Appendix A Supplementary Methods

Total outflow length

To estimate Porphyrion's total length from its projected length, we perform statistical deprojection. Equation 9 of Oei et al. [48] stipulates the probability density function (PDF) of an outflow's total length random variable (RV) L in case its projected length RV L_p is known to equal some value l_p. This PDF is parametrised by the tail index ξ of the Pareto distribution assumed to describe L. We calculate the median and expectation value of $L \mid L_\mathrm{p} = l_\mathrm{p}$ for tail indices $\xi = -3$ and $\xi = -4$, the integer values closest to the observationally favoured $\xi = -3.5 \pm 0.5$ [48].

First, we determine the cumulative distribution function (CDF) of $L \mid L_\mathrm{p} = l_\mathrm{p}$ through integration:

$$\begin{aligned} F_{L|L_\mathrm{p}=l_\mathrm{p}}(l) &\coloneqq \int_{-\infty}^{l} f_{L|L_\mathrm{p}=l_\mathrm{p}}(l')\ \mathrm{d}l' \\ &= \frac{-\xi}{2^{1+\xi}\pi} \frac{\Gamma^2\left(-\frac{\xi}{2}\right)}{\Gamma(-\xi)} \int_1^{\max\{x,1\}} \frac{x'^{\xi-1}}{\sqrt{x'^2-1}}\ \mathrm{d}x', \end{aligned} \tag{A1}$$

where $x \coloneqq \frac{l}{l_\mathrm{p}}$ and $x' \coloneqq \frac{l'}{l_\mathrm{p}}$.

For $\xi = -3$, the CDF concretises to

$$\begin{aligned} F_{L|L_\mathrm{p}=l_\mathrm{p}}(l) &= \frac{3}{2} \int_1^{\max\{x,1\}} \frac{\mathrm{d}x'}{x'^4\sqrt{x'^2-1}} \\ &= \begin{cases} 0 & \text{if } x < 1; \\ \frac{(2x^2+1)\sqrt{x^2-1}}{2x^3} & \text{if } x \geq 1. \end{cases} \end{aligned} \tag{A2}$$

The median conditional total length, l_m, is defined by $F_{L|L_\mathrm{p}=l_\mathrm{p}}(l_\mathrm{m}) \coloneqq \frac{1}{2}$. Numerically, we obtain $x_\mathrm{m} \coloneqq \frac{l_\mathrm{m}}{l_\mathrm{p}} \approx 1.0664$, or $l_\mathrm{m} \approx 1.0664\ l_\mathrm{p}$. As $l_\mathrm{p} = 6.43 \pm 0.05$ Mpc, we find $l_\mathrm{m} = 6.86 \pm 0.05$ Mpc. An analogous numerical determination of the 16-th and 84-th percentiles then yields $l = 6.9^{+1.6}_{-0.4}$ Mpc.

For $\xi = -4$, the CDF concretises to

$$\begin{aligned} F_{L|L_\mathrm{p}=l_\mathrm{p}}(l) &= \frac{16}{3\pi} \int_1^{\max\{x,1\}} \frac{\mathrm{d}x'}{x'^5\sqrt{x'^2-1}} \\ &= \begin{cases} 0 & \text{if } x < 1; \\ \frac{2}{3\pi}\left(\frac{(3x^2+2)\sqrt{x^2-1}}{x^4} + 3\arccos\frac{1}{x}\right) & \text{if } x \geq 1. \end{cases} \end{aligned} \tag{A3}$$

Numerically, we obtain $x_\mathrm{m} \approx 1.0515$, or $l_\mathrm{m} \approx 1.0515\ l_\mathrm{p}$, and thus $l_\mathrm{m} = 6.76 \pm 0.05$ Mpc. In the same way as before, we find $l = 6.8^{+1.2}_{-0.3}$ Mpc.

Equation 10 of Oei et al. [48] gives a closed-form expression for $\mathbb{E}\left[L \mid L_\mathrm{p} = l_\mathrm{p}\right](\xi)$. Table 1 of the same work lists $\mathbb{E}[L \mid L_\mathrm{p} = l_\mathrm{p}](\xi = -3) = \frac{3\pi}{8} l_\mathrm{p}$ and $\mathbb{E}[L \mid L_\mathrm{p} = l_\mathrm{p}](\xi = -4) = \frac{32}{9\pi} l_\mathrm{p}$. In the case of Porphyrion, these expressions concretise to $\mathbb{E}[L \mid L_\mathrm{p} = l_\mathrm{p}](\xi = -3) = 7.58 \pm 0.06$ Mpc and $\mathbb{E}[L \mid L_\mathrm{p} = l_\mathrm{p}](\xi = -4) = 7.28 \pm 0.06$ Mpc.

By conditioning L on more knowledge than a value for L_p alone, statistical deprojection could be made more precise. For example, one could additionally condition on the fact that Porphyrion is generated by a Type 2 radiatively efficient (RE) AGN. If Type 1 RE AGN are seen mostly face-on and Type 2 RE AGN are seen mostly edge-on, as proposed by the unification model [e.g. 27], then the detection of a Type 2 RE AGN would imply that the jets make a small angle with the sky plane. Extending the formulae to include this knowledge is beyond the scope of this work; however, mindful of the associated deprojection factor–reducing effect, we choose $\xi = -4$ as our fiducial tail index.

To assess Porphyrion's transport capabilities in a cosmological context, it is instructive to calculate its length relative to Cosmic Web length scales. In particular, the outflow's total length relative to the typical cosmic void radius at its epoch is $f_\mathrm{v} \coloneqq l(1+z)R_\mathrm{v}^{-1}$, where R_v is the typical comoving cosmic void radius. For $l = 6.8^{+1.2}_{-0.3}$ Mpc, $z = 0.896 \pm 0.001$, and $R_\mathrm{v} = 20$ Mpc [13], we find $f_\mathrm{v} = 64^{+12}_{-2}$ %. For our fiducial total length $l = 7$ Mpc, we find $f_\mathrm{v} = 66\%$.

