

Supplementary Information for: Current-driven dynamics of antiferromagnetic skyrmions: from skyrmion Hall effects to hybrid inter-skyrmion scattering

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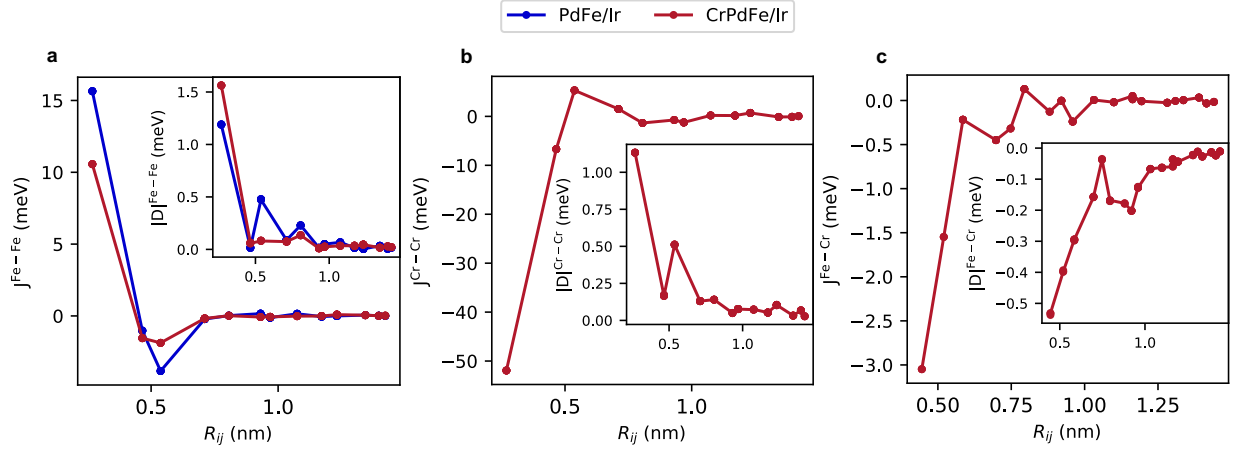


Figure 1: Distance-dependent magnetic interactions in CrPdFe and PdFe deposited on Ir(111). **a** The Heisenberg exchange interactions as function of distance among Fe atoms ($J^{\text{Fe-Fe}}$) in PdFe/Ir(111) (blue) and in CrPdFe/Ir(111) (red), with inset indicating the corresponding magnitude of DMI. **b** The Heisenberg exchange interactions between Cr atoms ($J^{\text{Cr-Cr}}$) with DMI depicted in the inset. Similarly to **b**, the interactions between Cr and Fe atoms are shown in **c**. The magnetic anisotropy of Cr atoms, $K_{\text{Cr}} = 0.4$ meV, and for Fe atoms $K_{\text{Fe}} = 0.5$ meV.

1 Ellipticity of AFM skyrmion residing on triangular lattice

The AFM skyrmions emerging in the triangular lattice of Cr deposited on PdFe bilayer grown on fcc Ir(111) surface are of distinctive elliptical shape. The skyrmions are hosted by the row-wise AFM (RW-AFM) state. The ellipticity stems from the competition among the nearest neighboring (nn) exchange interactions namely (J_1 , J_2 , and J_3). As shown in supplementary Figure 2, the RW-AFM spins are distributed among four sublattices (L_1 , L_2 , L_3 , and L_4) where the spins at each sublattice are interacting via J_3 which is of FM nature, facilitating the formation of FM skyrmion at each sublattice. The interactions with spins of the adjacent sublattices, which host the other FM skyrmions, are governed by J_1 and J_2 . These have a significant impact on shaping the resulting AFM skyrmion. In the case of a single AFM skyrmion, the two FM skyrmions building up the AFM skyrmion are hosted by sublattices L_1 and L_2 as shown in Supplementary Figure 2b, with their centers located at positions **M** and **N**. As depicted in Supplementary Figure 2a, **M** interacts with **N** through J_1 and with **N'** through J_2 , while its response to **P**, and **O** will cancel out since sublattices L_3 and L_4 are oppositely collinearly oriented. As a result, this unbalanced interaction leads to a deformation of the skyrmion along **MN** when compared to **MN'**, **MP** and **MO**, which induces an elongation along **MN** direction. One can control the shape of the skyrmion by tuning the ratio between J_1 and J_2 , as discussed in details in Ref. ¹.

