SUPPLEMENTARY INFORMATION

Perturbation-induced nonreciprocal transmission in nonlinear parity-time-symmetric silicon micromechanical resonators

Rui Wang, Lei Han*, Man-Na Zhang, Li-Feng Wang, and Qing-An Huang*

Key Laboratory of MEMS of the Ministry of Education, Southeast University, Nanjing 210096, China.
*e-mail: haneli@seu.edu.cn; hqa@seu.edu.cn

Contents

Supplementary Note 1

Eigenfrequencies for PT-symmetric silicon micromechanical resonators without perturbation

The dynamic of PT-symmetric micromechanical resonators without perturbation is expressed as

$$\begin{cases} m\frac{d^2x_L}{dt^2}+c\frac{dx_L}{dt}+kx_L+k_c(x_L-x_G)=0 \\ m\frac{d^2x_G}{dt^2}-c_S\frac{dx_G}{dt}+kx_G+k_c(x_G-x_L)=0 \end{cases}. \quad \text{(S1)}$$

Taking $x_{L,G}=a_{L,G}e^{i\omega t}=a_{L,G}e^{i\lambda\tau}$, Eq. (S1) is rewritten as

$$\begin{bmatrix} -\lambda^2+i\lambda\gamma+1+\mu & -\mu \\ -\mu & -\lambda^2-i\lambda g_s+1+\mu \end{bmatrix}\begin{bmatrix} a_L \\ a_G \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{(S2)}$$

where $\lambda=\omega/\omega_0$ is the normalized frequency ($\omega_0=\sqrt{k/m}$ is the uncoupled fundamental resonant frequency), and $\tau=\omega_0 t$ is the normalized time, respectively. In the case of weakly coupling ($\mu \ll 1$), we take the approximation that $\lambda^2\approx 2\lambda-1$, $\gamma\lambda\approx\gamma$ and $g_s\lambda\approx g_s$, Eq. (S2) is approximately given by

$$\begin{bmatrix} -\lambda+i\frac{\gamma}{2}+1+\frac{\mu}{2} & -\frac{\mu}{2} \\ -\frac{\mu}{2} & -\lambda-i\frac{g_s}{2}+1+\frac{\mu}{2} \end{bmatrix}\begin{bmatrix} a_L \\ a_G \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{(S3)}$$

The eigenfrequencies are then solved as

$$\lambda_\pm=1+\frac{\mu}{2}+i\frac{\gamma-g_s}{4}\pm\frac{1}{2}\sqrt{\mu^2-\left(\frac{\gamma+g_s}{2}\right)^2} \quad \text{(S4)}$$

Therefore, EPs are located at $\mu_{EP}=(g_s+\gamma)/2$, where $\lambda_+=\lambda_-$.

Supplementary Note 2

Eigenfrequencies for PT-symmetric silicon micromechanical resonators with perturbation

As the perturbation Δk is applied to the loss resonator, the dynamic of the system is rewritten as

$$\begin{cases} m\frac{d^2x_L}{dt^2}+c\frac{dx_L}{dt}+(k+\Delta k)x_L+k_c(x_L-x_G)=0 \\ m\frac{d^2x_G}{dt^2}-c_S\frac{dx_G}{dt}+kx_G+k_c(x_G-x_L)=0 \end{cases} \quad \text{(S5)}$$

Similar to Eqs. (S2) and (S3), it yields

$$\begin{bmatrix} -\lambda+i\frac{\gamma}{2}+1+\frac{\mu}{2}+\frac{\delta}{2} & -\frac{\mu}{2} \\ -\frac{\mu}{2} & -\lambda-i\frac{g_s}{2}+1+\frac{\mu}{2} \end{bmatrix}\begin{bmatrix} a_L \\ a_G \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{(S6)}$$

Therefore, the eigenfrequencies are solved as

$$\lambda_\pm=1+\frac{\mu}{2}+\frac{\delta}{4}+i\frac{\gamma-g_s}{4}\ \pm\frac{1}{2}\sqrt{\mu^2-\left(\frac{\gamma+g_s}{2}\right)^2+\frac{\delta^2}{4}+i\frac{\delta}{2}(g_s+\gamma)} \quad \text{(S7)}$$

