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Nonlinear coexistence phenomenons and FPGA implementation with the hybrid of memristive-memcapacitive hyperchaotic system

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Abstract

A hybrid circuit based on the magnetic-controlled memristor and charge-controlled memcapacitor is proposed in this paper. The circuit has hyperchaotic behavior in a large scale parameter range. By studying the influence of system parameters and initial conditions on the hybrid circuit, a variety of complex dynamic behaviors can be found: chaos turns into period, spike bursting, chaos turns into spike bursting and so on. Interestingly, different basins of attraction show different shapes. Selecting different basins of attraction positions, the direction of the chaotic attractor changes, but the shape does not change, thus forming a symmetrical coexistence attractor. The coexistence bifurcation diagram and coexistence time-domain waveform diagram under different initial conditions are studied, in the case of positive and negative changes in the initial value $v_0$, the chaotic characteristics of the two cases are consistent. Meanwhile, a series of spectral entropy complexity distributions are analyzed, and find that special shapes are consistent with the attractive basins. Finally, on the platform of Cyclone IV E series, the FPGA main chip of EP4CE115F29C7 is used to complete the hardware implementation of the hybrid memristive-memcapacitive hyperchaotic circuit.

Keywords: Memcapacitor, Memristor, Bursting, Attraction basin, Coexistence attractor, FPGA

1 Introduction

In 1971, Professor Chua predicted the existence of memristors [1]. In 1976, he elaborated on the composition principle and application characteristics of memristors, then proposed charge-controlled memristors and magnetical-controlled memristors [2]. HP laboratory completed the realization of the memristor in 2008 [3, 4], and then the memristor has a deeper research. Professor Chua pointed out that memory elements should not be limited to memristors, but should be extended to memcapacitors and meminductors. In 2009, he formally defined two new storage elements, memcapacitors and meminductors, and pointed out that they have the same properties as memristors [5]. In recent years, many researchers have proposed some new memcapacitor circuit models and designed chaotic oscillators based on the proposed models. Yuan et al. established the equivalent circuit of memcapacitor and meminductor, and then built a chaotic circuit based on meminductor and memcapacitor with basic circuit components. In addition, the application of memory elements such as meminductors and memcapacitors in chaotic systems is also studied [6–9]. Mou Jun et al. designed a chaotic circuit based on two memcapacitors [10]. Subsequently, Professor Wang proposed and designed a chaotic circuit based on memristor and memcapacitor, which replaces the capacitance and nonlinear components in Chua’s circuit with memcapacitor and memristor respectively [11]. In summary, most of the literature so far has involved placing a single memory element into different oscillating circuits to generate chaos. In recent years, two identical or different memory elements have also been placed in a circuit to generate chaotic oscillations. However, in general, there are few related studies on the hybrid circuit model of memristor and memcapacitor.
Bursting is an alternating periodic oscillation between the large amplitude and the small amplitude with a large amplitude difference in a dynamic system. In 1985, literature \[12\] was the first to systematically analyze the bursting patterns in neuronal models. In recent years, the bursting behavior of neurons and other nonlinear systems has been widely explored by many scholars. Different types of bursting patterns and their generation mechanisms in different non-autonomous and autonomous systems have also been revealed one by one \[13–15\]. Because of the different scales in the frequency domain between the external excitation and the system itself, the non-autonomous system is more likely to produce bursting oscillation. As a classical non-autonomous system with an important application background, the Duffing system has been widely studied by scholars because of its simple structure and rich behavior \[16–18\]. The third-order Holmes-type Duffing chaotic oscillator system proposed by Tamasevicius et al. was systematically analyzed \[19\], and the coexistence of multiple pairs of symmetric periodic chaos generated when the three initial conditions were changed in Ref. \[20\], it is pointed out that the further increase of the amplitude of the exponential change rate of time delay may lead to the breakdown of the channel between the two symmetrical bursting oscillations in the original double-branch bursting, resulting in two symmetrically coexisting single-branch bursting. Ref. \[21\] studied that when the excitation frequency was doubled in the Duffing system with external excitation, although the system could double the number of spikes of symmetrical bursts and the time interval between adjacent spikes was halved, the spike peak did not change. At present, there are relatively few studies on the bursting oscillation of memristive-memcapacitive systems, and there are few reports on the behavior of bursting coexistence \[22\]. Therefore, it is of great research value to construct new memristive and memcapacitive systems and explore novel dynamic behaviors such as bursting coexistence and spike bursting.

