*S*upplementary material

Output feedback control of a fractional-order model of the lung impedance

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Proof S1. Note that $f_{\mathfrak{D}}(z) = L + zM + \bar{z}M^T$ (Definition 1) e $\mathscr{M}_{\mathfrak{D}}(A,X) = L \otimes X + M \otimes (AX) + M^T \otimes (AX)^T$ (Theorem 2), have the equivalence $(1,z,\bar{z}) \equiv (X,AX,X)$[1]. Using this equivalence in (7), (8) and (9), the LMIs (10), (11) and (12) are obtained, respectively. The stability region for order $\alpha \in (0,1)$ is non-convex (it is not an LMI region), because it includes conjugate sectors in the complex right half-plane, as already shown in Theorem 1. Thus, inequality (13) limits θ to the complex left half-plane, which is a convex region. The stability region for the order $\alpha \in (1,2)$ is a convex region (it is a sector) contained in the complex left half-plane, according to Theorem 1. Thus, inequality (14) limits θ to this region. □

Proof S2. Assume the existence of solutions $X = X^T \in \mathbb{R}^{n\times n}$ and $M \in \mathbb{R}^{p\times n}$. As $M = KX$ and $X = X^T$, from (16):

$$AX + XA^T - BKX - XK^TB^T + 2\gamma X \prec 0, \tag{S1}$$

$$(A-BK)X + X(A-BK)^T + 2\gamma X \prec 0, \tag{S2}$$

which is the LMI (10) of Lemma 1 applied to the feedback system (15). From (17):

$$\begin{bmatrix} -rX & AX - BKX \\ XA^T - XK^TB^T & -rX \end{bmatrix} \prec 0, \tag{S3}$$

$$\begin{bmatrix} -rX & (A-BK)X \\ X(A-BK)^T & -rX \end{bmatrix} \prec 0, \tag{S4}$$

which is the LMI (11) applied to (15). From (18):

$$\begin{bmatrix} (AX + XA^T - BKX - XK^TB^T)sin(\theta) & (AX - XA^T - BKX + XK^TB^T)cos(\theta) \\ (-AX + XA^T + BKX - XK^TB^T)cos(\theta) & (AX + XA^T - BKX - XK^TB^T)sin(\theta) \end{bmatrix} \prec 0, \tag{S5}$$

$$\begin{bmatrix} ((A-BK)X + X(A-BK)^T)sin(\theta) & ((A-BK)X - X(A-BK)^T)cos(\theta) \\ (-(A-BK)X + X(A-BK)^T)cos(\theta) & ((A-BK)X + X(A-BK)^T)sin(\theta) \end{bmatrix} \prec 0, \tag{S6}$$

which is the LMI (12) applied to (15). □

The calculated gains K_{xw} and L were

$$K_{xw} = [\ -1.2948 \quad 0.5113 \quad -0.2848 \quad 0.0832 \quad -0.0152 \quad 0.0018 \quad 0.0002 \quad 0.0005 \quad 0.0007 \quad 0.0008 \quad 0.0009 \quad 0.0010 \quad 0.0011 \quad 0.0013 \quad 0.0017 \quad 0.0021 \quad 0.0026 \quad 0.0032 \quad 0.0039 \quad 0.0047 \quad 0.0055 \quad 0.0062 \quad 0.0068 \quad 0.0075 \quad -0.0578 \quad -0.0257\ ], \tag{S7}$$

$$L = [\ 10.9235 \quad -8.2037 \quad 2.0765 \quad -1.5701 \quad -0.4355 \quad -0.5560 \quad -0.4800 \quad -0.4313 \quad -0.3999 \quad -0.3717 \quad -0.3370 \quad -0.3060 \quad -0.2808 \quad -0.2511 \quad -0.2193 \quad -0.1941 \quad -0.1758 \quad -0.1551 \quad -0.1325 \quad -0.1184 \quad -0.1127 \quad -0.1075 \quad -0.1007 \quad -0.0968\ ]^T. \tag{S8}$$

References

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Competing interests

The authors declare no competing interests.