On the energy of high-frequency characteristics and the lock-in phenomena of transonic buffet

Lianyi Wei
weilianyi@imech.ac.cn

Chinese Academy of Sciences Institute of Mechanics

Guannan Zheng
Chinese Academy of Sciences Institute of Mechanics

Weishuang Lu
Chinese Academy of Sciences Institute of Mechanics

Chang Yan
Chinese Academy of Sciences Institute of Mechanics

Guowei Yang
Chinese Academy of Sciences Institute of Mechanics

Research Article

Keywords: transonic buffet, limit cycle oscillation, vortex shedding, energy exchange, buffet lock-in phenomena

Posted Date: November 2nd, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3509474/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License
On the energy of high-frequency characteristics and the lock-in phenomena of transonic buffet

Lianyi Wei \textsuperscript{1,2}, Weishuang Lu \textsuperscript{1}, Guannan Zheng \textsuperscript{1,2,3,*}, Chang Yan\textsuperscript{1,2}, Guowei Yang \textsuperscript{1,2}

\textsuperscript{1}Institute of Mechanics, Chinese Academy of Sciences, Beijing, 10090, China
\textsuperscript{2}School of Future Technology, University of Chinese Academy of Sciences, Beijing, 10091, China
\textsuperscript{3}School of Engineering, University of Chinese Academy of Sciences, Beijing, 10091, China

Abstract: The limit cycle oscillation (LCO) in the transonic buffet on the fixed supercritical airfoil OAT15A under $Ma = 0.73$, $AoA = 3.5^\circ$ and $Re \approx 3 \times 10^6$, is successfully simulated by means of the Reynolds Stress Model. Higher-frequency characteristics related to vortex shedding is investigated through the sparsity-promoting dynamic mode decomposition (SP-DMD) and the wavenumber-frequency spectrum analysis. The modal analysis on the LCO pressure field reveals that the vortex modes whose frequencies gather around 3000 Hz correspond to acoustic waves and vortex shedding, and these vortex modes contribute quite a bit to the LCO behaviour and interact with the shock according to the energy criterion. On the other hand, the velocities of the shock-induced perturbation, Kelvin-Helmholtz vortices and acoustic waves are clearly identified in the wavenumber-frequency spectrum. Further, the buffet lock-in phenomena under prescribed pitch conditions are also studied by evaluating the energy exchange between the flow and the airfoil. From the energetic point of view, the trough of the energy map indicates the buffet lock-in onset while the upper zero energy exchange boundary offers the exit of the lock-in. The energy exchange between the flow and the airfoil alters the intensity of the shock and triggers the lock-in. Under the coupled interaction between the buffet and the airfoil motion, the onset of the lock-in is lowered compared to the flutter boundary, narrowing the flight envelope.

Keyword: transonic buffet, limit cycle oscillation, vortex shedding, energy exchange, buffet lock-in phenomena

1 INTRODUCTION

The transonic buffet refers to the non-linear phenomena that the shock moves back and forth in a specific period on the upper surface of the airfoil under the interaction between shock and the boundary layer, which gives rise to the drastic fluctuation of the lift and the moment. Various physics mechanisms such as shock wave, K-H instability, shock-induced intermittent flow separation, the generation and propagation of sound and so on are involved \cite{1}. When aircrafts go through constant low-frequency buffet conditions that combine specific upstream Mach numbers (\textit{Ma}) with angles of attack (\textit{AoA}), the fatigue life of the structures can be attenuated that poses risks to the performance of aircrafts. As a consequence, it is of great importance to study the mechanisms of transonic buffets in aerospace.
Tijdeman observed three types of transonic buffet phenomena [2]. Type A oscillation refers to a small and sinusoidal motion of shock on the upper surface of an airfoil; Type B oscillation refers to a larger motion of the shock with the shock vanishing at some particular phases during the downstream excursion; Type C oscillation refers to the behavior that compression waves accumulate somewhere downstream so as to form a shock, while on the other hand the shock gradually evolves into a weak shock or compression wave until leaving the leading edge of the airfoil when the shock travels upstream. Experimental and numerical results show that the main difference among these three types of transonic buffets is due to the Reynolds number \(\text{Re}\) effects. The higher the \(\text{Re}\), the more significant the Type A motion.

There are mainly two theories to shed light upon the buffet phenomena despite the fact that the mechanisms behind the buffet are still not fully understood. Lee proposed a self-sustained oscillation model, as Figure 1 illustrates, that defines a close loop that perturbations emanating from the shock travel downstream at the speed of \(u_p\). When they reach the trailing edge, upward Kutta waves propagate within the subsonic area above the separated shear layer at the speed of \(u'_c\) until they reach the shock. According to Lee, the estimated buffet frequency can be expressed as Eq. (1):

\[
f_b = \left(\frac{c-x_s}{u'_c} + \frac{c-x_s}{u_p}\right)^{-1}.
\]

