The dynamics of fire activity in Brazilian Pantanal: A LGCP-based structural decomposition

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The dynamics of fire activity in Brazilian Pantanal: A LGCP-based structural decomposition

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Abstract We analyzed changes in fire occurrence patterns in the Brazilian Pantanal using remote sensing data resources. To estimate the spatio-temporal dynamics of fire occurrence, we employed a structural representation using a log-Gaussian Cox process (LGCP), which decomposes the intensity function into components of trend, seasonality, cycle, covariates, and time-varying spatial effects. The results obtained indicate significant variations in the trends of fire occurrence in the Brazilian Pantanal. The intensity of fire occurrence is statistically higher in natural vegetation. A counterfactual decomposition indicates that a significant portion of the recorded fires observed in the first three quarters of 2020 cannot be explained by climatic factors alone, suggesting possible causation by intentional human actions.

Keywords Brazilian Pantanal · Fire modelling · Spatio-Temporal Point Process

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1 Introduction

The Pantanal biome is the major wetland ecosystem of the world, located in the Upper Paraguay River Basin (UPRB) in South America, between the Cerrado and Amazon biomes. The Brazilian Pantanal is located in the southwest region, mostly in the state of Mato Grosso do Sul (65%), but also in the state of Mato Grosso (35%) (da Silva and de Moura Abdon 1998). The biome is mainly characterized by the well-defined dry and wet seasons, producing periodic fluctuations in the water level (flood pulse), shaping the scope of terrestrial and aquatic places on the lowland, and influencing the fauna and flora. The vegetation in the Pantanal is heterogenous, with several vegetation classes identified, and also serving as the habitat for substantial populations of animals, including threatened species (Tomas et al. 2019).

This high Pantanal's biological diversity has attracted great attention, making this biome more susceptible to anthropological threats. In the last few decades, the Pantanal has experienced a rapid evolution of the agricultural and livestock systems, replacing areas of natural vegetation by production zones (Harris et al. 2005; Silva et al. 2011; Arvor et al. 2012). The monitoring activities of 2012-2014 to assess the environmental impact in the UPRB have identified that 58% of the original vegetation in the plateau areas were converted to anthropic uses, whereas in the lowlands this conversion corresponds to 42%. In addition, the report also has found that 99% of all converted areas have been used as pastureland, while 0.6% for agriculture, and 0.4% for mining and urban areas (SOS Pantanal and WWF-Brasil 2015).

Related to agriculture and cattle ranching, the inclusion of exotic grass species and the burning practice are important threats in the region. As a consequence of replacing the original vegetation by cultivated pastures and uncontrolled fires, severe erosion has led to changes in the hydrological regimes and the patterns of water flow (Harris et al. 2005). As discussed by Ivory et al. (2019), changes in the vegetation productivity in these landscapes are likely linked to changes in rainfall and the flood-pulse, with different responses based on position relative to inundated areas. As a consequence, these changes in the vegetation cycle can lead to changes in the fire activity. Although occasional fire plays an important ecological role in the wetland, with positive impacts on some vegetation structure and nutrient recycling (de Oliveira et al. 2014; Arruda et al. 2016), fire is one of the most important environmental disturbances, affecting the variations of flood and dry periods, and, as a consequence, changing the required time of plants and animals to recover after the dry periods (Junk et al. 2006).

Extensive and more frequent fire events have been reported in the Brazilian Pantanal in the last few years. For instance, in 2020, more than 22 thousand fire outbreaks were registered in the Pantanal, with a burned area of 33,000km², exceeding by 176% the historical record of fire outbreaks registered in 2005 since the beginning of the monitoring by the Instituto Nacional de Pesquisas Espaciais (INPE) in 1998. In addition, the area burnt in 2019 in the Brazilian Pantanal was surprisingly 996% higher than in 2018, which is particularly high
when compared with the neighbor biomes (Cerrado and Amazon) that have recorded an increase around 40% and 65% of the burned area from 2018 to 2019, respectively. In addition, it is worth noting that 95.72% of the fire events in the Pantanal have occurred in native vegetation, whereas only 4.28% was in anthropized areas (MapBiomas, 2020). One reason for this increasing may be related to land use and climate changes, which has the potential to affect the rainfall intensity and the dry period, favoring the frequency of fire events, mostly human-induced (accidentally or deliberately), which tends to start in grasslands and then move to woodlands (Arruda et al., 2016). In the absence of specific legislation requiring that landowners restrict a minimum percentage of native vegetation cover in the farms, it is expected an average vegetation loss around 10% for the plateau and 3% for the lowland by 2050 (Guerra et al., 2020).

Given the widely-expected trend of agriculture and livestock expansion, and the importance of the Brazilian Pantanal to provide ecosystem services and the economic valuation of the region (Costanza et al., 1997; Steidl and Moraes, 2000; Costanza et al., 2014; Wantzen et al., 2023), there is an urgency to evaluate the possible changes in the patterns of fire occurrence in the region, in order to find solutions to minimize the impacts. Despite the existent long-term studies evaluating the variations in rainfall patterns in the Pantanal and the related consequences for fire frequency, the study of climate landscape dynamics is underdeveloped in the Pantanal (Schulz et al., 2019). In particular, for fire occurrences, there is a gap in research that our study aims to help fill. Additionally, the development of remote sensing technology has become an important tool to assess fire occurrences and monitoring the possible changes in the patterns of these events.

In this sense, to monitor the patterns of fire occurrence in the Brazilian Pantanal, we propose to model the spatial dynamics of point process of geolocated events (fire spots) based on remote sensing data resources through a spatio-temporal decomposition for spatio-temporal for point process, for the period 1999-2022. In particular, we use a novel dynamic representation of a log Gaussian Cox process (LGCP), where the intensity function is modeled through decomposition of components in trend, seasonality, cycles, covariates and spatial effects (Laurini, 2019; Valente and Laurini, 2020, 2021a,b), assuming that spatial effects are time varying, based on an autoregressive functional structure.

The novel approach introduced here, along with the associated statistical inference techniques, builds upon the Bayesian framework for log Gaussian Cox processes originally advanced by Simpson et al. (2016). This innovation empowers us to discern and estimate both permanent and transitory patterns in fire occurrences while effectively accounting for climatic influences. Consequently, it provides a robust means for investigating potential anthropogenic factors contributing to the spatio-temporal distribution of fires. This novel modeling approach provides a robust foundation for understanding and analyzing the spatio-temporal dynamics of fire occurrences in the Pantanal.
We also introduce a causal inference methodology aimed at identifying intervals characterized by rising trends in fire occurrences due to non-climatic factors. This methodology is applied to the recent period of heightened fire activity in the Pantanal, specifically in the year 2020. Through this analysis, we aim to explore and establish the possible anthropogenic drivers behind this surge in fires. Furthermore, we explore an alternative non-separable spatial random effects representation, allowing a direct interaction between spatial and temporal effects in the model.

