

$\text{In[*]} := L = \text{Min}[a x_3 + a (1 - \epsilon) x_4 + x_1, 1];$

$$x_{1n} = x_1 \frac{1}{x_1 (L - L s + s - s d) + L x_2 g} \left(s (1 - d) + L (1 - s) \frac{x_1 + \frac{x_3}{2} a + \frac{x_4}{2} a (1 - \epsilon)}{x_1 + x_3 a + x_4 a (1 - \epsilon)} \right);$$

(* Hermaphrodite *)

$$x_{2n} = x_2 \frac{L g}{x_1 (L - L s + s - s d) + L x_2 g} \frac{x_1 + \frac{x_3}{2} a + \frac{x_4}{2} a (1 - \epsilon)}{x_1 + x_3 a + x_4 a (1 - \epsilon)}; (* \text{Female} *)$$

$$x_{3n} = x_1 \frac{L (1 - s)}{x_1 (L - L s + s - s d) + L x_2 g} \frac{\frac{x_3}{2} a + \frac{x_4}{2} a (1 - \epsilon)}{x_1 + x_3 a + x_4 a (1 - \epsilon)}; (* \text{Male} *)$$

$$x_{4n} = x_2 \frac{L g}{x_1 (L - L s + s - s d) + L x_2 g} \frac{\frac{x_3}{2} a + \frac{x_4}{2} a (1 - \epsilon)}{x_1 + x_3 a + x_4 a (1 - \epsilon)}; (* \text{Male with CMS} *)$$

$\text{In[*]} := J = \{\{D[x_{1n}, x_1], D[x_{1n}, x_2], D[x_{1n}, x_3], D[x_{1n}, x_4]\},$
 $\{D[x_{2n}, x_1], D[x_{2n}, x_2], D[x_{2n}, x_3], D[x_{2n}, x_4]\},$
 $\{D[x_{3n}, x_1], D[x_{3n}, x_2], D[x_{3n}, x_3], D[x_{3n}, x_4]\},$
 $\{D[x_{4n}, x_1], D[x_{4n}, x_2], D[x_{4n}, x_3], D[x_{4n}, x_4]\}\}; (* \text{Jacobian} *)$

$\text{In[*]} := \text{eig} = \text{Eigenvalues}[J]; (* \lambda \text{ is the leading eigenvalue,}$
that is the largest numerically *)

$\lambda = \text{eig}[[4]]; (* \text{We need } -1 < \lambda < 1 \text{ for the considered}$
point to be stable (i.e., resistant to invasion) *)

$\text{In[*]} := \text{eig} /. x_1 \rightarrow 0.25 /. x_2 \rightarrow 0.25 /. x_3 \rightarrow 0.25 /. x_4 \rightarrow 0.25 /. a \rightarrow 5 /. s \rightarrow 0.3 /. d \rightarrow 0.1 /.$
 $g \rightarrow 1.3 /. \epsilon \rightarrow 0.2$

$\text{Out[*]} = \{0, 0., 0.163395, 1.16813\}$

Assuming[$a > 0 \ \&\& \ 0 < d < 1 \ \&\& \ 0 < s < 1 \ \&\& \ 0 < \epsilon < 1 \ \&\& \ 0 < g,$
 $\text{eig}[[4]] /. x_1 \rightarrow 1 /. x_3 \rightarrow 0 /. x_2 \rightarrow 0 /. x_4 \rightarrow 0 // \text{Simplify} (* \text{Condition}$
for males and CMS not to invade hermaphroditism is this term < 1 *)

$$\text{Out[*]} = \frac{1}{2 (-1 + d s)^2} \text{Root}\left[\left(2 a g - 2 a g s - 4 a d g s + 4 a d g s^2 + 2 a d^2 g s^2 - 2 a d^2 g s^3\right) \#1 + \right. \\ \left. (-a - 2 g + a s + a d s + 2 d g s - a d s^2) \#1^2 + \#1^3 \ \&, 3\right]$$

$\text{In[*]} := \text{Reduce}\left[\frac{1}{2 (-1 + d s)^2} \text{Root}\left[\left(2 a g - 2 a g s - 4 a d g s + 4 a d g s^2 + 2 a d^2 g s^2 - 2 a d^2 g s^3\right) \#1 + \right. \right. \\ \left. \left. (-a - 2 g + a s + a d s + 2 d g s - a d s^2) \#1^2 + \#1^3 \ \&, 3\right] < 1 \ \&\& \right. \\ \left. 0 < d < 1 \ \&\& \ 0 < s < 1 \ \&\& \ a > 0 \ \&\& \ g > 0\right] // \text{FullSimplify}$

