Supplementary Information

A tied-weight autoencoder for the linear dimensionality reduction of sample data

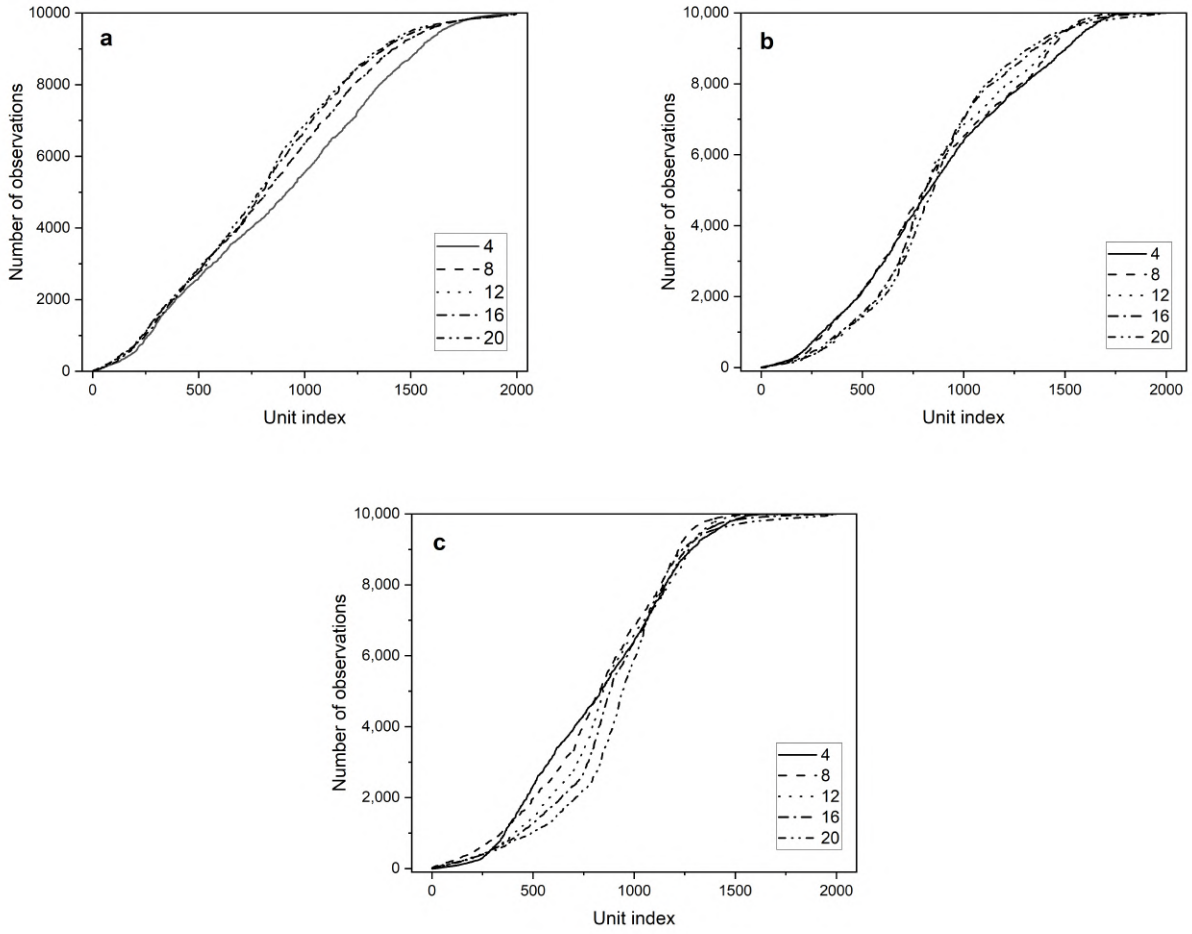

Figure S1. Plots of the number of observations versus the unit indices in ascending order of their observations. The number of units used and the total number of observations are 2,000 and 10,000, respectively. Each plot is drawn with different code sizes: 4, 8, 12, 16, and 20. (a) FMNIST, (b) SVHN, and (c) CIFAR10.

