

1 **Mathematical Programming Approaches for Modeling a Sustainable**
2 **Cropping Pattern under Uncertainty: A Case Study in Southern Iran**

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4 Mostafa Mardani Najafabadi^a and Niloofar Ashktorab^{b*}

5
6 ^{a,b} *Department of Agricultural Economics, Faculty of Agricultural Engineering and Rural*
7 *Development, Agricultural Sciences and Natural Resources University of Khuzestan.*

8
9 ^a Assistance professor of agricultural economics

10 Mobil phone number: +98 913 2309540

11 E-mail address: mostaf.korg@yahoo.com or m.mardani@asnrukh.ac.ir

12 Linkedin: <https://www.linkedin.com/in/mostafa-mardani-najafabadi-a995691bb/>

13 P.O. Box: 6341773637, Mollasani, Iran.

14
15 ^{b*} Corresponding author: Assistance professor of agricultural economics

16 E-mail address: nilo.ashktorab@gmail.com or ashktorab@asnrukh.ac.ir

17 Mobil phone number: +98 9171074484

18 Linkedin: <https://www.linkedin.com/in/niloofar-ashktorab-a4315680/>

19 P.O. Box: 6341773637, Mollasani, Iran.

20 ORCID: 0000-0002-2855-879X

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Mathematical Programming Approaches for Modeling a Sustainable Cropping Pattern under Uncertainty: A Case Study in Southern Iran

Abstract

In recent years, the excessive and unreasonable use of chemicals, the occasional use of water, and the use of improper irrigation methods have created a worrying and unstable situation in developing countries' agricultural activities. In the present study, the robust multi-objective fractional linear programming model (RMOLFP) was introduced to determine the sustainable optimal cropping pattern. This model was presented in the Gotvand irrigation and drainage network located in Khuzestan province, southern Iran, under two scenarios with and without considering the uncertainty to evaluate the ability of the model. The results showed that in the first scenario, the consumption of critical disruptive inputs of sustainable agriculture such as fertilizers and chemical pesticides decreased by 5.9% and 8.19%, respectively. On the other hand, the model's uncertainty condition was applied in the second scenario in which the increase in gross margin was reduced. There is a trade-off between protecting the optimization model against system uncertainty and gross margin. Finally, the ability of the proposed model to apply uncertainty conditions was verified by the Monte Carlo simulation method. The results of this simulation confirmed the use of the RMOLFP method in determining the sustainable optimal cropping pattern for the study area.

Keywords: Sustainable Cropping Pattern, Uncertainty, Robust Optimization, Controlled Estimation Method, Khuzestan.

Subject classification codes: C02, C61, C83, Q1, Q56.

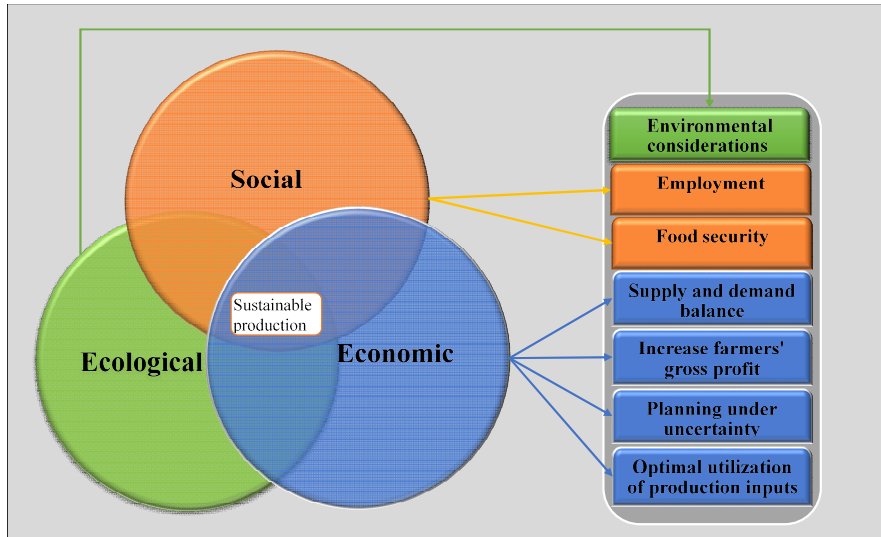
1. Introduction

Over the last five decades, agricultural development policies have increased agricultural production; however, the use of disruptive inputs such as pesticides, fertilizers and agricultural machinery has gradually replaced natural resources available inside farm M. Li et al. (2020). The issue of sustainable agriculture has been emphasized by numerous international organizations and has led to positive environmental, economic and social impacts (Tseng, Chiu, Tan, & Siriban-Manalang, 2013). Due to

64 the economic perspective, the emphasis on sustainable production has been increasing agricultural
65 inputs' productivity and improving and diversifying market opportunities (M. Li et al., 2020).

66 One of the approaches to achieve the optimum use of off-farm inputs and disrupt the sustainable
67 agriculture process is to determine the optimum utilization of inputs by establishing a sustainable
68 optimal cropping pattern (X. Li et al., 2017). Figure 1 shows the main purposes of studies that focus on
69 the sustainable production aspect of agricultural products also implicitly consider other objectives of
70 crop pattern studies. The explanation for this claim lies in the definition of sustainability provided in
71 Guideline 21 at the United Nations Conference of Environment and Development (UNCED) in 1992.
72 In this definition, sustainability was presented in ecological, economic and social dimensions (Mardani
73 Najafabadi, Abdesahi, & Shirzadi Laskookalayeh, 2020).

74 Ecological sustainability means maintaining the ability of the system to adapt to the environment
75 fully and its changes. Therefore, Studies focusing primarily on environmental considerations emphasize
76 this aspect of sustainability (Mardani Najafabadi, Ziaee, Nikouei, & Ahmadpour Borazjani, 2019).
77 Sustainability in the economic dimension is a crucial concept in sustainable development literature. One
78 of the most important objectives to be achieved in this dimension is to increase farmers' gross profit. In
79 most developing countries, the agricultural sector tends to increase agricultural output while reducing
80 unemployment (Mardani, Ziaee, & Nikouei, 2018). This is due to the country's food security and the
81 reduction of the adverse social effects of unemployment (Mardani Najafabadi et al., 2019). Thus, the
82 two remaining goals (food security and employment) can be considered in the social dimension of
83 sustainability.



84

85 Figure 1. Tree dimensions of sustainable optimal cropping pattern

86 Considerable studies have been carried out to optimize the allocation of arable land using
 87 mathematical programming techniques in different regions of the world. Most of the studies have used
 88 linear programming models (Wineman & Crawford, 2017), goal programming (Mardani, Esfanjari
 89 Kenari, Babaei, & Asemani, 2013) and multi-objective programming (Ren, Li, & Zhang, 2019).

90 On many practical issues such as quantitative sustainability indicators, optimizing
 91 the ratio of criteria than each criterion alone provides a better vision (Mardani
 92 Najafabadi et al., 2020). Fractional programming is the most common mathematical
 93 programming with relative objectives. On the other hand, the existence of different
 94 criteria in relational objectives have also led to the development of multi-objective
 95 linear fractional programming models. This approach has been used in various studies
 96 in agricultural management. For example, in the study of Zhang et al. (2017), the
 97 GFCCFP method was used to optimally allocate irrigation water in the Heihe basin in
 98 northwest China. CONNISE method has been used in their study to find optimal
 99 solutions. This method is defined based on the estimation of weak efficient boundaries
 100 in the objectives space, including some points of feasible set (Gorissen, 2015). The
 101 distance between these points should not exceed the value previously provided. This

102 method is beneficial for finding efficient solutions, but in problems with more than two
103 objectives, the optimal solutions lose their convergence, and the final solution is
104 challenging to find. This problem can be solved by controlling the distance considered
105 for a predetermined error using the Controlled Estimation Method (CEM) method
106 proposed by Caballero and Hernández (2004).

107 The agricultural uncertainties are traditionally classified in areas such as production
108 management, marketing and investment (Mardani Najafabadi et al., 2020). The
109 application of fractional programming under uncertainty in agricultural management is
110 widespread. In the meantime, the use of fuzzy, interval, gray, stochastic or a
111 combination of these methods are more common (Tan & Zhang, 2018; Wang, Liu, Guo,
112 Yue, & Guo, 2019).