$\text{Out[*]} = s > 0 \ \&\& \ d > 0 \ \&\& \ g > 0 \ \&\& \ a > 0 \ \&\& \ s < 1 \ \&\& \ d < 1 \ \&\& \ g + d s < 1 \ \&\& \ a + 2 d s < 2 + a s$

(* Condition for CMS not to invade hermaphroditism: $g + d s < 1$,
Condition for males not to invade hermaphroditism: $a + 2 d s < 2 + a s$ *)

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Assuming[a >  $\frac{2(-1+ds)}{-1+s}$  && 0 < d < 1 && 0 < s < 1 && 0 < e < 1 && 0 < g,
  eig[[4]] /. x1 -> 0 /. x3 -> 0 /. x2 -> 1/2 /. x4 -> 1/2 // Simplify]
(* Condition for CMS fixation is this term < 1 *)
Out[4]= 
$$\left( 8 \sqrt{-2 a^2 g s \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right] + 2 a^2 d g s \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right] + 4 a^2 g s e \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right] - 4 a^2 d g s e \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right] - 2 a^2 g s e^2 \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right] + 2 a^2 d g s e^2 \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right] - a^2 g \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right]^2 + a^2 g s \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right]^2 + 2 a^2 g e \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right]^2 - 2 a^2 g s e \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right]^2 - a^2 g e^2 \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right]^2 + a^2 g s e^2 \operatorname{Min}\left[1, \frac{1}{2} a (1-e)\right]^2} \sqrt{1+8 \sqrt{1^3} \&, 3} \right) / \left( a^2 g^2 (-1+e)^2 \operatorname{Min}\left[1, -\frac{1}{2} a (-1+e)\right]^2 \right)$$


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Reduce[
  (8 Root[(-2 a^2 g s Min[1, 1/2 a (1 - ε)] + 2 a^2 d g s Min[1, 1/2 a (1 - ε)] + 4 a^2 g s ε Min[
    1, 1/2 a (1 - ε)] - 4 a^2 d g s ε Min[1, 1/2 a (1 - ε)] -
    2 a^2 g s ε^2 Min[1, 1/2 a (1 - ε)] + 2 a^2 d g s ε^2 Min[1, 1/2 a (1 - ε)] -
    a^2 g Min[1, 1/2 a (1 - ε)]^2 + a^2 g s Min[1, 1/2 a (1 - ε)]^2 +
    2 a^2 g ε Min[1, 1/2 a (1 - ε)]^2 - 2 a^2 g s ε Min[1, 1/2 a (1 - ε)]^2 - a^2 g ε^2
    Min[1, 1/2 a (1 - ε)]^2 + a^2 g s ε^2 Min[1, 1/2 a (1 - ε)]^2) #1^2 + 8 #1^3 &, 3]) /
  (a^2 g^2 (-1 + ε)^2 Min[1, -1/2 a (-1 + ε)]^2) < 1 && 0 < d < 1 && 0 <
  s < 1 && a > 2 (-1 + d s) / (-1 + s) && 0 <
  ε <
  1 && 0 <
  g,
g] // Simplify (* Condition
for
CMS
fixation *)

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Out[4]= $0 < d < 1 \&\& 0 < s < \frac{-2 + a}{a - 2d} \&\& 0 < \epsilon < 1 \&\&$

$$\left(\left(a (-4 (-1 + d) s + a (-1 + g + s) (-1 + \epsilon)) (-1 + \epsilon) > 0 \&\& 2 < a \leq -\frac{2}{-1 + \epsilon} \right) \mid \mid \right. \\ \left. \left(a > -\frac{2}{-1 + \epsilon} \&\& g > 1 + s - 2 d s \right) \right)$$

(* Condition for CMS fixation if the dioecious
population is not pollen limited $g > 1 + s - 2 d s$,
if the dioecious population is pollen limited a
 $(-4 (-1 + d) s + a (-1 + g + s) (-1 + \epsilon)) (-1 + \epsilon) > 0$ or,
equivalently, $a (-1 + g + s) (-1 + \epsilon) < 4 (-1 + d) s$ *)

(* We do not analyse the condition for CMS not to invade androdioecy for 2
reasons. (1) It is the same as in the case of no pollen limitation -
model 1, because no pollen limitation should arise in our model if
females did not invade and all plants could produce pollen. Hence,
this condition is already analysed in the script of model
1. (2) The analysis of the condition for CMS to invade
androdioecy takes too much time to run because estimating
equilibrium frequencies is too complicated for this model *)