2 Data and Methods

Our main goal is to analyze the changes in the permanent and transitory patterns of fire occurrence in the Brazilian Pantanal biome, which was delimited based on The Map of Biomes and Coastal-Marine System of Brazil from Instituto Brasileiro de Geografia e Estatística (IBGE). For that, we use daily data of fire spots in the Brazilian Pantanal from July-1998 to December-2022, provided by the Programa Queimadas from Brazilian National Institute of Spatial Research (Instituto Nacional de Pesquisa Espacial-INPE). We also included covariates that could be important in the fire observations since our data set includes fire occurrences of different causes, such as human sources and natural causes, which can be influenced by climate variables. In particular, we included information on maximum temperature, rainfall, and land use/land cover (LULC) from 1998 to 2022. A detailed description can be found in the Online Appendix.

To properly deal with point pattern data we use a spatio-temporal decomposition based on a LGCP, a doubly stochastic version of the Poisson process, where the log intensity function is given by a Gaussian Markov random field (Rue and Held (2005)). This framework is a flexible way to overcome the limited structure of the Poisson process, by allowing to introduce more complex stochastic structures in the intensity function, controlling for general processes of spatial dependence.

However, inference procedures on the LGCP are difficult given the fact that the likelihood of these processes is analytically intractable. To bypass this problem, Simpson et al. (2016) proposed to approximate the LGCP likelihood through the stochastic partial differential equation (SPDE) approach (Lindgren et al. 2011) representation of the latent random field, which is a computationally effective way to deal with spatio-temporal models in the context of point pattern data. In addition, as proposed by Laurini (2017) and Valente and Laurini (2020), the LCGP structure allows us to estimate long-term changes and transient components through a structural decomposition (à la Harvey (1990)) of the intensity function. In particular, we proposed to

1 Available at https://www.ibge.gov.br/geociencias/cartas-e-mapas/informacoes-ambientais/15842-biomas.html?&t=o-que-e
2 Data and more information available at http://queimadas.dgi.inpe.br/queimadas/portal
decompose the intensity function into latent factors of trend, seasonality, and cycle, along with covariates and spatial effects.

In the analysis of the temporal pattern, the main object is the trend component, which shows the evolution of the average level of occurrences over time, and thus shows the persistent patterns of fire occurrence in the Pantanal. Variations in this component may indicate variations that are possibly related to changes in the patterns of land use management in agricultural activities, such as the use of burning for the removal of native vegetation and later use in pastures and plantations. This interpretation is possible by controlling the climatic effects through covariates, and also by controlling other possible non-permanent effects by including the seasonality and cycle components. In addition to the inclusion of these common temporal and covariate components, the model used in the analysis includes a structure of time-varying spatial random effects, which allow the capture of the remaining spatial patterns, allowing to analyze also if there are other effects in the fire patterns that have spatial dependence.

To provide a clearer idea of the method employed to reach our goals, we present a brief description of the likelihood approximation proposed by Simpson et al. (2016) in Online Appendix A.1, and the SPDE approach in Online Appendix A.2. Our approach is based on the extension of the static approach to log-Gaussian Cox processes proposed in Simpson et al. (2016) for a spatio-temporal dynamic structure, modeling the pattern of fire occurrences over time. Herein we provide some details about the model structure used in our analysis. Assuming a bounded region \( \Omega \subset \mathbb{R}^2 \), the number of points within a region \( D \subset \Omega \) in period \( t \) is Poisson distributed with mean \( \Lambda_t(D) = \int_D \lambda(s, t) \, ds \), where \( \lambda(s, t) \) is the intensity surface function of the point process. Under this structure the likelihood of the Poisson process \( Y_t \) is given by

\[
\pi(Y_t | \lambda) = \exp\left\{ -|\Omega| - \int_{\Omega} \lambda(s, t) \, ds \, dt \right\} \prod_{s \in Y_t} \lambda(s, t). \tag{1}
\]

We assume the structure of a Log-Gaussian Cox Process, decomposing the spatio-temporal log-intensity function \( \log \lambda(s, t) \) as a latent random field given by the sum of covariates and latent stochastic components:

\[
\log \lambda(s, t) = \mu_t + s_t + c_t + z(s, t)\beta + \xi(s, t)
\]

\[
\mu_t = \mu_{t-1} + \eta_{\mu}
\]

\[
s_t = s_{t-1} + s_{t-2} + \ldots + s_{t-m} + \eta_s
\]

\[
c_t = \theta_1 c_{t-1} + \theta_2 c_{t-2} + \eta_c
\]

\[
\xi(s, t) = \Theta \xi(s, t-1) + \omega(s, t) \tag{2}
\]

where \( \mu_t \) is the long term trend, \( s_t \) represents the seasonal components, \( c_t \) is a cycle component represented by an second-order autoregressive process with complex roots, \( z(s, t) \) is a set of covariates observed in the location \( s \) and
period \( t \), and \( \xi(s, t) \) are the spatial random effects represented by the Gaussian process \( \omega(s, t) \) continuously projected in space and given by

\[
\text{Cov}[\omega(s, t), \omega(s', t')] = \begin{cases} 
0 & \text{if } t \neq t' \\
\sigma^2 C(h) & \text{if } t = t' \quad \text{for } s \neq s'
\end{cases} (3)
\]

In this structure, the trend component is modelled as a first order random walk, which is a way widely used to model persistent components in time series models. The seasonality component is given by components that add up to zero within the year, which incorporate seasonal deviations from the series average in each period. The cyclic component is modeled with a second-order stationary autoregressive process, which is a parsimonious way of recovering periodic patterns in time series.

The spatial component is defined by a spatially continuous covariance function. By assumption, \( C(h) \) is a covariance function of the Matérn class, which can be written as

\[
C(h) = \frac{2^{1-\nu}}{\Gamma(\nu)} (\kappa ||h||)^\nu K_\nu(\kappa ||h||) (4)
\]

where \( h = ||s - s'|| \) is the Euclidean distance between locations \( s \) and \( s' \), \( \kappa > 0 \) is a spatial scale parameter, \( \nu > 0 \) is the smoothness parameter and \( K_\nu \) is a modified Bessel function.

To sum up, our goal is to obtain the intensity function of a latent spatio-temporal points process model, which is decomposed into components of trend, seasonality, cycle, plus the effect of covariates and the so-called spatial random effect, which captures the local effects not captured by the other components. By using this structure, we are able to assess possible changes in the patterns of fire occurrence in the Brazilian Pantanal. In addition, the reason to incorporate covariates in the analysis is to control the impact of climate variables and other effects related to land use and land cover.