At present, the generation and application of multistability and extreme multistability have become very popular topic in chaotic circuit systems \[23–27\]. Compared with other chaotic systems, the combination of a memristor or memcapacitor with chaotic systems can generate chaotic attractors with complex dynamic properties. However, the multistability of chaotic systems depends on the initial state of the system. In fact, multistable dynamical systems usually have very complex basins of attraction, which can be defined by fractal boundaries \[28\]. The attractive basin, also known as the domain of attraction, is a powerful method for analyzing the multi-stability of chaos in recent years. This method selects two sensitive initial values in the system as variables to draw a color map. Different colors in the map represent that the system is in different states \[29\]. In the attractive basin, in addition to the invariant manifold, there will be different bifurcations and different forms of attractive basin \[30\].

Many electronic circuits and nonlinear systems exhibit the coexistence of multiple attractors \[31–33\]. Generally, the emergence of coexisting attractors is not only closely related to the initial conditions of the system but also involves the symmetry of the system. Chaotic systems with multiple attractors can provide greater complexity in random number generators, control systems, image encryption, neural networks, and other chaos-based engineering applications. Therefore, chaotic systems with coexisting attractors have received extensive attention. However, some recent studies mainly focus on memristors, and there are few studies on hybrid memristive-memcapacitive circuit models. Symmetric attractor is an important topic of multistability phenomenon, and the chaotic system has important research value in the dynamic study of multi-stability phenomena.

In this paper, the hyperchaotic circuit based on a magnetic-controlled memristor and a charge-controlled memcapacitor is designed. Compared with the existing chaotic systems, the proposed system has the following characteristics:

1. A nonlinear circuit containing a magnetic-controlled memristor and a charge-controlled memcapacitor is designed. Two different dynamic models contain complex functions, which verify the existence of hybrid memory elements.

2. The bifurcation diagram of the system has a good correspondence with the Lyapunov exponent, and always maintains the hyperchaotic characteristics of the ultra-large parameter range. And the system has a variety of complex dynamic behaviors: chaos turns into period, spike bursting, and chaos turns into spike bursting.

3. Interestingly, the attractive basin and Spectral Entropy (SE) complexity distribution of the system have the same special shape, and the coexistence attractor is symmetric about the initial value $v_0 = 0$.

4. The digital memristive-memcapacitive chaotic circuit is built in DSP Builder, and the simulation is carried out. Based on this, the hardware implementation of the system FPGA is designed, which shows the feasibility and correctness of the system.

This paper focuses on a hybrid memristive-memcapacitive hyperchaotic system. In Sect. 2, a circuit model composed of a magnetic-controlled memristor and a charge-controlled memcapacitor is proposed, and some basic properties of the system are analyzed. Section 3 discusses the dynamic characteristics of the system, such as bursting behavior, basin of attraction and symmetric coexistence
4 gives the hardware implementation of FPGA. The paper is summarized in Sect. 5.

