The novel numerical solutions for Caputo-Fabruzo fractional Newell–Whitehead–SegelEquation by using Aboodh-ADM

Mehmet Merdan
mmerdan@gumushane.edu.tr

Yasin Şahin
Pınar Açıkgöz

Research Article

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Posted Date: April 24th, 2024

DOI: https://doi.org/10.21203/rs.3.rs-4287125/v1

Additional Declarations: No competing interests reported.

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[1]Mehmet Merdan [1]Yasin Şahin [1]Pınar Açıkgöz

[1]Gumushane University, Mathematical Engineering Department, Gumushane, Turkey

mmerdan@gumushane.edu.tr, y.sahin80@gmail.com, poral@gmail.com

Abstract

This article aims to solve Caputo-Fabruzo fractional differential equations using the Aboodh transform together with the Adomian Decomposition method (A-ADM). Since the Aboodh transform can only be applied to linear equations, ADM is an effective technique for approximating solutions of nonlinear differential equations. In nonlinear systems, the Newell-Whitehead-Segel equation plays an important role, explaining the emergence of stripes in 2-dimensional systems. The findings show that the results obtained from the tables provide superior results compared to the existing conformable q-Shehu homotopy analysis transform method (Cq-SHATM) in the literature. With the help of Matlab package program, numerical values were found to depict three-dimensional surfaces and displayed in a table.

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1. Introduction

Leibnitz developed arbitrary order derivatives shortly after integer derivatives, and this topic has recently piqued the interest of many experts. Many specialists have recently expressed interest in his unique explanation for complicated nonlinear systems. The advantage of using fractional differential equation models is that they are non-local and may be used to any model. Integer order derivatives preserve locality, whereas fractional order derivatives maintain nonlocality. This demonstrates that the future state of a physical system is determined not just by its current state, but also by its previous states. Models developed with fractional order derivatives are thus more accurate representations of reality. Fractional calculus is a science that has applications in many scientific and technical domains [1].

However, fractional order fluctuates over time and space. The situation results in a rapidly increasing field of FPDEs including variable order fractional operators. Several powerful computational approaches have been established in the scientific literature, with many notable scientists contributing to the discussion. These techniques include adomian decomposition method (ADM)[2-4], homotopy perturbation method (HPM)[5-6], homotopy analysis method (HAM)[7-10], Aboodh transform method[11-15], sorting method[16], Sumudu transform method (STM)[17-20], conformable Shehu homotopy perturbation method (CSHPM)[21-22], conformal q-Shehu homotopy analysis transform method (Cq-SHATM)[23-24], conformable sumudu decomposition method[25-26], conformable Laplace decomposition method[27-28], and differential transform method (DTM) [29-33].

Ripples in sand, lines in seashells, and many other similar striped patterns can be modeled with amplitude equations.

Corresponding author: Mehmet MERDAN

NWSE is one of the most important amplitude equations in applied sciences. It shows how stripes appear in two-dimensional systems [27,34-36]. NWSE is: [27,34, 37]

$$ {}_{0}^{CF}D_t^{\alpha} u(x,t) = a \frac{\partial^2 u(x,t)}{\partial x^2} + bu(x,t) - cu(x,t)^m, 0 < \alpha < 1, t \geq 0, \mathrm{x} \in \mathrm{R} \quad (1) $$

where m is a positive integer, a, b and c real numbers.

Similar to the Laplace transform, the Aboodh transform is used to solve linear equations. Nevertheless, the Adomian decomposition approach needs to be combined with a numerical technique, like the separation method, in order to solve nonlinear equations. Consequently, the Adomian decomposition method and the Aboodh transform are coupled in this study. In order to provide fresh numerical solutions for a NWS equation, this research will introduce a novel approach called A-ADM[38-39]. The second goal is to use the recently developed hybrid method to solve the time fraction Newell-Whitehead-Segel equation for the first time. Numerous solutions previously unreported in the literature have been identified, and their entire graphical features have been rendered. The primary goal of this paper is to use a new hybrid technique called the Aboodh Adomian decomposition method (A-ADM) to obtain new numerical solutions of the time fractional Newell-Whitehead-Segel equation. Furthermore, it was observed that the table's outcomes outperformed, ATHPM[40], q-HSATM[24] and LTDM [27] in the literature.

This article is organized as follows. Section 2 is on basic definitions and notations. Section 3 is about the Aboodh Adomian transform method for the components and Caputo-Fabruzo fractional order Newell-Whitehead-Segel equation. Section 4 provides insight into numerical applications of the method to linear and nonlinear Newell-Whitehead-Segel equations. Section 5 is on conclusion.

2. Preliminaries

Several fundamental definitions are provided in this section.
Definition 1.1. Caputo-Fabruzo fractional derivative[41] of order $f(t)$

$$ {}_{a}^{CF}D_t^{\alpha} f(t) = \frac{M(\alpha)}{1-\alpha} \int_a^t f'(\tau) e^{\left[\frac{-\alpha(t-\tau)}{1-\alpha}\right]} d\tau \ , \quad 0 < \alpha \leq 1 \quad (2) $$

$f' \in H'(a,b), b > 0$ and $M(\alpha)$ is the normalization constant depending on α where $M(0) = M(1) = 1$.

Definition 1.2. Caputo fractional derivative of $f(t)$, where $\alpha > 0$ is the order of the Caputo fractional derivative[40-43]:

$$ {}^{c}D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(p-\alpha)} \displaystyle\int_0^t (t-s)^{p-\alpha-1} f^{(p)}(s) ds, p-1 < \alpha \leq p \\ \dfrac{\partial^n}{\partial t^n} f(t) \ , \qquad \alpha = n \epsilon \mathbb{N} \end{cases} \quad (3) $$

It is defined as.