\(f_b\) refers to the buffet frequency, \(x_s\) refers to the mean location of the shock on the upper surface, \(c\) refers to the chord, and \((\cdot)'\) denotes the velocity propagating upward. \(u_p\) and \(u'_c\) can be derived using various approaches like the cross-correlation method, phase shift, wavenumber-frequency spectrum and so on. However, some researchers argued that Lee’s model may not be suitable. Paladin et al. [3] pointed out that the path between the trailing edge and the shock outside the boundary layer is not strictly necessary for the instability even though the acoustic waves can be superimposed on the buffet phenomena based on unsteady Reynolds-averaged Navier–Stokes equations simulations (URANS) locally filtered with a selective frequency damping (SFD) technique. Moise et al. [4] found that the wave propagation speeds do not match their simulations and that these waves are associated with vortex shedding, which plays no major role in buffet. In contrast to Lee’s model, Crouch et al. [5,6] established that the transonic buffet is actually a global linear instability. The global instability linearizes the Reynolds averaged Navier–Stokes equations (RANS) to study the most unstable eigenmode of the flow field and determine the buffet onset, which is consistent with a supercritical Hopf bifurcation.
The dynamic mode decomposition (DMD) is a data-based technique which identifies the dynamic modes in the sequence of flow data snapshots [7]. The dynamic modes are equivalent to the global modes in global instability analysis when the flow is linear. DMD can relate the characteristic frequencies to dominant features in buffet, which makes this method more suitable than the proper orthogonal decomposition (POD) whose modes are multi-frequencies and ranked in the energy content. The transonic buffet pressure snapshots acquired by URANS in the transonic buffet are used as DMD input data in several studies and the dominant frequencies and corresponding coherent structures are achieved [8–11]. However, as Poplingher et al. [8] stated, the higher-frequency behaviour concerning trailing edge vortex shedding was not reproduced due to the defect of URANS with the Spalart-Allmaras turbulence model (S-A) closure and the grid resolution in the wake. The present study addresses this problem with the Reynolds Stress Model (RSM) and further applies DMD to analyze the high-frequency characteristics in the buffet with the pressure field as input data.

As a peculiar case of buffet, the lock-in phenomenon is generally defined as the synchronisation of the buffet frequency with the airfoil excitation frequency. It occurs when the excitation frequency and the fixed airfoil buffet frequency are close about the ratio of 1 with sufficient moving amplitudes. In the lock-in region the lift coefficient oscillates more drastically than that of the buffet on the fixed airfoil. Davis & Malcolm discovered that the airfoil aerodynamic coefficients resonated tremendously when the NACA 64A010 airfoil was moving at a certain pitch frequency in their experiments [12], and Despré found similar resonances when the OAT15A airfoil flaps were in harmonic vibration at an excitation frequency near the buffet frequency [13]. Given a prescribed oscillating motion of the airfoil, Raveh and fellows conducted simulations on the NACA0012 lock-in phenomena and owed the LCO of the system to the phase
lag/lead between the moment/lift and the excitation movement, and their further works on aeroelastic airfoils also supported this point of view [14–17]. Gao et al. established a linear reduced-order model (ROM) to study the frequency lock-in phenomena and found that the coupling of one structural mode and one fluid mode leads to frequency shift [18]. Hartmann et al. carried out experiments on the prescribed oscillating airfoils and discovered a reduced level of the shock oscillation and that the lock-in is only related to the major changes in the flow conditions and phase lagged by the time it takes until the altered pressure waves generated in the trailing-edge region reach the shock, which proves that Lee’s model is still fit in the lock-in phenomena [19]. Note that in spite of considerable numerical simulations that deprive Lee’s model of its feasibility in the transonic buffet, experimental results from Hartmann and Feldhusen-Hoffman all conform that the buffet is driven by an acoustic feedback loop [20–23].

Recently energy maps have been adopted to the flow-induced pitching airfoil or flutter analysis [24–26]. These studies focus on fabricating the energy extraction as a function of pitch oscillation’s amplitudes and frequencies or a function of pitch oscillation’s amplitudes and $Ma$ that provides convenience to quantitively determine the flutter boundary. In the transonic flutter of one degree-of-freedom (pitch) and two degree-of-freedom (pitch and heave), the lower and the upper boundary are pinned down for positive energy transfer from the flow, where the instantaneous moment is in phase with the airfoil’s angular velocity [26]. But for the transonic buffet lock-in where there is a phase lag between the moment and the pitch motion, the boundary can be altered due to the complex shock/boundary interaction, which has not yet been illustrated from the view of the energy exchange. The utilization of the energy maps or energy extraction in the present study may give new insights into the mechanism of the lock-in phenomena.

This study first resorts to relate the higher-frequency characteristics to their own spatial distributions and velocities in buffet of the fixed airfoil through DMD and the wavenumber-frequency spectrum, and then analyzes the energy exchange between the airfoil and the flow when the buffet lock-in occurs under given pitch oscillations. The paper is structured as follows. The instruction is followed by the analysis methods used in this study in Sec. 2 and the numerical set up and validation in Sec. 3. Sec. 4 exhibits and explains the results in detail, with the fixed airfoil in Sec. 4.1 and the prescribed airfoil motion in Sec. 4.2. The concluding remarks are given in Sec. 5.

2 ANALYSIS METHODS

2.1 The wavenumber-frequency spectrum

The wave number-frequency spectrum analysis method converts the spatial-temporal signal into wave number-frequency domain and extracts the fluctuation propagation information from the frequency spectrum. The wave number-frequency relationship of pressure disturbances can be obtained by the two-dimensional Fourier transformation. The distribution of the amplitude of a wave component in the wave number and the frequency is shown in the diagram, and then the wave propagation direction and the velocity can be accordingly calculated. The expression of the two-dimensional Fourier transform is given by
where \( K \) is wave number, and \( \omega \) is the angular frequency. The frequency \( f \) is then obtained from \( \omega = 2\pi f \). In practice the number of probes and the time span are limited, and the discrete two-dimensional Fourier transform is often used to deal with these discrete spatial-temporal signals of pressure pulsations, with the expression

\[
P(K,\omega) = \frac{2|\sum_{n=1}^{M} \sum_{m=1}^{N} W(x_m, t_n) p(x_m, t_n) e^{-i(Kx_m + \omega t_n)}|}{MN}
\]  

(3)

where \( N \) is the total number of time points, and \( M \) is the number of pressure pulsation monitor probes. \( W(x_m) \) and \( W(t_n) \) are window functions, and the Hanning window is applied in this paper. \( p \) is the monitored pressure perturbation value. Notably the discrete 2-D Fourier transform has restrictions on the intervals of frequency and wave number. The frequency range \( F_{\text{max}} \) is half of the sampling frequency (inverse of the sampling time interval), i.e., \( F_{\text{max}} = 0.5/dT \). The wave number range is half of the reciprocal of the distance between two neighbouring probes, i.e., \( K_{\text{max}} = 2\pi \times 0.5/ dX \).

