Supplementary Information to: Vulnerability to Climate Change: Evidence from a Dynamic Factor Model*

Haroon Mumtaz[1] and Fulvia Marotta[2]

[1]Queen Mary University of London
[2]University of Oxford

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1 Robustness checks

1.0.1 Fat tails

First, we relax the assumption of normal disturbances in the benchmark model. The idiosyncratic components are defined as $\Upsilon_{it} \sim N\left(0, \frac{\sigma_{it}^2}{\lambda_{it}}\right)$. The variance σ_{it}^2 is allowed to evolve slowly over time and follows an AR model as described above. Higher frequency movements in volatility are captured by $\frac{1}{\lambda_{it}}$ that is independent over time. Following Geweke (1993), we assume a Gamma prior for λ_{it} of the form $p(\lambda_i) = \prod_{i=1}^{T} \Gamma(1, v_i)$, where $\Gamma(a, b)$ is the gamma density with mean a and degrees of freedom b. This leads to a scale mixture of normal distributions for Υ_{it} implying that these residuals follow a student's T density with degrees of freedom v_i and variance σ_{it}^2. The same specification is assumed for the shocks to the transition equation **??**. As discussed below, the covariance matrix of E_t is assumed to be diagonal for the purpose of identification. Each element of E_t is distributed normally: $E_{kt}\tilde{}N\left(0, \frac{h_{kt}}{\gamma_{kt}}\right)$ for $k = 1, .., N$. The weights $\frac{1}{\gamma_{kt}}$ capture high frequency movements in volatility. As above, we assume a Gamma prior for γ_{kt}:

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$p(\gamma_k) = \prod_{i=1}^{T} \Gamma(1, v_j)$. The degrees of freedom parameters ν are set to 10 implying tails that are fatter than the normal density. We add steps in the Gibbs sampler to draw λ_{it} and γ_{kt}. The conditional posterior distribution for this parameter is described in Koop (2003). Figure S1 shows that the estimated factors are close to the benchmark case. In additionn, the variance decomposition has a pattern that is similar to benchmark with the world factor playing a major role in explaining temperature fluctuations.

1.1 Two factors

We extend the benchmark model and add two world and regional factors. The top panels of Figure S2 display the estimated world factors. The first temperature factor displays the temporal pattern seen in the benchmark case with a sharp increase evident in the post-1980 period. Similarly, the first precipitation factor displays a decline after the mid-1970s. The second temperature and precipitation factors are more volatile and do not have a clear interpretation. In terms of the variance decomposition, the role of the world temperature factors is magnified in this extended model as some of the higher frequency movements in temperature are also explained by the second factor. As in the benchmark case, the regional factor remains important for precipitation volatility.

1.2 Longer Lags

We re-estimate the model using 4 lags in the transition equation for the factors. Figure S3 shows that the results regarding the factors and variance decomposition are very similar to the benchmark case.

1.3 Gibbs Sampling Algorithm

The empirical model is defined as:

$$\begin{pmatrix} T_{it} \\ P_{it} \end{pmatrix} = \begin{pmatrix} B_i^{W,T} & 0 \\ 0 & B_i^{W,P} \end{pmatrix} \begin{pmatrix} F_t^{W,T} \\ F_t^{W,P} \end{pmatrix} + \begin{pmatrix} B_i^{R,T} & 0 \\ 0 & B_i^{R,P} \end{pmatrix} \begin{pmatrix} F_t^{R,T} \\ F_t^{R,P} \end{pmatrix} + \begin{pmatrix} v_{it} \\ e_{it} \end{pmatrix} \quad (1)$$

where T denotes temperature series and P denotes precipitation series. The idiosyncratic components $u_{it} = \begin{pmatrix} v_{it} \\ e_{it} \end{pmatrix}$ are assumed to follow AR(1) processes:

$$u_{it} = \rho_i u_{it-1} + \Upsilon_{it} \quad (2)$$

The shocks Υ_{it} are assumed to have the following distribution:$\Upsilon_{it} \sim N(0, \sigma_{it})$. The variance σ_{it}^2 is allowed to evolve over time:

$$\ln \sigma_{it}^2 = \tilde{c}_i + \tilde{d}_i \ln \sigma_{it-1}^2 + g_i v_{it} \tag{3}$$

