

# Supplementary Material for Emergent nonlinear modes in coherent magnon circuits detected at ultrahigh frequency resolution

K. An<sup>1,2†</sup>, M. Xu<sup>1†</sup>, A. Mucchietto<sup>1</sup>, C. Kim<sup>2</sup>, K.-W. Moon<sup>2</sup>, C. Hwang<sup>2</sup>,  
D. Grundler<sup>1,3\*</sup>

<sup>1</sup>Laboratory of Nanoscale Magnetic Materials and Magnonics, Institute of Materials  
(IMX), School of Engineering, École Polytechnique Fédérale de Lausanne (EPFL),  
Lausanne, 1015, Switzerland.

<sup>2</sup>Quantum Technology Institute, Korea Research Institute of Standards and Science,  
Daejeon, 34113, Republic of Korea.

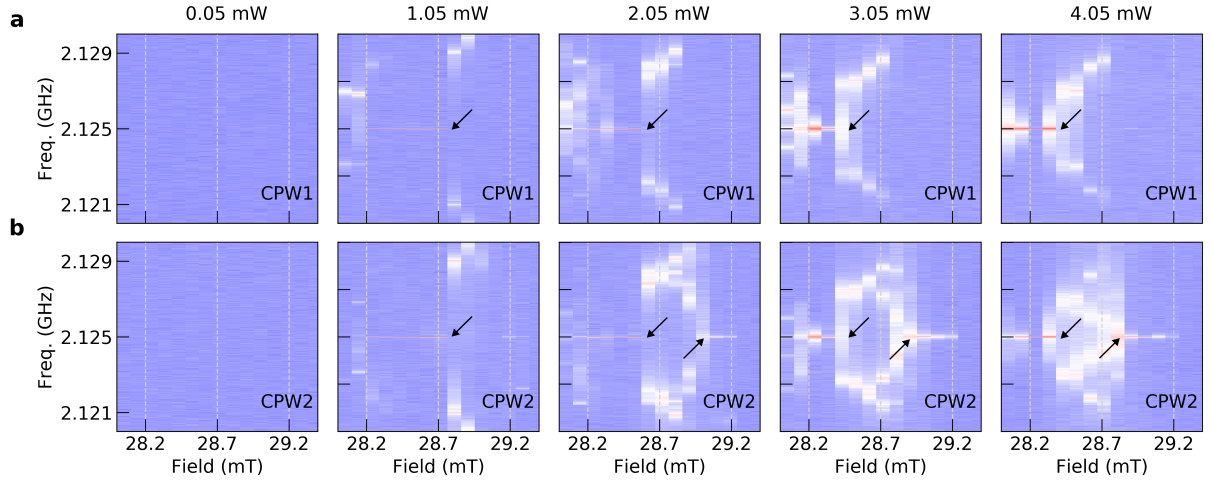
<sup>3</sup>Institute of Electrical and Micro Engineering, School of Engineering, École  
Polytechnique Fédérale de Lausanne (EPFL), Lausanne, 1015, Switzerland.

\*Corresponding author(s). E-mail(s): [dirk.grundler@epfl.ch](mailto:dirk.grundler@epfl.ch);

<sup>†</sup>These authors contributed equally to this work.

