p-y Curves for Piles in Uniform Sand and Normally Consolidated Clay

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**ABSTRACT**

The design of laterally loaded piles has commonly been done using the *p-y* method. The results of a *p-y* analysis are highly dependent on the *p-y* curves used in the analysis. Since the *p-y* curves routinely used in *p-y* analyses have been traditionally derived empirically from data obtained from a limited number of lateral pile load tests and *in situ* test data, the domain of applicability of the *p-y* method would necessarily be limited. An alternative path for *p-y* curve development is to obtain them from realistic numerical analyses, an approach explored in this paper. The paper presents the results of a series of three-dimensional (3D) finite element (FE) analyses of single piles in uniform soil profiles of sand and clay using advanced two-surface-plasticity constitutive models. The analyses were performed for different values of pile diameter, pile length, lateral load eccentricity, and sand relative density. We extracted *p-y* curves from the simulation results, and studied the factors that would be expected to have an impact on the *p-y* curves: relative density, soil type, pile diameter, load eccentricity, and the depth to the water table. Based on these analyses, we propose a new set of *p-y* curve equations that can be used for sand or clay in the design of laterally loaded piles.

*Keywords*: Finite Element Analysis; *p-y* analysis; *p-y* curves.
1. INTRODUCTION

To design piles for lateral loads, the \( p-y \) method – a method originated from the subgrade reaction method (Banerjee and Davies 1978; Madhav et al. 1971; Terzaghi 1995; Valsangkar et al. 1973; Winkler 1867) – is most often used. In a \( p-y \) analysis, the pile is modeled as a 1D Bernoulli beam, and the lateral pile-soil interaction is modeled through a series of springs distributed along the pile length (Salgado 2022). Both the stiffness of the pile and of the spring have an impact on the lateral capacity of the pile. The simulation can be done using the finite element method (FEM), the finite difference method (FDM), or any other reliable numerical method. By performing \( p-y \) analysis, we can obtain the global lateral load response curve for the pile and estimate, based on the results of the analysis, the pile capacity.

A number of formulations have been proposed for the \( p-y \) curves to represent the stiffness of different types of soils and piles. For sand, Reese et al. (1974) proposed equations for \( p-y \) curves based on lateral load test results on two 0.61-m-diameter open-ended pipe piles in dense sand. Murchison & O’Neill (1983) modified the equations proposed in Reese et al. (1974) and proposed \( p-y \) curve equations for loose sand. Wesselink et al. (1988) proposed \( p-y \) curves for calcareous sand based on centrifuge model test data on 13-23 mm-diameter scale models of prototype piles with diameters of 0.356, 1.22, and 2.14 meters.

For clay, Matlock (1970) proposed \( p-y \) curves for both static and cyclic loading conditions for steel pipe piles based on the results of load tests on piles with a diameter of 0.32m. Reese et al. (1975) proposed \( p-y \) curves for stiff clay below the water table based on test data on open-ended steel pipe piles with a 0.61-m diameter, and Welch & Reese (1972) proposed \( p-y \) curves for stiff clay above the water table based on load test results on drilled shafts with a diameter of 0.762m. Dunnavant & O’Neill (1989) proposed another set of \( p-y \) curve equations for submerged stiff clay based on a series of full-scale lateral load tests on open-ended steel pipe piles with outer diameters of 0.27m and 1.22m and bored reinforced concrete piles. These curves would in theory be applicable only to the conditions under which they were obtained, with pile type and geometry and method of pile installation being important factors. At present, the most used \( p-y \) curves for both sand and clay are those in API (1993), which would strictly apply to driven piles.

The \( p-y \) method has its limitations. First, \( p-y \) curves often do not capture the actual mechanics of the interaction between the soil and the pile (Basu & Salgado, 2007a, 2008; Choi et
Second, it is only applicable to specific pile geometries. For example, researchers have reported discrepancies between predictions using the $p$-$y$ method and the finite element method for monopiles with large diameters (Hu et al. 2022; Naser et al. 2022). Additionally, most $p$-$y$ curves available today are for piles with circular cross-sections, but piles with non-circular cross-sections are also frequently used (Basu and Salgado 2008; Choi et al. 2014). Lastly, some of the well-known $p$-$y$ relationships involve parameters that are arbitrarily determined by the users based on their experience, such as the $J$ parameter in API curves for sand, which introduces some errors into the results (see, e.g., Han et al., 2015). Due to these reasons, the $p$-$y$ method often fails to predict pile response with sufficient accuracy (Anderson et al. 2003; Tak Kim et al. 2004).

There are other methods developed to predict the lateral capacity of piles. Continuum-based methods that can be used as efficiently as the $p$-$y$ method have been proposed in which the soil is regarded as an elastic continuum (Basu & Salgado, 2007a; Guo & Lee, 2001; Han et al., 2015; Hu et al., 2022), which is conceptually more sound than a discrete, soil-pile spring-based approach. However, the $p$-$y$ method continues to be the industry's preferred method of laterally loaded pile analysis. Recognizing both its continued use and its weaknesses, researchers have tried to improve the $p$-$y$ design method, most recently for marine sand and glacial clay. The pile soil analysis (PISA) research project improved the $p$-$y$ method by considering the vertical shear tractions at the soil-pile interface and the resistances at the pile base (Burd et al. 2020b; a; Byrne et al. 2019, 2020; b; McAdam et al. 2020; Taborda et al. 2020; Zdravković et al. 2015; Zdravković et al. 2020). In the PISA design method, the pile is assumed to be a Timoshenko beam, instead of an Euler-Bernoulli beam, as in the original $p$-$y$ method, to obtain more accurate results. As for the soil reaction, a 4-parameter conic function describes the relationship between the pile displacement or rotation and the soil resistance. The soil reaction curves play the same role in the PISA design approach as $p$-$y$ curves in $p$-$y$ analysis. Burd et al. (2020b) proposed soil reaction curves for marine sand, and Byrne et al. (2020a) proposed soil reaction curves for glacial clay. Both extracted the soil reaction curves from the 3D finite element analysis results and fitted them into the form used in the PISA design method to find that the resultant global response curves fit well with those extracted from the 3D finite element analysis results.

