both control signals is applied in real time. For the first stage, the algorithm starts with an initial set of values of the control parameters, for example, the gains resulting from applying the Ziegler-Nichols rule [9], and then iteratively updates the control parameters by computing the cost function gradient with respect to each gain of the PID. This procedure continues until a given iteration number is reached. In the last iteration, the optimized gains of the PID are obtained. In the second stage, the computation of the gradient of the same cost function considered in the first stage is used to compute an optimized control sequence at each stage of the process (in closed loop with the optimized PID), but now the minimization is made with respect to the control sequence. The procedure continues until a specified number of iterations is reached. Please note that in this stage, when this control sequence is implemented, the dimensionality issue [19] could appear for a relatively large finite time horizon and/or for multivariable systems. However, some advantages of the proposal are that it just needs gains re-tuning when the PID is optimized and applied to discrete nonlinear system. Notice that, to the best of our knowledge, the methodology for tuning PID controllers for nonlinear systems is incipient. Another advantage is that no significant hardware changes are needed. Since the additional control is calculated offline and in open loop, it is simple to implement online with a low-cost digital device.

The control algorithm is implemented using the LabVIEW programming language on an industrial controller cDAQ-9132 which remotely monitors a cDAQ-9184 of National Instruments. The selected industrial controller has industry standard certifications, is embedded, robust, dependable, and has high performance. It is connected to the Ethernet network via the remote chassis cDAQ-9184 and is equipped with high-precision I/O modules, NI9211 and NI9263. It is important to mention that this paper gives experimental evidence of the viability of programming advanced control in an industrial controller. It represents an effort to bridge the knowledge gap between theoretical outcomes and their potential application to industrial operations.

The system over which the control is applied is an atmospheric dehydration system for fish meat (modeled as a discrete scalar nonlinear system with input delay and in closed loop with a PID controller as a neutral type time delay system), whose actuator is an electric grid that demands a lot of current, hence the importance to apply control techniques that not only optimize the system response but also energy consumption.

In order to give experimental evidence of the feasibility of the proposed control scheme, the synthesized control laws are tested to control the temperature of a dehydration fish meat process.

The main contributions of the paper are summarized as follows:

• The extension of the gradient descent method to the case of nonlinear time delay systems (pointwise input and state delays). It allows to compute optimized gains for a PID controller, when a quadratic cost function penalizes both convergence and control signal.

• The real time application of the gradient descent algorithm for nonlinear discrete time delay systems.

• The application of optimized control parameters in an industrial PID. It represents a potential impact on the industry because the use or implementation of new hardware in the control loop is not necessary.

• The numerical optimization of discretized neutral time delay systems. Please notice that this optimization problem in general remains open.

This paper is organized as follows: In Section 2 some preliminary results are presented, then in Section 3 the problem formulation is described and the main result is presented. The experimental results are reported in Section 4, and finally Section 5 presents concluding remarks.

2 Preliminaries

This section presents the previous results, useful for the optimization of delay free discrete systems [1].

Consider a discrete system of the form:

$$\overline{x}(i+1) = F\left(\overline{x}\left(i\right), \overline{u}\left(i\right), \overline{\alpha}\right), \ i = 0, 1, .., N-1 \tag{1}$$

where $F = \begin{bmatrix} F_1 & F_2 & ,..., & F_n \end{bmatrix}^T$. Define the performance index as

$$J = \phi\left(\overline{x}(N), \overline{\alpha}\right) + \sum_{i=0}^{N-1} L\left(\overline{x}(i), \overline{u}\left(i\right), \overline{\alpha}\right) \tag{2}$$

where L and ϕ are scalar functions, for some $N \in \mathbb{R}$, $\overline{x} \in \mathbb{R}^n$, $\overline{u} \in \mathbb{R}^r$, $\overline{\alpha} \in \mathbb{R}^m$

The classical optimization problem consists to find the sequence of controls $\overline{u}(i), i = 0, 1, .., N-1$, and the control parameters $\overline{\alpha}$ which minimize the performance index given by (2) subject to the trajectories of the plant (1).

In this paper, the control parameters $\overline{\alpha}$ represent the gains of a discrete PID controller, when the discrete system given by (1) presents deviations in its arguments.

The principle of Causality. The state and the control parameter $\overline{\alpha}$ at the kth stage, together with the sequence of controls $\overline{u}(k, r-1) = [\overline{u}(k), \overline{u}(k+1), ..., \overline{u}(r-1)]$ uniquely determine the state of the rth stage $\overline{x}(r)$. Thus the initial state $\overline{x}(0)$, control parameters $\overline{\alpha}$, and the control sequence $\overline{u}\,(0, N-1)$ uniquely determine the trajectory $\overline{x}\,(1, N) = [\overline{x}(1), \overline{x}(2), ..., \overline{x}(N)]\,.$

The principle of optimality. If the control sequence $\overline{u}(0, k-1)$ and the control parameter $\overline{\alpha}$ have been chosen in an optimal manner. Then due to the principle of causality, the trajectory $\overline{x}(0, k)$ is also determined. The performance index J may be written as $J = J_1 + J_2$ where

$$J_1 = \sum_{i=0}^{k-1} L(\overline{x}(i), \overline{\alpha}, \overline{u}(i))$$

$$J_2 = \sum_{k}^{N-1} \left\{ L(\overline{x}(i), \overline{\alpha}, \overline{u}(i)) + \phi(\overline{x}(N), \overline{\alpha}) \right\}$$

The first term has been determined from the assumption that the optimal control sequence and parameters are known. Therefore, optimizing the performance index J is necessary and sufficient to determine the sequence $\overline{u}(k, N-1)$ to minimize J_2. This was summarized by Bellman as the principle of optimality [19].

The performance index J can be rewritten as a function of $\overline{x}(k)$ and $\overline{u}\,[k, N-1]$ as follows:

$$J = V\left(\overline{x}(k), \overline{u}\,[k, N-1]\right),$$

that corresponds to the control sequence $\overline{u}\,[k, N-1]$, if the control sequence is selected as the optimal control sequence, the return function becomes the optimal return function, required for the dynamic programming solution.

The gradient method is motivated by a consideration of a first order expansion of the return function around a nominal control sequence and control parameter as:

$$V\left(\overline{x}(0), \overline{u}^{j+1}\,[0, N-1]\,, \overline{\alpha}^{j+1}\right) = V\left(\overline{x}(0), \overline{u}^{j}\,[0, N-1]\,, \overline{\alpha}^{j}\right) + \frac{\partial V\left(\overline{x}(0), \overline{u}^{j}\,[0, N-1]\,, \overline{\alpha}^{j}\right)}{\partial \overline{\alpha}} \delta\overline{\alpha}$$
$$+ \frac{\partial V\left(\overline{x}(0), \overline{u}^{j}\,[0, N-1]\,, \overline{\alpha}^{j}\right)}{\partial \overline{u}(k)} \delta\overline{u}\,[0, N-1]$$

where the variations $\delta\overline{\alpha} = \overline{\alpha}^{j+1} - \overline{\alpha}^{j}$ and $\delta\overline{u} = \overline{u}^{j+1} - \overline{u}^{j}$ are small enough to ensure the validity of the expansion. If these variations are chosen as:

$$\delta\overline{u} = -\varepsilon \left[\frac{\partial V\left(\overline{x}(0), \overline{u}^{j}\left[0, N-1\right], \overline{\alpha}^{j}\right)}{\partial \overline{u}[0, N-1]} \right]^{T}$$

and

$$\delta\overline{\alpha} = -\varepsilon \left[\frac{\partial V\left(\overline{x}(0), \overline{u}^{j}\left[0, N-1\right], \overline{\alpha}^{j}\right)}{\partial \overline{\alpha}} \right]^{T}$$

with $\varepsilon > 0$. Therefore, it is clear that the return function evaluated with the new control sequence and new control parameters must be minimized at each step of the optimization algorithm.