The world and regional factors are assumed to follow a VAR process:

$$Z_t = c + \sum_{j=1}^{P} b_j Z_{t-j} + E_t \tag{4}$$

where $\underbrace{Z_t}_{N \times 1} = \left(F_t^{W,J}, F_t^{R,J}\right)$ for $J = [T, P]$ The covariance matrix of E_t is assumed to be diagonal for the purpose of identification. Each element of E_t is distributed normally: $E_{kt} \sim N(0,\ h_{kt})$ for $k = 1, .., N$. The stochastic volatilities h_{kt} evolve as random walks:

$$\ln h_{kt} = \bar{c}_k + \bar{D}_k \ln h_{kt-1} + d_k u_{kt} \tag{5}$$

1.4 Priors

We use the following prior distributions

1. The factor loadings are denoted by $b_i = [B_i^{W,J}, B_i^{R,J}]$ for for $J = [T, P]$. The prior is normal: $P(b_i) \sim N(\tilde{b}_i, \Sigma_b)$ where $\tilde{b}_i$ denotes principal component estimates of factor loadings and Σ_b is a diagonal matrix with diagonal elements equal to 0.1.

2. The prior for the persistence of the idiosyncratic components ρ_i is normal: $N(\rho_0, \Sigma_p)$ where $\rho_0 = 0.5$ and $\Sigma_p = 0.01$

3. The prior for the coefficients $\tilde{A}_i = [\tilde{c}_i, \tilde{d}_i]$ is normal with mean $A_0 = [0, 0.95]$ and variance Σ_A where Σ_A is a diagonal matrix with 0.1 on the main diagonal.

4. The prior for g_i is inverse Gamma with scale $g_0 = 0.001$ and degrees of freedom $T_0 = 1$

5. We use a Minnesota type prior (see Bańbura *et al.* (2010) for the coefficients of the VAR in equation 4 which are denoted by $\bar{\beta}$ in vectorised form. The tightness parameter is set to 0.1

6. The prior for $\bar{A}_k = [\bar{c}_i, \bar{d}_i]$ is normal with mean $\bar{A}_0 = [0, 0.95]$ and with variance $\bar{\Sigma}_A$, a diagonal matrix with diagonal elements equal to 0.1.

7. The prior for d_k is inverse Gamma with scale $d_0 = 0.001$ and degrees of freedom $T_{d,0} = 1$

The Gibbs sampling algorithm draws from the following conditional posterior distributions in each iteration (Ψ denotes all other parameters):

1. Factor loadings $G(b_i|\Psi)$. For each series y_{it} the model can be written as:

$$y_{it} = b_i z_t + u_{it} \tag{6}$$
$$u_{it} = \rho_i u_{it-1} + \Upsilon_{it} \tag{7}$$
$$var(\Upsilon_{it}) = \sigma_{it} \tag{8}$$

   where $z_t = [F_t^{W,J}, F_t^{R,J}]$ for $J = T$ or $J = P$. A GLS transformation can be used to remove the autocorrelation and heteroscedasticity. Define $y_{it}^* = \frac{y_{it} - \rho y_{it-1}}{\sigma_{it}^{0.5}}$ and $x_{it}^* = \frac{z_{it} - \rho z_{it-1}}{\sigma_{it}^{0.5}}$. The conditional posterior is normal with variance $V = \left(\Sigma_b^{-1} + x_i^{*\prime} x_i^*\right)^{-1}$ and mean $M = V\left(\Sigma_b^{-1}\tilde{b}_i + x_i^{*\prime} y_i^*\right)$.

