Supplementary material for:
Inferring community structure in attributed hypergraphs using stochastic block models

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S1 Inference of community structure in synthetic hypergraphs

S1.1 Methods

We generate hypergraphs with node attributes as follows. We fix $N = 1,000$ nodes, $K = 2$ communities, and $Z = 2$ categories of the node attribute. We introduce the set of parameters $\{p_{\boldsymbol{U}}, w_{\text{in}}, D, |E|/N\}$ to control community structure and node attributes. First, we generate a hypergraph with community structure using the algorithm presented in Ref. [8]. To this end, we input $\boldsymbol{U}$, $\boldsymbol{W}$, and the distribution of the hyperedge size to the algorithm to sample a hypergraph conditioned on them as follows. We input $\boldsymbol{U}$ using a given probability $p_{\boldsymbol{U}}$: we assign the membership vector $[p_{\boldsymbol{U}}, 1-p_{\boldsymbol{U}}]$ to 500 nodes chosen uniformly at random and the membership vector $[1-p_{\boldsymbol{U}}, p_{\boldsymbol{U}}]$ to other nodes. We input $\boldsymbol{W}$, setting its diagonal entries to the value $w_{\text{in}} > 0$ and all other entries to 1. We input the uniform distribution of the hyperedge size given the maximum size of the hyperedge, D, and the hyperedge sparsity (i.e., the number of hyperedges divided by that of nodes), $|E|/N$. Afterward, we generate node attributes $\boldsymbol{X}$ according to Eqs. (7)–(9), where we set $\boldsymbol{\beta}$ as the identity matrix of size two; in this case, the attribute of a node explicitly encodes the propensity to which the node belongs to each community. We independently generate 100 hypergraphs using this procedure for a given combination of parameter values $(p_{\boldsymbol{U}}, w_{\text{in}}, D, |E|/N)$.

We examine the 41 unique combinations of parameter values $(p_{\boldsymbol{U}}, w_{\text{in}}, D, |E|/N)$ as follows: (i) we vary $p_{\boldsymbol{U}}$ from 0.5 to 1.0 in increments of 0.1 while fixing $(w_{\text{in}}, D, |E|/N) = (10, 10, 10)$; (ii) we vary w_{in} from 0.1 to 0.9 in increments of 0.1 and from 1.0 to 10.0 in increments of 1.0 while fixing $(p_{\boldsymbol{U}}, D, |E|/N) = (0.8, 10, 10)$; (iii) we vary D from 2 to 10 in increments of 1 while fixing $(p_{\boldsymbol{U}}, w_{\text{in}}, |E|/N) = (0.8, 10, 10)$; and (iv) we vary $|E|/N$ from 2 to 20 in increments of 2 while fixing $(p_{\boldsymbol{U}}, w_{\text{in}}, D) = (0.8, 10, 10)$.

To measure the quality of inferred communities in synthetic hypergraphs, we calculate the cosine similarity between the ground-truth membership matrix, denoted by $(u^{\text{g}}_{ik})_{1\le i\le N,\ 1\le k\le K}$, and an inferred membership matrix, denoted by $(u_{ik})_{1\le i\le N,\ 1\le k\le K}$, as in Refs. [4]:

$$\frac{1}{N}\sum_{i=1}^{N} \frac{\sum_{k=1}^{K} u_{ik} u^{\text{g}}_{ik}}{\sqrt{\left[\sum_{k=1}^{K}(u_{ik})^2\right]\left[\sum_{k=1}^{K}(u^{\text{g}}_{ik})^2\right]}}. \tag{S1}$$

The order of the indices of the K ground-truth communities may not align with that in the K inferred communities. Therefore, we compute the cosine similarity for all $K!$ permutations of the indices of the K inferred communities, and we regard the highest one as the cosine similarity [4].

S1.2 Results

We begin by assessing the proposed model on synthetic hypergraphs. For a given combination of parameter values $(p_{\boldsymbol{U}}, w_{\text{in}}, D, |E|/N)$, we compute the average of the cosine similarity for the proposed model over the 100 synthetic hypergraphs generated for a given combination $(p_{\boldsymbol{U}}, w_{\mathrm{in}}, D, |E|/N)$. We set the hyperparameter $K = 2$ in the proposed model.