Definition1. 3. A new transformation for exponential functions, called the Aboodh transformation[11-15], takes into account the functions in the set $\mathcal{A}$ defined as follows.

$$\mathcal{A} = \{f(t) \colon \exists\, M, k_1\,, k_2 > 0\,, |\, f(t)| < Me^{-st}\} \tag{4}$$

Aboodh transform of function $f(t)$

$$\mathcal{A}\{f(t)\} = \mathcal{A}\{y(s)\} \; = \frac{1}{s}\int_0^\infty f(t)\, e^{-st}\, dt \;, \qquad t \; \geq 0\,, \qquad k_1 \; \leq s \; \leq \; k_2 \tag{5}$$

It is defined as.

Definition 1.4. Laplace Transform of Caputo-Fabruzo fractional derivative[44-46]:

$$\boldsymbol{L}\{{}_{0}^{CF}D_t^{\alpha+n} f(t)\}(s) = \frac{s^{n+1}\, L\{f(t)\} - f(0)s^n - \; f'(0)s^{n-1} - \cdots - \; f^{(n)}(0)}{\alpha + s(1-\alpha)} \;,$$

$$0 < \alpha \leq 1 \tag{6}$$

for $n = 0$

$$\boldsymbol{L}\{{}_{0}^{CF}D_t^{\alpha} f(t)\}(s) = \frac{s\, L\{f(t)\} - f(0)}{\alpha + s(1-\alpha)} \tag{7}$$

Theorem 1.1. Aboodh transform of the Caputo-Fabruzo fractional derivative[47-48] of order α, where the Aboodh transform of the function $f(t)$ is $A\{y(s)\}$;

$$\boldsymbol{\mathcal{A}}\{{}_{0}^{CF}D_t^{\alpha} f(t)\}(s) = \frac{s\, \mathcal{A}\{y(s)\} - \frac{y(0)}{s}}{\alpha + s(1-\alpha)} \tag{8}$$

3. Methods and Metarials

To present the Aboodh decomposition method, we consider the more general nonhomogeneous nonlinear fractional partial differential equation

$${}_{0}^{CF}D_t^{\alpha} u(x,t) + Ru(x,t) + Nu(x,t) = h(x,t) \tag{9}$$

with the initial condition

$$u(x,0) = f(x) \tag{10}$$

where

$R$is the remaining linear operator less than N is the nonlinear operator and $h(x,t)$ is the nonhomogeneous term.
Let's take the Aboodh transformation of both sides of equation (9)

$$\mathcal{A}\{{}^{CF}_{0}D^{\alpha}_{t}u(x,t)\} = \mathcal{A}\{-Ru(x,t) - Nu(x,t) + h(x,t)\}$$

$$\frac{s\mathcal{A}\{u(x,t)\} - \frac{u(x,0)}{s}}{\alpha + s(1-\alpha)} = \mathcal{A}\{-Ru(x,t) - Nu(x,t) + h(x,t)\}$$

$$\mathcal{A}\{u(x,t)\} = \frac{u(x,0)}{s^2} + \left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{-Ru(x,t) - Nu(x,t) + h(x,t)\} \quad (11)$$

Now, replacing the unknown function $u(x,t)$ by an infinite series of u_m's i.e.

$$u(x,t) = \sum_{m=0}^{\infty} u_m(x,t) \quad (12)$$

and the nonlinear term by an infinite series of the Adomian polynomial B_m's given by

$$Nu(x,t) = \sum_{m=0}^{\infty} B_m(u_0, u_1, u_2, \dots), m = 0,1,2, \dots \quad (13)$$

Where

$$B_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[F\left(\sum_{m=0}^{\infty} \lambda^i u_i\right)\right]_{\lambda=0}, m = 0,1,2, \dots \quad (14)$$

Substituting equations (11) and (12) into equation (10) we obtain

Let's take the inverse Aboodh transform of each side in equation (11).

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) + \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left\{\mathcal{A}\sum_{m=0}^{\infty} Ru_m(x,t) - \mathcal{A}\left\{\sum_{n=0}^{\infty} B_m(x,t)\right\} + \mathcal{A}h(x,t)\right\}\right\} \quad (15)$$

$$\begin{cases} n = 0;\ u_0(x,t) = u(x,0) = f(x) \\ n \geq 0;\ u_{n+1}(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left\{\mathcal{A}\sum_{m=0}^{\infty} Ru_m(x,t) - \mathcal{A}\left\{\sum_{n=0}^{\infty} B_m(x,t)\right\} + \mathcal{A}h(x,t)\right\}\right\} \end{cases} \quad (16)$$

(16) in the equation $n = 0,1,2, \dots$ using their values;

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0 + u_1 + u_2 + \cdots \tag{17}$$

4. Convergence analysis

Consider the Banach space $C[0,T]$ of all continuous functions on $[0,T]$ with supremum norm. Throughout this section, we consider $u(x,t), u_n(x,t) \in C[0,T]$.

Theorem 1.2 (Uniqueness Theorem [49-50]) The solution for the nonlinear fractional differential Eq. (11) obtained by A-ADM is unique for $0 < \gamma < 1$

Proof The solution of nonlinear FPDEs Eq. (11) is presented as

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{18}$$

Here,

$$u_{n+1}(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left\{\mathcal{A}\sum_{m=0}^{\infty} Ru_m(x,t) - \mathcal{A}\left\{\sum_{n=0}^{\infty} B_m(x,t)\right\} + \mathcal{A}h(x,t)\right\}\right\}$$

Assume that *u* and *w* are two different solutions of Eq. (11), then with the help of the aforementioned equation, we obtain

$$|u - v| = \left|-\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\{\mathcal{A}[R(u-v) + N(u-v]\}\right\}\right| \tag{19}$$

Now, using the convolution theorem for the Aboodh transform, it is obtained as

$$|u - v| \le \int_0^t (|R(u-v)| + |\mathrm{N}(u) - \mathrm{N}(v)|) \frac{(t-\tau)^{\beta}}{\Gamma(1+\beta)}\, d\tau \le \int_0^t (\xi|u-v| + \zeta|u-v|) \frac{(t-\tau)^{\beta}}{\Gamma(1+\beta)}\, d\tau \tag{20}$$