2 Design of memristive and memcapacitive models

2.1 Magnetic-controlled memristive model

Memristor is a two-port basic circuit element, which can be divided into two types: magnetic-controlled and charge-controlled. According to the definition of Zhang [34], a new magnetic-controlled memristive model is proposed,

\[
\begin{align*}
  i(t) &= W(\varphi)V(t) = d(e-g|\varphi|)V \\
  \varphi &= m\varphi(1-|V|)
\end{align*}
\]

where \( W(\varphi) \) is called the memristive value of the flux-controlled memristor, \( \varphi \) is the magnetic flux. \( i(t) \) and \( v(t) \) are the current and voltage of the circuit variables, and \( d, e, g, m \) (\( d = e = g = m = 1 \)) are the actual parameters. The input current \( i(t) = A\sin(2\pi ft) \), the \( v-i \) characteristic curve of the memristor is shown in Fig. 1. We can observe that the trajectory of the \( v-i \) characteristic curve is similar to the tilted '8'.

- When the frequency \( f = 1\text{Hz} \), the voltage \( A = 2\text{V} \), \( 2.2\text{V} \), and \( 2.4\text{V} \), respectively, the tight hysteresis loop is expanded in Fig. 1(a).
- When the voltage \( A = 3\text{V} \) and the frequencies \( f \) are \( 1\text{Hz} \), \( 2\text{Hz} \) and \( 3\text{Hz} \), respectively, the tight hysteresis loop shrinks in Fig. 1(b).

This is consistent with the definition of memristor.

2.2 Charge-controlled memcapacitive model

The memcapacitor can also be divided into magnetic-control and charge-control according to the control quantity. According to the definition of

\[
\begin{align*}
  u(t) &= C_M^{-1}(\sigma)q(t) = b\sigma^2q \\
  \sigma &= f q^2 - na^2
\end{align*}
\]

Akif [35], based on the definition of the ideal charge-control memcapacitor, the expression of the new memcapacitor is defined:

\[
\begin{align*}
  u(t) &= C_M^{-1}(\sigma)q(t) = b\sigma^2q \\
  \sigma &= f q^2 - na^2
\end{align*}
\]

where \( q(t) \) and \( u(t) \) are the charge and voltage corresponding to the memcapacitor at time \( t \), respectively, \( \sigma \) is the time continuous integral of the charge \( g(t) \) through the memcapacitor, \( \phi(\sigma) \) is the magnetic flux through the memcapacitor, is a function of \( \sigma \), \( C_M^{-1} \) is the reciprocal of the capacitance of the memcapacitor, \( b, f, n \) (\( b = 2 \), \( f = 0.01 \), \( n = 1 \)) are the actual parameters. The input signal of the memcapacitor is set to \( q(t) = Q\sin(2\pi ft) \), and the curve of the memcapacitor \( q-v \) shows an '8' hysteresis characteristic.

- As shown in Fig. 2(a), when the given frequency \( f = 1\text{Hz} \), it can be seen that the typical \( q-v \) tight hysteresis loops with charge \( Q \) of \( 10\text{C} \), \( 13\text{C} \) and \( 15\text{C} \) gradually expand, respectively.
- As shown in Fig. 2(b), under the condition of given \( Q = 20\text{C} \) (C is the charge unit Coulomb), when the frequency \( f \) is \( 1\text{Hz} \), \( 2\text{Hz} \) and \( 3\text{Hz} \), respectively, the tight hysteresis loop shrinks with the increase of frequency \( f \).

The above phenomenon is consistent with the definition of memcapacitor.

3 Design and dynamic analysis of the hybrid memristive-memcapacitive chaotic circuit

3.1 Design of the hybrid chaotic circuit based on memristor and memcapacitor

Based on the proposed memristor and memcapacitor models, a simple hybrid memristive-memcapacitive chaotic circuit model including memcapacitor \( (C_M) \), memristor \( (M) \), inductance \( (L) \) and capacitance \( (C) \) is constructed, as shown in
Fig. 3 Structure of the memristive-memcapacitive circuit with 1 charge-controlled memcapacitor (left-CM) and 1 magnetic-controlled memristor (right-M)

Fig. 3. According to Kirchhoff’s law, the differential equation of the circuit is as follows:

\[
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L}(-V + V_{CM}) \\
\frac{dV}{dt} &= \frac{1}{C}(i_L - i_M) \\
\frac{dq}{dt} &= -i_L \\
\frac{d\sigma}{dt} &= fq^2 - n\sigma^2 \\
\frac{d\varphi}{dt} &= m\varphi(1 - |V|)
\end{align*}
\]