Void penetration probability

Porphyrion's orientation relative to its native Cosmic Web filament is currently unknown. We calculate the probability that an outflow breaches its filament, thus penetrating the surrounding voids, by assuming that jet orientations are independent from filament orientations. We furthermore assume that the jets are straight and of equal length, that the filament is of cylindrical shape, and that the host galaxy resides at the filament's spine, where the gravitational potential is lowest. The RV Θ_f denotes the angle between the jet axis and the filament axis, whilst the constants R_f and $D_\mathrm{f} \coloneqq 2R_\mathrm{f}$ denote the filament radius and diameter, respectively. An outflow of total length l penetrates voids with probability

$$\begin{aligned}\mathbb{P}\left(L \sin\Theta_\mathrm{f} > D_\mathrm{f} \mid L = l\right) &= \mathbb{P}\left(\sin\Theta_\mathrm{f} > \frac{D_\mathrm{f}}{l}\right) \\ &= 1 - F_{\sin\Theta_\mathrm{f}}\left(\frac{D_\mathrm{f}}{l}\right). \end{aligned} \tag{A4}$$

The RV Θ_f has support on the interval $0 < \theta \leq \frac{\pi}{2}$. On this interval, the CDF is

$$\begin{aligned} F_{\Theta_\mathrm{f}}(\theta) \coloneqq \mathbb{P}(\Theta_\mathrm{f} \leq \theta) &= \frac{1}{2\pi}\int_0^\theta \int_0^{2\pi} \sin\theta' \, \mathrm{d}\varphi \, \mathrm{d}\theta' \\ &= 1 - \cos\theta. \end{aligned} \tag{A5}$$

The RV $\sin\Theta_\mathrm{f}$ has support on the interval $0 < y \leq 1$. On this interval, the CDF is

$$\begin{aligned} F_{\sin\Theta_\mathrm{f}}(y) &\coloneqq \mathbb{P}\left(\sin\Theta_\mathrm{f} \leq y\right) = \mathbb{P}\left(\Theta_\mathrm{f} \leq \arcsin y\right) \\ &= 1 - \cos\arcsin y = 1 - \sqrt{1-y^2}, \end{aligned} \tag{A6}$$

where we use the fact that the arcsine function is monotonically increasing. By combining Eqs. A4 and A6, we obtain

$$\mathbb{P}\left(L\sin\Theta_\mathrm{f} > D_\mathrm{f} \mid L = l\right) = \begin{cases} 0 & \text{if } l \leq D_\mathrm{f}; \\ \sqrt{1 - \left(\frac{D_\mathrm{f}}{l}\right)^2} & \text{if } l > D_\mathrm{f}. \end{cases} \tag{A7}$$

Typically, however, we only know an outflow's *projected* length — not its *total* length. The quantity of highest practical interest therefore is

$$\begin{aligned} p_\mathrm{v} &\coloneqq \mathbb{P}\left(L\sin\Theta_\mathrm{f} > D_\mathrm{f} \mid L_\mathrm{p} = l_\mathrm{p}\right) \\ &= \mathbb{P}\left(L_\mathrm{p}\frac{\sin\Theta_\mathrm{f}}{\sin\Theta_\mathrm{o}} > D_\mathrm{f} \mid L_\mathrm{p} = l_\mathrm{p}\right) \\ &= \mathbb{P}\left(\frac{\sin\Theta_\mathrm{f}}{\sin\Theta_\mathrm{o}} > \frac{D_\mathrm{f}}{l_\mathrm{p}}\right) = 1 - F_X\left(\frac{D_\mathrm{f}}{l_\mathrm{p}}\right), \end{aligned} \tag{A8}$$

where the RV Θ_o is the angle between the jet axis and the line of sight, and the RV X is the ratio of the RVs $\sin\Theta_\mathrm{f}$ and $\sin\Theta_\mathrm{o}$. These latter RVs are independent and identically distributed: $F_{\sin\Theta_\mathrm{f}}(y) = F_{\sin\Theta_\mathrm{o}}(y)$. We derive the PDF f_X by using the standard formula for the ratio distribution PDF (for independent RVs). This formula demands the determination of $f_{\sin\Theta_\mathrm{f}}(y) = f_{\sin\Theta_\mathrm{o}}(y)$. From Eq. A6, we find that on the interval $0 < y \leq 1$, the PDF is

$$f_{\sin\Theta_\mathrm{f}}(y) = \frac{\mathrm{d}}{\mathrm{d}y}F_{\sin\Theta_\mathrm{f}}(y) = \frac{y}{\sqrt{1-y^2}}. \tag{A9}$$

To find the distribution of X, it is helpful to distinguish three intervals. For $x \leq 0$, $f_X(x) = 0$, because X is the ratio of two positive RVs. Then, for $0 < x < 1$,

$$f_X(x) = x\int_0^1 \frac{y^3}{\sqrt{1-x^2y^2}\sqrt{1-y^2}}\,\mathrm{d}y, \tag{A10}$$

while for $x > 1$,

$$f_X(x) = x\int_0^{\frac{1}{x}} \frac{y^3}{\sqrt{1-x^2y^2}\sqrt{1-y^2}}\,\mathrm{d}y. \tag{A11}$$