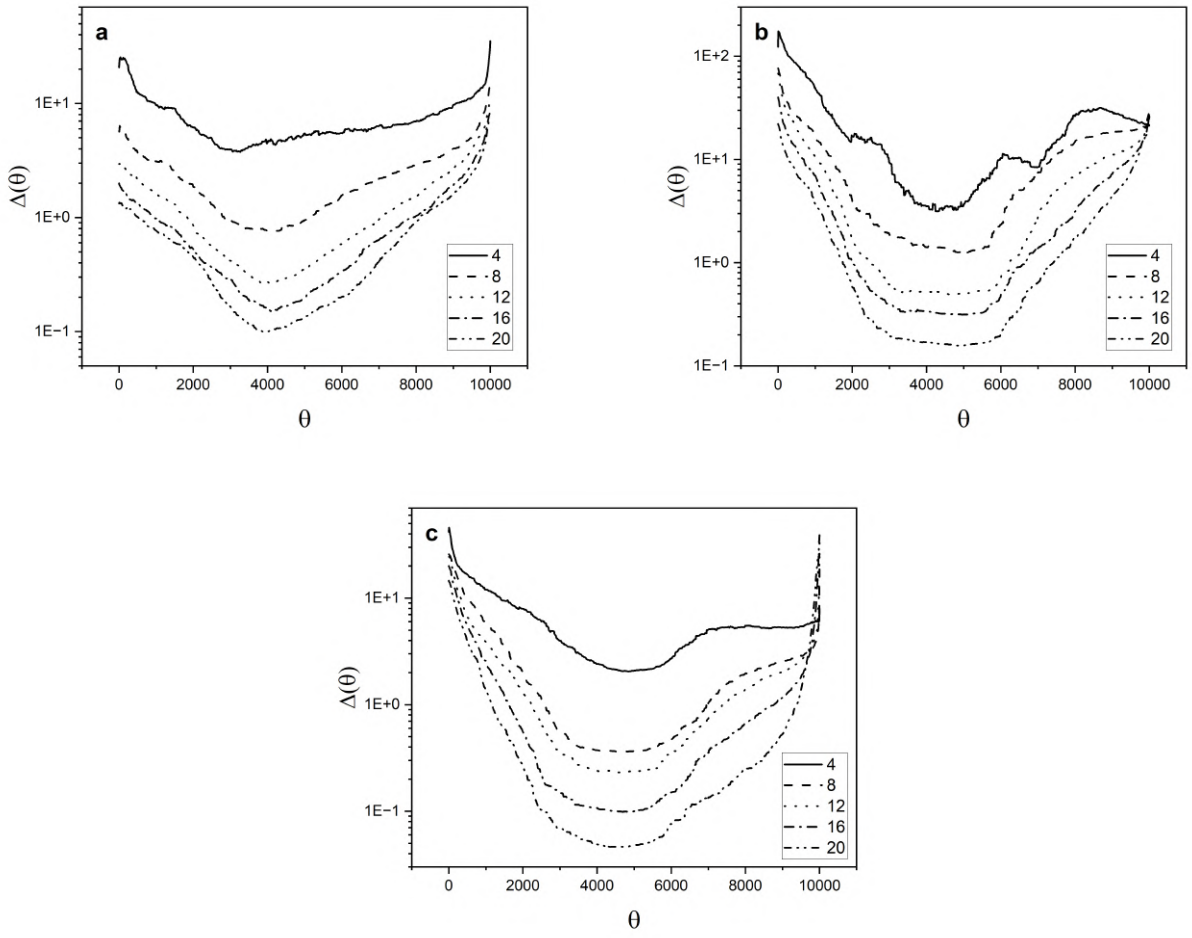

Figure S2. Plots of $\Delta(\theta)$ versus θ drawn with different code sizes: 4, 8, 12, 16, and 20. (a) FMNIST, (b) SVHN, and (c) CIFAR10.