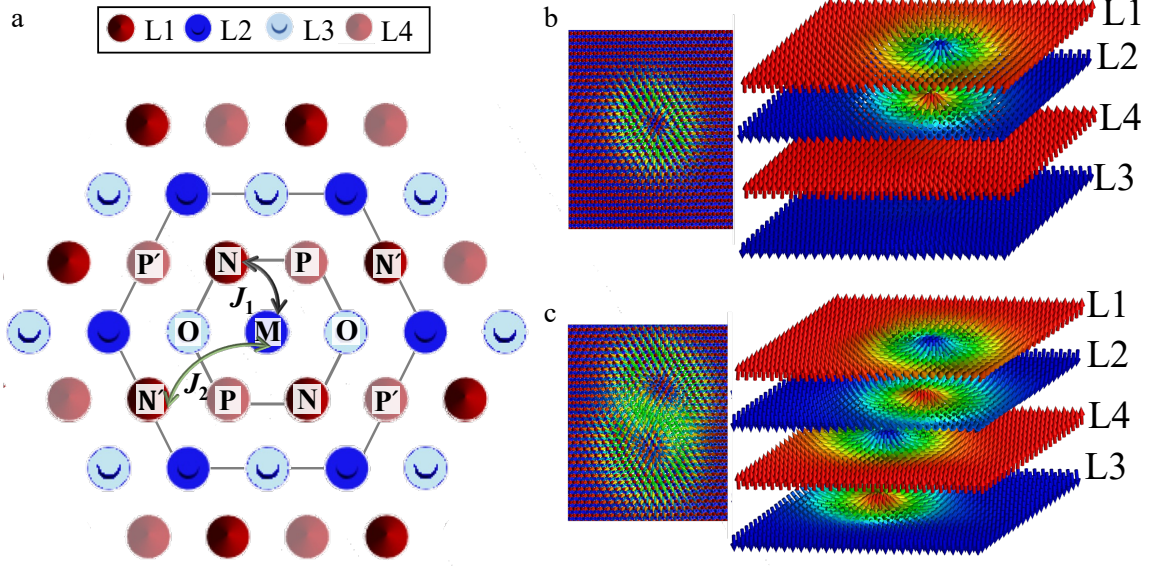


Figure 2: Spin distribution of single and double AFM skyrmions on RW-AFM spin alignment. **a** Schematic representation of the four sublattices distribution on the triangular lattice. **b** Schematic representation of the building blocks for single AFM skyrmion **b**, and double AFM skyrmions **c** illustrating that a single AFM skyrmion is made of two FM skyrmions, each hosted by sublattices L1 and L2, while L3 and L4 remain collinear, whereas the latter can host a second AFM skyrmion in case of the overlapped skyrmion scenario.

2 Thiele equation skyrmions driven by CPP

Thiele equation for CPP injection^{2-4,6,7}:

$$\mathbf{G} \times \mathbf{v} - \alpha \mathcal{D} \cdot \mathbf{v} + \mathcal{B} \cdot \mathbf{S}_P = 0, \quad (1)$$

where, $\mathbf{G} = (0, 0, Q)$, with Q is the topological charge of the skyrmion being $+1, -1$ for core up, core down FM skyrmions, respectively. $\mathbf{v}$ is the skyrmion velocity, $\mathcal{D}$ is the dissipative tensor, where $\mathcal{D}_{\mu\nu} = \int d^2r (\partial_\mu \mathbf{m} \cdot \partial_\nu \mathbf{m}) / 4\pi$, and $\mathbf{m}(r)$ is the magnetic skyrmion profile can be expressed as, $\mathbf{m}(r) = \mathbf{m}(\theta, \phi) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$. $\mathbf{S}_P$ stands for unit spin polarization direction, where for current polarised at angle θ_{js} from the skyrmion main axis is given by $(\cos \theta_{js}, \sin \theta_{js})$. $\mathcal{B}$ is the driving force related tensor, where $\mathcal{B}_{\mu\nu} = \gamma\eta \int (\partial_\mu \mathbf{m} \times \mathbf{m})_\nu d^2r / 4\pi$, with γ is the gyro-magnetic ratio, and η is the applied current amplitude parameter.

