Supplemental materials: Beating Carnot efficiency with periodically driven chiral conductors

PHOTOTRANSITION AMPLITUDE a_n

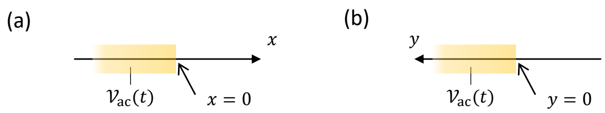

Fig. S1. Chiral channel with AC driven contact. Panel (a) is when an electron propagates out of the AC driven contact and (b) when an electron propagates into the contact.

For completeness, we summarize the derivation [26, 36] that an electron propagating in a chiral channel out of an AC driven contact [see Fig. S1 (a)] absorbs (or emits) $|n|$ photons with the transition amplitudes a_n of Eq. (2), for $n > 0$ ($n < 0$).

When an electron of initial energy $\mathcal{E}$ propagates from the AC driven contact [see Fig. S1 (a)], its wave function solution of the time-dependent Schrödinger equation is

$$\psi_{\mathcal{E}}(x,t) = e^{-i\mathcal{E}(t-x/v)}\Big[\Theta(-x)e^{i\phi_{\rm ac}(t)} + \Theta(x)e^{i\phi_{\rm ac}(t-x/v)}\Big]. \tag{S1}$$

Here, v is propagation velocity for the chiral channel, $\phi_{\rm ac}(t) = -e\int_{-\infty}^{t} dt'\mathcal{V}_{\rm ac}(t')/\hbar$, and $\Theta(x)$ is 0 for $x < 0$ and 1 otherwise. This solution can be verified by plugging it into the Schrödinger equation for the Hamiltonian of linear dispersion, $[i\hbar(\partial/\partial t + v\partial/\partial x) - e\mathcal{V}_{\rm ac}(t)\Theta(-x)]\psi_{\mathcal{E}}(x,t) = 0$. When an electron is in the contact, $x < 0$, the phase factor $e^{i\phi_{\rm ac}(t)}$ does not induce any change of energy as it is a spatially independent phase shift. When an electron is out of the contact, $x > 0$, the spatially dependent phase factor $e^{i\phi_{\rm ac}(t-x/v)}$ describes the electronic energy change due to the photoassisted transition. By definition of a_n in Eq. (2), $e^{i\phi_{\rm ac}(t-x/v)} = \sum_n a_n e^{-in\Omega(t-x/v)}$, thus the transition amplitude with which the electron changes its energy by $n\hbar\Omega$ is a_n.

We also remark the case when an electron propagates into the AC driven contact [see Fig. S1 (b)], which is used for the results of nonchiral conductors shown below. The wave function solution of the time-dependent Schrödinger equation with initial energy $\mathcal{E}$ is

$$\psi_{\mathcal{E}}(y,t) = e^{-i\mathcal{E}(t-y/v)}\Big[\Theta(-y)+\Theta(y)e^{i\{\phi_{\rm ac}(t)-\phi_{\rm ac}(t-y/v)\}}\Big]. \tag{S2}$$

This can be verified by plugging it into the Schrödinger equation $[i\hbar(\partial/\partial t + v\partial/\partial y) - e\mathcal{V}_{\rm ac}(t)\Theta(y)]\psi_{\mathcal{E}}(y,t) = 0$. According to the solution and $e^{-i\phi_{\rm ac}(t-y/v)} = \sum_n a_n^* e^{in\Omega(t-y/v)} = \sum_n a_{-n}^* e^{-in\Omega(t-y/v)}$, the transition amplitude with which the electron changes its energy by $n\hbar\Omega$ is a_{-n}^*.

UNITARITY OF FLOQUET SCATTERING MATRIX OF EQ. (3)

Here we show that the Floquet scattering matrix of Eq. (3) satisfies the unitarity conditions, $\sum_{n\alpha}|\mathcal{S}_{\alpha\beta}(\mathcal{E}_n,\mathcal{E})|^2 = 1$ and $\sum_{n\beta}|\mathcal{S}_{\alpha\beta}(\mathcal{E},\mathcal{E}_{-n})|^2 = 1$.