113 In the mid-1990s, the issue of creating conservatism in mathematical programming
114 models was introduced by limiting uncertain parameters (Ben-Tal & Nemirovski,
115 2000). The benefits of using robust optimization are reporting point-based optimal
116 solution (confronting the optimal interval solutions in interval programming) (M.
117 Sabouhi & Mardani, 2017) and lack of awareness of the data distribution (the challenge
118 of coping with the need to be aware of the data distribution in the stochastic
119 programming) (Mardani Najafabadi & Taki, 2020).

120 Recently, a wide range of robust optimization applications in the management of
121 water resources and agricultural land have been implemented. For the first time, this
122 method was used in fractional programming by Gorissen (2015). Tan and Zhang (2018)
123 optimized the allocation of water resources and agricultural land using robust fractional
124 programming to increase water-use efficiency in the arid northwest region of China. It

125 should be noted, however, that in both studies, single-objective fractional programming
126 is considered, and the problem of drastically losing the convergence of the optimal
127 solutions in multi-objective fractional programming in their methods remains. Using
128 CEM can be an appropriate approach to address this defect and enable the application
129 of robust multi-objective fractional programming to provide a sustainable optimal
130 cropping pattern.

131 Gotvand irrigation and drainage network is located in southwestern Iran in Khuzestan
132 province. According to official statistics from government agencies, fertilizer and
133 pesticides used in the lands covered by this network is 3.6 times the average in Iran
134 (Anonymous, 2018). The surplus irrigation water in the network is returned to the rivers
135 by drainage, contaminate water with a surplus of fertilizers and pesticides in
136 downstream of the network (Mardani Najafabadi et al., 2020). Due to the mentioned
137 issue, selecting this region to determine the cropping pattern that leads to optimal use
138 of disturbing inputs to sustainable agriculture, seems appropriate. Besides, the irrigation
139 water consumption is better managed in this network.

140 Therefore, the purpose of the present study was to propose a mathematical
141 programming model to determine the sustainable optimal cropping pattern for the
142 Gotvand irrigation and drainage network covered land. For this purpose, a robust multi-
143 objective fractional linear programming model (RMOLFP) was proposed. Thus, the
144 innovation and importance of this study can be examined in two respects; first, a new
145 mathematical programming model for optimal cropping pattern with emphasis on
146 sustainability has been proposed, and second, a detailed plan has been prepared for an
147 area that desperately needs a revision of the cropping pattern.

148 **2. Methodology**

149 **2.1 Study Area**

150 Gotvand irrigation and drainage network is designed for irrigation of land in three
151 areas of Gotvand, Aghili and Dimcheh, confined between two rivers, Karoun and Lor
152 (Figure 2). The purpose of this network was to manage water resources better and
153 distribute the water equitably among the farmers in the area. The land covered by this
154 network is 43930 hectares, of which 34144 hectares with over 4300 plots are used. This
155 irrigation network directs irrigation water to 1,076 plots of land in the Gotvand region,
156 using 56 km of canals and 121 active valves with an average discharge of 68 m³/s.

157 Aghili area is smaller so that by using 44 km of canals and 165 active valves, 11.23
158 m³/s of irrigation water can irrigate 1118 plots of land. Both Gotvand and Aghili regions
159 receive their required irrigation water from the Gotvand Dam constructed on the Karoun
160 River. The most considerable amount of infrastructure is in the Dimecheh area. The
161 length of the canals in this area is 179 km and uses 241 valves to deliver 43 cubic meters
162 of river water to 2006 arable land. In addition to the Gotvand Dam, the Dimcheh area
163 feeds directly on several alternative dams from the Lor River, a tributary of the Dez
164 River (Anonymous, 2017).

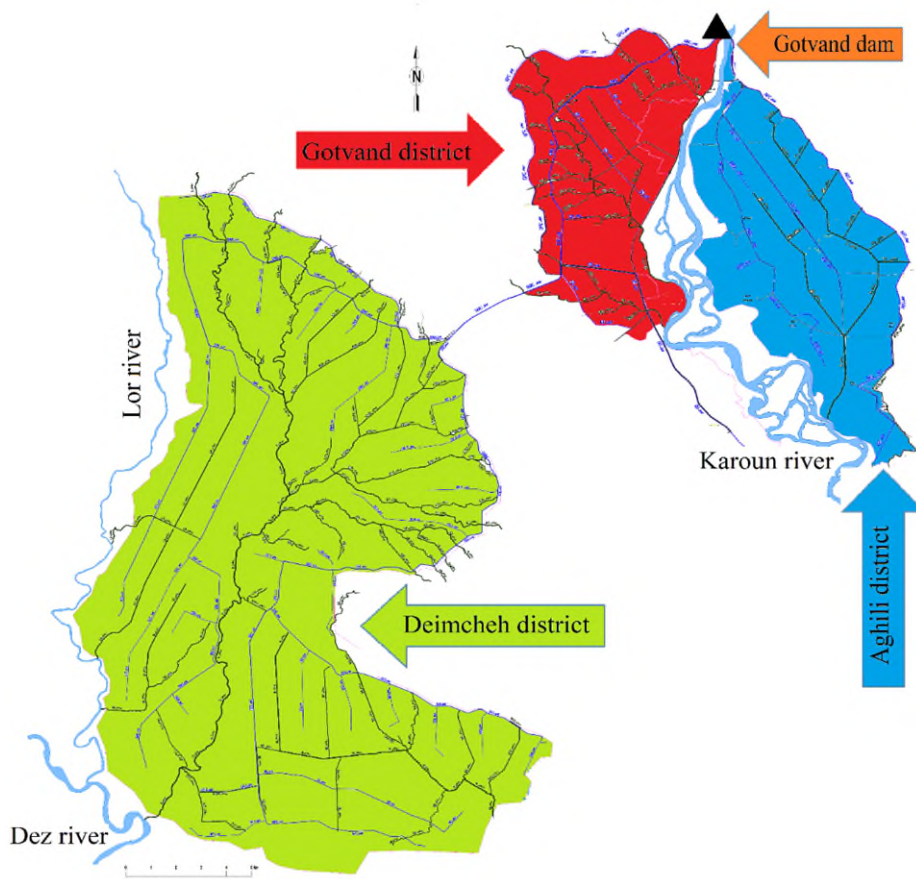


Figure 2. Schematic of the Gotvand irrigation and drainage network

2.2 Multi-objective Fractional Linear Programming

Fractional linear programming is a type of fractional programming, and the numerator and denominator of the objective function is affine Function (AF), and the possible set is a convex polyhedron. A multi-objective linear fractional programming is presented as Eq. (1):

$$\text{Max} \quad \left\{ \varphi_1(x) = \frac{c_1^t x + \alpha_1}{d_1^t x + \beta_1}, \dots, \varphi_p(x) = \frac{c_p^t x + \alpha_p}{d_p^t x + \beta_p} \right\} \quad (1)$$

$$\text{s.t.} \quad Ax \leq b,$$

$$x \geq 0,$$

Where c and d are the coefficients of the objective function, α and β are the constant components, A is the technical coefficient matrix, x is the decision variable, and b is right-sided values.

The CONNISE method, which combines the constraint method (CON) and the Non-inferior Set method

179 (NISE), can be used to find optimal solutions to multi-objective optimization problems. This method is
 180 also used for multi-objective fractional linear programming.

181 Assuming that in multi-objective programming, E is efficient set while E^w is weak efficient set.
 182 The CONNISE method is defined based on the estimation of weak efficient boundaries including points
 183 of the set of possible zones in the objective space ($\varphi(E^w)$). The distance between possible points
 184 should not exceed a certain value.

185 Although the CONNISE method was very useful for finding efficient solution but in problems with
 186 more than two aims, the solutions lose their convergence so finding the final solution is very difficult.
 187 This problem can be solved using the Controlled estimation method (CEM). In this method, the intended
 188 interval for the predetermined error is controlled. The CEM algorithm which is consists of 7 steps,
 189 proposed by Caballero and Hernandez (2004).