R is a bounded operator i.e. $|R(u) - R(v)| \le \xi|u-v|$, N satisfies Lipschitz condition with $\delta > 0$ such that $|\mathrm{N}(u) - \mathrm{N}(v)| \le \zeta|u-v|$

$$|u - v| \leq \int_0^t (\xi + \zeta)|u - v| \frac{(t-\tau)^\beta}{\Gamma(1+\beta)}\, d\tau \tag{21}$$

Then, using the integral mean-value theorem, it yields

$|u - v| \leq [(\xi + \zeta)|u - v|]MT$, where $M = max(t-\tau)^\beta$ and $t \in [0,T]$.
Hence, $|u - v| \leq \gamma|u - v|$, where $\gamma = (\xi + \zeta)MT$. So $(1-\gamma)|u - v| \leq 0$, implies $u = v$ whenever, $0 < \gamma < 1$. Hence, the solution is unique.

Theorem 1.3 (Convergence theorem [50-51]) Assume that X is a Banach space and $H : X \to X$ is a nonlinear mapping. If the inequality

$$\|H(u) - H(v)\| \leq \varphi\| u - v\| \qquad , \forall u, v \in X \tag{22}$$

exists, then H has a fixed point in view of Banach fixed point theory [51]. Furthermore, for the arbitrary choice $of\ u_0, u_1 \in X$, the sequence created by the A-ADM converges to a fixed point of H and

$$\| u_m - u_n\| \leq \frac{\gamma^n}{1-\gamma} \| u_1 - u_0\| \quad , \forall u, v \in X. \tag{23}$$

Proof: Let us take a Banach space $(C[J], \|.\|)$ of all continuous functions on J with the norm expressed as $\|h(\psi)\| = \max_{\psi \in J} |h(\psi)|$.
Now, we demonstrate that the sequence $\{u_n\}$ is a Cauchy sequence in the Banach space:

$$\| u_m - u_n\| = \max_{\psi \in J} |u_m - u_n|$$

$$= \max_{\psi \in J} \left| \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\{\mathcal{A}[R(u_{m-1} - u_{n-1}) + N(u_{m-1} - u_{n-1}]\}\right\}\right|$$

$$\leq \max_{\psi \in J} \left[\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\{\mathcal{A}[R|u_{m-1} - u_{n-1}| + N|u_{m-1} - u_{n-1}|]\}\right\}\right]$$

Now, making use of the convolution theorem for Laplace transform, it gives

$$\| u_m - u_n\| \leq \max_{\psi \in J} \left[\int_0^\psi [|R(u_{m-1} - u_{n-1})| + |N(u_{m-1} - u_{n-1})|] \frac{(\psi - \xi)^\beta}{\Gamma(1+\beta)}\, d\xi\right]$$

$$\leq \max_{\psi \in J} \left[\int_0^\psi [\xi|u_{m-1} - u_{n-1}| + \zeta|u_{m-1} - u_{n-1}|] \frac{(\psi - \phi)^\beta}{\Gamma(1+\beta)}\, d\phi\right]$$

.
Next, by the application of the integral mean value theorem [51], we obtain

$$\| u_m - u_n\| \leq \max_{\psi \in J} [(\xi|u_{m-1} - u_{n-1}| + \zeta|u_{m-1} - u_{n-1}|)T] \leq \gamma\|u_{m-1} - u_{n-1}\|$$

Let $m = n + 1$, then we have

$$\| u_{n+1} - u_n\| \leq \gamma\|u_n - u_{n-1}\| \leq \gamma^2\|u_{n-1} - u_{n-2}\| \leq \cdots \leq \gamma^n\|u_1 - u_0\|$$

On using the triangular inequality, it yields

$$\begin{aligned}\| u_m - u_n\| &\le \| u_{n+1} - u_n\| + \| u_{n+2} - u_{n+1}\| + \cdots + \| u_m - u_{m-1}\| \\ &\le [\gamma^n + \gamma^{n+1} + \cdots + \gamma^{m-1}]\| u_1 - u_0\| \\ &\le \gamma^n[1 + \gamma + \gamma^2 + \cdots + \gamma^{m-n-1}]\| u_1 - u_0\| \le \gamma^n \left[\frac{1-\gamma^{m-n-1}}{1-\gamma}\right]\| u_1 - u_0\|.\end{aligned}$$

Because $\gamma \in (0,1)$, so $1 - \gamma^{m-n-1} < 1$, then we have

$$\| u_m - u_n\| \le \frac{\gamma^n}{1-\gamma}\| u_1 - u_0\|$$

But $\| u_1 - u_0\| < \infty$, so as $m \to \infty$ then $\| u_m - u_n\| \to 0$. Therefore, the sequence $\{u_n\}$ is Cauchy sequence in $C[J]$, and so the sequence is convergent.

5. Application

Example-1.

$${}_0^{CF}D_t^{\alpha} u(x,t) = u_{xx} - 2u \,, 0 < \alpha \le 1 \qquad (22)$$

Caputo-Fabruzo fractional partial differential equation $u(x,0) = e^x$ Solve with the help of Aboodh transformation method for the initial condition.