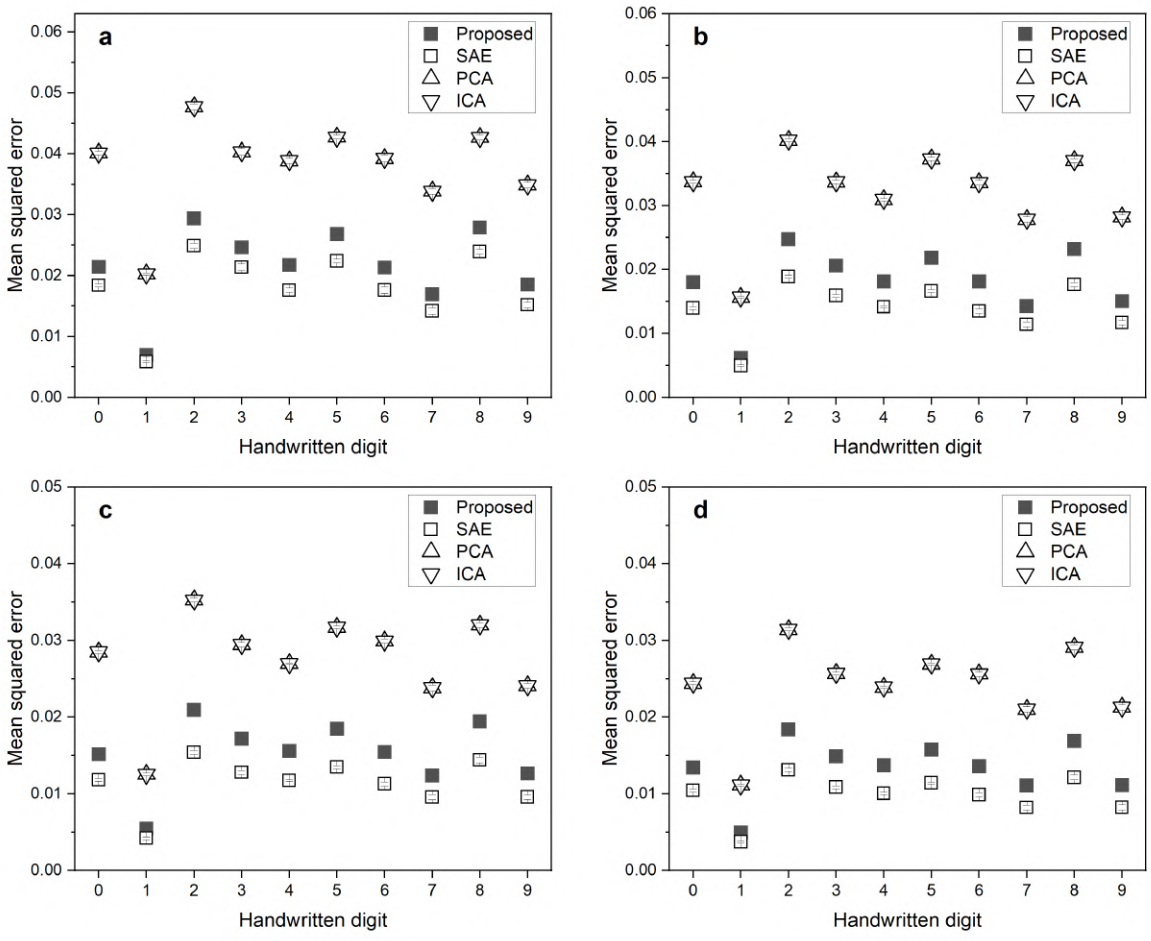

Figure S3. Plots of the mean squared error for each class label of MNIST when the number of codes is (a) 8, (b) 12, (c) 16, and (d) 20.

(a) FMNIST: Mean squared error averaged over independent trials

code size 8

Model	Class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.015	0.009	0.015	0.015	0.014	0.023	0.015	0.012	0.024	0.017
SAE	0.013	0.007	0.012	0.012	0.012	0.020	0.013	0.011	0.021	0.014
PCA	0.023	0.023	0.025	0.027	0.024	0.034	0.024	0.021	0.036	0.030
ICA	0.023	0.023	0.025	0.027	0.024	0.034	0.024	0.021	0.036	0.030

code size 12

Model	Class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.013	0.008	0.013	0.013	0.012	0.021	0.014	0.011	0.022	0.015
SAE	0.011	0.006	0.010	0.011	0.010	0.017	0.012	0.010	0.018	0.013
PCA	0.020	0.017	0.021	0.021	0.020	0.032	0.020	0.019	0.032	0.026
ICA	0.020	0.017	0.021	0.021	0.020	0.032	0.020	0.019	0.032	0.026

code size 16

Model	Class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.012	0.008	0.012	0.012	0.011	0.019	0.013	0.010	0.021	0.014
SAE	0.010	0.005	0.009	0.010	0.009	0.016	0.011	0.009	0.016	0.011
PCA	0.018	0.015	0.019	0.018	0.018	0.029	0.018	0.017	0.029	0.023
ICA	0.018	0.015	0.019	0.018	0.018	0.029	0.018	0.017	0.029	0.023

code size 20

Model	Class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.011	0.007	0.011	0.011	0.011	0.018	0.012	0.010	0.020	0.013
SAE	0.009	0.005	0.008	0.009	0.008	0.014	0.010	0.008	0.015	0.011
PCA	0.016	0.014	0.016	0.017	0.016	0.027	0.016	0.016	0.026	0.021
ICA	0.016	0.014	0.016	0.017	0.016	0.027	0.016	0.016	0.026	0.021

(b) SVHN: Mean squared error averaged over independent trials

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.120	0.138	0.138	0.133	0.141	0.137	0.132	0.133	0.139	0.137
SAE	0.114	0.128	0.130	0.125	0.132	0.128	0.122	0.125	0.130	0.127
PCA	0.134	0.155	0.151	0.146	0.155	0.152	0.150	0.147	0.151	0.153
ICA	0.134	0.155	0.151	0.146	0.155	0.152	0.150	0.147	0.151	0.153

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.107	0.119	0.119	0.116	0.122	0.118	0.114	0.116	0.122	0.118
SAE	0.102	0.112	0.114	0.110	0.115	0.112	0.107	0.111	0.116	0.112
PCA	0.121	0.137	0.135	0.132	0.138	0.135	0.131	0.131	0.136	0.135
ICA	0.121	0.137	0.135	0.132	0.138	0.135	0.131	0.131	0.136	0.135

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.097	0.108	0.108	0.105	0.110	0.107	0.103	0.106	0.109	0.107
SAE	0.093	0.102	0.103	0.100	0.105	0.102	0.098	0.101	0.103	0.102
PCA	0.112	0.124	0.124	0.120	0.126	0.123	0.118	0.121	0.125	0.124
ICA	0.112	0.124	0.124	0.120	0.126	0.123	0.118	0.121	0.125	0.124