Due to the fact that the propagation speed of the buffet perturbation \( u_p \) is much smaller than the acoustic wave speed \( u_c' \), the wave number of buffet perturbations \( k_p = 2\pi f / u_p \) is consequently much larger than the wave number of the acoustic waves \( k_c = 2\pi f / u_c' \). The wavenumber-frequency analysis clearly separates these speeds apart without the aid of filtering compared to the cross-correlation method.

### 2.2 Dynamic mode decomposition

The snapshot sequence such as the pressure, velocity and density sequence with an identical time interval \( \Delta t_1 \) between two neighbouring snapshots can be written as matrices \( X \) and \( Y \)

\[
X = [v_1, v_2, ..., v_{N-1}]
\]  

(4)

\[
Y = [v_2, v_3, ..., v_N]
\]  

(5)

where the spatial dimension of the flow field is \( m \), i.e., \( v_i \in \mathbb{R}^m \). A linear mapping is assumed which is approximately satisfied over the full sampling period

\[
v_{i+1} = Av_i, \quad i \in \{1, ..., N - 1\}
\]  

(6)

If the dynamic system is nonlinear, such as the LCO in the fully developed buffet, the assumption is a linear approximation. According to this assumption, it further yields

\[
Y = [Av_1, Av_2, ..., Av_{N-1}] = AX
\]  

(7)

that described the sequence as a Krylov sequence. The next step is to find the matrix \( A \) and calculate its eigenvectors and eigenvalues to extract the characteristics in the dynamic system as it represents the time evolution. An approximation of \( A \) is requisite due to the vast size of the original matrix as \( m \gg N \). According to Schmid, the low-dimensional representation of \( A \) can be derived by

\[
F_{dmd} \equiv U^*AU
\]  

(8)

where \( F_{dmd} \in \mathbb{C}^{N-1 \times N-1} \). \((\cdot)^*\) denotes the conjugate transpose of the matrix and \( UU^* = I \). The basis \( U \) is achieved by the singular value decomposition (SVD) of the snapshot sequence.
\[ X = UV^* \] (9)

where \( U \in \mathbb{C}^{m \times (N-1)} \) and \( V \in \mathbb{C}^{(N-1) \times (N-1)} \) are the left and right singular vectors and the matrix \( \Sigma \in \mathbb{C}^{(N-1) \times (N-1)} \) contains the non-zero singular values of the snapshot sequence in its diagonal. The approximated form of \( A \) can eventually be written as
\[ A \approx F_{dmd} = UYV^* \Sigma^{-1} \] (10)

The Ritz eigenvalues of \( F_{dmd} \) are part of the eigenvalues of \( A \). For the eigenvalue \( \mu_j \) of the \( j \)th mode, the corresponding eigenvector is \( y_j \). For the snapshot sequence \( X \) the \( j \)th eigenvector is \( \phi_j := Uy_j \), with the corresponding amplitude being \( a_j := z_j v_1 \), \( \{z_1, ..., z_{N-1}\} \) being the eigenvector of \( F_{dmd}^* \) and \( v_1 \) being the initial condition. The linear combination of the dynamic modes yields the approximated snapshots
\[ v_i = \sum_{j=1}^{N-1} a_j \mu_j^{i-1} \phi_j, \quad i \in 1, ..., N - 1 \] (11)

which can be rewritten in the matrix form
\[ X = [\phi_1, \phi_2, ..., \phi_{N-1}] \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{N-1} \end{pmatrix} \begin{pmatrix} 1 & \mu_1 & \cdots & \mu_1^{N-1} \\ 1 & \mu_2 & \cdots & \mu_2^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mu_{N-1} & \cdots & \mu_{N-1}^{N-1} \end{pmatrix} \begin{pmatrix} b_a \\ V_{and} \end{pmatrix} \] (12)

By minimizing the objective function
\[ \min_a \| X - \Phi D_a V_{and} \|_F^2 \] (13)

the optimal amplitudes \( a_j \) of all modes can be found, with \( \| \cdot \|_F^2 \) being the Frobenius norm. The growth rate and the frequency of the \( j \)th mode are calculated from
\[ g_j = Re((log \mu_j)/\Delta t) \] (14)
\[ \omega_j = Im((log \mu_j)/\Delta t) \] (15)

respectively.

The SP-DMD is a variant of the standard DMD algorithm [27] that introduces a penalty term indicating the non-zero elements in the amplitude vector \( a \) into Eq. (13), thus transforming the objective function into a convex optimization problem:
\[ \min_a \| X - \Phi D_a V_{and} \|_F^2 + \gamma \sum_{j=1}^{N-1} |a_j| \] (16)

forcing a sparse solution. The tradeoff between the approximation error
\[ I_{loss} = 100 \frac{\| X - \Phi D_a V_{and} \|_F^2}{\| X \|_F^2} \] (17)

and the number of extracted modes is selected by the users. Javanović et al. [27] proposed an algorithm that uses alternating direction method of multipliers (ADMM) to massively promote the rate of convergence. The user has to deal with the augmented-Lagrangian parameter \( \rho \) and the sparsity parameter \( \gamma \) to achieve the desired results and save the computational cost. The approach is utilized in this paper and details of the algorithm are given in Javanović et al. [27].
3 NUMERICAL SETUP AND VALIDATION