Eq. (3)

Compare Fig. (3) with the expression of Eqs. (1) and (2), among \( V_{CM} = u(t) = b\sigma^2 q \), \( i_M = i(t) = d(e - g|\varphi|)V \), so Eq. (3) becomes,

\[
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L}(-V + b\sigma^2 q) \\
\frac{dV}{dt} &= \frac{1}{C}(i_L - d(e - g|\varphi|)V) \\
\frac{dq}{dt} &= -i_L \\
\frac{d\sigma}{dt} &= fq^2 - n\sigma^2 \\
\frac{d\varphi}{dt} &= m\varphi(1 - |V|)
\end{align*}
\]

Eq. (4)

By normalizing \( x = i_L, y = V, z = q, u = \sigma, v = \varphi, a = 1/L, c = 1/C \) the state equation of the system (4) can be expressed as:

\[
\begin{align*}
\dot{x} &= a(-y + bz u^2) \\
\dot{y} &= c(x - dy(e - g|v|)) \\
\dot{z} &= -x \\
\dot{u} &= f z^2 - nu^2 \\
\dot{v} &= m\varphi(1 - |V|)
\end{align*}
\]

Eq. (5)

In the formula, \( x, y, z, u, v \) are system state variables, \( a, b, c, d, e, f, g, m, n \) are system parameters, and all of them are positive numbers. When \( a = 1, b = 2, c = 1, d = 1, e = 1, f = 0.01, g = 1, m = 1, n = 1 \), the divergence of the system is

\[
\nabla V = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} + \frac{\partial u}{\partial u} + \frac{\partial v}{\partial v} = |v| - |y| - 2u
\]

Eq. (6)

When \(|v| - |y| - 2u < 0 \), \( \nabla V < 0 \), system (5) belongs to the dissipative system.

3.2 Equilibrium point set and stability

To obtain the specific data of the equilibrium point of the system, let \( \dot{x} = \dot{y} = \dot{z} = \dot{u} = \dot{v} = 0 \) in the system (5), we can get the following equations:

\[
\begin{align*}
\alpha(-y + bz u^2) &= 0 \\
c(x - dy(e - g|v|)) &= 0 \\
-z &= 0 \\
f z^2 - nu^2 &= 0 \\
m\varphi(1 - |V|) &= 0
\end{align*}
\]

Eq. (7)

Obviously, according to the equation of state (7), one of the equilibrium points is \((0, 0, 0, 0, 0)\). After linearization of Eq. (5), the Jacobi matrix \( J \) of the system can be obtained as:

\[
J = \begin{bmatrix}
0 & -a & J_{13} & J_{14} & 0 \\
0 & c & J_{22} & 0 & 0 \\
J_{32} & 0 & J_{43} & J_{44} & 0 \\
0 & J_{52} & 0 & J_{55} & 0
\end{bmatrix}
\]

Eq. (8)

\[
\begin{align*}
J_{13} &= abu^2 \\
J_{14} &= 2abzu \\
J_{22} &= -cd(e - g|v|) \\
J_{25} &= cd|g|sign(v) \\
J_{43} &= 2fz \\
J_{44} &= -2nu \\
J_{52} &= -m|\varphi(1 - |V|)| \\
J_{55} &= m(1 - |y|)
\end{align*}
\]

Eq. (9)

Let the characteristic equation \(|J - \lambda \mathbf{E}| = 0\), the equilibrium point and the set parameters are substituted, and the corresponding eigenvalues can be obtained. The eigenvalue corresponding to the equilibrium point \( A (0, 0, 0, 0, 0) \) is \((0, -0.5 + 0.8660i, -0.5 - 0.8660i, 0, 1)\). From the eigenvalue point of view, the system is unstable at the equilibrium point \((0, 0, 0, 0, 0)\), and the system will produce chaotic or hyperchaotic behavior. At the same time, there is a pair of conjugate eigenvalues in the obtained eigenvalues. This kind of equilibrium point is a saddle-focus equilibrium point [36], which is very important for chaotic systems.