3 Problem statement and main results

Consider a discrete time delay system with state and input delay of the form

$$x(i+1) = F\left(x\left(i\right), x\left(i-M_1\right)\right) + G\left(x\left(i\right), x\left(i-M_1\right)\right) u\left(i-M_2\right), i = 0, 1, ..., N-1 \tag{3}$$

where N represents the number of the system stages, M_1 represents a discrete delay in the state, M_2 represents a discrete delay in the input and $N, M_1, M_2 \in \mathbb{N}$, satisfying $N > M_1, M_2$, the initial condition of the state is $\varphi\left[-M_1, 0\right] = \left[\varphi\left(-M_1\right), \varphi\left(1-M_1\right), ..., \varphi\left(-1\right), \varphi\left(0\right)\right]$, and the initial condition for the control is $\psi\left[-M_2, 0\right] = \left[\psi\left(-M_2\right), \psi\left(1-M_2\right), ..., \psi\left(-1\right), \psi\left(0\right)\right]$, and $x(i)$, $F\left(x\left(i\right), x\left(i-M_1\right)\right) \in \mathbb{R}^n$, $u\left(i\right) \in \mathbb{R}^m$, $G\left(x\left(i\right), x\left(i-M_1\right)\right) \in \mathbb{R}^{n\times m}$, with sampling time T_s and the discrete time is $t = iT_s$.

Let the following performance index

$$J = \phi\left(x(N), \alpha\right) + \sum_{i=0}^{N-1} L\left(x\left(i\right), u\left(i\right), \alpha\right), \tag{4}$$

where L and ϕ are scalar functions, $J > 0$.

Now, consider the following assumption:

Assumption 1. *Assume that the state $x(k)$, $x(k-M_1)$ on the kth and $(k-M_1)$ th stages together with the control sequence $u\left[k, r-1\right] = \left[u\left(k\right), .., u\left(r-1\right)\right]$ and the control parameters α uniquely determine the state in the rth stage $x(r)$. So, the initial states $x(-M_1)$, $x(1-M_1)$, $x(2-M_1), ..., x(-1)$, $x(0)$ and the control sequence $u\left[0, N-1\right]$ uniquely determine the state in the trajectory*

$$x\left[1, N\right] = \left[x\left(1\right), .., x(M_1), ..., x\left(N\right)\right].$$

Problem. To find the vector of parameters α and the control sequence $u(i - M_2)$ such that the performance index J given by (4) reaches a minimum, subject to the trajectories of system (3).

To address this issue, the following Theorem is proposed.
Theorem 1. *The performance index (4) subject to the system (3) reaches a local minimum with the following control sequence:*

$$u^{j+1}(k) = u^j(k) - \bar{\varepsilon}\left[\frac{\partial V\left(\varphi\left[-M_1, 0\right], u^j\left[0, N-1\right], \alpha^j\right)}{\partial u^j(k)}\right]^T,$$

where $\bar{\varepsilon} > 0$, and the control parameters computed at the initial stage are given by

$$\alpha^{j+1} = \alpha^j - \varepsilon\left[\frac{\partial V\left(\varphi\left[-M_1, 0\right], u^j\left[0, N-1\right], \alpha^j\right)}{\partial \alpha}\right]^T,$$

where $\varepsilon > 0$, and j is the step of the gradient descent optimization algorithm.

Proof. As in the free delay case, it is needed to find the control sequence $u(i)$, $i = 0, .., N-1$, and the vector of parameters α such that the performance index J given by (4) is minimized subject to the trajectory given by dynamic equations (3). As a consequence of the Assumption 1, the performance index J can be rewritten as a function of $\varphi\left[-M_1, 0\right]$ and $u\left[0, N-1\right]$ as follows:

$$J = V\left(\varphi\left[-M_1, 0\right], u\left[0, N-1\right]\right),$$

it means that if the sequence $\varphi\left[-M_1, 0\right]$ is given, it is only necessary to determine $u\left[0, N-1\right]$ to minimize the performance index J. The function $V\left(\varphi\left[-M_1, 0\right], u\left[0, N-1\right]\right)$ is called *return function*.

Define the optimal return function $V^{opt}\left(x\left(k\right), k\right)$ as the numerical optimal value of performance index when the optimal control sequence is substituted in the cost:

$$V^{opt}\left(x\left(k\right), k\right) = V\left(x\left(k\right), u^{opt}\left[k, N-1\right], k\right).$$

Let be $u\left(x\left(i\right), i\right)$ (we assume that the control explicitly depends of the discrete time), $i = k, ..., N-1$, an arbitrary control law and $V(x\left(k\right), k)$ its return function, so:

$$V\left(x\left(k\right), k\right) = \phi\left(x(N)\right) + \sum_{i=k}^{N-1} L\left(x\left(i\right), u\left(x(i), i\right)\right),$$

where $x\left(k, N\right)$ is chosen such that

$$x\left(i+1\right) = F\left(x\left(i\right), x\left(i-M_1\right)\right) + G\left(x\left(i\right), x\left(i-M_1\right)\right) u\left(x\left(i\right), i\right).$$

$$i = k, k+1, ..., N-1.$$

Now, according with the definition of the return function, it follows that

$$V(x(k), k) = L(x(k), u(x(k), k)) + V(x(k+1), k+1),$$
$$\text{with } V(x(N), N) = \phi(x(N)).$$

These concepts are useful in order to extend the gradient algorithm for time delay systems.

As in the case of free delay systems [1], the gradient algorithm can be used to obtain an approximation of the optimal control sequence $u^{opt}[0, N-1]$ and control parameters α. The expansion in a Taylor series, considering a first order approximation, of the return function $V(\varphi[-M_1, 0], u[0, N-1])$ around of a nominal control sequence $u^j[0, N-1]$ and parameters α^j, where superindex j is the iteration of the gradient algorithm:

$$V\left(\varphi[-M_1,0], u^{j+1}[0,N-1], \alpha^{j+1}\right) = V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)$$
$$+\frac{\partial V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)}{\partial \alpha}\delta\alpha + \frac{\partial V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)}{\partial u(k)}\delta u[0,N-1]$$

where

$$\frac{\partial V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)}{\partial \alpha} = \left.\frac{\partial V\left(\varphi[-M_1,0], u[0,N-1], \bar{\alpha}\right)}{\partial \alpha}\right|_{\substack{\alpha=\alpha^j \\ u=u^j}}$$
$$\delta\alpha = \alpha^{j+1} - \alpha^j$$
$$\delta u[0,N-1] = u^{j+1} - u^j$$

The differences $\|\delta\alpha\|$ and $\|\delta u[0, N-1]\|$ has to be sufficiently small in order to guarantee the convergence of the series. Now, choose the difference $\delta\alpha$ as

$$\delta\alpha = -\varepsilon\left[\frac{\partial V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)}{\partial \alpha}\right]^T$$

and the difference $\delta u[0, N-1]$ as

$$\delta u[0,N-1] = -\bar{\varepsilon}\left[\frac{\partial V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)}{\partial u[0,N-1]}\right]^T$$

for some $\varepsilon, \bar{\varepsilon} > 0$ and the return function is $V\left(\varphi[-M_1,0], u^{j+1}[0,N-1], \alpha^{j+1}\right)$, and it is clear that is smaller than $V\left(\varphi[-M_1,0], u^j[0,N-1], \alpha^j\right)$.