3.1 Test case description

Molton and Jacquin conducted experiments on the supercritical airfoil OAT15A in the S3Ch wind tunnel in Onera’s Chalais Meudon Center [28]. The test section of the wind tunnel has a square cross section and the size of test chamber is $0.78 \times 0.78 \text{ m}^2$. The chord length of the OAT15A airfoil is $c = 230 \text{ mm}$. The thickness of the airfoil is 12.5%, and the thickness of the trailing edge is 0.5% of the chord length. The transition locations are fixed at 0.07c on both the upper and lower surfaces. The incoming Mach number is $M_a = 0.73$, Reynolds number $R_e \approx 3 \times 10^6$, the stagnation point pressure $P_\infty = 10,000 \text{ Pa}$, the far field temperature $T_\infty = 300 \text{ K}$. The AoA ranges from 2.5° to 3.9°. The experiments showed that the buffet onset was 3.25° at the designed condition, and the larger the AoA the more drastic the shock oscillation. Buffet frequencies of all cases remained around 69-70 Hz. Further LDV measurements were carried out at an AoA of 3.5°.

3.2 Numerical setup

In this study, ANSYS Fluent software is utilized to solve the test cases. The two-dimensional (2D) mesh demonstrated in Figure 2 is used to simulate the buffet flow of the OAT15A airfoil. The height of the first layer is $5 \times 10^{-6}c$ which ensures all $y+ < 1$ on the first layer. The pressure far field boundary is at least 30c away from the airfoil. The surface of the airfoil is distributed with $n_x = 508$ grid points, and around the shock oscillation region the mesh is refined to achieve $\Delta x = 0.004c$. The number of normal grids is $n_y = 144$ and the streamwise grid number in the wake region is 121, with a total grid number of around 110,400 for the whole domain. In this study the Stress-Omega Reynolds Stress Model (RSM) is chosen as the turbulence model in the flow solver. The Stress-Omega Reynolds Model is constructed by taking the second-moments of the exact momentum equations, yielding four excess transport equations for the Reynolds stresses, as well as an equation for the dissipation rate for the closure. RSM has proved its applicability in the buffet reproduction. In contrast, the Menter’s $k - \omega$ Sher-Stress Transport (SST) model is also applied in the test case to evaluate its performance against RSM.

![Fig 2](image)

Figure 2 Computational domain of the test case: (a) the whole domain; (b) grids around the airfoil.

For the two flow solvers the solution process goes as: by means of the density-based method, the steady flow field is calculated in advance in each turbulence model. After
the solution is converged, the simulation is switched to the unsteady solution in
RSM/SST model, with the ROE flux construction form, the third order MUSCL
discretization in the flow, the second order upwind discretization in the turbulent kinetic
energy and dissipation rate, the Least Square Cell based method in gradient calculation
and the second order implicit time advancement enabled. Specifically, the $\alpha_1$
coefficient of the baseline model in the SST model, which indicates the eddy viscosity
level, is mildly reduced to promote flow unsteadiness as Giannelis et al. [29] suggested.

In this study, the time step is chosen as $\Delta t = 5e^{-6} \text{s} \ (\Delta \tau = 0.0055)$ with a minimum
of 20 inner iterations in each physical time step.

The wave number-frequency spectrum analysis is performed in a set of probes
distributed on streamline 1 in Figure 3. Streamline 1 is located about 0.02c away from
the separation zone, guaranteeing that the separation zone will not pass within this
streamline as suggested by Hartmann et al. [23]. The set contains 615 probes, which
are evenly distributed in the range $[-0.5 \text{ m}, 0.73 \text{ m}]$ with an increment of about 2mm
between each probe. The total number of probes and the increment of each probe meet
the demand of needed resolution [30]. The set of probes is recorded in 20000 continuous
time steps for spectral analysis.

![Figure 3 Probe distributions to monitor pressure fluctuations.](image)

The SP-DMD analysis is performed in the pressure data sequence of the domain,
with the time interval between two snapshots being $\Delta t_1 = 0.0001 \text{s}$, allowing the
method to capture the characteristic frequency as high as to 5000 Hz. The total
snapshots taken are 1500 ensuring at least 15 complete buffet cycles are recorded. Note
that the sampled time covers only the LCO phase instead of the linear phase, so the
DMD reconstruction is only the linear estimation of the LCO phenomena.

### 3.3 Numerical verification

The time-averaged lift coefficient $c_p$ and the root-mean-square error (RMSE) of
the suction side pressure derived by the RSM and the SST are compared with the experiment in Figure 4. The trend and slope of $c_p$ from the RSM match well with the experiment, while the RMS slightly underestimates the shock motion for the simulated upmost position of the shock is downward of the upmost position measured in the experiment as Figure 4(a) exhibits. The mean shock location of the RMS is at 0.49$c$, close to the experimental mean location at 0.45$c$. The magnitude of pressure fluctuation is well predicted by the RSM compared with the experiment. On the other hand, however, though the SST yields the shock oscillation, the SST presents an inferior performance as the $c_p$ curve deviates a lot from the experiment and the shock motion is much overestimated as shown in Figure 4(b). The magnitude of pressure fluctuation is also underestimated by the SST. The shape of both the $c_p$ and the pressure RMSE cannot be seen as similar to those in the experiment, probably because the SST may generate another flow separation phenomenon that sustains the shock oscillation. Therefore, the RSM is adopted in the further investigation in this paper.

![Figure 4](image)

Figure 4 (a) The mean pressure coefficient of simulations and the experiment; (b) the pressure RMSE of simulations and the experiment.