## Field dependence at different power levels

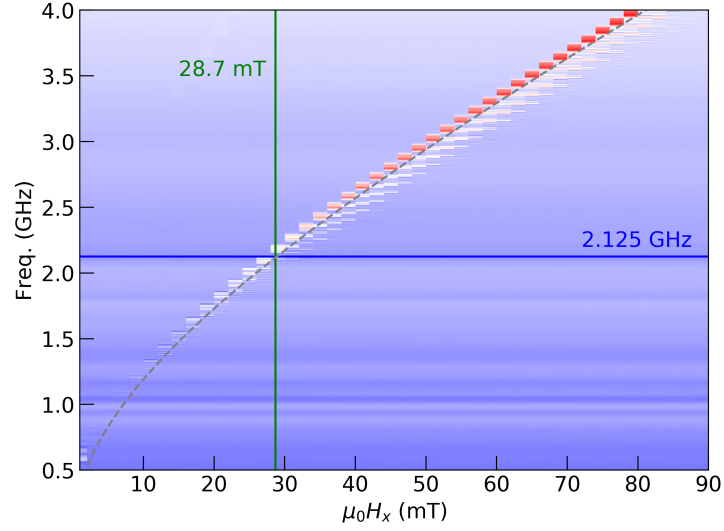
We present the dependence of the field on various power levels in Fig. S1. At powers exceeding 0.05 mW, we observe an exceptionally narrow  $f_p/2$  peak. As the field increases, this peak begins to split, marked by the downward arrows in Fig. S1, and this splitting progressively shifts to lower fields with an increase in power. This trend follows a field-power relationship of  $-0.18$  mT/mW, consistent with the observations noted in Fig. 2 of the main text. Additionally, the converging branch observed in CPW2 demonstrates a similar downward shift with an increase in power, as indicated by the upward arrows in Fig. S1. We also note that the amount of frequency splitting decreases with increasing power.



**Fig. S1 Field dependent spectra with various excitation powers.** **a** CPW1 and **b** CPW2 signals detected by the VNA for nonlinear spin waves at different power levels as a function of in-plane field applied perpendicular to the CPWs. The excitation frequency was  $f_p = 4.25$  GHz. The nonlocal magnon branch starts to emerge above 1 mW as seen in the CPW2 spectra. The field for splitting shifts down with increasing power (indicated by downward arrows), consistent with the shift shown in Fig. 2 of the main text. Correspondingly the second converging point in the CPW2 spectra also shifts down with power as indicated by upward arrows in **b**

## S-parameter measurement

The  $S$ -parameter measurement was taken to extract the basic magnetic properties of our film. Figure S2 shows the measured  $S_{21}$  amplitude. The maximum in the color plot is best-fitted with the well-known backward volume mode with  $k_y = \pi/w_{\text{YIG}}$  and  $k_x = 0.5 \text{ rad}/\mu\text{m}$  with the following parameters :  $\gamma/(2\pi) = 28 \text{ GHz/T}$ ,  $\mu_0 M_s = 0.176 \text{ T}$ , and  $D_{\text{ex}} = 5.4 \times 10^{-17} \text{ T/m}^2$ . The vertical green line shows that 28.7 mT is required to place the resonance at 2.125 GHz, which agrees reasonably well with the measured field value in the frequency offset measurement. The extracted parameters were used to calculate the magnon dispersion curves in the discussion section.

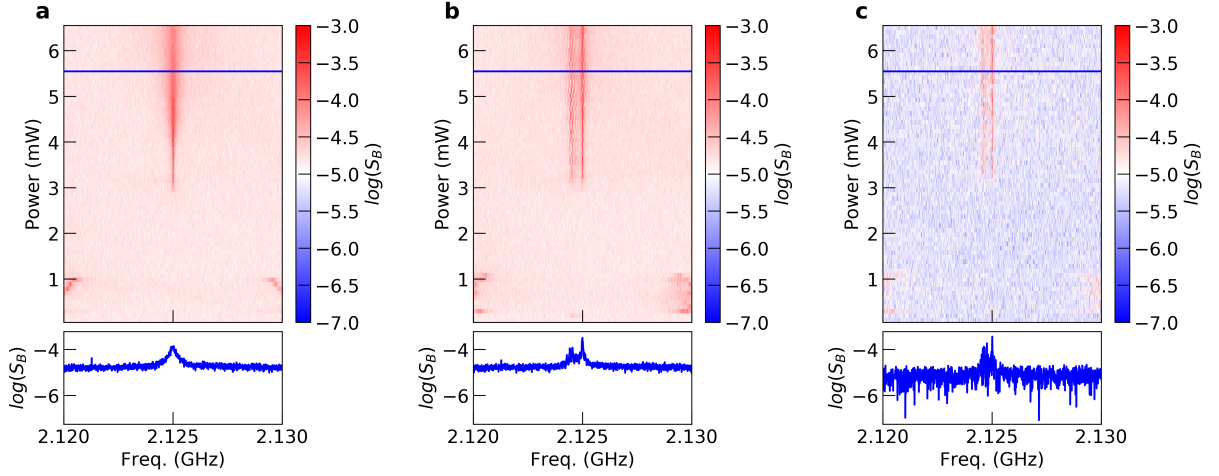


**Fig. S2**  $S$  parameter measurement. Field dependent transmission signal intensity ( $S_{21}$ ). The dashed grey line represents the calculated field dependence with the given material parameters for spin waves propagating with  $k_x = 0.5 \text{ rad}/\mu\text{m}$ . To excite  $f_p/2 = 2.125 \text{ GHz}$ , the required magnetic field is 28.7 mT, close to the field value in the frequency offset measurement.