The first condition is satisfied as,

$$\begin{aligned}\sum_{n\alpha}|\mathcal{S}_{\alpha L}(\mathcal{E}_n,\mathcal{E})|^2 &= \sum_n |a_n|^2\Big(|t_{\rm st}(\mathcal{E}_n)|^2 + |r_{\rm st}(\mathcal{E}_n)|^2\Big) \\ &= \sum_n |a_n|^2 = 1,\end{aligned} \tag{S3}$$

$$\sum_{n\alpha}|\mathcal{S}_{\alpha R}(\mathcal{E}_n,\mathcal{E})|^2 = |t'_{\rm st}(\mathcal{E})|^2 + |r'_{\rm st}(\mathcal{E})|^2 = 1. \tag{S4}$$

The second condition is satisfied as,

$$\begin{aligned}\sum_{n\beta}|\mathcal{S}_{L\beta}(\mathcal{E},\mathcal{E}_{-n})|^2 &= \sum_n |a_n|^2|r_{\rm st}(\mathcal{E})|^2 + |t'_{\rm st}(\mathcal{E})|^2 \\ &= |r_{\rm st}(\mathcal{E})|^2 + |t'_{\rm st}(\mathcal{E})|^2 = 1,\end{aligned} \tag{S5}$$

$$\begin{aligned}\sum_{n\beta}|\mathcal{S}_{R\beta}(\mathcal{E},\mathcal{E}_{-n})|^2 &= \sum_n |a_n|^2|t_{\rm st}(\mathcal{E})|^2 + |r'_{\rm st}(\mathcal{E})|^2 \\ &= |t_{\rm st}(\mathcal{E})|^2 + |r'_{\rm st}(\mathcal{E})|^2 = 1.\end{aligned} \tag{S6}$$

DERIVATION OF $\langle n\rangle = 0$

Here we prove that the mean number of photons $\langle n\rangle = \sum_n n|a_n|^2$ involved in the photoassisted transitions by the AC driving is zero.

Let $a(t)$ be the Fourier transform of a_n,

$$a(t) \equiv \sum_n a_n e^{-in\Omega t}, \tag{S7}$$

$$= \exp\left[-i\frac{e}{\hbar}\int_{-\infty}^{t} dt'\,\mathcal{V}_{\rm ac}(t')\right]. \tag{S8}$$

The mean number $\langle n\rangle$ is expressed in terms of $a(t)$ as

$$\sum_n n|a_n|^2 = \frac{i}{2\pi}\int_0^{2\pi/\Omega} dt\,\frac{\partial a}{\partial t}a^*(t)\,. \tag{S9}$$

This can be verified when using Eq. (S7) and $\int_0^{2\pi/\Omega} dt\, e^{i(n-n')\Omega t} = (2\pi/\Omega)\delta_{nn'}$ for integers n and n'. Using that $|a(t')|^2 = 1$,

$$\frac{\partial a}{\partial t}a^*(t) = \frac{1}{a(t)}\frac{\partial a}{\partial t} = \frac{\partial \ln a}{\partial t} = -i\frac{e\mathcal{V}_{\rm ac}(t)}{\hbar}. \tag{S10}$$

In the last equality we used Eq. (S8). Hence, the mean number $\langle n\rangle$ is determined by the time-averaged AC voltage $\overline{\mathcal{V}_{\rm ac}}$

$$\langle n\rangle = \frac{e\overline{\mathcal{V}_{\rm ac}(t)}}{\hbar\Omega}. \tag{S11}$$

This vanishes because $\overline{\mathcal{V}_{\rm ac}} = 0$.

PHOTON NUMBER UNCERTAINTY δn IN TERMS OF AC VOLTAGE PROFILE $\mathcal{V}_{\rm ac}(t)$

Here, we derive that the photon number uncertainty δn is equal to the ratio between the root mean square of AC voltage and the photon energy quantum

$$\delta n = \frac{e\left\{\overline{\mathcal{V}_{\rm ac}^2(t)}\right\}^{1/2}}{\hbar\Omega}. \tag{S12}$$

Proof. We start by the fact that $\langle n^2\rangle$ is written in terms of $a(t)$ (see Eq. (S7)- (S8)) as

$$\sum_n n^2|a_n|^2 = -\frac{1}{2\pi\Omega}\int_0^{2\pi/\Omega} dt\, \frac{\partial^2 a}{\partial t^2}a^*(t). \tag{S13}$$

This can be verified when using Eq. (S7) and $\int_0^{2\pi/\Omega} dt\, e^{i(n-n')\Omega t} = (2\pi/\Omega)\delta_{nn'}$ for integers n and n'. Then, we use the integration by parts,

$$\sum_n n^2|a_n|^2 = -\frac{1}{2\pi\Omega}\Big[\frac{\partial a}{\partial t}a^*(t)\Big]_0^{2\pi/\Omega} + \frac{1}{2\pi\Omega}\int_0^{2\pi/\Omega} dt\, \frac{\partial a}{\partial t}\frac{\partial a^*}{\partial t}. \tag{S14}$$