Figure 5 shows the transient flow velocity contours and streamlines in the vicinity of the airfoil derived by the RSM within a complete buffet period. Phase 1 and phase 5 mark the most upstream and downstream positions during the oscillation of the shock wave respectively. In the downstream excursion (phase 1 to phase 5), the shock gradually strengthens, and the separation shear layer grows thinner and gradually attaches to the airfoil surface. When the shock travels from the downstream to the upstream (phase 5 to phase 8), the shock weakens and the shear layer becomes thick again making the separation significant. At this moment the vortex shedding in the wake area is also obvious, while in the downstream excursion such shedding is suppressed even quenched due to the attachment of the separation shear layer to the airfoil surface. Figure 5 demonstrates that the RSM used in this study is able to reproduce the oscillation of the shock on the upper surface of the airfoil and the vortex shedding at the trailing edge. The buffet frequency calculated by the RSM is 80 Hz, which is acceptable compared to the experiment frequency 69-70 Hz.
RESULTS AND DISCUSSIONS

4.1 Higher-frequency characteristics of the buffet on the fixed airfoil

4.1.1 DMD analysis

According to the aforementioned SP-DMD method, the relation between the sparsity parameter $\gamma$ against the number of reduced modes and the approximation error $I_{\text{loss}}$
are plotted in Figure 6. As Figure 6 indicates, the number of reduced modes and the approximation error remain stable before $\gamma$ reaches about $10^6$, with the number of reduced modes almost equal to the number of the input snapshot sequence and the approximation precision being high. When $\gamma$ exceeds $10^6$ the number of reduced modes drops and the estimation error rises as $\gamma$ increases, resulting in sparser solution to the decomposition. After $\gamma$ is large enough, the number of reduced modes and the approximation error should plateau again, with the number of reduced modes being the minimum and the approximation error maximum. Note that in this study these second plateau phases are not calculated and presented as $\gamma$ has already been large enough ($10^8$) due to the fine resolution in the DMD snapshots and even larger $\gamma$ will substantially add to the computational cost while the corresponding solution may not be suitable any longer due to the collapse of the reconstruction precision. One may try to lower the magnitude of $\gamma$ by interpolating a coarser grid on the domain, in other words, reducing the number of spatial measuring probes or the snapshot resolution. The more probes, the greater dimensions and complexity of the problem, and the larger $\gamma$ needed to seek sparse solution.

![Figure 6](image)

(a) The relation between $\gamma$ and the number of reduced modes; (b) the relation between $\gamma$ and the approximation error.

The sparsity parameter $\gamma$ is chosen as $3 \times 10^7$ as the approximation error falls below 5% while the number of reduced modes is the least (367 modes) under the circumstance. The Ritz eigenvalues with the unit circle and the mode frequencies and their amplitudes are plotted in Figure 7. Eigenvalues on the unit circle are stable throughout the snapshot sequence. In the diagram of the frequency and the amplitude, two kinds of peaks are conspicuous. The peak with larger magnitude of the amplitude gathers around buffet frequency with the time-invariant mode included. The secondary peak accumulates about 3000 Hz which matches the vortex shedding frequency in many literatures[31,32], indicating that this peak corresponds to the vortex modes.
Figure 7 (a) The amplitude magnitude of all DMD modes; (b) the Ritz eigenvalue of all DMD modes.

The growth rates of each mode are also plotted as in Figure 8. The traditional DMD mode selecting method is based on the initial amplitudes, yet this approach might put forward some transient modes with large amplitudes at the beginning but decaying drastically soon afterwards. The initial snapshot chosen also has a great impact on the amplitudes of these less important transient modes. Kou et al. [9] proposed an energy criterion

\[ I_j = \int |b_j(t)| dt \approx \int |b_{ij}| \Delta t = \sum_{i=1}^{N} |a_{ij}(\mu_j)^{i-1}| \| \Phi_j \|_F^2 \times \Delta t_{\text{ref}} \] (18)

that ranks each mode in their contribution to the system over the sampled time and filtered the less important transient modes. The modes are sorted according to this criterion, which is established through the integral of the absolute time coefficient

\[ |b_j(t)| = |a_{ij}(\mu_j)^{i-1}| \] (19)

at all instants. The growth rate of mode 1 (buffet mode) is below zero, implying that this mode is stable, while the growth rates of mode 2 to mode 5 locate above zero, resulting in unstable characteristics. From mode 1 to mode 7 their frequencies follow the linear relation, as has been listed in Table 1. Within the dominant 20 DMD modes, only one high-frequency mode appears that is assessed of order \(10^3\) Hz (two orders of magnitude larger than the buffet mode). The existence of this mode means it has quite a lot energy contribution to the dynamic system over the sampled time. From Figure 8 it is also clear that such contribution may result from the positive growth rate, as this mode is unstable.

