Supplementary Information

Title:

Another Mechanism for Gait Generation: Mechanical Stabilization Can Spontaneously Realize Walking in a Two-Legged Robot

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List of Movies:

Supplementary Movie 1. Gait generation at 12 V

Supplementary Movie 2. Gait generation at 8 V

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Supplementary Figure 1. Design of the legged robot. In deferential modules, TAMIYAINC. SP.1568 is adopted because of the lower frictions, where no oil is injected into the gear room to minimize the damping effect. The BLDD (DJI, RoboMaster GM6020) is used as the drive unit. The direct drive provides high back-drivability and transfers the GRF to the shaft, ensuring that the angular velocity of the shaft can be varied. The robot has a yaw joint in the middle position between the drive unit and the leg shaft. The joints are equipped with a soft material (Smooth-On, Dragon Skin 10 Medium) at both ends to maintain the home position. In addition, the motor sets the torque value from the microcontroller (Arduino, Arduino Uno Rev3) via CAN communication. Simultaneously, the microcontroller reads the value of the pressure sensor (ALPHA, MF02-N-221-A01) via the pressure divider circuit.

Supplementary Figure 2. Experimental environment. The robot moves on a treadmill. Here, the white points in the figure are drawn at 100-mm intervals. Based on this scale, the distance traveled by the robot in the walking experiment was measured on the image. The speed of the treadmill was adjusted appropriately for each experiment to prevent the robot from falling. The cables were long enough for the robot to move, and the tension transmitted from the cables to the robot was minimized.

Supplementary Figure 3. Gait data extraction by waveform after the lowpass filter. The experimental data are shown when the input voltage to the motor was 8 V (**a**) and 12 V (**b**). In each figure, the blue line represents the experimental data for the left leg and the yellow line, for the right leg. The frequency at which the one-sided spectrum of the FFT reached its maximum was considered as the ground motion from the swing leg to the support leg, and the higher frequency components were removed.

Supplementary Figure 4. Definition of the phase in walking obtained by wavelet transforms. Experimental data are shown for motor input torques of 8 V (**a**, **c**) and 12 V (**b**, **d**). Figure (**a**, **b**) ((**c**, **d**)) shows the data of the left (right) leg. Since the highlighted areas are the main frequency components of the acquired data, they are extracted and used for the phase difference analysis. The Morlet function expressed as a wavelet was used: $mw(x) = \exp(-x^2/2)\cos 5x$, where $x = (t-\beta)/\alpha$. t: time of experimental data, α and β: parameters of the wavelet. In the presented results, we extracted data at $\alpha = 0.9$ for 8 V and $\alpha = 0.5$ for 12 V.

Supplementary Figure 5. Extractions of gait frequency by Fast Fourier Transform (FFT). Experimental data are shown for motor input torques of 8 V (**a**,**c**) and 12 V (**b**,**d**). In the data extracted by the wavelet transform, a larger input voltage value results in increased noise. The main frequencies are visualized by FFT to reduce noise. The red dashed line shows the frequency when the correction value is multiplied by 1.2 times the frequency when the one-sided spectrum is at its maximum. Note that these cutoff frequencies can take different values in each data.

Supplementary Note 1. Calculation of the phase difference

The phase difference was calculated by focusing only on the maximum value (" + " in **Supplementary Figure 3**) of each waveform. First, the average period T_{L_ave} and T_{R_ave} of each waveform obtained is calculated by the following equation.

$$T_{j\_ave} = \frac{1}{m-1}\sum_{n=1}^{m-1} T_{j,n+1} - T_{j,n}, \tag{S1}$$

where $j = \{\mathrm{L}, \mathrm{R}\}$, m is the number of waveform maxima, $T_{j,n}$ is the time at which the nth maximum occurs in either the left or right data. The above equation is used to calculate the average period of the entire waveform.

Next, the average difference in the timing of the maxima between the left and right leg data is determined by the following equation:

$$T_{\mathrm{LR}\_ave} = \frac{1}{m}\left|\sum_{n=1}^{m} T_{\mathrm{L},n} - T_{\mathrm{R},n}\right|, \tag{S2}$$

where $T_{\mathrm{L},n}$ is the time of the nth maximum in the waveform of the data acquired from the left leg, $T_{\mathrm{R},n}$ is the time of the nth maximum in the waveform of the data acquired from the right leg. The above equation is used to calculate the average of the peak timing differences between the left and right legs.

From the above, the phase difference is obtained from the ratio of the average time difference of each maximum value to the average of the average period of the waveforms obtained from the left and right legs.

$$Phase\ difference = 2\pi \times \frac{T_{\mathrm{LR}\_ave}}{(T_{\mathrm{L}\_ave} + T_{\mathrm{R}\_ave})/2} \tag{S3}$$

The above equation calculates the phase difference by multiplying the ratio of the phase difference in one period by the phase of one period, 2π.