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.091	0.102	0.101	0.099	0.103	0.101	0.097	0.100	0.102	0.099
SAE	0.086	0.095	0.095	0.092	0.097	0.095	0.091	0.094	0.096	0.093
PCA	0.104	0.116	0.114	0.112	0.117	0.115	0.110	0.113	0.116	0.113
ICA	0.104	0.116	0.114	0.112	0.117	0.115	0.110	0.113	0.116	0.113

(c) CIFAR10: Mean squared error averaged over independent trials

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.159	0.182	0.192	0.198	0.201	0.197	0.206	0.192	0.164	0.187
SAE	0.149	0.168	0.179	0.184	0.190	0.182	0.195	0.178	0.153	0.173
PCA	0.165	0.187	0.196	0.203	0.203	0.202	0.209	0.198	0.169	0.191
ICA	0.165	0.187	0.196	0.203	0.203	0.202	0.209	0.198	0.169	0.191

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.146	0.166	0.176	0.181	0.187	0.180	0.193	0.176	0.151	0.170
SAE	0.137	0.156	0.164	0.167	0.176	0.166	0.183	0.164	0.141	0.159
PCA	0.152	0.175	0.182	0.187	0.192	0.187	0.197	0.183	0.157	0.177
ICA	0.152	0.175	0.182	0.187	0.192	0.187	0.197	0.183	0.157	0.177

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.136	0.156	0.163	0.167	0.176	0.167	0.184	0.165	0.140	0.158
SAE	0.127	0.146	0.153	0.155	0.167	0.154	0.175	0.154	0.132	0.149
PCA	0.141	0.163	0.169	0.173	0.181	0.173	0.188	0.171	0.146	0.165
ICA	0.141	0.163	0.169	0.173	0.181	0.173	0.188	0.171	0.146	0.165

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.127	0.147	0.154	0.156	0.168	0.156	0.176	0.155	0.132	0.148
SAE	0.120	0.139	0.144	0.146	0.159	0.145	0.167	0.146	0.124	0.140
PCA	0.132	0.153	0.159	0.162	0.172	0.161	0.179	0.161	0.137	0.154
ICA	0.132	0.153	0.159	0.162	0.172	0.161	0.179	0.161	0.137	0.154

Figure S4. Results of mean squared error for each class label when the number of codes is 8, 12, 16, and 20. (a) FMNIST, (b) SVHN, and (c) CIFAR10