As discussed before we extend the likelihood approximation proposed by Simpson et al. (2016) to represent the likelihood function of a spatio-temporal LGCP and perform inference for hyperparameters and latent components using a Bayesian method by means of Integrated Nested Laplace Approximations introduced in Rue et al. (2009) using the SPDE representation for the latent log-intensity function proposed by Lindgren et al. (2011) and the approximation of the likelihood function introduced by Simpson et al. (2016). We present the main details of these methods in Online Appendix A.1 and A.2.

A relevant concern about the formulation given by Eq. 2 is about the identification of the model, composed by the sum of several latent components. A formal analysis on the identification of this model would require analyzing the spectral representation of the LGCP process with the log-intensity function used in our analysis, since the identification of the latent components of trend, cycle and seasonality in state space representations is based on the spectral generating function, as discussed by Hotta (1989). As we are using a direct representation in the time and space domains for this process, we do not have
a spectral representation available, and thus the identification is based on the numerical optimization properties of the inference procedures.

Since we are using Bayesian inference methods through Integrated Nested Laplace Approximations, it is possible to verify signs of identification problems through the numerical procedures for estimating the posterior mode necessary for the application of Laplace approximations. Signals of identification problems can be noted through the presence of singularities and negative eigenvalues in the numerical Hessian matrix evaluations used in the optimization procedure used for mode finding. In estimating all the models used in this work, there were no signs of numerical problems related to non-identification conditions, which shows that empirically the model seems to be identified. We assumed additional restrictions on the definition of random effects, assuming that the cycle (AR(2)), seasonality and the spatial random effect sum to zero in time, avoiding problems of identification with the process average. It is also important to note that the estimates are robust to the initial values used in the numerical optimization, indicating the absence of multiple modes, which would also be evidence of lack of identification. Thus, although we do not have a formal proof of identification in our model, the evidence of numerical stability is favorable to the parameterization used.

3 Results and Discussion

In this section, we discuss the results of the final specification for the analyzed model, where the statistically significant covariates were rainfall, given by the rainfall accumulated in the quarter, and land use/cover variables in each location analyzed. Regarding the maximum temperature variable, it did not provide a statistically significant pattern to explain the variation in the fire events, even though there was a long-term increase in maximum temperature trends in the Pantanal region. From the obtained results (see Table 2), it is possible to observe a negative relation between rainfall and the intensity of fire events, i.e., the lower precipitation leads to higher fire intensity. On the other hand, natural forests such as forest formation, savanna, and natural grassland have a positive effect on fire intensity. It is important to emphasize that other types of land use/cover were not statistically significant in terms of the intensity of the process, especially land use in agricultural activities.

It is not surprising that ecosystems formed by savannas and grasslands are positively related to fire intensity since they are mostly fire-dependent ecosystems (Hardesty et al., 2005). Indeed, fires in this type of formation are typically mild and frequent, often occurring in the transitional months between seasons and providing benefits to the fauna and flora (Pivello, 2011; Pivello et al., 2021). However, in this kind of vegetation there is also the occurrence of anthropogenic fires, which is used to clean the field, control pests, and to stimulate the regrowth of grasses to cattle, increasing the fire frequency. Natural fires are usually controlled by the rainy season, while human-induced fires usually also occur in dry season and are more intense than natural fires, spread-
ing more easily and without rain to extinguish it. As a consequence, recurrent human-induced fires can affect the spatial pattern and intensity of the fire activity even in the fire-dependent environments. In combination with drought, these alterations can cause severe and catastrophic fires, as those recorded in 2020 in the Pantanal (Pivello et al., 2021). Additionally, it is important to emphasize that riparian and gallery forests along water bodies in the grassland and savanna formation are classified as fire-sensitive environments, and can be gradually reduced when in contact with recurrent fires (Arruda et al., 2016). However, it is important to highlight that our data source do not distinguish whether the fire occurrence is natural or caused by human actions, thus additional studies are needed to better elucidate the role of human influences in the fire activity in this kind of vegetation.

On the other hand, forest formation is considered fire-sensitive, i.e., are not capable to adapt to natural fire regimes and where fire disrupts the ecosystem (Hardesty et al., 2005). However, according to our results, this formation is positively related with the intensity of fire activity in the Pantanal, which may be due to a combination of factors, such as recurrent droughts, human activities, and the lack of environmental policies to cope with illegal fires (Pivello et al., 2021).

The precision parameters of the seasonality, cycle, and trend cannot be directly interpreted, and they are better interpreted through the time series of posterior distribution of these components shown in Figure 1. Thus, we decided to focus on the interpretation of the figures with the time series containing the posterior averages (solid lines) and the estimated 95% Bayesian credibility intervals (shaded areas) for each component. These components are interpreted as the contribution to the log-intensity function of the process for each period in time.

Regarding the trend component (sub-figure (a) of Figure 1), we can observe a constant growth between 1998 and 2005, a rapid reduction between 2005 and 2008, with a stabilization in the values of the trend between 2008 and 2014. From 2015 until 2023, we can see two peaks, with the most significant between 2018 and 2021. Finally, at the end of the observed period, the trend reverts to a lower average level between 1998 and 2022.

The estimated seasonal component (sub-figure (b) of Figure 1) is quite stable, and the model does not indicate relevant changes in this pattern. It is also important to note that the estimated cyclical component (sub-figure (c) of Figure 1) has a stable behavior over time.

The quarterly spatial heterogeneity of the fire events over 1998 and 2022 in the Brazilian Pantanal is clearer through the estimated spatial random effects (see Figure 2), which capture the observed variability of dry (with higher risk of fire occurrence) and wet periods in the region, with higher variability in the most fire-susceptible areas, i.e., mainly in the south and central areas, but also in the northeast and north portion of the Pantanal.

Figure 3 shows the predicted values for the model for the log-intensity of the LGCP process, between 1998 and 2022. We can observe that this function adequately explains the variations observed in the occurrences of fires in each
quarter of the sample, in particular the periods with the record of occurrences of events in 2005Q3, 2007Q3 and 2020Q3.

As discussed by Marengo et al. (2021), the Pantanal region has suffered a prolonged drought since 2019. During both the summer of 2019 and 2020, there were significant changes in upper, middle and lower-level circulation and moisture transport in South America (Cunha et al. (2023)), causing rainfall anomalies, which has caused a decrease in the hydrometric levels throughout the Paraguay River (Silva et al. (2023)). These unusual drought and warm conditions favoring the fire spread, causing unprecedented wildfires in the region in the year 2020 (Libonati et al., 2020). However, by analyzing the results of the estimated model and, in particular the growing trend, the hypothesis
that the record of fires is explained only by the low accumulated rainfall is not supported. Low rainfall plays an important role in the rainfall record, but there is also a permanent change in the average pattern that is captured by the trend component.