2. Persistence of idiosyncratic components $G(\rho_i|\Psi)$. Conditional on the remaining parameters, the idiosyncratic components can be written as:

$$u_{it} = \rho_i u_{it-1} + \Upsilon_{it} \tag{9}$$
$$var(\Upsilon_{it}) = \sigma_{it} \tag{10}$$

   A GLS transformation can be used to remove heteroscedasticity. Define $y_{it}^* = \frac{u_{it}}{\sigma_{it}^{0.5}}$ and $x_{it}^* = \frac{u_{it-1}}{\sigma_{it}^{0.5}}$. The conditional posterior is normal with mean and variance defined as in Step 1 above.

3. Stochastic volatility of idiosyncratic components $G(\sigma_{it}|\Psi)$. Conditional on the remaining parameters, a non-linear state-space model applies to each idiosyncratic component:

$$u_{it} = \rho_i u_{it-1} + \Upsilon_{it} \tag{11}$$
$$var(\Upsilon_{it}) = \sigma_{it} \tag{12}$$
$$\ln \sigma_{it}^2 = \tilde{c}_i + \tilde{d}_i \ln \sigma_{it-1}^2 + g_i v_{it} \tag{13}$$

   We draw σ_{it} from the conditional posterior using the particle Gibbs sampler with ancestor sampling introduced by Lindsten *et al.* (2014).

4. $G(g_i|\Psi)$. This conditional posterior is inverse Gamma with scale parameter $g_0 + \left(\ln\sigma_{it}^2 - \tilde{c}_i - \tilde{d}_i \ln\sigma_{it-1}^2\right)' \left(\ln\sigma_{it}^2 - \tilde{c}_i - \tilde{d}_i \ln\sigma_{it-1}^2\right)$ and degrees of freedom $T_i + T_0$ where T_i denotes the time-series for the ith cross-section.

5. $G(\tilde{A}_i|\Psi)$. The transition equation for the stochastic volatility is a linear regression model. Conditional on $ln\sigma_{it}^2, g_i$, the conditional posterior is normal. Let $y = ln\sigma_{it}^2$ and $x = [1, ln\sigma_{it-1}^2]$. Then the variance of the conditional posterior is defined as: $V = \left(\Sigma_A^{-1} + \frac{1}{g_i}x'x\right)^{-1}$ and mean $M = V\left(\Sigma_A^{-1}A_0 + \frac{1}{g_i}x'y\right)$

6. VAR coefficients $G(\bar{\beta}|\Psi)$. Conditional on the remaining parameters 4 is a VAR with heteroscedastic disturbances. As we assume that the covariance matrix of E_t is diagonal, one can draw from the conditional posterior of the coefficients equation by equation. The conditional posterior is normal after a GLS transformation. Carriero *et al.* (2022) describe an efficient algorithm to implement this draw and we follow their approach.

7. Stochastic volatility $G(h_k|\Psi)$. The transition equations can be cast as a non-linear state-space system:

$$Z_t = c + \sum_{j=1}^{P} b_j Z_{t-j} + E_t \tag{14}$$

$$\ln h_{kt} = \bar{c}_k + \bar{D}_k \ln h_{kt-1} + d_k u_{kt} \tag{15}$$

   As in step 3, we employ a particle Gibbs sampler to draw h_{kt} from the conditional posterior distribution.

8. $G(d_k|\Psi)$. This step is identical to Step 4.

9. $G(\bar{A}_k|\Psi)$. This step is identical to Step 5.

1.5 Contribution of regional factor to temperature variance

Figure S4 shows the contribution of the regional factor to temperature volatility while Figure S5 shows the contribution of the regional factor to precipitation volatility. See main text for details.