According with Assumption 1, a changue on $u(k)$ will not affect the value of $L\left(x\left(i\right),u\left(i\right)\right)$ for $i<k$. So, the following relation is satisfied

$$\frac{\partial V\left(\varphi\left[-M_1,0\right],u\left[0,N-1\right]\right)}{\partial u(k)}=\frac{\partial V\left(x\left(k\right),u\left[k,N-1\right]\right)}{\partial u(k)},$$

Then, the return function can be rewritten as:

$$V\left(x\left(k\right),u\left[k,N-1\right]\right)=L\left(x(k),u\left(k\right)\right)+V\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)$$

Now, the derivative of $V\left(x\left(k\right),u\left[k,N-1\right]\right)$ respect to α is

$$\frac{\partial V\left(x\left(k\right),u\left[k,N-1\right]\right)}{\partial\alpha}=\frac{\partial L\left(x(k),u\left(k\right)\right)}{\partial\alpha}+\frac{\partial V\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)}{\partial x\left(k+1\right)}\frac{\partial F\left(x\left(k\right)\right)}{\partial\alpha}$$
$$+\frac{\partial V\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)}{\partial\alpha},$$

and the derivative of $V\left(x\left(k\right),u\left[k,N-1\right]\right)$ respect to $u(k)$ is

$$\frac{\partial V\left(x\left(k\right),u\left[k,N-1\right]\right)}{\partial u(k)}=\frac{\partial L\left(x(k),u\left(k\right)\right)}{\partial u(k)}+\frac{\partial V\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)}{\partial x\left(k+1\right)}\frac{\partial F\left(x(k)\right)}{\partial u(k)}$$

In order to obtain $V_x\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)$ for $k=0,1,...,N-1$ consider the following

$$\frac{\partial V\left(x\left(N\right)\right)}{\partial x\left(N\right)}=\frac{\partial\phi\left(x(N)\right)}{\partial x\left(N\right)}$$
$$\frac{\partial V\left(x\left(k\right),u\left[k,N-1\right]\right)}{\partial x\left(k\right)}=\frac{\partial L\left(x(k),u\left(k\right)\right)}{\partial x\left(k\right)}+\frac{\partial V\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)}{\partial x\left(k+1\right)}\frac{\partial F\left(x\left(k\right)\right)}{\partial x\left(k\right)}$$

□

3.1 Algorithm summary

In this section, we summarize the numerical algorithm for discrete nonlinear time delay systems.

1. Propose a nominal control sequence $u^j\left[0,N-1\right]$. Build and storage the nominal trajectory $x\left[0,N\right]$ with the state equation

$$x(i+1)=F\left(x\left(i\right),x\left(i-M_1\right)\right)+G\left(x\left(i\right),x\left(i-M_1\right)\right)u\left(i-M_2\right),i=0,1,...,N-1$$

Calculate the numerical value of the cost J :

$$J = \phi(N) + \sum_{i=0}^{N-1} L(x(i), u(i), \alpha).$$

2. Calculate the partial derivatives respect to the control parameters and the control sequence.
3. Calculate the new values of the control parameters and control sequence.
4. Calculate the new trajectory of the discrete system with the control parameters and control sequence obtained of the previous step.
5. Verify that the new numerical value of the performance index is less than of the previous value of J. If not, reduce the value of ε.
6. Repeat from the step 1, until a fixed number of iterations is reached.

Figure 1 shows the flow diagram for the implementation of the numerical algorithm described in the algorithm.

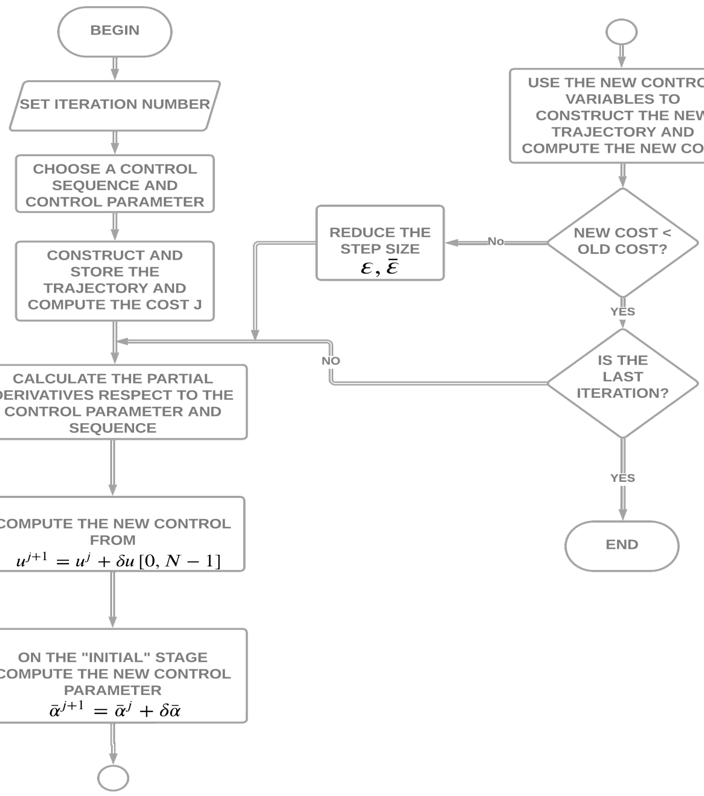

Fig. 1 Flow diagram of the optimization method.

3.2 Optimized control parameters for dehydration process

As presented in [2]. Consider the discrete scalar system:

$$x\left(i+1\right)=a_0x\left(i\right)+\gamma_0x^2\left(i\right)+\gamma_1x^3\left(i\right)+\beta u\left(i-M\right) \tag{5}$$

Assume that a nominal control $u\left[0,N-1\right]$ is given; in this case, it is given by the PID algorithm proposed in [16]

$$u(i)=u(i-1)+k_p\left\{\left[1+\frac{T_d}{T_s}+\frac{T_s}{T_i}\right]e(i)-\left[1+2\frac{T_d}{T_s}\right]e(i-1)+\frac{T_d}{T_s}e(i-2)\right\}$$

where k_p is the proportional gain T_d is the derivative time and T_i is the integral time of the PID controller. Now consider $e(t)=SP-x(t)$, where SP is fixed, if $SP=0$, then $e(t)=-x(t)$. So, the control equation can be rewritten as:

$$u(i)=u(i-1)-x(i)\alpha_1+x(i-1)\alpha_2-x(i-2)\alpha_3,$$

and in a compact form as:

$$u(i)=u(i-1)+\overline{\chi}(i)\overline{\alpha}. \tag{6}$$

Where $\overline{\chi}(k)=[-x(i),x(i-1),-x(i-2)]$ and $\overline{\alpha}=[\alpha_1,\alpha_2,\alpha_3]^T$. So the PID equation is parameterized with $\alpha_1=kp+\frac{k_d}{T_s}+k_iT_s$, $\alpha_2=k_p+2\frac{k_d}{T_s}$, $\alpha_3=\frac{k_d}{T_s}$.