Supplementary Note 3

Non-reciprocal ratio for PT-symmetric micromechanical resonators with perturbation

In order to achieve transmission, the time-harmonic input force $f(t)=Fe^{i\lambda\tau}$, where F is the amplitude, is applied to the loss resonator for the forward transmission and to the gain resonator for the backward transmission, respectively. The output amplitude of the forward and backward transmission are labeled as a and b, respectively. Eq. (S6) with input forces is then rewritten as, for the forward and backward transmission, respectively,

$$\begin{bmatrix} -\lambda + i\frac{\gamma}{2} + 1 + \frac{\mu}{2} + \frac{\delta}{2} & -\frac{\mu}{2} \\ -\frac{\mu}{2} & -\lambda - i\frac{g_f}{2} + 1 + \frac{\mu}{2} \end{bmatrix} \begin{bmatrix} a_L \\ a_G \end{bmatrix} = \begin{bmatrix} \frac{F}{2k} \\ 0 \end{bmatrix} \tag{S8}$$

$$\begin{bmatrix} -\lambda - i\frac{g_b}{2} + 1 + \frac{\mu}{2} & -\frac{\mu}{2} \\ -\frac{\mu}{2} & -\lambda + i\frac{\gamma}{2} + 1 + \frac{\mu}{2} + \frac{\delta}{2} \end{bmatrix} \begin{bmatrix} b_G \\ b_L \end{bmatrix} = \begin{bmatrix} \frac{F}{2k} \\ 0 \end{bmatrix} \tag{S9}$$

where g_f and g_b are the gain in the forward and backward transmission, respectively. The output amplitude can be solved as, for the forward and backward transmission, respectively,

$$a_{out} = a_G = \left| \frac{iF\mu}{4i + 2g_f - 2\gamma + ig_f\gamma + 2i\delta + g_f\delta + 4i\mu + g_f\mu - \gamma\mu + i\delta\mu - 8i\lambda - 2g_f\lambda + 2\gamma\lambda - 2i\delta\lambda - 4i\mu\lambda + 4i\lambda^2} \right| \tag{S10}$$

$$b_{out} = b_L = \left| \frac{iF\mu}{4i + 2g_b - 2\gamma + ig_b\gamma + 2i\delta + g_b\delta + 4i\mu + g_b\mu - \gamma\mu + i\delta\mu - 8i\lambda - 2g_b\lambda + 2\gamma\lambda - 2i\delta\lambda - 4i\mu\lambda + 4i\lambda^2} \right| \tag{S11}$$

The transmission T is defined as the ratio of output to input, and the non-reciprocal ratio R_{nr} is defined as the ratio of T_a to T_b. With the operating frequency at the exponential growth mode λ_-, the non-reciprocal ratio is given by

$$R_{nr} = \left| \frac{a_{out}/F}{b_{out}/F} \right| = \left| \frac{a_{out}}{b_{out}} \right| = \left| \frac{(2s - \delta - 2ig_b)(2s + \delta + 2i\gamma) - 4\mu^2}{(2s - \delta - 2ig_f)(2s + \delta + 2i\gamma) - 4\mu^2} \right| \tag{S12}$$

with

$$s = Re\left[\sqrt{\mu^2 - \left(\frac{\gamma + g_s}{2}\right)^2 + \frac{\delta^2}{4} + i\frac{\delta}{2}(g_s + \gamma)} \right]$$

where $g_s = g_f$ for the forward transmission and $g_s = g_b$ for the backward transmission, respectively.

Supplementary Note 4

Realization of nonlinear gain

The schematic of the nonlinear gain is shown in Fig. S1(a). A positive feedback loop consists of trans-impedance amplifier (TIA), voltage amplifier (VA), bandpass filter (BPF), automatic gain control (AGC), and phase shifter (PS). The feedback circuit collects the motional current from the gain resonator, which is a velocity-dependent signal. It is then converted into voltage by the TIA and passed through the VA to ensure adequate gain. BPF is used to filter out unwanted oscillator modes. AGC provides the nonlinearity, by which the fitting result follows $A_n = A_0/(1+\left(\frac{V_{in}}{v_0}\right)^{\beta})$, as shown in Fig. S1(b). PS is used to adjust the phase of the entire circuit. The output voltage of the circuit is then fed back to the external electrode of the gain resonator, producing additional electrostatic force. The models of components in the feedback loop are listed in Table S1.

