

Supplementary Materials:

Engineering Complex Vectorial Light Field in Leaky-Wave Metasurfaces: Harnessing Structural Parameter-Independent Detour and Geometric Phases

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1 Azimuth and ellipticity of orthogonal polarization states on a Poincaré sphere

Arbitrary polarized light $\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} A \cdot \cos \alpha \\ A \cdot \sin \alpha e^{i\delta} \end{bmatrix}$ can be synthesized from linearly polarized components of x - and y -polarization, which have a phase difference denoted by δ . Defining $\tan \alpha = \frac{A_y}{A_x}$, the azimuth angle (ψ) and ellipticity (χ) of an arbitrary polarization state can be described by relationships $\tan 2\psi = \tan 2\alpha \cdot \cos \delta$ and $\sin 2\chi = \sin 2\alpha \cdot \sin \delta$, respectively. Different polarization states are represented as points on the Poincaré sphere using Stokes parameters ($S_0 = E_x^2 + E_y^2 = 1$, $S_1 = E_x^2 - E_y^2 = \cos 2\chi \cdot \cos 2\psi$, $S_2 = 2E_x E_y \cos \delta = \cos 2\chi \cdot \sin 2\psi$ and $S_3 = 2E_x E_y \sin \delta = \sin 2\chi$), as illustrated in Fig. S1. The azimuth angle (ψ) is defined as the angle between the main axis of the polarization ellipse and the positive x -axis direction, indicating the ellipse's orientation within the plane, as detailed in $-\pi/2 < \psi < \pi/2$. Ellipticity (χ) is characterized by $\tan \chi = \pm \frac{b}{a}$, where a is the half major axis and b is the half minor axis of the ellipse. The sign of χ differentiates between right-handed (positive) and left-handed (negative) polarizations.

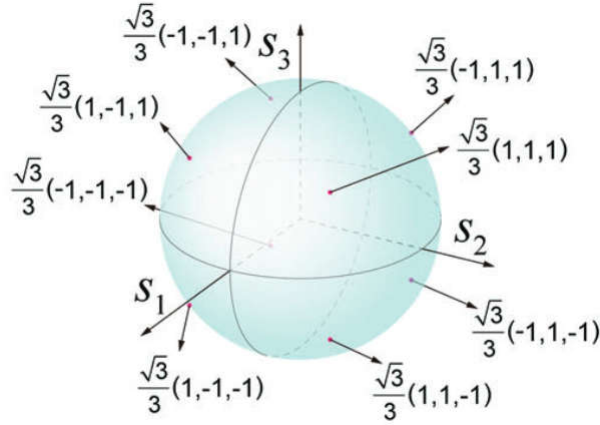


Figure S1: Different polarization states are represented as points on the Poincaré sphere using Stokes parameters.

Arbitrary polarization states can be decomposed into a pair of orthogonal polarization states. These orthogonal states may include vertical versus horizontal polarization, positive or negative 45-degree polarization, left- or right-handed circular polarization, or polarization states with any specific azimuth angle. Table S2 presents a summary of the azimuth angles and ellipticity values for various orthogonal polarization states, culminating in a conclusion of perpendicular azimuthal angles and opposite ellipticity values.

To delve deeper into the limitations placed on the azimuth angle and ellipticity during the decomposition of orthogonal polarization, we set the azimuth angles at $\pi/4$ and $-\pi/4$ respectively, varied the phase difference δ , and computed the ellipticity (χ) for different polarization states. Table S3 compiles the outcomes of these theoretical calculations, affirming the conclusion that perpendicular azimuth angles are associated with opposing ellipticity.

