

# Supplementary Information

## Stochastic Properties of Musical Timeseries

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### Representation of rests

In the methods section we explained that we treated rests (absence of notes) by introducing a pitch of 0 for each time unit of silence. Even though this appears as a suitable procedure to treat rests, which has been used in previous work [1, 2], the introduction of zeros in place of missing values in a timeseries might create artifacts depending on the statistical distribution of the missing values. In order to check that our results are not due to such artifacts, we applied the Lomb-Scargle periodogram to a number of pieces from our corpus and compared the results to their multitaper equivalent. The Lomb-Scargle periodogram is able to deal with missing values as it does not require uniform data sampling. It is subject, however, to a larger bias and variance than the multitaper method. The Lomb-Scargle estimation is expressed as:

$$P_{LS}^{\hat{}}(f) = \frac{a^2}{2} \left( \sum_{t=0}^{N-1} x(t) \cos(2\pi f[t - \tau]) \right)^2 + \frac{b^2}{2} \left( \sum_{t=0}^{N-1} x(t) \sin(2\pi f[t - \tau]) \right)^2 \quad (1)$$

where

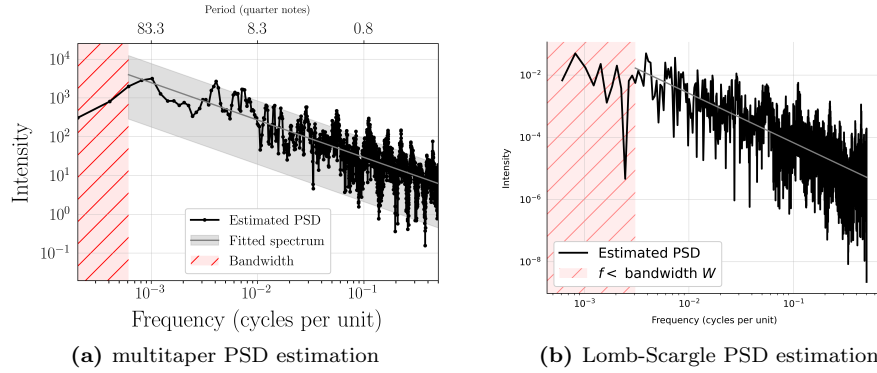
$$a = \frac{\sum_{n=0}^{N-1} x(t_n) \cos(2\pi f[t_n - \tau])}{\sum_{n=0}^{N-1} \cos^2(2\pi f[t_n - \tau])} \quad (2)$$

$$b = \frac{\sum_{n=0}^{N-1} x(t_n) \sin(2\pi f[t_n - \tau])}{\sum_{n=0}^{N-1} \sin^2(2\pi f[t_n - \tau])} \quad (3)$$

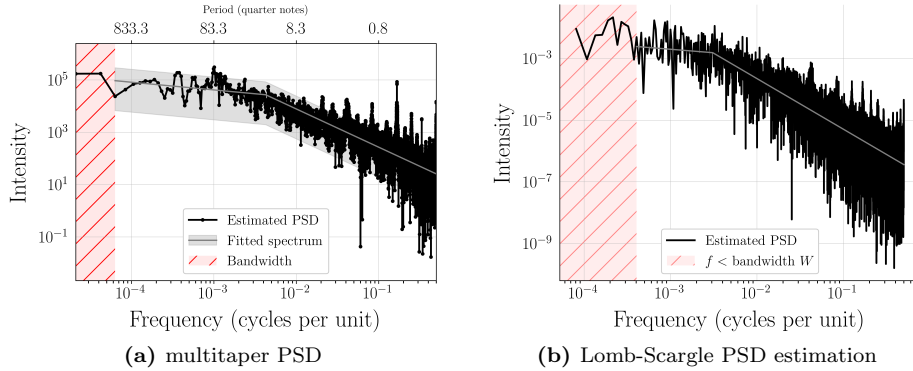
$$\tau = \frac{1}{4\pi f} \tan^{-1} \left( \frac{\sum \sin(2\pi f t_n)}{\sum \cos(2\pi f t_n)} \right) \quad (4)$$

The PSD estimations obtained by the Lomb-Scargle periodogram confirmed our results by revealing identical shapes (PL or PL+P), similar slopes, and cutoff periods, as the multitaper estimation. See e.g. Fig 1 and Fig 2 for a direct comparison of the two methods for a movement of a sonata by J.S. Bach and a string quartet by Brahms.