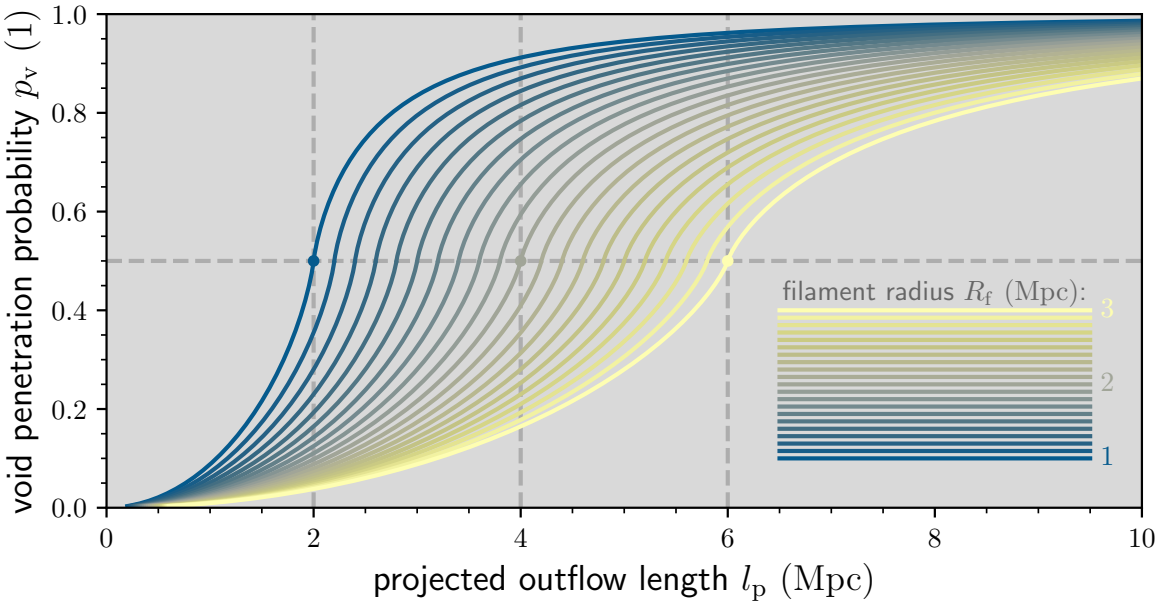

Fig. A1: **From its projected length and an estimated radius for its native filament, one can calculate the probability that an outflow penetrates the surrounding voids.** Outflows with projected lengths equalling the diameters of their filaments penetrate voids with 50% probability (as the dots exemplify).

Solving the integrals leads to

$$f_X(x) = \begin{cases} 0 & \text{if } x \leq 0; \\ \frac{(x^2+1)\ln\frac{1+x}{1-x} - 2x}{4x^2} & \text{if } 0 < x < 1; \\ \frac{(x^2+1)\ln\frac{x+1}{x-1} - 2x}{4x^2} & \text{if } x > 1. \end{cases} \tag{A12}$$

At $x = 1$, f_X is undefined. We calculate F_X through integration, yielding

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq 0; \\ \frac{1}{2} + \frac{1}{4}\left(\left(x + 1 - \frac{1}{x}\right)\ln\frac{1+x}{1-x} + \ln\frac{(1-x)^2}{1-x^2}\right) & \text{if } 0 < x < 1; \\ \frac{1}{2} + \frac{1}{4}\frac{x^2-1}{x}\ln\frac{x+1}{x-1} & \text{if } x > 1. \end{cases} \tag{A13}$$

F_X is continuously extendable at $x = 1$ by defining $F_X(1) := \frac{1}{2}$. Finally, we define $x := \frac{D_\mathrm{f}}{l_\mathrm{p}}$, so that $p_\mathrm{v}(x) = 1 - F_X(x)$:

$$p_\mathrm{v}(x) = \begin{cases} \frac{1}{2} - \frac{1}{4}\left(\left(x + 1 - \frac{1}{x}\right)\ln\frac{1+x}{1-x} + \ln\frac{(1-x)^2}{1-x^2}\right) & \text{if } 0 < x < 1; \\ \frac{1}{2} & \text{if } x = 1; \\ \frac{1}{2} - \frac{1}{4}\frac{x^2-1}{x}\ln\frac{x+1}{x-1} & \text{if } x > 1. \end{cases} \tag{A14}$$

The median of X equals unity. Therefore, half of the outflows with projected lengths $l_\mathrm{p} = D_\mathrm{f}$ penetrate voids. For outflows with larger projected lengths, void penetration is more likely than not. Figure A1 shows void penetration probabilities for physically relevant parameter ranges.

For Porphyrion, we take l_p as before and adopt a filament (core) radius $R_\mathrm{f} = 1.2$ Mpc [68]; this yields $x = 0.373 \pm 0.003$ and therefore $p_\mathrm{v} = 95.2 \pm 0.1\%$.

Filament shape modification

We predict that powerful, long-lived outflows like Porphyrion cause their host galaxies' filaments to expand thermally. Through lateral shocks, the jets distribute an amount of heat Q over the warm–hot IGM. This medium is sufficiently dilute that plasma interactions can be neglected; as a result, the ideal gas law, $pV = Nk_\mathrm{B}T$, may be adopted as the equation of state. Here, p, V, N, and T are the filament's pressure, volume, plasma particle number, and temperature, respectively; k_B is Boltzmann's constant. Assuming a thermodynamic process at constant pressure and particle number, the work W is

$$W = p\Delta V = Nk_\mathrm{B}\Delta T. \tag{A15}$$

Before the outflow's emergence, the filament's equation of state is $pV_\mathrm{i} = Nk_\mathrm{B}T_\mathrm{i}$, where V_i and T_i are its initial volume and temperature, respectively. Upon dividing Eq. A15 by this equation of state, one obtains

$$\frac{\Delta V}{V_\mathrm{i}} = \frac{\Delta T}{T_\mathrm{i}}. \tag{A16}$$

Assuming that the filament retains a cylindrical shape, initially with radius r_i and finally with radius r_f, and using that $\Delta V \coloneqq V_\mathrm{f} - V_\mathrm{i}$, one obtains

$$\frac{r_\mathrm{f}}{r_\mathrm{i}} = \sqrt{1 + \frac{\Delta T}{T_\mathrm{i}}}. \tag{A17}$$

The radius ratio, $\frac{r_\mathrm{f}}{r_\mathrm{i}}$, depends only on the ratio between the temperature increase $\Delta T \coloneqq T_\mathrm{f} - T_\mathrm{i}$ and the initial temperature. The temperature increase is

$$\Delta T = \frac{Q}{NC_\mathrm{p,m}}, \tag{A18}$$

where $C_\mathrm{p,m}$ is the molar heat capacity at constant pressure. For a monatomic gas or a hydrogen plasma, $C_\mathrm{p,m} = \frac{5}{2}R$, where R is the molar gas constant. The number of filamentary electrons and atomic nuclei

affected by the outflow is

$$N = \frac{\pi r_{\mathrm{i}}^2 L \rho_{\mathrm{i}}}{\mu m_{\mathrm{p}}}, \tag{A19}$$

where L is the length of the cylindrical segment affected, ρ_{i} is the initial baryonic mass density, μ is the average mass of a plasma particle relative to the proton mass, and m_{p} is the proton mass. We estimate $\frac{L}{2}$ by multiplying the typical speed of lateral shocks with the outflow's lifetime. We decompose $\rho_{\mathrm{i}} = \rho_{\mathrm{c},0}\Omega_{\mathrm{BM},0}(1+z)^3(1+\delta)$, where z and δ are the filament's cosmological redshift and baryonic overdensity, respectively.