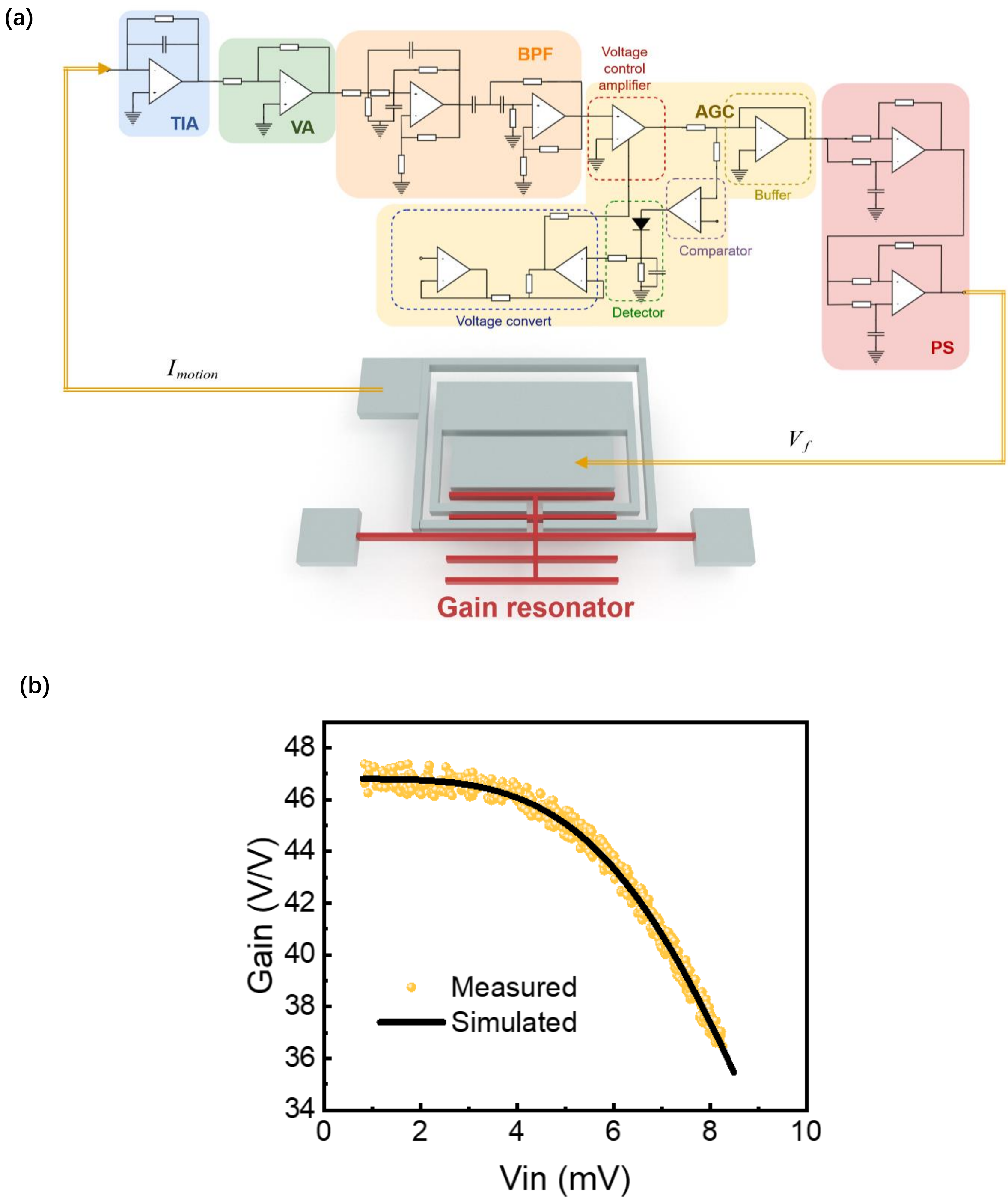

FIG. S1 (a) Schematic of feedback loop: The gain resonator generates the motional current I_{motion}, and the output signal V_f is applied to the feedback electrode of the gain resonator. Blue region: trans-impedance amplifier (TIA); green region: voltage amplifier (VA); tangerine region: bandpass filter (BPF); yellow region: automatic gain control (AGC) consisting of voltage control amplifier, comparator, detector and voltage convert; red region: phase shifter (PS). (b) Measured and simulated nonlinear gain of AGC as a function of input voltage amplitude. Simulation parameter: $A_n = A_0/(1+\left(\frac{V_{in}}{v_0}\right)^{\beta})$, where the pre-factor gain A_0=46.8, saturation amplitude v_0=0.011 mV, nonlinear coefficient, β=4.

TABLE S1. Components in feedback loop.

Symbol	Model
TIA	Zurich HF2TA
VA	OPA627
BPF, PS	NE5532
Voltage control amplifier	VCA810
Buffer	OPA690
Comparator	AD8561
Voltage convert	TL082

Supplementary Note 5

Capacitive transduction of micromechanical resonators

In experiments, the AC voltage signal v_{in} applied on the electrode with DC bias V_{dc} generates the input force, which is expressed as

$$f(t) = \frac{1}{2}\frac{\partial C}{\partial x}(V_{dc} + v_{in})^2 \approx \frac{1}{2}\frac{\varepsilon A}{d^2}\left(V_{dc}^2 + 2V_{dc}v_{in} + v_{in}^2\right) \qquad (S13)$$

where $\frac{\partial C}{\partial x}$ is the change in capacitance per unit displacement, d is the initial gap between the driven electrode and micromechanical resonator, A is the area of the electrode, and ε is the dielectric constant. Ignoring the constant term V_{dc}^2 and the high-order components v_{in}^2, the effective input force is $\frac{\varepsilon A}{d^2}V_{dc}v_{in}$.

