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Model robust optimal designs for dose response models (Supplementary Material)

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1 Formulations to determine the optimal allocation via SDP

Here we present SDP formulation for the D-optimality criteria. They maximize the n_θ^{th} root of the determinant of the FIM and were introduced in Vandenberghe and Boyd (1996, 1999) and Ben-Tal and Nemirovski (2001).

$$
\begin{aligned}
Z^* = \max_{\xi,B} \; & t && \text{(1a)}\\
\text{s.t.} \; & \begin{pmatrix} M(\xi) & B^\intercal \\ B & \operatorname{diag}(B) \end{pmatrix} \succeq 0_{n_\theta} && \text{(1b)}\\
& t \leqslant \prod_{i=1}^{n_\theta} B_{i,i}^{1/n_\theta} && \text{(1c)}\\
& \sum_{i=1}^{k} w_i = 1 && \text{(1d)}
\end{aligned}
$$

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$$0 \leqslant w_i \leqslant 1, \quad i \in [\![k]\!]. \tag{1e}$$

2 D-optimal designs for single models

Table presents the D-optimal designs obtained for dose response models considered in the study.

Table 1: D-optimal designs for all the dose response models, $\mathbf{X} = [0, 150]$ and $\Delta x = 0.5$.

Models													
$\xi_{\mathcal{M}_1}$		$\xi_{\mathcal{M}_2}$		$\xi_{\mathcal{M}_3}$		$\xi_{\mathcal{M}_4}$		$\xi_{\mathcal{M}_5}$		$\xi_{\mathcal{M}_6}$		$\xi_{\mathcal{M}_7}$	
x	**w**	**x**	**w**	**x**	**w**	**x**	**w**	**x**	**w**	**x**	**w**	**x**	**w**
0.0000	0.4998	0.0000	0.3333	0.0000	0.3334	0.0000	0.5000	0.0000	0.2501	0.0000	0.1996	0.0000	0.1996
150.0000	0.5002	18.5000	0.1643	96.0000	0.3331	150.0000	0.5000	39.5000	0.2113	0.5000	0.0843	0.5000	0.0843
		19.0000	0.1690	150.0000	0.3334			40.0000	0.0389	1.0000	0.1171	1.0000	0.1171
		150.0000	0.3333					62.0000	0.2500	20.0000	0.0101	20.0000	0.0101
								150.0000	0.2497	20.5000	0.1892	20.5000	0.1892
										80.0000	0.1602	80.0000	0.1602
										80.5000	0.0393	80.5000	0.0393
										150.0000	0.2001		
$n_{s_1} = 2$		$n_{s_2} = 4$		$n_{s_3} = 3$		$n_{s_4} = 2$		$n_{s_5} = 5$		$n_{s_6} = 8$		$n_{s_7} = 7$	
$Z^* = 7.4999 \times 10^{1}$		$Z^* = 6.8388 \times 10^{-3}$		$Z^* = 9.1364 \times 10^{-3}$		$Z^* = 2.5086$		$Z^* = 2.4660 \times 10^{-3}$		$Z^* = 2.7966 \times 10^{-3}$		$Z^* = 2.2223 \times 10^{-3}$	

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