190 In the first stage, the ideal and non-ideal points (the pay-off matrix) are obtained. The boundaries of
 191 objective space are formed between ideal (D_i^*) and non-ideal (D_{i-}) values. In the second stage,
 192 $\varepsilon = \max_{i=1, \dots, p} |\varphi_i^* - \varphi_{i-}|$, is calculated, where φ_i^* and φ_{i-} are ideal and non-ideal values in the objective
 193 space, respectively. At this stage, acceptable percentage of error for the interval between possible points
 194 (d) is considered by the decision maker and finally, $\delta = \frac{1}{d}$ is calculated. Then, $s = \emptyset$ and $A = \emptyset$ sets are
 195 defined that s is points of current estimation and A is loose points. In the third stage, $m = E(\frac{1}{d})$ is
 196 calculated in which $E(\frac{1}{d})$ is integer part of $\frac{1}{d}$. By using Parametric Constraints Method (PCM) and
 197 m number, set s iteration is achieved.

198 In the fourth stage $\theta = \alpha\delta$ is calculated where $\alpha \in [0, 1]$, assume that $s = \{x^1, \dots, x^k\}$, $K=1, \dots, N$,
 199 and $K \neq J$. If for any $i=1, \dots, p$, $|\varphi_i(x^j) - \varphi_i(x^k)| < \theta / |\varphi_i^* - \varphi_{i-}|$, inequality is established, x^k point
 200 is eliminated from S set. This process will continue until many points left in the set S . This step is known
 201 as the filtering step. In the fifth step, it is assumed that $s = \{x^i, \dots, x^N\}$ and for each x^j , $J=1, \dots, N$, the

202 endpoint (x^k) corresponding to (x^j) is found in the objective space. If $\max_{i=1,\dots,p} |\varphi_i(x^j) - \varphi_i(x^k)| > d\varepsilon$,
 203 x^j is unstable point and then x^j is placed in set A . This stage is repeated for all x^j . If A is null, this
 204 process is over, otherwise the sixth step is started and m that is obtained from third stage, considered as
 205 $m+1$. If $A = \{x^1, \dots, x^m\}$, for each x^j belongs to A , considered instead x^j point, which is closer to x^k
 206 (in stage four). This stage is repeated for all points in set A . Ultimately, in stage seventh, set $A = \emptyset$ is
 207 obtained and it goes to step three.

208 **2.3 Robust Optimization**

209 In the present study, robust optimization applies the uncertain conditions to the proposed parameters.
 210 The linear form of the robust optimization model can be written as Eq. (2) (Ben-Tal & Nemirovski,
 211 2000):

212 Maximize cx

213 Subject to $\sum_{j=1}^n \delta_{ij} x_j \leq b_i, \quad \forall i, j \in J_i$ (2)

214 $l \leq X \leq u.$

215 Where J_i is a subset of the parameters associated with the uncertain parameter δ_{ij} which is specified
 216 for each constraint i . Assume that δ_{ij} are independent, symmetric and their boundaries are in the range
 217 of $[-1, 1]$.

218 Eq. (2) can be rewritten as robust optimization model that improves the reliability of systems under
 219 uncertainty (Bertsimas, Iancu, & Parrilo, 2010):

220 Maximize cx

221 Subject to $a_{ij} x + B_i(x, \Gamma_i) \leq b_i \quad \forall i$ (3)

222 $x \geq 0$

223 Where

224 $B_i(x, \Gamma_i) = \text{Max} \sum_{j=1}^n x_j \hat{a}_{ij} u_{ij}$

225 *Subject to*
$$\sum_{j=1}^n u_{ij} \leq \Gamma_i \quad \forall i \quad (4)$$

226
$$0 \leq u_{ij} \leq 1$$

227 Where for each j , $y_j = |x_j^*|$. In Model (5), $\sum_i^n a_{ij} x_j \leq b_i$ is presents i th constraint in terms of
 228 certainty. This model is based on a nonlinear form of robust optimization model. The reason for the
 229 nonlinearity of this model is additional maximization expression. This maximization statement by the
 230 controlling parameter of the amount of conservatism (Γ_i) guarantees the model's reliability against
 231 uncertainty. calculating, the linear form of Model 5 is used to avoid the complexity of calculations in
 232 the maximization expression as follows (Bertsimas et al., 2010):

233 And finally, the form of robust optimization is model (5):

234 *Maximize*
$$cx \text{ subject to } \sum_i^n a_{ij} x_j +$$

 235
$$\max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = [\Gamma_i], t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} \hat{a}_{ij} y_j + (\Gamma_i - [\Gamma_i]) \hat{a}_{it_i} y_{t_i} \right\} \leq b_i, \quad \forall i \quad (5)$$

236
$$-y_j \leq x_j \leq y_j, \quad \forall j \in J_i$$

237
$$l \leq X \leq u, \quad y \geq 0$$

238

239 *Maximize*
$$z = cx$$

240 *subject to*
$$\sum_j a_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i \quad \forall i$$

241
$$z_i + p_{ij} \geq \hat{a}_{ij} y_j \quad \forall i, j, \forall J_i$$

242
$$-y_j \leq x_j \leq y_j \quad \forall j \quad (6)$$

243
$$l_j \leq x_j \leq u_j \quad \forall j$$

244
$$(p_{ij}, y_j, z_i) \geq 0 \quad \forall i, j, \forall J_i$$

245 Where z , f and p are non-negative additional variables and ϵ is a given uncertainty level in the model.

246 In model 7, which is a linear form of optimization, $n + k + 1$ variables and $m + k + n$ constraints are

247 existed. The degree of confidence in the model against uncertainty depends on the parameters Γ_i . If

248 $\Gamma_i = 0$, the maximization statement is eliminated and the constraint condition is changed from
 249 uncertainty to certainty. If $\Gamma_i = |j_i|$ the degree of system protection against uncertainty is maximized
 250 and fully implemented. There are different values for Γ_i and it depends on the probability level of the
 251 deviation of i th constraint (p_i) and the number of uncertain parameters (n). Eq. (7) shows this
 252 dependence (Bertsimas et al., 2010).

$$253 \quad \Gamma_j = 1 + \Phi^{-1}(1 - p_i) \sqrt{n} \quad (7)$$

254 Where Φ is cumulative distribution of the standard Gaussian variable and n is uncertainty resources
 255 in each constraint.

256 **2.4 Application of RMOLFP Method to Optimal Sustainable Cropping Pattern**

257 **2.4.1 Objective Function**

258 In order to optimize the cropping pattern and align this model with the relative objectives, the
 259 objective functions in the present study are defined as Eq. (8):

$$260 \quad Min : \varphi_{1-3} = \frac{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R Fer_{tjrs} A_{jsr}}{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R A_{jsr}} \quad t = 1, 2, 3; \quad (8)$$

261 The objective function ϕ_1 to ϕ_3 are related to the sustainable use of nitrogen (N), phosphate (P) and
 262 potash (K) fertilizers. In these functions, A_{jsr} represents the cultivation area crop j in season s for region
 263 r , and Fer_{tjs} represents the required amount of fertilizer by type t (N, P or K) to cultivate each hectare
 264 of crop j in season s for region r .

265 Functions ϕ_4 to ϕ_6 are related to sustainability to herbicides, fungicides and insecticides.

$$266 \quad Min : \varphi_{4-6} = \frac{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R Pes_{zjrs} A_{jsr}}{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R A_{jsr}} \quad z = 1, 2, 3 \quad (9)$$

267 In these equations, Pes_{zjs} represents the required amount of pesticide by the type z per hectare of
 268 product j in season s for the region r .

269 Water resource productivity is another ecological objective of the optimal cropping pattern model
 270 that has been addressed in the objective function ϕ_7 .

$$271 \quad \text{Min} : \varphi_7 = \frac{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R \frac{NetW_{jsr}}{Eff_r} A_{jsr}}{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R A_{jsr}} \quad (10)$$

272 $NetW_{jsr}$ is the net water requirement for crop j in season s and region r (m^3 / ha), and Eff_r is irrigation
273 efficiency in the region r .