In this paper, we performed 3D finite element analyses (FEA), and extracted $p$-$y$ curves from the results of these analyses. The 3D finite element analyses were performed for single
piles in both normally consolidated sand and clay. We investigated the impact of factors such as soil type, relative density, pile diameter, depth of water table, and load eccentricity on the pile response. The $p-y$ curves extracted from the analyses may be used in the design of piles in soil profiles with soils similar to those assumed in the analyses.

In section 2 of this paper, we discuss the finite element analysis configuration, including mesh configuration, boundary conditions, and analysis steps. In section 3, we describe the global response curves of laterally loaded piles extracted from the finite element analyses for the cases examined, and the key factors that impact the global response curves. Analyses using the $p-y$ curves proposed are then validated in sections 4 and 5.

2. **FINITE ELEMENT ANALYSES: MESH, BOUNDARY CONDITIONS, MATERIAL MODELS AND LOADING**

Analyses of single piles with diameter $B$ ranging from 0.36m to 1.0 m were considered to quantify the effect of pile stiffness on the response of single piles subjected to lateral loads. In this study, only long piles were considered. These are piles with length $L$ sufficiently long to have zero deflection at a depth shallower than the pile base when the pile is loaded laterally. The length $L$ of the piles is 10 m in sand profiles, 10m or 20m in uniform normally consolidated (NC) clay, and 20 m in multilayered soil profiles with NC clay. The cases considered are shown in Figure 1.

Figure 1 Analysis cases.

2.1. **Finite Element Mesh and Boundary Conditions**

Figure 2 shows the mesh configuration used in the three-dimensional FE analyses for a single pile with $B = 0.36$ m and $L = 10$ m installed in a uniform soil profile. As shown in the figure, only half of the problem domain is considered in the analyses because of the symmetry of the
problem domain. The width and length of the soil domain are taken as more than 25 times the pile diameter, and the thickness of the soil domain exceeds 1.5 times the pile length to avoid boundary effects.

For the case shown in Figure 2, the soil domain, with dimensions of 50 m × 25 m × 15 m, is discretized using 17,643 linear, 8-noded hexahedral elements with reduced integration. The bottom of the problem domain is fixed in the vertical and horizontal directions, and the sides of the problem domain are fixed in their normal directions, as shown in Figure 2.

![Figure 2 Boundary conditions applied in the three-dimensional FE analysis for a single pile with diameter \( B = 0.36 \text{ m} \) length \( L = 10 \text{ m} \) in a uniform soil profile: (a) front view and (b) side view.](image)

The size of the soil elements that are right next to the pile should be small, as shown in Figure 3, because gradients in this region are greatest, and the soil there provides the most soil resistance when piles are loaded laterally. We investigated the impact of the smallest element size on the \( p-y \) curves. Figure 4 shows a comparison of \( p-y \) curves extracted from different mesh configurations. The mesh is marked with the dimension \( d \) of the smallest element contained in the whole domain. The results show that the smallest element size has very limited impact on the \( p-y \) curves for the values considered.
Figure 3 Mesh configuration with detail of fine zone next to and around the pile head.

Figure 4 The impact of mesh size on $p$-$y$ curves for single piles in (a) normally consolidated clay, and (b) dense sand ($D_r=80\%$).

Figure 5 shows the impact of the smallest element size on the global response curves. The vertical axis is the lateral load applied on the pile head, and the horizontal axis is the lateral deflection of the pile head. The difference in the lateral capacity of the pile, which is defined as the lateral load on the pile head when the lateral deflection at the pile head is 0.05m, is less than 5%. Because of the limited impact of smallest element size on the results for elements smaller...
than 50 mm, this is the minimum element size used in all analyses whose results are reported in this paper.

![Figure 5](image)

Figure 5 Impact of the smallest element size on global response curve of a 10-m-long single pile in uniform dense sand ($D_0=80\%$)

### 2.2. Analysis Steps

The FEA consisted of two explicit analysis steps. In the first step, gravity is applied to the problem domain after assigning the initial stress field to the Gauss points of elements representing the soil and pile. The initial stress field is one with total vertical stress equal to $\gamma_{sat}z$, where $\gamma_{sat}$ is the saturated soil unit weight and $z$ is the depth, and total horizontal stress equal to $[K_0(\gamma_{sat} - \gamma_d) + \gamma_w]z$, where $\gamma_d$ is the dry soil unit weight, $\gamma_w$ is the unit weight of water, and $K_0$ is the coefficient of lateral earth pressure at rest. The first step continues until static equilibrium is reached, which means that the computed stress field matches the predefined stress field. In the second step, the lateral load $H$ is applied at the pile head as a gradually increasing horizontal force.

