Supplementary information

Adiabatic Quantum Power Flow

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Supplementary tables

For justifying the partitioned AQPF algorithm applied on the 14-bus test system, the obtained $\vec{\mu}$ and $\vec{\omega}$ with $\epsilon = 1 \times 10^{-3}$ using DA are presented in the following table. The corresponding $\vec{\mu}$ and $\vec{\omega}$ obtained by NR are also included in the table.

Table 1: Comparison of $\vec{\mu}$ and $\vec{\omega}$ obtained by DA using a partitioned AQPF and NR for the 14-bus test system.

	DA		NR	
i	μ_i	ω_i	μ_i	ω_i
0	1.0	0.0	1.0	0.0
1	1.045	-0.106	1.038	-0.118
2	0.986	-0.239	0.978	-0.249
3	1.001	-0.197	0.996	-0.203
4	1.007	-0.154	0.999	-0.176
5	1.051	-0.280	1.029	-0.292
6	1.030	-0.284	1.012	-0.265
7	1.075	-0.257	1.054	-0.276
8	1.010	-0.290	0.989	-0.288
9	1.001	-0.289	0.987	-0.291
10	1.028	-0.296	1.014	-0.293
11	1.063	-0.312	1.028	-0.302
12	1.002	-0.297	1.002	-0.301
13	0.979	-0.302	0.972	-0.306

Supplementary equations

For the QUBO formulation of the net active power, substitution of equations (7)–(8) into equation (5) yields

$$
\begin{aligned}
p_i = \sum_{j=0}^{n} \mu_i^0 g_{ij} \mu_j^0 + \mu_i^0 g_{ij} x_{j,0}^{\mu} \Delta\mu_j - \mu_i^0 g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ x_{i,0}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 + x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ x_{i,1}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 - x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ \omega_i^0 g_{ij} \omega_j^0 + \omega_i^0 g_{ij} x_{j,0}^{\omega} \Delta\omega_j - \omega_i^0 g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ x_{i,0}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 + x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ x_{i,1}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 - x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ \omega_i^0 b_{ij} \mu_j^0 + \omega_i^0 b_{ij} x_{j,0}^{\mu} \Delta\mu_j - \omega_i^0 b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ x_{i,0}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 + x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
- x_{i,1}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 - x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ \mu_i^0 b_{ij} \omega_j^0 - \mu_i^0 b_{ij} x_{j,0}^{\omega} \Delta\omega_j + \mu_i^0 b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ x_{i,0}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 - x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ x_{i,1}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 + x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j .
\end{aligned} \tag{1}
$$

A similar expression can also be derived for the QUBO formulation of the net reactive power injection by substituting (7)–(8) into equation (6).

$$
\begin{aligned}
q_i = \sum_{j=0}^{n} \omega_i^0 g_{ij} \mu_j^0 + \omega_i^0 g_{ij} x_{j,0}^{\mu} \Delta\mu_j - \omega_i^0 g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ x_{i,0}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 + x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,0}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ x_{i,1}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 - x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,1}^{\omega} \Delta\omega_i g_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ \mu_i^0 g_{ij} \omega_j^0 - \mu_i^0 g_{ij} x_{j,0}^{\omega} \Delta\omega_j + \mu_i^0 g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ x_{i,0}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 - x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,0}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ x_{i,1}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 + x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,1}^{\mu} \Delta\mu_i g_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ \mu_i^0 b_{ij} \mu_j^0 - \mu_i^0 b_{ij} x_{j,0}^{\mu} \Delta\mu_j + \mu_i^0 b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ x_{i,0}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 - x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j + x_{i,0}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
+\ x_{i,1}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 + x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,0}^{\mu} \Delta\mu_j - x_{i,1}^{\mu} \Delta\mu_i b_{ij} x_{j,1}^{\mu} \Delta\mu_j \\
-\ \omega_i^0 b_{ij} \omega_j^0 - \omega_i^0 b_{ij} x_{j,0}^{\omega} \Delta\omega_j + \omega_i^0 b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
-\ x_{i,0}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 - x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j + x_{i,0}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j \\
+\ x_{i,1}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 + x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,0}^{\omega} \Delta\omega_j - x_{i,1}^{\omega} \Delta\omega_i b_{ij} x_{j,1}^{\omega} \Delta\omega_j.
\end{aligned}
\tag{2}
$$

Supplementary equations (1)–(2) are extended forms of equations (9)–(10) presented in the paper, respectively.