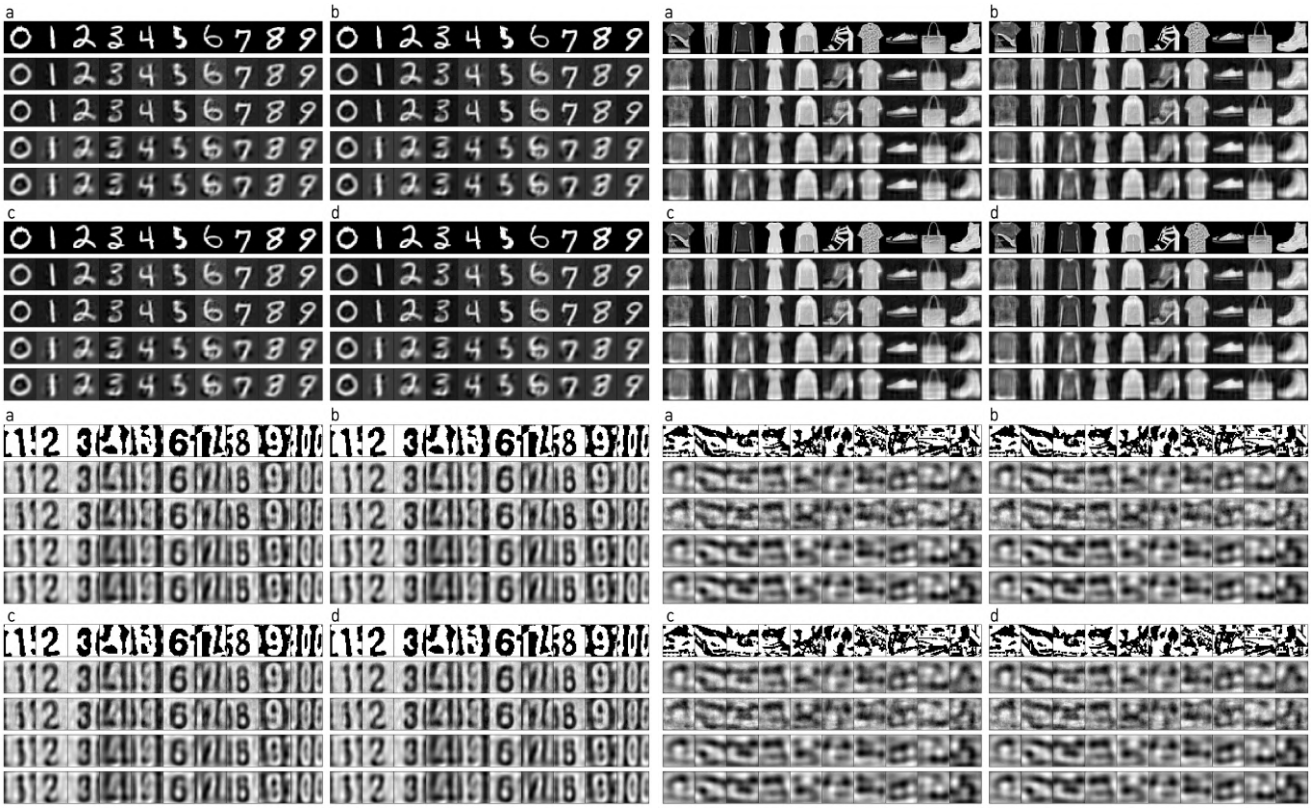

Figure S5. The reconstructed input images when the code size is (a) 8, (b) 12, (c) 16, and (d) 20. From top to bottom and left to right: MNIST, FMNIST, SVHN, and CIFAR10.

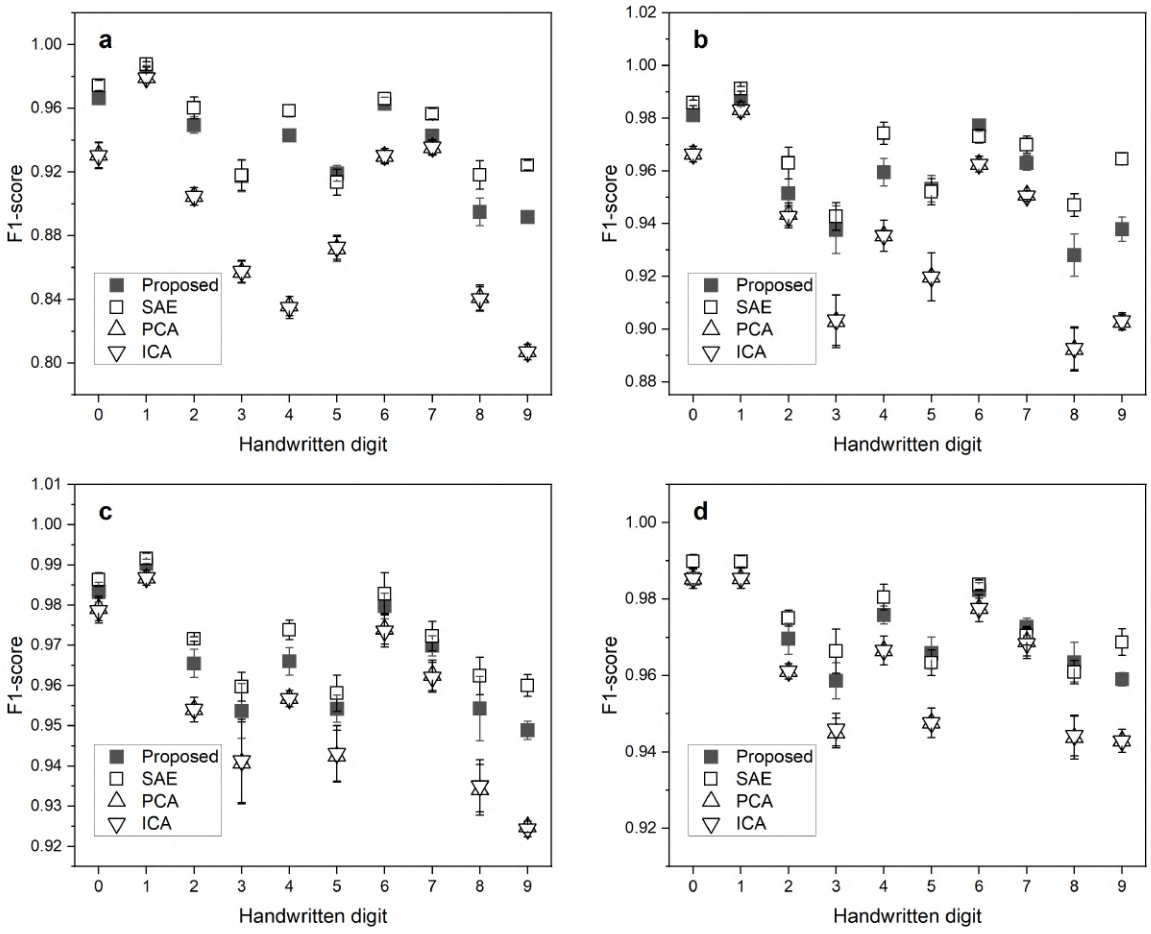

Figure S6. Plots of the F1 score for each class label of MNIST when the number of codes is (a) 8, (b) 12, (c) 16, and (d) 20.