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Figure S1: Robustness: Fat tails

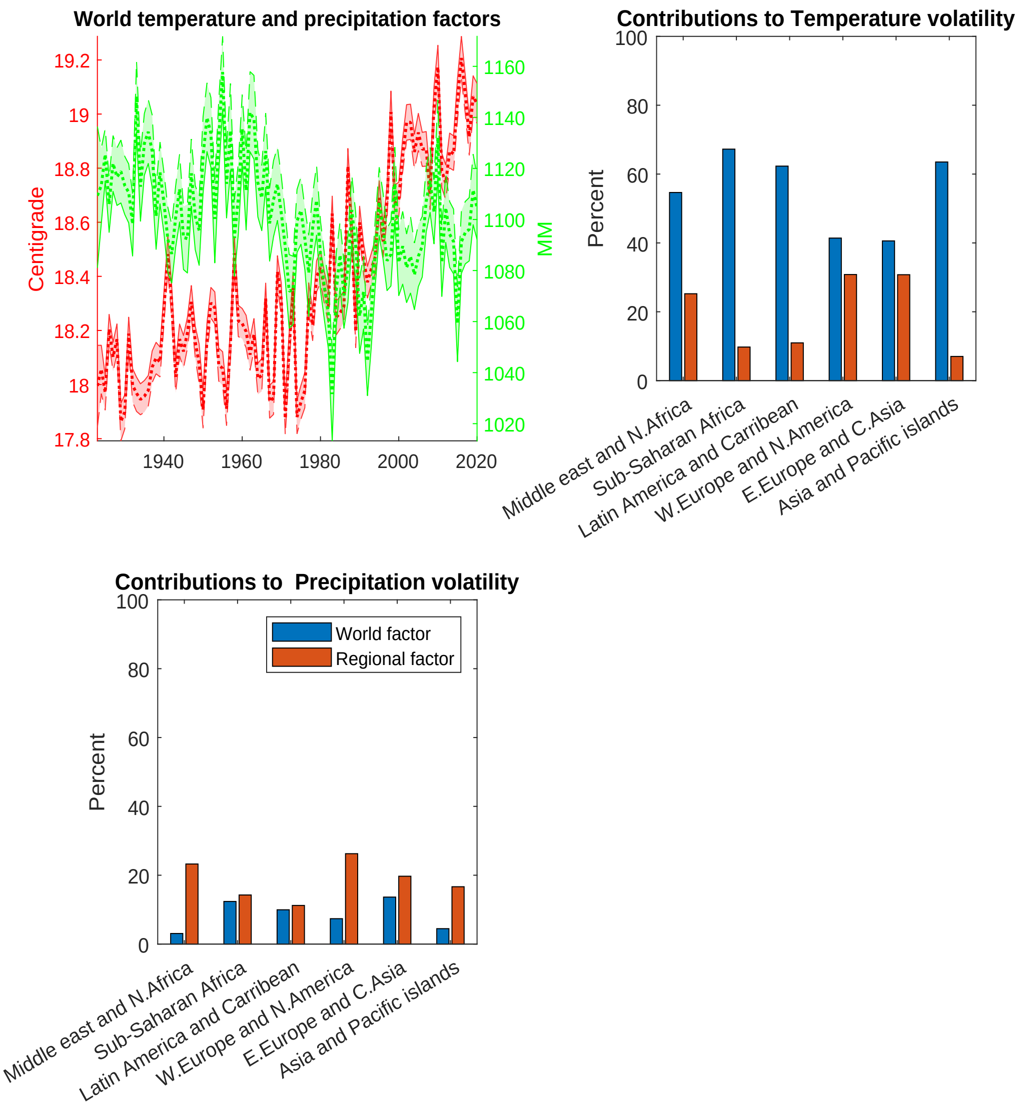