3.3 Characteristic analysis of hyperchaos in the hybrid memristive-memristive system

When the initial condition is set to \( I_0 = (1, 0.15, 0.15, 0.15) \) and the parameters \( a = 1, b = 2, c = 1, d = 1, e = 1, f = 0.01, g = 1, m = 1, n = 1 \), the chaotic attractor shown in Fig. 4 can be obtained.
When only one global Lyapunov exponent is positive, the system is defined as a chaotic system. When there is more than a positive Lyapunov exponent, the system is defined as a hyperchaotic system [37]. In this section, the sampling interval as 0.01, the observation time is 2000, and the parameters and initial conditions are unchanged. The Jacobi matrix method is used to find the Lyapunov exponent.

Fig. 5 shows the Lyapunov exponent spectrum of the system (5) changing with time. The Lyapunov exponent spectrum $LE_1 = 1.0132$, $LE_2 = 0.8288$, $LE_3 = 0.3574$, $LE_4 = 0.0847$, $LE_5 = -0.0926$, which satisfies the conditions of hyperchaotic system.

**Fig. 4** The phase diagram of system (5) with fixed parameters $(a, b, c, d, e, f, g, m, n) = (1, 2, 1, 1, 0.01, 1, 1, 1)$, initial condition $I_0 = (1, 0.15, 0.15, 0.15, 1)$

**Fig. 5** Lyapunov exponents spectrum of the system (5) with fixed parameters $(a, b, c, d, e, f, g, m, n) = (1, 2, 1, 1, 1, 0.01, 1, 1, 1)$, initial condition $I_0 = (1, 0.15, 0.15, 0.15, 1)$
Fig. 6 The Lyapunov exponents and bifurcation diagrams varying with the parameters \( m \) and \( n \), respectively, and initial conditions \( I_0 \), other parameters \((a, b, c, d, e, f, g) = (1, 2, 1, 1, 0.01, 1)\). 

(a) Lyapunov exponent with the change of \( m \); (b) Bifurcation diagram with the change of \( m \); (c) Lyapunov exponent with the change of \( n \); (d) Bifurcation diagram with the change of \( n \)

Fig. 7 Two-dimensional Lyapunov exponent spectrum with the changes of parameters \((m, n)\), \((a, b, c, d, e, f, g) = (1, 2, 1, 1, 0.01, 1)\), initial condition \( I_0 = (1, 0.15, 0.15, 0.15, 1)\). 

(a) Largest Lyapunov (LLE); (b) Second Lyapunov (LE₂)
The influence of system parameters on the system is further analyzed. The bifurcation diagram and its corresponding Lyapunov exponent spectrum are shown in Fig. 6. The dynamic results of the Lyapunov exponent spectrum are consistent with the results of the bifurcation diagram.

It can be found in Fig. 7 that the largest LE and the second LE of the Lyapunov exponents of the change with two parameters $m$ and $n$ are greater than 0, indicating that the system is always in a hyperchaotic state under the influence of the two parameters $m$ and $n$.

### 3.4 Attractor evolution and transient chaotic behavior

A special phenomenon is found when studying the influence of parameter $b \in (0, 110)$ on the system. As shown in Fig. 8, when $b = 95$, the bifurcation diagram begins to show a wide range of chaos.

In order to further study this special phenomenon, the values of different parameters $b$ are selected. As shown in Fig. 9, the time-domain waveform gradually changes from chaotic oscillation to periodic oscillation, and the attractor continues to evolve. This phenomenon is called transient chaos[38].