	Azimuth/2 (ψ)	Ellipticity/2 (χ)	Stokes Vector	Jones Basis	Orthogonal Polarization States	
Horizontal Polarization	0	0	(1,0,0)	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$		
Vertical Polarization	$\pi/2$	0	(-1,0,0)	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$		
+45° Polarization	$\pi/4$	0	(0,1,0)	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$		
-45° Polarization	$-\pi/4$	0	(0,-1,0)	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$		
RCP	NA	$\pi/4$	(0,0,1)	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ i \end{bmatrix}$		
LCP	NA	$-\pi/4$	(0,0,-1)	$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ -i \end{bmatrix}$		
λ_1^+	$\pi/4$	$\pi/8$	$\frac{\sqrt{2}}{2}(0,1,1)$	$\begin{bmatrix} \cos \frac{\pi}{4} \\ \sin \frac{\pi}{4} e^{i\pi/4} \end{bmatrix}$		
λ_1^-	$-\pi/4$	$-\pi/8$	$\frac{\sqrt{2}}{2}(0,-1,-1)$	$\begin{bmatrix} \cos -\frac{\pi}{4} \\ \sin -\frac{\pi}{4} e^{i\pi/4} \end{bmatrix}$		
λ_2^+	$-\pi/8$	$\sin^{-1} \frac{\sqrt{3}}{3}/2$	$\frac{\sqrt{3}}{3}(1,-1,1)$	$\begin{bmatrix} \cos(-\frac{\tan^{-1} \sqrt{2}}{2}) \\ \sin(-\frac{\tan^{-1} \sqrt{2}}{2}) e^{i\pi/4} \end{bmatrix}$		
λ_2^-	$3\pi/8$	$-\sin^{-1} \frac{\sqrt{3}}{3}/2$	$\frac{\sqrt{3}}{3}(-1,1,-1)$	$\begin{bmatrix} \cos(-\frac{\tan^{-1} \sqrt{2}}{2} + \frac{\pi}{2}) \\ \sin(-\frac{\tan^{-1} \sqrt{2}}{2} + \frac{\pi}{2}) e^{i\pi/4} \end{bmatrix}$		

Figure S2: Various orthogonal polarization states.

Phase difference (δ)	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$	π
Azimuth Angle (ψ)	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$
Ellipticity (χ)	0	$\frac{\pi}{16}$	$\frac{\pi}{8}$	$\frac{3\pi}{16}$	$\frac{\pi}{4}$	$\frac{3\pi}{16}$	$\frac{\pi}{8}$	$\frac{\pi}{16}$	0
									
Phase difference (δ)	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$	$\frac{5\pi}{8}$	$\frac{3\pi}{4}$	$\frac{7\pi}{8}$	π
Azimuth Angle (ψ)	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$	$-\frac{\pi}{4}$
Ellipticity (χ)	0	$-\frac{\pi}{16}$	$-\frac{\pi}{8}$	$-\frac{3\pi}{16}$	$-\frac{\pi}{4}$	$-\frac{3\pi}{16}$	$-\frac{\pi}{8}$	$-\frac{\pi}{16}$	0
									

Figure S3: Ellipticity and azimuth angle corresponding to various polarization states.

2 Matrix representation of interference metasurfaces

We first illuminate a single rectangular nanorod structure with linearly polarized wave, and its scattered field is expressed as:

$$\tilde{E} = [\hat{x}, \hat{y}] \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \cos(2\theta) \\ \frac{\sqrt{2}}{2} \sin(2\theta) \end{bmatrix} \cdot e^{i\phi} \quad (1)$$

, where θ and ϕ represent the rotation angle and the transmission phase of the nanorod. Replacing the single structure with a pair of nanorods, the scattering field is:

$$\tilde{E} = [\hat{x}, \hat{y}] \cdot \frac{\sqrt{2}}{4} \begin{bmatrix} \cos(2\theta_1) \cdot e^{i\phi_1} + \cos(2\theta_2) \cdot e^{i\phi_2} \\ \sin(2\theta_1) \cdot e^{i\phi_1} + \sin(2\theta_2) \cdot e^{i\phi_2} \end{bmatrix}. \quad (2)$$

The first case: when the transmission phases are equal, that is, $\phi_1 - \phi_2 = 0$, and using the cosine function relation $\cos \theta_1 + \cos \theta_2 = 2 \cos((\theta_1 + \theta_2)/2) \times \cos((\theta_1 - \theta_2)/2)$, Eq. 2 is rewritten as:

$$\tilde{E} = [\hat{x}, \hat{y}] \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} \cos(\theta_1 + \theta_2) \cdot \cos(\theta_1 - \theta_2) \cdot e^{i\phi} \\ \frac{\sqrt{2}}{2} \sin(\theta_1 + \theta_2) \cdot \cos(\theta_1 - \theta_2) \cdot e^{i\phi} \end{bmatrix} = [\hat{x}, \hat{y}] \cdot \frac{\sqrt{2}}{2} \cos(\theta_1 - \theta_2) \begin{bmatrix} \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) \end{bmatrix} \cdot e^{i\phi} \quad (3)$$

Equation 5 shows that the intensity and polarization direction of the interference field obtained by a pair of identical metasurfaces depends on the respective rotation angles.