We assess the outflow-induced morphological change to Porphyrion's filament by evaluating Eq. A17, taking $Q = 10^{55}$ J, $r_{\mathrm{i}} = 1.2$ Mpc, $L = 7$ Mpc, $z = 0.9$, $1+\delta = 10$, $\mu = 0.5$, and $T_{\mathrm{i}} = 10^7$ K; we find $\Delta T = 9 \cdot 10^6$ K and $r_{\mathrm{f}} = 1.7$ Mpc (an increase of ∼40%). Porphyrion's heat dissipation renders the outflow's native filament much hotter and thicker than it would have otherwise been.

Transport of heavy elements to voids

Outflows from RE AGN could contain more heavy atoms than outflows from RI AGN: RE AGN tend to reside in galaxies with higher star formation rates and thus more vigorous stellar winds, suggesting increased entrainment of wind-borne atomic nuclei into jets [77]. The order-of-magnitude calculations presented here, to be verified by future simulations, indicate that Mpc-scale outflows could supply Mpc^3-scale volumes in cosmic voids with significant heavy atoms, though consistent with upper limits [65].

We calculated the heavy element enrichment of the IGM in voids due to the deposition of atomic nuclei initially entrained in the jets of Mpc-scale outflows. In particular, we estimated the final metallicity in the deposition region considering both internal and external entrainment. Internal entrainment refers to the entrainment into jets of atomic nuclei from stellar winds, *internal* to the host galaxy. External entrainment refers to the entrainment into jets of atomic nuclei dwelling in the IGM, *external* to the host galaxy.

Denoting the (two-sided) internal mass entrainment rate by $\dot{M}_{\mathrm{i}}$, which we assumed constant through time, the total internally entrained mass by an outflow of age T is

$$M_{\mathrm{i}} = \dot{M}_{\mathrm{i}} T. \tag{A20}$$

The internal mass entrainment rates of Centaurus A and 3C 31, both Fanaroff–Riley I (FR I) outflows in the Local Universe, are estimated to be $\dot{M}_{\mathrm{i}} = 2 \cdot 10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$ [77] and $\dot{M}_{\mathrm{i}} = 8 \cdot 10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$ [35], respectively. At $M_\star = 6.3 \cdot 10^{11}\ M_\odot$, 3C 31's host stellar mass is similar to Porphyrion's. However, Porphyrion is a Fanaroff–Riley II (FR II) outflow, suggesting a smaller jet opening angle ω and thus a

smaller $\dot{M}_\mathrm{i}$. If ω is a factor of order unity smaller for FR II outflows than for FR I outflows [e.g. 36, 59], and $\dot{M}_\mathrm{i} \propto \omega^2$, then $\dot{M}_\mathrm{FR\,II} \sim 10^{-1}\dot{M}_\mathrm{FR\,I}$. On the other hand, Porphyrion's host is seen much closer to the cosmic heyday of star formation, suggesting a larger SFR and thus a *larger* $\dot{M}_\mathrm{i}$. After $z = 1.9$, SFRs S typically decayed exponentially with an e-folding time of 3.9 Gyr [e.g. 40]. If $\dot{M}_\mathrm{i} \propto S$, then $\dot{M}_\mathrm{i}(z = z_\mathrm{P}) \sim e^2\dot{M}_\mathrm{i}(z = 0) \sim 10^1\dot{M}_\mathrm{i}(z = 0)$, where z_P is Porphyrion's redshift. Assuming that both effects are indeed of comparable importance, we provisionally adopted 3C 31's $\dot{M}_\mathrm{i} = 8 \cdot 10^{-3}\ M_\odot\ \mathrm{yr}^{-1}$ as our fiducial value. Taking T as before, we obtained $M_\mathrm{i} = 2 \cdot 10^7\ M_\odot$. The total externally entrained mass is

$$M_\mathrm{e} = 2\int_0^{\frac{l}{2}} \rho_\mathrm{e}(r)A_\mathrm{e}(r)\ \mathrm{d}r, \tag{A21}$$

where $A_\mathrm{e}(r)$ is the entrainment cross-section at a distance r from the AGN. Perhaps the simplest approach is to parametrise $A_\mathrm{e} =: \pi R_\mathrm{e}^2$, where R_e is a (constant) effective radius defined such that all baryons closer to the jet axis than R_e are entrained. Taking l and ρ_e as before, and $R_\mathrm{e} = 1$ kpc [e.g. 55], we obtained $M_\mathrm{e} = 4 \cdot 10^7\ M_\odot$. Although highly uncertain, these estimates suggest that M_i and M_e can be of the same order of magnitude.

The total internally entrained mass in heavy elements is $Z_\mathrm{i}M_\mathrm{i}$, where Z_i is the mass-weighted mean metallicity of the galaxy's stellar winds. The total externally entrained mass in heavy elements is $Z_\mathrm{e}M_\mathrm{e}$, where Z_e is the mass-weighted metallicity of the IGM along the jet. Assuming that the IGM in the voids is initially pristine, its final metallicity is

$$Z_\mathrm{v} = \frac{Z_\mathrm{i}M_\mathrm{i} + Z_\mathrm{e}M_\mathrm{e}}{M_\mathrm{i} + M_\mathrm{e} + \rho_\mathrm{v}V}, \tag{A22}$$

where ρ_v is the baryon mass density within a deposition region of volume V. Taking a spherical deposition region with a diameter of 1 Mpc, and ρ_v as before, we obtained $\rho_\mathrm{v}V = 7 \cdot 10^9\ M_\odot$. Assuming $Z_\mathrm{i} = Z_\odot$ [e.g. 77] and $Z_\mathrm{e} = 10^{-1}\ Z_\odot$ [e.g. 42], we found $Z_\mathrm{v} = 3 \cdot 10^{-3}\ Z_\odot$. In conclusion, order-of-magnitude arguments suggest that void-penetrating Mpc-scale outflows can endow the local IGM with metallicities $Z_\mathrm{v} \sim 10^{-3}$–$10^{-2}\ Z_\odot$.