274 The ecological dimension of sustainability of crop production in the proposed model is provided by
275 objective functions ϕ_1 to ϕ_7 .

276 The objective function ϕ_8 ensures the economic dimension of the sustainability of agricultural
277 production (Eq. 11).

$$278 \quad \text{Max} : \varphi_8 = \frac{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R (Mp_{jr} + Sp_{jr} - Cp_{jr}) A_{jsr}}{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R A_{jsr}} \quad (11)$$

279 In this equation, Mp_{jr} , Sp_{jr} and Cp_{jr} are the main crops value, the sub-crop value and the production
280 cost for cultivating one hectare of crop j in region r , respectively.

281 It should be noted that the result of $Mp_{jr} + Sp_{jr} - Cp_{jr}$ is equal to the gross margin of crop j in
282 region r (\$/ha) and from now on, it will be shown with the symbol GM_{jr} .

283 The social dimension of a sustainable cropping pattern model can be considered in workforce
284 recruitment for the agricultural labor force. In this regard, the objective function ϕ_9 (Eq. 12) tries to
285 increase the workforce recruitment in the optimal cropping pattern.

$$286 \quad \text{Max} : \varphi_9 = \frac{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R Lab_{jsr} A_{jsr}}{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R A_{jsr}} \quad (12)$$

287 Where Lab_{jsr} represents the amount of required labor to cultivate each hectare of crop j in season s
288 and region r (man-day / ha).

289 2.4.2 Constraint sets

290 The scarcity of production inputs limits crop cultivation. Equations 13 to 19 clearly define these
291 limitations. The set of irrigation water constraint is related to the amount of available water. Eq. (13)
292 determines the allowable limit for the amount of water available for each area.

293
$$\sum_{j=1}^J \frac{NetW_{jsr}}{Eff_r} A_{jsr} \leq TW_{sr} \quad \forall s, r \quad (13)$$

294 TW_{sr} is the total amount of available water in season s and region r (m3).

295 The set of constraints (14) to (17) are related to the allowable limit for the amount of labor, agricultural
296 machinery, fertilizer and pesticide, respectively.

297
$$\sum_{j=1}^J Lab_{jsr} A_{jsr} \leq TLab_{sr} \quad \forall s, r \quad (14)$$

298
$$\sum_{j=1}^J Mec_{jsr} A_{jsr} \leq TMec_{sr} \quad \forall s, r \quad (15)$$

299
$$\sum_{j=1}^J Fer_{tjsr} A_{jsr} \leq TFer_{tsr} \quad \forall t, s, r \quad (16)$$

300
$$\sum_{j=1}^J Pes_{zjsr} A_{jsr} \leq TPes_{zsr} \quad \forall z, s, r \quad (17)$$

301 Where, $TLab_{sr}$, $TMec_{sr}$, $TFer_{tsr}$ and $TPes_{zsr}$, represents the total amount of available labor (man-day),
302 available agricultural machinery (hour), Total available fertilizer of type t (kg) and amount of available
303 pesticide of type z (kg or liter), respectively.

304 Eq. (18) relates to land and in this set, the cultivation area of crops is limited by the amount of
305 available land in each region.

306
$$\sum_{j=1}^J \sum_{s=1}^S Sch_{jsr} A_{jsr} \leq TA_r \quad \forall r \quad (18)$$

307 The TA_r represents the amount of available land in the region r (ha) for all crops and Sch_{jsr} indicates
308 the land occupancy coefficient. If the crops are cultivated each month (from planting to harvesting), the
309 land occupancy coefficient of that crop in those months is one, and otherwise zero.

310 In order to rely on the optimal solutions and to accept the model presented in this study, the current
311 profit of crop cultivation in each region should be provided (Eq. 19).

312
$$\sum_{j=1}^J \sum_{s=1}^S GM_{jr} A_{jsr} \geq Cur Profit_r \quad \forall r \quad (19)$$

313 In this equation $CurProfit_r$, is the current profit in the region r .

314 The set of constraints 20 and 21 specifies the maximum and minimum production of agricultural
315 products, respectively.

316
$$\sum_{s=1}^S Yield_{jr} A_{jsr} \leq MaxProd_{jr} \quad \forall jr \quad (20)$$

317
$$\sum_{s=1}^S Yield_{jr} A_{jsr} \geq MinProd_{jr} \quad \forall jr \quad (21)$$

318 Where $Yield_{jr}$ is the yield of crop j in the region r and $MaxProd_{jr}$ and $MinProd_{jr}$ are the
319 maximum and minimum allowable amount of production of agricultural products, respectively. The
320 amount of maximum production in Eq. (20) is determined based on the potential of each region and
321 macroeconomic considerations. The amount of minimum production in Eq. (21) is calculated based on
322 self-consumption needs and food security policies set by the government for each region.

323 2.4.3 Uncertain data

324 There are many uncertain parameters in an optimization model for cropping pattern. In this study,
325 the total amount of available water (TW_{sr}) and the gross margin (GM_{jr}) are considered as uncertain
326 parameters. Therefore, by defining the parameter Γ_{sr}^{TW} , variables z_{sr}^{TW} and p_{sr}^{TW} and using model 6, the
327 constraint of the total amount of available water (Eq. 13) is converted into two Eqs. 22 and 23.

$$328 \quad E \sum_{j=1}^J \frac{NetW_{jsr}}{Eff_r} A_{jsr} + z_{sr}^{TW} \Gamma_{sr}^{TW} TW_{sr} + p_{sr}^{TW} TW_{sr} \leq T\bar{W}_{sr} \quad \forall s,r \quad (22)$$

$$329 \quad z_{sr}^{TW} + p_{sr}^{TW} \geq \varepsilon T\bar{W}_{sr} \quad \forall s,r \quad (23)$$

330 Where $T\bar{W}_{sr}$ represents the nominal value of the total amount of available water.

331 Considering GM_{jr} as an uncertain parameter, the two Eqs. 11 and 19 change. These equations, like
332 Eqs. (22) and (23), are converted to robust form by defining new parameters and variables. Eq. (24) and
333 (25) are formulated to robustification the objective function φ_8 (maximizing gross margin) and Eq.
334 (26) and (27) are formulated to robustification the Eq. 19.

$$335 \quad \varphi_8 = \frac{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R (G\bar{M}_{jr}) A_{jsr}}{\sum_{j=1}^J \sum_{s=1}^S \sum_{r=1}^R A_{jsr}} - z^{GM1} \Gamma^{GM1} - \sum_{j=1}^J \sum_{r=1}^R p_{jr}^{GM1} \quad (24)$$

$$336 \quad z^{GM1} + p_{jr}^{GM1} \geq \varepsilon G\bar{M}_{jr} \quad \forall j,r \quad (25)$$