The recent observed increase in the number of fire outbreaks in the Brazilian Pantanal may be linked to the recent Brazilian environmental policies. Indeed, we do not have a singular government intervention that could affect the fire occurrence, but rather a set of interventions that could affect the Brazil’s environmental system. Although the weakening of Brazilian environmental laws is not entirely new, the election of Brazil’s former president, Jair Bolsonaro, has weakened environmental protect measures. Supported by the ruralist groups, Bolsonaro has introduced several measures that encourage the expansion of agriculture and livestock, such as drastic reduction in funds for controlling and monitoring biomes and freer use of agrochemicals
and pesticides, leading to substantial environmental damage (Abessa et al., 2019; Menezes and Barbosa Jr., 2021).

Therefore, considering this scenario where along with the relaxation of environmental legislation, we have an historical record of fire outbreaks registered in 2020 (Silva et al., 2022; Shimabukuro et al., 2023), we aimed to estimate the difference of the observed and predicted trajectory of the trend of fire events in the Brazilian Pantanal. One way to analyze this issue is through a counterfactual decomposition, which can also be interpreted as a form of causal effect given by some intervention. In this work we use a generalization of the causal decomposition mechanism for time series proposed in Brodersen et al., (2015). The idea is to use a state space model with the inclusion of covariates to perform the analysis of a possible causal effect given by intervention,

3 See also https://www.science.org/content/article/brazil-s-new-president-has-scientists-worried-heres-why
controlling for the effect of covariates that are not affected by the intervention. More details about the method used to construct the counterfactual can be found in the Online Appendix.

From this decomposition, we map the difference between the estimated and predicted trend values for the event occurrence scale, which is the variable of interest in our counterfactual decomposition. In this analysis the interpretation is what is the difference in the number of fires with the trend estimated with the entire sample in relation to the trend forecast before the intervention. The Figure 4 shows the (a) posterior predictive expectation of the counterfactual (solid line) and the estimated 95% Bayesian credibility interval (shaded area), and (b) the cumulative difference between estimated trend based on the intensity function decomposition and the counterfactual prediction (solid line) and the estimated 95% Bayesian credibility interval (shaded area). It is important to note that this mechanism considers the effect of covariates on the model, especially the rainfall observed in the year 2020. Thus, this decomposition gives the counterfactual effect on the number of fires if there had not been a change in the behavior of the model trend.

The posterior mean of the counterfactual difference in the number of occurrences is 2221.758 for the first quarter of 2020, 2791.228 for the second quarter and 2899.734 for the third quarter of 2020, totaling an estimated value of 7912.72 fire spots given by the difference in the trend estimated by the model, for a total of 18259 observed fires. This result indicates that about 43% of fires occurrences cannot be attributed to climatic factors (rainfall) or other transient factors, and are permanent changes in the pattern of fires in the Pantanal. This result supports the interpretation that changes in the occurrence patterns of fires in the Pantanal are partly permanent changes, and
cannot be explained by climatic factors and other non-permanent effects, supporting the hypothesis of an increase in the number of intentional fires due to human actions.

Additionally, in the Online Appendix A.6 we provide a non-separable version of our spatio-temporal model, which allows us to analyze a more complex dependency structure, despite the computational cost involved in this type of analysis.

4 Conclusion

As a contribution to assess the changes in the patterns of fire occurrences in the Brazilian Pantanal, we propose to use a dynamic version of a Log Gaussian Cox process where the intensity function is decomposed into latent components such as trend, seasonality, cycles, and covariates and spatial effects, allowing us to identify long-term changes in the intensity of occurrences over time, and also to capture mean-reverting effects, taking into account the spatial heterogeneity. Within this framework, our findings suggest the existence of a variability in the trend component, which exhibits a growth pattern between 1998 and 2005, and after 2019, whereas it remained relatively stable between 2008 and 2019, and also a statistically significant major incidence of fires in natural vegetation areas. By analyzing the historical record on fire spots in the first three quarters of 2020, our results suggest that it cannot be totally attributed to climate variables, providing evidence of human-induced events.

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The authors reports no conflict of interests or competing interests of any type.

References


A Appendix

A.1 LGCP likelihood approximation

The method proposed by Simpson et al. (2016) is based on constructing an approximation to the intractable likelihood function for a LGCP process with a stochastic intensity function in the form

\[ \pi(Y \mid \lambda) = \exp \left\{ |\Omega| - \int_{\Omega} \lambda(s) \, ds \right\} \prod_{s_i \in Y} \lambda(s_i). \]  

(5)

The main idea is to construct a continuously specified random field using a basis expansion:

\[ Z(s) = \sum_{i=1}^{n} z_i \phi_i(s), \]  

(6)

with \( z = (z_1, \ldots, z_n)^T \) being a multivariate Gaussian random vector and \( \{\phi_i(s)\}_{i=1}^{n} \), is a set of linearly independent deterministic basis functions. The log-likelihood \( \log \pi(y \mid Z) = |\Omega| - \int_{\Omega} \exp(Z(s)) \, ds + \sum_{i=1}^{N} Z(s_i) \) is the sum of two components: the stochastic integral, and the field evaluated at the data points. Using a deterministic integration rule \( \int_{\Omega} f(s) \, ds \approx \sum_{i=1}^{P} \hat{\alpha}_i f(\hat{s}_i) \), for fixed nodes \( \{\hat{s}_i\}_{i=1}^{P} \) and weights \( \{\hat{\alpha}_i\}_{i=1}^{P} \) (Simpson et al. 2016) proposed the approximation

\[ \log \{\pi(y \mid z)\} \approx C - \sum_{i=1}^{P} \hat{\alpha}_i \exp \left\{ \sum_{j=1}^{n} z_j \phi_j(\hat{s}_i) \right\} + \sum_{i=1}^{N} \sum_{j=1}^{n} z_j \phi_j(s_i) \]

\[ = C - \hat{\alpha}^T \exp(A_1 z) + 1^T A_2 z, \]  

(7)

with \( C \) is a constant, \( [A_1]_{ij} = \phi_j(\hat{s}_i) \) is a matrix containing the values of the latent Gaussian model at the integration nodes \( \{\hat{s}_i\} \), and \( [A_2]_{ij} = \phi_j(s_i) \) evaluates the latent Gaussian field at the observed points \( \{s_i\} \). As discussed by Simpson et al. (2016) the main advantage of the approximation (7) follows a Poisson representation. Given \( z \) and \( \theta \), the approximate likelihood is composed of \( N + p \) independent Poisson random variables. To show this property, Simpson et al. (2016) write \( \log \eta = (z^T A_1^T, z^T A_2^T)^T \) and \( \alpha = (\hat{\alpha}^T, \hat{\theta})^T \). For pseudo-observations \( y = (y_1 \cdots y_N) \), the approximate likelihood factors is given by

\[ \pi(y \mid z) \approx C \prod_{i=1}^{N+p} \eta_i^{y_i} e^{-\alpha_i \theta_i}, \]  

(8)

which is analogous to the likelihood for observing \( N + p \) conditionally independent Poisson random variables with means \( \alpha_i \eta_i \) and observed values \( y_i \). The properties of this approximation are discussed in Simpson et al. (2016). The essential results about the convergence properties of the approximations for the stochastic integral and the random field are demonstrated in Section 6 and the Appendix of Simpson et al. (2016).