The applied loading rate ranged from 10 kN/s to 25 kN/s. These loading rates produce a quasi-static response and are high enough to produce an undrained response for clay, but still sufficiently low to produce a drained response for sand. When the load eccentricity $h$ is not zero, an equivalent moment $M (= H \times h)$ is applied together with the lateral load $H$ at the pile head as a gradually increasing moment to model the eccentricity of the lateral load.
2.3. Constitutive Models

Since the mechanical responses of sand and clay are highly nonlinear and depend on the soil’s initial state, loading conditions, and intrinsic variables, it is very important to use advanced constitutive models for soil to accurately simulate its responses under various conditions. In the FEA, sand and clay layers are modeled using the constitutive models developed by Loukidis & Salgado (2009) and Chakraborty et al. (2013), respectively. Both models were developed based on critical-state soil mechanics and two-surface plasticity concepts; they can capture the mechanical responses of sand and clay realistically. The properties of Ottawa sand and Boston Blue Clay (BBC) were used in the analyses, and Table 2-1 and Table 2-2 provide the input parameters used in the analyses for sand and clay, respectively. In the analyses, both dry and saturated sands and fully saturated clay were considered.

<table>
<thead>
<tr>
<th>Table 2-1 Model parameters used for Ottawa sand.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter symbol</strong></td>
</tr>
<tr>
<td>Small-strain parameters</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
<tr>
<td>$C_\theta$</td>
</tr>
<tr>
<td>$n_\theta$</td>
</tr>
<tr>
<td>$\gamma_1$</td>
</tr>
<tr>
<td>$\alpha_1$</td>
</tr>
<tr>
<td>Critical state</td>
</tr>
<tr>
<td>$\Gamma_c$</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\xi$</td>
</tr>
<tr>
<td>$M_{cc}$</td>
</tr>
<tr>
<td>Bounding surface</td>
</tr>
<tr>
<td>$k_0$</td>
</tr>
<tr>
<td>Dilatancy</td>
</tr>
<tr>
<td>$D_0$</td>
</tr>
<tr>
<td>$k_d$</td>
</tr>
<tr>
<td>Plastic modulus</td>
</tr>
<tr>
<td>$h_1$</td>
</tr>
<tr>
<td>$h_2$</td>
</tr>
<tr>
<td>$\varepsilon_{lim}$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>Stress-induced anisotropy</td>
</tr>
<tr>
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<tr>
<td>$c_2$</td>
</tr>
<tr>
<td>$n_s$</td>
</tr>
<tr>
<td>Inherent anisotropy</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$k_h$</td>
</tr>
<tr>
<td>Yield surface radius</td>
</tr>
<tr>
<td>$m$</td>
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</table>
### Table 2-2 Model parameters used for Boston Blue Clay.

<table>
<thead>
<tr>
<th>Parameter symbol</th>
<th>Parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Small-strain parameters</strong></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.25</td>
</tr>
<tr>
<td>$C_g$</td>
<td>250</td>
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<tr>
<td>$\zeta$</td>
<td>5</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.036</td>
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<tr>
<td><strong>Normal consolidation line</strong></td>
<td></td>
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<tr>
<td>$N$</td>
<td>1.138</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.187</td>
</tr>
<tr>
<td><strong>Stress anisotropy</strong></td>
<td>$K_{0,NC}$</td>
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<tr>
<td><strong>Shear strength</strong></td>
<td>$M_{cc}$</td>
</tr>
<tr>
<td>$n_s$</td>
<td>0.2</td>
</tr>
<tr>
<td>$k_b$</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>2.7</td>
</tr>
<tr>
<td><strong>Dilatancy surface</strong></td>
<td>$D_0$</td>
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<tr>
<td><strong>Flow rule</strong></td>
<td>$c_2$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.31</td>
</tr>
<tr>
<td><strong>Plastic modulus</strong></td>
<td>$h_0$</td>
</tr>
<tr>
<td><strong>Yield surface radius</strong></td>
<td>$m$</td>
</tr>
</tbody>
</table>
The simulations were performed using the commercial software ABAQUS/Explicit (SIMULIA 2021). A user-defined model subroutine—VUMAT—was coded in FORTRAN to implement the soil models in ABAQUS. The piles were modeled as elastic with Young’s modulus $E$ equal to 200 GPa and Poisson’s ratio $\nu$ equal to 0.2. The pile geometry is a cylinder with the same outer diameter as that of the pipe pile considered in the analyses. The Young’s modulus of the cylindrical pile is determined such that it has the same bending stiffness as the pipe pile. In the analyses, we consider wall thicknesses of 6 mm and 9 mm for the outer pile diameter $B$ of 0.36 m (14.17 inches), wall thicknesses of 16 mm for $B = 0.6$ m, and wall thicknesses of 20 mm for $B = 1.0$ m. Table 2-3 shows the calculated Young’s modulus of the cylindrical piles having the same bending stiffnesses as the pipe piles considered in the analyses.