The output amplitude is detected by the time-varying capacitor between the detection electrode and resonator. The motion current is converted to voltage by trans-impedance amplifier (TIA). The converted output voltage v_{out} is expressed as

$$v_{out} = A_T V_{dc}\frac{\partial C}{\partial x}\frac{\partial x}{\partial t} \approx A_T V_{dc}\frac{\varepsilon A}{d^2}\frac{\partial x}{\partial t} \qquad (S14)$$

where A_T is the gain coefficient of TIA. Hence, the amplitude detection of the resonator is replaced by the measurement in voltage. Accordingly, the non-reciprocal ratio is calculated by the ratio of output voltage in forward (v_{out-a}) and backward (v_{out-b}) transmission, as follows:

$$R_{nr} = \left|\frac{a_{out}}{b_{out}}\right| = \left|\frac{v_{out-a}}{v_{out-b}}\right| \qquad (S15)$$

Supplementary Note 6

Electrostatic coupling between the loss and gain resonator

In experiments, the coupling strength of the electrostatically-coupled resonators is determined by the voltage difference V_{cp} across the loss and gain resonator. As shown in Fig. 1b and c in the main text, the two resonators are separated by a small gap, and the coupling voltage is applied on the resonators inducing a voltage difference V_{cp}. The electrostatic force between the two resonators results in an elastic stiffness, which is given by

$$k_c = -V_{cp}^2\frac{\varepsilon A}{d^3} \qquad (S16)$$

The negative sign shows that the electrostatic coupling is opposite in direction compared to the conventional mechanical spring.