The second case: the transmission phase difference of a pair of metasurfaces is π , that is, $\phi_1 - \phi_2 = \pi$. Equation 2 can be simplified to:

$$\begin{aligned} \tilde{E} &= [\hat{x}, \hat{y}] \cdot \begin{bmatrix} -\frac{\sqrt{2}}{2} \sin(\theta_1 + \theta_2) \cdot \sin(\theta_1 - \theta_2) \cdot e^{i\phi} \\ \frac{\sqrt{2}}{2} \cos(\theta_1 + \theta_2) \cdot \sin(\theta_1 - \theta_2) \cdot e^{i\phi} \end{bmatrix} \\ &= [\hat{x}, \hat{y}] \cdot \frac{\sqrt{2}}{2} \sin(\theta_1 - \theta_2) \begin{bmatrix} \cos(\theta_1 + \theta_2 + \frac{\pi}{2}) \\ \sin(\theta_1 + \theta_2 + \frac{\pi}{2}) \end{bmatrix} \cdot e^{i\phi} \end{aligned} \quad (4)$$

3 A general design flow for leaky-wave metasurfaces

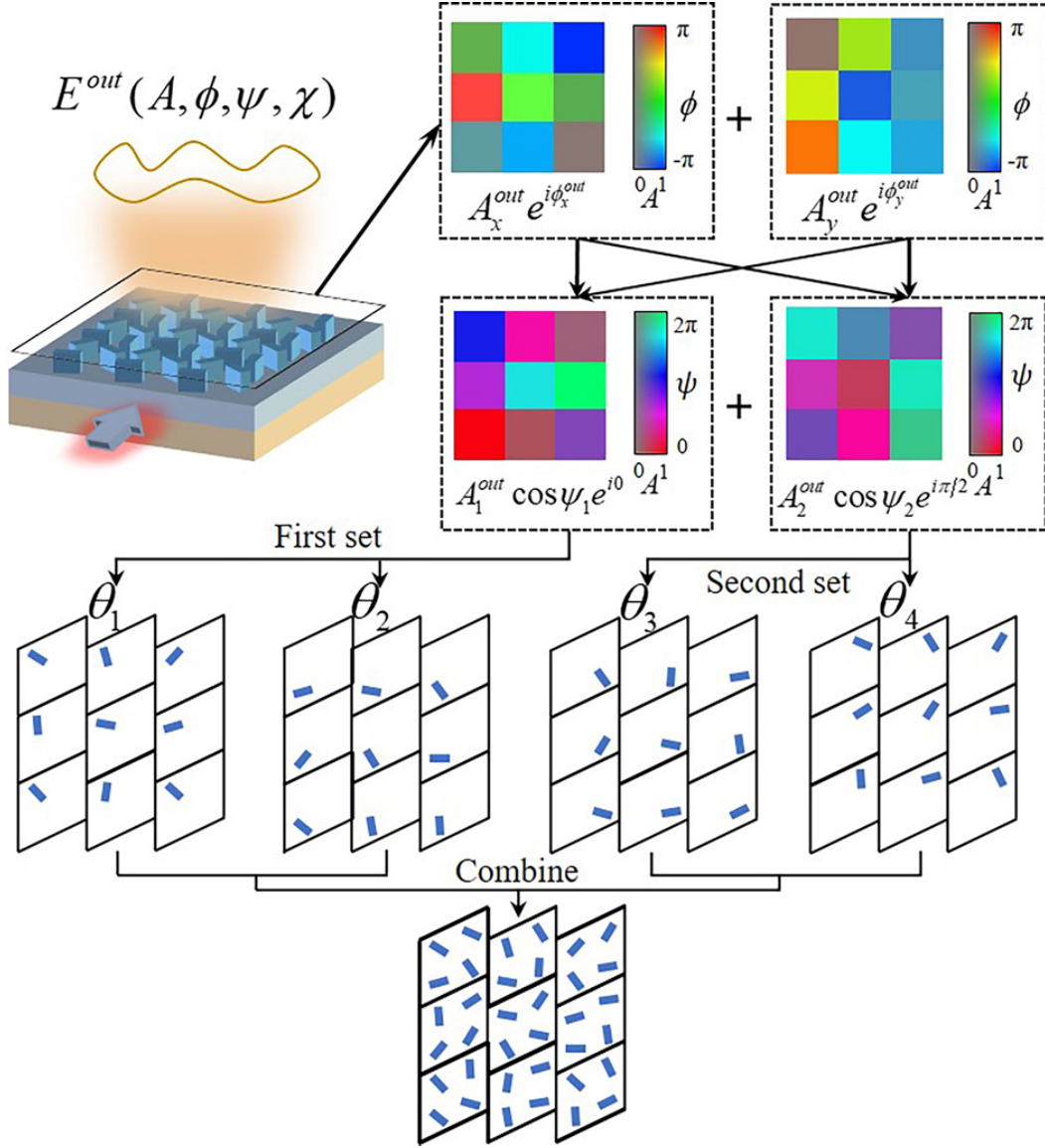


Figure S4: A general design flow for leaky-wave metasurfaces enabling comprehensive amplitude, phase, and polarizations control.