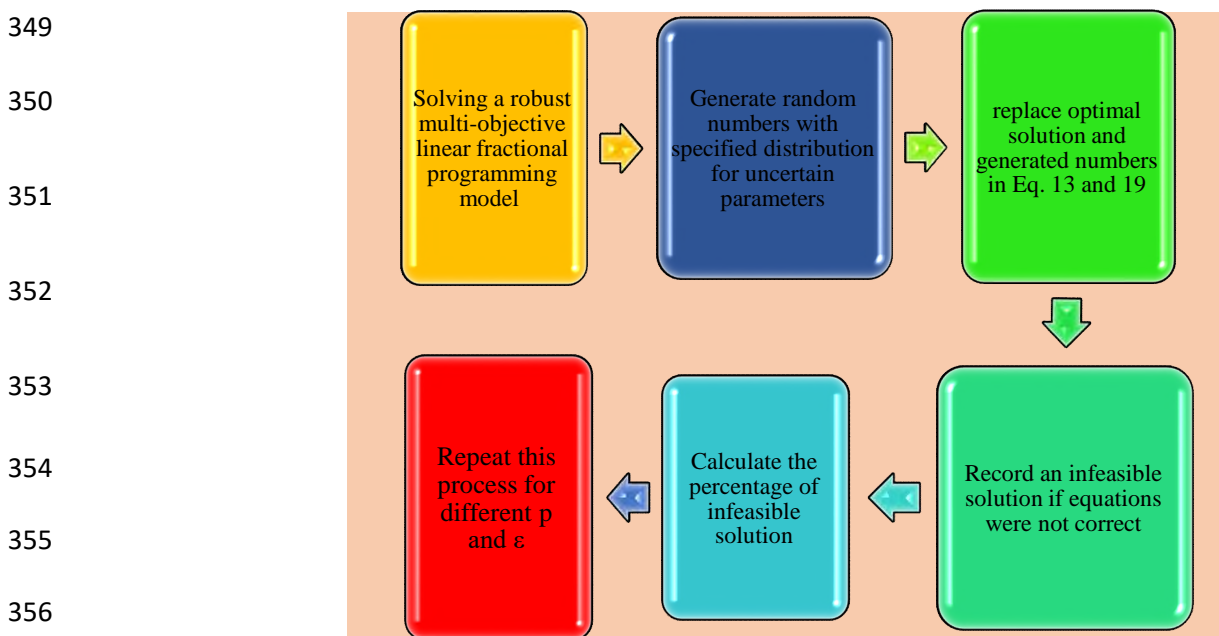
$$337 \quad \sum_{j=1}^J \sum_{s=1}^S GM_{jr} A_{jsr} - z^{GM2} \Gamma^{GM2} - \sum_{j=1}^J p_{jr}^{GM2} \geq CurProfit_r \quad \forall r \quad (26)$$

$$338 \quad z^{GM2} + p_{jr}^{GM2} \geq \varepsilon G\bar{M}_{jr} \quad \forall j,r \quad (27)$$

339 2.4.4 Model evaluation

340 Monte Carlo simulation method is used to evaluate the proposed model (Figure 3). For this purpose,
341 after solving the robust multi-objective linear fractional programming, 10,000 random numbers with
342 specified distribution for the uncertain parameters (in this study, the amount of available water and
343 gross margin) are generated. After solving the proposed mathematical programming model, the values
344 of the optimal cultivation area (A_{jsr}^*) and generated numbers are replaced in Eq. 13 and 19. If the
345 equations were not correct after these replacements, they are recorded as infeasible solution indicating
346 model's inability to apply uncertain data.

347 This process is performed at different levels of probability deviation of constraint (p) and given
348 uncertainty level (ϵ).



357 Figure 3. Monte Carlo simulation process to evaluate the capability of a robust multi-objective linear
358 fractional programming model

359 **3. Results**

360 **3.1 Primary data processing**

361 In this study, all the required data were obtained from various governmental agencies such as the
362 Jihad-e-Keshavarzi organization (Anonymous, 2018), Khuzestan Water and Power Authority and the
363 Great Karoun Irrigation Network Operating Company (Anonymous, 2017) in the year 2017. In this

364 network, there are generally winter and summer agricultural products in which 12 types of crops are
 365 cultivated. Some of these crops are perennial (such as alfalfa). Table 1 lists these crops along with the
 366 current cultivation area. Wheat is one of the most popular crops among farmers in this irrigation and
 367 drainage network due to its guaranteed price and elimination of income risk, with about 17,000 hectares
 368 cultivation area. Although the availability of water and canal length in the Aghili area is less than the
 369 Dimech area, the presence of more modern irrigation equipment in the Aghili area has made its current
 370 cultivation area (about 2800 hectares) the same as the area of Dimcheh.

371 Table 1. Crop cultivation area in the Gotvand irrigated and drainage network by operation zones (ha)

Crop	Gotvand	Aghili	Dimcheh	total	The share of the cultivation area (Percent)
Corn	0	1015	75	1090	3.19
Eggplant	212	251	1088	1551	4.54
Rice	736	1977	1902	4615	13.52
Beans	859	0	881	1740	5.1
Okra	0	580	293	873	2.56
Barley	32	61	90	183	0.54
Onion	10	110	0	120	0.35
Wheat	4468	6695	5782	16945	49.63
Canola	0	428	360	788	2.31
Vegetables	967	0	1837	2804	8.21
Broad bean	943	0	1015	1958	5.73
Alfalfa	631	119	725	1475	4.32
Total	8858	11238	14048	34144	100

372

373 Table (2) summarizes the important parameters used in the proposed model. Table (1) and
 374 Table (2) provide exciting findings on the relationship between the cultivation area and input
 375 consumption. Although the Corn cultivation area is lower than wheat and rice, the gross margin
 376 for corn (\$ 5864 per hectare) is more than the mentioned crops (Table 1). The government has
 377 led a restriction on corn cultivation in the study area since the consumption of fertilizer and
 378 pesticide for its cultivation is more the other two crops. However, this coercion was

379 accompanied by fierce opposition from farmers (due to poverty) and led to severe tensions
380 between government bodies and farmers. It is also evident that despite the low consumption of
381 these inputs in barley, the cultivation area of this crop is just about 183 hectares. The simplest
382 reason is the low gross profit of this product (\$ 192 per hectare) compared to other products,
383 so the low gross margin has led to farmers' unwillingness to cultivate this crop.

384 Tables (1) and (2) show that the study area desperately needs a revision of the current
385 cropping pattern. This change should be made in light of the problems already mentioned.
386 Firstly, the farmers tend to participate in this change, and it is related to how much income,
387 they can earn (more or equal than their current income) and secondly, due to the critical
388 conditions of fertilizers and chemical pesticides consumption in the area, their consumption is
389 drastically reduced. In other words, it is necessary to make a compromise between the different
390 objectives in the area.

391

392

393 Table 2- Average technical and economic coefficients of agricultural product in the Gotvand irrigation and drainage network (per hectare)

	Rice	Corn	Beans	Broad bean	Alfalfa	Onion	Barley	Wheat	Canola	Okra	Vegetables	Eggplant
Gross margin (\$)	1842	5864	100	401	1106	1003	192	412	214	2904	3481	2958
Production Cost (\$)	1603	1378	1031	817	2146	1910	563	768	740	1824	1620	954
Phosphate (kg)	239	109	112	60	238	114	22	98	88	55	124	36
Nitrogen (kg)	395	472	224	30	16	248	123	353	279	324	367	152
Potash (kg)	0	14	0	60	17	9	0	6	25	46	53	17
Total fertilizer (kg)	634	595	336	150	272	370	146	457	393	344	444	371
Herbicide (kg)	0.88	3.07	0	0	0.19	1.86	0.28	1.48	3.08	1.18	1.44	1.8
Insecticides (kg)	1.52	0.58	0	0	0.14	1.68	0.01	0.12	0.76	0.7	0	1.6
Fungicides (kg)	0	0	0	0	0.06	1.24	0.01	0.01	0.14	0	0.11	0.12
Total pesticide(kg)	2.4	3.65	0	0	0.4	4.78	0.3	1.61	3.98	1.2	0	3.24
Net water requirement(m ³)	10260	3084	6252	5868	14448	5052	2148	2724	4106	13973	9502	4523
Machinery (hours)	25	31	34	40	38	24	28	26	36	28	24	40
Labor (man-day)	121	15	31	32	60	43	8	7	8	85	24	16

394

395 **3.2 Software and model solving algorithm**

396 The proposed mathematical algorithms are modeled by a robust multi-objective fractional
397 linear programming method and are fully coded in GAMS optimization software and solved
398 for different levels of p and ε and generate different scenarios to apply conservatism against
399 uncertainty. The optimization method chosen for solving this model was CONOPT4, which is
400 an optimizer for solving large-scale nonlinear programming problems. CONOPT4 is based on
401 the Generalized Reduced Gradient (GRG) method and is developed in GAMS software by the
402 Danish Consulting and development firm ARKI (GAMS/CONOPT4, 2015).

403 **3.3 Results of the RMOLFP Model Solution**

404 **3.3.1 Cultivation area**

405 With the implementation of this model, a massive difference in the current and proposed
406 patterns was observed. Referring to Table (3), the details of these changes are identified, and
407 the crop cultivation level is presented at $p = 1$ and 0 (scenario 1), which in fact, models the
408 certainty conditions. Also, the model results at $p = 0.1$ and $\varepsilon = 0.1$ (scenario 2) are presented
409 for applying the uncertainty conditions, which creates an excellent opportunity for comparing
410 these two scenarios.