<table>
<thead>
<tr>
<th>Outer diameter of the pile (m)</th>
<th>Wall thickness (mm)</th>
<th>Young’s modulus of cylindrical pile (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>6</td>
<td>25.4</td>
</tr>
<tr>
<td>0.36</td>
<td>9</td>
<td>37.1</td>
</tr>
<tr>
<td>0.6</td>
<td>16</td>
<td>39.4</td>
</tr>
<tr>
<td>1.0</td>
<td>20</td>
<td>30.1</td>
</tr>
</tbody>
</table>

According to Hu et al. (2021), when the lateral capacity of a pile obtained from FEA by modeling the pile-soil interface using the Coulomb friction model with an interface friction angle of 21° is compared with the one obtained from FEA by considering perfect contact between the soil and the pile, the difference in the results is less than 2.5%. Given that the effect of an interface model choice for the soil-pile interface is not significant, we considered only perfect contact between the soil and the pile in the analyses reported here.

### 3. GLOBAL RESPONSE OF SINGLE LATERALLY LOADED PILES IN UNIFORM SOIL

The lateral load response of a single pile in sandy soil is affected by the relative density of the sand. Figure 6(a) shows the lateral load $H$ applied at the pile head versus the lateral deflection $y_{\text{top}}$ at the pile head obtained from the FE simulations for a 0.36-m-diameter, 10-m-long pile loaded laterally in dense sand ($D_r = 80\%$) and loose sand ($D_r = 40\%$). The lateral load is applied with an eccentricity $h$—the distance from the pile head at which the lateral load would...
need to be applied to produce the moment applied at the pile head—of 10 m, representing a scenario in which the lateral load is applied, for example, at the top of a bridge pier and then transferred to the pile head. The lateral pile deflection at the ground surface in loose sand (= 36 mm) is twice as much as that in dense sand (= 18 mm) for a small load (= 13 kN).

Figure 6(b) shows the same load-deflection response, under both drained and undrained conditions, also obtained from 3D FEA of a 0.36-m-diameter, 10-m-long single pile with a load eccentricity of 10 m installed in a saturated normally consolidated (NC) clay. The response of single piles in NC clay is generally softer than that in sandy soil. The lateral capacity of the pile under undrained conditions is less than that under drained conditions. Considering that lateral loads are often transient in nature, leading to an undrained response in the clay next to the pile, undrained conditions should normally be used to estimate the lateral capacity of piles in clay.

![Figure 6 Response of a 0.36-m-diameter, 10-m-long single pile to lateral loads in: (a) dense and loose sand, and (b) NC clay under drained and undrained conditions.](image)

The load eccentricity $h$ has significant impact on the lateral capacity of single piles. In this research, the load eccentricity is applied to the pile by applying a moment together with the lateral load. The equivalent moment $M$ that must be applied at the pile head is equal to the product of the load eccentricity $h$ by the lateral load $H$ applied at the pile head. The pile head is always considered at the same level as the ground surface. Figure 7 compares the lateral load-deflection responses of a 0.36-m-diameter, 10-m-long single pile loaded laterally in dense sand ($D_R = 80\%$) with two other eccentricities: $h = 2$ m and 5 m. For the same load magnitude $- 44$ kN $-$ the pile lateral deflection at the ground level is over 76 mm if the load is applied at $h = 10$ m,
whereas the lateral deflection at the ground level is only 18 mm if the load is applied at \( h = 2 \text{ m} \). The lateral load capacity of a single pile decreases significantly with increasing load eccentricity.

![Figure 7](image)

Figure 7 Effect of load eccentricity \( h \) on the lateral load response of single piles: FE analysis results for a 0.36-m-diameter, 10-m-long single pile loaded laterally with three different load eccentricities \( h \) in dense sand (\( D_R = 80\% \)).

4. PROPOSED \( p-y \) RELATIONSHIPS FOR SAND

4.1. General Discussion

From the 3D FEA results, it is possible to extract \( p-y \) curves at different depths along the pile length. Figure 8 shows the \( p-y \) curves derived from the FEA at different depths for a single pile with \( B = 0.36 \text{ m} \) and \( L = 10 \text{ m} \) in sand for two different relative densities. For the same normalized pile deflection \( y/B \), the normalized unit soil resistance \( p/B \) increases as the depth increases because of the increasing confining stress with increasing depth. The \( p-y \) curve is stiffer for dense sand (\( D_R = 80\% \)) than for loose sand (\( D_R = 40\% \)) at the same depth, leading to a stiffer global load-deflection response for a single pile in dense sand, as shown in Figure 6(a).
Figure 8 The $p-y$ curves obtained from the FE analyses at different depths: (a) in dense ($D_R = 80\%$) sand and (b) in loose ($D_R = 40\%$) sand.

Figure 9 shows $p-y$ curves obtained from the FEA at different depths for single piles ($B = 0.36$ m and $L = 10$ m) in uniform loose and dense sand when the lateral load is applied with load eccentricities $h$ equal to $0$ m and $10$ m. Unlike what is observed for the global lateral load capacity of a single pile, which decreases significantly with increasing load eccentricity, as shown in Figure 7, the $p-y$ curves at different depths for a single pile in sand are almost the same for different load eccentricities. This is an important result, confirming that the same $p-y$ curves can be used regardless of the load eccentricity.
Figure 9 The $p$-$y$ curves obtained from the FEA at different depths for single piles with values of load eccentricity $h = 0$ m (0 ft) and 10 m (32.81 ft) (a) in loose ($D_r = 40\%$) sand, and (2) in dense ($D_r = 80\%$) sand.