The first term vanishes using Eq. (S10). We relate the second term to the AC voltage using $|a(t)|^2 = 1$ and

$$\frac{\partial a^*}{\partial t} = \frac{\partial}{\partial t}\frac{1}{a} = -\frac{1}{a^2}\frac{\partial a}{\partial t}. \tag{S15}$$

Using this and (S10), we obtain

$$\sum_n n^2|a_n|^2 = \frac{1}{2\pi\Omega}\int_0^{2\pi/\Omega} dt\,\Big(\frac{e\mathcal{V}_{\rm ac}(t)}{\hbar}\Big)^2. \tag{S16}$$

Then Eq. (S16) is equal to Eq. (S12) squared. □

DERIVATION OF EQ. (11)

Here we derive the deviation of entropy from the Clausius relation, Eq. (11), in the regime of small biases $k_B|\theta_L-\theta_R|$, $k_B|\mu_L-\mu_R| \ll k_B\theta_L$, and small energy uncertainty induced by the AC voltage, $\delta n\,\hbar\Omega \ll k_B\theta_L$.

First, we approximate the outgoing distribution $f_\alpha^{\rm (out)}(\mathcal{E})$ in the small driving frequency. We use the relation of the outgoing distribution to the ingoing distribution in terms of the Floquet scattering matrix,

$$f_\alpha^{\rm (out)}(\mathcal{E}) = \sum_{\beta n}|\mathcal{S}_{\alpha\beta}(\mathcal{E},\mathcal{E}_{-n})|^2 f_\beta(\mathcal{E}_{-n}). \tag{S17}$$

Due to the small energy uncertainty $\delta n\,\hbar\Omega \ll k_B\theta_L$, the ingoing distribution shifted by energy quanta (much smaller than thermal broadening) is similar to the original distribution. Therefore, we approximate

$$f_\beta(\mathcal{E}_{-n}) = f_\beta(\mathcal{E}) - n\,\hbar\Omega f_\beta'(\mathcal{E}) + \frac{(n\,\hbar\Omega)^2}{2}f_\beta''(\mathcal{E}). \tag{S18}$$

Here $f_\beta'(\mathcal{E})$ and $f_\beta''(\mathcal{E})$ are the first and second derivative of the ingoing distribution $f_\beta(\mathcal{E})$. Then, plugging Eq. (S18) into Eq. (S17), we obtain an approximation for the output distribution,

$$\begin{aligned} f_\alpha^{\rm (out)}(\mathcal{E}) = &\sum_{\beta n}|\mathcal{S}_{\alpha\beta}(\mathcal{E},\mathcal{E}_{-n})|^2 f_\beta(\mathcal{E}) \\ &- \sum_{\beta n} n|\mathcal{S}_{\alpha\beta}(\mathcal{E},\mathcal{E}_{-n})|^2\,\hbar\Omega\, f_\beta'(\mathcal{E}) \\ &+ \frac{1}{2}\sum_{\beta n} n^2|\mathcal{S}_{\alpha\beta}(\mathcal{E},\mathcal{E}_{-n})|^2\,(\hbar\Omega)^2\, f_\beta''(\mathcal{E})\,. \end{aligned} \tag{S19}$$

We use Eq. (3) for the Floquet scattering matrix and sum over the photon number. Using $\langle n\rangle = 0$, we have

$$f_\alpha^{\rm (out)}(\mathcal{E}) = f_\alpha^{\rm (out,st)}(\mathcal{E}) + \frac{1}{2}(\delta n)^2\,|\mathcal{S}_{\alpha L}^{\rm (st)}(\mathcal{E})|^2\, f_L''(\mathcal{E})\,(\hbar\Omega)^2\,. \tag{S20}$$

Here, $f_\alpha^{\rm (out,st)}(\mathcal{E}) \equiv \sum_\beta |\mathcal{S}_{\alpha\beta}^{\rm (st)}(\mathcal{E})|^2 f_\beta(\mathcal{E})$ is the outgoing distribution in the static case.