In equation (9), the constant clause is

$$
\sum_{j=0}^{n} \mu_i^0 g_{ij} \mu_j^0 + \omega_i^0 g_{ij} \omega_j^0 + \omega_i^0 b_{ij} \mu_j^0 - \mu_i^0 b_{ij} \omega_j^0, \tag{3}
$$

the linear clause is

$$
\begin{aligned}
\sum_{j=0}^{n} \sum_{k=0}^{1} (-1)^k \mu_i^0 g_{ij} x_{j,k}^{\mu} \Delta\mu_j + (-1)^k x_{i,k}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 \\
+(-1)^k \omega_i^0 g_{ij} x_{j,k}^{\omega} \Delta\omega_j + (-1)^k x_{i,k}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 \\
+(-1)^k \omega_i^0 b_{ij} x_{j,k}^{\mu} \Delta\mu_j + (-1)^k x_{i,k}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 \\
-(-1)^k \mu_i^0 b_{ij} x_{j,k}^{\omega} \Delta\omega_j - (-1)^k x_{i,k}^{\mu} \Delta\mu_i b_{ij} \omega_j^0,
\end{aligned}
\tag{4}
$$

and the quadratic clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{1}\sum_{l=0}^{1}&(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i g_{ij}x_{j,l}^{\mu}\Delta\mu_j\\ &+(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i g_{ij}x_{j,l}^{\omega}\Delta\omega_j\\ &+(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i b_{ij}x_{j,l}^{\mu}\Delta\mu_j\\ &-(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i b_{ij}x_{j,l}^{\omega}\Delta\omega_j,\end{aligned}\tag{5}$$

Similarly, in equation (10), the constant clause is

$$\sum_{j=0}^{n}\omega_i^0 g_{ij}\mu_j^0-\mu_i^0 g_{ij}\omega_j^0-\mu_i^0 b_{ij}\mu_j^0-\omega_i^0 b_{ij}\omega_j^0,\tag{6}$$

the linear clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{1}&(-1)^{k}\omega_i^0 g_{ij}x_{j,k}^{\mu}\Delta\mu_j+(-1)^{k}x_{i,k}^{\omega}\Delta\omega_i g_{ij}\mu_j^0\\ &-(-1)^{k}\mu_i^0 g_{ij}x_{j,k}^{\omega}\Delta\omega_j-(-1)^{k}x_{i,k}^{\mu}\Delta\mu_i g_{ij}\omega_j^0\\ &-(-1)^{k}\mu_i^0 b_{ij}x_{j,k}^{\mu}\Delta\mu_j-(-1)^{k}x_{i,k}^{\mu}\Delta\mu_i b_{ij}\mu_j^0\\ &-(-1)^{k}\omega_i^0 b_{ij}x_{j,k}^{\omega}\Delta\omega_j-(-1)^{k}x_{i,k}^{\omega}\Delta\omega_i b_{ij}\omega_j^0,\end{aligned}\tag{7}$$

and the quadratic clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{1}\sum_{l=0}^{1}&(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i g_{ij}x_{j,l}^{\mu}\Delta\mu_j\\ &-(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i g_{ij}x_{j,l}^{\omega}\Delta\omega_j\\ &-(-1)^{k+l}x_{i,k}^{\mu}\Delta\mu_i b_{ij}x_{j,l}^{\mu}\Delta\mu_j\\ &-(-1)^{k+l}x_{i,k}^{\omega}\Delta\omega_i b_{ij}x_{j,l}^{\omega}\Delta\omega_j,\end{aligned}\tag{8}$$

For the Ising model formulation of the net active power, the substitution of equations (11)–(12) into equation (5) yields