4 Intuitive demonstration of complete control over four degrees of freedom

Figure (a) shows the field intensity distribution on the $z - x$ plane in free space under the excitation of the TE fundamental mode of 0.6THz propagating along the z -axis. The guided wave mode is scattered into free space, and its energy is focused to specific positions (-2mm, 12mm) and (2mm, 12mm) respectively, which are very close to the theoretical design positions. It can also be observed that both metalens achieve better focusing effects, but the field strength at the focus is different. To conduct a more detailed analysis of the field strength at the focus, $x = 12\text{mm}$ on the focal plane is selected. The field strength information is extracted and plotted in Figure (b). According to the results in Figure (b), the ratio of the electric field strengths of the two metalens at the focus is 0.4971: 1, which is consistent with the designed field strength ratio of 0.5: 1. At the same time, our work also extracted the electric field components at the two focal points and analyzed their respective polarization states, as shown in Figure (c). The results show that compared with the designed left-handed circular polarization state and the linear polarization state along the 45-degree direction, the polarization state obtained by simulation and the theoretical polarization state show quite good consistency.

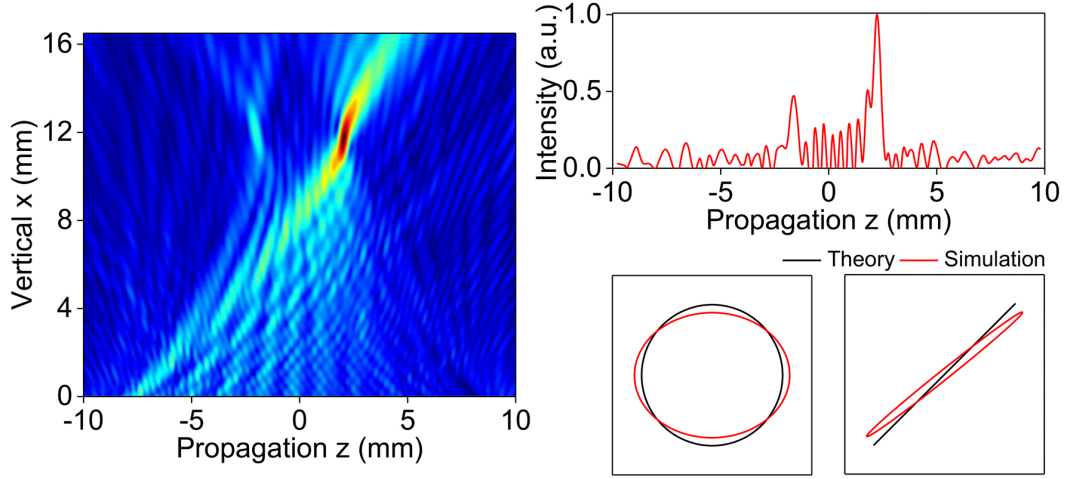


Figure S5: Simulation results of leaky-wave metasurface fully controlling amplitude, phase and polarization. (a) Field intensity on the cross section z - x ; (b) Field intensity on the focal plane $x=12\text{mm}$; (c) Polarization states at the two foci.

5 Simulation results of the second case

For the second case, the two metasurface structures within each set possess identical transmission phases (detour phases), but differ in their rotation angles θ_1 and θ_2 (or θ_3 and θ_4), while maintaining a complete *PCR* and equal diffraction ratios of $T_1 = T_2 = 1$ (or $T_3 = T_4 = 1$). The resulting polarization states are represented as follows (for a detailed derivation, see the Supplemental Material):

$$E_{\theta_1\theta_2} = \frac{\sqrt{2}}{2} \cos(\theta_1 - \theta_2) e^{i\phi_{12}} \begin{bmatrix} \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) \end{bmatrix}. \quad (5)$$

$$E_{\theta_3\theta_4} = \frac{\sqrt{2}}{2} \cos(\theta_3 - \theta_4) e^{i\phi_{34}} \begin{bmatrix} \cos(\theta_3 + \theta_4) \\ \sin(\theta_3 + \theta_4) \end{bmatrix}. \quad (6)$$

The Eqs. (5) and (6) correspond to the polarized vectors $\vec{A}_1$ and $\vec{A}_2$, respectively. From this, we deduce:

$$\begin{aligned} A_1 &= \frac{\sqrt{2}}{2} \cos(\theta_1 - \theta_2) \\ \psi_1 &= \theta_1 + \theta_2 \\ A_2 &= \frac{\sqrt{2}}{2} \cos(\theta_3 - \theta_4) \\ \psi_2 &= \theta_3 + \theta_4 \end{aligned} \quad (7)$$