Figure S2: Robustness: Two factors

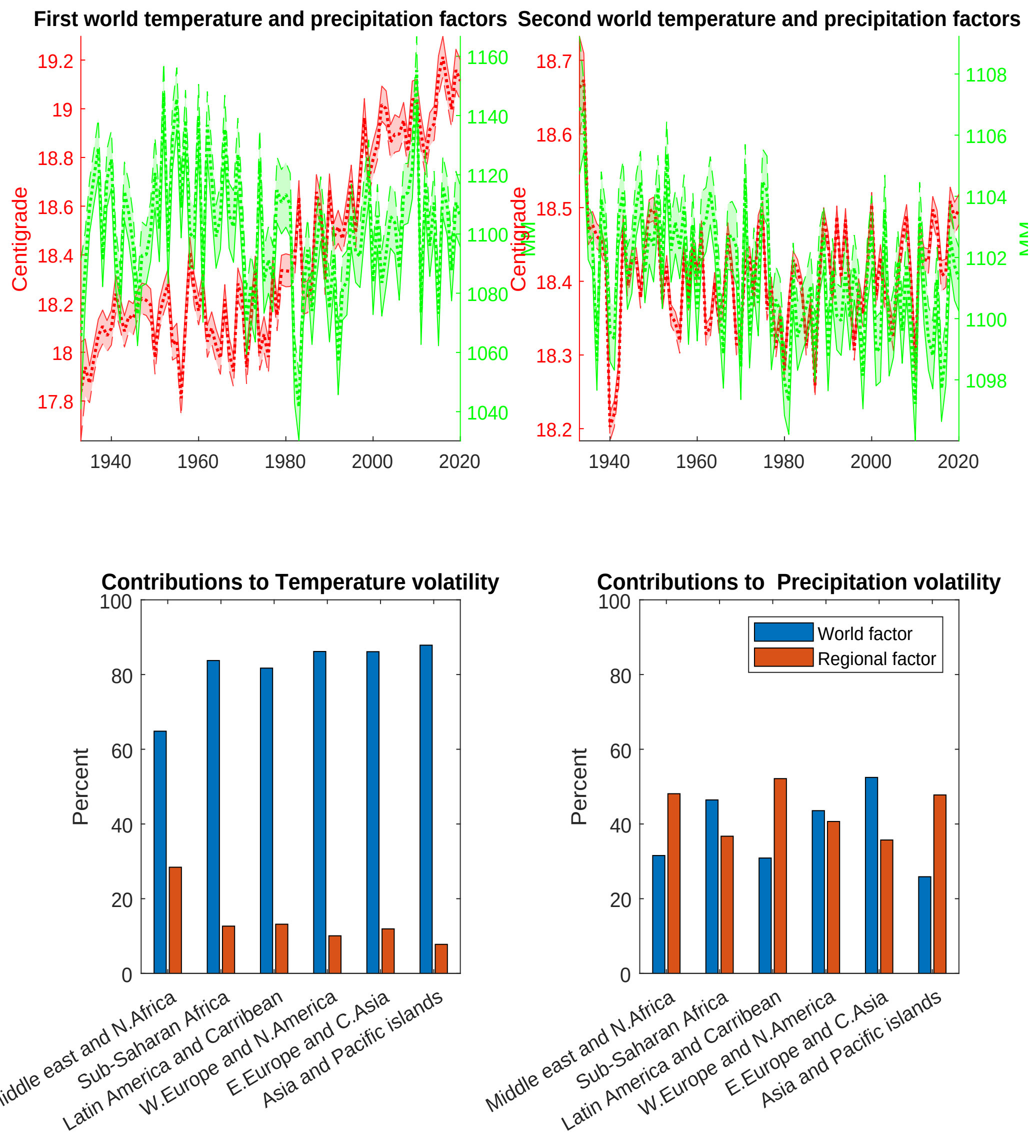


Figure S3: Robustness: Longer lags