Quasar mass–based host galaxy candidate elimination

SDSS J152933.03+601552.5 is the quasar-hosting galaxy 19″ north-northeast of J152932.16+601534.4, the galaxy we have identified as Porphyrion's host. We initially also considered SDSS J152933.03+601552.5 as a host galaxy candidate. However, aforementioned arguments involving the presence of jets and their orientation and, to a lesser degree, arguments involving core radio luminosity and core synchrotron self-absorption all favour J152932.16+601534.4. We now discuss how our results would

change if, instead, SDSS J152933.03+601552.5 were Porphyrion's host galaxy. Doing so will lead to a contradiction that disproves this alternative hypothesis.

First, we discuss results that do not require dynamical modelling. To start with, Porphyrion would remain generated by an RE AGN. The host galaxy redshift would decrease from $z = 0.896 \pm 0.001$ to $z = 0.799 \pm 0.001$, decreasing Porphyrion's projected length from $l_\mathrm{p} = 6.43 \pm 0.05$ Mpc to $l_\mathrm{p} = 6.21 \pm 0.05$ Mpc. Again using $\xi = -4$, the total length would decrease from $l = 6.8^{+1.2}_{-0.3}$ Mpc to $l = 6.5^{+1.2}_{-0.3}$ Mpc and its conditional expectation from $\mathbb{E}[L \mid L_\mathrm{p} = l_\mathrm{p}] = 7.28 \pm 0.06$ Mpc to $\mathbb{E}[L \mid L_\mathrm{p} = l_\mathrm{p}] = 7.03 \pm 0.06$ Mpc. If orientation distinguishes Type 1 from Type 2 RE AGN, as the unification model supposes, then these statistical deprojection results may underestimate Porphyrion's total length. Porphyrion would remain the projectively largest galaxy-made structure identified so far. Porphyrion's total radio luminosity at rest-frame wavelength $\lambda_\mathrm{r} = 2$ m would decrease from $L_\nu = 2.8 \pm 0.3 \cdot 10^{26}$ W Hz^{-1} to $L_\nu = 2.2 \pm 0.2 \cdot 10^{26}$ W Hz^{-1}.

Next, we discuss results that come from dynamical modelling. The jet power would decrease from $Q = 1.3 \pm 0.1 \cdot 10^{39}$ W to $Q = 1.0 \pm 0.1 \cdot 10^{39}$ W, while the age would slightly increase from $T = 1.9^{+0.7}_{-0.2}$ Gyr to $T = 1.9^{+0.7}_{-0.1}$ Gyr.[6] The transported energy would decrease from $E = 7.6^{+2.1}_{-0.7} \cdot 10^{55}$ J to $E = 6.4^{+1.8}_{-0.6} \cdot 10^{55}$ J, and the black hole mass gain from $\Delta M_\bullet > 8.5^{+2.4}_{-0.8} \cdot 10^8\ M_\odot$ to $\Delta M_\bullet > 7.2^{+2.0}_{-0.7} \cdot 10^8\ M_\odot$.

Finally, we arrive at a contradiction, as the quasar's SMBH mass (measured from its SDSS BOSS spectrum) $M_\bullet = 2.5 \pm 0.3 \cdot 10^8\ M_\odot$ [11]. This mass is lower than the minimum mass gain associated to the fuelling of Porphyrion's jets. Thus, assuming that SDSS J152933.03+601552.5 is the outflow's host galaxy leads to a contradiction. This argument reaffirms that J152932.16+601534.4 is Porphyrion's host.

Diffusion of lobe plasma through voids

When cosmic rays move through the jumbled magnetic fields of galaxy clusters and filaments of the Cosmic Web, the Lorentz force scatters them repeatedly. The mean free path of the ensuing random walk is so short that the CRs radiate away their energy before they are able to travel a cosmologically significant distance [e.g. 6]. Clusters and filaments thus effectively lock into place the CRs that are injected into them. By contrast, magnetic fields with Mpc-scale coherence lengths in voids are orders of magnitude weaker than those in clusters and filaments [e.g. 10], and as a result, CRs that are released into voids might diffuse through their entirety within a few gigayears. Void-filling diffusion of CRs might be especially rapid at early epochs: during Porphyrion's lifetime, for instance, the proper volumes of voids were on average an order of magnitude smaller than they are today.

Consider a void region filled with relativistic particles, so that their velocity components obey

$$v_x^2 + v_y^2 + v_z^2 \approx c^2. \tag{A23}$$

[6] Significant jet-mediated transport of heavy elements to the IGM would remain plausible. The host's stellar mass would decrease from $M_\star = 6.7 \pm 1.4 \cdot 10^{11}\ M_\odot$ to $M_\star = 4.0^{+0.3}_{-0.3} \cdot 10^{11}\ M_\odot$, while the SFR would become $S = 4.9^{+0.3}_{-0.4} \cdot 10^1\ M_\odot\ \mathrm{yr}^{-1}$ [3].

We treat v_x, v_y, and v_z as random variables subject to the above constraint. If the particles have no bulk motion, and move in all directions with equal probability density,

$$\mathbb{E}[v_x^N] = \mathbb{E}[v_y^N] = \mathbb{E}[v_z^N] \tag{A24}$$

for any $N \in \mathbb{R}$. In particular, given the absence of bulk motion, $\mathbb{E}[v_x] = \mathbb{E}[v_y] = \mathbb{E}[v_z] = 0$. By taking expectations on both sides of Eq. A23, using the linearity of expectation, and invoking Eq. A24, we find

$$\mathbb{E}[v_x^2] = \mathbb{E}[v_y^2] = \mathbb{E}[v_z^2] = \frac{c^2}{3}. \tag{A25}$$

Without loss of generality, we assume the region's magnetic field $\vec{B}$ to be oriented along the z-axis. The speed perpendicular to $\vec{B}$ is $v_\perp = \sqrt{v_x^2 + v_y^2}$, so that, upon invoking Eq. A25, we find $\mathbb{E}[v_\perp^2] = \frac{2}{3}c^2$. A typical speed for relativistic particles perpendicular to a magnetic field thus is

$$\sqrt{\mathbb{E}[v_\perp^2]} = \sqrt{\frac{2}{3}}c \approx 0.8165\ c. \tag{A26}$$