Solution-1. (22) If the Aboodh transformation of both sides of the equation is taken;

$$\mathcal{A}\{{}_0^{CF}D_t^{\alpha} u(x,t)\} = \mathcal{A}\{u_{xx} - 2u\}$$

$$\frac{s\mathcal{A}\{u(x,t)\} - \dfrac{u(x,0)}{s}}{\alpha + s(1-\alpha)} = \mathcal{A}\{u_{xx} - 2u\}$$

$$\mathcal{A}\{u(x,t)\} = \frac{u(x,0)}{s^2} + \left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{u_{xx} - 2u\} \qquad (23)$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \quad , u_{xx} = \sum_{n=0}^{\infty} u_{n_{xx}} \quad , u = \sum_{n=0}^{\infty} u_n$$

(23) substituted in the equation,

$$\mathcal{A}\left\{\sum_{n=0}^{\infty} u_n(x,t)\right\} = \frac{u(x,0)}{s^2} + \left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{\sum_{n=0}^{\infty} u_{n_{xx}} - 2\sum_{n=0}^{\infty} u_n\right\} \qquad (24)$$

(24) If the inverse Aboodh transformation of the equation is taken

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) + \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{\sum_{n=0}^{\infty} u_{n_{xx}} - 2\sum_{n=0}^{\infty} u_n\right\}\right\}$$

$$\begin{cases} n = 0; \ u_0(x,t) = u(x,0) = e^x \\ n \geq 0; \ u_{n+1}(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{\sum_{n=0}^{\infty} u_{n_{xx}} - 2\sum_{n=0}^{\infty} u_n\right\}\right\} \end{cases} \quad (25)$$

(25) in the equation $n = 0,1,2, \ldots$ using the values;

$$u_1(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{u_{0_{xx}} - 2u_0\right\}\right\}, \ \ u_{0_{xx}} = e^x$$

$$u_1(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{e^x - 2e^x\}\right\} = -e^x\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{1\}\right\}$$

$$= -e^x\mathcal{A}^{-1}\left\{\frac{\alpha}{s^3} + \frac{1-\alpha}{s^2}\right\} = -e^x(\alpha t + 1 - \alpha\ )$$

$$u_1(x,t) = -\alpha e^x t - (1-\alpha)e^x$$

$$u_2(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{u_{1_{xx}} - 2u_1\right\}\right\}, \ \ u_{1_{xx}} = -\alpha e^x t - (1-\alpha)e^x$$

$$u_2(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{-\alpha e^x t - (1-\alpha)e^x - 2(-\alpha e^x t - (1-\alpha)e^x)\}\right\}$$

$$= e^x\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{\alpha t + (1-\alpha)\}\right\}$$

$$= e^x\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left(\frac{\alpha}{s^3} + \frac{1-\alpha}{s^2}\right)\right\}$$

$$= e^x\mathcal{A}^{-1}\left\{\frac{\alpha^2}{s^4} + \frac{2\alpha(1-\alpha)}{s^3} + \frac{(1-\alpha)^2}{s^2}\right\}$$

$$= e^x\left\{\frac{\alpha^2 t^2}{2!} + 2\alpha(1-\alpha)t + (1-\alpha)^2\right\}$$

$$u_2(x,t) = \alpha^2 e^x\ \frac{t^2}{2!}\ + 2\alpha(1-\alpha)e^x t + (1-\alpha)^2 e^x$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{u_{2_{xx}} - 2u_2\right\}\right\},$$

$$u_{2_{xx}} = \alpha^2 e^x\ \frac{t^2}{2!}\ + 2\alpha(1-\alpha)e^x t + (1-\alpha)^2 e^x$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s}+1-\alpha\right)\mathcal{A}\left\{\alpha^2 e^x\ \frac{t^2}{2!}\ +2\alpha(1-\alpha)e^x t+(1-\alpha)^2 e^x-2(\alpha^2 e^x\ \frac{t^2}{2!}\right.\right.$$

$$\left.\left.+2\alpha(1-\alpha)e^x t+(1-\alpha)^2 e^x)\right\}\right\}$$

$$= -e^x\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s}+1-\alpha\right)\mathcal{A}\left\{\alpha^2\ \frac{t^2}{2!}\ +2\alpha(1-\alpha)t+(1-\alpha)^2\right\}\right\}$$

$$= -e^x\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s}+1-\alpha\right)\left(\frac{\alpha^2}{s^4}+\frac{2\alpha(1-\alpha)}{s^3}+\frac{(1-\alpha)^2}{s^2}\right)\right\}$$

$$= -e^x\mathcal{A}^{-1}\left\{\left(\frac{\alpha^3}{s^5}+\frac{3\alpha^2(1-\alpha)}{s^4}+\frac{3\alpha(1-\alpha)^2}{s^3}+\frac{(1-\alpha)^2}{s^2}\right)\right\}$$

$$= \ -e^x\ \left\{\frac{\alpha^3 t^3}{3!}+3\alpha^2(1-\alpha)\frac{t^2}{2!}+3\alpha(1-\alpha)^2 t+(1-\alpha)^3\right\}$$

$$u_3(x,t) = -\alpha^3 e^x\ \frac{t^3}{3!}-3\alpha^2(1-\alpha)e^x\frac{t^2}{2!}-3\alpha(1-\alpha)^2 e^x t-(1-\alpha)^3 e^x$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0+u_1+u_2+\cdots$$

$$u(x,t) = e^x-\alpha e^x t-(1-\alpha)e^x+\alpha^2 e^x\ \frac{t^2}{2!}\ +2\alpha(1-\alpha)e^x t+(1-\alpha)^2 e^x-\alpha^3 e^x\ \frac{t^3}{3!}$$

$$-3\alpha^2(1-\alpha)e^x\frac{t^2}{2!}-3\alpha(1-\alpha)^2 e^x t-(1-\alpha)^3 e^x+\cdots$$

is found as. And if the necessary arrangements are made;

$$u(x,t) = (-\alpha^3+4\alpha^2-4\alpha+2)e^x+(-3\alpha^3+4\alpha^2-2\alpha)e^x t\ +\frac{1}{2}(3\alpha^3-2\alpha^2)e^x t^2$$

$$-\frac{1}{6}\alpha^3 e^x t^3+\cdots\ (5)$$

$$\alpha = 1\ i\c{c}in;\ u(x,t) = e^x\left(1-t\ +\frac{t^2}{2!}-\frac{t^3}{3!}+\cdots\right) = e^{x-t} \qquad (26)$$

is found as.