$$
\begin{aligned}
p_i = \sum_{j=0}^{n} & \mu_i^0 g_{ij} \mu_j^0 + \mu_i^0 g_{ij} s_{j,0}^{\mu} \Delta\mu_j + \mu_i^0 g_{ij} s_{j,1}^{\mu} \Delta\mu_j + \mu_i^0 g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,0}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 + s_{i,0}^{\mu} \Delta\mu_i g_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,0}^{\mu} \Delta\mu_i g_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,0}^{\mu} \Delta\mu_i g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,1}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 + s_{i,1}^{\mu} \Delta\mu_i g_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,1}^{\mu} \Delta\mu_i g_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,1}^{\mu} \Delta\mu_i g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,2}^{\mu} \Delta\mu_i g_{ij} \mu_j^0 + s_{i,2}^{\mu} \Delta\mu_i g_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,2}^{\mu} \Delta\mu_i g_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,2}^{\mu} \Delta\mu_i g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & \omega_i^0 g_{ij} \omega_j^0 + \omega_i^0 g_{ij} s_{j,0}^{\omega} \Delta\omega_j + \omega_i^0 g_{ij} s_{j,1}^{\omega} \Delta\omega_j + \omega_i^0 g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
+ \; & s_{i,0}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 + s_{i,0}^{\omega} \Delta\omega_i g_{ij} s_{j,0}^{\omega} \Delta\omega_j + s_{i,0}^{\omega} \Delta\omega_i g_{ij} s_{j,1}^{\omega} \Delta\omega_j + s_{i,0}^{\omega} \Delta\omega_i g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
+ \; & s_{i,1}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 + s_{i,1}^{\omega} \Delta\omega_i g_{ij} s_{j,0}^{\omega} \Delta\omega_j + s_{i,1}^{\omega} \Delta\omega_i g_{ij} s_{j,1}^{\omega} \Delta\omega_j + s_{i,1}^{\omega} \Delta\omega_i g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
+ \; & s_{i,2}^{\omega} \Delta\omega_i g_{ij} \omega_j^0 + s_{i,2}^{\omega} \Delta\omega_i g_{ij} s_{j,0}^{\omega} \Delta\omega_j + s_{i,2}^{\omega} \Delta\omega_i g_{ij} s_{j,1}^{\omega} \Delta\omega_j + s_{i,2}^{\omega} \Delta\omega_i g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
+ \; & \omega_i^0 b_{ij} \mu_j^0 + \omega_i^0 b_{ij} s_{j,0}^{\mu} \Delta\mu_j + \omega_i^0 b_{ij} s_{j,1}^{\mu} \Delta\mu_j + \omega_i^0 b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,0}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 + s_{i,0}^{\omega} \Delta\omega_i b_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,0}^{\omega} \Delta\omega_i b_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,0}^{\omega} \Delta\omega_i b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,1}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 + s_{i,1}^{\omega} \Delta\omega_i b_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,1}^{\omega} \Delta\omega_i b_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,1}^{\omega} \Delta\omega_i b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,2}^{\omega} \Delta\omega_i b_{ij} \mu_j^0 + s_{i,2}^{\omega} \Delta\omega_i b_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,2}^{\omega} \Delta\omega_i b_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,2}^{\omega} \Delta\omega_i b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
- \; & \mu_i^0 b_{ij} \omega_j^0 - \mu_i^0 b_{ij} s_{j,0}^{\omega} \Delta\omega_j - \mu_i^0 b_{ij} s_{j,1}^{\omega} \Delta\omega_j - \mu_i^0 b_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,0}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 - s_{i,0}^{\mu} \Delta\mu_i b_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,0}^{\mu} \Delta\mu_i b_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,0}^{\mu} \Delta\mu_i b_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,1}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 - s_{i,1}^{\mu} \Delta\mu_i b_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,1}^{\mu} \Delta\mu_i b_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,1}^{\mu} \Delta\mu_i b_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,2}^{\mu} \Delta\mu_i b_{ij} \omega_j^0 - s_{i,2}^{\mu} \Delta\mu_i b_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,2}^{\mu} \Delta\mu_i b_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,2}^{\mu} \Delta\mu_i b_{ij} s_{j,2}^{\omega} \Delta\omega_j .
\end{aligned}
\tag{9}
$$

A similar expression can also be derived for the Ising model formulation of the net reactive power injection by substituting (11)–(12) into equation (6).