Figure S1(a) shows the cosine similarity for the proposed model with a given value of the hyperparameter γ when we vary the parameter $p_{\boldsymbol{U}}$ for synthetic hypergraphs. The hyperparameter γ controls the extent to which the proposed model incorporates node attribute data into the inference of community structure in a given hypergraph. In our synthetic hypergraphs, the propensity to which a node belongs to different communities is controlled by $p_{\boldsymbol{U}}$ and the attribute of a node explicitly encodes the propensity of the node. Therefore, the value of γ at which the proposed model achieves the highest cosine similarity is expected to depend on $p_{\boldsymbol{U}}$. Indeed, when $p_{\boldsymbol{U}} = 0.5$, the proposed model with a smaller value of γ yields a higher cosine similarity (see Fig. S1(a)), indicating that the node attributes little contribute to the inference of community structure. This is because each node belongs to the two communities with the same propensity when $p_{\boldsymbol{U}} = 0.5$, making it difficult to associate the node attributes with the communities. On the other hand, when $p_{\boldsymbol{U}} = 1.0$, the proposed model with a higher value of γ yields a higher cosine similarity (see Fig. S1(a)). This is because each node belongs to one of the two communities when $p_{\boldsymbol{U}} = 1.0$, and the community to which each node belongs is directly linked with the attribute of the node.

We next compare the proposed model with the two baseline models (i.e., Hy-MMSBM [9] and HyCoSBM [1]) in terms of the cosine similarity in synthetic hypergraphs. We set the hyperparameter $K = 2$ in all the models and set the hyperparameter $\gamma = 0.5$ in the HyCoSBM and the proposed model. We compute the average of the cosine similarity for each model over the 100 hypergraphs generated for a given combination $(p_{\boldsymbol{U}}, w_{\mathrm{in}}, D, |E|/N)$.

We found that the cosine similarity for the proposed model is higher than that for the Hy-MMSBM when $0.7 \leq p_{\boldsymbol{U}} \leq 1.0$ and that for the HyCoSBM when $0.6 \leq p_{\boldsymbol{U}} \leq 1.0$ (see Fig. S1(b)). We also found that the cosine similarity for the proposed model usually is higher than that for the two baseline models when we fix $p_{\boldsymbol{U}} = 0.8$ and vary another structural parameter (i.e., w_{in}, D, or $|E|/N$; see Figs. S1(c)–S1(f)).

We compare the average of the cosine similarity between the HyCoSBM [1] and the proposed model when we vary the value of γ from 0.1 to 0.9 in increments of 0.1 in both models. We set $K = 2$ in both models.

Tables S1–S5 show the cosine similarity for the HyCoSBM and the HyperNEO with a given value of γ when we vary one of the parameters for synthetic hypergraphs. First, the proposed model with $\gamma = 0.4$ or $\gamma = 0.5$ achieves a high value of the average cosine similarity over the different values of $p_{\boldsymbol{U}}$ (See Table S1). Second, when we vary any parameter for synthetic hypergraphs, the proposed model with $\gamma = 0.5$ achieves a higher value of the average cosine similarity than the HyCoSBM with any value of γ (see Tables S1–S5).

These results for synthetic hypergraphs indicate the following. First, node attribute data contributes to the inference of community structure in hypergraphs when node attributes are sufficiently associated with the communities. Second, not imposing any constraint on the membership matrix in inferring it yields a comparable or higher inference accuracy than imposing it.

S2 Pseudocode of the HyperNEO

Algorithm S1 shows the pseudocode of our algorithm. HyperNEO takes a hypergraph with node attributes G and the hyperparameters (K, γ) as inputs and returns a set of inferred variables $(\boldsymbol{U}, \boldsymbol{W}, \boldsymbol{\beta})$.

S3 Empirical hypergraphs

Table S6 shows the properties of the empirical hypergraphs. For any empirical hypergraph, the set E contains a unique hyperedge, e, composed of the same set of nodes, and the number of times e appears in the data set is stored in A_e.