For elliptical skyrmion⁵, evaluating the integrals for $\mathcal{D}_{\mu\nu}^i$, and $\mathcal{B}_{\mu\nu}^i$ for each building FM skyrmion building block i , where μ, ν run over x and y and assuming skyrmions of identical size and shape, are given by:

$$(\mathcal{D}_{xx}^i, \mathcal{D}_{yy}^i) = \frac{\pi^2}{8} \left(\frac{b}{a}, \frac{a}{b} \right), \text{ where } a, \text{ and } b \text{ are the semi-major and semi-minor ellipse axes, while } \mathcal{D}_{yx}^i = \mathcal{D}_{xy}^i = 0. \text{ The components of the driving force tensor are } (\mathcal{B}_{xy}^i, \mathcal{B}_{yx}^i) = \frac{\gamma\pi}{8}\eta(-b, a) \text{ and } \mathcal{B}_{xx}^i = \mathcal{B}_{yy}^i = 0.$$

So, equation 1 for elliptical AFM skyrmion reads,

$$-\alpha \mathcal{D}_{xx}^i v_x \hat{i} - \alpha \mathcal{D}_{yy}^i v_y \hat{j} + \mathcal{B}_{xy}^i \sin \theta_{js} \hat{i} + \mathcal{B}_{yx}^i \cos \theta_{js} \hat{j} = 0, \quad (2)$$

which gives,

$$\begin{aligned} v_y &= \frac{\mathcal{B}_{yx} \cos \theta_{js}}{\alpha \mathcal{D}_{yy}}, \\ v_x &= \frac{\mathcal{B}_{xy} \sin \theta_{js}}{\alpha \mathcal{D}_{xx}}. \end{aligned} \quad (3)$$

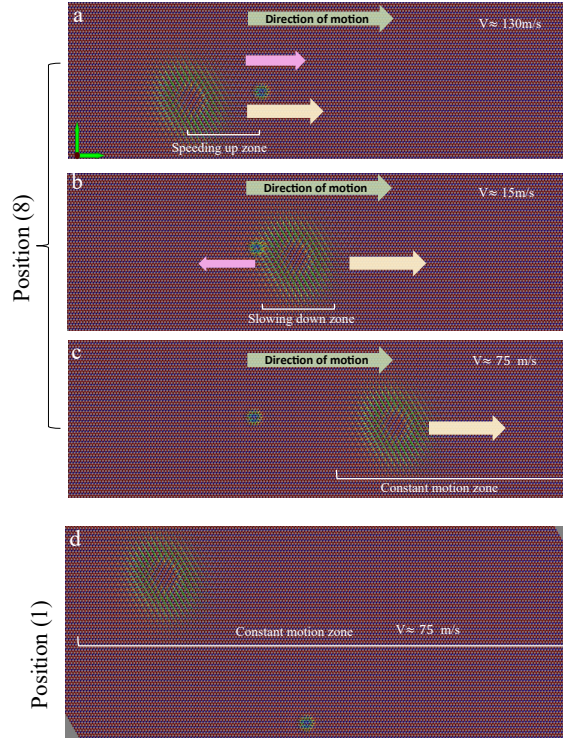


Figure 3: Impact of a pinned FM skyrmion on the translational motion of AFM skyrmion in a synthetic AFM-FM skyrmion scenario. a-c Snapshots showing the different regions where a current-driven AFM skyrmion ($\eta/\alpha = 0.0167$) interact with single FM skyrmion at Fe layer when located at position (1) in main Figure 4. When the FM skyrmions lays ahead of the AFM skyrmions and it lays in its vicinity, both the attraction force works with the drag force causing the velocity increase of the AFM skyrmion (a), while when the AFM skyrmion passes the FM skyrmion, and it lays behind it, the attraction force acts against the drag force causing deceleration of the AFM skyrmion b. When the AFM skyrmion passes the interaction area, it gains constant motion speed c, similar to the velocity of the AFM skyrmion when moving away from the FM skyrmion d which represents the AFM skyrmion when located at position(8) in main Figure 4.