Starting from Fick's first law of diffusion, and solving the case of Brownian motion in three dimensions, one obtains

$$r = \sqrt{6Dt}, \tag{A27}$$

where r is the typical proper distance to the particles' origin after a time t. To find the diffusion coefficient D, we consider Bohm diffusion, in which charged particles diffuse through a turbulent magnetic field as a result of the Lorentz force. Whereas predicting the trajectory of any single charged particle requires knowledge of the specific magnetic field structure in its surroundings, the statistical properties of Bohm diffusion are determined solely by the statistical properties of the magnetic field and the charge and energy of the diffusing particles. The Larmor radius for a particle with Lorentz factor γ, total velocity v, rest mass m, and charge q, is

$$r_\mathrm{L} = \frac{\gamma(v) m v_\perp}{|q|B}. \tag{A28}$$

For a relativistic particle whose $v_\perp$ is given by Eq. A26, we obtain a Larmor radius

$$r_\mathrm{L}(E) = \sqrt{\frac{2}{3}}\frac{E}{c|q|B} = 8.8 \cdot 10^2\ \mathrm{pc} \cdot \frac{E}{1\ \mathrm{GeV}} \cdot \frac{10^{-15}\ \mathrm{G}}{B}, \tag{A29}$$

where E is the total (i.e. rest plus kinetic) energy of the particle. The diffusion coefficient for charged particles in a magnetic field with a Kolmogorov turbulence spectrum is well approximated [21] by

$$D(E) \approx D_{\mathrm{Bohm}}(E_0) \left(\frac{E}{E_0}\right)^{\frac{1}{3}} + D_{\mathrm{Bohm}}(E_1) \left(\frac{E}{E_1}\right)^{2}. \tag{A30}$$

Here, E_0 is the energy for which the circumference of gyration equals the magnetic field coherence length λ_{c}:

$$2\pi r_{\mathrm{L}}(E_0) = \lambda_{\mathrm{c}}. \tag{A31}$$

For $q = \pm e$, where e is the elementary charge, this equation implies that

$$E_0 = \sqrt{\frac{3}{2}} \frac{\lambda_{\mathrm{c}} c |q| B}{2\pi} = 1.8 \cdot 10^2 \text{ GeV} \cdot \frac{\lambda_{\mathrm{c}}}{1 \text{ Mpc}} \cdot \frac{B}{10^{-15} \text{ G}}. \tag{A32}$$

Furthermore, $E_1 = \frac{3}{2} E_0$. The Bohm diffusion coefficient [e.g. 21] D_{Bohm} is

$$D_{\mathrm{Bohm}}(E) = \frac{c}{3} r_{\mathrm{L}}(E) \tag{A33}$$

$$= 9.0 \cdot 10^{-2} \frac{\mathrm{Mpc}^2}{\mathrm{Gyr}} \cdot \frac{E}{1 \text{ GeV}} \cdot \frac{10^{-15} \text{ G}}{B}. \tag{A34}$$

We note that

$$D_{\mathrm{Bohm}}(E_0) = \frac{c}{6\pi} \lambda_{\mathrm{c}} = 1.6 \cdot 10^1 \frac{\mathrm{Mpc}^2}{\mathrm{Gyr}} \cdot \frac{\lambda_{\mathrm{c}}}{1 \text{ Mpc}} \tag{A35}$$

is independent of the void's magnetic field strength. Because $D_{\mathrm{Bohm}} \propto r_{\mathrm{L}} \propto E$, we have $D_{\mathrm{Bohm}}(E_1) = \frac{3}{2} D_{\mathrm{Bohm}}(E_0)$.

For $E = 1$ GeV, $\lambda_{\mathrm{c}} = 1$ Mpc, and $B = 10^{-15}$ G, we find $D(E) = 2.9 \, \frac{\mathrm{Mpc}^2}{\mathrm{Gyr}}$. After $t = 1$ Gyr, the typical displacement of cosmic rays that escaped from the outflow's lobes is $r = 4.2$ Mpc. We note that r scales slowly with particle energy, magnetic field strength, and (to a lesser degree) with coherence length:

$$r \propto E^{\frac{1}{6}} B^{-\frac{1}{6}} \lambda_{\mathrm{c}}^{\frac{1}{3}} t^{\frac{1}{2}}. \tag{A36}$$

For short time intervals t, we can ignore the expansion of the Universe; defining $r_\mathrm{c} \coloneqq r(1+z)$, the void volume-filling fraction $\mathcal{V}$ of a single lobe becomes

$$\mathcal{V} = \left(\frac{2r_\mathrm{c}}{D_\mathrm{c}}\right)^3. \tag{A37}$$

For sufficiently short time intervals t, particles move in rectilinear fashion, and the typical proper displacement of a relativistic particle within t is $r = ct$, not $r = \sqrt{6Dt}$. Diffusion can only possibly provide an accurate description of the typical displacement for sufficiently large t. As superluminal motion is impossible,

$$\sqrt{6Dt} < ct, \text{ or } t > \frac{6D}{c^2} =: \tau_\mathrm{d}, \tag{A38}$$

where τ_d is the diffusion timescale (as in Globus et al. [21], but with a factor 6 instead of 4). Diffusion only has a role to play in the description of particle movement through voids when $\tau_\mathrm{d} < \tau_\mathrm{b}$, the ballistic timescale for particle movement through voids. We define

$$\tau_\mathrm{b} \coloneqq \frac{R_\mathrm{c}}{(1+z)c}, \tag{A39}$$

where R_c is the comoving void radius. As $\tau_\mathrm{d} \propto D$, there is a maximum diffusion coefficient, D_max, above which the diffusive description is invalid. Solving $\tau_\mathrm{d}(D_\mathrm{max}) = \tau_\mathrm{b}$ for D_max, we obtain

$$D_\mathrm{max} = \frac{R_\mathrm{c} c}{6(1+z)}. \tag{A40}$$

This maximum diffusion coefficient corresponds to a minimum magnetic field strength, B_min. Approximating $D_\mathrm{max} \approx D_\mathrm{Bohm}(E, B_\mathrm{min})$, we find

$$B_\mathrm{min} = 2\sqrt{\frac{2}{3}}\frac{(1+z)E}{c|q|R_\mathrm{c}} \tag{A41}$$

$$= 8.8 \cdot 10^{-20}\ \mathrm{G} \cdot \frac{E}{1\ \mathrm{GeV}} \cdot \frac{20\ \mathrm{Mpc}}{R_\mathrm{c}} \cdot \frac{1+z}{1} \cdot \frac{e}{|q|}. \tag{A42}$$

We should only apply diffusion theory to the problem of particle movement through voids for void magnetic field strengths $B \gg B_\mathrm{min}$. For particle energies of 1 GeV, we therefore only consider diffusion for $B \gtrsim 10^{-18}$ G.