## Detection without establishing phase sensitivity

Stimulated by the phase sensitive detection of nonlinear waves, we further explored the influence of coherency on the measured spectrum at CPW2 by replacing VNA port 1 with an external microwave generator (compare Fig.S3a (VNA only, 10 average) with Fig.S3b (10 average) and Fig.S3c (single shot)). These studies were conducted under very similar environmental conditions, without remounting the sample between measurements. Nonetheless, these tests were carried out subsequent to the remounting of the sample following the initial experiments discussed in the main text, which may account for some discrepancies in the power evolution spectrum.

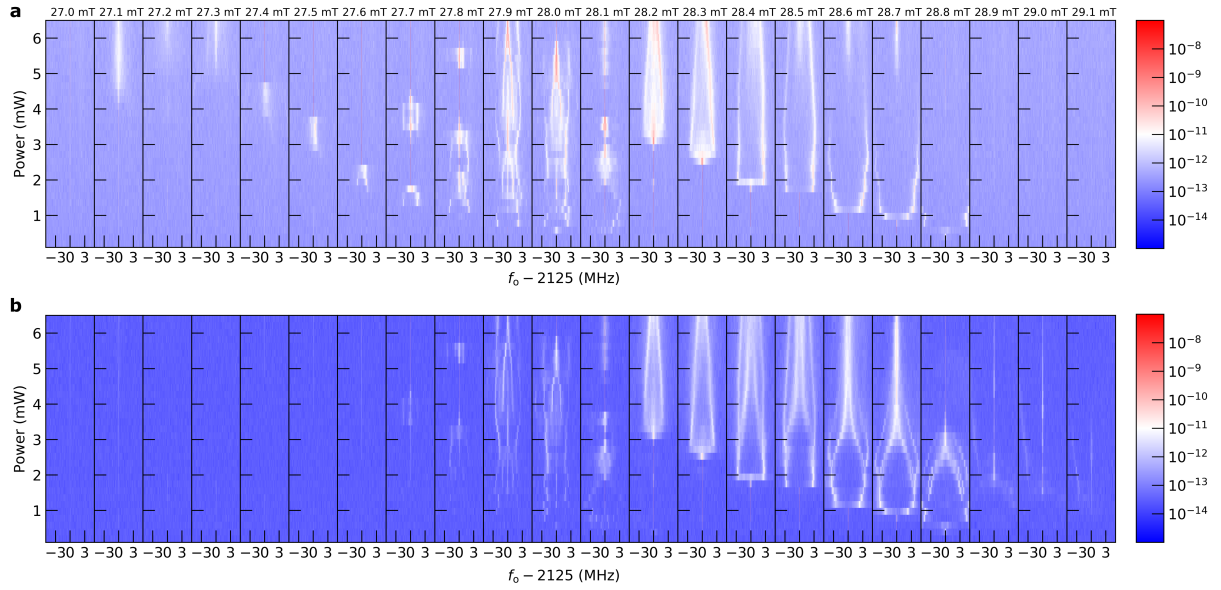
Despite these slight variations, the side peak structures and half-frequency peaks are evident in all three measurement configurations. A substantial intensity fluctuation is observed in the single shot data from the external generator, as depicted in Fig.S3c. This fluctuation ranges from baseline noise levels to peak intensities due to phase randomness. Such variability is significantly subdued in the 10-times averaged data (Fig.S3b), resulting in a comparative reduction of peak intensity against the spectrum presented in Fig.S3a. This highlights the superiority of the phase-stable, all-electrical approach introduced in this work over traditional scalar detection methods.