The closed loop system equation of (5) with control (6) is

$$\begin{aligned}x\left(i+1\right)&=a_0x\left(i\right)+\gamma_0x^2\left(i\right)+\gamma_1x^3\left(i\right)+\beta u\left(i-1-M\right)+\beta\overline{\chi}(i-M)\overline{\alpha}\\&=f\left(x\left(i\right)\right)+\beta u\left(i-M\right),\end{aligned} \tag{7}$$

where $i=0,1,...,N$, $f\left(x\left(i\right),x\left(i-M\right)\right)=a_0x\left(i\right)+\gamma_0x^2\left(i\right)+\gamma_1x^3\left(i\right)+\beta\overline{\chi}(i-M)\overline{\alpha}$, with initial conditions $x\left(i-M\right)=\varphi=T_{env}$, and $\varphi\left[-M,0\right]=\left[\varphi\left(-M\right),\varphi\left(1-M\right),...,\varphi\left(-1\right),\varphi\left(0\right)\right]$, $x(k)=\varphi\left(k\right),k=-M,1-M,...,-1,0.$ $N>M$. And for the control $u(i-1-M)=\psi=0$, $\psi\left[-M-1,0\right]=\left[\psi\left(-M-1\right),\psi\left(-M\right),...,\psi\left(-1\right),\psi\left(0\right)\right]$.

It is needed to find the optimal values for the control parameters which minimize the performance index:

$$J=\frac{1}{2}x\left(N\right)Hx\left(N\right)+\frac{1}{2}\sum_{i=0}^{N-1}\left\{x\left(i\right)Qx\left(i\right)+u\left(i\right)Ru\left(i\right)\right\}, \tag{8}$$

with $H,Q\geq 0$ and $R>0$, and $T=NT_s$, for some integer N.

Proposition 2. *Consider the system given by equation (7) and the quadratic performance index given by (8). The performance index (8) subject to (7) reaches a local minimum if the control parameters $\overline{\alpha}$ are optimized as*

$$\overline{\alpha}^{j+1} = \overline{\alpha}^{j} - \varepsilon \left[\frac{\partial V\left(\varphi\left[-M,0\right], u^{j}\left[0,N-1\right], \overline{\alpha}^{j}\right)}{\partial \overline{\alpha}} \right]^{T}, \varepsilon > 0$$

where j is the step of the gradient algorithm.

Proof. For the proof of Proposition 2, the parameter optimization part of Theorem 1 is used. In this case, the return function is

$$V\left(x\left(k\right), \overline{\alpha}, k\right) = \frac{1}{2} x\left(N\right) H x\left(N\right) + \frac{1}{2} \sum_{i=k}^{N-1} \left\{ x\left(i\right) Q x\left(i\right) + u\left(i\right) R u\left(i\right) \right\},$$

then

$$\begin{aligned} V\left(x\left(k\right), \overline{\alpha}, k\right) &= \frac{1}{2}\left\{ x\left(k\right) Q x\left(k\right) + u\left(k\right) R u\left(k\right) \right\} + V\left(x\left(k+1\right), k+1\right), \\ \text{with } V\left(x\left(N\right), N\right) &= \frac{1}{2} x\left(N\right) H x\left(N\right). \end{aligned}$$

With the control sequence $u\left[0,N-1\right]$, it is possible to construct the trajectory $x\left[0,N\right]$.

According with the algorithm exposed in the proof of Theorem 1, follows that:

$$\begin{aligned} \frac{\partial V\left(x\left(N\right), N\right)}{\partial x(N)} &= \frac{\partial}{\partial x(N)} \left[\frac{1}{2} x\left(N\right) H x\left(N\right) \right] = H x\left(N\right) \\ \frac{\partial V\left(x\left(k\right), k\right)}{\partial x(k)} &= \frac{\partial L\left(x(k), u\left(k\right)\right)}{\partial x\left(k\right)} + \frac{\partial V\left(x\left(k+1\right), k+1\right)}{\partial x\left(k+1\right)} \frac{\partial\left[F\left(x\left(k\right)\right)\right]}{\partial x\left(k\right)} \end{aligned}$$

in a similar way

$$\begin{aligned} &\frac{\partial V\left(x\left(N\right), N\right)}{\partial \overline{\alpha}} = \frac{\partial}{\partial \overline{\alpha}} \left[\frac{1}{2} x\left(N\right) H x\left(N\right) \right] \\ &= \frac{\partial}{\partial \overline{\alpha}} \{ [a_0 x\left(N-1\right) + \gamma_0 x^2\left(N-1\right) + \gamma_1 x^3\left(N-1\right) + \beta u\left(N-2-M\right) \\ &\quad + \beta \overline{\chi}(N-1-M)\overline{\alpha}] H [a_0 x\left(N-1\right) + \gamma_0 x^2\left(N-1\right) + \gamma_1 x^3\left(N-1\right) \\ &\quad + \beta u\left(N-2-M\right) + \beta \overline{\chi}(N-1-M)\overline{\alpha}] \} \\ &= a_0 x\left(N-1\right) H \beta \overline{\chi}^{T}\left(N-1-M\right) + \gamma_0 x^2\left(N-1\right) H \beta \overline{\chi}^{T}\left(N-1-M\right) ... \end{aligned}$$

$$+\gamma_1 x^3 (N-1) H\beta\overline{\chi}^T (N-1-M) + \beta u (N-2-M) H\beta\overline{\chi}^T (N-1-M) ...$$
$$+\beta\overline{\chi}^T (N-1-M) H\beta\overline{\chi} (N-1-M) \overline{\alpha}$$

and the derivative of the return function respect to the control parameters is

$$\frac{\partial V(x(k),k)}{\partial\overline{\alpha}} = \frac{\partial L(x(k),u(k))}{\partial\overline{\alpha}} + \frac{\partial V(x(k+1),k+1)}{\partial x(k+1)} \frac{\partial [F(x(k))]}{\partial x(k)} + \frac{\partial V(x(k+1),k+1)}{\partial\overline{\alpha}}$$

where

$$\begin{aligned}
\frac{\partial L(x(k),u(k))}{\partial\overline{\alpha}} &= \frac{1}{2}\{\frac{\partial}{\partial\overline{\alpha}}[[a_0 x(k-1) + \gamma_0 x^2 (k-1) + \gamma_1 x^3 (k-1) + \beta u (k-2-M) ... \\
&+\beta\overline{\chi}(k-1-M)\overline{\alpha}]Q[a_0 x(k-1) + \gamma_0 x^2 (k-1) + \gamma_1 x^3 (k-1) + \beta u (k-2-M) ... \\
&+\beta\overline{\chi}(k-1-M)\overline{\alpha}] + [u(k-1) + \overline{\chi}(k)\overline{\alpha}] R [u(k-1) + \overline{\chi}(k)\overline{\alpha}]]\} \\
&= a_0 x(k-1) Q\beta\overline{\chi}^T (k-1-M) + \gamma_0 x^2 (k-1) Q\beta\overline{\chi}^T (k-1-M) + \gamma_1 x^3 (k-1) Q\beta\overline{\chi}^T (k-1-M) ... \\
&+\beta u (k-2-M) Q\beta\overline{\chi}^T (k-1-M) + \beta\overline{\chi}^T (k-1-M) Q\beta\overline{\chi}(k-1-M)\overline{\alpha} + u(k-1) R\overline{\chi}^T (k) ... \\
&+\overline{\chi}^T (k) R\overline{\chi}(k)\overline{\alpha}
\end{aligned}$$

and

$$\begin{aligned}
F_{\overline{\alpha}}(k) &= \frac{\partial}{\partial\overline{\alpha}}\{a_0 x(k-1) + \gamma_0 x^2 (k-1) + \gamma_1 x^3 (k-1) + \beta u (k-2-M) + \beta\overline{\chi}(k-1-M)\overline{\alpha}\} \\
&= \beta\overline{\chi}^T (k-1-M)
\end{aligned}$$

with the above equations, it is possible to compute the gradient of the return function respect to the control parameters $\overline{\alpha}$

$$V_{\overline{\alpha}}(k) = \frac{\partial V(x(k), u[k,N-1], \overline{\alpha})}{\partial\overline{\alpha}},$$