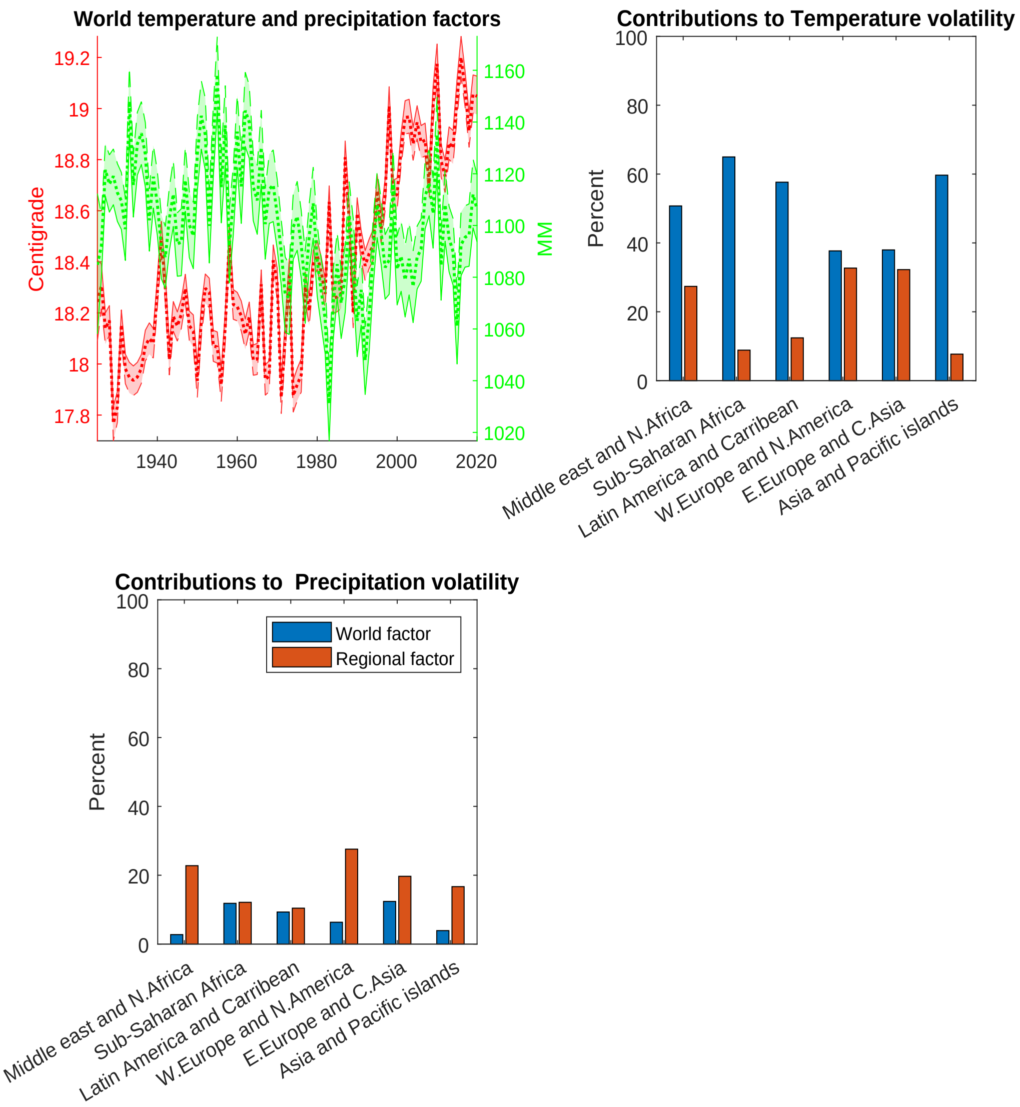
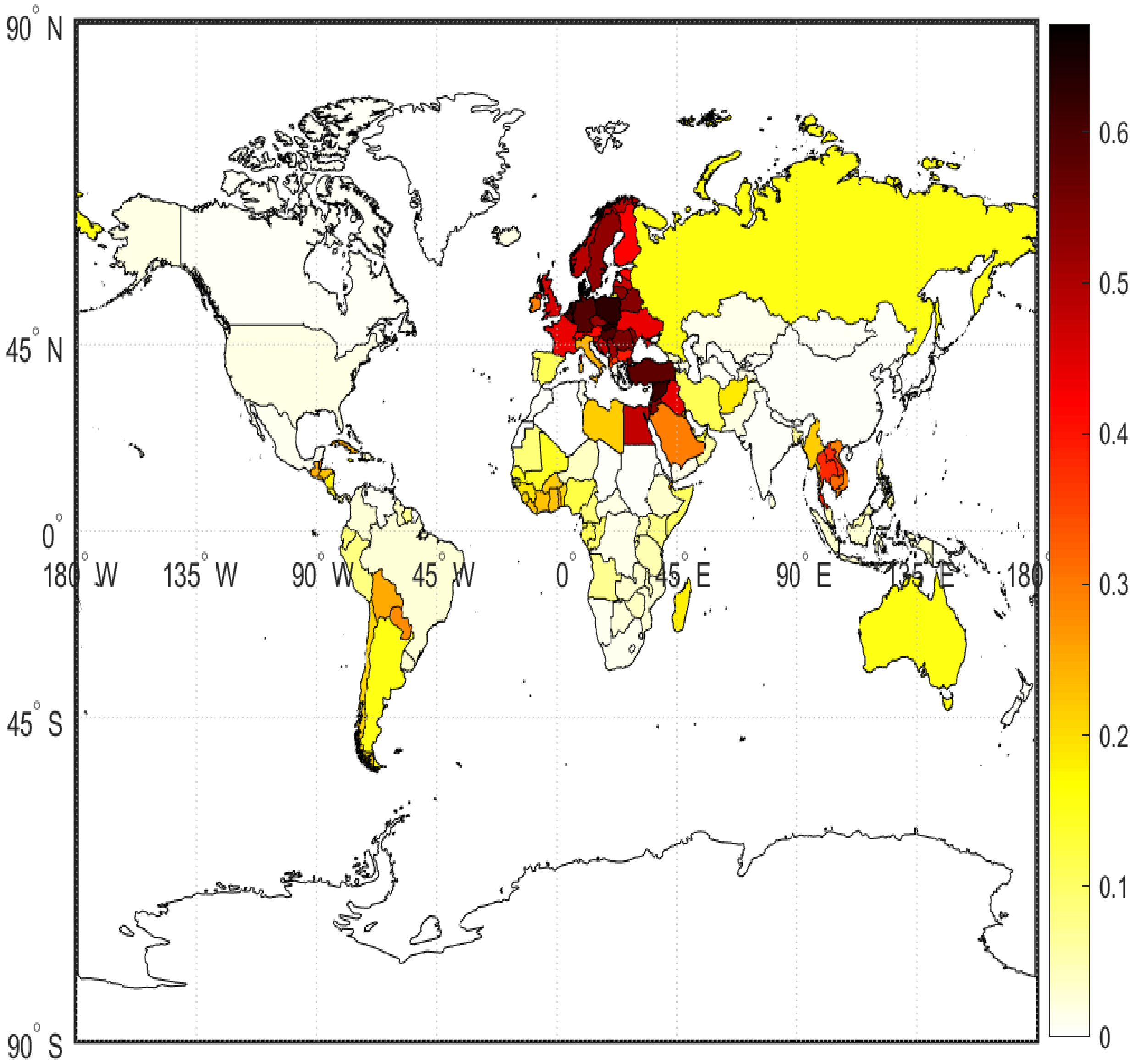
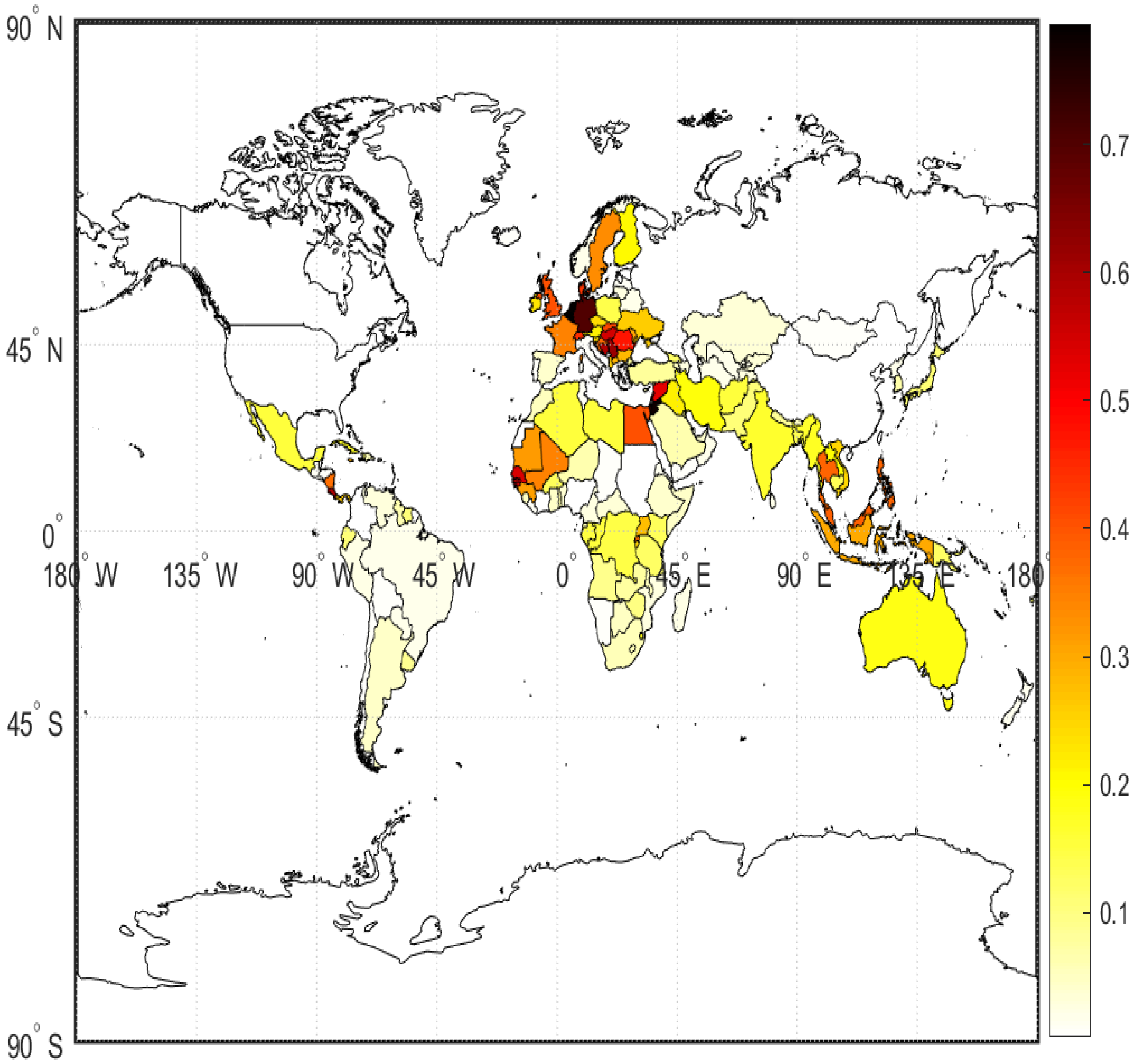


Figure S4: Contribution of the regional factor to temperature volatility



Darker colours the heat map indicates a larger contribution

Figure S5: Contribution of the regional factor to precipitation volatility



Darker colours the heat map indicates a larger contribution