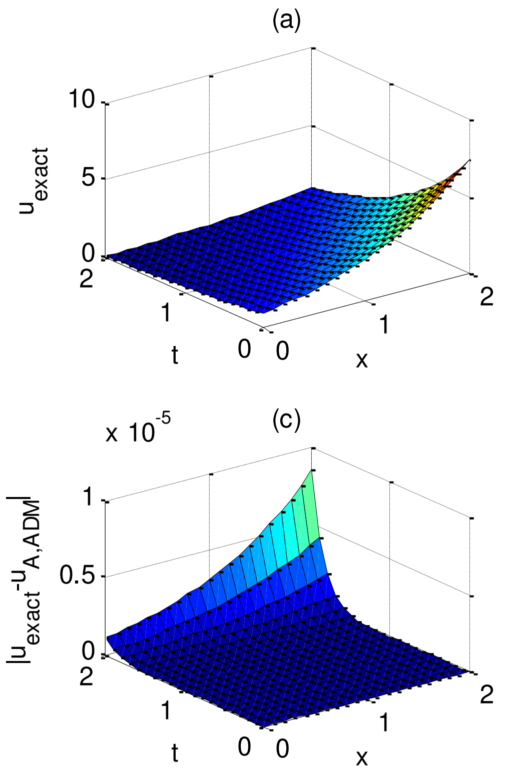

Figure 1. (a) Exact solution of $u(x,t)$ (b) $u(x,t)$ solution of A-ADM (c) Nature of absolute error = $|u_{exact} - u_{A-ADM}|$ at $\alpha = 1$

Table 1. Absolute errors of $u(x,t)$ at $\alpha = 1$ and $n = 8$ for Example 1.

x	t	Exact Solution	Approximation	Absolute Error
-3	0.15	0.04285212687	0.04285212687	0
	0.30	0.03688316740	0.03688316740	0
	0.45	0.03174563638	0.03174563648	0.1000000000 10^{-9}
	0.60	0.02732372245	0.02732372375	0.1300000000 10^{-8}
	0.75	0.02351774586	0.02351775544	0.9580000000 10^{-8}
	0.90	0.02024191145	0.02024196018	0.4873000000 10^{-7}
3	0.15	17.28778184	17.28778184	0.
	0.30	14.87973172	14.87973172	0.
	0.45	12.80710378	12.80710382	0.4000000000 10^{-7}
	0.60	11.02317638	11.02317691	0.4000000000 10^{-6}
	0.75	9.487735836	9.487739700	0.3864000000 10^{-5}
	0.90	8.166169913	8.166189572	0.00001965900000

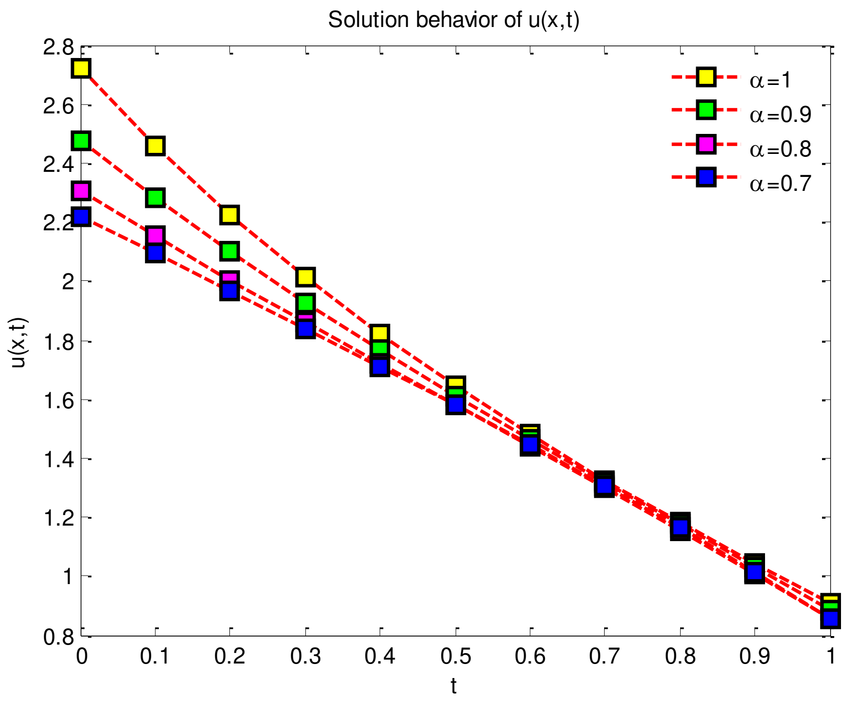

Figure 2. The solution behavior of the approximate solutions $u(x,t)$ for $\alpha \in \{0.7,0.8,0.9,1\}$

Example-2.

$$ {}^{CF}_{0}D_t^{\alpha} u(x,t) = u_{xx} + 2u - 3u^2, 0 < \alpha \leq 1 \tag{27}$$

Caputo-Fabruzo fractional partial differential equation $u(x,0) = \lambda$ Solve for the initial condition with the help of Aboodh-Adomian decomposition method.

Çözüm. (27) Let us take the Aboodh transformation of both sides of the equation.

$$\frac{s\mathcal{A}\{u(x,t)\} - \dfrac{u(x,0)}{s}}{\alpha + s(1-\alpha)} = \mathcal{A}\{u_{xx} + 2u - 3u^2\}$$

$$\mathcal{A}\{u(x,t)\} = \frac{u(x,0)}{s^2} + \left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{u_{xx} + 2u - 3u^2\} \tag{28}$$

$$u_{xx} = \sum_{n=0}^{\infty} u_{n_{xx}} \quad , u = \sum_{n=0}^{\infty} u_n \ , \sum_{n=0}^{\infty} B_n = \sum_{r=0}^{n} u_r u_{n-r}$$

B_n 's Adomian polynomials

$$B_0 = u_0 u_0 = \lambda^2 , B_1 = u_0 u_1 + u_1 u_0 = 2u_0 u_1 , \; B_2 = u_0 u_2 + u_1 u_1 + u_2 u_0 = 2u_0 u_2 + u_1^2$$