Supplementary Note 7

Perturbations to the loss resonator

Similar to electrostatic coupling, the perturbation is also applied by electrostatic force. In the initial state (the stiffness of the two resonators is approximately equal), a voltage difference V_{bias} is observed between the disturbed electrode and the resonator. Then, the DC voltage on the disturbance electrode is modified, resulting in a voltage difference V_{bias}-V_{pert} between the electrode and loss resonator. Therefore, the perturbation δ is expressed as

$$\delta = \frac{\frac{\epsilon A}{d^3}\left(V_{bias}^2 - \left(V_{bias} - V_{pert}\right)^2\right)}{k} \tag{S16}$$

Supplementary Note 8

Equivalent circuit representation and its simulation

The schematic diagram of equivalent circuit for the simulation of non-reciprocal transmission is shown in Fig. S2. The simulation was performed by SPICE in Cadence ORCAD. Here, the micromechanical resonators are equivalent to a pair of resistor–inductor–capacitor (RLC) series circuits with capacitive coupling. The inductance, capacitance, and resistance of circuit correspond to the equivalent lumped parameters of mass, stiffness, and damping of the resonators. The values of mass and stiffness were calculated by finite element simulation in COMSOL Multiphysics, showing that m=7.6×10-10 kg, k=340 N/m. The controlled source is used to represent the electromechanical transduction coefficient. The input transduction and output transduction are equivalent to voltage control voltage source and current control current source, respectively. The current amplifier U1 converts the current into voltage and feeds into the analog behavior model (ABM) as the feedback for a nonlinear feedback voltage. The output voltage of the ABM is pumped into the gain resonator to complete the positive feedback and form the equivalent nonlinear gain. The parameters of the circuit are given in Table. S2. The non-reciprocal ratio was given according to Eq. (S15).

TABLE S2. Parameters in the simulations

Lumped parameter	Value
L1, L2	7.6×10^{-10} H
C1	0.00293 ~ 0.00294 F
C2	0.00294 F
C3	6.84 F
R1, R2	6.1×10^{-7} Ω
p1, p4	2.5×10^{-7}
p2	1.1×10^{-7}
R31, R32	1 MΩ
ABM	$\frac{6\times10^{-6}V_{in}}{1+\left(\frac{V_{in}}{0.011}\right)^4}$