(a) FMNIST: F1 score obtained using FMNIST

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.76	0.96	0.67	0.81	0.65	0.90	0.56	0.89	0.94	0.91
SAE	0.76	0.96	0.69	0.82	0.67	0.90	0.56	0.89	0.95	0.91
PCA	0.73	0.93	0.58	0.76	0.58	0.88	0.49	0.87	0.91	0.90
ICA	0.73	0.93	0.59	0.76	0.57	0.88	0.48	0.87	0.91	0.90

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.79	0.97	0.74	0.85	0.74	0.92	0.61	0.91	0.96	0.93
SAE	0.79	0.97	0.74	0.86	0.75	0.93	0.61	0.91	0.97	0.93
PCA	0.77	0.97	0.69	0.84	0.70	0.91	0.59	0.90	0.95	0.92
ICA	0.77	0.97	0.69	0.84	0.70	0.91	0.59	0.90	0.95	0.92

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.79	0.97	0.76	0.85	0.75	0.94	0.61	0.92	0.96	0.94
SAE	0.79	0.98	0.75	0.86	0.75	0.93	0.63	0.92	0.96	0.95
PCA	0.79	0.97	0.73	0.85	0.75	0.92	0.61	0.91	0.95	0.93
ICA	0.79	0.97	0.73	0.85	0.75	0.92	0.61	0.91	0.95	0.93

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.79	0.98	0.74	0.87	0.74	0.94	0.64	0.92	0.96	0.94
SAE	0.81	0.97	0.76	0.87	0.75	0.94	0.64	0.93	0.95	0.94
PCA	0.79	0.96	0.73	0.84	0.74	0.92	0.61	0.91	0.95	0.93
ICA	0.79	0.96	0.73	0.84	0.74	0.92	0.61	0.91	0.95	0.93

(b) SVHN: F1 score obtained using SVHN dataset

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.64	0.54	0.42	0.49	0.26	0.27	0.46	0.18	0.15	0.31
SAE	0.65	0.57	0.46	0.53	0.34	0.27	0.53	0.18	0.20	0.36
PCA	0.60	0.42	0.32	0.41	0.20	0.25	0.33	0.14	0.08	0.20
ICA	0.60	0.42	0.32	0.41	0.20	0.25	0.34	0.14	0.08	0.20

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.73	0.64	0.54	0.65	0.41	0.44	0.62	0.31	0.31	0.46
SAE	0.74	0.68	0.54	0.67	0.46	0.47	0.64	0.34	0.35	0.50
PCA	0.71	0.57	0.46	0.60	0.38	0.40	0.55	0.28	0.28	0.40
ICA	0.71	0.57	0.46	0.60	0.38	0.40	0.55	0.29	0.29	0.40

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.75	0.69	0.55	0.67	0.54	0.49	0.68	0.43	0.46	0.52
SAE	0.74	0.70	0.55	0.69	0.51	0.48	0.68	0.40	0.50	0.54
PCA	0.74	0.66	0.52	0.68	0.45	0.45	0.68	0.38	0.39	0.49
ICA	0.74	0.66	0.52	0.68	0.45	0.45	0.68	0.38	0.40	0.49

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.77	0.71	0.61	0.73	0.57	0.55	0.70	0.51	0.53	0.59
SAE	0.76	0.73	0.59	0.72	0.56	0.53	0.69	0.47	0.53	0.58
PCA	0.77	0.69	0.59	0.72	0.54	0.53	0.71	0.47	0.47	0.57
ICA	0.76	0.69	0.59	0.72	0.54	0.53	0.71	0.47	0.47	0.57

(c) CIFAR10: F1 score obtained using CIFAR10 dataset

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.38	0.36	0.27	0.13	0.25	0.28	0.36	0.36	0.41	0.35
SAE	0.38	0.35	0.25	0.12	0.23	0.27	0.34	0.37	0.40	0.34
PCA	0.34	0.34	0.26	0.08	0.23	0.26	0.33	0.37	0.41	0.33
ICA	0.34	0.34	0.26	0.08	0.23	0.26	0.33	0.37	0.41	0.33

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.41	0.40	0.26	0.14	0.27	0.30	0.39	0.40	0.42	0.36
SAE	0.37	0.38	0.26	0.17	0.24	0.30	0.39	0.39	0.43	0.34
PCA	0.40	0.36	0.26	0.13	0.25	0.29	0.39	0.39	0.42	0.35
ICA	0.40	0.36	0.26	0.13	0.25	0.29	0.39	0.39	0.42	0.35

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.43	0.42	0.29	0.17	0.29	0.32	0.40	0.41	0.43	0.37
SAE	0.39	0.40	0.28	0.20	0.27	0.31	0.41	0.41	0.42	0.35
PCA	0.43	0.40	0.27	0.19	0.27	0.31	0.42	0.41	0.44	0.36
ICA	0.43	0.40	0.27	0.19	0.27	0.31	0.42	0.41	0.44	0.36