The effect of the groundwater level on the $p$-$y$ curves for single piles in sand was also evaluated. Two different levels of the groundwater table were considered in the FEAs: one with the groundwater level at a depth greater than the height of the soil domain (= 20 m) with the entire soil domain under fully dry conditions; and the other with the groundwater level at the ground surface, with the entire soil domain fully saturated. Figure 10 shows the $p$-$y$ curves obtained from the FE analysis for a single pile ($B = 0.36$ m and $L = 10$ m) in loose sand with the groundwater table at different depths. When the $p$-$y$ curves at the same depths are compared for the cases with different groundwater levels, the case with the groundwater table at the ground surface provides a lower unit soil resistance $p$ at the same pile deflection $y$, as shown in Figure 10(a), because of the correspondingly lower vertical effective stress at that depth for the high water table case. When the $p$-$y$ curves at the same level of initial vertical effective stress $\sigma'_v0$ are compared for the cases with different groundwater levels, it is seen that they are almost the same, as shown in Figure 10(b). This confirms that we can generally use effective stresses to normalize $p$-$y$ curves.
Figure 10 The p-y curves obtained from the FE analysis of a single pile in loose ($D_w = 40\%$) sand with the groundwater table at the ground surface (fully saturated case) and at a depth greater than 20 m (fully dry case): (a) p-y curves at different depths, and (b) p-y curves at different values of initial vertical effective stress $\sigma'_v$.

Figure 11 shows the effect of pile diameter on the p-y curves for single piles in uniform dense and loose sands. In Figure 11, the normalized unit soil resistance $p/B$ is plotted versus normalized pile deflection $y/B$ for different depths for 10-m-long single piles with two different diameters: $B = 0.36$ m and 0.6 m. As shown in the figure, a slightly lower $p/B$ results for a given value of normalized pile deflection $y/B$ for the pile with the larger diameter at shallow depths — depths less than 1 m — for both dense and loose sands, but the opposite result is obtained for depths greater than 1 m. The effect of pile diameter on derived p-y curves for these two routinely used diameters appears to be small. Accordingly, the results presented here, which are based on the analysis for piles with a diameter of 0.36 m, are expected to apply to piles with diameters in the range considered in the present analyses.
Figure 11 The $p$-$y$ curves obtained from the FEA at different depths for single piles with a diameter $B$ of 0.36 m and 0.6 m (a) in dense ($D_R = 80\%$) sand, and (b) in loose ($D_R = 40\%$) sand.

4.2. $p$-$y$ Curves for Sand

Based on the results of the FEA performed for laterally loaded single piles with a diameter of 0.36 m, new $p$-$y$ curves are proposed for sand. The relationship between the unit soil resistance $p$ (with units of force per length) and the pile deflection $y$ (with units of length) is given by

$$p = p_u \tanh \left( b \frac{y}{L_R} \right)^c$$

(1)

where the limit unit soil resistance $p_u$ (with units of force per length) is given by

$$p_u = \left( \frac{D_R}{100\%} \right)^{1.4} \min \left( 46.6, 13.8 + 7.0 \left( \frac{p_A}{\sigma'_{v_0}} \right) \right) \sigma'_{v_0} B$$

(2)

with

$$b = \frac{32.2}{\left( \frac{p_u}{p_A B} \right)^{0.86}} \exp \left[ 4.1 \left( \frac{D_R}{100\%} \right)^5 \right]$$

(3)

and

$$\sigma'_0 = \frac{100}{D_R} \left( \frac{C_F}{C_D} + 1 \right)$$
with $D_R =$ relative density of the sand as a percentage (0 % $\leq D_R \leq 100$ %), $\sigma'_{vo} =$ initial vertical effective stress, $B =$ pile diameter, $p_A (= 100$ kPa $\approx 1$ tsf) = reference stress, and $L_R (= 1$ m $\approx 3.281$ ft or equivalent in other units) = reference length. The equation is fitted to the data extracted from the FEA results using the least squares method.

Figure 12 shows the pile lateral load-deflection responses obtained by performing $p$-$y$ analyses on a 0.36-m-diameter and 10-m-long pile using the $p$-$y$ curves proposed in this study. The commercial $p$-$y$ analysis software PYGMY (Stewart 2000) was used to perform the $p$-$y$ analyses for single piles in uniform loose and dense sands. As shown in Figure 12, the $p$-$y$ curves obtained using Equations (1) to (4) generate lateral load-deflection pile responses that are in good agreement with the ones from the 3D FE analyses.

![Figure 12 Results of p-y analyses using the p-y curves proposed in this study for (a) uniform loose ($D_R = 40\%$) sand, and (b) uniform dense ($D_R = 80\%$) sand.](image)

The proposed $p$-$y$ curves are tested on a single pile with the pile length $L=10$m and $B=0.36$m sand with a relative density of 50%. Figure 13 compares the $p$-$y$ analysis results using the proposed equations and the lateral load-deflection curve extracted from the FEA results. The proposed equations yield good predictions of the load deflection curve of the pile.
The proposed $p$-$y$ curves for sand also work well for layered sand profiles. Figure 14 shows the results of $p$-$y$ analyses for a single pile ($B = 0.36$ m and $L = 10$ m) performed using PYGMY (Stewart 2000) with the proposed $p$-$y$ curves for two different layered sand profiles: dense-over-loose ($D_R = 80\%$ over $D_R = 40\%$) sand profile with the thickness of the upper layer equal to $3B$, and loose-over-dense ($D_R = 40\%$ over $D_R = 80\%$) sand profile with the thickness of the upper layer equal to $3B$. As shown in the figure, the pile lateral load-deflection responses obtained from the $p$-$y$ analyses are in close agreement with those obtained from the 3D FE analyses.
80% over \(D_R = 40\%) \) sand profile and (b) loose-over-dense \(D_R = 40\% \) over \(D_R = 80\%) \) sand profile.