Algorithm S1 HyperNEO

Require: Hypergraph with node attributes: $(V, E, \boldsymbol{X})$; number of communities: K; scaling parameter: γ.

Ensure: Inferred parameters: $(\boldsymbol{U}, \boldsymbol{W}, \boldsymbol{\beta})$.

BestLoglik $= -\infty$
BestParams = None
$N_{\mathrm{R}} = 10$ // Number of random initializations.
$N_{\mathrm{I}} = 20$ // Number of iterations.
for $r = 1, \ldots, N_{\mathrm{R}}$ **do**
  Initialize $\boldsymbol{U}$, $\boldsymbol{W}$, and $\boldsymbol{\beta}$ uniformly at random.
  for $i = 1, \ldots, N_{\mathrm{I}}$ **do**
    Calculate ρ using Eq. (13) and h using Eq. (16).
    Update $\boldsymbol{U}$ using Eq. (21).
    Update $\boldsymbol{W}$ using Eq. (22).
    Update $\boldsymbol{\beta}$ using Eq. (23).
  L $= (1-\gamma)\mathcal{L}_{\mathcal{A}}(\boldsymbol{U}, \boldsymbol{W}) + \gamma \mathcal{L}_{\boldsymbol{X}}(\boldsymbol{U}, \boldsymbol{W})$
  if L $>$ BestLoglik **then**
    BestLoglik $\leftarrow$ L
    BestParams $\leftarrow (\boldsymbol{U}, \boldsymbol{W}, \boldsymbol{\beta})$
return BestParams

S4 Hyperparameter tuning in empirical hypergraphs

Table S7 shows the tuned hyperparameter set for each model in each empirical hypergraph.

S5 Visualization of inferred membership and affinity matrices

Suppose that a given model produces inferred membership and affinity matrices, denoted by $(u_{ik})_{1 \le i \le N,\ 1 \le k \le K}$ and $(w_{kq})_{1 \le k \le K,\ 1 \le q \le K}$, respectively.

We perform the visualization of the inferred membership matrix as follows. First, we construct the normalized membership matrix $(u'_{ik})_{1 \le i \le N,\ 1 \le k \le K}$, where we define $u'_{ik} = u_{ik} / \sum_{k'=1}^{K} u_{ik'}$. This normalization is performed only for visualization purposes. Second, we arbitrarily fix the order of the indices of the Z categories of the node attribute. Suppose that the order is given by $z_1, \ldots, z_Z$, where $z_l \in \{1, \ldots, Z\}$ for any $l = 1, \ldots, Z$. Third, we arrange the indices of the N nodes according to their attributes as follows. We first arrange arbitrarily the indices of the nodes that have the attribute z_1. Then, for $l = 2, \ldots, Z$ in order, we arrange arbitrarily the indices of the nodes that have the attribute z_l, and then, we place these indices behind the indices of the nodes that have the attribute z_{l-1}. In this way, we obtain the order of the indices of the N nodes, denoted by $i_1, \ldots, i_N$, where $i_m \in \{1, \ldots, N\}$ for any $m = 1, \ldots, N$. Fourth, we arbitrarily fix the order of the indices of the K communities. Suppose that the order is given by $k_1, \ldots, k_K$, where $k_n \in \{1, \ldots, K\}$ for any $n = 1, \ldots, K$. Finally, we visualize a heat map of the matrix $(u'_{i_m k_n})_{1 \le m \le N,\ 1 \le n \le K}$.

We perform the visualization of the inferred affinity matrix as follows. We denote by $w_{\max}$ is the largest element of the matrix. First, we calculate the normalized affinity matrix $(w'_{kq})_{1 \le k \le K,\ 1 \le q \le K}$, where we define $w'_{kq} = w_{kq} / w_{\max}$. Then, we fix the same order of the indices of the K communities, i.e., $k_1, \ldots, k_K$, as that for the inferred membership matrix. We visualize a heat map of the matrix $(w'_{k_n k_{n'}})_{1 \le n \le K,\ 1 \le n' \le K}$.