Table 1 The frequency and the energy criterion index for some most dominant modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency ( f_j ) [Hz]</th>
<th>( St = f c/U_m )</th>
<th>Energy criterion ( I_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean flow</td>
<td>0</td>
<td>0</td>
<td>( 1.14 \times 10^5 )</td>
</tr>
<tr>
<td>Mode 1</td>
<td>76.60</td>
<td>0.069</td>
<td>( 1.09 \times 10^4 )</td>
</tr>
<tr>
<td>Mode 2</td>
<td>153.07</td>
<td>0.139</td>
<td>( 4.77 \times 10^3 )</td>
</tr>
<tr>
<td>Mode 3</td>
<td>230.22</td>
<td>0.208</td>
<td>( 2.69 \times 10^3 )</td>
</tr>
<tr>
<td>Mode 4</td>
<td>307.10</td>
<td>0.278</td>
<td>( 2.16 \times 10^3 )</td>
</tr>
<tr>
<td>Mode 5</td>
<td>382.43</td>
<td>0.346</td>
<td>( 1.55 \times 10^3 )</td>
</tr>
<tr>
<td>Mode</td>
<td>Growth Rate</td>
<td>Frequency</td>
<td>Amplitude</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>-----------</td>
<td>-----------</td>
</tr>
<tr>
<td>Mode 6</td>
<td>458.99</td>
<td>0.416</td>
<td>$1.36 \times 10^3$</td>
</tr>
<tr>
<td>Mode 7</td>
<td>538.31</td>
<td>0.487</td>
<td>$1.07 \times 10^3$</td>
</tr>
<tr>
<td>Mode 8</td>
<td>49.26</td>
<td>0.045</td>
<td>$8.79 \times 10^2$</td>
</tr>
<tr>
<td>Mode 9</td>
<td>25.52</td>
<td>0.023</td>
<td>$8.55 \times 10^2$</td>
</tr>
<tr>
<td>Mode 18 (vortex)</td>
<td>3103</td>
<td>2.810</td>
<td>$2.68 \times 10^2$</td>
</tr>
</tbody>
</table>

Figure 8 The growth rates of the DMD modes.

Figure 9 demonstrates the pressure contours of the time-invariant mode (the mean flow), mode 1 to mode 5. From Figure 9(a) the mean shock location is given, and from Figure 9(b) to Figure 9(f) the most drastically varied zones of the pressure phase occur about the shock oscillation area, with secondary vertically symmetric structures in the wake. Consequently, mode 1 to mode 5 exert their impacts mainly on the buffet oscillation characterized with the low frequencies.
Figure 9 The pressure reconstruction of DMD modes: (a) the mean flow; (b) mode 1; (c) mode 2; (d) mode 3; (e) mode 4; (f) mode 5.

To examine Figure 10 that represents the most dominant vortex mode, differences emerge with the buffet modes. Firstly, the most massive spatial structures are not buffet-related anymore. Instead, the pressure structure pair sequence in the wake rules. Secondly, the pressure structure sequence in the wake is vertically antisymmetric rather than symmetric as the buffet mode, indicating an alternating shedding behaviour. Thirdly, the acoustic field is illustrated by the pressure field around the airfoil. The acoustic wave fronts are aligned with the alternating pressure contour, and the propagation direction is perpendicular to the contour. However, the acoustic fields below and above the airfoil are distinct. The acoustic waves travel all way along the pressure side of the airfoil till they dissipate around the leading edge. The acoustic waves above the suction side, terminate about the shock area and seem to not diffuse further into the leading edge. Apart from that the wave fronts above the suction side exhibit different shapes as those below the pressure side. Two discontinuous gaps are observed in the upper pressure field that alter the propagation direction of the waves. These changes should result from the reflection of the acoustic waves by the shock. These rebounded waves are superimposed on the original acoustic field. Such propagation and reflection can be seen from the pressure gradient field (x-component) in Figure 11. As a result, the vortex mode is related to the vortex shedding and the acoustic field, suggesting that these phenomena have energy contribution to the buffet flow field.
4.1.2 Wavenumber-frequency analysis

To better reveal different velocities of disturbances corresponding to distinct mechanisms in the same order of magnitude, the index of Eq. (3) is rescaled to the form of

$$\hat{P}(k, \omega) = \log_{10}(P(k, \omega))$$

The wavenumber-frequency spectrum of the measured probe is presented in Figure 12(a). The upward and downward velocities of disturbances generated from the shock above the separated zone are nearly the same, with the upgoing one slightly larger than the downward one. In Figure 12(a), the downward vortices with a much higher velocity ($u_v$) is also spotted. Yet in Figure 12(a) the information of the up-travelling sound waves is overwhelmed by the low-frequency perturbations when the sampled section covers the shock oscillation zone. So, the section $[-0.19\,m, 0.73\,m]$ of the streamline 1 which falls behind the shock oscillation zone is exploited to extract the up-propagating sound speed, as Figure 12(b) exhibits. According to the wavenumber-frequency spectra, the velocities of disturbances are calculated and listed in Table 2. Based on Eq. (1), the estimated buffet frequency is 71 Hz, smaller than the simulated result (80 Hz) but closer to the experimental result (69-70 Hz).
Figure 12 (a) The wavenumber-frequency spectrum of all probes on streamline 1; (b) the wavenumber-frequency spectrum of probes on streamline 1 behind the shock.

Table 2 Velocities derived from the wavenumber-frequency spectrum.

<table>
<thead>
<tr>
<th>Velocity (m/s)</th>
<th>Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_p$</td>
<td>9.6</td>
</tr>
<tr>
<td>$u_p'$</td>
<td>11.3</td>
</tr>
<tr>
<td>$u_v$</td>
<td>196.6</td>
</tr>
<tr>
<td>$u_c'$</td>
<td>65.8</td>
</tr>
</tbody>
</table>

Let’s further examine the vorticity contour (Figure 13) of the fixed airfoil. During the downstream excursion of the shock, the shear layer remains attached to the upper surface of the airfoil, and no apparent vortex sequence is found in the airfoil aft section. When the shock travels upward, a sequence of vortices above the thickened separation zone is generated and dissipates into the wake. These vortices are the Kelvin-Helmholtz (K-H) vortices initialised by the upgoing acoustic waves or Kutta waves. The distance between two neighbouring K-H vortices is almost identical with the averaged value of $\lambda = 0.026m$ or $\lambda/c = 0.113$. These vortices are additionally tracked in snapshots with the time interval of 5μs and the vortex velocity turns out about 180 m/s which is almost the same with $u_v$ measured in the wavenumber-frequency analysis. The frequency of the K-H instability is estimated about 7000 Hz in this case, which can be clearly identified in the power spectrum of density (PSD) of the wall pressure perturbations at the probe in the airfoil aft section, where the high-frequency peaks including vortex shedding and K-H instability are more pronounced [31], as Figure 14 shows. Due to the low-frequency dynamics of the flow field that interact with the high-frequency structures, the frequency of K-H instabilities should vary significantly. Despite the fact that this study captures the characteristic frequency and the general zone of the K-H instability, the fine-grained development and break-down of these vortices are not acquired because of the restriction of URANS. This can be solved by the large eddy simulation (LES) [4], the detached-eddy simulation (DES) [33] or direct numerical simulations (DNS) [34,35].
4.2 Energy exchange of the buffet on the prescribed pitching airfoil