411 Table (3) reports the area of each crop for cultivation under two scenarios. It is observed that
412 onion in both scenarios was excluded from the model. This can be traced to the large
413 consumption of inputs and the low gross margin for this product. For example, the total
414 pesticide use (herbicides, insecticides and fungicides) for this product was 4.78 liters per
415 hectare, which is more than the other crops. However, the share of this product is insignificant,
416 and its overall elimination due to regional conditions seems reasonable. Due to the significant
417 share of wheat and rice in the current cropping pattern (about 63%), the cultivation area
418 variation of both crops in two scenarios is analyzed.

419 Table (3) shows that there is an increase (in scenario 1) in the cultivation area of wheat and
420 rice, which increases the cultivation area to 2635 hectares in the entire area. With the increase
421 in model protection against uncertainty, the proposed cultivation area has decreased
422 significantly. The main reason for this reduction compared to scenario 1 is the extensive water
423 use of these two products. This result seems reasonable due to the increased conservatism
424 leading to a decrease in the amount of available water.

425 The cultivation area of some crops such as rice and corn have been increased in the proposed
426 models to offset the reduction in profits in both scenarios. So, the policy that is currently being
427 implemented by the government to limit corn cultivation area is not appropriate. This policy
428 increases the dissatisfaction of farmers and, on the other hand, loses the alternatives. For
429 example, the government could authorize the cultivation of corn along with barley, which has
430 an increased cultivation area on both scenarios, which will increase farmers' interest in
431 cultivating barley (Table 2). However, this policy should be done with caution if the whole
432 proposed pattern is implemented .

433 It can be observed that by increasing the model's protection against uncertainty (decreasing the
434 probability level p), the total cultivation area decreases; at $p = 1$, the cultivation area increased
435 by 9.34%, and at $p = 0.1$ the mentioned area decreased by 3.63%.

Table 3. Cultivation area of optimal cropping pattern of Gotvand irrigation and drainage network at different levels of p and ϵ

Region	Rice	Corn	Beans	Broad bean	Alfalfa	Onion	Barley	Wheat	Canola	Okra	Vegetables	Eggplant	Total
(Scenario 1)													
Gotvand	1128	0	403	852	879	0	65	4849	0	0	1605	76	9857
Aghili	2731	1446	0	0	229	0	0	6897	95	1090	0	350	12838
Dimcheh	2610	424	273	1021	635	0	329	5970	112	286	2627	678	14965
Total	6469	1870	676	1873	1743	0	394	17716	207	1376	4232	1104	37660
Percentage change	28.66	41.72	-157.3	-4.53	15.38	-100	53.55	4.35	-280.6	36.54	33.74	-40.47	9.34
(Scenario 2)													
Gotvand	1128	0	403	852	879	0	65	4849	0	0	1605	76	9857
Aghili	2270	1446	0	0	119	0	0	6816	95	1049	0	350	12145
Dimcheh	2045	424	273	922	232	0	329	4217	112	91	1905	393	10943
Total	5442	1870	676	1774	1229	0	394	15883	207	1139	3510	819	32943
Percentage change	15.2	41.72	-157.3	-10.36	-19.97	-100	53.55	-6.69	-280.6	23.38	20.12	-89.28	-3.63

438 3.3.2 *Consumption of chemical inputs (ecological dimension)*

439 Table (4) presents the results of the changes in fertilizer and pesticide in the study area. However,
440 the absolute reduction in the number of consumed inputs in optimal models does not necessarily indicate
441 an improvement in the use of these production resources, and it just happened by reducing the
442 cultivation area. Therefore, the average fertilizer application per hectare can be a good criterion for
443 examining the effect of cropping pattern change on the optimal use of these resources. The results show
444 that the fertilizers and pesticides used has decreased by approximately 6% and 8%, respectively, in both
445 scenarios. Due to the current challenging economic situations in Iran (Poor economic growth, sanctions
446 and unemployment), reducing this amount of fertilizer is crucial for the agriculture sector.

447

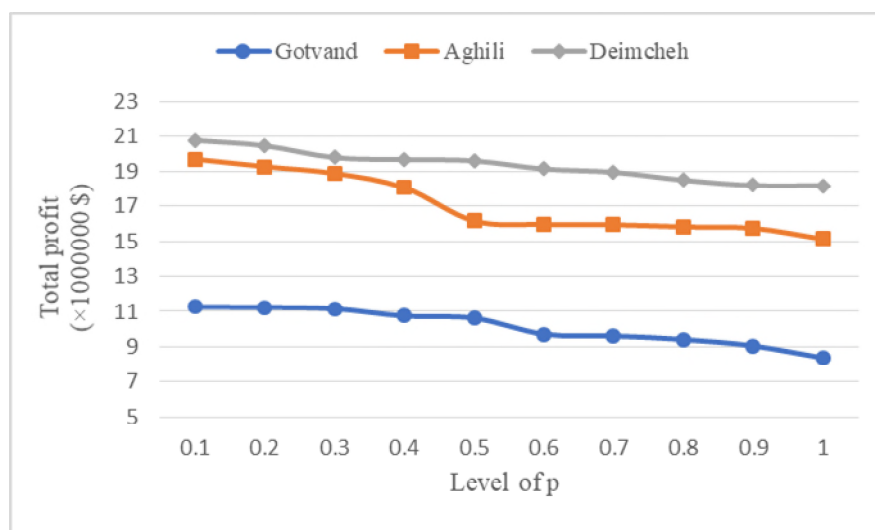
448 Table 4. Average current and optimal consumption of pesticides and Fertilizers in the Gotvand Irrigation and Drainage Network (Unit: Kg or L/ha)

Model	Region	Phosphate	Nitrogen	Potash	Total fertilizer	Herbicide	insecticide	Fungicides	Total pesticides
Current	Gotvand	129.9	309.5	18.5	457.9	1.14	0.26	0.02	1.43
	Aghili	136.8	402.8	10	549.6	1.76	0.58	0.03	2.36
	Dimcheh	131.4	322.1	19.4	472.8	1.31	0.47	0.03	1.81
	Total	132.7	344.8	15.9	493.4	1.4	0.43	0.03	1.86
Scenario 1	Gotvand	127.2	293.8	18.4	439.3	1.1	0.26	0.02	1.38
	Aghili	126.3	360.8	9.7	496.8	1.5	0.56	0.01	2.08
	Dimcheh	125.7	312.2	18.7	456.6	1.22	0.43	0.03	1.68
	Total	126.4	322.3	15.6	464.3	1.27	0.42	0.02	1.71
	Percentage change	-4.8	-6.5	-2.1	-5.9	-9	-4.13	-29.63	-8.19
Scenario 2	Gotvand	127.2	293.8	18.4	439.3	1.1	0.26	0.02	1.38
	Aghili	121.5	362.8	9.9	494.1	1.54	0.53	0.01	2.09
	Dimcheh	124.4	315.2	18.7	458.4	1.22	0.43	0.03	1.67
	Total	124.4	323.9	15.7	464	1.29	0.41	0.02	1.71
	Percentage change	-6.3	-6.1	-1.7	-6	-8.17	-6.43	-32.09	-8.15

449

450 3.3.3 System profit (economic dimension)

451 The vital point in examining the proposed model is that with increasing system protection against
 452 uncertainty (decreasing level of p from 1 to 0.1), the Total profit has increased. Figure 4 is presented to
 453 investigate this issue better and analyze its sensitivity at different levels of p ranging from 0.1 to 1 with
 454 steps 0.1 for each region. It is observed that as the level of p increases, the amount of total profit
 455 decreased in all three regions. In other words, there is a trade-off correlation between the degree of
 456 system protection against uncertainty (robustness) and the amount of total profit.