We conclude that the replacement of rests by pitches of 0 does not change the overall slope of the PSDs.



**Fig 1. Comparison of PSD estimation with and without missing values for the Sonata for solo Violin in G minor, BWV 1001, presto, by J.S. Bach** (A) presents the PSD estimation using the multitaper method and replacing rests by a pitch of 0, (B) shows the Lomb-Scargle periodogram omitting rests. The inclusion or exclusion of missing notes as 0 pitch does not change the overall slope of the PSD. The Lomb-Scargle periodogram shows more scattering, as expected by the fact that this method does not perform any averaging.



**Fig 2. Comparison of PSD estimation with and without missing values for the String Quartet Op.51, No.1 in C minor by J. Brahms.**

(A) presents the PSD estimation using the multitaper method and replacing non-played notes by a pitch of 0, (B) shows the Lomb-Scargle periodogram omitting rests. The inclusion or exclusion of rests as 0 pitch does not change the overall slope of the PSD. The Lomb-Scargle periodogram shows more scattering, as expected by the fact that this method does not perform any averaging.

## Data

### Classical music scores

The following is a list of all classical music pieces used in our analysis, sorted by composer. We used MIDI transcriptions mostly from the database [kunstderfuge.com](http://kunstderfuge.com) [3], but also from other databases [4, 5].

#### Single movements

**J.S. Bach:** Kunst der Fuge 07 (top voice), Violin Sonata No.1 in G minor, BWV 1001, adagio; Violin Sonata No.1 in G minor, BWV 1001 presto; Gavotte; Chaconne (top notes); Partita for unaccompanied flute 1013, movements 1, 2, 3 & 4.

**L. van Beethoven:** String Trio in C minor, Op.9 No.3, movement 1 & 2; String Quartet No.13, Op.130, 1st movement; String Quartet No.6, Op.18 No.6, 1st movement; Symphony No.2, Op.36, 1st movement; Symphony No.2, Op.36, 3rd movement; Symphony No.1, Op.21, 3rd movement; Symphony No.4, Op.61, movements 1, 3 & 4; Violin Sonata No.4, Op.23 movements 1, 2 & 3; Violin Sonata No.8, Op.30 No.3, 3rd movement; Cello Sonata No.4, Op.102 No.1, 1st movement;

**J. Brahms:** String Quartet No.1, Op.51 No.1, 1st movement.

**A. Dvorak:** String Quartet No.10, Op.51, 1st movement; String Quartet No.11, Op.61, 1st movement; Serenade for Strings, Op.22, movements 1, 2, 3, 4 & 5.

**J. Haydn:** String Quartet in G major, Hob.III:41, 1st & 3rd movement; String

Quartet in E-flat major, Hob.III:31, movement 1, 2, 3 & 4; String Quartet in D major, Op. 1 No. 3, Hob. III:3, 1st movement; String Quartet in G major, Op. 1 No. 4, Hob. III:4, 2nd movement.

**D. Shostakovich:** Prelude and Fugue No. 11 in B major (top voice); Prelude and Fugue No. 13 in F sharp major (top voice).

**R. Schumann:** String Quartet No.1, Op.41 No.1, movements 1 & 2.

**W. Mozart:**String Quartet No.17 in B-flat major, K.458, 3rd movement; String Quartet No.18 in A major, K.464 movements 1, 2, 3 & 4; Sonata for Violin and Piano in C major, KV 6, 1st movement.

