$$(\theta_m + f\rho_b K_d)\frac{\partial^2 c_m}{\partial x^2} = \theta_m D_m \frac{\partial^2 c_m}{\partial x^2} - J_W \frac{\partial c_m}{\partial x} - \alpha(c_m - c_{im}) - (\theta_m \mu_{\ell,m} + f\rho_b K_d \mu_{im})c_m + \theta_m \gamma_{\ell,m}(x) + f\rho_b \gamma_{s,m}(x)$$

$$\beta R\frac{\partial C_1}{\partial T} = \frac{1}{P}\frac{\partial^2 C_1}{\partial Z^2} - \frac{\partial C_1}{\partial Z} - \omega(C_1 - C_2) - \mu_1 C_1$$