The airfoil is mounted in one degree-of-freedom (1-DOF) pitch operating at an initial zero angle of attack. The pivot point is fixed at \( x = 0.45c, y = 0 \) of the airfoil around the mean shock location. The pitch motion of the airfoil is defined as:

\[
\theta(t) = 2\sin(2\pi f_p t) = A_{\theta} \sin(2\pi f_p t)
\]

(21)

where \( f_p \) denotes the excitation frequency of the pitch motion and \( A_{\theta} = 2^\circ \) denotes the amplitude the pitch oscillation. Raveh et al. [17] suggested that the amplitude of the pitch motion should not be too small otherwise the aerodynamic buffet response cannot be affected. Furthermore, to allow the lock-in and LCO to occur, the system’s damping must approach zero so that the system is on the brink of instability. The pitch system in this study thus has no damping. The energy extraction from the flow by the airfoil is defined as

\[
E^* = \int_{t_n}^{t_n + T} M \dot{\theta} dt \tag{22}
\]

where \( M \) is the aerodynamic moment about the elastic axis and \( T \) is the period of the pitch oscillation. In this section the evolution of the energy extraction

\[
E(t) = \int_0^t M \dot{\theta} dt \tag{23}
\]

is also investigated. The prescribed pitch frequencies are given in Table 3, and the pitch cases are all calculated after the same timestep of the buffet on the fixed airfoil.
Table 3 Prescribed pitch motion frequencies.

<table>
<thead>
<tr>
<th>Case</th>
<th>$f_p$ [Hz]</th>
<th>$f_p/f_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>0.75</td>
</tr>
<tr>
<td>3</td>
<td>72</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>1.0</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>1.125</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>1.25</td>
</tr>
<tr>
<td>7</td>
<td>110</td>
<td>1.375</td>
</tr>
<tr>
<td>8</td>
<td>120</td>
<td>1.5</td>
</tr>
<tr>
<td>9</td>
<td>140</td>
<td>1.75</td>
</tr>
</tbody>
</table>

In Figure 15 all the energy extraction $E(t)$ from the flow and the moment coefficient $c_m$ are plotted. Leaving out the first several transient cycles, the evolution of $E(t)$ from Case 1 to Case 3 reveals a negative energy transfer from the flow to the airfoil. As the pitch frequency grows larger the flow transfers positive energy to the airfoil in Case 4 where $f_p/f_b = 1$. Then in Case 5, the energy exchange reaches an equilibrium that nearly no energy is transferred. The energy extraction evolution $E(t)$ from Case 1 to Case 6 is defined as the linear stage in this paper as the energy exchange within each cycle is approximately the same. From Case 6 to Case 9, the energy evolution exhibits an oscillating characteristic compared to the former cases referred to as the linear stage. The energy evolution in Case 6 exhibits a dynamic equilibrium that the energy transferred from the flow to the airfoil is approximately zero over the sampled time but the specific energy exchange within each cycle during the sampled time can be positive or negative. From Case 7 to Case 9, overall, the energy evolution indicates a negative energy transfer from the flow to the airfoil over the sampled time, with the exact energy exchange within each cycle varying from negative to positive, forming a secondary oscillation as in Case 6. The energy evolution in Case 6 to Case 9 is thus defined as the non-linear stage. The average $E^*$ representing the energy extraction in each period for all test cases is plotted in Figure 16, where the averaged $E^*$ crossed zero from negative to positive around $f_p/f_b = 0.925$ and from positive to negative around $f_p/f_b = 1.125$. 

(a) Case 1  
(b) Case 2
Figure 15 The energy extraction evolution $E(t)$ of all cases.

Figure 16 The averaged energy extraction $E^*$ for pitch at $A_{\theta} = 2^\circ$

The frequency contents of the energy extraction evolution $E(t)$ and the moment coefficient $c_m$ are also discussed in this section by Fast-Fourier Transform (FFT) as Figure 17 demonstrates. From Case 3 to Case 4 both $E(t)$ and $c_m$ present a monofrequency characteristic in the low-frequency zone that the buffet frequency locks onto the pitch motion frequency. From Case 1 to Case 2 and Case 5 to Case 9, however, $E(t)$ and $c_m$ display a response in the airfoil motion and the buffet frequency.
indicating the disappearance of the lock-in phenomena. Particularly in Case 6 to Case 9, for $E(t)$ apart from the pitch frequency and its harmonics, there is a minor peak before the pitch frequency representing the gap between the pitch frequency and the buffet frequency. It can be seen from Figure 17(f) that the frequency gap dominates the secondary oscillation of the $E(t)$ evolution. Though the frequency content of $E(t)$ shows mainly the pitch motion, this minor peak indirectly shows the buffet frequency. Note that this secondary peak only appears when the pitch frequency is greater than the buffet frequency.

