

$$\text{In[35]:= } x1n = x1 \frac{1}{x1 (1 - s d) + x2 g} \left(s (1 - d) + (1 - s) \frac{x1 + \frac{x3}{2} a + \frac{x4}{2} a (1 - \epsilon)}{x1 + x3 a + x4 a (1 - \epsilon)} \right);$$

(* Hermaphrodite *)

$$x2n = x2 \frac{g}{x1 (1 - s d) + x2 g} \frac{x1 + \frac{x3}{2} a + \frac{x4}{2} a (1 - \epsilon)}{x1 + x3 a + x4 a (1 - \epsilon)}; (* Female *)$$

$$x3n = x1 \frac{1 - s}{x1 (1 - s d) + x2 g} \frac{\frac{x3}{2} a + \frac{x4}{2} a (1 - \epsilon)}{x1 + x3 a + x4 a (1 - \epsilon)}; (* Male *)$$

$$x4n = x2 \frac{g}{x1 (1 - s d) + x2 g} \frac{\frac{x3}{2} a + \frac{x4}{2} a (1 - \epsilon)}{x1 + x3 a + x4 a (1 - \epsilon)}; (* Male with CMS *)$$

In[39]:= J = {{D[x1n, x1], D[x1n, x2], D[x1n, x3], D[x1n, x4]},
 {D[x2n, x1], D[x2n, x2], D[x2n, x3], D[x2n, x4]},
 {D[x3n, x1], D[x3n, x2], D[x3n, x3], D[x3n, x4]},
 {D[x4n, x1], D[x4n, x2], D[x4n, x3], D[x4n, x4]}}; (* Jacobian *)

In[40]:= eig = Eigenvalues[J]; (* λ is the leading eigenvalue,
 that is the largest numerically *)
 λ = eig[[4]]; (* We need -1 < λ < 1 for the considered
 point to be stable (i.e., resistant to invasion) *)

In[6]:= %4

In[7]:= eig /. x1 → 0.25 /. x2 → 0.25 /. x3 → 0.25 /. x4 → 0.25 /. a → 4 /. s → 0.3 /. d → 0.1 /.
 g → 4 /. ε → 0.2

Assuming[a > 0 && 0 < d < 1 && 0 < s < 1 && 0 < ε < 1 && 0 < g ,
 eig[[4] /. x1 → 1 /. x3 → 0 /. x2 → 0 /. x4 → 0 // Simplify] (* Condition
 for males and CMS not to invade hermaphroditism is this term < 1 *)

$$\text{Out[12]:= } \frac{a + 2 g - a s + \text{Abs}[a - 2 g - a s]}{4 - 4 d s}$$

In[30]:= Reduce[$\frac{a + 2 g - a s + \text{Abs}[a - 2 g - a s]}{4 - 4 d s} < 1 \&\& 0 < d < 1 \&\& 0 < s < 1 \&\& a > 0 \&\& g > 0$] //
 FullSimplify

Out[30]:= s > 0 && d > 0 && g > 0 && a > 0 && s < 1 && d < 1 && g + d s < 1 && a + 2 d s < 2 + a s

(* Condition for CMS not to invade hermaphroditism g+d s<1 ,
 Condition for males not to invade hermaphroditism a+2 d s<2+a s *)

Solve[{x1n == x1 /. x2 → 0 /. x4 → 0, x3n == x3 /. x2 → 0 /. x4 → 0}, {x1, x3}] //
 Simplify(* Androdioecious equilibria
 frequencies when CMS cannot invade but males did *)

$$\text{Out[26]:= } \left\{ \{x1 \rightarrow 1, x3 \rightarrow 0\}, \left\{ x1 \rightarrow \frac{a (-1 + (-1 + 2 d) s)}{2 (-1 + a) (-1 + d s)}, x3 \rightarrow \frac{2 + a (-1 + s) - 2 d s}{2 (-1 + a) (-1 + d s)} \right\} \right\}$$

In[31]:= Assuming[$a + 2 d s < 2 + a s \ \&\& \ 0 < d < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < \epsilon < 1 \ \&\& \ 0 < g,$
 $\lambda /. x1 \rightarrow \frac{a (-1 + (-1 + 2 d) s)}{2 (-1 + a) (-1 + d s)} /. x2 \rightarrow 0 /. x3 \rightarrow \frac{2 + a (-1 + s) - 2 d s}{2 (-1 + a) (-1 + d s)} /. x4 \rightarrow 0 //$

Simplify]

(* Condition for CMS not invaded androdioecy are these terms < 1 *)

Out[31]=
$$\left[\begin{array}{l} \frac{2-2 d s}{a-a s} \quad ! \left(a < 0 \vee a g (-1+s) + 2 s (1-g+d (-3+g-s) + 2 d^2 s) < -2 \right) \\ \frac{g (a-a s-2 (-1+d) s)}{a (-1+s) (-1+(-1+2 d) s)} \quad \text{True} \end{array} \right]$$

In[34]:= Reduce[

$$\frac{g (a-a s-2 (-1+d) s)}{a (-1+s) (-1+(-1+2 d) s)} < 1 \ \&\& \ 0 < d < 1 \ \&\& \ 0 < s < 1 \ \&\& \ a + 2 d s > 2 + a s \ \&\& \ 0 < g]$$

(* Condition for CMS NOT to invade androdioecious population *)

Out[34]=
$$0 < d < 1 \ \&\& \ a > 2 \ \&\& \ 0 < s < \frac{-2+a}{a-2 d} \ \&\& \ 0 < g < \frac{-a+2 a d s+a s^2-2 a d s^2}{-a-2 s+a s+2 d s}$$

In[25]:= Assuming[$a > \frac{2 (-1+d s)}{-1+s} \ \&\& \ 0 < d < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < \epsilon < 1 \ \&\& \ 0 < g,$

eig[[4]] /. x1 → 0 /. x3 → 0 /. x2 → 1/2 /. x4 → 1/2 // Simplify]

(* Condition for CMS fixation is this term < 1 *)

Out[25]=
$$\frac{1+s-2 d s}{g}$$

(* Condition for CMS fixation $\frac{1+s-2 d s}{g} < 1$ *)