From Eq. (S20), we use that the second term is much smaller than the first term, due to the condition $\delta n\,\hbar\Omega \ll k_B\theta_L$, and we approximate the Shannon entropy of the outgoing distribution up to leading order in Ω,

$$\sigma[f_\alpha^{\rm (out)}(\mathcal{E})] = \sigma[f_\alpha^{\rm (out,st)}(\mathcal{E})] + \sigma'[f_\alpha^{\rm (out,st)}(\mathcal{E})]\,\frac{(\delta n)^2}{2}\,|\mathcal{S}_{\alpha L}^{\rm (st)}(\mathcal{E})|^2\, f_L''(\mathcal{E})\,(\hbar\Omega)^2\,. \tag{S21}$$

Here $\sigma'[x] \equiv d\sigma[x]/dx$. Then, we obtain the approximation of the entropy production with the condition $\delta n\,\hbar\Omega \ll k_B\theta_L$,

$$\overline{\dot{S}} = \overline{\dot{S}^{\rm (st)}} + \sum_\alpha \frac{(\delta n)^2}{2h}\int d\mathcal{E}\, k_B\,\sigma'[f_\alpha^{\rm (out,st)}(\mathcal{E})]\,|\mathcal{S}_{\alpha L}^{\rm (st)}(\mathcal{E})|^2\, f_L''(\mathcal{E})(\hbar\Omega)^2, \tag{S22}$$

where $\overline{\dot{S}^{(\mathrm{st})}}$ is the time-averaged entropy production in the static case.

Now, we apply the condition of the small biases to Eq. (S22). In the leading order of the small biases, $\overline{\dot{S}^{(\mathrm{st})}} = \sum_\alpha \overline{I_h^\alpha}/\theta_\alpha$ (see below) and $\sigma'[f_\alpha^{(\mathrm{out,st})}(\mathcal{E})] \approx \sigma'[f_L^{(\mathrm{in})}(\mathcal{E})]$. Using $\sigma'[f_L^{(\mathrm{in})}(\mathcal{E})] = (\mathcal{E}-\mu_L)/(k_B\theta_L)$ and $\sum_\alpha |\mathcal{S}_{\alpha L}^{(\mathrm{st})}(\mathcal{E})|^2 = 1$, we obtain the deviation of the entropy production from the Clausius relation, in the case of small biases and small energy uncertainty

$$\delta\overline{\dot{S}} = \frac{(\delta n\,\hbar\Omega)^2}{2h\theta_L}\int d\mathcal{E}\, f_L''(\mathcal{E})(\mathcal{E}-\mu_L). \tag{S23}$$

This yields Eq. (11), because using the integration by parts, $\int d\mathcal{E}\, f_L''(\mathcal{E})(\mathcal{E}-\mu_L) = -\int d\mathcal{E}\, f_L'(\mathcal{E}) = 1$.

For completeness, we show the derivation for $\overline{\dot{S}^{(\mathrm{st})}} = \sum_\alpha \overline{I_h^\alpha}/\theta_\alpha$ in the leading order of the small biases. The outgoing distribution in the static case is expanded for small biases as,

$$\begin{aligned} f_\alpha^{(\mathrm{out,st})}(\mathcal{E}) &= \sum_\beta |S_{\alpha\beta}^{(\mathrm{st})}(\mathcal{E})|^2 f_\beta(\mathcal{E}) \\ &= f_\alpha(\mathcal{E}) + \sum_\beta |S_{\alpha\beta}^{(\mathrm{st})}(\mathcal{E})|^2 \{f_\beta(\mathcal{E}) - f_\alpha(\mathcal{E})\}\,. \end{aligned} \tag{S24}$$

Then, Shannon entropy of the outgoing distribution is expanded,

$$\begin{aligned} \sigma[f_\alpha^{(\mathrm{out,st})}(\mathcal{E})] &= \sigma[f_\alpha(\mathcal{E})] \\ &+ \sigma'[f_\alpha(\mathcal{E})]\sum_\beta |S_{\alpha\beta}^{(\mathrm{st})}(\mathcal{E})|^2\{f_\beta(\mathcal{E}) - f_\alpha(\mathcal{E})\}\,. \end{aligned} \tag{S25}$$

Using $\sigma'[f_\alpha(\mathcal{E})] = (\mathcal{E}-\mu_\alpha)/\theta_\alpha$, the entropy production is found to be,

$$\overline{\dot{S}^{(\mathrm{st})}} = \sum_{\alpha\beta}\frac{1}{h}\int d\mathcal{E}\,\frac{\mathcal{E}-\mu_\alpha}{\theta_\alpha}|S_{\alpha\beta}^{(\mathrm{st})}(\mathcal{E})|^2\{f_\beta(\mathcal{E}) - f_\alpha(\mathcal{E})\}\,. \tag{S26}$$

This equals $\sum_\alpha \overline{I_h^\alpha}/\theta_\alpha$.