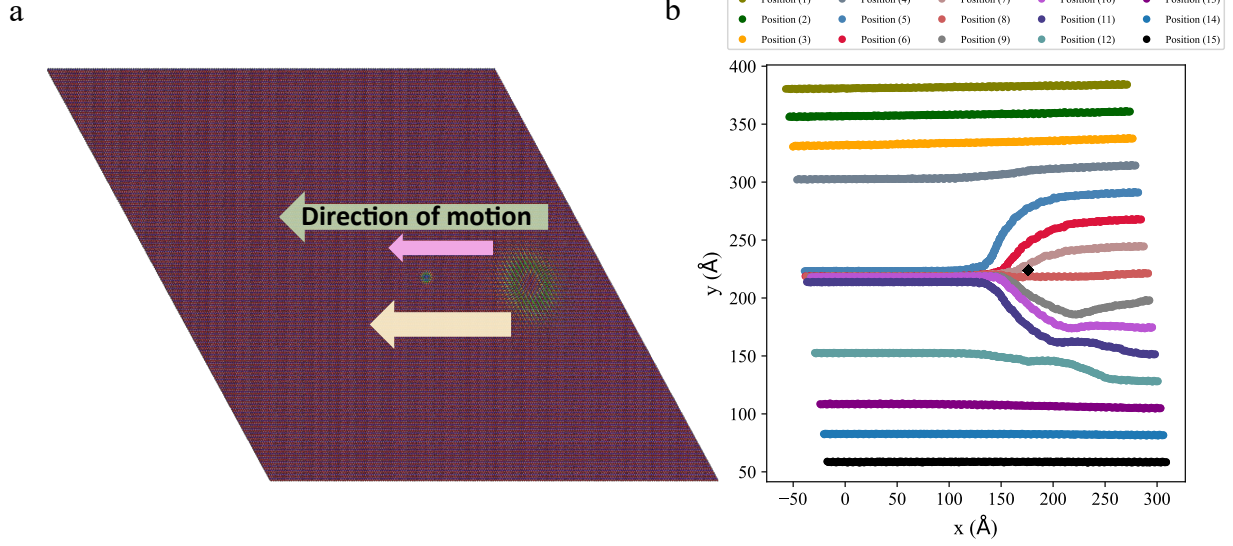


Figure 4: Position dependent deflection of AFM skyrmions due to the presence of FM skyrmions at Fe layer assuming a driving motion from right to left. **a** In contrast to main Figure 4 , the AFM skyrmion is positioned on the right hand side with respect to the FM skyrmion. **b** The trajectory of the single AFM skyrmion when moved toward the pinned FM skyrmion at Fe layer from different initial positions from right to left. Deflections similar to those reported in the main manuscript occur depending on the relative positions of the AFM and FM skyrmions.