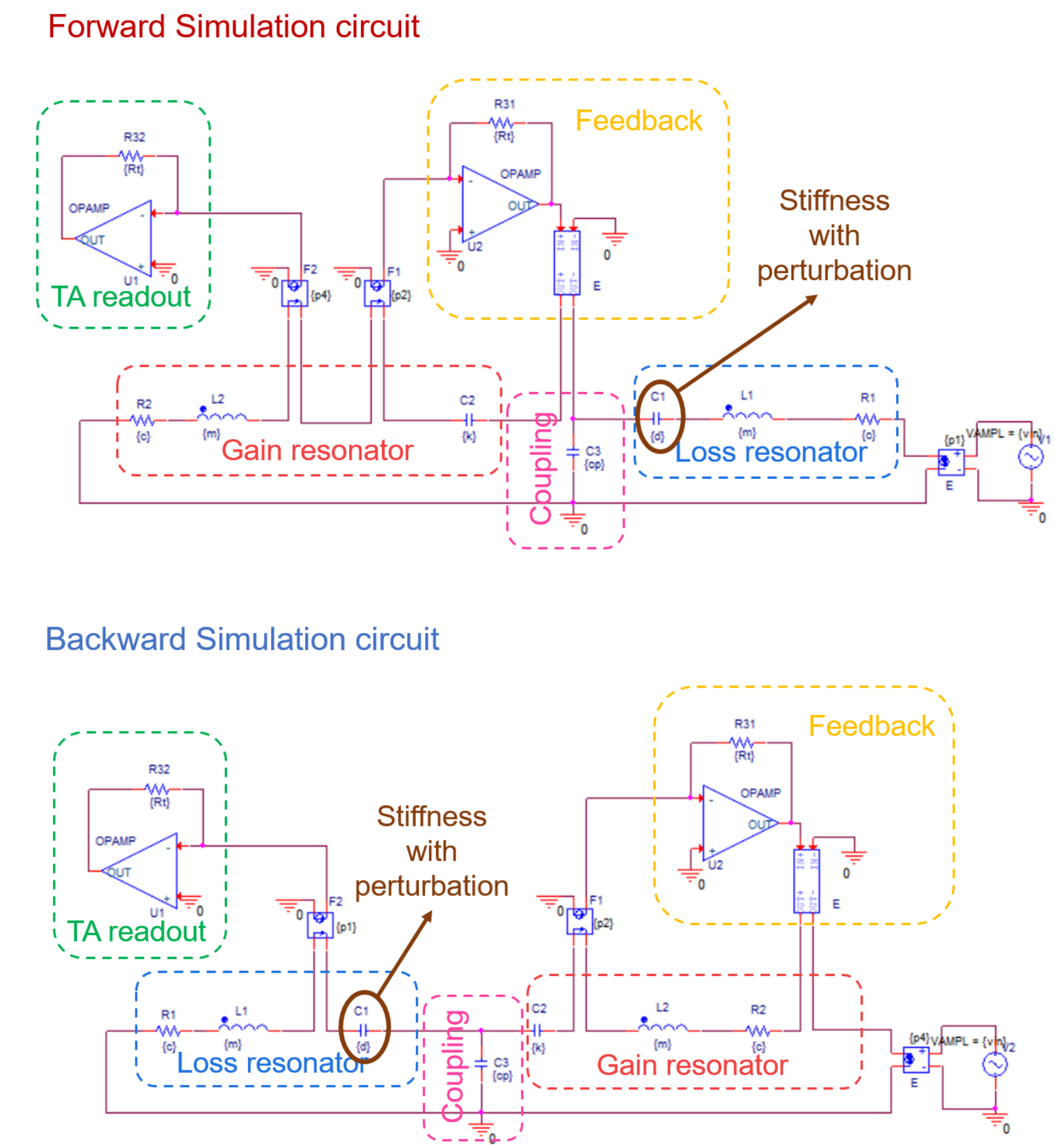

FIG. S2 Equivalent lumped circuit of PT-symmetric micromechanical resonators with perturbation in forward and backward transmission for simulation, respectively: the resistor–inductor–capacitor (RLC) series circuit in the red square and blue square is the equivalent model of the gain and loss resonator, respectively; the capacitance in the pink square is the equivalent model of the electrostatic coupling, the ABM in the yellow square represents the feedback circuit.