as a backward sequence as follows:

if $k = N$

$$\begin{aligned}
V_{\overline{\alpha}}(N) &= a_0 x(N-1) H\beta\overline{\chi}^T (N-1-M) + \gamma_0 x^2 (N-1) H\beta\overline{\chi}^T (N-1-M) ... \\
&+\gamma_1 x^3 (N-1) H\beta\overline{\chi}^T (N-1-M) + \beta u (N-2-M) H\beta\overline{\chi}^T (N-1-M) ... \\
&+\beta\overline{\chi}^T (N-1-M) H\beta\overline{\chi}(N-1-M)\overline{\alpha}
\end{aligned}$$

and

$$V_x(N) = \frac{\partial V(x(N),N)}{\partial x(N)} = \frac{\partial}{\partial x(N)}\left[\frac{1}{2} x(N) H x(N)\right] = H x(N)$$

if $k = N-1$

$$
\begin{aligned}
V_{\overline{\alpha}}(N-1) &= L_{\overline{\alpha}}(N-1) + V_x(N)F_{\overline{\alpha}}(N-1) + V_{\overline{\alpha}}(N) \\
&= a_0 x\left(N-2\right)Q\beta\overline{\chi}^T\left(N-2-M\right) + \gamma_0 x^2\left(N-2\right)Q\beta\overline{\chi}^T\left(N-2-M\right)... \\
&\quad + \gamma_1 x^3\left(N-2\right)Q\beta\overline{\chi}^T\left(N-2-M\right) + \beta u\left(N-3-M\right)Q\beta\overline{\chi}^T\left(N-2-M\right)... \\
&\quad + \beta\overline{\chi}^T\left(N-2-M\right)Q\beta\overline{\chi}\left(N-2-M\right)\overline{\alpha} \\
&\quad + u(N-2)R\overline{\chi}^T(N-1) + \overline{\chi}^T(N-1)R\overline{\chi}(N-1)\overline{\alpha}... \\
&\quad + Hx\left(N\right)\beta\overline{\chi}^T\left(N-2-M\right) + a_0 x\left(N-1\right)H\beta\overline{\chi}^T(N-1-M)... \\
&\quad + \gamma_0 x^2\left(N-1\right)H\beta\overline{\chi}^T(N-1-M) + \gamma_1 x^3\left(N-1\right)H\beta\overline{\chi}^T(N-1-M)... \\
&\quad + \beta u\left(N-2-M\right)H\beta\overline{\chi}^T\left(N-1-M\right) + \beta\overline{\chi}^T\left(N-1-M\right)H\beta\overline{\chi}\left(N-1-M\right)\overline{\alpha}
\end{aligned}
$$

and

$$
\begin{aligned}
V_x(N-1) &= V_x(N)F_x(N-1) + L_x(N-1) \\
&= Hx\left(N\right)\left[a_0 + 2\gamma_0 x\left(N-1\right) + 3\gamma_1 x^2\left(N-1\right)\right] + 2x\left(N-1\right)Q
\end{aligned}
$$

if $k = N-2$

$$
\begin{aligned}
V_{\overline{\alpha}}(N-2) &= L_{\overline{\alpha}}(N-2) + V_x(N-1)F_{\overline{\alpha}}(N-2) + V_{\overline{\alpha}}(N-1) \\
&= a_0 x\left(N-3\right)Q\beta\overline{\chi}^T\left(N-3-M\right)... \\
&\quad + \gamma_0 x^2\left(N-3\right)Q\beta\overline{\chi}^T\left(N-3-M\right)... \\
&\quad + \gamma_1 x^3\left(N-3\right)Q\beta\overline{\chi}^T\left(N-3-M\right)... \\
&\quad + \beta u\left(N-4-M\right)Q\beta\overline{\chi}^T\left(N-3-M\right)... \\
&\quad + \beta\overline{\chi}^T\left(N-3-M\right)Q\beta\overline{\chi}\left(N-3-M\right)\overline{\alpha}... \\
&\quad + u(N-3)R\overline{\chi}^T(N-2) + \overline{\chi}^T(N-2)R\overline{\chi}(N-2)\overline{\alpha}... \\
&\quad + [Hx\left(N\right)\left[a_0 + 2\gamma_0 x\left(k-1\right) + 3\gamma_1 x^2\left(k-1\right)\right]... \\
&\quad + 2x\left(N-1\right)]\beta\overline{\chi}^T\left(N-3-M\right) + a_0 x\left(N-2\right)Q\beta\overline{\chi}^T\left(N-2-M\right)... \\
&\quad + \gamma_0 x^2\left(N-2\right)Q\beta\overline{\chi}^T\left(N-2-M\right) + \gamma_1 x^3\left(N-2\right)Q\beta\overline{\chi}^T\left(N-2-M\right)... \\
&\quad + \beta u\left(N-3-M\right)Q\beta\overline{\chi}^T\left(N-2-M\right) + \beta\overline{\chi}^T\left(N-2-M\right)Q\beta\overline{\chi}\left(N-2-M\right)\overline{\alpha}... \\
&\quad + u(N-2)R\overline{\chi}^T(N-1) + \overline{\chi}^T(N-1)R\overline{\chi}(N-1)\overline{\alpha}... \\
&\quad + Hx\left(N\right)\beta\overline{\chi}^T\left(N-2-M\right) + a_0 x\left(N-1\right)H\beta\overline{\chi}^T(N-1-M)... \\
&\quad + \gamma_0 x^2\left(N-1\right)H\beta\overline{\chi}^T(N-1-M) + \gamma_1 x^3\left(N-1\right)H\beta\overline{\chi}^T(N-1-M)... \\
&\quad + \beta u\left(N-2-M\right)H\beta\overline{\chi}^T\left(N-1-M\right) + \beta\overline{\chi}^T\left(N-1-M\right)H\beta\overline{\chi}\left(N-1-M\right)\overline{\alpha}
\end{aligned}
$$

and

$$
\begin{aligned}
V_x(N-2) &= V_x(N-1)F_x(N-2) + L_x(N-2) \\
&= [Hx(N)[a_0 + 2\gamma_0 x(N-1) + 3\gamma_1 x^2(N-1)] + 2x(N-1)Q] \times [a_0 + 2\gamma_0 x(N-2) \ldots \\
&\quad + 3\gamma_1 x^2(N-2)] + 2x(N-2)Q
\end{aligned}
$$

the sequence continues until $k = 0$, and in that stage the new control parameters are calculated as mentioned before as

$$
\begin{aligned}
\overline{\alpha}^{j+1} &= \overline{\alpha}^{j} - \varepsilon \left[\frac{\partial V\left(\varphi[-M,0], u^j[0,N-1], \overline{\alpha}^j\right)}{\partial \overline{\alpha}} \right]^T \\
&= \overline{\alpha}^{j} - \varepsilon V_{\overline{\alpha}}(0)^T, \varepsilon > 0
\end{aligned}
$$

Remark 1. *It is clear that the return function decreases on value at each step of the optimization algorithm with the election of the new control parameters. This local optimization can be seen considering the first order approximation of the return function* $V(\varphi[-M,0], u[0,N-1])$ *around the control parameters* $\overline{\alpha}^j$,

$$
\begin{aligned}
& V\left(\varphi[-M,0], u[0,N-1], \overline{\alpha}^{j+1}\right) \\
= & V\left(\varphi[-M,0], u[0,N-1], \overline{\alpha}^{j}\right) + \frac{\partial V\left(\varphi[-M,0], u[0,N-1], \overline{\alpha}^j\right)}{\partial \overline{\alpha}} \delta\overline{\alpha}
\end{aligned}
$$

with the difference $\delta\overline{\alpha} = \overline{\alpha}^{j+1} - \overline{\alpha}^{j}$ *selected as*

$$
\delta\overline{\alpha} = -\varepsilon \left[\frac{\partial V\left(\varphi[-M,0], u^j[0,N-1], \overline{\alpha}^j\right)}{\partial \overline{\alpha}} \right]^T
$$

for some $\varepsilon > 0$.