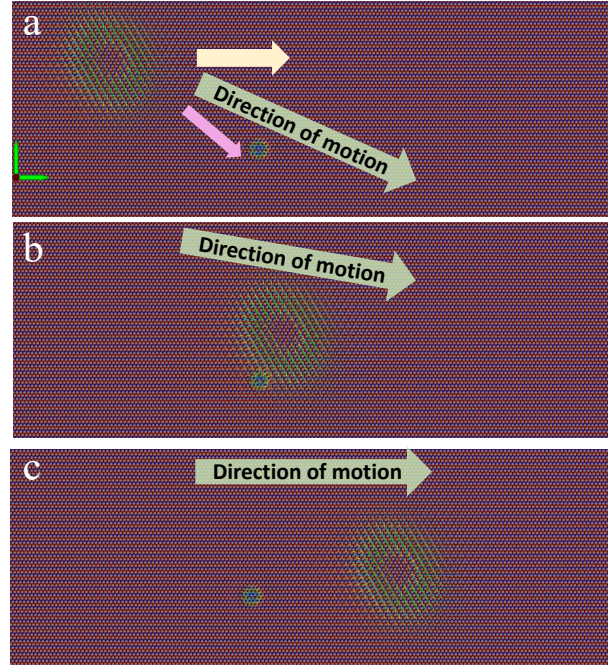


Figure 5: Direction of motion of AFM skyrmion interacting with a FM skyrmion in a synthetic AFM-FM skyrmion scenario. a-c Snapshots showing the trajectory of the AFM skyrmion when the FM skyrmion is not located along the path. A deflection occurs, followed by a straight-forward motion. Here, the FM skyrmion is pinned.

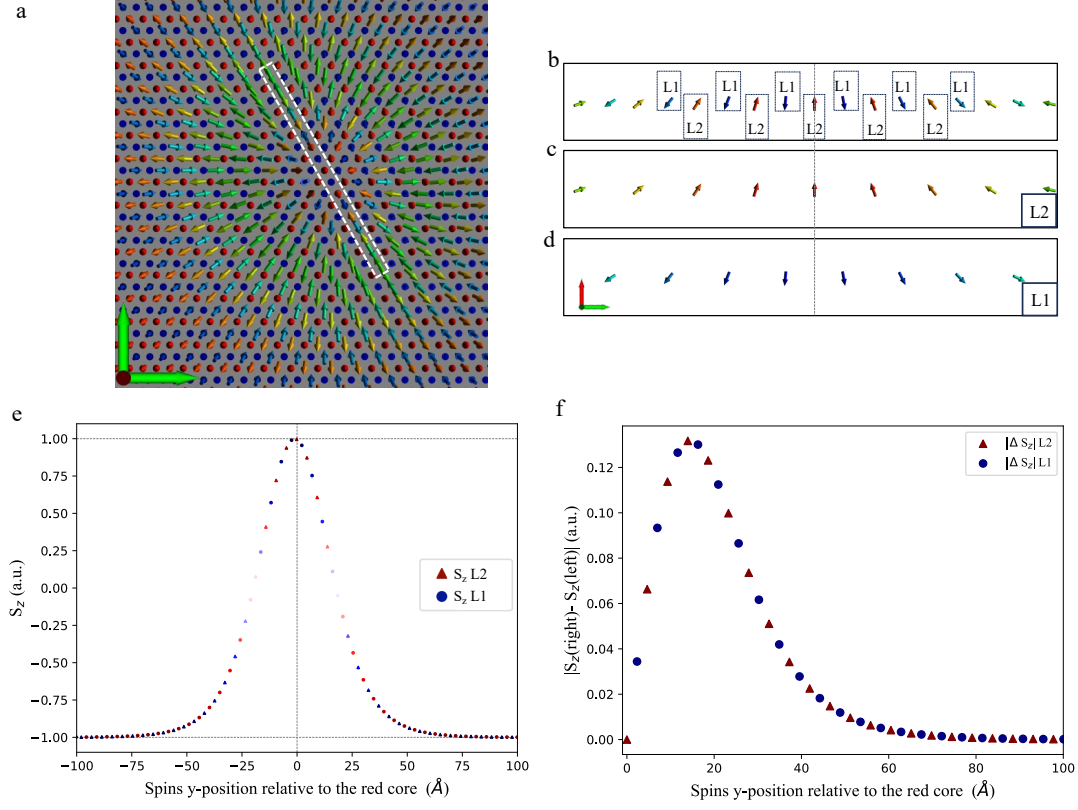


Figure 6: Spin analysis along the line connecting the two FM building-block skyrmions of the single AFM skyrmion. **a** Snapshot showing the AFM skyrmion. **b** Snapshot of the spins along the line connecting two centers of the AFM skyrmion. Two spirals belong to two sublattices L1 and L2 are shown separately in respectively **c** and **d**. Once, one sublattice hosts a skyrmion core, the adjacent spins rearrange themselves to accommodate the different magnetic interactions and produce a natural spin asymmetry with respect to the skyrmion core. This asymmetry is quantified in the S_z profile **e**. Taking the difference between right and left S_z highlights how the spin asymmetry develops away from the skyrmion core. It reaches a maximum at a distance of 1.4 nm.