The diffusing cosmic rays lose energy over time. In voids, losses by inverse Compton scattering to CMB photons are by far more important than losses by synchrotron radiation, because

$$\frac{P_{\mathrm{IC,CMB}}}{P_{\mathrm{s}}} = \frac{B_{\mathrm{CMB}}^2(z)}{B^2}, \tag{A43}$$

and $B_{\mathrm{CMB}}^2(z) \gg B^2$ in voids. (Here, P_{IC} and P_{s} are, respectively, the inverse Compton and synchrotron powers of a single cosmic ray.) The inverse Compton loss timescale for an electron or positron of total energy E at cosmological redshift z is

$$\begin{aligned}
\tau_{\mathrm{IC,CMB}}(E,z) \coloneqq& \frac{E}{P_{\mathrm{IC,CMB}}(E,z)} && \text{(A44)}\\
=& \frac{6m_e^2c^3\mu_0}{4\beta^2 E\sigma_{\mathrm{T}}B_{\mathrm{CMB}}^2(0)(1+z)^4} && \text{(A45)}\\
=& 1.2\ \mathrm{Gyr}\cdot\frac{1}{\beta^2}\cdot\frac{1\ \mathrm{GeV}}{E}\cdot\frac{1}{(1+z)^4}, && \text{(A46)}
\end{aligned}$$

where σ_{T} is the Thomson cross-section for electrons and positrons. For protons, the inverse Compton loss timescale equals the above multiplied by a factor $\left(\frac{m_p}{m_e}\right)^4 \approx 1.1{\cdot}10^{13}$. Therefore, for non–ultra-high-energy cosmic ray protons, both synchrotron *and* inverse Compton losses are negligible.

Because $\frac{\mathrm{d}D}{\mathrm{d}E} > 0$, diffusion slows down as particles lose energy; in other words, a particle's *highest* diffusion coefficient is its *initial* diffusion coefficient.

Let X_{c} be the comoving displacement along the x-direction. We consider N time steps, each of length $\tau \coloneqq \frac{t}{N}$. Let $X_{\mathrm{c},i}$ be the comoving displacement along the x-direction achieved in the i-th time step, and let X_i be the corresponding proper displacement.

$$X_{\mathrm{c}} \coloneqq \sum_{i=1}^{N} X_{\mathrm{c},i} \tag{A47}$$

Because $\mathbb{E}[X_{\mathrm{c},i}] = 0$, $\mathbb{E}[X_{\mathrm{c}}] = \sum_{i=1}^{N}\mathbb{E}[X_{\mathrm{c},i}] = 0$. Therefore

$$\mathbb{E}[X_{\mathrm{c}}^2] = \mathbb{V}[X_{\mathrm{c}}] = \sum_{i=1}^{N}\mathbb{V}[X_{\mathrm{c},i}] = \sum_{i=1}^{N}\mathbb{E}[X_{\mathrm{c},i}^2]. \tag{A48}$$

Because $X_{\mathrm{c},i} = (1+z_i)X_i$, $\mathbb{E}[X_{\mathrm{c},i}^2] = (1+z_i)^2\mathbb{E}[X_i^2]$. Therefore

$$\mathbb{E}[X_{\mathrm{c}}^2] = \sum_{i=1}^{N}(1+z_i)^2\frac{\mathbb{E}[X_i^2]}{2\tau}\cdot 2\tau. \tag{A49}$$

Following Einstein's definition of the diffusion coefficient, the proper diffusion coefficient in the x-direction for the i-th time step, $D_{x,i}$, is

$$D_{x,i} \coloneqq \frac{\mathbb{E}[X_i^2]}{2\tau}. \tag{A50}$$

We can then write

$$\mathbb{E}[X_\mathrm{c}^2] = 2\tau \sum_{i=1}^{N} (1+z_i)^2 D_{x,i}. \tag{A51}$$

Proceeding analogously for the y- and z-directions, and defining $R_\mathrm{c}^2 \coloneqq X_\mathrm{c}^2 + Y_\mathrm{c}^2 + Z_\mathrm{c}^2$, we find

$$\mathbb{E}[R_\mathrm{c}^2] = \mathbb{E}[X_\mathrm{c}^2] + \mathbb{E}[Y_\mathrm{c}^2] + \mathbb{E}[Z_\mathrm{c}^2] \tag{A52}$$

$$= 2\tau \sum_{i=1}^{N} (1+z_i)^2 (D_{x,i} + D_{y,i} + D_{z,i}). \tag{A53}$$

In the isotropic case, $D_{x,i} = D_{y,i} = D_{z,i} =: D_i$, so that

$$\mathbb{E}[R_\mathrm{c}^2] = 6\tau \sum_{i=1}^{N} (1+z_i)^2 D_i = 6t \cdot \frac{1}{N} \sum_{i=1}^{N} (1+z_i)^2 D_i. \tag{A54}$$

Denoting the (time) average of a function $f(t)$ by $\langle f \rangle$, we have

$$r_\mathrm{c} \coloneqq \sqrt{\mathbb{E}[R_\mathrm{c}^2]} = \sqrt{6\langle (1+z)^2 D \rangle t}. \tag{A55}$$

Let E be an RV denoting particle energy, and let f_E be its PDF. Let n_l be the lobe particle number density, and let R_l be the lobe radius. The number of particles in an outer shell with thickness ΔR with energies between E and $E + \mathrm{d}E$ is

$$\mathrm{d}N(E) = 4\pi R_\mathrm{l}^2 \Delta R \cdot n_\mathrm{l} f_E(E) \mathrm{d}E. \tag{A56}$$