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.42	0.42	0.27	0.19	0.26	0.30	0.41	0.40	0.40	0.37
SAE	0.39	0.41	0.26	0.18	0.25	0.28	0.38	0.39	0.41	0.35
PCA	0.41	0.41	0.26	0.20	0.25	0.29	0.40	0.41	0.40	0.35
ICA	0.41	0.41	0.26	0.20	0.25	0.29	0.40	0.41	0.40	0.35

Figure S7. Results of F1 score for each class label when the number of codes is 8, 12, 16, and 20. (a) FMNIST, (b) SVHN, and (c) CIFAR10

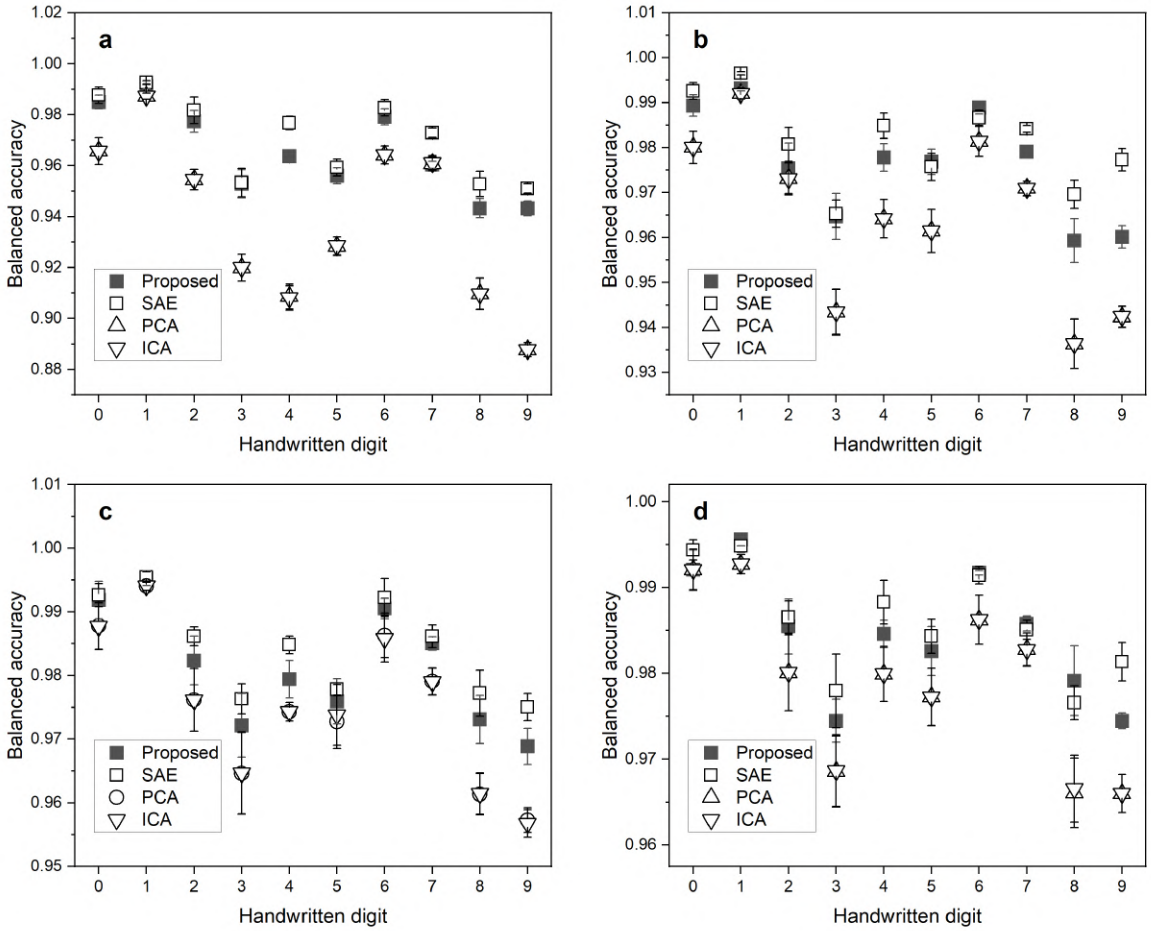

Figure S8. Plots of balanced accuracy for each class label of MNIST when the number of codes is (a) 8, (b) 12, (c) 16, and (d) 20.

(a) FMNIST: Balanced accuracy obtained using FMNIST dataset

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.86	0.97	0.82	0.90	0.80	0.94	0.76	0.94	0.97	0.95
SAE	0.86	0.98	0.83	0.91	0.81	0.94	0.76	0.94	0.97	0.95
PCA	0.84	0.95	0.76	0.88	0.76	0.93	0.72	0.94	0.96	0.94
ICA	0.84	0.95	0.77	0.88	0.76	0.93	0.72	0.94	0.96	0.94

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.88	0.98	0.86	0.92	0.86	0.95	0.78	0.95	0.98	0.96
SAE	0.89	0.98	0.86	0.93	0.86	0.95	0.78	0.96	0.98	0.96
PCA	0.87	0.98	0.83	0.92	0.83	0.95	0.77	0.95	0.97	0.95
ICA	0.87	0.98	0.83	0.92	0.83	0.95	0.77	0.95	0.97	0.95