CHOICE OF DC VOLTAGE BIAS TO MAXIMIZE THE POWER

Here we discuss the choice of DC voltage bias which maximizes the generated power, for given temperatures of the reservoirs and average chemical potential μ. Then, this bias is used for obtaining the results of Figs. 2, 3 and 4. We followed the approach of the linear thermoelectricity [5], deriving response coefficients in the linear regime, namely when the thermal and voltage biases are small compared to the average temperature, $k_B|\Delta\theta|, |e\Delta V| \ll k_B\theta$, where $\Delta\theta \equiv \theta_L - \theta_R$, $\Delta V \equiv (\mu_L - \mu_R)/e$, $\theta \equiv (\theta_L + \theta_R)/2$.

To expand the generated power $-\Delta V\overline{I_e^R}$, ($\overline{P_{\mathrm{in}}} = 0$ for the setup of chiral conductors) we consider the linear regime for the current up to the first order of the biases ΔV and $\Delta\theta$. Using Eq. (4) and expanding the Fermi distributions in the small biases, we obtain $\overline{I_e^R} = \overline{I_e^R}|_{\Delta V=\Delta\theta=0} + G\Delta V + L\Delta\theta$,

$$G = \frac{e}{h}\int d\mathcal{E}\,\frac{\mathcal{T}(\mathcal{E})+\mathcal{T}'(\mathcal{E})}{2}\big(-f'(\mathcal{E})\big), \tag{S27}$$

$$L = \frac{1}{h}\int d\mathcal{E}\,\frac{\mathcal{T}(\mathcal{E})+\mathcal{T}'(\mathcal{E})}{2}\big(-f'(\mathcal{E})\big)\frac{\mathcal{E}-\mu}{\theta}. \tag{S28}$$

Here, G and L are the response coefficients of the time-averaged electrical and thermoelectrical currents. $\overline{I_e^R}|_{\Delta V=\Delta\theta=0}$ is the electric pump current in the absence of the biases. $f(\mathcal{E})$ is the Fermi-Dirac distribution of the average temperature $\theta = (\theta_L+\theta_R)/2$ and average chemical potential $\mu = (\mu_L+\mu_R)/2$. Now, the generated power is in a quadratic form of ΔV, hence generated power is maximal for the voltage bias

$$\Delta V = -\frac{L}{2G}\Delta\theta - \frac{\overline{I_e^R}|_{\Delta V=\Delta\theta=0}}{2G}. \tag{S29}$$

PHOTOASISTED TRANSITION AMPLITUDES FOR THE LORENTZIAN VOLTAGE BIAS

For completeness, we give the photoassisted transition amplitudes a_n in the chiral conductor driven by the Lorentzian voltage pulses described in the main text, which is obtained in Ref. [36],

$$\begin{aligned} a_n &= -e^{-n\Omega w}(1-e^{-2\Omega w})\,, \quad (n \ge 0) \\ a_{-1} &= e^{-\Omega w}\,, \\ a_n &= 0\,. \quad (n \le -2) \end{aligned} \tag{S30}$$

DETAILED RESULTS FOR FIG. 2–3

Fig. S2 shows detailed results used to obtain Figs. 2–3.

NONCHIRAL CONDUCTORS

Here, we show that for nonchiral conductors, a large power injection $\overline{P}_{\mathrm{in}}$ from the AC driving into the system diminishes the heat engine efficiency, in contrast to the case of chiral conductors. By analyzing the mean number of photons involved in the photoassisted scattering, we show that the AC power injection is positive and determined by Joule's law in the slow driving and zero temperature regime. We also show numerical results proving the efficiency decrease.

Figure S3 shows the setup of nonchiral conductors driven by AC voltage bias. In this case, the Floquet