5. PROPOSED p-y RELATIONSHIPS FOR CLAY

5.1. General Discussion

As done for sand, p-y curves for clay can be derived from the results of the 3D FE analyses. Figure 15(a) shows the p-y curves obtained at different depths for a single pile \((B = 0.36 \text{ m and } L = 10 \text{ m})\) in normally consolidated (NC) clay loaded under undrained conditions. The normalized unit soil resistance \(p/B\) increases with increasing depth. Referring to the p-y curves for the same pile in loose sand \((D_R = 40\%)\), shown in Figure 15(b), the p-y curves at a fixed depth are softer for the NC clay than for the loose sand, leading to a softer global load-deflection response in NC clay (refer to Figure 6).

Figure 15 p-y curves obtained at different depths from the FE analyses of single piles \((B = 0.36 \text{ m and } L = 10 \text{ m})\) in: (a) NC clay under undrained conditions and (b) loose sand \((D_R = 40\%)\).

Figure 16 shows the p-y curves obtained from the FEA at different depths for a single pile \((B = 0.36 \text{ m and } L = 20 \text{ m})\) in uniform NC clay under undrained conditions when the lateral load is applied with load eccentricity \(h = 0 \text{ m and } 10 \text{ m}\). Similarly, to the results for sand shown in Figure 9, the p-y curves at different depths for a single pile in NC clay are almost the same for the different load eccentricities.
Figure 16 $p$-$y$ curves obtained at different depths from the FE analyses of single piles ($B = 0.36$ m and $L = 20$ m) with load eccentricities $h = 0$ m and 10 m in NC clay under undrained conditions.

Figure 17 shows the effect of the pile diameter on the $p$-$y$ curves for a single pile in NC clay under undrained conditions. Three diameters are considered, 0.36 m, 0.6 m and 1.0 m for 20-m-long piles. As shown in Figure 17, a single pile with a larger pile diameter in NC clay provides slightly lower $p/B$ for the same pile deflection $y$ at the same depth, but the difference in the $p$-$y$ curves for different pile diameters becomes negligible as the depth increases.
Figure 17 The $p$-$y$ curves obtained from the FE analyses for single piles with a diameter $B$ of 0.36 m, 0.6 m and 1.0 m in NC clay under undrained condition (a) at shallow depths less than 3 m, and (b) at depths greater than 3 m.

5.2. $p$-$y$ Curves for Normally Consolidated Clay Under Undrained Conditions

Based on the results of the FEA performed for laterally loaded single piles, new $p$-$y$ curves are proposed for NC clay. The relationship between the unit soil resistance $p$ (with units of force per length) and the pile deflection $y$ (with units of length) is given by

$$p = p_u \begin{cases} \frac{y}{y_c} & \text{when } y < y_c, \text{ and} \\ a \left( \frac{y}{y_c} \right) + 1 - a & \end{cases}$$

when $y \geq y_c$

where the limit unit soil resistance $p_u$ (with units of force per length) and $y_c$ are given by

$$p_u = \min \left[ 64.0 \left( \frac{s_u}{p_A} \right) + 1.75 \frac{L_R}{B} + 5.05 \right] \times 11.67 s_u B$$

and

$$y_c = 0.0579B + \left( 0.35 \frac{B}{L_R} \right) \exp \left[ -23.0 \frac{B}{L_R} - 80.0 \right] \frac{s_u}{p_A} L_R$$

with
In these equations, $s_u$ = undrained shear strength of the clay, $B$ = pile diameter, $p_a$ (= 100 kPa ≈ 1t every) = reference stress, and $L$ (= 1 m ≈ 3.281 ft or equivalent in other units) = reference length. The equation is fitted to the data extracted from the FEA results using the least squares method.

