Appendix

Peaks-over-threshold (POT), focuses on the characteristics of high threshold exceedance with GP model (Coles, 2001; Opitz, Huser, Bakka, & Rue, 2018). Given high threshold u, the distribution function of threshold excess $Y_{[u]} = (X-u)|X>u$ equals approximately

$$H(y;\ \sigma,\xi) = 1 - \left[1+\xi\left(\frac{y}{\sigma}\right)\right]_{+}^{-1/\xi}, \quad y>0, \tag{1}$$

where $x_+ = \max(x,0)$ and scale parameter $\sigma = \sigma(u) > 0$ depends on the threshold u and shape parameter $\xi \in \mathrm{R}$. The case with $\xi = 0$ is interpreted as the limit of $H(y;\ \xi,\sigma)$ as $\xi \to 0$, leading to the exponential distribution. Moreover, to select an appropriate threshold and reduce the potential bias, we apply the mean residual life plot and the parameter estimates plot in Figure 1, where $u = 12.63$ is the selected downward excess threshold, transformed via $\max(\boldsymbol{y}(\boldsymbol{s},t)) - 35$, confirms a linear trend in the MRL plot and rather stable estimates for scale/tail parameters.

We also utilize the reparametrization of the GP distribution (Opitz et al., 2018) with the tail index ξ and a specific q-th quantile κ_q for some fixed probability of interest $q \in (0,1)$, and $y > 0$

$$\mathrm{GP}\left(y;\ \kappa_q,\xi\right) = \begin{cases} 1-\left[1+\left\{(1-q)^{-\xi}-1\right\}y/\kappa_q\right]_{+}^{-1/\xi}, & \xi \neq 0, \\ 1-(1-q)^{y/\kappa_q}, & \xi = 0, \end{cases}$$

which is commonly applied in INLA to avoid problems due to the correlation among estimated GP parameters. In particular, we use $q = 0.65$ in our research.

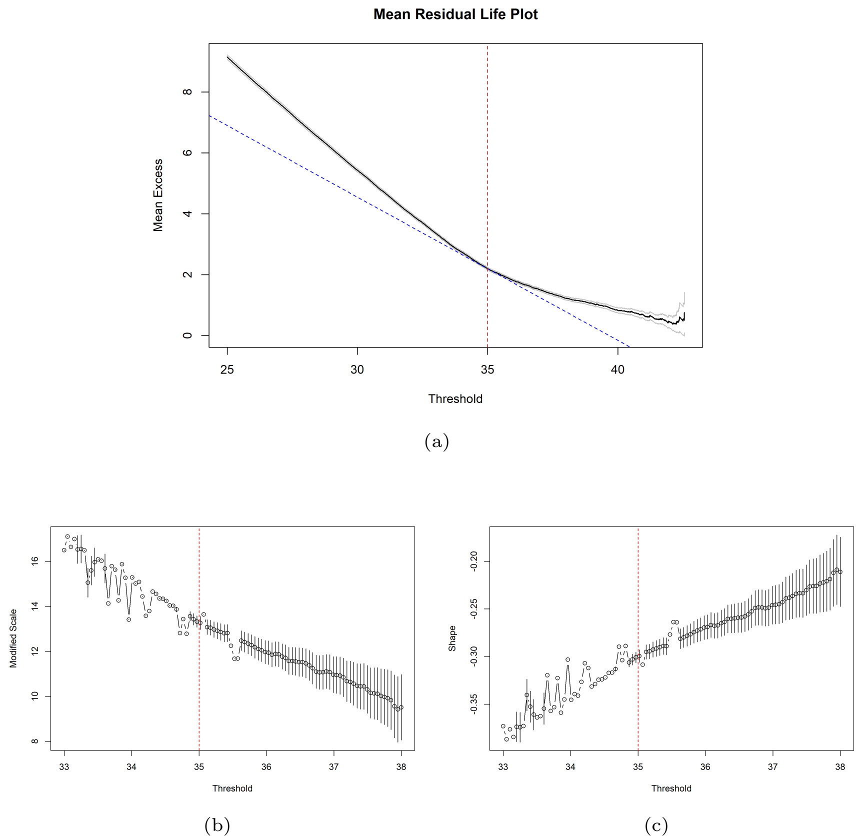

Fig. 1: Threshold selection plots for transformed birth rate. The red vertical line for transformed $u = 35$ corresponds to the original downward threshold $\widetilde{u} = \max(\boldsymbol{y}(\boldsymbol{s}, t)) - 35 = 12.63$, which is the potential selection. The blue line confirms a linear trend in the MRL plot.

References

Coles, S. (2001). *An introduction to statistical modeling of extreme values.* Springer.

Opitz, T., Huser, R., Bakka, H., Rue, H. (2018). INLA goes extreme: Bayesian tail regression for the estimation of high spatio-temporal quantiles. *Extremes*, *21*(3), 441–462.