Our parameterization is based on decomposing the random field representation of the intensity function of a spatio-temporal LGCP as the composition of the static representation of for each period \( t \), and in each period the log-intensity function is given by the sum of the common latent components of trend, seasonality and cycle and the effects of covariates, given by Eq[2]. The random field generated by intensity function of the LGCP approach of Simpson et al. (2016) is represented using the SPDE framework of Lindgren et al. (2011). We describe this method in the next section.
A.2 The spatial covariance function and model details

In this section we provide a brief description of the SPDE approach proposed by Lindgren et al. (2011). The spatial structure of the model is given by the Matérn family, as discussed in the section Data and Methods. The marginal variance of the covariance function \( \sigma^2 \) is given by:

\[
\sigma^2 = \frac{\Gamma(\nu)}{4\pi\kappa^{2\nu} \tau^2 \Gamma(\nu + \frac{d}{2})}
\]  

(9)

where \( \tau \) is a scaling parameter and \( d \) is the space dimension. In order to easier obtain the results, we adopt a parameterization in terms of \( \log \tau \) and \( \log \kappa \) for the covariance function, following Lindgren et al. (2011):

\[
\log \tau = \frac{1}{2} \log \left( \frac{\Gamma(\nu)}{\Gamma(\alpha)(4\pi)^{d/2}} \right) - \log \rho - \nu \log \rho
\]

(10)

\[
\log \kappa = \log \left( \frac{8\nu}{\Gamma(\alpha)} \right) - \log \rho
\]

where \( \rho = \left( \frac{8\nu}{\Gamma(\alpha)} \right)^{1/2} \). To approximate the LGCP likelihood, we adopt SPDE approach, using the fact that the term \( \omega(s, t) \) corresponds to a random field with a Matérn covariance, which allows to approximate this structure with a Gaussian Markov Random Field (GMRF).

Thus, the first main result for the SPDE approach is that the GF \( \omega(s) \) with the Matérn covariance function is a stationary solution to the linear fractional SPDE (Whittle, 1954; Lindgren et al., 2011):

\[
(\kappa - \Delta)\frac{\alpha}{2} x(s) = W(s), \quad s \in \mathbb{R}^d, \quad \alpha = \nu + d/2, \quad \kappa > 0, \quad \nu > 0
\]  

(11)

where \( \Delta = \sum_{i=1}^d \partial^2 \partial_{s_i}^2 \) is the Laplacian operator and \( W(s) \) is a spatial white noise. Therefore, in order to find a GMRF approximation of a GF, we need to find the stochastic weak solution of SPDE (11). Using Finite Elements Method (FEM), it is possible to construct an approximated solution of SPDE (Lindgren et al., 2011), which is given by

\[
x(s, t) \approx \tilde{x}(s, t) = \sum_{j=1}^n w_j \phi_j(s, t)
\]  

(12)

where \( n \) is the number of vertices of the triangulation, \( \{w_j\} \) are the weights with Gaussian distribution and \( \{\phi_j\}_{j=1}^n \) are the basis functions defined for each node on the mesh. In summary, the idea is to calculate the weights \( \{w_j\} \), which determine the values of the field at the vertices, while the values inside the triangles are determined by linear interpolation (Lindgren et al., 2011), and the equation (12) represents a link between the GF and GMRF, where \( \{\omega_j\} \) has a Markovian structure (Lindgren et al., 2011).

By replacing the GF by the GMRF approximation, we obtain an approximation of the LGCP likelihood, which consists of \( n + n_t \) independent Poisson random variables, where \( n \) is the number of vertices and \( n_t \) is the number of observed fires (Simpson et al., 2016). Under the GMRF structure, it is possible to estimate the model within the Bayesian framework using the Integrated Nested Laplace Approximation (INLA) framework, which allows the use of deterministic approximations to perform the estimation of latent parameters and components in models with an additive structure. More detailed description of the INLA method can be found in Rue et al. (2009). In all analyzes we use the standard reference prior structure described in Martino and Rue (2020). We use log-gamma priors for the precision parameters of the trend and seasonal components, and penalized complexity priors for the precision and the autoregressive parameters of the cycle components, Gaussian priors for the parameters of covariates, and a multivariate Gaussian prior for the parameters of the Matérn covariance function. Details can be obtained from the authors.
A.3 Data details

The daily data of fire spots used in the paper is provided by the Programa Queimadas from Brazilian National Institute of Spatial Research (Instituto Nacional de Pesquisa Espacial-INPE), which uses two different sensors as the main source of information, namely Moderate Resolution Imaging Spectroradiometer (MODIS) Aqua and Terra products, and the Advanced Very High Resolution Radiometer (AVHRR) from National Oceanic and Atmospheric Administration (NOAA). In particular, the NOAA/AVHRR was used as reference satellite for INPE from 1998 to 2002, when it was replaced by MODIS sensor, causing inhomogeneities in the observed time series, which can be considered an important limitation of our analysis.

However, despite the discontinuity in the observed data, it is worth noting that even indicating a fraction of the actual number of fires and forest fires, for using the same detection method and collecting images at close times over the years, the results obtained from both reference satellite allows us to analyze the spatial and temporal trends of the fires. In this sense, to reach our goal, we use the data from the MODIS/NASA and AVHRR/NOAA satellites, with the data validation carried out by the Queimadas system. The data set provides geographical information, and time and period of fire spots within Brazilian Pantanal, which is located within the states of Mato Grosso do Sul and Mato Grosso, in the southwest region. In addition, in order to provide a clearer interpretation of the results obtained, we used a quarterly aggregation of the daily data, which is the sum of the observed fire events in each quarter of the year.

We have plotted the number of quarterly observed fire spots (see Figure 5), from 1998 to 2022, where it is possible to see the unprecedented fire breakout in 2020. To emphasize the large number of fire outbreaks detected in 2020, in Figure 6, we show the observed fire events (black dots) in the Brazilian Pantanal in the third quarters of 2018 and 2020 (on the top of the figure). Also, in the bottom of Figure 6, it is possible to observe the non-parametric kernel density estimate of the intensity function of the occurrence process. We choose to compare the fire outbreak in 2020 with 2018 since the latter presented fire patterns close to the average in comparison to the past 20 years. Based on Figure 6, we can see a notable increase in the intensity and spatial distribution of fire outbreaks in 2020 compared to 2018.