Most currently used $p$-$y$ curves are derived by back-calculation from field test data. The pile installation method has a considerable influence on the soil reaction. Thus, the back-calculated $p$-$y$ curves are also dependent on the method of pile installation. Currently, API curves are the most often used $p$-$y$ curves when designing laterally loaded piles, and the API curves would strictly apply only to driven piles. The $p$-$y$ curves extracted from the simulation results presented here are for wished-in-place piles, in which the installation effect is neglected. Because modeling of pile installation is still challenging, if possible at all, comparison of results from analysis performed for wished-in-place piles to results of field tests is not uncommon (see, e.g., Burd et al. 2020b). Figure 18 shows a comparison between $p$-$y$ curves obtained by using Equations (5) to (8) and $p$-$y$ curves obtained from the API method for the 0.36-m-diameter single pile in NC clay. As shown in the figure, the $p$-$y$ curves proposed in this study provide substantially stiffer responses than those from the API (API 1993) method. This raises the question of which set of curves provides a more realistic estimate of the response of a real pile. Comparisons of $p$-$y$ analysis results with finite element results and both centrifuge and field load test results are helpful in arriving at an answer.
We compared the results from an FEA and a p-y analysis of a 0.36-m-diameter, 10-m-long pile in uniform NC clay using the p-y curves proposed in this study for clay and the API curves. The p-y analyses were performed using PYGMY (Stewart 2000). In the p-y analysis using the proposed p-y curves, the top 6 m of the soil profile was discretized into 19 layers, each with a thickness equal to approximately 0.32 m. As shown in Figure 19, the load-deflection response obtained from the p-y analysis using the proposed p-y curves is in close agreement with that obtained from the 3D FE analysis of the same problem. The p-y curves given by Equations (5) to (8) are used to represent the mechanical response of each layer. The load-deflection response obtained from the p-y analyses using the API (API 1993) p-y curves for clay is conservative when compared with that obtained from the 3D FE analysis, regardless of the value of the empirical parameter $J$ that appears in the API relationships. This conclusion is consistent with the findings of Jeanjean (2009), who performed a series of centrifuge tests on a laterally loaded pipe instrumented with a series of strain gauges in fine Alwhite kaolin under monotonic and cyclic load conditions, as well as an FEA analysis matching the prototype geometry of the centrifuge test. By comparing the measured soil resistance, the FEA-derived soil resistance, and the API curves, they concluded that the API p-y curves were conservative.
Figure 19 Results of p-y analysis using the p-y curves proposed in this study for NC clay under undrained conditions. The load-deflection responses obtained using the API (API 1993) p-y curves with the empirical parameter \( J = 0.25 \) and \( J = 1 \) are also plotted for comparison.

To perform a comparison of analyses using the different p-y formulations with experimental data, we referred to the centrifuge lateral load tests performed by Liu et al. (2022) on piles in NC and OC clay, by Yu et al. (2017) on piles in NC clay and by Truong and Lehane (2018) on piles in NC clay. Centrifuge data on lateral loading of piles in clay are limited. The three publications to which we refer here contain the most comprehensive records, with all the information required for calculations.

The model piles that used by Liu et al. (2022) were made of aluminum, with Young’s modulus of 75 GPa. The piles were embedded 330mm in the soil, corresponding to 13.2m in prototype scale. The pile heads were 50mm above the soil surface, corresponding to 2m for the prototype. The pile diameter was 19mm (0.8m for the prototype), and the wall thickness was 1mm (0.023m for the prototype). The undrained shear strength \( s_u \) is zero at the soil surface and grows approximately linearly with depth. We performed p-y analyses using the proposed equations based on the data provided by Liu et al. (2022) and compared the results with those calculated using the API p-y curves. When performing the analyses, we estimated \( s_u \) using a linear equation fitted to the original profile provided by Liu et al. (2022). The equation for \( s_u \) is:

\[
s_u = 1.6667 \times z \tag{9}
\]
where $s_u$ is in kPa and $z$ is the depth from the ground surface to the calculation point in meters. The results are shown in Figure 20. The proposed equations tend to overestimate the stiffness of the pile-soil interaction system, while the API curves tend to underestimate it substantially. The pile capacity calculated using the proposed equation overestimates measured values by 27%, whereas the pile capacity calculated using API curves underestimates measured values by 70%.

![Figure 20 Comparison between p-y analysis results using proposed equations and API curves based on the data provided by Liu et al. (2022).](image)

In Yu et al. (2017), the model piles were open-ended pipe piles made of fabricated hollow stainless-steel pipe coated with a thin layer of epoxy. In prototype scale, the pile was 2 m in diameter and 14 m in length, and was embedded 12 m into the soil. The flexural rigidity $EI$ of the pile was 30 GNm$^2$. The pile was pushed into the soil at a constant rate of 1 mm/s. The soil sample was normally consolidated Kaolin clay with a water content of 120%. The undrained shear strength profile of the soil was estimated using a T-bar penetrometer. The relationship between the undrained shear strength $s_u$ and the depth to the mudline $z$ can be described as:

$$s_u = 1.39z$$

(10)

where $s_u$ is in kPa, and $z$ is in m. The unit weight of the soil is 16.4 kN/m$^3$. We performed p-y analyses using the proposed equations and the API curves. The results are shown in Figure 21. We can see that the proposed equations overestimate the measured data by 5% to 50%, while the API curves underestimate the measured data by 40% to 75%.
Truong and Lehane (2018) performed a centrifuge test to investigate the effects of pile shape and pile end conditions on the lateral response of displacement piles in soft clay. Both closed-ended pipe piles and open-ended pipe piles were used in the test, and the results showed that the lateral response for open-ended pipe piles and closed-ended pipe piles with the same circular cross sections and the same flexure rigidity are almost the same when the lateral displacement at the pile head is small. The piles are jacked into the sample at a rate of 2 mm/s using the actuator. We compared the $p-y$ analysis results using the proposed equations and the API curves with the measured data for their open-ended pipe pile results. The model pile was 132 mm long, corresponding to 10.56 m in prototype scale. The outer diameter of the model pile was 11 mm, corresponding to 0.88 m in prototype scale. The load was applied to the pile with a load eccentricity of 17 mm, corresponding to 1.32 m in prototype scale. The flexural rigidity $EI$ of the pile in prototype scale is $1 \times 10^6$ kNm$^2$. The soil was normally consolidated kaolin clay, and the undrained shear strength profile was measured using isotropically consolidated triaxial compression tests and T-bar tests. The T-bar test results showed that the value of $s_u$ at the surface was minimal. The average rate of increase of $s_u$ with depth measured from the T-bar test for normally consolidated clay was given as 1.4 kPa/m. With all the information above, the variation of $s_u$ with depth $z$ from the mudline can be described by

$$s_u = 1.4z \quad (11)$$
where $s_u$ is in kPa, and $z$ is in m. The results are shown in Figure 22. We can see that the $p-y$ analysis results obtained using the proposed equations at worst overestimate the capacity of the pile by less than 10%, but the $p-y$ analysis using the API curves tends to underestimate the capacity of the pile by more than 60%.