Supplementary Note 9
Fabrication process

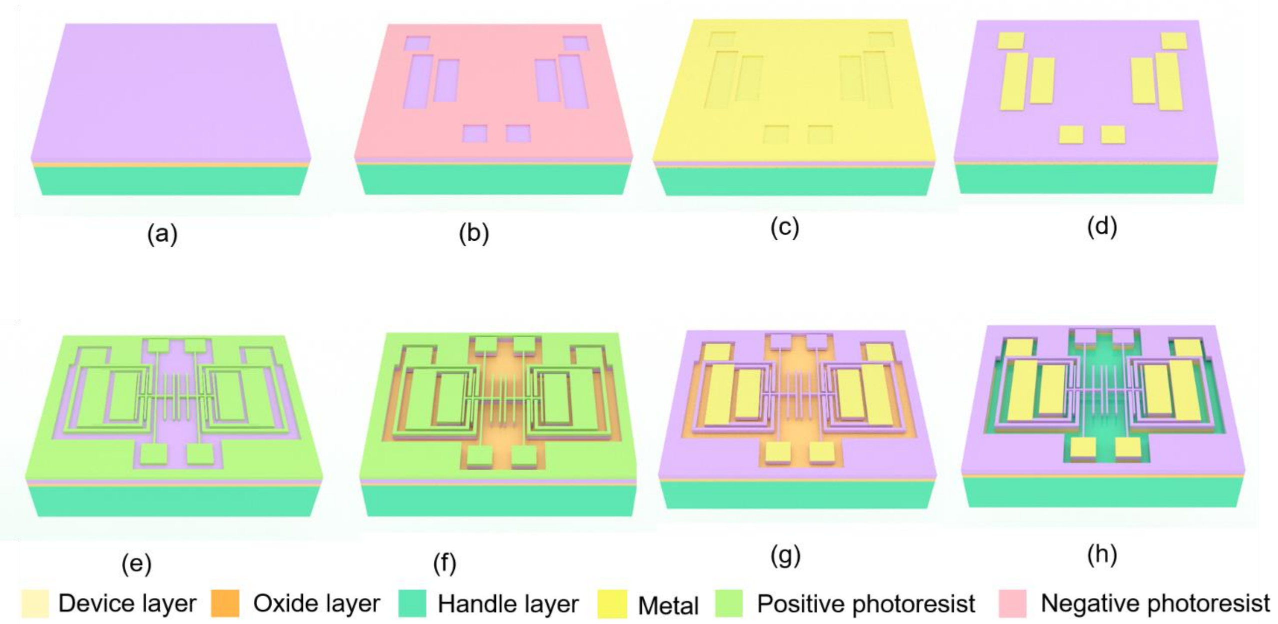

FIG. S3 Fabrication process: (a) SOI wafer, (b) Lithography for metal layer, (c) Metal deposited, (d) Negative photoresist lifted off, (e) Lithography for device layer, (f) Silicon etching, (g) Positive photoresist lifted off, (h) Oxide layer lifted off.

The process consists of two photolithography processes that define the metal electrode and device structure, respectively. The fabrication process is illustrated in Fig.S3 and described in detail as follows:

(a) A SOI wafer was prepared with a n-type dope device layer. The oxide and handle layers are 4 μm and 400 μm thick, respectively.
(b) After cleaning, the wafer was coated with negative photoresist and exposed by the first-level mask. It was then developed by photoresist developer to form the desired pattern for the electrode.
(c) A thin layer of titanium (20 nm) and gold (200 nm) was deposited on the photoresist by electron beam (e-beam) evaporation.
(d) The remaining photoresist was lifted off by organic solvent and oxygen plasma to expose the metal electrodes.
(e) A layer of positive photoresist was then coated on the wafer with metal electrodes and lithographically patterned by the second-level mask. After developing, the exposed device layer was prepared for etching by deep reactive ion etching (DRIE).
(f) The DRIE process was conducted through a series of alternating etching and passivation steps with etching gas SF_6 and passivation gas C_4F_8 to etch silicon down to the oxide layer. The repeating application of these steps ensures the formation of vertical sidewalls.
(g) The layer of photoresist was removed by organic solvent and oxygen plasma to expose the electrodes.
(h) Finally, the silicon micromechanical structure wasreleased by an HF wet etch, which removes the oxide layer.

The parameters of the fabricated device in detail are listed in Table S3.

TABLE S3. Parameters for silicon micromechanical resonators

Parameter	Value
Beam length	500μm
Beam width	8μm
Electrode length	260μm
Electrode width	8μm
Device layer thickness	25μm
Electrode gaps	2.8μm
Computed resonator natural frequency	107kHz
Computed resonator stiffness (k)	346N/m
Computed resonator mass (m)	7.7×10^{-10}kg