These particles escape from the shell over a timescale

$$\tau_\mathrm{e}(E) = \frac{\Delta R^2}{2D_{\perp,\mathrm{c}}(E)}, \tag{A57}$$

where $D_{\perp,\mathrm{c}}$ is the compound cross-field diffusion coefficient [19]. The number of particles with energies between E and $E + \mathrm{d}E$ escaping from the shell per unit of time thus is

$$\frac{\mathrm{d}N(E)}{\tau_{\mathrm{e}}(E)} = \frac{8\pi R_{\mathrm{l}}^2 \cdot D_{\perp,\mathrm{c}}(E) \cdot n_{\mathrm{l}} f_E(E)\mathrm{d}E}{\Delta R}. \tag{A58}$$

The particulate energy escaping from the shell per unit of time and unit of energy, which we shall call the power density P_E, is

$$P_E(E) \coloneqq E \cdot \frac{\mathrm{d}N(E)}{\tau_{\mathrm{e}}(E)\mathrm{d}E} = \frac{8\pi R_{\mathrm{l}}^2 \cdot D_{\perp,\mathrm{c}}(E) \cdot n_{\mathrm{l}} f_E(E) E}{\Delta R}. \tag{A59}$$

Finally, the total power P is

$$P \coloneqq \int_E P_E(E)\ \mathrm{d}E = \frac{8\pi R_{\mathrm{l}}^2 n_{\mathrm{l}}}{\Delta R} \int_E D_{\perp,\mathrm{c}}(E) E f_E(E)\ \mathrm{d}E \tag{A60}$$

$$= \frac{8\pi R_{\mathrm{l}}^2 n_{\mathrm{l}}}{\Delta R} \mathbb{E}_E[D_{\perp,\mathrm{c}}(E)E]. \tag{A61}$$

The compound cross-field diffusion coefficient is [17]

$$D_{\perp,\mathrm{c}}(E) \approx D_{\perp}(E) \left(1 + \frac{\Lambda^2(E)}{\ln \Lambda(E)}\right), \tag{A62}$$

where $D_{\perp}$ is the cross-field diffusion coefficient, given by

$$D_{\perp}(E) \approx \frac{c}{3} r_L(E) \delta_B(r_L(E)). \tag{A63}$$

Here

$$\delta_B(l) \approx f_{\mathrm{i}} \cdot \left(\frac{l}{l_{\mathrm{i}}}\right)^{\frac{2}{3}}, \tag{A64}$$

where l_{i} is the turbulence injection scale and f_{i} is the total turbulence energy density up to this scale, relative to the energy density of the thermal medium surrounding the lobe [19]. Additionally,

$$\Lambda(E) = \frac{1}{\sqrt{2}} \frac{\delta_B(\lambda_{\mathrm{c,l}})}{\delta_B(r_{\mathrm{L}}(E))} = \frac{1}{\sqrt{2}} \left(\frac{\lambda_{\mathrm{c,l}}}{r_{\mathrm{L}}(E)}\right)^{\frac{2}{3}}, \tag{A65}$$

where $\lambda_{\mathrm{c,l}}$ is the lobe's magnetic field correlation length. We assume that E has a Pareto distribution [e.g. 73], so that its PDF, f_E, (for $p \neq -1$) is given by

$$f_E(E) = \begin{cases} \frac{p+1}{E_{\max}^{p+1} - E_{\min}^{p+1}} E^p & \text{if } E_{\min} < E < E_{\max}; \\ 0 & \text{otherwise;} \end{cases} \tag{A66}$$

and $E_{\min} \coloneqq \gamma_{\min} mc^2$ and $E_{\max} \coloneqq \gamma_{\max} mc^2$. We calculated the total power assuming $R_{\mathrm{l}} = 100$ kpc, $\Delta R = \lambda_{\mathrm{c,l}} = 10$ kpc, $l_{\mathrm{i}} = 10$ kpc, $f_{\mathrm{i}} = 10^{-2}$, $B = B_{\mathrm{l}} = 10^{-7}$ G, $|q| = e$, $m = m_e$, $p = -2.4$, $\gamma_{\min} = 10$, $\gamma_{\max} = 10^5$, and $n_{\mathrm{l}} = 10^{-10}$ cm^{-3}. We find $P = 10^{30}$ W.

To estimate the final void magnetic field strength B_{v}, we followed an argument akin to that in Beck et al. [4]. If the lobe would expand to fill the entire void, then magnetic flux conservation yields

$$B_{\mathrm{v}} = B_{\mathrm{l}} \left(\frac{R_{\mathrm{l}}}{R_{\mathrm{v}}} \right)^2. \tag{A67}$$

By squaring and dividing both sides of this equation by $2\mu_0$, one recasts it in terms of magnetic energy densities and obtains

$$u_{B_{\mathrm{v}}} = u_{B_{\mathrm{l}}} \left(\frac{R_{\mathrm{l}}}{R_{\mathrm{v}}} \right)^4. \tag{A68}$$

However, only a fraction of the lobe's magnetic energy can escape, and it is only this fraction that we should consider in our calculation. If we assume that the magnetic energy that is carried out of the lobe is comparable to the energy of the escaped particles, which equals Pt, then

$$u_{B_{\mathrm{v}}} = \frac{Pt}{E_{\mathrm{l}}} u_{B_{\mathrm{l}}} \left(\frac{R_{\mathrm{l}}}{R_{\mathrm{v}}} \right)^4, \tag{A69}$$

where E_{l} is the total magnetic energy of the lobe. Recasting this equation back to magnetic field strengths, we obtain

$$B_{\mathrm{v}} = \sqrt{\frac{Pt}{E_{\mathrm{l}}}} \left(\frac{R_{\mathrm{l}}}{R_{\mathrm{v}}} \right)^2 B_{\mathrm{l}}. \tag{A70}$$

The energy ratio in Beck et al. [4]'s analogous Eq. 4 should likewise appear under a square root. This is a matter of typography only: the authors did take the square root to obtain their results (private communication with M. Hanasz). For $t = 10^0$ Gyr, $E_{\mathrm{l}} = 10^{55}$ J, $R_{\mathrm{v}} = 10^1$ Mpc, and P, R_{l}, and B_{l} as before, we obtained $B_{\mathrm{v}} = 6 \cdot 10^{-16}$ G.