Regarding the covariates, it is worth discussing a meaningful limitation related to the selected covariates in our analysis. Since the proposed model performs a spatio-temporal analysis for the occurrences of a process observed continuously in space, the covariates must be available at every location of the interest region within the observation window. Due to this methodological constraint, the number of available covariates is limited, and we were able to include limited information in climatic patterns and land use. In particular, regarding the rainfall and maximum temperature data, we calculated the spatially continuous projections from weather station data, following the methodology of Laurini (2017), as discussed below.

The rainfall data were constructed using the time series of the monitoring stations provided by Brazilian National Agency of Waters (Agência Nacional de Águas-ANA) and National Institute of Meteorology (Instituto Nacional de Meteorologia - INMET), whereas the maximum temperature data were obtained based on the information provided by the INMET. For both data, we calculated the spatially continuous projections for each period in the sample, following the methodology proposed by Laurini (2017). By adopting this methodology, we were able to avoid some common problems faced for the analysis of the data sources used in climatology, namely, the dimensionality of the spatio-temporal dataset, the importance of the spatial features, and missing data. In particular, by combining a structure of trend-cycle decomposition with the continuous spatial formulation, the approach allows us not only to estimate the patterns throughout the spatial continuum and how it propagates throughout the area of interest, but also provides a way to solve the missing data problems by adding the latent components with the prediction obtained for the spatial effect in the geographic position of the weather station using the continuous projection of the

Data and other information available at http://queimadas.dgi.inpe.br/queimadas/portal
spatial effect, without the necessity of additional treatments for missing data or interpolation methods. In summary, this methodology allows us to control possible changes in weather patterns, and is also based on possible changes in trends, seasonality and cycles in climate data.

In addition, we included, as categorical variables, yearly information on LULC provided by the Landsat-based MapBiomas project (Collection 5). The database includes annual historical maps of each biome, which contains a hierarchical system of classification of land use/land cover following the Food and Agriculture Organization (FAO) and Instituto Brasileiro de Geografia e Estatística (IBGE) classification systems. The first level contains six classes, namely, forest, non-forest formation, farming, non vegetated area, water, and non-observed. Forest constitutes natural forest and forest plantation, whereas the non forest natural formation includes wetland, grassland, salt flat, rocky outcrop, and other non forest formations. The farming class includes pasture and agricultural land and mosaic. The non vegetated areas are defined by beach and dune, urban area, mining e and other non vegetated areas. Finally, water class includes rivers, lakes, ocean, and aquaculture. The accuracy statistics varies according to the level and biome. In particular, considering the Pantanal
Fig. 6: Observed fires (top left and top right) and nonparametric intensity estimation (bottom left and bottom right) - 2018Q3 (top and bottom left) and 2020Q3 (top and bottom right)

biome, the first level has 81.6% of overall accuracy with 12.9% of allocation mismatch and 5.6% of quantity mismatch. For the second and third levels, the overall accuracy is 73.5%, whereas the allocation and quantity mismatch are 17.5% and 9%, respectively. The methodology overview of the MapBiomas project is available at [https://mapbiomas.org/](https://mapbiomas.org/) whereas the accuracy assessment for the Brazilian biomes is available in the MapBiomas accuracy statistics web page. A detailed description of the MapBiomas land use/land cover classification can be found in Table 1, while Figure 7 shows a map with the MapBiomas classifications for the year 2019.

A.4 Additional results of the spatio-temporal model

As discussed in the Data and Methods section, the estimation of the spatio-temporal model is based on the construction of a mesh, which represents a discretization of the continuous space for the evaluation of the likelihood function of the Log Gaussian Cox Process using the spde approximation to numerically evaluate the latent random field. In this work, we use a mesh with 1002 triangles, as shown in Figure 8. Through the INLA method, we estimated the posterior distribution of the parameters described in the Equation (2). Thus, the estimated parameters (see Table 2) are the precision of the trend component ($1/\eta_t$), seasonal component ($1/\eta_s$), and cycle component ($1/\eta_c$), the parameters of the second-order autoregressive process of the cycle (PACF1 and PACF2), the parameters associated with the set of observed covariates ($\beta$), the parameters of spatial covariance ($\log \tau$ and $\log \kappa$), and the parameter of spatial time dependence ($\Phi$).

The estimated cycle parameters indicate low persistence in this component, with low values of partial autocorrelations, but with a relevant variance over time, as shown in Figure 4. The parameters $\log \tau$, $\log \kappa$, and $\Phi$ are linked to the representation of the Matérn spatial covariance matrix used in the representation of the model, and which are
Table 1: MapBiomas Land Use/land cover Classification

<table>
<thead>
<tr>
<th>LULC ID</th>
<th>LULC Description</th>
<th>ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Forest</td>
<td></td>
</tr>
<tr>
<td>1.1.</td>
<td>Natural Forest</td>
<td>2</td>
</tr>
<tr>
<td>1.1.1.</td>
<td>Forest Formation</td>
<td>3</td>
</tr>
<tr>
<td>1.1.2.</td>
<td>Savanna Formation</td>
<td>4</td>
</tr>
<tr>
<td>1.1.3.</td>
<td>Mangrove</td>
<td>5</td>
</tr>
<tr>
<td>1.2.</td>
<td>Forest Plantation</td>
<td>9</td>
</tr>
<tr>
<td>2.</td>
<td>Non Forest Natural Formation</td>
<td></td>
</tr>
<tr>
<td>2.1.</td>
<td>Wetland</td>
<td>11</td>
</tr>
<tr>
<td>2.2.</td>
<td>Grassland Formation</td>
<td>12</td>
</tr>
<tr>
<td>2.3.</td>
<td>Salt Flat</td>
<td>32</td>
</tr>
<tr>
<td>2.4.</td>
<td>Rocky Outcrop</td>
<td>29</td>
</tr>
<tr>
<td>2.5.</td>
<td>Other non Forest Formation</td>
<td>13</td>
</tr>
<tr>
<td>3.</td>
<td>Farming</td>
<td></td>
</tr>
<tr>
<td>3.1.</td>
<td>Pasture</td>
<td>15</td>
</tr>
<tr>
<td>3.2.</td>
<td>Agriculture</td>
<td>18</td>
</tr>
<tr>
<td>3.2.1.</td>
<td>Temporary Crop</td>
<td></td>
</tr>
<tr>
<td>3.2.1.1.</td>
<td>Soybean</td>
<td>39</td>
</tr>
<tr>
<td>3.2.1.2.</td>
<td>Sugar cane</td>
<td>20</td>
</tr>
<tr>
<td>3.2.1.3.</td>
<td>Other Temporary Crops</td>
<td>41</td>
</tr>
<tr>
<td>3.2.2.</td>
<td>Perennial Crop</td>
<td>36</td>
</tr>
<tr>
<td>3.3.</td>
<td>Mosaic of Agriculture and Pasture</td>
<td>21</td>
</tr>
<tr>
<td>4.</td>
<td>Non vegetated area</td>
<td></td>
</tr>
<tr>
<td>4.1.</td>
<td>Beach and Dune</td>
<td>23</td>
</tr>
<tr>
<td>4.2.</td>
<td>Urban Infrastructure</td>
<td>24</td>
</tr>
<tr>
<td>4.3.</td>
<td>Mining</td>
<td>30</td>
</tr>
<tr>
<td>4.4.</td>
<td>Other non Vegetated Areas</td>
<td>25</td>
</tr>
<tr>
<td>5.</td>
<td>Water</td>
<td></td>
</tr>
<tr>
<td>5.1.</td>
<td>River, Lake and Ocean</td>
<td>33</td>
</tr>
<tr>
<td>5.2.</td>
<td>Aquaculture</td>
<td>31</td>
</tr>
<tr>
<td>6.</td>
<td>Non Observed</td>
<td></td>
</tr>
</tbody>
</table>