(28) If the inverse Aboodh transformation of the equation is taken;

$$\sum_{n=0}^{\infty} u_n(x,t) = u(x,0) + \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{\sum_{n=0}^{\infty} u_{n_{xx}} + 2\sum_{n=0}^{\infty} u_n - 3\sum_{n=0}^{\infty} B_n\right\}\right\}$$

$$\begin{cases} n = 0;\; u_0(x,t) = u(x,0) = \lambda \\ n \geq 0;\; u_{n+1}(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{\sum_{n=0}^{\infty} u_{n_{xx}} + 2\sum_{n=0}^{\infty} u_n - 3\sum_{n=0}^{\infty} B_n\right\}\right\} \end{cases} \quad (29)$$

(29) in the equation $n = 0,1,2, \ldots$ using their values;

$$u_1(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{u_{0_{xx}} + 2u_0 - 3B_0\}\right\} , u_0 = \lambda, u_{0_{xx}} = 0 , B_0 = \lambda^2$$

$$u_1(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{0 + 2\lambda - 3\lambda^2\}\right\} = (2\lambda - 3\lambda^2)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{1\}\right\}$$

$$= (2\lambda - 3\lambda^2)\mathcal{A}^{-1}\left\{\frac{\alpha}{s^3} + \frac{1-\alpha}{s^2}\right\}$$

$$u_1(x,t) = (2\lambda - 3\lambda^2)(t + 1 - \alpha)$$

$$u_2(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{u_{1_{xx}} + 2u_1 - 3B_1\}\right\}, \;\; u_{1_{xx}} = 0 , B_1 = 2\lambda u_1$$

$$u_2(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{0 + (2 - 6\lambda)u_1\}\right\}$$

$$u_2(x,t) = (2 - 6\lambda)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{u_1\}\right\}$$

$$= (2 - 6\lambda)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{(2\lambda - 3\lambda^2)(t + 1 - \alpha)\}\right\}$$

$$= (2\lambda - 3\lambda^2)(2 - 6\lambda)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}(t + 1 - \alpha)\right\}$$

$$= (2\lambda - 3\lambda^2)(2 - 6\lambda)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left(\frac{1}{s^3} + \frac{1-\alpha}{s^2}\right)\right\}$$

$$= (2\lambda - 3\lambda^2)(2 - 6\lambda)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s^4} + \frac{1-\alpha^2}{s^3} + \frac{(1-\alpha)^2}{s^2}\right)\right\}$$

$$u_2(x,t) = (2\lambda - 3\lambda^2)(2-6\lambda)\left(\alpha\frac{t^2}{2!} + (1-\alpha^2)t + (1-\alpha)^2\right)$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{u_{2_{xx}} + 2u_2 - 3B_2\}\right\}, u_{2_{xx}} = 0, B_2 = 2\lambda u_2 + u_1^2$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{0 + 2u_2 - 6\lambda u_2 - 3u_1^2\}\right\}$$

$$= \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\{(2-6\lambda)u_2 - 3u_1^2\}\right\}$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{(2 - 6\lambda)\left((2\lambda - 3\lambda^2)(2-6\lambda)\left(\alpha\frac{t^2}{2!} + (1-\alpha^2)t + (1-\alpha)^2\right)\right) - 3((2\lambda - 3\lambda^2)^2(t + 1 - \alpha)^2)\right\}\right\}$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\mathcal{A}\left\{(2-6\lambda)^2(2\lambda - 3\lambda^2)\left(\alpha\frac{t^2}{2!} + (1-\alpha^2)t + (1-\alpha)^2\right) - 3((2\lambda - 3\lambda^2)^2(t^2 + 2(1-\alpha)t + (1-\alpha)^2)\right\}\right\}$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left[(2-6\lambda)^2(2\lambda - 3\lambda^2)\mathcal{A}\left\{\alpha\frac{t^2}{2!} + (1-\alpha^2)t + (1-\alpha)^2\right\} - \left(3(2\lambda - 3\lambda^2)^2\mathcal{A}(\frac{2t^2}{2} + 2(1-\alpha)t + (1-\alpha)^2\right)\right]\right\}$$

$$u_3(x,t) = \mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left[(2-6\lambda)^2(2\lambda - 3\lambda^2)\left\{\frac{\alpha}{s^4} + \frac{(1-\alpha^2)}{s^3} + \frac{(1-\alpha)^2}{s^2}\right\} - 6(2\lambda - 3\lambda^2)^2\left\{\frac{1}{s^4} + \frac{2(1-\alpha)}{s^3} + \frac{(1-\alpha)^2}{s^2}\right\}\right]\right\}$$

$$u_3(x,t) = (2-6\lambda)^2(2\lambda - 3\lambda^2)\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left\{\frac{\alpha}{s^4} + \frac{(1-\alpha^2)}{s^3} + \frac{(1-\alpha)^2}{s^2}\right\} - 6(2\lambda - 3\lambda^2)^2\mathcal{A}^{-1}\left\{\left(\frac{\alpha}{s} + 1 - \alpha\right)\left(\frac{1}{s^4} + \frac{2(1-\alpha)}{s^3} + \frac{(1-\alpha)^2}{s^2}\right)\right\}\right\}$$

$$u_3(x,t) = (2-6\lambda)^2(2\lambda-3\lambda^2)\mathcal{A}^{-1}\left\{\frac{\alpha^2}{s^5} + \frac{\alpha(1-\alpha^2)}{s^4} + \frac{\alpha(1-\alpha)^2}{s^3} + \frac{\alpha(1-\alpha)}{s^4} + \frac{(1+\alpha)(1-\alpha)^2}{s^3} + \frac{(1-\alpha)^3}{s^2}\right\}$$

$$- 6(2\lambda-3\lambda^2)^2\mathcal{A}^{-1}\left\{\frac{\alpha}{s^5} + \frac{2\alpha(1-\alpha)}{s^4} + \frac{\alpha(1-\alpha)^2}{s^3} + \frac{(1-\alpha)}{s^4} + \frac{2(1-\alpha)^2}{s^3} + \frac{(1-\alpha)^3}{s^2}\right\}$$