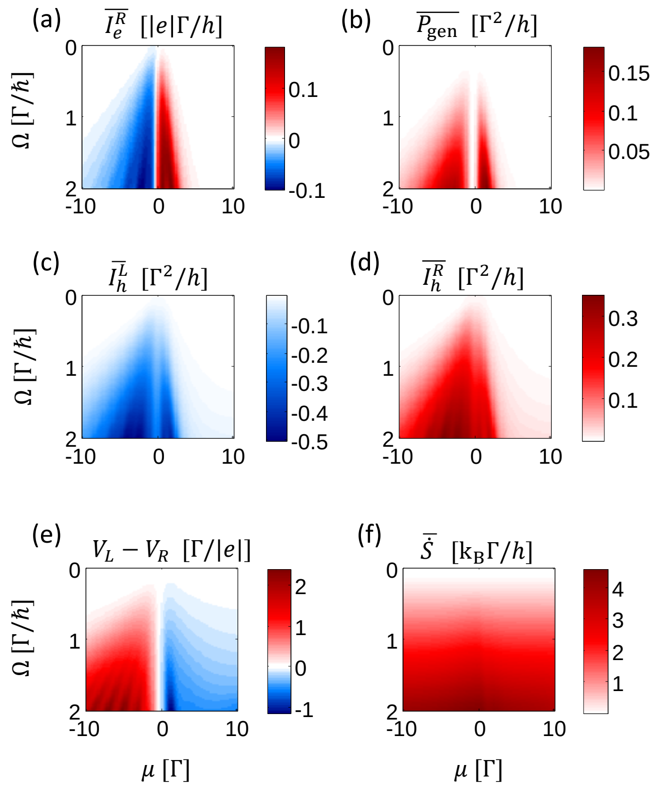

Fig. S2. Detailed results used to obtain Figs. 2 and 3, such as charge current (a), generated power (b), heat current into the left reservoir (c), heat current into the right reservoir (d), DC bias voltage choices $(\mu_L - \mu_R)/e$ maximizing generated power [see Eq. (S29)] (e), and entropy production rate (f). In all the panels, horizontal axes are average chemical potential μ in units of Γ and vertical axes are angular frequency of the ac voltage Ω in units of $\Gamma/\hbar$.

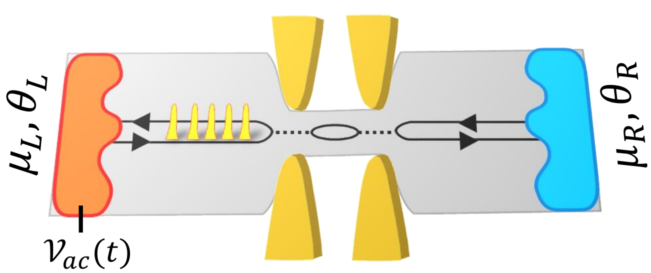

Fig. S3. Setup of nonchiral conductor. In contrast to the chiral conductor, the left-going and right-going modes are not spatially separated.

scattering matrix (including the photoassisted processes due to the AC voltage) becomes different from that of the chiral case, due to an additional process that an electron absorbs ($n > 0$) or emits ($n < 0$) $|n|$ photons when it is back scattered into the left contact with transition amplitude a^*_{-n}; see Fig. S1 and related text. Therefore, the Floquet scattering matrix becomes

$$\begin{aligned}
S_{RL}(\mathcal{E}_n, \mathcal{E}) &= a_n t_{\rm st}(\mathcal{E}_n), \\
S_{LL}(\mathcal{E}_n, \mathcal{E}) &= \sum_m a_m a^*_{-n+m} r_{\rm st}(\mathcal{E}_m), \\
S_{LR}(\mathcal{E}_n, \mathcal{E}) &= a^*_{-n} t'_{\rm st}(\mathcal{E}), \\
S_{RR}(\mathcal{E}_n, \mathcal{E}) &= \delta_{n0} r'_{\rm st}(\mathcal{E}).
\end{aligned} \tag{S31}$$

Note that the reflection amplitude $S_{LL}(\mathcal{E}_n, \mathcal{E})$ from left input channel is determined by three steps rather than the two steps of the chiral case (see Eq. (3)), due to the additional photon absorption/emission process which occurs when the electron finally enters the left reservoir. As before, the Floquet scattering matrix satisfies the unitarity condition $\sum_{n\alpha} |S_{\alpha\beta}(\mathcal{E}_n, \mathcal{E})|^2 = 1$ and $\sum_{n\beta} |S_{\alpha\beta}(\mathcal{E}, \mathcal{E}_{-n})|^2 = 1$.

The power injection from AC driving $\overline{P}_{\rm in}$ does not vanish in the nonchiral setup, in contrast to the chiral setup. We show this by investigating the mean photon number involved in the photoassisted scattering. They fulfill

$$\begin{aligned}
\langle n(\mathcal{E})\rangle_{RL} + \langle n(\mathcal{E})\rangle_{LL} &= \sum_n n|a_n|^2 \, |t_{\rm st}(\mathcal{E}_n)|^2 \\
&+ \sum_{n,m,l} n\{a_m a^*_{-n+m} r_{\rm st}(\mathcal{E}_m)\}^* \, a_l a^*_{-n+l} \, r_{\rm st}(\mathcal{E}_l) \, .
\end{aligned} \tag{S32}$$

$$\langle n(\mathcal{E})\rangle_{LR} + \langle n(\mathcal{E})\rangle_{RR} = 0 \, . \tag{S33}$$

The last term of Eq. (S32) is the result of interferences of all possible processes in which, e.g., an electron from the left reservoir first absorbs m photons, reflects at the resonance level, and absorbs $m - n$ photons. The appearance of such term, contrarily to the case of chiral conductor, is key factor for finite AC power injection in the nonchiral setup.