Figure S2 shows inferred affinity matrices of the three models in each empirical hypergraph.

References

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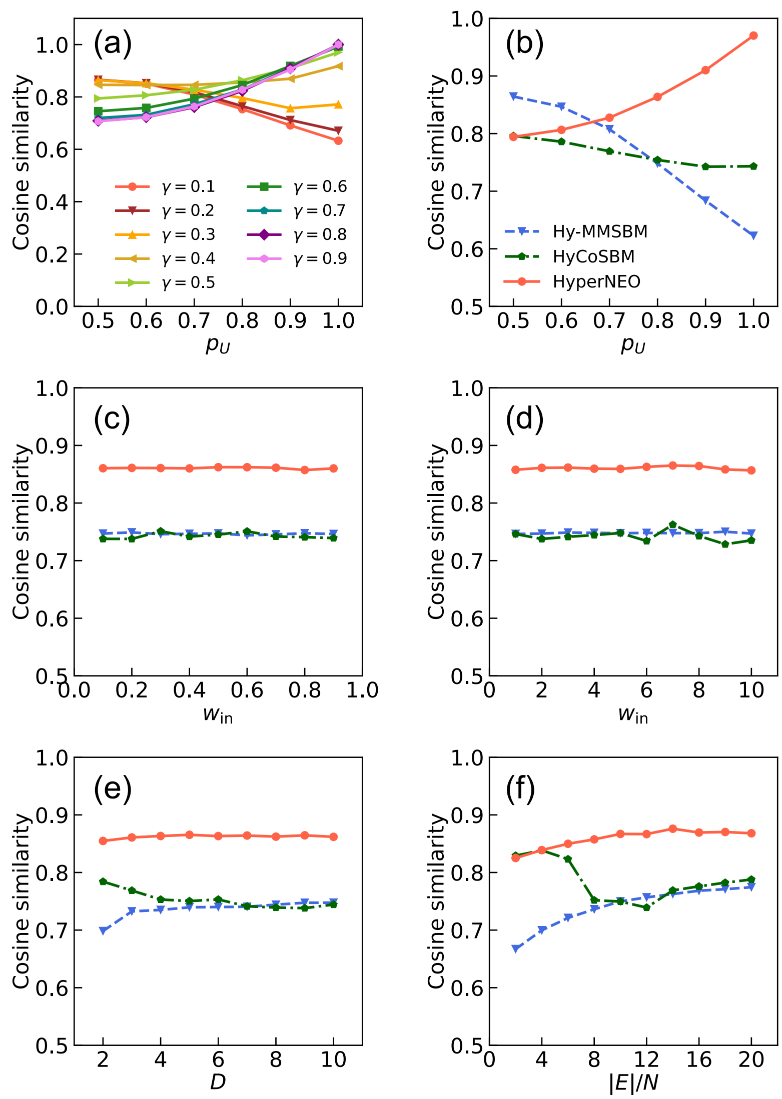

Figure S1: Inference accuracy of the proposed model on synthetic hypergraphs. (a) Cosine similarity for the proposed model with a given value of γ. (b)–(f) Comparison of the cosine similarity between the three models when we vary a parameter for synthetic hypergraphs.