### Full compositions

**J.S. Bach:** Flute Sonata BWV 1013; Violin Sonata No.2 in A minor, BWV 1003; Violin Sonata No.1 in G minor, BWV 1001; Cello Suite No.1 in G major, BWV 1007; Prelude and Fugue in C-sharp major, BWV 848.

**L. van Beethoven:** String Trio in C minor, Op.9 No.3; Symphony No.2, Op.36; Symphony No.1, Op.21,, Violin Sonata No.4, Op.23 (top voice).

**A. Dvorak:** String Quartet No.10, Op.51; String Quartet No.11, Op.61; Sere-nade for Strings, Op.22

**J. Haydn:** Violin Sonata in D major; String Quartet in E flat major, Op. 20 No. 1, Hob. III:31; String Quartet Op. 33; String Quartet Op. 76.

**W. Mozart:** String Quartet No. 2 in D major, KV 155; Concerto for Piano and Orchestra No. 18 in B flat major, KV 456; String Quartet No. 20 in D major, KV 499; String Quartet No. 18 in A major, KV 464 ; String Quartet No. 14 in G major, KV 387; String Quartet No. 15 in D minor, KV 421; String Quartet No. 16 in E flat major, KV 428 (1783).

**G. Handel:** Violin Sonata in F major, HWV 370; Violin Sonata in A major, HWV 361; Violin Sonata in G minor, HWV 368; Violin Sonata in D major, HWV 371;

**R. Schumann:** Violin Sonata No.1, Op.105; Violin Sonata No.2, Op.121

**P. Tchaikovsky:** String Quartet No.1, Op.11

**Wagner:** Symphony in C.

**A. Vivaldi:** Trio Sonata “La Folia”.

### Jazz Improvisations

As mentioned in the materials section, all the improvised music data that we used were obtained from the Weimar Jazz Database [6]. We removed pieces that were too short (less than 500 data points) or non-standard MIDI files. The following is the complete list of pieces we used sorted by artist.

**Art Pepper:** Stardust-1, Stardust-2

**Benny Carter:** I Got It Bad, It’s a Wonderful World-1

**Ben Webster:** Did You Call Her Today

**Bob Berg:** I Didn’t Know What Time It Was, Nature of The Beast, Angels.

**Branford Marsalis:** Gut Bucket Steepy, Housed From Edward-1, The Near-



ness of You, Three Little Words

**Chris Potter:** Arjuna, D.I.T., PopTune#1, Rumples, Togo

**Clifford Brown:** A Night In Tunisia, Jordu

**Coleman Hawkins:** Body and Soul, Sophisticated Lady

**Curtis Fuller:** Blue Train

**David Liebman:** Nica's Dream, Pablo's Story, Softly as In A Morning Sunrise,  
There Will Never Be Another You

**David Murray:** Blues for Two-1

**Dexter Gordon:** Society Red, The Rainbow People

**Don Byas:** Body and Soul

**Eric Dolphy:** Dahomey Dance

**Freddie Hubbard:** Dolphin Dance, Maiden Voyage

**Hank Mobley:** Remember, Soul Station

**JJ Johnson:** Walkin', Yesterdays

**Joe Henderson:** In'N Out-1, The Sidewinder, Totem Pole

**Joe Lovano:** Body and Soul-1, Central Park West, I Can't Get Started, Little  
Willie Leaps in

**John Coltrane:** Blue Train, Body and Soul – Alternate Take, Impressions -  
1961, Impressions - 1963, My Favorite Things-1, My Favorite Things-2, Nature  
Boy, Nutty, So What