### 3.5 Complexity analysis

The complexity of chaotic systems is crucial for chaos-based information security applications. Chaotic signals are widely used in secure communication due to their high complexity. The core idea of the SE algorithm is to perform the discrete Fourier transform on the chaotic sequence [39], to know the energy distribution of the system in the frequency domain, and then combine the Shannon entropy to obtain the corresponding SE value. Therefore, this paper uses it to measure the complexity of chaotic systems.

To analyze the complexity of the system more deeply, the chaotic diagram of Spectral Entropy (SE) complexity based on two parameters and two initial values is shown in Fig. 10.

- White and yellow represents the lower SE complexity, and red and black represents the higher SE complexity. The deeper the color, the greater the complexity of the system, indicating that the higher the security of the system.
- A series of SE complexity distributions with special shapes can be observed in Fig. 10(a-f). In addition, the corresponding color changes obviously with the change of the system parameters, and the area occupied by red and black is the largest, indicating that the system is very sensitive to the parameters.
- The SE complexity map in Fig. 10(h) is consistent with the distribution of the two-parameter lyapunov exponential spectrum in Fig. 7.

These phenomena mean that in the memristive -memcapacitive hyperchaotic system, the existence of multistability is further verified, reflecting the rich complexity characteristics of the memristor chaotic circuit, and its complexity is not only affected by the system parameters but also by the initial conditions of the system.
3.6 Analyze the attractive basin, bursting oscillation and coexisting attractors

We further analyze the characteristics of attractive basins, and consider the Lyapunov exponential function to analyze the stability of the system. When the system is running in the initial state, it will move in the direction of decreasing the Lyapunov exponential function until it reaches its local minimum [40, 41]. The local minima of the Lyapunov exponential function refer to the stable points in the phase space, and each attractor will surround a considerable basin of attraction. In this sense, these points are also called attractors. These attractive basins represent a stable chaotic state. When the stable point enters the lowest region of the attractive basins, the solution of the chaotic system can be obtained. In dynamical systems with multiple attractors, the corresponding basins may have fractal boundaries or even more complex structures.

(1) Basins of attraction:
- From the attractive basin map in Fig. 11(a-c), the attractive basin map is rotationally symmetric, evenly distributed, and has the same appearance. From the attractive basin map in Fig. 11(d-f), the attractive basin map is axisymmetric about x-axis, which is consistent with the attractive basin in Fig. 10(d-f).
- According to the analysis of this phenomenon, the red and orange regions represent that the system is an attractive basin of infinitely distant attractors, and the cyan region indicates that the system is in chaotic motion. In the orange region, there is a line composed of many yellow dots, which fall in the center of the orange region. They
Fig. 11 Attractor basins with different initial values, parameters \((a, b, c, d, e, f, g, m, n) = (1, 2, 1, 1, 1, 0.01, 1, 1, 1)\). a-c: rotational symmetry, d-f: Symmetry to \(x\)-axis

are the local minima of the Lyapunov exponential function and represent the stable points in the phase space. They are also the solution of the chaotic system and the symmetry point of the attractive basins.

(2) Bursting behavior:

- The attractors with different states can be obtained by selecting different position coordinates in the basin of attraction. When the initial conditions of the red or orange basin area are selected, as shown in Fig. 12(b), the attractors of the two are the same in shape and opposite in orientation. When the cyan and orange mixed interlaced parts are selected as the initial conditions, as shown in Fig. 12(d), the attractors are also in the same shape and opposite in orientation. In the whole basin diagram, it can be found that the steady-state regions represented by cyan are separated. Therefore, they are multi-stable. Under different initial conditions, the basin of system (5) shows different states, especially in the cyan region.

- Bursting is one of the important activities of neurons to send information, and it is also a phenomenon of alternation between resting state and spike state[42]. When studying the influence of initial conditions on the system, not only the bursting oscillation is found but also a more complex dynamic behavior is found, which is transformed from a chaotic state to the bursting oscillation:

(i) As shown in Fig. 12(a), the time-domain waveform is a typical spike bursting oscillation state. The corresponding attractor is shown in Fig. 12(b).