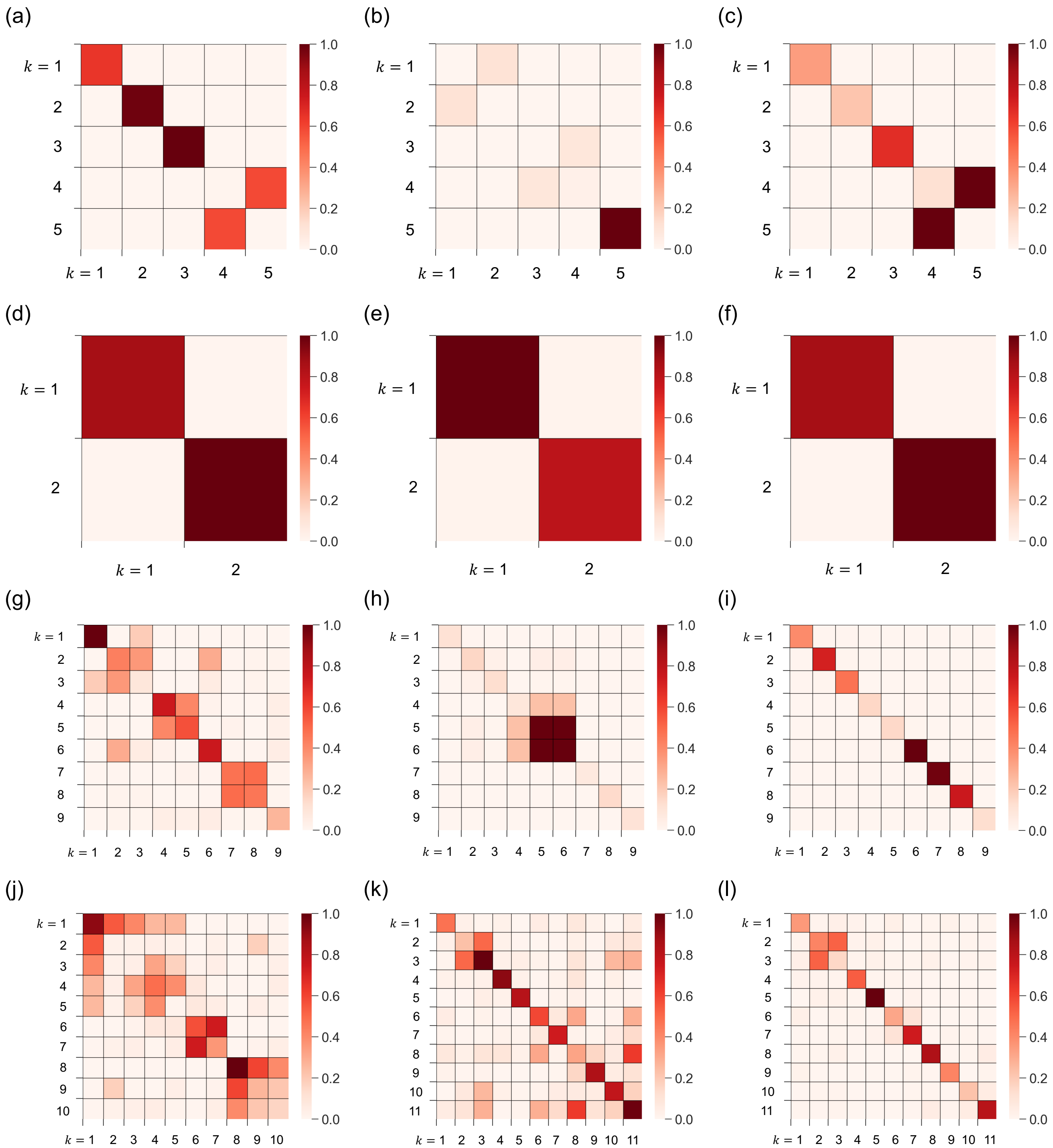

Figure S2: Inferred affinity matrices in each empirical hypergraph. Panels (a)–(c) show the results for the workplace hypergraph, panels (d)–(f) show the results for the hospital hypergraph, panels (g)–(i) show the results for the high-school hypergraph, and panels (j)–(l) show the results for the primary-school hypergraph. The results for the Hy-MMSBM are shown in (a), (d), (g), and (j), those for the HyCoSBM are shown in (b), (e), (h), and (k), and those for the proposed model are shown in (c), (f), (i), and (l).

Table S1: Cosine similarity for the HyCoSBM and HyperNEO with a given value of the hyperparameter γ when we vary $p_{\boldsymbol{U}}$. In Tables S1–S5, the rightmost column shows the average of the cosine similarity of a model with a given value of γ across the different values of a given parameter for synthetic hypergraphs, and the highest value in each column is indicated in bold.