**Kenny Dorham:** Punjab

**Lee Morgan:** Blue Train

**Michael Brecker:** Confirmation, Midnight Voyage, Naked Soul

**Miles Davis:** Miles Runs the Voodoo Down-1. Miles Runs the Voodoo Down-  
2, Walkin'

**Milt Jackson:** Softly as in a Morning Sunrise

**Ornette Coleman:** Peace

**Pat Metheny:** All the Things You Are

**Pepper Adams:** A night In Tunisia

**Sidney Bechet:** Summertime

**Sonny Rollins:** Blue Seven-1, Vierd Blues

**Stan Getz:** Body and Soul, Crazy rhythm

**Steve Coleman:** The oracle-1, The oracle-2

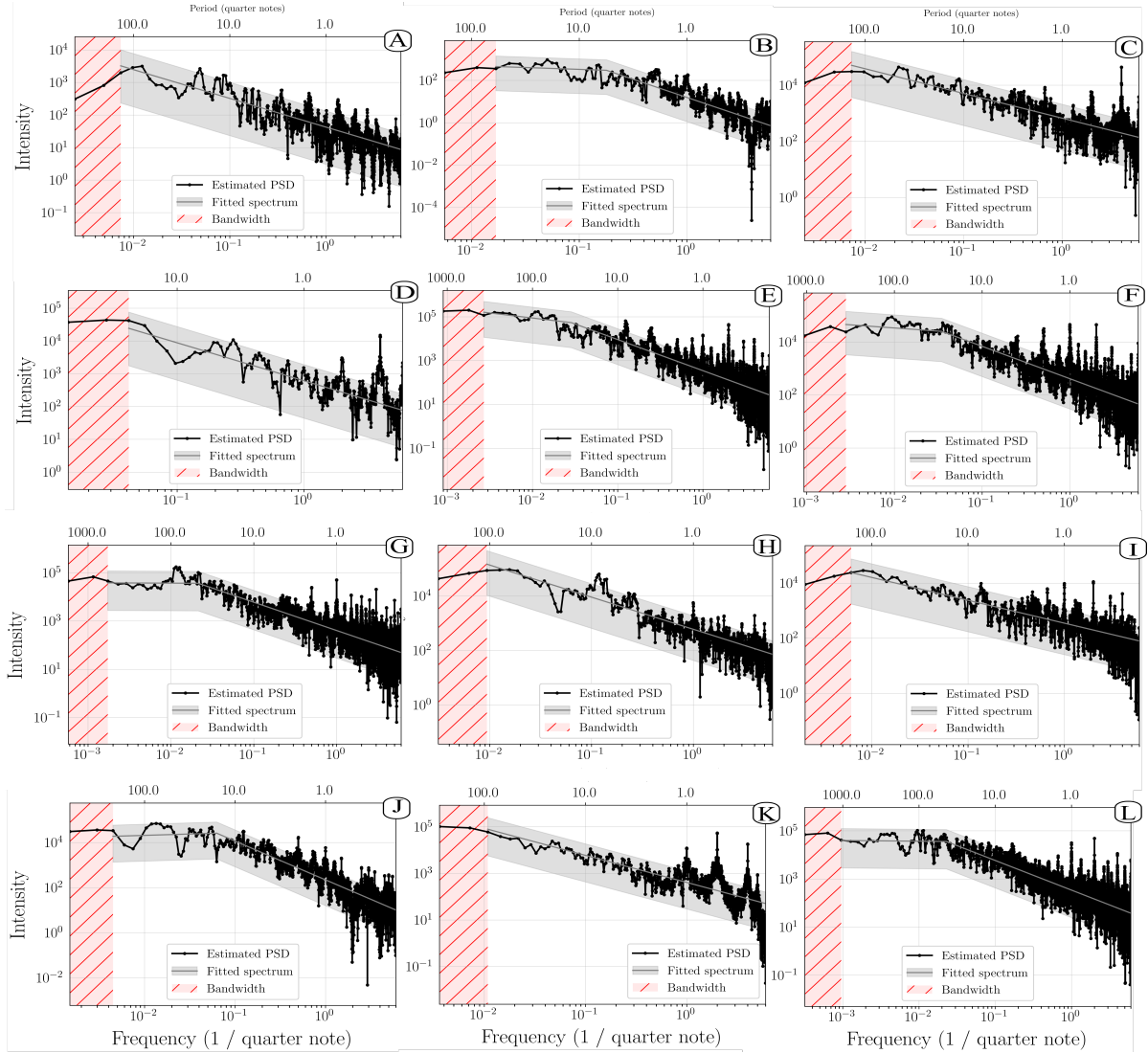
**Steve Lacy:** Let's cool One

**Steve Turre:** Dat Dere

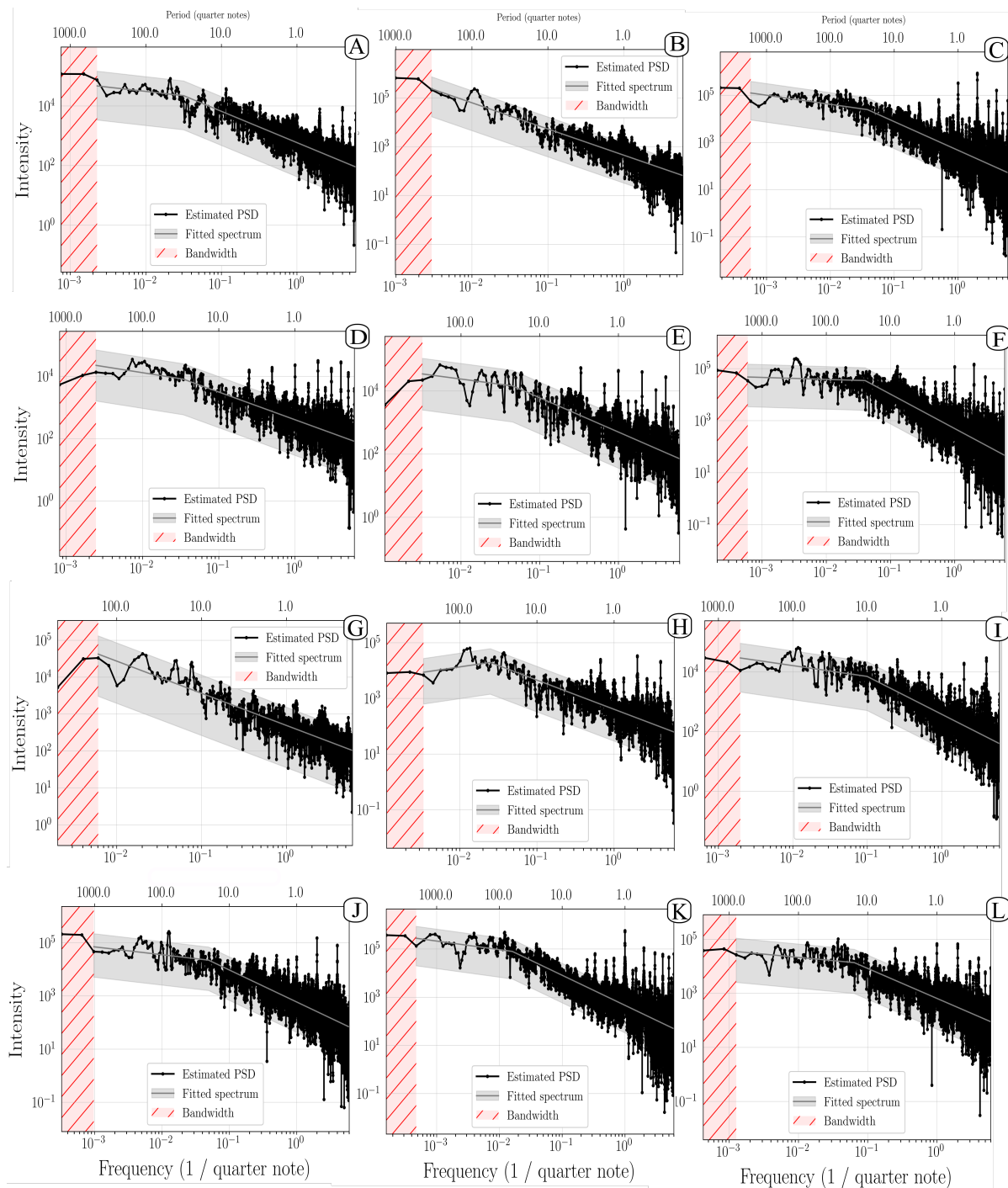
**Wayne Shorter:** Eighty one, Footprints