457
 458 Figure 4. Total profit at different levels of p in the Gotvand irrigation and drainage network regions

459 3.3.4 Other inputs (social and ecological dimensions)

460 Table (5) reports the utilization of other inputs generated by implementing the two scenarios of
 461 optimal cropping patterns and the current situation. In both scenarios and all three regions, irrigation
 462 water use per hectare decreased. In the Gotvand region, the reduction of water use per hectare is
 463 increased with increasing system protection against uncertainty, and in other regions, it remains
 464 unchanged. In other words, there is no particular trend (such as total profit) for water consumption by
 465 increasing the system protection against uncertainty. In Gotvand and Dimcheh regions, as the level of
 466 p decreases, the utilization of labour and machinery does not have a significant change, while in the
 467 Agheli region, by decreasing the level of p, the increase in the labour is about 10% lower than the first
 468 scenario.

469 Table 5. Average, current and optimal consumption of other agricultural inputs per hectare in different
 470 scenarios

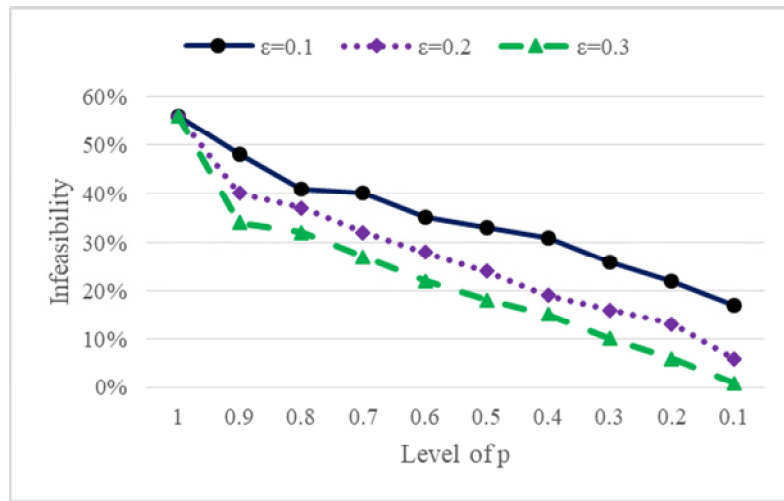
		Scenario 1			Scenario 2	
		Current amount	Optimal amount	Percentage Change	Optimal amount	Percentage Change
Gotvand	Irrigation water (Cubic meters)	6403.7	5804.8	-9.4	5804.8	-36.9
	Machinery (hour)	32.1	28.3	-11.8	28.3	-11.8
	Labor (man-day)	30.1	30.8	2.2	30.8	2.2
Aghili	Irrigation water (Cubic meters)	6040.4	5519.8	-8.6	5284.6	-7.4
	Machinery (hour)	27.2	27.2	0	27.2	0
	Labor (man-day)	33	40	21.2	36.8	11.6
Dimcheh	Irrigation water (Cubic meters)	6598.7	6138.9	-9.1	5997.5	-9.1
	Machinery (hour)	32.1	28	-12.6	28	-12.8
	Labor (man-day)	36.4	36.4	0	36.4	0

471

472 **3.4 Evaluation the capability of the proposed model against uncertainty**

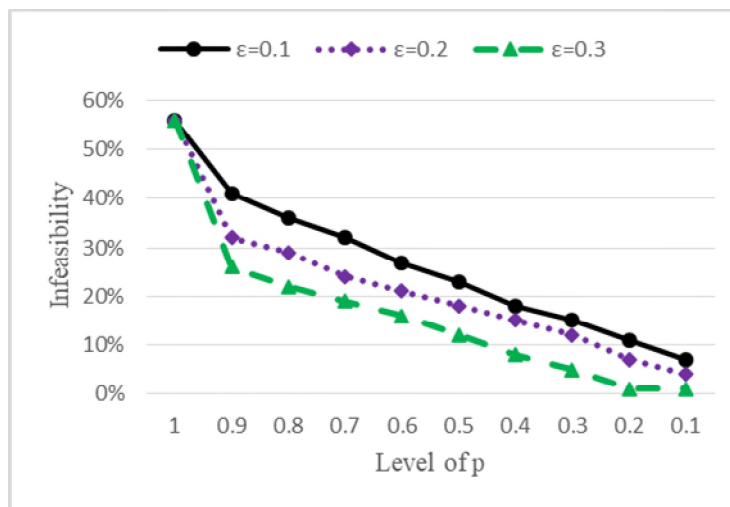
473 Monte Carlo simulation method was used to validate the proposed model, as shown in Figure 3. In
 474 this regard, 10,000 random data were generated for the available water parameter with uniform (at a
 475 given distance) and normal (with 95% convergence) distributions for uncertain parameters GM_{jr} and
 476 TW_{sr} . Then, the feasibility of randomly generated numbers was investigated by using the simulation
 477 method. Figures (5) and (6) show the results of this simulation method for different levels of p and ϵ
 478 for random numbers generated using uniform and normal distributions, respectively. It is observed that
 479 in both distributions, with the decrease in the level of p , the percentage of infeasibility is reduced; So
 480 that at the 10% given uncertainty level ($\epsilon=0.1$) with a uniform distribution, with the decrease level of p
 481 from 100% to 10%, the infeasibility percentage is decreased from 56% to 17%. It is also found that by
 482 increasing the given uncertainty level in both distributions, the average infeasibility is decreased. The

483 lower infeasibility at high levels of conservatism (lower levels of p) indicates the considerable
 484 robustness of the proposed optimal cropping pattern model to overcome the uncertainty.



485

486 Figure 4. Infeasibility percentage of the proposed model assuming uniform distribution over a given
 487 interval



488

489 Figure 5. Infeasibility percentage of the proposed model assuming normal distribution with 95%
 490 convergence

491 **4. Discussion**

492 In this section, the results of the present study and previous studies are compared based on three
 493 ecological, economic and social dimensions of the optimal sustainable cropping pattern. A summary of
 494 this discussion is provided as follows:

495 ***4.1 Ecological dimension***

496 Increasing the cultivation area in the Gotvand irrigation and drainage network led to a decrease in
497 the total area under the proposed model, improperly. Reducing the cultivation area in optimal cropping
498 pattern models is not limited to this study and has been recommended in most similar studies for
499 different regions of Iran (M. S. Sabouhi & Mardani, 2013). For example, Mardani Najafabadi et al.
500 (2019), used the multi-objective robust optimization method in their cropping pattern programming
501 model and they found that about 16.5% of the total cultivation area in Isfahan province has been
502 reduced.

503 According to statistics published by the Khuzestan Agricultural Jihad Organization, the use of
504 fertilizers and pesticides in the Gotvand irrigation and drainage network has an upward trend and the
505 consumption of these two destructive inputs will almost double. It is crucial to prevent the increase in
506 the use of fertilizers and pesticides for this region, however, the reduction in their use in the proposed
507 model is less than 10% (Anonymous, 2017).

508 In contrast, Neamatollahi, Vafabakhshi, Jahansuz, and Sharifzadeh (2017) applied different
509 scenarios by using a fuzzy multi-objective planning model, they found that the reduction rate of
510 fertilizer and chemical pesticide consumption are 38% and 35%, respectively. These percentages are
511 realized only when the demand for the industrial and livestock sectors is not taken into account. If this
512 demand is calculated, the consumption of fertilizers and pesticides will increase sharply by 52 and 11
513 percent, respectively. The important point in the present study is that the need for self-consumption and
514 food safety considerations are considered in Equation 21 and this has led to more input consumption of
515 fertilizers and pesticides. However, contrary to the study of Neamatollahi et al. (2017), the consumption
516 of these two destructive inputs has decreased.

517 In many studies that have tried to optimize the cropping pattern, the ecological dimension has been
518 considered only for the optimal use of irrigation water (C. Zhang et al., 2018; F. Zhang et al., 2018).
519 Tan and Zhang (2018) optimized irrigation water use efficiency using robust fraction programming.
520 The result of their study is that with increasing conservatism (reducing the level of probability of
521 deviation), the efficiency of water consumption decreases.

522 ***4.2 Economic dimension***

523 It should be noted that the economic and ecological dimensions are closely related. For example, in
524 most studies that consider environmental objectives, optimal use of inputs that lead to disruption of the
525 ecological system has been suggested as a viable solution for economic (lower cost on the consumption
526 of production inputs) and ecological (optimal use of chemical inputs) sustainability (Mardani
527 Najafabadi et al., 2019).