To clarify the meaning of the last term of Eq. (S32), we change the variable n to a new variable $p \equiv -n + m$ in the last term in Eq. (S32),

$$\begin{aligned}
\langle n(\mathcal{E})\rangle_{RL} + \langle n(\mathcal{E})\rangle_{LL} &= \sum_n n|a_n|^2 \, |t_{\rm st}(\mathcal{E}_n)|^2 \\
&- \sum_{p,m,l} p \, a_p a^*_{p+l-m} a^*_m a_l \, r^*_{\rm st}(\mathcal{E}_m) \, r_{\rm st}(\mathcal{E}_l) \\
&+ \sum_{p,m,l} m \, a_p a^*_{p+l-m} a^*_m a_l \, r^*_{\rm st}(\mathcal{E}_m) \, r_{\rm st}(\mathcal{E}_l) \, .
\end{aligned} \tag{S34}$$

For the last term, we use

$$\sum_p a_p a^*_{p+l-m} = \delta_{m,l}. \tag{S35}$$

This property follows from the fact that the left hand side is the $(m-l)$-th Fourier coefficient of $e^{i\phi_{\rm ac}(t)} e^{-i\phi_{\rm ac}(t)} = 1$. Using Eq. (S35) the last term and the first term of the right hand side of Eq. (S34) vanish due to $\langle n \rangle = 0$. Then, we obtain

$$\langle n(\mathcal{E})\rangle_{RL} + \langle n(\mathcal{E})\rangle_{LL} = -\sum_{p,m,l} p\, a_p a^*_{p+l-m} \times a^*_m a_l\, r^*_{\rm st}(\mathcal{E}_m)\, r_{\rm st}(\mathcal{E}_l)\,. \tag{S36}$$

We utilize the fact that [$a(t)$ is the Fourier transform of the phototransition probabilities, see Eq. (S7)]

$$\sum_p p\, a_p a^*_{p+l-m} = \frac{i}{2\pi}\int_0^{2\pi/\Omega} dt\, \frac{\partial a}{\partial t} a^*(t) e^{i(m-l)\Omega t} = \frac{1}{2\pi}\int_0^{2\pi/\Omega} dt\, \frac{e\mathcal{V}_{\rm ac}(t)}{\hbar} e^{i(m-l)\Omega t}\,. \tag{S37}$$

The first equality can be verified when employing the definition of $a(t)$, Eq. (S7). In the second equality, Eq. (S10) is used. Using Eqs. (S36), (S37), we obtain the mean number

$$\langle n(\mathcal{E})\rangle_{RL} + \langle n(\mathcal{E})\rangle_{LL} = -\frac{1}{2\pi}\int_0^{2\pi/\Omega} dt\, \frac{e\mathcal{V}_{\rm ac}(t)}{\hbar} |\psi_r(t;\mathcal{E})|^2, \tag{S38}$$

where $\psi_r(t;\mathcal{E}) \equiv \sum_l a_l r_{\rm st}(\mathcal{E}_l) e^{-il\Omega t}$ is the wave function immediately after the reflection at time t for an electron of initial energy $\mathcal{E}$. The mean number of Eq. (S38) is generally nonzero and contributes to the AC power injection as explicitly demonstrated for the slow driving and zero temperature limits.

In the slow driving regime, $\hbar\Omega \ll \Gamma$, we further simplify Eq. (S38). Expanding the wave function in the driving frequency with $r_{\rm st}(\mathcal{E}_l) = r_{\rm st}(\mathcal{E}) + (\partial r_{\rm st}/\partial\mathcal{E}) l\hbar\Omega$, we find

$$\psi_r(t;\mathcal{E}) = a(t) r_{\rm st}(\mathcal{E}) + i\hbar \frac{\partial r_{\rm st}}{\partial \mathcal{E}} \frac{\partial a}{\partial t}. \tag{S39}$$

The first term of Eq. (S39) does not contribute to the mean number $\langle n(\mathcal{E})\rangle_{RL} + \langle n(\mathcal{E})\rangle_{LL}$, as the contribution is proportional to $\int_0^{2\pi/\Omega} dt\, \mathcal{V}_{\rm ac}(t)|a(t)|^2 = \int_0^{2\pi/\Omega} dt\, \mathcal{V}_{\rm ac}(t) = 0$. We now consider the second term,

$$\langle n(\mathcal{E})\rangle_{RL} + \langle n(\mathcal{E})\rangle_{LL} = -\frac{1}{2\pi}\int_0^{2\pi/\Omega} dt\, e\mathcal{V}_{\rm ac}(t) \times 2\mathrm{Re}\Big[i r^*_{\rm st}(\mathcal{E}) \frac{\partial r_{\rm st}}{\partial\mathcal{E}} a^*(t) \frac{\partial a}{\partial t}\Big]. \tag{S40}$$

