Supplementary Information: Edge modes in 1D microwave photonic crystal

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ABSTRACT

The microstrip of modulated width is a realization of a one-dimensional photonic crystal operating in the microwave regime. Like any photonic crystal, the periodic microstrip is characterised by the presence of frequency bands and band gaps that enable and prohibit wave propagation, respectively. The frequency bands for microstrip of symmetric unit cell can be distinguished by 0 or π Zak phase. The sum of these topological parameters for all bands below a given frequency gap determines the value of the surface impedance and whether or not edge modes are present at the end of the microstrip. We demonstrate that edge modes are absent in a finite microstrip terminated at both ends in the centres of unit cells, but they can be induced by adding the defected cells. Edge modes present at both ends of the microstrip enable microwave tunneling with high transitivity in the frequency gap with or without a change in phase. This has been demonstrated experimentally and developed in detail using numerical simulations and model calculations. The investigated system, with a doublet of edge modes in the frequency gap, can be considered as a narrow passband filter of high selectivity.

S1. Impedance and effective permittivity for uniform microstrip

The effective permittivities and impedances for wider (ε_1, Z_1) and narrower (ε_2, Z_2) segments of the microstrip (see Fig.1 in the manuscript) were calculated using the approximate formulas[1]:

$$\varepsilon_{1,2} = \frac{\varepsilon+1}{2} + \frac{\varepsilon-1}{2\sqrt{1+12\,r_{1,2}}}, \tag{1}$$

and

$$Z_{1,2} = \frac{120\pi}{\sqrt{\varepsilon_{1,2}}\left(r_{1,2}+1.393+\frac{2}{3}\left(r_{1,2}+1.444\right)\right)}, \tag{2}$$

where $r_{1,2} = w_{1,2}/u$. In periodic microstrip the effective permittivity and impedance can be considered as functions of $z-$coordinate: $\varepsilon(z)$, $Z(z)$, taking two alternative values: ε_1, ε_2 and Z_1, Z_2 as z is increasing along the microstrip.

S2. Bloch impedance

The electric and magnetic fields in a selected section of the waveguide can be expressed in terms of left and right propagating plane waves $E_k = E_+ + E_-$, $H_k = H_+ + H_- = 1/Z(E_+ - E_-)$:

$$\begin{aligned} Z_B &= Z\frac{E_+ + E_-}{E_+ - E_-} = \\ &= Z\frac{1+r_-}{1-r_-} = -Z\frac{1+r_+}{1-r_+}, \end{aligned} \tag{3}$$

where r_-, r_+ are the reflection coefficients for the waves reflected at the left and right sides, respectively. The symbol Z is the wave impedance for the selected section of the waveguide $Z = Z_1$ or $Z = Z_2$.

In the general case, the Bloch impedance can also be expressed in terms of the logarithmic derivatives $\rho(k,x)$ of the Bloch function $E_k(z)$ and $H_k(z)$, expressed as $\partial_z \ln(E_k(z))$ or $\partial_z \ln(H_k(z))$. To prove this, we can use the following relations resulting from Maxwell's equations: $\partial_z E_k(z) = i\omega\mu_0 H_k(z)$, $\partial_z H_k(z) = i\omega\varepsilon\varepsilon_0 E_k(z)$:

$$Z_B(z) = -i\frac{\partial_z \ln(H_k(z))}{\mu_0\omega} = i\frac{\varepsilon_0\varepsilon(z)}{\partial_z \ln(E_k(z))}. \tag{4}$$

In the frequency gaps, the logarithmic derivative of the Bloch function is a real-valued function of the frequency and reaches the zeros and pols at the edges of the gaps (depending on the symmetry of the Bloch function)[2]. These properties justify the characteristics of the Bloch impedance in the frequency gaps $Z_B = i\xi$.

It is also known that the logarithmic derivative of the Bloch function $\rho(k,x)$ has the following property[2,3]:

$$\rho(k,-z) = -\rho(-k,z)^*, \tag{5}$$

where the asterisk '*' denotes complex conjugation. This property means that in a frequency gap where ρ is real and $k = n\pi/d + ik_I$, changing the side of the interface/surface ($x \leftrightarrow -x$) and the direction of the decay for the interface/surface mode ($k_I \leftrightarrow -k_I$) results in the fillip of the logarithmic derivative and the imaginary part of the Bloch impedance ($\xi \leftrightarrow -\xi$).

The logarithmic derivative taken at the boundary of the centrosymmetric unit cell $x = x_0$ can change its sign if we select the centrosymmetric cell alternatively (i.e. for $z_0 \to z_0 + d/2$). The change of sign is observed in every second gap, i.e. for the gaps $n = 1,3,5,\ldots$, which are located at the edge of the 1$^{\text{st}}$ Brillouin zone:

$$\rho(\pi/d + ik_I, z_0) = -\rho(\pi/d + ik_I, z_0 + d/2). \tag{6}$$

Changing $z_0 \to z_0 + d/2$ results in reversing the sign of the imaginary part of the Bloch impedance ($\xi \leftrightarrow -\xi$) in every second gap: $n = 1,3,5,\ldots$.

S3. Lumped element model for periodic microstrip

The formal relationship between output and input voltages (V_j) and currents (I_j) for each segment (j^{th}) of the lumped-element model network is described by the ABCD-matrix $\mathbf{M}$:

$$\begin{pmatrix} V_j \\ I_j \end{pmatrix} = \mathbf{M} \begin{pmatrix} V_{j+1} \\ I_{j+1} \end{pmatrix}. \tag{7}$$

For our system the matrix has a form:

$$\mathbf{M} = \begin{pmatrix} \cos\Omega - p\Omega\sin(\Omega) & i\left(p\left(\cos\Omega - 1\right) + \sin\Omega\right) \\ i\left(p\left(\cos\Omega + 1\right) + \sin\Omega\right) & \cos\Omega - p\Omega\sin(\Omega) \end{pmatrix}, \tag{8}$$

where $\omega = k_0 d$ is the dimensionless angular frequency, the parameter $k_0 = \omega/(c/\sqrt{\varepsilon_{\text{eff}}})$ is the wave number in uniform space characterized by the dielectric constant ε_{eff}. The symbol p is defined as $p = q(c/\sqrt{\varepsilon_{\text{eff}}}(1/(2d))$, where $q = Z_0 C$.

In an infinite network, the voltages and currents in successive cells are related by the wavenumber k:

$$V_j = e^{ik_z d} V_{j+1}, \quad I_j = e^{ik_z d} I_{j+1}. \tag{9}$$

The lumped-element model contains two parameters: ε_{eff} and p, which are related to the geometric and material parameters of the real MSTL. We have tuned the values of ε_{eff} and p to obtain positions of the stop bands that are the same as those found from the experiment and numerical simulations.

The resulting transmission for the finite structure can be calculated as a product of the $\mathbf{M}$ matrices Eq. (8) for successive networks. This matrix can then be converted into the corresponding scattering matrix $\mathbf{S}$, which allows to obtain the transmission spectrum by taking its S_{21} element[1].

References

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