Supplementary Note 10
Experimental setup

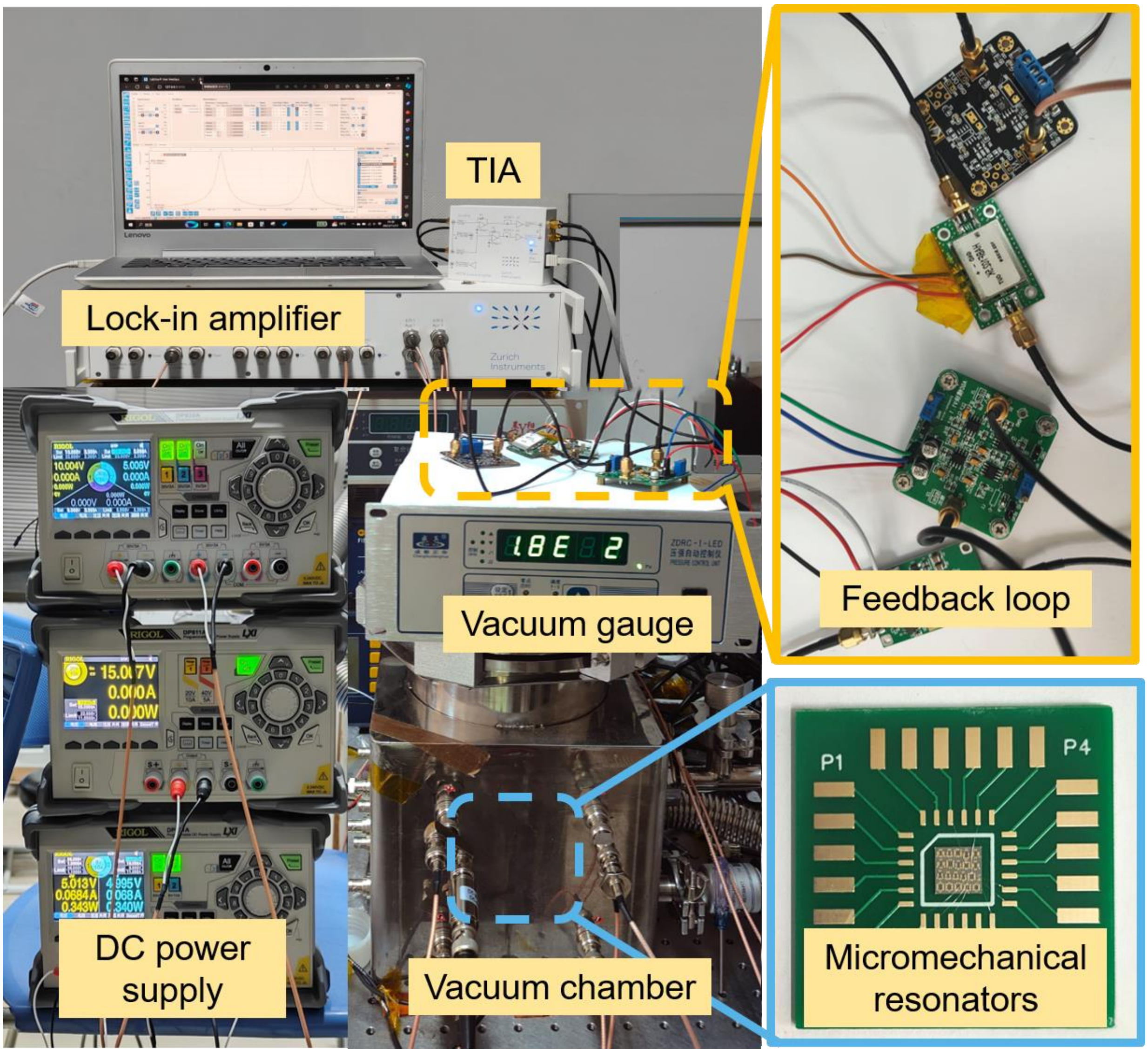

FIG.S4 Photograph of experimental setup, including lock-in amplifier, feedback loop, vacuum chamber, vacuum gauge, DC power supply, and the micromechanical resonators.

As shown in Fig.S4, a unit of prepared SOI die contains 24 pairs of micromechanical resonators. The SOI die was placed on a printed circuit board (PCB) and gold wire bonded from the electrodes of the resonators to the metal pad on the PCB to form an electrical connection to the test device. The PCB was placed in a vacuum chamber equipped with a pressure-controlled vacuum gauge to adjust pressure in order to regulate damping. The vacuum gauge can be set in the range of $8\sim8\times10^5$ mtorr. In experiments, a loss parameter of $\gamma=9\times10^{-4}$ corresponds to the pressure of 2.2 torr. The high-precision linear DC power supply (RIGOL DP800) provides DC voltage to the micromechanical resonator for the purpose of electrostatic coupling, input bias, and perturbation, respectively. Furthermore, the power supply is employed to energize the active device in the feedback loop. The Zurich HF2LI lock-in amplifier provides driven signal and measures the output signal from resonators. Each point was sampled 20 times and then averaged to improve the accuracy by the lock-in amplifier. All the devices were placed on a damping isolation platform (Newport United States).