(ii) As shown in Fig. 12(c), the time-domain waveform changes from chaotic oscillation to spike bursting oscillation, the attractor orbit is divided into two states: chaotic attractor moves for some time and then transforms into a bursting state Fig. 12(d).

(3) Coexisting attractors:

- As shown in Fig. 13, the trajectory of the attractor is always in a chaotic state with initial conditions \(I_5\) and \(I_6\). When the initial value \(v_0\) is changed to \(-v_0\), the attractor is symmetric about \(v = 0\), which means that by setting the initial conditions about \(v_0 = 0\) symmetry, the coexistence attractor about \(v = 0\) symmetry is obtained.

- For the convenience of observation, the bifurcation diagram and time-domain waveform diagram drawn with the initial condition \(I_5\) are half of the bifurcation diagram and time-domain waveform diagram drawn with the initial condition \(I_6\), and it can be seen this half is completely coincident in Fig. 14.
The above phenomena show that in the cyan basin, the chaotic characteristics of the region concerning $v_0 = 0$ symmetry are the same, and only the directions of the attractors are different.

4 FPGA design and implementation of the memristive-memcapacitive chaotic circuit

4.1 Discretization of the memristive-memcapacitive chaotic circuits

Using FPGA to design continuous chaotic systems requires discretization of the system, and considering the accuracy problem, the improved Euler algorithm [43] is selected to discretize the system (5). The discretized memristive-memcapacitive chaotic circuit can be described as

\[
\begin{align*}
\bar{x}(n+1) &= x(n) + \frac{\Delta t}{2} [a(-(y(n) + y(n+1)) + b(z(n) + z(n+1))(u(n) + u(n+1))^2)] \\
\bar{y}(n+1) &= y(n) + \frac{\Delta t}{2} [c(x(n) + x(n+1)) - d(y(n) + y(n+1))(e - g(v(n) + v(n+1)))] \\
\bar{z}(n+1) &= z(n) + \frac{\Delta t}{2} [-x(n) + x(n+1)] \\
\bar{u}(n+1) &= u(n) + \frac{\Delta t}{2} [f(z(n) + z(n+1))^2 - n(u(n) + u(n+1))^2] \\
\bar{v}(n+1) &= v(n) + \frac{\Delta t}{2} [m(v(n) + v(n+1))(1 - (y(n) + y(n+1)))]
\end{align*}
\]

where $\Delta t$ is the iteration step, take $\Delta t = 2^{-11}$. 

Fig. 12 Dynamic behavior of different initial conditions. a Coexistence time-domain waveform with bursting oscillation, initial conditions $I_1$ and $I_2$; b Coexistence attractor corresponding to the time domain waveform subgraph(a); c Coexistence time-domain waveform from chaos to bursting oscillation, initial conditions $I_3$ and $I_4$; d Coexistence attractor corresponding to the time domain waveform subgraph(c)
Fig. 13 Coexistence attractor with initial conditions of $I_5 = (1.1, 0.15, 0.15, 0.15, 1.1)$ and $I_6 = (1.1, 0.15, 0.15, -1.1)$, parameters $(a, b, c, d, e, f, g, m, n) = (1, 2, 1, 1, 0.01, 1, 1, 1)$

Fig. 14 Coexistence bifurcation diagram and coexistence time domain waveform with the initial conditions $I_5, I_6$. a Coexistence bifurcation diagram; b coexistence time domain waveform
Fig. 15: Digital memristive-memcapacitive hyperchaotic system circuit in DSP Builder.
4.2 Design and implementation of the memristive-memcapacitive chaotic circuit based on DSP Builder