Model	$p_{\boldsymbol{U}}$						Mean
	0.5	0.6	0.7	0.8	0.9	1.0	
HyCoSBM							
$\gamma = 0.1$	0.869	0.853	0.815	0.754	0.693	0.641	0.771
0.2	**0.873**	**0.855**	0.821	0.769	0.714	0.681	0.785
0.3	0.854	0.847	0.810	0.773	0.715	0.699	0.783
0.4	0.826	0.812	0.795	0.759	0.699	0.675	0.761
0.5	0.796	0.786	0.769	0.754	0.743	0.743	0.765
0.6	0.770	0.783	0.805	0.845	0.892	0.971	0.844
0.7	0.728	0.738	0.778	0.837	0.910	0.998	0.831
0.8	0.719	0.731	0.772	0.827	0.906	**0.999**	0.826
0.9	0.709	0.723	0.766	0.825	0.904	**0.999**	0.821
HyperNEO							
$\gamma = 0.1$	0.865	0.851	0.809	0.753	0.691	0.633	0.767
0.2	0.866	0.853	0.818	0.765	0.712	0.671	0.781
0.3	0.864	0.852	0.828	0.796	0.757	0.772	0.811
0.4	0.846	0.845	**0.846**	0.855	0.869	0.918	**0.863**
0.5	0.794	0.806	0.828	**0.864**	0.910	0.970	0.862
0.6	0.745	0.758	0.793	0.845	**0.918**	0.992	0.842
0.7	0.719	0.732	0.772	0.828	0.910	**0.999**	0.827
0.8	0.709	0.723	0.761	0.823	0.909	**0.999**	0.821
0.9	0.707	0.723	0.764	0.828	0.905	**0.999**	0.821

Table S2: Cosine similarity for the HyCoSBM and HyperNEO when we vary γ and w_{in}, where $0 < w_{\text{in}} < 1.0$.

Model	w_{in}									Mean
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
HyCoSBM										
$\gamma = 0.1$	0.758	0.755	0.756	0.753	0.754	0.753	0.756	0.754	0.754	0.755
0.2	0.767	0.769	0.765	0.768	0.765	0.764	0.765	0.765	0.770	0.766
0.3	0.766	0.765	0.765	0.765	0.767	0.760	0.767	0.770	0.766	0.766
0.4	0.746	0.743	0.758	0.749	0.754	0.733	0.753	0.742	0.754	0.748
0.5	0.738	0.738	0.751	0.742	0.745	0.751	0.742	0.741	0.739	0.743
0.6	0.840	0.838	0.840	0.850	0.846	0.847	0.850	0.844	0.836	0.843
0.7	0.832	0.835	0.837	0.832	0.830	0.835	0.835	0.832	0.834	0.834
0.8	0.834	0.830	0.832	0.830	0.829	0.830	0.832	0.827	0.829	0.830
0.9	0.823	0.823	0.827	0.824	0.828	0.825	0.835	0.827	0.828	0.827
HyperNEO										
$\gamma = 0.1$	0.751	0.751	0.751	0.753	0.752	0.754	0.751	0.750	0.751	0.752
0.2	0.763	0.761	0.766	0.761	0.763	0.763	0.760	0.762	0.761	0.762
0.3	0.794	0.794	0.793	0.794	0.793	0.795	0.798	0.789	0.791	0.793
0.4	0.853	0.855	0.858	0.851	0.855	0.856	0.853	0.841	0.847	0.852
0.5	**0.860**	**0.861**	**0.861**	**0.860**	**0.862**	**0.862**	**0.861**	**0.857**	**0.860**	**0.861**
0.6	0.841	0.846	0.845	0.846	0.849	0.842	0.845	0.845	0.845	0.845
0.7	0.833	0.833	0.835	0.833	0.827	0.826	0.829	0.834	0.832	0.831
0.8	0.825	0.832	0.825	0.823	0.825	0.831	0.828	0.823	0.828	0.827
0.9	0.826	0.828	0.823	0.823	0.820	0.824	0.826	0.822	0.823	0.824

Table S3: Cosine similarity for the HyCoSBM and HyperNEO when we vary γ and w_{in}, where $1.0 \leq w_{\text{in}} \leq 10.0$.