**Fig. S3 Spectra comparison with and without phase synchronization.** Transmission spectra comparison from Receiver B at VNA Port 2 **a** using input from VNA port 1 (averaged over 10 measurements). **b** using an external microwave generator (10 averages). **c** with single shot detection using the external generator. Despite the external generator's lack of phase synchronization with the VNA Receiver B, consistent spectral features are noted. However, significant signal fluctuations due to non-synchronized phases are evident in **c**, reduced by averaging in **b**. This averaging results in lower intensity compared to the VNA-only setup (**a**). The bottom panel linecut profiles at 5.6 mW highlight the signal-to-noise ratio variations.

## Power dependent spectra over an extended field range

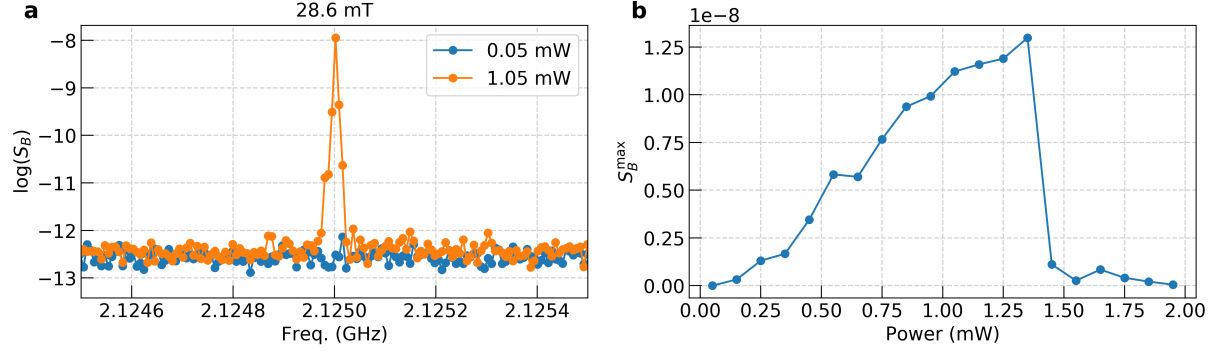
In Fig. 2 of the main text, we showed the evolution of spectra over the 1.2 mT field range, where the co-propagating magnons are collected nonlocally in the CPW2 spectrum. Here we show the measurements over an extended field range of 27-29.1 mT. It should be noted that, although the same sample was used for these measurements, it was dismounted and remounted in the process. We attribute the observed variations in the low-field frequency structure to this procedure, which indicates the sensitivity of the system to the sample's precise orientation with respect to the external magnetic field. Nonetheless, the prominent branch unique to the CPW2 spectrum remains distinctly observable.



**Fig. S4 Power dependent spectra over an extended field range.** Power evolution spectra over an extended field range from **a** CPW1 and **b** CPW2. Below 28 mT the sharp  $f_p/2$  peak is still observable. More complicated side peak features develop at lower fields. Despite minor differences in low field characteristics, we observe consistently similar power evolution spectra from CPW1 and CPW2 spectra including the emerging branch in CPW2 attributed to the co-propagating magnons.