□

3.3 Optimal control sequence for dehydration process

Proposition 3. *Consider the system given by Equation (7) and the quadratic performance index given by (8). A suboptimal sequence* $u[0,N-1]$ *which causes that the performance index (8) subject to (7) reaches a local minimum is computed as:*

$$
u^{j+1}(k) = u^j(k) - \bar{\varepsilon} \left[\frac{\partial V\left(\varphi[-M,0], u^j[0,N-1], \overline{\alpha}^j\right)}{\partial u^j(k)} \right]^T, \bar{\varepsilon} > 0
$$

where j *is the step of the gradient algorithm.*

Proof. For the computation of the optimized control sequence, the derivative of the return function respect to the control sequence is needed and has the form

$$\frac{\partial V\left(x\left(k\right),u\left[k,N-1\right]\right)}{\partial u(k)}=\frac{\partial L\left(x(k),u\left(k\right)\right)}{\partial u(k)}+\frac{\partial V\left(x\left(k+1\right),u\left[k+1,N-1\right]\right)}{\partial x\left(k+1\right)}\frac{\partial F\left(x(k)\right)}{\partial u(k)}$$

In the proof of Proposition 2, the calculation of the derivative of $V\left(x\left(k\right),u\left[k,N-1\right]\right)$ respect to $x(k+1)$ was achieved, so this information together with the next derivatives will be of great help to find the optimized control sequence.

$$\frac{\partial L\left(x(k),u\left(k\right)\right)}{\partial u(k)}=Ru(k)$$

$$\frac{\partial F\left(x(k)\right)}{\partial u(k)}=\beta$$

With the above equations it is now possible to compute the partial derivative of the cost function respect to the control sequence in each stage of the trajectory $x[0,N-1]$ as follows:

If $k=N-1$

$$V_{\overline{u}}(N-1)=L_u(N-1)+V_x(N)F_u(N-1)$$

if $k=N-2$

$$V_{\overline{u}}(N-2)=L_u(N-2)+V_x(N-1)F_u(N-2)$$

if $k=N-3$

$$V_{\overline{u}}(N-3)=L_u(N-3)+V_x(N-2)F_u(N-3)$$

and the computing continues until $k=0$. This information is useful to generate the new control sequence as:

$$u^{j+1}(k)=u^j(k)-\bar{\varepsilon}\left[\frac{\partial V\left(\varphi\left[-M,0\right],u^j\left[0,N-1\right],\overline{\alpha}^j\right)}{\partial u^j(k)}\right]^T,\bar{\varepsilon}>0$$

Remark 2. *As on the proof of Proposition 2 it is clear that the return function decreases on value with the election of the new control sequence. This local optimization can be seen considering the first order approximation of the return function* $V\left(\varphi\left[-M,0\right],u\left[0,N-1\right]\right)$ *around the control sequence* $u^j\left[0,N-1\right]$*,*

$$\begin{aligned}&V\left(\varphi\left[-M,0\right],u^{j+1}\left[0,N-1\right],\overline{\alpha}\right)\\&=V\left(\varphi\left[-M,0\right],u^{j}\left[0,N-1\right],\overline{\alpha}\right)+\frac{\partial V\left(\varphi\left[-M,0\right],u^{j}\left[0,N-1\right],\overline{\alpha}\right)}{\partial u(k)}\delta u\left[0,N-1\right]\end{aligned}$$

with the difference $\delta u\left[0, N-1\right] = u^{j+1}\left[0, N-1\right] - u^{j}\left[0, N-1\right]$ *selected as*

$$\delta u\left[0, N-1\right] = -\bar{\varepsilon}\left[\frac{\partial V\left(\varphi\left[-M, 0\right], u^{j}\left[0, N-1\right], \overline{\alpha}\right)}{\partial u[0, N-1]}\right]^{T}$$

for some $\bar{\varepsilon} > 0$.

□

In the next section, the experimental results of the gains optimization of a PID control and the application of the optimized control sequence is showed.

4 Experimental results

This section describes the atmospheric dehydration system and explains the drying process.

4.1 Platform description and modeling

The experimental platform consists on an atmospheric dehydration system, similar to the one presented on [5] and [18]. The model considered for this system only includes an input delay. This system consists of a drying chamber in which the product is placed and the controlled temperature is measured. The air flow is produced by a DC fan with a constant velocity of 2.1m/s, this air flow is then heated while it passes through an electrical grid. The temperature is measured by using a LM35 sensor and the control law calculates the voltage to be applied to the actuator every 200ms. The control law is implemented by using the LabVIEW software of National Instruments on the cDAQ-9132 controller and the remote chassis cDAQ-9184 where the analog input module NI9207 and the analog output module NI9263 are connected as shown in Figure 2.

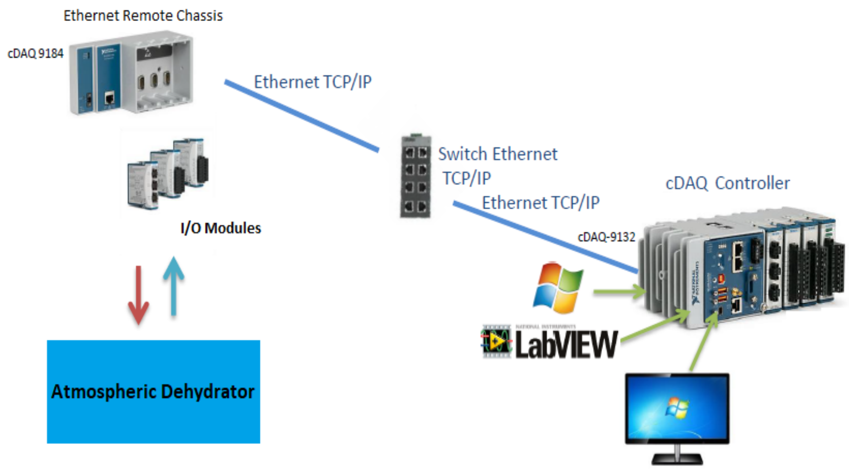

Fig. 2 Industrial PAC connection.

As mentioned above, the actuator for this system is an electrical grid fed by AC power whose nominal value varies between 0-120Vrms (power stage). The analog output of the industrial controller module provides a direct current voltage that varies between 0-5VDC (control signal). The process is modeled as the scalar system:

$$x\,(i+1)\; = a_0 x\,(i) + \gamma_0 x^2\,(i) + \gamma_1 x^3\,(i) + \beta u\,(i - M)$$

The purpose of the control consists in regulating the temperature of the dehydration chamber, represented by $x(i)$ by controlling the applied voltage on the power stage board, to achieve this it is necessary to compute the control signal (0-5VDC) represented by $u\,(i - M)$, that is mapped to 0-120VRMS by the power stage board. The inherent delay in the input h is a consequence of the distance between the actuator and the drying chamber. The system parameters for the setpoint of $(60°)$ $a_0 = 1.000146377925391$, $\gamma_0 = -9.359303656000177e - 04$, $\gamma_1 = 7.952239373253265e - 04$ and $\beta = 3.665347874142847e - 05$, were obtained by means of the least squares recursive method for this operation region, and the input delay was experimentally measured, from the step response, as $h = 50$s. Figure 3 shows the proposed scheme for the implementation of the control in the industrial controller cDAQ-9132.