![Figure 22](image)

**Figure 22** The $p-y$ analysis results of centrifuge test performed in Truong and Lehane (2018).

Results of well documented lateral load pile tests performed in the field are scarce, but provide the best reference data for comparison when they are of sufficient quality and the soil has been well characterized. Matlock et al. (1980) performed such tests on piles installed in Harvey, Louisiana. The tests were performed several decades ago using the technology available at the time. One limitation of the data is the lack of CPT test results or high-quality laboratory test data. The top 2.44 m of soil were excavated prior to the test to remove organic materials, and thus the mudline of this test was set at 2.44 m below the soil surface. The piles were steel open-ended pipe piles with an outer diameter of 16.83 cm and a wall thickness of 0.71 cm. The piles were driven into the soil to a depth of 11.58 m, and the load was applied to the pile at an elevation 0.23 m above the mudline. The lateral deflection was measured at a distance of 0.99 m above the mudline. The soil was very soft clay.

*In situ* vane shear test and triaxial compression tests were performed to estimate the undrained shear strength $s_u$ of the soil. However, the significant difference between the results of these different types of tests highlights some uncertainty as to the $s_u$ profile to use to perform
the $p-y$ analyses. We set as the upper bounds to the $s_u$ profile the profile resulting from the vane shear test. As the lower bound to the $s_u$ profile, we used the results from the triaxial tests, as shown in Figure 23. According to Brown (1985), the water table is above the soil surface in this case. Assuming the soil's saturated unit weight to be 20.4 kN/m$^3$ and the ratio between the incremental undrained shear strength and the incremental vertical effective stress to be equal to 0.3 for normally consolidated clay, we can also estimate the $s_u$ profile using:

$$\frac{s_u}{\sigma_v'} = \left(\frac{s_u}{\sigma_v'}\right)_{NC} \times OCR^{0.8}$$  \hspace{1cm} (12)

The resulting profile will be referred to as the "estimated $s_u$ profile." It lies slightly above the lower bound profile. We performed $p-y$ analyses based on the maximum $s_u$ profile, the minimum $s_u$ profile, and the estimated $s_u$ profile using both the proposed equations and the API curves. Figure 24 shows the results of these analyses: the lateral load test data lies between the upper and lower bound of the $p-y$ analysis results calculated using the proposed equation. The load-deflection curve calculated using API curves is again much lower than that resulting from the test results. The proposed $p-y$ curves perform better, even if the uncertainty about undrained shear strength profile prevents definitive conclusions.

![Figure 23 Undrained shear strength $s_u$ profile used in $p-y$ analyses for lateral load test conducted in Harvey, Louisiana.](image)
Figure 24 $p$-$y$ analysis results for the lateral load test conducted in Harvey, Louisiana.

6. SUMMARY AND CONCLUSIONS

In this paper, we examined the factors that have an impact on $p$-$y$ curves for laterally loaded piles, and proposed a new set of $p$-$y$ curves for use in practice. For this purpose, we performed a series of finite element analyses for single piles in sand (using a two-surface constitutive model calibrated for Ottawa sand) as well as in normally consolidated clay (using a two-surface constitutive model calibrated for Boston Blue Clay) under undrained conditions. The simulations cover a wide range of pile diameters (0.36 m to 1 m) and load eccentricities. For sand, we also considered different relative densities. The analyses are for "wished-in-place" piles.

For sand, we found that the $p$-$y$ curves depend on the relative density of the sand. The greater the relative density of the sand, the stiffer the $p$-$y$ curves. Furthermore, the load eccentricity has negligible impact on the $p$-$y$ curves. Additionally, when the $p$-$y$ curves for the same sand relative density and the same pile diameter for the same initial vertical effective stress $\sigma'_{v0}$ are compared for the cases with different groundwater level elevations, we find that groundwater level or cross section depth has minimum impact on the $p$-$y$ curves.

For normally consolidated clay, we found that the $p$-$y$ curves depend on the pile diameter. A single pile with a larger pile diameter in NC clay develops slightly lower $p/B$ for the same pile deflection $y$ at the same depth, but the difference in $p$-$y$ curves for different pile
diameters becomes negligible as the depth increases. The extracted $p-y$ curves were fitted using the least squares method, and equations for the $p-y$ curves were proposed. Pile geometry parameters—such as pile diameter—and soil variables—such as relative density, initial vertical effective stress, and undrained shear strength—appear in the $p-y$ equations proposed.

Comparison of the proposed $p-y$ curves for clay with the corresponding $p-y$ curves proposed by API (API 1993) shows that the proposed $p-y$ curves are generally stiffer. Comparisons of lateral load versus lateral deflection plots from $p-y$ analyses using both the proposed $p-y$ curves and those from API show that API may significantly underpredict lateral load capacity of piles in clay.

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