## Linewidth of $f_p/2$ peak

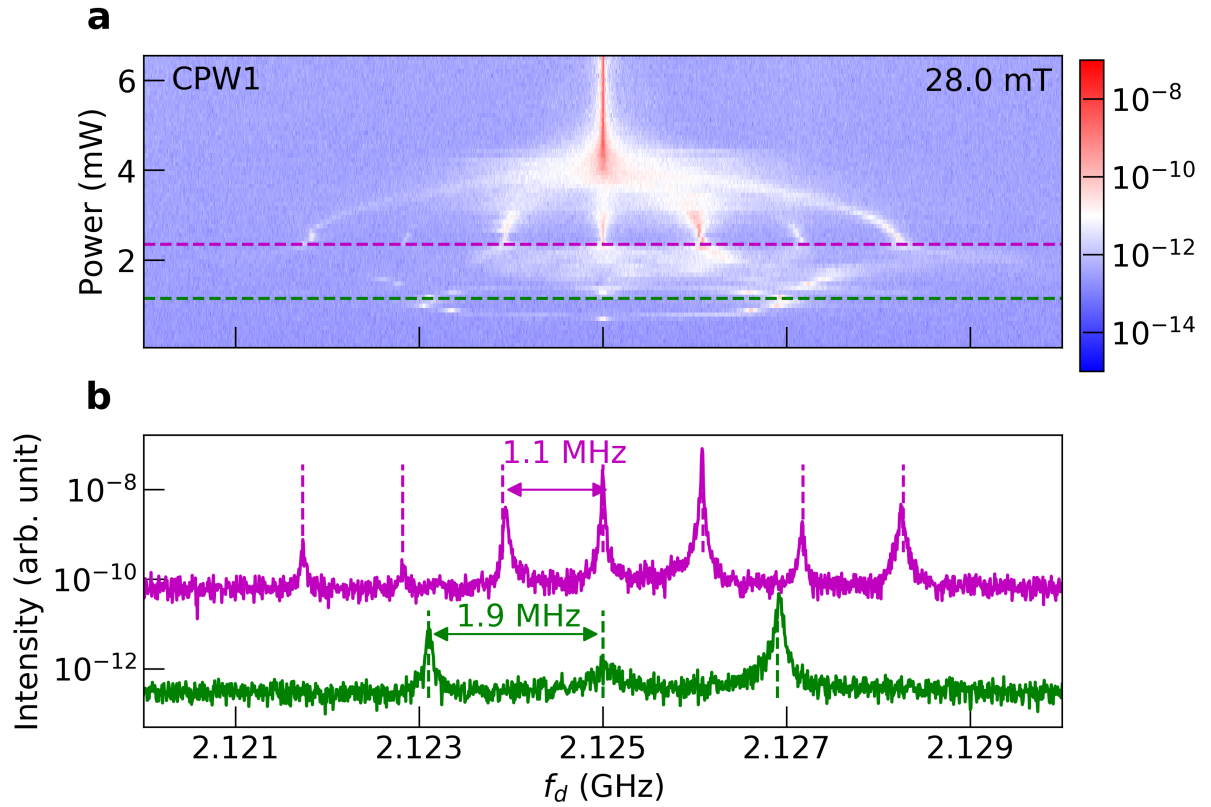
Here we provide a detailed view of the  $f_p/2$  peak, characterized by an exceptionally narrow linewidth on the order of 10 kHz. This narrow linewidth is indicative of the highly selective nature of the parametric pumping process and is limited by the instrumental bandwidth. Above 0.1 mW, the peak grows near-linearly with power and decreases at the onset of frequency splitting, which occurs at approximately 1.45 mW.



**Fig. S5 Detailed view of the  $f_p/2$  peak.** Linewidth of  $f_p/2$  peak at 28.6 mT **a** The half-frequency peak spans over only a few frequency points in the semi-log plot indicating an exceptionally narrow linewidth of about 10 kHz. **b** Power dependence shows that the peak exhibits near-linear growth with power until the frequency splitting point, beyond which its intensity rapidly decreases.

## Detailed magnon spectrum at low field

At a magnetic field of 28 mT, multiple discrete side peaks become evident. A closer look of the spectrum is presented in Fig. S6a. Around 1 mW, we observe a variable frequency spacing, reaching a maximum of approximately 1.9 MHz (indicated by the green line in Fig. S6b). As power increases towards 2 mW, this spacing contracts to 1.1 MHz, with additional peaks manifesting at integer multiples of this spacing. Further power increase results in a convergence at the half-frequency. Although we suspect that this phenomenon originates from the scattering among higher-order quantized modes along the  $y$  direction, a theoretical model that reproduces this behavior has yet to be developed.



**Fig. S6 Formation of the magnon frequency comb-like feature at low field.** **a** Measured evolution of parametrically excited spectrum as a function of power at CPW1. Magnon frequency spacing sensitively changes with source power at 28 mT. **b** Color-coded horizontal linecuts are taken at 1.15 and 2.35 mW for the green and purple dashed lines in **a**. The frequency spacings are equidistant and reduces by almost a factor of 2 when increasing power twice.