Supplementary Figure 6. Motor specifications. (a) Rotational speed of the motor versus voltage with load and no load. The characteristics of the motor used were investigated and used to estimate the output torque of the motor. The motor rotation speed at each voltage value was measured under no load and load, respectively. The equations in the figure are the result of the linear approximation of the respective results. **(b)** Relationship of the motor rotation speed to the load torque at each voltage value using the speed/torque gradient of 156 rpm/Nm specified in the specifications. The straight line at each voltage is expressed using the following equation: $n = -156 * \tau + \omega_0$, where n is the motor rotation speed, τ is the load torque, ω_0 is the motor rotation speed under no load (calculated from the linear approximation in **(a)**. The rotational speed under load at each voltage value was calculated (red “×” in **(b)**, and an approximate straight line (black line) was obtained.

This equation represents the increase in rotational speed due to an increase in input voltage or input torque. Based on this approximation, the value of the ratio of the dimensionless torque to the dimensionless friction coefficient, $\bar{\tau}_{\mathrm{c}}/\bar{f}$, is determined. Since $\bar{\tau}_{\mathrm{c}}/\bar{f} = \tau_{\mathrm{c}}/(r_{\mathrm{LR}}f)$ from **Supplementary Table 2**, the input torque τ_{c} at each voltage value is determined based on the approximation formula shown above. In addition, $r_{\mathrm{LR}}f$ is the moment generated by friction. Therefore, the value of input torque τ_{c} at 4 V when the robot starts walking is the value of τ_{c} determined to be a constant value, $r_{\mathrm{LR}}f = \tau_{\mathrm{c}}$ (voltage = 4 V), in balance with the load due to friction. The above values were used for nondimensionalization.

Supplementary Note 2. Calculation of the cost of transport

To calculate the energy efficiency, we measured the distance traveled by the robot and the current value for each constant voltage value during a measurement time of 10 s. This allows us to calculate CoT [1,2], which is expressed by the following equation:

$$\mathrm{CoT} = \frac{\int_0^T P(t)dt}{mgD} = \frac{V\int_0^{10} I(t)dt}{mgD}. \tag{S4}$$

The voltage value V [V] is a constant input, $I(t)$[A] is the current value, mg is the weight of the actual machine, 2.59 kg, and D[m] is the travel distance.

Supplementary Note 3. Measurement systems

The resistance of a pressure-sensitive sensor is measured using a voltage divider circuit. The voltage divider circuit (**Supplementary Figure 1**) comprises the pull-up resistor R_1 and the pull-down resistor R_2 and can generate V_{out} proportional to a certain voltage, V_{cc}. By attaching a pressure-sensitive sensor to the pull-down resistor R_2, the resistance value of the sensor can be obtained from the generated V_{out}. When the pull-up resistor $R_1 =$ 1 kΩ and the pull-down resistor $R_2 =$ 20 MΩ are connected in series and between the power supply V_{cc} and the ground (0 V), the divided V_{out} is expressed using the following equation by the resistive voltage divider rule:

$$V_{\text{out}} = \frac{R_2 V_{\text{cc}}}{R_1 + R_2}. \tag{S5}$$

Using equation S5, R_2 can be obtained as

$$R_2 = \frac{R_1 V_{\text{out}}}{V_{\text{cc}} - V_{\text{out}}}. \tag{S6}$$

However, when V_{cc} and V_{out} are equal, the denominator becomes zero, and the value of R_2 becomes infinite. Therefore, if the reciprocal of R_2 is the conductance G, it can be expressed using the following equation:

$$G = \frac{1}{R_2} = \frac{V_{\text{cc}} - V_{\text{out}}}{R_1 V_{\text{out}}}. \tag{S7}$$

This allows the magnitude of the force applied to the pressure-sensitive sensor to be compared from the magnitude of the value of G. The power supply voltage V_{cc} is 5 V. V_{out} from the pressure-sensitive sensors at the backs of the left and right legs was obtained using the analog pins of the microcontroller.

Supplementary Note 4. Details of mathematical analysis

The linearization is shown below using the Jacobi matrix obtained around each immobile point:

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{c_{\text{LR}}}{I_{\text{LR}}} & \frac{-fr_{\text{LR}}\cos\left(\sin^{-1}\left(\frac{\tau_{\text{c}} - fr_{\text{LR}}}{fr_{\text{LR}}}\right)\right)}{2I_{\text{LR}}} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{c_{\text{c}} + 2c_{\text{LR}}}{I_{\text{c}} + 2I_{\text{LR}}} & \frac{-fr_{\text{LR}}\cos\left(\sin^{-1}\left(\frac{\tau_{\text{c}} - fr_{\text{LR}}}{fr_{\text{LR}}}\right)\right)}{I_{\text{c}} + 2I_{\text{LR}}} \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{c_{\text{LR}}}{I_{\text{LR}}} & \frac{-fr_{\text{LR}}\cos\left(\sin^{-1}\left(\frac{fr_{\text{LR}} - \tau_{\text{c}}}{fr_{\text{LR}}}\right)\right)}{2I_{\text{LR}}} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{c_{\text{c}} + 2c_{\text{LR}}}{I_{\text{c}} + 2I_{\text{LR}}} & \frac{-fr_{\text{LR}}\cos\left(\sin^{-1}\left(\frac{fr_{\text{LR}} - \tau_{\text{c}}}{fr_{\text{LR}}}\right)\right)}{I_{\text{c}} + 2I_{\text{LR}}} \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} -\frac{c_{\mathrm{LR}}}{I_{\mathrm{LR}}} & 0 & 0 & \frac{f r_{\mathrm{LR}} \sin\left(\cos^{-1}\left(\frac{\tau_{\mathrm{c}} - f r_{\mathrm{LR}}}{f r_{\mathrm{LR}}}\right)\right)}{I_{\mathrm{LR}}} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{f r_{\mathrm{LR}} \sin\left(\cos^{-1}\left(\frac{\tau_{\mathrm{c}} - f r_{\mathrm{LR}}}{f r_{\mathrm{LR}}}\right)\right)}{2(I_{\mathrm{c}} + 2I_{\mathrm{LR}})} & -\frac{c_{\mathrm{c}} + 2c_{\mathrm{LR}}}{I_{\mathrm{c}} + 2I_{\mathrm{LR}}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \text{and}$$