$$
\begin{aligned}
q_i = \sum_{j=0}^{n} & \omega_i^0 g_{ij} \mu_j^0 + \omega_i^0 g_{ij} s_{j,0}^{\mu} \Delta\mu_j + \omega_i^0 g_{ij} s_{j,1}^{\mu} \Delta\mu_j + \omega_i^0 g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,0}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 + s_{i,0}^{\omega} \Delta\omega_i g_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,0}^{\omega} \Delta\omega_i g_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,0}^{\omega} \Delta\omega_i g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,1}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 + s_{i,1}^{\omega} \Delta\omega_i g_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,1}^{\omega} \Delta\omega_i g_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,1}^{\omega} \Delta\omega_i g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
+ \; & s_{i,2}^{\omega} \Delta\omega_i g_{ij} \mu_j^0 + s_{i,2}^{\omega} \Delta\omega_i g_{ij} s_{j,0}^{\mu} \Delta\mu_j + s_{i,2}^{\omega} \Delta\omega_i g_{ij} s_{j,1}^{\mu} \Delta\mu_j + s_{i,2}^{\omega} \Delta\omega_i g_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
- \; & \mu_i^0 g_{ij} \omega_j^0 - \mu_i^0 g_{ij} s_{j,0}^{\omega} \Delta\omega_j - \mu_i^0 g_{ij} s_{j,1}^{\omega} \Delta\omega_j - \mu_i^0 g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,0}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 - s_{i,0}^{\mu} \Delta\mu_i g_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,0}^{\mu} \Delta\mu_i g_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,0}^{\mu} \Delta\mu_i g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,1}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 - s_{i,1}^{\mu} \Delta\mu_i g_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,1}^{\mu} \Delta\mu_i g_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,1}^{\mu} \Delta\mu_i g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,2}^{\mu} \Delta\mu_i g_{ij} \omega_j^0 - s_{i,2}^{\mu} \Delta\mu_i g_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,2}^{\mu} \Delta\mu_i g_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,2}^{\mu} \Delta\mu_i g_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & \mu_i^0 b_{ij} \mu_j^0 - \mu_i^0 b_{ij} s_{j,0}^{\mu} \Delta\mu_j - \mu_i^0 b_{ij} s_{j,1}^{\mu} \Delta\mu_j - \mu_i^0 b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
- \; & s_{i,0}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 - s_{i,0}^{\mu} \Delta\mu_i b_{ij} s_{j,0}^{\mu} \Delta\mu_j - s_{i,0}^{\mu} \Delta\mu_i b_{ij} s_{j,1}^{\mu} \Delta\mu_j - s_{i,0}^{\mu} \Delta\mu_i b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
- \; & s_{i,1}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 - s_{i,1}^{\mu} \Delta\mu_i b_{ij} s_{j,0}^{\mu} \Delta\mu_j - s_{i,1}^{\mu} \Delta\mu_i b_{ij} s_{j,1}^{\mu} \Delta\mu_j - s_{i,1}^{\mu} \Delta\mu_i b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
- \; & s_{i,2}^{\mu} \Delta\mu_i b_{ij} \mu_j^0 - s_{i,2}^{\mu} \Delta\mu_i b_{ij} s_{j,0}^{\mu} \Delta\mu_j - s_{i,2}^{\mu} \Delta\mu_i b_{ij} s_{j,1}^{\mu} \Delta\mu_j - s_{i,2}^{\mu} \Delta\mu_i b_{ij} s_{j,2}^{\mu} \Delta\mu_j \\
- \; & \omega_i^0 b_{ij} \omega_j^0 - \omega_i^0 b_{ij} s_{j,0}^{\omega} \Delta\omega_j - \omega_i^0 b_{ij} s_{j,1}^{\omega} \Delta\omega_j - \omega_i^0 b_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,0}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 - s_{i,0}^{\omega} \Delta\omega_i b_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,0}^{\omega} \Delta\omega_i b_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,0}^{\omega} \Delta\omega_i b_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,1}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 - s_{i,1}^{\omega} \Delta\omega_i b_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,1}^{\omega} \Delta\omega_i b_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,1}^{\omega} \Delta\omega_i b_{ij} s_{j,2}^{\omega} \Delta\omega_j \\
- \; & s_{i,2}^{\omega} \Delta\omega_i b_{ij} \omega_j^0 - s_{i,2}^{\omega} \Delta\omega_i b_{ij} s_{j,0}^{\omega} \Delta\omega_j - s_{i,2}^{\omega} \Delta\omega_i b_{ij} s_{j,1}^{\omega} \Delta\omega_j - s_{i,2}^{\omega} \Delta\omega_i b_{ij} s_{j,2}^{\omega} \Delta\omega_j .
\end{aligned}
\tag{10}
$$

Supplementary equations (9)–(10) are extended forms of equations (13)–(14) presented in the paper, respectively.