Model	w_{in}										Mean
	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	
HyCoSBM											
$\gamma = 0.1$	0.757	0.755	0.753	0.755	0.755	0.756	0.754	0.755	0.756	0.754	0.755
0.2	0.767	0.771	0.766	0.769	0.767	0.765	0.772	0.768	0.769	0.769	0.768
0.3	0.765	0.765	0.767	0.766	0.765	0.767	0.763	0.767	0.768	0.770	0.766
0.4	0.747	0.759	0.760	0.772	0.747	0.746	0.758	0.769	0.748	0.747	0.755
0.5	0.746	0.738	0.741	0.744	0.748	0.734	0.762	0.743	0.728	0.735	0.742
0.6	0.841	0.841	0.850	0.845	0.847	0.845	0.845	0.855	0.847	0.845	0.846
0.7	0.835	0.834	0.841	0.836	0.834	0.837	0.836	0.836	0.836	0.838	0.836
0.8	0.838	0.832	0.832	0.830	0.830	0.834	0.826	0.827	0.827	0.828	0.830
0.9	0.827	0.830	0.825	0.829	0.825	0.826	0.826	0.826	0.825	0.831	0.827
HyperNEO											
$\gamma = 0.1$	0.752	0.751	0.753	0.751	0.751	0.752	0.753	0.752	0.751	0.752	0.752
0.2	0.764	0.766	0.764	0.765	0.766	0.764	0.761	0.766	0.761	0.763	0.764
0.3	0.796	0.794	0.797	0.798	0.798	0.793	0.795	0.791	0.802	0.795	0.796
0.4	0.850	0.848	0.850	0.850	0.851	0.843	0.854	0.855	**0.859**	0.847	0.851
0.5	**0.858**	**0.861**	**0.862**	**0.860**	**0.859**	**0.863**	**0.865**	**0.864**	0.858	**0.857**	**0.861**
0.6	0.846	0.842	0.845	0.840	0.847	0.846	0.846	0.846	0.839	0.848	0.844
0.7	0.833	0.832	0.832	0.835	0.829	0.832	0.831	0.835	0.828	0.833	0.832
0.8	0.822	0.825	0.829	0.824	0.828	0.826	0.828	0.827	0.822	0.829	0.826
0.9	0.824	0.830	0.826	0.829	0.825	0.820	0.828	0.826	0.818	0.820	0.825

Table S4: Cosine similarity for the HyCoSBM and HyperNEO when we vary γ and D.

Model	D									Mean
	2	3	4	5	6	7	8	9	10	
HyCoSBM										
$\gamma = 0.1$	0.748	0.741	0.744	0.743	0.746	0.750	0.752	0.755	0.758	0.749
0.2	0.755	0.753	0.755	0.758	0.758	0.765	0.763	0.770	0.772	0.761
0.3	0.753	0.754	0.759	0.760	0.759	0.756	0.770	0.757	0.768	0.760
0.4	0.741	0.738	0.743	0.743	0.753	0.753	0.773	0.736	0.760	0.749
0.5	0.784	0.769	0.753	0.750	0.753	0.741	0.739	0.738	0.744	0.752
0.6	**0.855**	0.844	0.851	0.854	0.841	0.850	0.847	0.843	0.846	0.848
0.7	0.848	0.843	0.844	0.833	0.835	0.836	0.835	0.830	0.830	0.837
0.8	0.843	0.838	0.841	0.834	0.834	0.827	0.835	0.831	0.832	0.835
0.9	0.842	0.833	0.827	0.828	0.821	0.828	0.827	0.825	0.829	0.829
HyperNEO										
$\gamma = 0.1$	0.749	0.740	0.741	0.741	0.745	0.746	0.748	0.750	0.751	0.745
0.2	0.755	0.750	0.749	0.755	0.754	0.757	0.759	0.762	0.766	0.756
0.3	0.777	0.767	0.769	0.773	0.776	0.784	0.789	0.791	0.794	0.780
0.4	0.798	0.799	0.807	0.816	0.826	0.839	0.840	0.842	0.847	0.824
0.5	0.855	**0.861**	**0.863**	**0.865**	**0.863**	**0.864**	**0.862**	**0.865**	**0.862**	**0.862**
0.6	0.847	0.854	0.841	0.848	0.844	0.846	0.848	0.848	0.849	0.847
0.7	0.832	0.835	0.834	0.829	0.830	0.834	0.833	0.833	0.830	0.832
0.8	0.827	0.823	0.827	0.825	0.830	0.829	0.825	0.825	0.828	0.827
0.9	0.825	0.824	0.830	0.825	0.827	0.825	0.824	0.826	0.824	0.825

Table S5: Cosine similarity for the HyCoSBM and HyperNEO when we vary γ and $|E|/N$.