## Power-spectral densities of additional examples

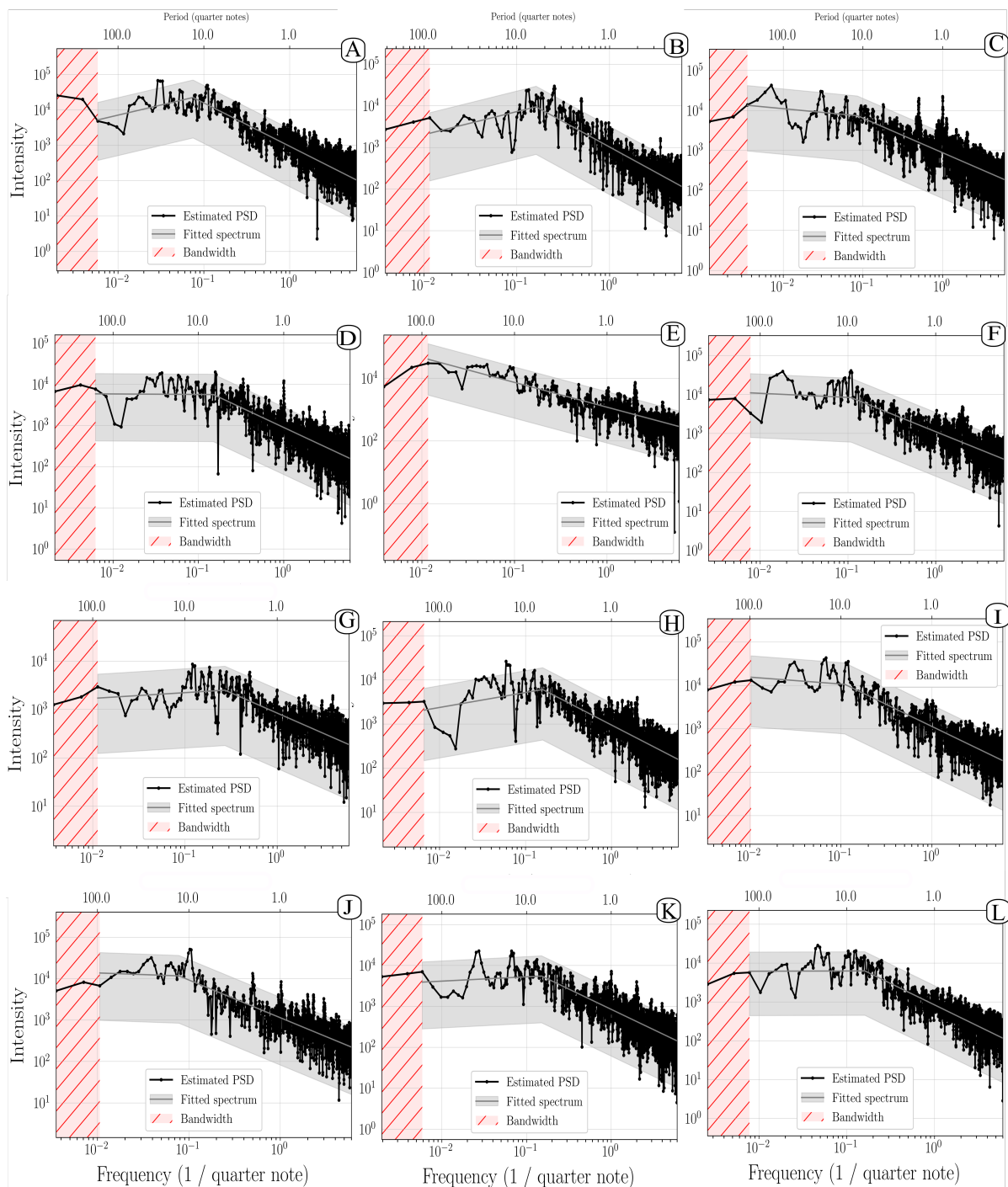
We carried out PSD analyses for a total of 553 musical pieces. The following Figs 3, 4 and 5 show examples in addition to the examples in the main text for single movements, full compositional works, and improvised jazz solos.