$$u_3(x,t) = (2-6\lambda)^2(2\lambda-3\lambda^2)\left\{\alpha^2\frac{t^3}{3!} + \alpha(-\alpha^2-\alpha+2)\frac{t^2}{2!} + (2\alpha+1)(1-\alpha)^2 t + (1-\alpha)^3\right\}$$

$$- 6(2\lambda-3\lambda^2)^2\left\{\alpha\frac{t^3}{3!} + (2\alpha+1)(1-\alpha)\frac{t^2}{2!} + (\alpha+2)(1-\alpha)^2 t + (1-\alpha)^3\right\}$$

$$u_3(x,t) = (2\lambda-3\lambda^2)((2-6\lambda)^2\alpha^2 - 6(2\lambda-3\lambda^2)\alpha)\frac{t^3}{3!} + (2\lambda-3\lambda^2)\big((2-6\lambda)^2(-\alpha^3-\alpha^2+2\alpha) - 6(2\lambda-3\lambda^2)(-2\alpha^2+\alpha+1)\big)\frac{t^2}{2!} + (1-\alpha)^2(2\lambda-3\lambda^2)\big((2-6\lambda)^2(2\alpha+1) - 6(2\lambda-3\lambda^2)(\alpha+2)\big)t + (1-\alpha)^3(2\lambda-3\lambda^2)\big((2-6\lambda)^2 - 6(2\lambda-3\lambda^2)\big)$$

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) = u_0 + u_1 + u_2 + \cdots$$

$$u(x,t) = \lambda + (2\lambda-3\lambda^2)(t+1-\alpha) + (2\lambda-3\lambda^2)(2-6\lambda)\left(\alpha\frac{t^2}{2!} + (1-\alpha^2)t + (1-\alpha)^2\right) + (2\lambda-3\lambda^2)((2-6\lambda)^2\alpha^2 - 6(2\lambda-3\lambda^2)\alpha)\frac{t^3}{3!} + (2\lambda-3\lambda^2)\big((2-6\lambda)^2(-\alpha^3-\alpha^2+2\alpha) - 6(2\lambda-3\lambda^2)(-2\alpha^2+\alpha+1)\big)\frac{t^2}{2!} + (1-\alpha)^2(2\lambda-3\lambda^2)\big((2-6\lambda)^2(2\alpha+1) - 6(2\lambda-3\lambda^2)(\alpha+2)\big)t + (1-\alpha)^3(2\lambda-3\lambda^2)\big((2-6\lambda)^2 - 6(2\lambda-3\lambda^2)\big) + \cdots$$

is found as.And if the necessary arrangements are made;

$$u(x,t)= \Big(\lambda+(2\lambda-3\lambda^2)(1-\alpha)+(1-\alpha)^2(2\lambda-3\lambda^2)(2-6\lambda)$$
$$+(1-\alpha)^3(2\lambda-3\lambda^2)\big((2-6\lambda)^2-6(2\lambda-3\lambda^2)\big)\Big)$$
$$+\Big(\Big((2\lambda-3\lambda^2)+(1-\alpha^2)(2\lambda-3\lambda^2)(2-6\lambda)$$
$$+(1-\alpha)^2(2\lambda-3\lambda^2)\big((2-6\lambda)^2(2\alpha+1)-6(2\lambda-3\lambda^2)(\alpha+2)\big)\Big)\Big)t$$
$$+\Big((2-6\lambda)(2\lambda-3\lambda^2)\alpha$$
$$+(2\lambda-3\lambda^2)\big((2-6\lambda)^2(-\alpha^3-\alpha^2+2\alpha)$$
$$-6(2\lambda-3\lambda^2)(-2\alpha^2+\alpha+1)\big)\Big)\frac{t^2}{2!}$$
$$+(2\lambda-3\lambda^2)((2-6\lambda)^2\alpha^2-6(2\lambda-3\lambda^2)\alpha)\frac{t^3}{3!}+\cdots$$

is found as. $\alpha=1$ for its special value,

$$u(x,t)=\lambda+(2\lambda-3\lambda^2)t+(2\lambda-9\lambda^2+9\lambda^3)t^2+\left(-14\lambda^2+36\lambda^3+\frac{4}{3}\lambda-27\lambda^4\right)t^3$$
$$+\left(-15\lambda^2+75\lambda^3+\frac{2}{3}\lambda-135\lambda^4+81\lambda^5\right)t^4$$
$$+\left(-\frac{62}{5}\lambda^2+108\lambda^3+\frac{4}{15}\lambda-351\lambda^4+486\lambda^5-243\lambda^6\right)t^5+O(t^6)$$

$$u(x,t)=\frac{-\frac{2}{3}\lambda e^{2t}}{-\frac{2}{3}+\lambda-\lambda e^{2t}} \qquad (30)$$

is found as.

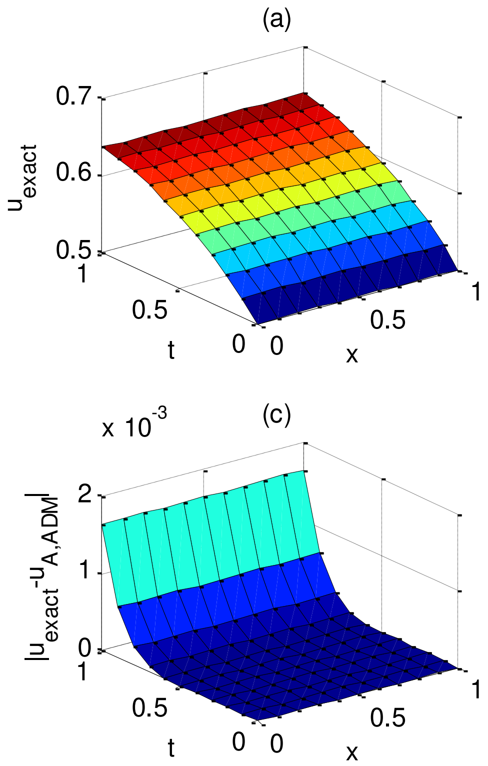

Figure 3. (a) Exact solution of $u(x,t)$ (b) $u(x,t)$ solution of A-ADM (c) Nature of absolute error $= |u_{exact} - u_{A-ADM}|$ at $\alpha = 1, \lambda = 0.5$