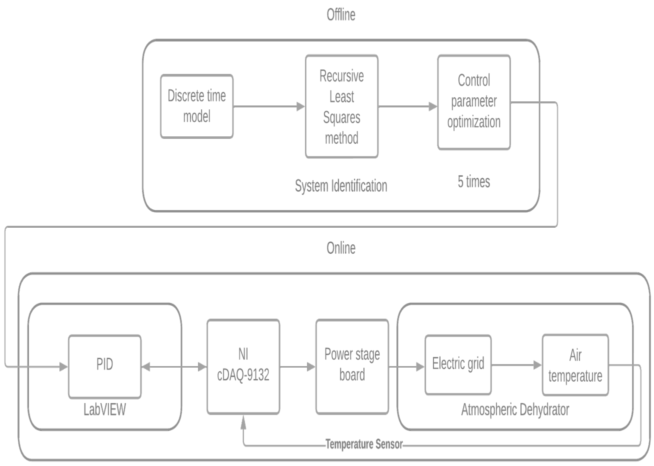

Fig. 3 Control application diagram.

4.2 Neutral system

As presented on [23] the dehydration process in closed loop with a PID controller can be seen as a neutral system. The steps followed to model the dehydration process as a neutral time delay system will be presented below.

To address the problem, on Figure 4 a schematic diagram of the dehydrator machine is presented.

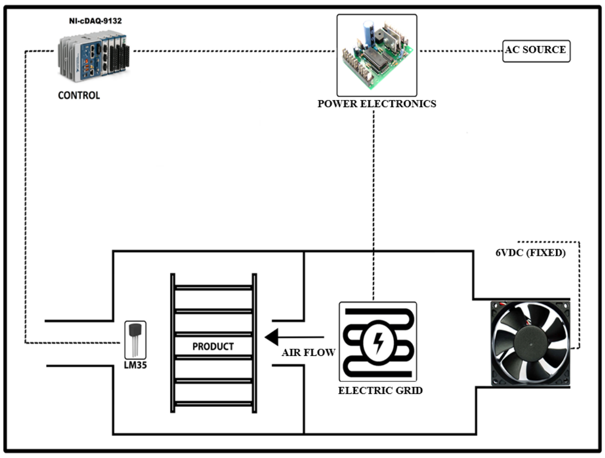

Fig. 4 Dehydrator machine schematic.

Considering that the differential equation that describes the dynamics of the dehydrator system can be modeled as

$$\dot{x}(t) = ax(t) + bu(t-\tau), \tag{9}$$

where a, b are scalars, $x(t)$ is the temperature, $u(t-\tau)$ is the voltage applied to an electric grid with a delay τ is considered due of the distance between the actuator (electric grid) and the dehydration chamber where the temperature is measured.

It is assumed that the error is $e(t) = x(t) - SP$, where SP is the desired temperature, then the error dynamics

$$\dot{e}(t) = a(e(t) + SP) + bu(t-\tau). \tag{10}$$

If $x(t) = SP$, then $aSP = -bu_{ss}$, so the previous equation can be seen as

$$\dot{e}(t) = ae(t) + b\hat{u}(t-\tau), \tag{11}$$

where $\hat{u}(t-\tau) = u(t-\tau) - u_{ss}$.

For the control $\hat{u}(t-\tau)$, a PID is proposed, therefore

$$\hat{u}(t-\tau) = -k_p e(t-\tau) - k_d \dot{e}(t-\tau) - k_i \int_0^{t-\tau} e(\xi)d\xi + v(t), \tag{12}$$

where $v(t)$ is an optimizing control signal, in the sense of minimizing a cuadratic performance indexwhich involves the trajectory and the control signal. The computation of the optimizing control signal is achieved by applying the gradient descent method described before.

System (11) in closed loop with (12) is

$$\dot{e}(t) = ae(t) - bk_p e(t-\tau) - bk_d \dot{e}(t-\tau) - bk_i \int_0^{t-\tau} e(\xi)d\xi + bv(t), \tag{13}$$

Considering the following

$$\sigma(t) = \int_0^{t-\tau} e(\xi)d\xi, \tag{14}$$

the augmented system is

$$\begin{pmatrix} \dot{e}(t) \\ \dot{\sigma}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} a & -bk_i \\ 0 & 0 \end{pmatrix}}_{A_0} \begin{pmatrix} e(t) \\ \sigma(t) \end{pmatrix} + \underbrace{\begin{pmatrix} -bk_p & 0 \\ 1 & 0 \end{pmatrix}}_{A_1} \begin{pmatrix} e(t-\tau) \\ \sigma(t-\tau) \end{pmatrix} + \underbrace{\begin{pmatrix} -bk_d & 0 \\ 0 & 0 \end{pmatrix}}_{D} \begin{pmatrix} \dot{e}(t-\tau) \\ \dot{\sigma}(t-\tau) \end{pmatrix} + \underbrace{\begin{pmatrix} b \\ 0 \end{pmatrix}}_{B} v(t).$$

Which can be seen as

$$\dot{\bar{x}}(t) = A_0\bar{x}(t) + A_1\bar{x}(t-\tau) + D\dot{\bar{x}}(t-\tau) + Bv(t), \tag{15}$$

where $\bar{x}(t) = \begin{bmatrix} e(t) \\ \sigma(t) \end{bmatrix}$, with initial condition $\varphi(\theta) = \begin{bmatrix} e(\theta) \\ \sigma(\theta) \end{bmatrix}$, $\theta \in [-\tau, 0]$, $\varphi(\theta) \in \mathcal{C}^1$:$[-\tau, 0]$. Note that the system (15) results in a neutral type delay system due to the presence of the derivative term of the PID equation.

For the algorithm implementation, it is needed to discretize system (15) as follows:

$$\frac{\bar{x}(k) - \bar{x}(k-1)}{T_s} = A_0\bar{x}(k-1) + A_1\bar{x}(k-M-1) + D\frac{\bar{x}(k-M) - \bar{x}(k-M-1)}{T_s} + Bv(k-1),$$

where T_s is the sampling time, k is discrete time and $M = \frac{\tau}{T_s}$ is the delay instants, rewriting it follows that:

$$\bar{x}(k) = \underbrace{[A_0Ts + I]}_{\widetilde{A_0}}\bar{x}(k-1) + \underbrace{[A_1Ts - D]}_{\widetilde{A_1}}\bar{x}(k-M-1) + D\bar{x}(k-M) + \underbrace{BTs}_{\widetilde{B}}v(k-1),$$

where $\widetilde{A_0} = [A_0T_s + I]$, $\widetilde{A_1} = [A_1T_s - D]$, $\widetilde{B} = BT_s$. Hence, previous equation can be rewritten as

$$\bar{x}(k) = \widetilde{A_0}\bar{x}(k-1) + \widetilde{A_1}\bar{x}(k-M-1) + D\bar{x}(k-M) + \widetilde{B}v(k-1), \qquad (16)$$

where the control sequence $v(k-1)$ is computed by means of the gradient descent algorithm described on the previous section.