Furthermore, by comparing Eqs. (1) and (2) in the main text with the results $A_1 \cos \psi_1 = \text{real}(E_x)$, $A_2 \cos \psi_2 = \text{imag}(E_x)$, $A_1 \sin \psi_1 = \text{real}(E_y)$ and $A_2 \sin \psi_2 = \text{imag}(E_y)$, we arrive at:

$$\begin{aligned} \theta_1 - \theta_2 &= \arccos \sqrt{2[\text{real}(E_x)]^2 + 2[\text{real}(E_y)]^2} \\ \theta_1 + \theta_2 &= \text{angle}[\text{real}(E_x) + 1i \cdot \text{real}(E_y)] \\ \theta_3 - \theta_4 &= \arccos \sqrt{2[\text{imag}(E_x)]^2 + 2[\text{imag}(E_y)]^2} \\ \theta_3 + \theta_4 &= \text{angle}[\text{imag}(E_x) + 1i \cdot \text{imag}(E_y)] \end{aligned} \quad (8)$$

Hence, we can establish the magnitude and phase distribution of E_x and E_y for the output wave relative to the intended target. By substituting these into Eq. (8), we can determine the rotation angles for the two pairs of metasurface structures within each cell. Importantly, the approach is supplemented by the strategic implementation of a phase shift of $\pi/2$ between the two sets of metasurfaces, which is adeptly accomplished through the application of detour phases.

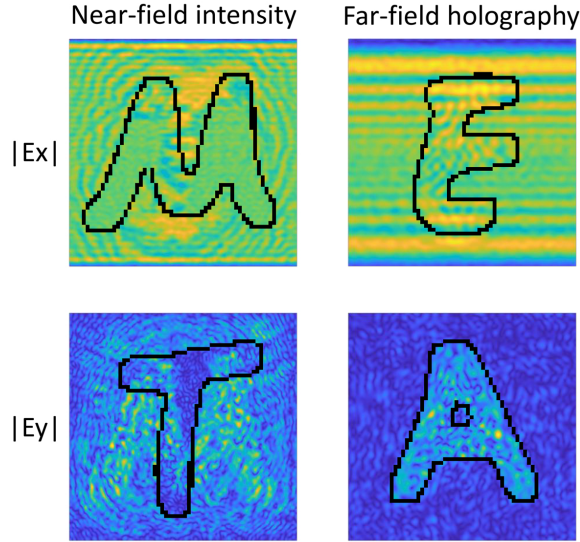


Figure S6: Near-field intensity imaging and far-field holography designed by the second scheme. The imaging effect is not perfect. The reason is that the structures that affect the light field are too close, resulting in the size of the scattering structure being too small, which further causes the scattering field intensity to be too weak.

6 Guide to the simulation data sheet

The data sheet comprises seven columns, detailing the x-direction cell number of the structure's location, y-direction cell number, x-coordinate, y-coordinate, rectangle width, rectangle length, and the structure's rotation angle. The simulation data consists of five sets: the 128x128 resolution 'META' four-channel image generated by the Gerchberg-Saxton (GS) algorithm, the 200x200 resolution far-field holographic image from the gradient optimization algorithm, and the 200x200 resolution zodiac image also produced using the gradient optimization algorithm.

Function 1: GS version of holographic letters "M", "E", "T" and "A"

Name: metaAtomArray-META-b-GS-neff3.3286f12mm-Ex0.5-Ey0.5-200x200.mat

Function 2: Descending version of holographic letters "M", "E", "T" and "A"

Name: metaAtomArray-META-b-MAPS-neff3.3286f12mm-Ex0.5-Ey0.5-200x200.mat

Function 3: GS version of the holographic zodiac signs "Rat", "Ox", "Tiger" and "Rabbit"

Name: metaAtomArray-Sign1234-b-GS-neff3.3286f12mm-Ex0.5-Ey0.5-200x200.mat

Function 4: GS version of the holographic zodiac signs "Dragon", "Snake", "Horse" and "Sheep"

Name: metaAtomArray-Sign5678-b-GS-neff3.3286f12mm-Ex0.5-Ey0.5-200x200.mat

Function 5: GS version of the holographic zodiac signs "monkey", "rooster", "dog" and "pig"

Name: metaAtomArray-Sign9101112-b-GS-neff3.3286f12mm-Ex0.5-Ey0.5-200x200.mat