Using Eq. (S10), we obtain

$$\langle n(\mathcal{E})\rangle_{LL} + \langle n(\mathcal{E})\rangle_{RL} = -\frac{\partial |r_{\rm st}(\mathcal{E})|^2}{\partial\mathcal{E}} \frac{e^2\overline{\mathcal{V}^2_{\rm ac}(t)}}{\hbar\Omega}. \tag{S41}$$

In the zero-temperature limit, $\theta_L \ll \Gamma$, we obtain a simple expression of the AC power injection, using Eq. (7) and (S41),

$$\overline{P_{\rm in}} = |t_{\rm st}(\mu_L)|^2 \frac{e^2}{h} \overline{\mathcal{V}^2_{\rm ac}(t)} \tag{S42}$$

This is the AC power determined by Joule's law with conductance $|t_{\rm st}(\mu)|^2 e^2/h$. It shows that the AC power injection is generally nonzero for nonchiral setups, in contrast to the chiral setup. One should not apply Eq. (S42) with $|t_{\rm st}(\mu)|^2 = 1$ to the situation when there is no resonance level, because the Floquet scattering matrix becomes different, as $\mathcal{S}_{RL}(\mathcal{E}_n,\mathcal{E}) = a_n$, $\mathcal{S}_{LR}(\mathcal{E}_n,\mathcal{E}) = a^*_{-n}$, $\mathcal{S}_{LL}(\mathcal{E}_n,\mathcal{E}) = \mathcal{S}_{RR}(\mathcal{E}_n,\mathcal{E}) = 0$, which leads to $\overline{P_{\rm in}} = 0$.

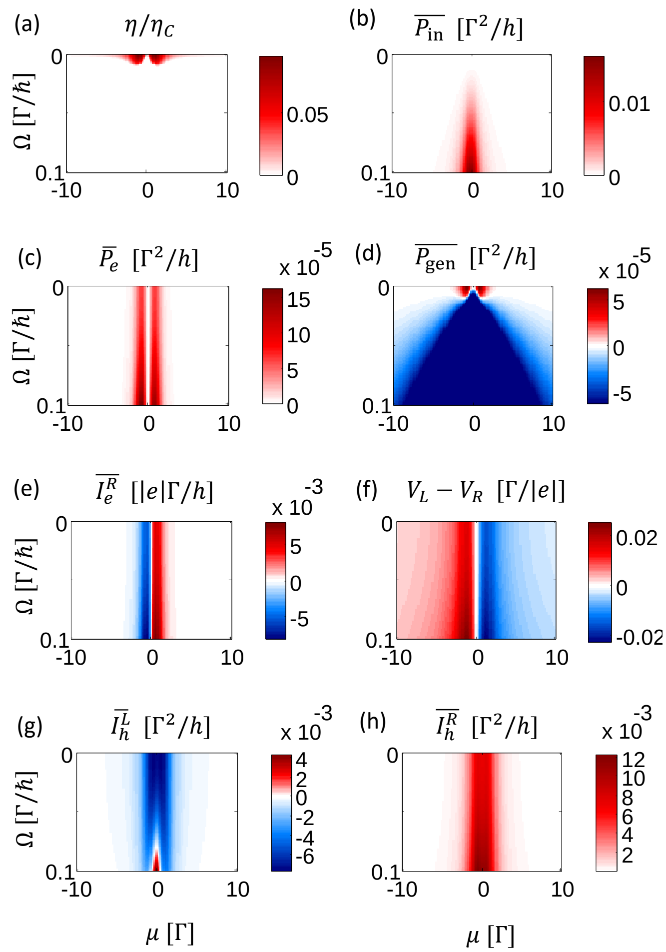

Fig. S4. Heat engine efficiency in the AC driven nonchiral conductor of Fig. S3. (a) The efficiency decreases when the AC frequency increases, satisfying the Carnot limit, in contrast to the chiral-conductor. This is due to large positive AC power injection (b) which dominates the electric power (c) and makes the generated power (d) negative. Here, the generated power lower than $5\times10^{-5}\Gamma^2/h$ is plotted with blue color. Additional details are also shown, such as charge current (e), DC voltage maximizing electric power chosen as in Eq. (S29) (f), and heat current into the left (g), and right (h) reservoir.

Fig. S4 shows the efficiency decrease by the AC driving in the case of nonchiral conductors. The parameters such as AC driving protocol, temperatures θ_L and θ_R are the same as in Fig. 2.