also better interpreted by the posterior distribution of the spatial random effect (Figure 2). It is important to note that the temporal persistence of the spatial effect is relatively low, with Group $\Phi$ parameter estimated with a posterior mean value of 0.345.

A.5 Counterfactual Inference

From the decomposition of the intensity function, we propose to use the Brodersen et al. (2015) approach to obtain the posterior predictive expectation of the counterfactual, which is estimated on the basis of a diffusion-regression state-space model that predicts the counterfactual response in a synthetic control that would have occurred without any intervention. The synthetic control method (Abadie and Gardeazabal 2003, Abadie et al. 2010) is a systematic way to construct the counterfactual. By constructing a synthetic control, Brodersen
Fig. 8: Spatial mesh of the Brazilian Pantanal

Table 2: Estimated Parameters

<table>
<thead>
<tr>
<th>Fixed effects</th>
<th>Mean</th>
<th>SD</th>
<th>0.025quant</th>
<th>0.5quant</th>
<th>0.975quant</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rainfall</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>Forest formation</td>
<td>0.071</td>
<td>0.036</td>
<td>0.000</td>
<td>0.071</td>
<td>0.141</td>
<td>0.071</td>
</tr>
<tr>
<td>Savanna formation</td>
<td>0.084</td>
<td>0.038</td>
<td>0.009</td>
<td>0.084</td>
<td>0.158</td>
<td>0.084</td>
</tr>
<tr>
<td>Grassland formation</td>
<td>0.185</td>
<td>0.036</td>
<td>0.115</td>
<td>0.185</td>
<td>0.255</td>
<td>0.185</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random effects</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision for trend</td>
<td>5.468</td>
<td>0.159</td>
<td>5.175</td>
<td>5.468</td>
<td>5.820</td>
<td>5.426</td>
</tr>
<tr>
<td>Precision for seasonality</td>
<td>13750.823</td>
<td>324.050</td>
<td>12486.171</td>
<td>13786.492</td>
<td>13095.810</td>
<td></td>
</tr>
<tr>
<td>Precision for cycle</td>
<td>1.220</td>
<td>0.033</td>
<td>1.149</td>
<td>1.219</td>
<td>1.284</td>
<td>1.227</td>
</tr>
<tr>
<td>PACF1 for cycle</td>
<td>0.069</td>
<td>0.012</td>
<td>0.044</td>
<td>0.069</td>
<td>0.093</td>
<td>0.069</td>
</tr>
<tr>
<td>PACF2 for cycle</td>
<td>-0.128</td>
<td>0.012</td>
<td>-0.154</td>
<td>-0.128</td>
<td>-0.105</td>
<td>-0.125</td>
</tr>
<tr>
<td>Log τ</td>
<td>-4.153</td>
<td>0.008</td>
<td>-4.171</td>
<td>-4.154</td>
<td>-4.137</td>
<td>-4.152</td>
</tr>
<tr>
<td>Log κ</td>
<td>2.168</td>
<td>0.007</td>
<td>2.153</td>
<td>2.168</td>
<td>2.183</td>
<td>2.168</td>
</tr>
<tr>
<td>Group Φ</td>
<td>0.345</td>
<td>0.004</td>
<td>0.337</td>
<td>0.345</td>
<td>0.355</td>
<td>0.345</td>
</tr>
</tbody>
</table>

et al. (2015) proposes to combine three sources of information, using a state-space time-series model: (1) the time series behavior of the response itself, prior to the intervention; (2) the behavior of other time series that were predictive of the target series prior to the intervention; (3) prior knowledge about the model parameters.

In the structure proposed in Brodersen et al. (2015), the permanent causal effect is given by the difference of the values adjusted by the model using all available information, with the values predicted by the model using the trend component forecast using the data.
before the intervention. Thus, the causal effect is given by the difference in the forecast using the trend observed using the entire sample, with the model using a trend forecast with the information before the intervention occurred. In this form, the model captures the causal effect as the permanent change given by the intervention, controlling for all other transient and covariate effects included in the model. Our generalization is based on the generalization of this model of time series to a spatio-temporal context, given by the inclusion of random spatial effects.

Through Bayesian approach, the inference procedure starts by specifying a prior distribution, \( p(\theta) \), on the model parameters as well as a distribution on the initial state values, \( p(\delta_0) \). According to [Brodersen et al. (2015)]\(^1\), the posterior inference can be broken down into three parts: simulate draws of the model parameters \( \theta \) and the state vector \( \delta \) given the observed data \( \mu_{1:n} \), using MCMC; from posterior simulations, simulate from the posterior predictive distribution \( p(\tilde{\mu}_{n+1:m}|\mu_{1:n}) \) over the counterfactual time series \( \tilde{\mu}_{n+1:m} \) given the observed pre-intervention \( \mu_{1:n} \); use the posterior predictive samples to compute the posterior distribution of the (pointwise) impact, which is defined as

\[
\phi^{(\tau)}_t := \mu_t - \tilde{\mu}^{(\tau)}_t
\]

for each draw \( \tau \) and each time point \( t = n+1, \ldots, m \). From the same samples it is possible to obtain posterior distribution of cumulative impact, estimated by

\[
\sum_{t'=n+1}^t \phi^{(\tau)}_{t'} \quad \forall t = n+1, \ldots, m.
\]