$$\mathbf{A}_4 = \begin{bmatrix} -\frac{c_{\mathrm{LR}}}{I_{\mathrm{LR}}} & 0 & 0 & \frac{f r_{\mathrm{LR}} \sin\left(\cos^{-1}\left(\frac{f r_{\mathrm{LR}} - \tau_{\mathrm{c}}}{f r_{\mathrm{LR}}}\right)\right)}{I_{\mathrm{LR}}} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{f r_{\mathrm{LR}} \sin\left(\cos^{-1}\left(\frac{f r_{\mathrm{LR}} - \tau_{\mathrm{c}}}{f r_{\mathrm{LR}}}\right)\right)}{2(I_{\mathrm{c}} + 2I_{\mathrm{LR}})} & -\frac{c_{\mathrm{c}} + 2c_{\mathrm{LR}}}{I_{\mathrm{c}} + 2I_{\mathrm{LR}}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Supplementary Note 5. Nondimensionalization

Equation (1) is nondimensionalized with representative scales as-M $= I_{\mathrm{LR}}/{r_{\mathrm{LR}}}^2$, L $= r_{\mathrm{LR}}$, and T $= I_{\mathrm{LR}}/c_{\mathrm{LR}}$. **Supplementary Table 1** shows the parameters before nondimensionalization, and **Supplementary Table 2** shows the parameters obtained upon nondimensionalization. After the nondimensionalization of equation (1), the following equation is obtained:

$$\begin{pmatrix} \ddot{\phi} \\ \dot{\phi} \\ \ddot{\theta}_{\mathrm{c}} \\ \dot{\theta}_{\mathrm{c}} \end{pmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\bar{c}_{\mathrm{c}} + 2}{\bar{I}_{\mathrm{c}} + 2} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} \dot{\phi} \\ \phi \\ \dot{\theta}_{\mathrm{c}} \\ \theta_{\mathrm{c}} \end{pmatrix} + \begin{pmatrix} -\bar{f} \sin\frac{\phi}{2} \cos\theta_{\mathrm{c}} \\ 0 \\ -\frac{1}{\bar{I}_{\mathrm{c}} + 2}\left(\bar{f} \sin\theta_{\mathrm{c}} \cos\frac{\phi}{2} - \bar{\tau}_{\mathrm{c}} + \bar{f}\right) \\ 0 \end{pmatrix}. \tag{S8}$$

The fixed points in the above equations are denoted as follows:

$$\boldsymbol{x}_1^* = \left[0 \quad 4n\pi \quad 0 \quad \sin^{-1}\left(\frac{\bar{\tau}_{\mathrm{c}}}{\bar{f}} - 1\right)\right]^T,$$

$$\boldsymbol{x}_2^* = \left[0 \quad (4n + 2)\pi \quad 0 \quad \sin^{-1}\left(1 - \frac{\bar{\tau}_{\mathrm{c}}}{\bar{f}}\right)\right]^T,$$

$$\boldsymbol{x}_3^* = \left[0 \quad 2\cos^{-1}\left(\frac{\bar{\tau}_{\mathrm{c}}}{\bar{f}} - 1\right) \quad 0 \quad \left(2n + \frac{1}{2}\right)\pi\right]^T, \text{ and}$$

$$\boldsymbol{x}_4^* = \left[0 \quad 2\cos^{-1}\left(1 - \frac{\bar{\tau}_{\mathrm{c}}}{\bar{f}}\right) \quad 0 \quad \left(2n + \frac{3}{2}\right)\pi\right]^T.$$

Supplementary References

[1] Gabrielli, G. & von Karman, T. What price speed? specific power required for propulsion of vehicles. *Mech. Eng.* **72**, 775-781 (1950).

[2] Tucker, V. A. The energetic cost of moving about : walking and running are extremely inefficient forms of locomotion. Much greater efficiency is achieved by birds, fish—and bicyclists. *Am. Sci.* **63**, 413-419 (1975).