In equation (13), the constant clause is

$$
\sum_{j=0}^{n} \mu_i^0 g_{ij} \mu_j^0 + \omega_i^0 g_{ij} \omega_j^0 + \omega_i^0 b_{ij} \mu_j^0 - \mu_i^0 b_{ij} \omega_j^0 , \tag{11}
$$

the linear clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{2}&(k+1)\mu_i^0 g_{ij}s_{j,k}^{\mu}\Delta\mu_j+(k+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}\mu_j^0\\ &+(k+1)\omega_i^0 g_{ij}s_{j,k}^{\omega}\Delta\omega_j+(k+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}\omega_j^0\\ &+(k+1)\omega_i^0 b_{ij}s_{j,k}^{\mu}\Delta\mu_j+(k+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}\mu_j^0\\ &-(k+1)\mu_i^0 b_{ij}s_{j,k}^{\omega}\Delta\omega_j-(k+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}\omega_j^0,\end{aligned}\tag{12}$$

and the quadratic clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{2}\sum_{l=0}^{2}&(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}s_{j,l}^{\mu}\Delta\mu_j\\ &+(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}s_{j,l}^{\omega}\Delta\omega_j\\ &+(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}s_{j,l}^{\mu}\Delta\mu_j\\ &-(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}s_{j,l}^{\omega}\Delta\omega_j.\end{aligned}\tag{13}$$

Similarly, in equation (14), the constant clause is

$$\sum_{j=0}^{n}\omega_i^0 g_{ij}\mu_j^0-\mu_i^0 g_{ij}\omega_j^0-\mu_i^0 b_{ij}\mu_j^0-\omega_i^0 b_{ij}\omega_j^0,\tag{14}$$

the linear clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{2}&(k+1)\omega_i^0 g_{ij}s_{j,k}^{\mu}\Delta\mu_j+(k+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}\mu_j^0\\ &-(k+1)\mu_i^0 g_{ij}s_{j,k}^{\omega}\Delta\omega_j-(k+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}\omega_j^0\\ &-(k+1)\mu_i^0 b_{ij}s_{j,k}^{\mu}\Delta\mu_j-(k+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}\mu_j^0\\ &-(k+1)\omega_i^0 b_{ij}s_{j,k}^{\omega}\Delta\omega_j-(k+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}\omega_j^0,\end{aligned}\tag{15}$$

and the quadratic clause is

$$\begin{aligned}\sum_{j=0}^{n}\sum_{k=0}^{2}\sum_{l=0}^{2}&(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i g_{ij}s_{j,l}^{\mu}\Delta\mu_j\\ &-(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i g_{ij}s_{j,l}^{\omega}\Delta\omega_j\\ &-(k+1)(l+1)s_{i,k}^{\mu}\Delta\mu_i b_{ij}s_{j,l}^{\mu}\Delta\mu_j\\ &-(k+1)(l+1)s_{i,k}^{\omega}\Delta\omega_i b_{ij}s_{j,l}^{\omega}\Delta\omega_j.\end{aligned}\tag{16}$$

Supplementary algorithms

The AQPF algorithm is a modification of Algorithm (1) for the combinatorial power flow algorithm, that is

Algorithm 1 Adiabatic power flow algorithm.

Initialize $\vec{p}^d = [p_1^d, p_2^d, \ldots, p_n^d]$
Initialize $\vec{q}^d = [q_1^d, q_2^d, \ldots, q_n^d]$
Initialize $Y \in \{(g_{ij} + \mathrm{j}b_{ij}) : i, j = 0, 1, \ldots, n\}$
$\Delta\mu \leftarrow 1 \times 10^{-2}$
$\Delta\omega \leftarrow 1 \times 10^{-3}$
$\vec{\mu} = [\mu_0, \mu_1, \ldots, \mu_n] \leftarrow 1$
$\vec{\omega} = [\omega_0, \omega_1, \ldots, \omega_n] \leftarrow 0$
Calculate $\vec{p} = [p_0, p_1, \ldots, p_n]$ from equation (5)
Calculate $\vec{q} = [q_0, q_1, \ldots, q_n]$ from equation (6)
Calculate Hamiltonian H from equation (15)
$\epsilon \leftarrow 1 \times 10^{-4}$
it $\leftarrow 0$
while $H > \epsilon$ and it $<$ $\mathrm{it}_{\max}$ **do**
  Minimize the QUBO/Ising model
  Update $\vec{\mu}$ from equations (7) or (11)
  Update $\vec{\omega}$ from equations (8) or (12)
  Recalculate $\vec{p}$ from equation (9) or (13)
  Recalculate $\vec{q}$ from equation (10) or (14)
  Recalculate H from equation (15)
  Update $\Delta\mu, \Delta\omega$
  it $\leftarrow$ it + 1
end while=0