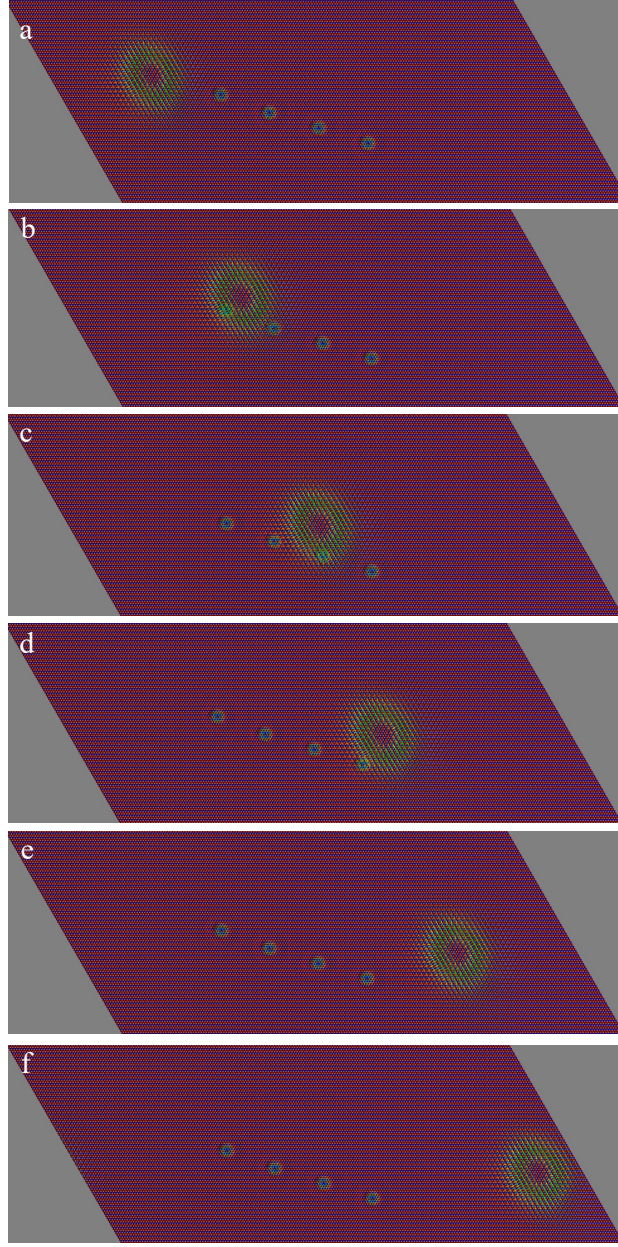


Figure 7: Seeding of FM skyrmions for the motion control of AFM skyrmions. Snapshots at different times (a-e) vividly demonstrating the capacity to precisely steer the motion of AFM skyrmions by precisely arranging the FM skyrmions within the Fe layer in a hybrid synthetic scenario of AFM and FM films. The figures obtained with a current parameter of $\eta/\alpha=0.0167$.

3 Supplementary Movie1

This Movie shows the trajectory of the single AFM skyrmion at the Cr layer when the Fe layer hosts FM state, with $\eta/\alpha = 0.02$ meV.

4 Supplementary Movie2

This Movie shows the trajectory of the single AFM skyrmion at the Cr layer when the Fe layer hosts SkX state, with $\eta/\alpha = 0.02$ meV.

5 Supplementary Movie3

This Movie shows the trajectory of the single AFM skyrmion at the Cr layer when the Fe layer hosts spirals state, with $\eta/\alpha = 0.02$ meV.

6 Supplementary Movie4

This Movie shows the single AFM skyrmion at the Cr layer gets pinned by the FM skyrmion residing at the Fe layer beneath, with $\eta/\alpha = 0.001$ meV.

References

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