(a) Case 1
(b) Case 2
(c) Case 3
(d) Case 4
(e) Case 5
(f) Case 6
The energy map as the function of the airfoil pitch amplitude of a wider range \(1^\circ \leq A_\theta \leq 5^\circ\) and the pitch frequency \(f_p\) is plotted in Figure 18, where the frequency responses in the test cases are also marked. The lock-in boundary is also outlined in the dashed line. Different from the flutter boundary in the energy map characterized with the \(E^* = 0\) (solid line in Figure 18), only the offset of the lock-in satisfies such condition. Instead, the onset boundary deviates from the equilibrium line and travels along the trough of the map where the gradient of the energy exchange is the maximum, as Figure 19 shows. Under the coupled interaction between the buffet and the airfoil motion, the onset of the lock-in is lowered compared to the flutter boundary, narrowing the flight envelope.
Figure 18 The energy map of the buffet lock-in on the pitch airfoil and the frequency response of the moment coefficient. Solid line: the flutter boundary at $E^* = 0$. Dashed line: the buffet lock-in boundary (the offset overlaps with the upper flutter boundary).
From the discussion above, we can postulate that the onset of the buffet lock-in occurs at the trough of the energy map and the exit of the lock-in lies at the equilibrium energy exchange. The bump encircled by these two boundaries is the buffet lock-in region. To support this assumption, contours of the root-mean square (RMS) of X-direction velocity of cases in Table 3 are given in Figure 20. When the pitch motion is under the buffet lock-in onset (Figure 20(a) to Figure 20(b)), the flow absorbs energy from the structure and the shock intensity is strengthened at the downstream location, with a less shock oscillation excursion and less velocity fluctuations in the separation zone. Under the circumstance the buffet has its influence so as not to be overwhelmed by the pitch motion. When the pitch motion crosses the onset into the lock-in region (Figure 20(c) to Figure 20(d)), however, the energy transferred from the airfoil to the flow substantially shrinks and the airfoil even absorbs energy from the flow. Consequently, the shock intensity is alleviated, leading to stronger velocity fluctuations in the separation zone due to the varied shock intensity during its extended excursion on the upper surface. The reduction of the shock intensity in the lock-in region implies a more complex interaction between the shock and the shear layer that may render the buffet more perceptible to the airfoil motion so as to synchronize with the airfoil pitch excitation. The trough of the energy map in the pre-lock-in region triggers the synchronization of the frequencies. The shock/boundary interaction in the buffet region also has a significant impact on the vortex shedding, and the vortex shedding that possesses noticeable energy in the flow field as has been mentioned in the precious section, should react upon the shock motion. Due to the restriction of RMS the fine-
grained vortex structures and their development are not able to be captured in this paper. The influence of the energy exchange on the vortex shedding characteristics remains to be further investigated with the aid of LES, DES or DNS based on a three-dimensional domain. After the pitch motion quits the lock-in offset (Figure 20(e) to Figure 20(i)), the shock intensity is again strengthened and the velocity fluctuation again mildly weakens due to the termination of the energy transfer from the flow to the airfoil. As a result, the buffet gets the upper hand and the buffet frequency returns in the spectrum. A comprehensive understanding of the buffet lock-in from the energetic point of view should be fabricated in a larger parameter space including other pivot point locations, pitch frequencies and angles of attack. The $Ma$ and the $Re$ effects should also be taken into account in the future work.

(a) Case 1  
(b) Case 2 

(c) Case 3  
(d) Case 4 

(e) Case 5  
(f) Case 6
CONCLUSION

In this paper, on the one hand the SP-DMD method combined with the energy ranking criterion and the wavenumber-frequency spectrum are used to study the high-frequency characteristics of the buffet. On the other hand, the energy exchange or the energy map as a function of the amplitude and the excitation frequency is applied to analyze the buffet lock-in phenomena on a prescribed pitch airfoil. The buffet lock-in boundary is extracted based on the energy map. Some useful conclusions can be drawn as follows:

1. High-frequency modes of the DMD analysis related to the vortex shedding and the acoustic waves are important for sustaining the LCO due to their contribution to the flow field according to the energy index. The vortex shedding velocity is nearly the same with the K-H vortex moving velocity, while the K-H frequency is much higher than the vortex shedding frequency.

2. The trough of the energy map indicates the buffet lock-in onset while the upper zero energy exchange boundary offers the exit of the lock-in. Under the coupled interaction between the buffet and the airfoil motion, the onset of the lock-in is lowered compared to the flutter boundary, narrowing the flight envelope. The offset of the lock-in boundary, however, is compatible with the flutter boundary at zero energy exchange.

3. In the pre-lock-in region, the flow absorbs energy from the structure and the shock is strengthened downstream with less oscillations and streamwise velocity fluctuations, which makes the buffet less inclined to be affected by the pitch motion. In the lock-in region, with reduced energy transferred to the flow and even positive energy export from the flow to the structure, the shock is weakened and the shock/boundary interaction is prominent compared to the pre-lock-in condition,

Figure 20 The RMS x velocity of all cases and the fixed airfoil.
resulting in the synchronization of the buffet with the airfoil. At the exit of the lock-in, the buffet gains strength again due to the termination of the energy export to the structure. The influence of the energy exchange on the vortex shedding in the lock-in region remains to be further studied with LES, DES or DNS. A comprehensive understanding of the buffet lock-in requires a larger parameter space including other elastic axis, pitch frequencies, angles of attack, $Ma$ and $Re$.

ACKNOWLEDGEMENTS
The authors give thanks to all co-working fellows for their generous help regarding this work.

FUNDING
The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

AUTHOR DECLARATIONS
Conflict of Interest
The authors have no conflicts to disclose.

DATA AVAILABILITY
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Reference


Conference (32nd AIAA Aeroacoustics Conference), American Institute of Aeronautics and Astronautics, Portland, Oregon.