**Fig 3. PSD estimations of single movements.** (A) Kunst der Fuge, BWV 1080 (Bach, Johann Sebastian); (B) Sarabande from Partita in A minor, BWV 1013 (Bach, Johann Sebastian); (C) Presto from Violin Sonata No.1 in G minor, BWV 1001 (Bach, Johann Sebastian); (D) Adagio molto from Symphony No.2, Op.36 (Beethoven, Ludwig van) (E) Presto from Violin Sonata No.1 in G minor, BWV 1001 (Bach, Johann Sebastian); (F) Tio in D major from 3 String Trios, Op.9 (Beethoven, Ludwig van); (G) Moderato from Serenade for Strings, Op.22 (Dvořák, Antonín); (H) Larghetto from Serenade for Strings, Op.22 (Dvořák, Antonín); (I) Allegro from String Quartet No.11, Op.61 (Dvořák, Antonín); (J) Andante from String Quartet No.18 in A major, K.464 (Mozart, Wolfgang Amadeus); (K) Allegro from String Quartet No.18 in A major, K.464 (Mozart, Wolfgang Amadeus); (L) String Quartet in E-flat major, Hob.III:31 from String Quartets, Op.20 (Haydn, Joseph).



**Fig 4. PSD estimations of full compositional works.** (A) Partita in A minor, BWV 1013 (Bach, Johann Sebastian); (B) Violin Sonata No.2 in A minor, BWV 1003 (Bach, Johann Sebastian); (C) Cello Suite No.1 in G major, BWV 1007 (Bach, Johann Sebastian); (D) String Trio in C minor, Op.9 No.3 (Beethoven, Ludwig van); (E) Symphony No.1, Op.21 (Beethoven, Ludwig van); (F) Symphony No.2, Op.36 (Beethoven, Ludwig van); (G) Serenade for Strings, Op.22 (Dvořák, Antonín); (H) Violin Sonata in A major, HWV 361 (Handel, George Frideric); (I) Violin Sonata in G minor, HWV 368 (Handel, George Frideric); (J) Sonata for Violin No.2 (Haydn, Joseph); (K) String Quartets, Op.20 (Hyadn, Joseph); (L) String Quartet No.2 in D major, K.155/134a (Mozart, Wolfgang Amadeus).



**Fig 5. PSD estimations of various jazz solos from the Weimar Jazz database [6].** (A) Anthropology (Art Pepper); (B) Stardust (Art Pepper); (C) Just Friends (Benny Carter); (D) It's a wonderful world (Benny Carter); (E) Did you call her today (Ben Webster); (F) When or where (Ben Webster); (G) Angels (Bob Berg); (H) Nature of the beast (Bob Berg); (I) House from Edward (Branford Marsalis); (J) U.M.M.G (Branford Marsalis); (K) Arjuna (Chris Potter), (L) Blues for two - take 1- (David Murray).

## References

- [1] Nigel Nettheim. On the spectral analysis of melody. *Interface*, 21(2):135–148, 1992.
- [2] Alfredo González-Espinoza, Hernán Larralde, Gustavo Martínez-Mekler, and Markus Müller. Multiple scaling behaviour and nonlinear traits in music scores. *Royal Society Open Science*, 4(12):171282, dec 2017.
- [3] [www.kunsterfuge.com](http://www.kunsterfuge.com).
- [4] [www.metronimo.com](http://www.metronimo.com).
- [5] Casey A. Mullin. International music score library project. *Notes*, 67(2):376–381, 2010.
- [6] Martin Pfeiderer, Klaus Frieler, Jakob Abeßer, Wolf-Georg Zaddach, and Benjamin Burkhart, editors. *Inside the Jazzomat - New Perspectives for Jazz Research*. Schott Campus, 2017.