4.3 Drying process

The purpose of the dehydration process is to reduce the amount of water under 10% of humidity of every fish meat sample. The fish meat sample is weighted every 60 minutes in order to measure the loss of water on the sample. The experimental results of the closed-loop system were obtained by applying different control strategies:

- Discrete PID tuned by the Ziegler-Nichols rules
- Discrete PID tuned by means of the gradient descent algorightm considering a linear system
- Discrete PID tuned by means of the gradient descent algorithm considering a non linear system
- Discrete PID plus an optimal control sequence obtained by considering a linear system
- Discrete PID plus an optimal control sequence obtained by considering a non linear system
- Discrete PID plus an optimal control sequence obtained by considering a neutral type time delay system

On Figure 5 the system responses under the action of a PID controller tuned with different methods are shown. In subplot 1 the temperature response is shown, the control signals are shown in subplot 2 and the error signals in subplot 3.

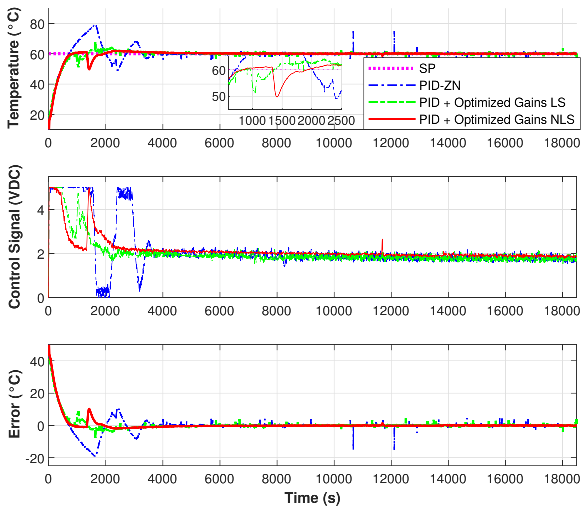

Fig. 5 System performance on the region of 60° with the PID tuned by the gradient descent method.

On Figure 6, a graphical comparison is shown between a PID tuned by the ZN rules and a PID tuned by the gradient algorithm considering a linear and nonlinear discrete time delay system. For this experiment not only the optimized gains were used, but also the computed local optimal control sequence, considering a linear and nonlinear discrete time delay system.

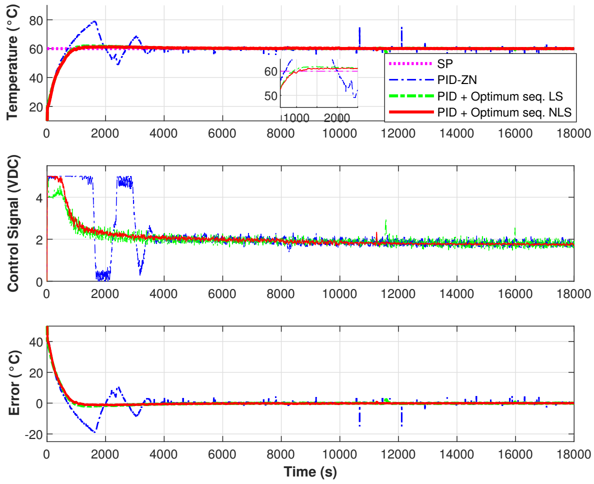

Fig. 6 System performance on the region of 60° with the PID tuned by the gradient descent method + optimal control sequence

On Figure 7, a graphical comparison is shown between the PID tuned by the gradient descent method + an optimal control sequence computed by the same algorithm considering a nonlinear discrete time delay system and a discrete neutral delay system. In this grapgic it can be seen that the system performance is improved in the sense of a faster settling time.

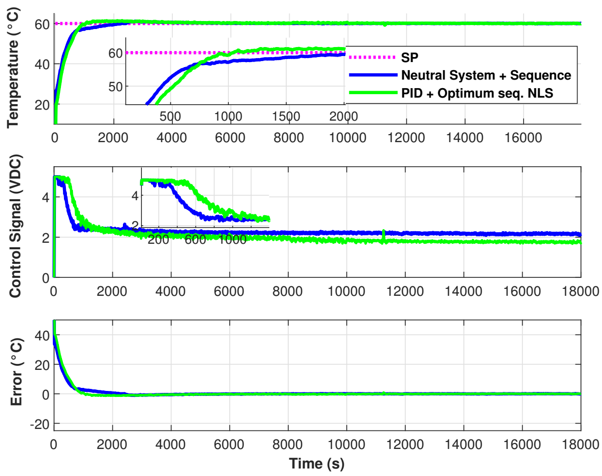

Fig. 7 Comparison of the system performance on the region of 60° between neutral system and PID + Optimum sequence NLS.

Figure 8 shows the lost of humidity by the dried product when the discrete neutral time delay system + optimum control sequence is applied, when the SP is adjusted to 60°C, 70°C and 80°C. It is only shown the humidity measured on the first 5 hours of the experiment.

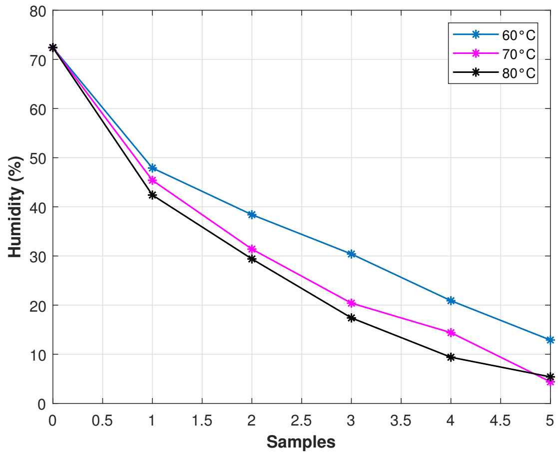

Fig. 8 Relative humidity of the fish meat when the PID + Optimized Gains and control sequence NLS is applied

In order to analyse and compare the different results, in the Table 1 some performance indicators are shown: integral absolute error (IAE), settling time (Tss) with the $\pm 2\%$ criteria, and the total energy consumption in kWh. The duration of all the experiments was $Tdur = 19,200s$.

Table 1 Performance indicators for the region of 60°.

Control	IAE	$M_p(\%)$	$T_{ss}(s)$	$E_c(kWh)$
PID-ZN	37,246	31.18	3735	0.806
PID + Optimized Gains LS	22,585	6.666	3500	0.782
PID + Optimized Gains NLS	20,292	3.2	3400	0.878
PID + Control sequence LS	19,467	3.41	2760	0.801
PID + Control sequence NLS	19,378	2.5	2570	0.803
Discrete Neutral system + Control sequence	15,902	1.83	1612	0.990

According to these experimental results, the implementation of the proposed optimization algorithm presents the following advantages when compared with a PID control tuned by the Ziegler Nichols rules:

- A lower numerical value for the integral absolute error (IAE) was obtained.
- A lower overshoot (M_p) was obtained.

•When only optimized gains were used, the settling times (Tss) were lower than when the Ziegler-Nichols gains were used, but were significantly lower when local optimal control sequence was added.

•A lower energy consumption was achieved when using the optimized gains and the optimized control parameters plus the local optimal control sequence (except when the nonlinear system and neutral system are considered for its calculation) for both linear and nonlinear model of the plant.

5 Conclusions

The optimization of the parameters of a PID controller and the computation of a local optimal control sequence applied to the temperature control of a discrete nonlinear time delay system is addressed in this paper. The development of a solution for this problem involves energy consumption optimization and quickness of the convergence of the process variable to the setpoint. According to the experimental results, the optimization of the gains for a PID controller applied to a discrete nonlinear time delay system, improves the performance of the closed loop in a meaningful way, not only helps to save energy but also reduces the maximum overshoot and the settling time. In addition of the previous statements, an optimal control sequence (local) for a discrete neutral type delay system is presented in this paper. This is achieved by using the gradient descent method, applied to a first order system (dehydration system) in closed loop with a PID controller (optimized by the same method).

The authors declare that they have no conflict of interest.

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