To realize the digital memristive-memcapacitive chaotic circuit, this paper uses Matlab / Simulink R2012b and DSP Builder 13 development platform. DSP Builder is an important part of the system on the programmable chip (SOPC). It connects the two fields of simulation and RTL (hardware implementation) and can complete the design of the DSP digital system based on FPGA. The digital memristive-memcapacitive chaotic circuit is designed by DSP Builder technology in Fig. 15. As a module in the FPGA circuit system, it is connected with other functional modules to form a digital hardware system. Finally, the implementation from the algorithm level to the RTL level is completed. This method uses graphical design, which greatly improves the development efficiency of digital circuits and increases the flexibility and reliability of digital circuit design.
4.3 The memristive-memcapacitive hardware circuit based on FPGA

It is well known that the circuit implementation based on the field programmable gate array (FPGA) has the advantages of high stability, fast operation and fast response. The Signal Compiler module is used to analyze and synthesize the memristive-memcapacitive chaotic circuit designed by DSP Builder. After RTL-level functional simulation with Modelsim, the Quartus II schematic sub-module generated by VHDL language is obtained. Based on this sub-module, combined with the corresponding hardware resources and software programming, the hardware circuit of the memristive-memcapacitive chaotic circuit is designed on FPGA. Fig.16 is the schematic diagram of the circuit, which is composed of the following parts:

(1) CM3_GN: CM3_GN is a circuit schematic module that is generated by analyzing and synthesizing the memristive-memcapacitive chaotic circuit designed by DSP Builder using the Signal Compiler tool. Clock is clock input pin and aclr is reset pin, which connect Clk signal and Rst signal respectively in Fig.16. The output pins correspond to the vector output ports of each order, which are connected to the 14-bit data bus.

(2) PLL module : PLL is a phase-locked loop, also known as a phase-locked loop. Since the external crystal oscillator of the development board is 50MHz, to make the DA chip AD9767 work in a high-speed state, it must be doubled to 100MHz. We call the PLL IP core provided by Intel FPGA and embed it into the FPGA chip hardware circuit, such as the PLL module in Fig.16. inclk0 is the PLL clock input pin, which is connected to Clk (50MHz). c0 is the PLL output pin, and the output frequency is 100MHz.

(3) F_Div module : F_Div is a frequency divider module, which provides the clock signal for the CM3_GN module. The clk is the clock input pin of the F_Div module, which is connected to the Clk (50MHz), the rst is the reset pin, which is connected to the external reset button, and the q is the divider output pin, which is connected to the subsequent clock input pin in Fig.16.

The main platform of the experiment adopts the Cyclone IV E series, the FPGA main chip of EP4CE115F29C7, the DAC chip of 14bits dual channel AD9767 and the oscilloscope. The AD9767 chip converts the generated digital signal into an analog signal and outputs it to the oscilloscope. The waveform is observed by a digital oscilloscope, and the chaotic attractors corresponding to the memristive-memcapacitive systems (5) are shown in Fig.17. Obviously, the phase trajectory captured by the experiment is consistent with the chaotic attractors in Fig.4 of MATLAB simulation, which verifies the correctness of the digital circuit.

5 Conclusion

In this paper, a hybrid hyperchaotic system based on the memristor-memcapacitor is designed, which is composed of only four components. Then the dynamic characteristics are analyzed, including the equilibrium point, Lyapunov exponent spectrum and bifurcation diagram. It is shown that the system is a hyperchaotic system with a large parameter range. Compared with the existing memristive-memcapacitive hybrid circuits, the proposed system has a variety of complex dynamic behaviors: transient chaotic behavior, spike bursting, chaos into spike bursting and so on. More interestingly, the cross-section of the attractive basin of the system has a variety of special shapes and the SE complexity distribution map corresponds to it. In addition, the system has symmetrical coexisting attractors. The above phenomena indicate that the system has rich chaotic characteristics. Finally, the improved Euler algorithm is used to discretize the circuit, and after circuit simulation, the hardware implementation of the memristive-memcapacitive hyperchaotic circuit is verified by FPGA technology.

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Data Availability Data used to support the study already included in the manuscript.

Declaration

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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