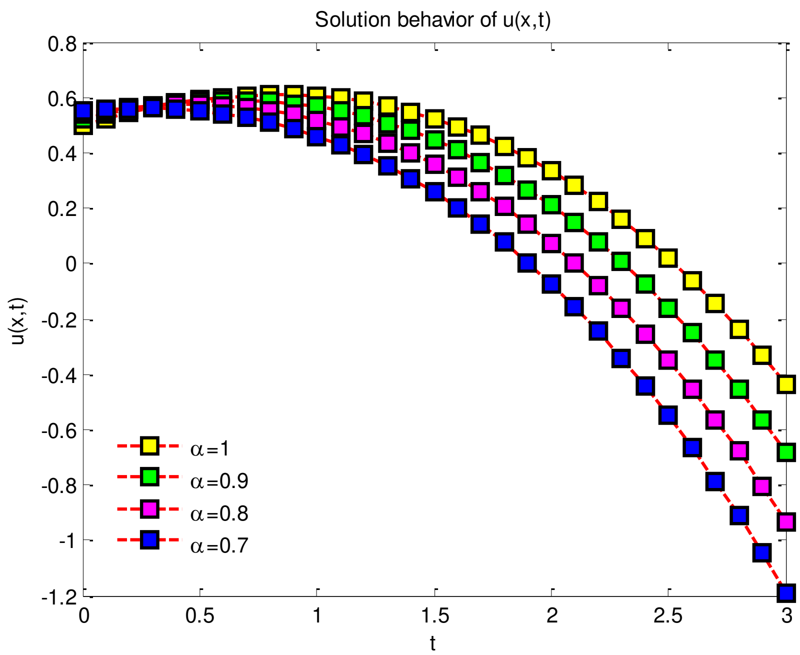

Figure 4. $u(x,t)$ solution behavior of A-ADM with respect to t when $\forall x \in R$ for $\beta = 0.5$ and $\alpha \in \{0.7,0.8,0.9,1\}$

Table 2 Comparative study between ATHPM [40], LTDM [27], q–HSATM[24] and A-ADM for the numerical solutions $u(x,t)$ at $\alpha = 1$ and $\beta = 0.01$

t	|uexact – uATHPM|	|uexact – uLTDM|	|uexact – uq–HSATM|	|uexact – uA–ADM|
0.001	1×10^{-12}	7.56×10^{-7}	1×10^{-12}	$5.2x10^{-15}$
0.002	9×10^{-11}	1.51×10^{-6}	9×10^{-11}	$8.3x10^{-14}$
0.003	3.2×10^{-10}	2.26×10^{-6}	3.2×10^{-10}	$4.2x10^{-13}$
0.004	7.7×10^{-10}	3.02×10^{-6}	7.7×10^{-10}	$1.3x10^{-12}$
0.005	1.5×10^{-9}	3.78×10^{-6}	1.5×10^{-9}	$3.2x10^{-12}$
0.006	2.6×10^{-9}	4.53×10^{-6}	2.6×10^{-9}	$6.8x10^{-12}$
0.007	4.1×10^{-9}	5.20×10^{-6}	4.1×10^{-9}	$1.2x10^{-11}$
0.008	6.1×10^{-9}	6.00×10^{-6}	6.1×10^{-9}	$2.1x10^{-11}$
0.009	8.7×10^{-9}	6.80×10^{-9}	8.7×10^{-9}	$3.4x10^{-11}$
0.010	1.2×10^{-8}	7.05×10^{-6}	1.2×10^{-8}	$5.2x10^{-11}$

6. Result and discussion

Figure 1 shows three-dimensional plots of the A-ADM solution, the exact solution, and the absolute error at $\alpha = 1$. The results obtained from the absolute errors of the eighth-order A-ADM solution are shown in Table 1. Table 1 shows that the absolute error increases significantly when any value of the space variable x and time value t increases. In Figure 3, graphs of $u(x,t)$ temperatures for different values of $\alpha = 0.7, \alpha = 0.8, \alpha = 0.9$ and $\alpha = 1$ are drawn for equation (14). Figure 3 shows three-dimensional plots of the A-ADM solution, the exact solution, and the absolute error at $\alpha = 1, \lambda = 0.5$. In Figure 4, graphs of $u(x,t)$ temperatures are drawn for different values of $\alpha = 0.7, \alpha = 0.8, \alpha = 0.9, \alpha = 1$ and $\beta = 0.5$ for equation (19). Table 2 shows that the robustness of A-ADM compared to q-HSATM, ATHPM, and LTDM yields much more robust results.

6. Conclusion

This study investigates the performance of NTFNWSE using A-ADM. It is imperative to show the effect of the Caputo-Fabruzo fractional operator included in the model under consideration. Additionally, MATLAB software was used to generate 2D and 3D. Graphs showing the solutions of equations (14) and (20) for different values of α were obtained by Maple software, and the variability of the general structure of the 3D created surface graphs for Equation solutions was also revealed.

Additionally, MATLAB software was used to obtain graphs of the numerical solutions. It can be seen that the general structure of the surface graphs of the NTFNWSE equation drawn in Maple software varies for various α and $\beta = 0.5$ values. Numerical solutions for NTFNWSE were obtained quickly and successfully. Therefore, it can be predicted that A-ADM is extremely effective and robust in obtaining numerical data by using A-ADM for the solutions of various fractional nonlinear partial differential equations**.**

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data sharing was not applicable to this publication because no datasets were generated or processed during the research.

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