Model	$\|E\|/N$										Mean
	2	4	6	8	10	12	14	16	18	20	
HyCoSBM											
$\gamma = 0.1$	0.713	0.720	0.733	0.747	0.755	0.762	0.767	0.774	0.775	0.778	0.752
0.2	0.681	0.734	0.753	0.760	0.767	0.776	0.779	0.783	0.784	0.788	0.760
0.3	0.768	0.704	0.732	0.752	0.762	0.776	0.780	0.790	0.786	0.787	0.764
0.4	0.832	0.766	0.718	0.737	0.746	0.766	0.780	0.788	0.782	0.790	0.771
0.5	0.829	0.838	0.823	0.752	0.750	0.739	0.769	0.776	0.782	0.788	0.785
0.6	0.829	0.834	0.831	0.842	0.834	0.823	0.802	0.792	0.787	0.784	0.816
0.7	0.827	0.827	0.833	0.829	0.832	0.837	0.849	0.850	0.857	0.848	0.839
0.8	0.824	0.824	0.831	0.828	0.829	0.830	0.837	0.833	0.834	0.840	0.831
0.9	0.821	0.825	0.825	0.826	0.829	0.826	0.829	0.829	0.827	0.826	0.826
HyperNEO											
$\gamma = 0.1$	0.705	0.715	0.730	0.741	0.753	0.758	0.766	0.769	0.774	0.777	0.749
0.2	0.793	0.758	0.756	0.760	0.764	0.769	0.773	0.774	0.778	0.780	0.771
0.3	0.832	0.826	0.814	0.802	0.799	0.791	0.788	0.792	0.791	0.790	0.802
0.4	**0.836**	**0.842**	**0.855**	0.857	0.850	0.841	0.833	0.835	0.823	0.820	0.839
0.5	0.825	0.839	0.850	**0.857**	**0.867**	**0.867**	**0.876**	**0.869**	**0.870**	0.868	**0.859**
0.6	0.825	0.827	0.832	0.832	0.843	0.847	0.861	0.868	0.870	**0.872**	0.848
0.7	0.828	0.824	0.826	0.824	0.832	0.838	0.837	0.845	0.848	0.857	0.836
0.8	0.827	0.826	0.823	0.828	0.822	0.826	0.825	0.835	0.825	0.835	0.827
0.9	0.829	0.820	0.824	0.822	0.825	0.821	0.821	0.827	0.828	0.822	0.824

Table S6: Properties of the empirical hypergraphs. N: number of nodes, $|E|$: number of hyperedges, $\bar{k}$: average degree of the node, $\bar{s}$: average size of the hyperedge, D: maximum size of the hyperedge, and Z: number of categories of the node attribute.

Data	N	$\|E\|$	$\bar{k}$	$\bar{s}$	D	Z	References
workplace	92	788	17.7	2.1	4	5	[6, 9]
hospital	75	1,825	59.1	2.4	5	4	[9, 11]
high-school	327	7,818	55.6	2.3	5	9	[2, 3, 7]
primary-school	242	12,704	127.0	2.4	5	11	[2, 3, 5, 10]

Table S7: Tuned hyperparameter set for each model in the empirical hypergraphs.

Data	Hy-MMSBM	HyCoSBM	HyperNEO
	K	(K, γ)	(K, γ)
workplace	5	(5, 0.9)	(5, 0.9)
hospital	2	(2, 0.2)	(2, 0.4)
high-school	9	(9, 0.9)	(9, 0.8)
primary-school	10	(11, 0.9)	(11, 0.8)