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.88	0.98	0.86	0.92	0.87	0.96	0.78	0.96	0.97	0.97
SAE	0.88	0.98	0.86	0.93	0.86	0.96	0.80	0.96	0.98	0.97
PCA	0.88	0.98	0.84	0.92	0.86	0.96	0.79	0.95	0.97	0.96
ICA	0.88	0.98	0.84	0.92	0.86	0.96	0.78	0.95	0.97	0.96

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.88	0.98	0.85	0.93	0.86	0.96	0.80	0.96	0.98	0.97
SAE	0.89	0.98	0.87	0.93	0.86	0.96	0.80	0.96	0.98	0.97
PCA	0.88	0.97	0.85	0.91	0.85	0.95	0.79	0.96	0.98	0.96
ICA	0.88	0.97	0.85	0.91	0.85	0.95	0.79	0.96	0.98	0.96

(b) SVHN: Balanced accuracy obtained by SVHN dataset

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.80	0.74	0.67	0.71	0.59	0.60	0.69	0.56	0.54	0.63
SAE	0.81	0.75	0.69	0.73	0.63	0.60	0.72	0.56	0.57	0.65
PCA	0.78	0.67	0.62	0.67	0.56	0.59	0.63	0.54	0.51	0.57
ICA	0.78	0.67	0.62	0.67	0.56	0.59	0.63	0.54	0.51	0.57

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.85	0.79	0.75	0.80	0.67	0.70	0.78	0.63	0.61	0.71
SAE	0.85	0.81	0.75	0.80	0.70	0.71	0.79	0.64	0.64	0.73
PCA	0.83	0.75	0.70	0.77	0.65	0.67	0.75	0.61	0.60	0.68
ICA	0.83	0.75	0.70	0.77	0.65	0.67	0.75	0.62	0.61	0.67

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.86	0.82	0.75	0.81	0.74	0.72	0.81	0.69	0.70	0.74
SAE	0.85	0.83	0.76	0.82	0.72	0.72	0.82	0.68	0.72	0.75
PCA	0.85	0.80	0.74	0.81	0.69	0.70	0.82	0.67	0.66	0.72
ICA	0.85	0.80	0.74	0.81	0.69	0.70	0.82	0.67	0.66	0.72

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.87	0.83	0.79	0.84	0.76	0.75	0.82	0.74	0.73	0.77
SAE	0.87	0.84	0.78	0.83	0.75	0.74	0.82	0.71	0.73	0.77
PCA	0.87	0.83	0.78	0.83	0.74	0.74	0.83	0.71	0.69	0.76
ICA	0.87	0.83	0.78	0.83	0.74	0.74	0.83	0.71	0.69	0.76

(c) CIFAR10: Balanced accuracy obtained using CIFAR10 dataset

code size 8

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.64	0.65	0.59	0.53	0.59	0.59	0.67	0.64	0.68	0.65
SAE	0.64	0.64	0.58	0.53	0.58	0.59	0.66	0.65	0.67	0.65
PCA	0.62	0.64	0.59	0.51	0.57	0.59	0.66	0.65	0.68	0.64
ICA	0.62	0.64	0.58	0.51	0.57	0.59	0.66	0.65	0.67	0.64

code size 12

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.66	0.67	0.58	0.53	0.60	0.61	0.69	0.67	0.68	0.65
SAE	0.64	0.67	0.59	0.54	0.58	0.61	0.69	0.66	0.68	0.64
PCA	0.65	0.65	0.58	0.52	0.59	0.60	0.69	0.66	0.68	0.65
ICA	0.65	0.65	0.58	0.52	0.59	0.60	0.69	0.66	0.68	0.65

code size 16

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.67	0.69	0.60	0.54	0.61	0.62	0.69	0.67	0.68	0.66
SAE	0.65	0.67	0.60	0.55	0.59	0.62	0.69	0.67	0.68	0.65
PCA	0.67	0.68	0.59	0.55	0.60	0.62	0.69	0.67	0.69	0.66
ICA	0.67	0.68	0.59	0.55	0.60	0.62	0.69	0.67	0.69	0.66

code size 20

Model	class label									
	0	1	2	3	4	5	6	7	8	9
Proposed	0.67	0.68	0.59	0.55	0.59	0.61	0.68	0.67	0.67	0.65
SAE	0.66	0.67	0.59	0.55	0.58	0.60	0.66	0.67	0.67	0.65
PCA	0.66	0.68	0.59	0.56	0.58	0.61	0.67	0.67	0.67	0.65
ICA	0.67	0.68	0.59	0.56	0.58	0.60	0.67	0.67	0.67	0.64

Figure S9. Results of balanced accuracy for each class label when the number of codes is 8, 12, 16, and 20. (a) FMNIST, (b) SVHN, and (c) CIFAR10