528 Sensitivity analysis of gross profit in the present study showed that with increasing system protection
529 against uncertainty, system profit decreases. According to a review of research, this sensitivity analysis
530 has been used in studies such as Tan & Zhang (2018) and M. S. Sabouhi and Mardani (2013) that used
531 a robust optimization approach. Depending on the number of uncertain parameters in the models, this
532 decrease in profit or increase in cost is distinctive.

533 Increasing the protection of the system against uncertainty is not limited to the robust optimization
534 method. For example, in the study of Zhang et al. (2017), the probability of occurrence for different
535 inflow is considered as different levels of conservatism. The profit of the system has increased by
536 increasing the conservatism which means the protection of the system against uncertainty is reduced.
537 Wang et al. (2019) used a bi-level multi-objective linear fractional programming model with the theory
538 of fuzzy sets and different levels of alpha (Uncertainty condition) to redistribute water to the
539 agricultural, industrial, urban, and ecological sectors of the Heihe River basin in China. They found that
540 increasing the level of conservatism (alpha levels) reduced the total gross profit, especially in the
541 agricultural sector.

542 Filippi et al., (2017) used the mixed integer linear programming model in similar conditions. The
543 objective function of their model was different, so they maximized the net return function with the
544 approach of Conditional Value-at-Risk as a safety measure. Increasing the risk-aversion parameter in
545 this model, which can be considered as increasing the protection of the system against uncertainty, led
546 to a decrease in profit.

547 **4.3 Social dimension**

548 In the present study increasing employment and food security has been considered as two aspects of
549 social dimensions. According to the results, by implementing the cultivation pattern introduced in this
550 study, the total employment rate will increase which shows the priority of ecological dimension in
551 Gotvand region (especially reducing the use of fertilizers and pesticides). Although the aspect of food
552 security in the present study is not considered as a main objective, however, in Equation 21, the need
553 for self-consumption as well as government policies on food security when estimating the right-hand
554 side of this constraint (the minimum production rate of each product) is calculated.

555 Mardni Najafabadi et al., (2019) considered Maximization of the use of labor (employment
556 aspect) and minimization of the net import of energy from agricultural products (food security aspect).
557 As the result, the employment rate in the whole region increased by 9%; But its relative rate (labor per
558 hectare) has decreased by 11 percent. In this study, the food security index did not improve due to a 2%
559 increase in net energy imports. Considering the increase in population and the consequent increase in
560 the need for calories, this result seems reasonable. However Wineman and Crawford (2017) studied
561 that due to the importance of the aspect of food security in a country, its only main objective is to
562 maximize calorie production through the cultivation of agricultural products.

563 Increasing social equality is one of the important indicators in some studies of water or land
564 redistribution pattern. Increase social equity in water redistribution with the aim of increasing the Gini
565 coefficient in the equitable distribution of irrigation water is another economic dimension that studied
566 by Wang et al. (2019). The results of this redistribution showed that if the proposed model is
567 implemented, the Gini coefficient of water distribution is reduced, which indicates a reduction in the
568 distribution inequality.

569 **4.4 Summery**

570 Due to many aspects of the optimal cropping pattern, Table 6 was designed to provide a summary
571 of the status of the studies previously discussed. Although, it should be noticed that these studies have

572 different objectives depending on the areas under study and the different priorities of each of them in
573 solving the challenges, so considering the different challenges is not the reason for the superiority of
574 the studies.

575 The proposed model in the present study does not apply only the supply-demand balance aspect
576 since the transportation equations in this model is not provided. In the study of Mardani Najafabadi et
577 al. (2019), agricultural products could be transferred from one county to another and also outside of
578 Isfahan province. For this reason, the lack of supply of products from outside the province or other
579 county was compensated. However, in this study, adding this feature required redesigning the model,
580 which was not necessary due to regional priorities. As can be seen, this aspect has not been considered
581 in most studies.

582

Table 6. Comparison of the results of the present study with others

NO.	Title	Mathematical programming model	Economic dimension							Ecological dimension		Social dimension		
			inputs	utilization of	Optimal	increase	Gross profit	balance	Supply - demand	condition	uncertainty	Applying	considerations	Environmental
1	(Mardani Najafabadi et al., 2019)	Robust Multi-Objective Programming	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
2	(C. Zhang et al., 2018)	inexact robust two-stage mixed-integer linear programming	✓	✓	✗	✓	✗	✓	✓	✓	✗	✗	✗	
3	(Wineman & Crawford, 2017)	Liner programming	✓	✓	✗	✗	✗	✗	✗	✗	✗	✗	✓	
4	(Zhang et al., 2017)	interval multistage joint-probabilistic chance constrained programming	✓	✓	✗	✓	✗	✓	✓	✓	✗	✗	✗	
5	(Zeng, Kang, Li, Zhang, & Guo, 2010)	Fuzzy multi-objective linear programming	✓	✓	✗	✓	✗	✓	✓	✓	✗	✗	✗	
6	(F. Zhang et al., 2018)	interval nonlinear multi-objective programming	✓	✓	✗	✓	✗	✓	✓	✓	✗	✗	✗	
7	(Filippi et al., 2017)	Mixed integer linear programming	✓	✓	✗	✓	✗	✓	✓	✗	✗	✗	✗	
8	(Tan & Zhang, 2018)	Robust fractional programming	✓	✓	✗	✓	✗	✓	✓	✗	✗	✗	✗	
9	(Wang et al., 2019)	bi-level multi-objective linear fractional programming	✓	✓	✗	✓	✗	✓	✓	✓	✗	✗	✓	

10 Proposed model

Robust Multi-Objective Fractional
Linear Programming



584

585 **5. Conclusion**

586 Over the past few decades, environmentalists have consistently emphasized the need to create a
587 sustainable agricultural system because of imbalances between water supply and demand, falling
588 aquifers, increasing fertilizer and pesticides use and depleting soil resources to conserve valuable
589 environmental resources. Therefore, assessing the sustainability of farming systems to prevent the
590 degradation of water and soil resources and reducing economic and social damages has become one of
591 the top priorities of agricultural policymakers. Hence, this study aims to achieve an optimal cropping
592 pattern for balancing the consumption of agricultural inputs (especially fertilizers and pesticides) by
593 using a robust multi-objective fractional programming approach based on two scenarios (with and
594 without uncertainty). The study area was located under the irrigation and drainage network of Gotvand
595 in Khuzestan province, Iran, while it is dissimilar in terms of the average consumption of fertilizers and
596 pesticides due to other provinces. The model was solved at different levels of deviation probability of
597 each constraint (p) in two scenarios. The results showed that using the optimal cropping pattern in both
598 scenarios made it possible to achieve the appropriate and optimal use of agricultural inputs (especially
599 fertilizers and pesticides), which is the main objective of the study. However, the gross profit of the
600 proposed model shows a slight increase in increasing system protection against uncertainty. Since, in
601 the real world, the probability of uncertainty in the utilized data is higher, it is, therefore, advisable to
602 use scenario (2) results to apply changes in the cropping pattern. Different and sometimes conflicting
603 objectives in the structural programming scheme of the cropping pattern have been considered. It is
604 possible to compromise between the considered objectives in the RMOLFP model, which is
605 recommended to decision-makers. On the one hand, farmers are encouraged to implement the proposed
606 model (due to the increase in gross profit) and on the other hand, the rate of water and chemical inputs
607 (fertilizer and pesticides) consumption, which are in line with the implementation of the sustainable
608 optimal cropping pattern, will be reduced.

609 **6. Declarations**

610 **Availability of data and materials**

611 All data generated or analyzed during this study are included in this published article.

612 **Competing interests**

613 The authors declare that they have no competing interests.

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617 **Authors' contributions**

618 **Mostafa Mardani Najafabadi:** Conceptualization, Methodology, Software, Formal analysis and
619 investigations, Writing - Original Draft, Writing - Review & Editing, Resources.

620 **Nilofar Ashktorab:** Conceptualization, Methodology, Software, Formal analysis and investigations,
621 Writing - Original Draft, Writing - Review & Editing, Resources.

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