A.6 Non-separable spatio-temporal model

The model proposed in this paper assumes a separable structure between the spatial and temporal effects, assuming a Kronecker product between the spatial and temporal covariates to obtain the spatio-temporal representation, which is advantageous due to its flexibility. Despite the computational cost, by assuming a non-separable structure for time and space, it is possible to analyze a more complex dependency structure. In order to provide an additional robustness analysis, we estimated a non-separable version of the proposed model presented in the previous section, following [Bakka et al. (2020)]\(^2\), which provides a non-separable representation for the spatio-temporal random effects using the generalization of the Matérn covariance structure. In particular, in this representation the structure of the spatio-temporal random effects is given by a diffusion-based extension of the Matérn field, i.e., the random field \( u(s,t) \) can be written as

\[
\left( \gamma \frac{d}{dt} + \Lambda \right)^{\alpha_1} u(s,t) = \xi(s,t)
\]

with \( \Lambda = \gamma^2 - \Delta \), \( \xi(s,t) \) is Gaussian noise that is white in time but correlated, with precision operator \( Q(\gamma_s, \gamma_e, \alpha_e) = \gamma^2 L^{\alpha_s} \), with \((\gamma_t, \gamma_s, \gamma_e)\) being a fixed scaling parameters, and \((\alpha_1, \alpha_s, \alpha_e)\) parameters in the model.

Defining \( \alpha = \alpha_s + (\alpha_t - 1/2) \), and assuming that \( \alpha_t, \alpha_s, \alpha_e \) satisfy \( \alpha > 1 \) the solution \( u(s,t) \) has marginal spatial covariance function given by:

\[
C(u(t, s_1), u(t, s_2)) = \frac{\sigma^2}{\Gamma(\nu_s)2^{\nu_s-1}(\gamma_s||s_1 - s_2||)^\nu_s K_{\nu_e}(\gamma_e||s_1 - s_2||),}
\]

where \( \nu_s = \alpha - 1 \) and

\[
\sigma^2 = \frac{\Gamma(\alpha - 1/2)\Gamma(\alpha - 1)}{\Gamma(\alpha t)\Gamma(\alpha s)8^\nu_s/\gamma_s^2\gamma_t^22^{2\nu_s-1}}.
\]

In order to carry on our analysis, we estimate a non-separable version of our LGCP model, replacing the random field structure, \( \xi(s,t) \), of the model defined in [2] with the random field \( u(s,t) \) previously defined.
We also assume the parametrization of the parameters defining $\sigma, r_s, r_t$ as:

\[
\begin{align*}
c_1 &= \frac{\Gamma(\alpha_t - 1/2)\Gamma(\alpha - 1)}{\Gamma(\alpha)\Gamma(\alpha)4\sqrt{\pi}} \\
\sigma &= \gamma_s^{-1} c_1 \frac{1/2}{\gamma_t} \frac{1/2}{\gamma_s} (\alpha - 1) \\
r_s &= \gamma_s^{-1} \sqrt{8}\nu_s \\
r_t &= \gamma_t \sqrt{8(\alpha_t - 1/2)}\gamma_s^{\alpha_s},
\end{align*}
\]

and the model is estimated using as parameters $\log \sigma, \log r_s, \log r_t$. Due to the higher computational cost and memory limitations in representing the model with the non-separable structure, we estimate the model with an alternative mesh with a lower resolution than the separable model, shown in Figure 9, with the same prior previously defined for the estimation of the model 2, and also adopting the INLA approximations to perform the Bayesian inference procedures. The estimated posterior distribution of parameters in the non-separable spatio-temporal LGCP model is presented in the Table 3, the estimated latent components of trend, seasonal and cycle components in the Figure 10 and the non-separable spatio-temporal random effect is presented in the Figures 11.

Fig. 9: Spatial mesh of the Brazilian Pantanal - Non-separable spatio-temporal model

The results of the estimated non-separable spatio-temporal model indicate similar effects for the covariates in relation to those obtained in the model with the separable spatial random effects. Additionally, this model was able to capture a wider range of values for the random effects when compared to those estimated by the separable model, as can be seen in...
Table 3: Estimated Parameters - Non-separable spatio-temporal model

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>0.025quant</th>
<th>0.5quant</th>
<th>0.975quant</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rainfall</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.001</td>
</tr>
<tr>
<td>Forest formation</td>
<td>0.148</td>
<td>0.044</td>
<td>0.062</td>
<td>0.148</td>
<td>0.235</td>
<td>0.148</td>
</tr>
<tr>
<td>Savanna formation</td>
<td>0.186</td>
<td>0.049</td>
<td>0.090</td>
<td>0.186</td>
<td>0.283</td>
<td>0.186</td>
</tr>
<tr>
<td>Grassland formation</td>
<td>0.295</td>
<td>0.049</td>
<td>0.199</td>
<td>0.295</td>
<td>0.391</td>
<td>0.295</td>
</tr>
<tr>
<td><strong>Random effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Precision for trend</td>
<td>6.048</td>
<td>0.278</td>
<td>5.539</td>
<td>6.038</td>
<td>6.686</td>
<td>6.003</td>
</tr>
<tr>
<td>Precision for seasonality</td>
<td>11785.288</td>
<td>710.372</td>
<td>10312.207</td>
<td>11752.653</td>
<td>13245.996</td>
<td>11827.200</td>
</tr>
<tr>
<td>PACF1 for cycle</td>
<td>0.077</td>
<td>0.013</td>
<td>0.050</td>
<td>0.077</td>
<td>0.105</td>
<td>0.077</td>
</tr>
<tr>
<td>PACF2 for cycle</td>
<td>-0.585</td>
<td>0.007</td>
<td>-0.598</td>
<td>-0.585</td>
<td>-0.569</td>
<td>-0.586</td>
</tr>
<tr>
<td>log σ</td>
<td>0.973</td>
<td>0.044</td>
<td>0.887</td>
<td>0.973</td>
<td>1.063</td>
<td>0.972</td>
</tr>
<tr>
<td>log r_s</td>
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<td>-0.981</td>
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<tr>
<td>log r_t</td>
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<td>0.003</td>
<td>0.592</td>
<td>0.599</td>
<td>0.606</td>
<td>0.599</td>
</tr>
</tbody>
</table>

Figure 11. Regarding the estimated components, in Figure 10, it is possible to observe that the non-separable model was able to capture an increase in the trend component during 2019 and 2020, when the historical fires occurred in the Pantanal biome. Despite that, the estimated cycle and seasonal components presented similar patterns to those obtained by the model with the separable structure.
Fig. 10: Trend, Seasonal and Cycle decomposition of fire occurrences in the Brazilian Pantanal - Non-separable spatio-temporal model
Fig. 11: Spatial Random Effects - Non-separable spatio-temporal model